SELMA
Technical Documentation

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SELMA
Svensk Ekonomisk Lineariserad Modell för samhällsekonomisk Analys
Technical Documentation*

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Abstract

This document contains a full description of The National Institute of Economic Research's (NIER) model SELMA, a DSGE model intended to support macroeconomic analysis and forecasting at the NIER and at the Ministry of Finance. The model consists of a small open economy; Sweden, and a large economy that represents the rest of the world; Foreign. Sweden consists of a household sector with Ricardian households that have access to financial markets and Non-Ricardian households that do not, a production sector that is subject to price and wage stickiness, and a central bank that conducts monetary policy according to a Taylor rule. In addition, the Swedish economy is characterized by a detailed fiscal sector, with a fiscal authority that conducts fiscal policy and has several fiscal instruments at its disposal. Sweden engages in trade with Foreign. The Foreign economy consists of a household sector, a production sector that is subject to price and wage stickiness, and a central bank that conducts monetary policy. This technical documentation entails a description of the model, a description of the calibration of the model parameters, and a presentation of impulse response functions for selected shocks. The document also contains a full list of the dynamic and steady state equations, and derivations of these equations.

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1 Introduction

The model described in this documentation is named SELMA. SELMA is a two country version of a New-Keynesian DSGE model which depicts the Swedish economy and a Foreign economy, where the latter represents the rest of the world. The non-fiscal blocks of the model build on well-known contributions by, among others, Christiano, Trabandt, and Walentin (2011), Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003) and Adolfson et al. (2008). The most closely related model for the non-fiscal part of SELMA is the model described in Corbo and Strid (2020), whereas the most closely related model for the fiscal part is the model described in Coenen, Straub, and Trabandt (2013). The Swedish economy in SELMA is characterized by a detailed fiscal sector, with a fiscal authority that conducts fiscal policy and has several fiscal instruments at its disposal as well as the ability to issue government debt. With a detailed fiscal sector, it is possible to analyze the effects of fiscal policy, taking into account general equilibrium effects as well as analyzing the interaction between monetary and fiscal policy. In addition, an energy sector similar to Corbo and Strid (2020) is included.

In the Swedish economy, unemployment is modelled following Galí (2011) and Galí, Smets, and Wouters (2012).

The main difference between SELMA and its earlier counterparts in the models in Adolfson et al. 2008 and Coenen, Straub, and Trabandt (2013), except for the presence of unemployment in SELMA, is the structure of the Foreign economy. Both Adolfson et al. (2008) and Coenen, Straub, and Trabandt (2013) assume a vector auto-regressive (VAR) representation of the Foreign economy, while in SELMA it is modelled as a structural economy with optimizing, forward-looking households and firms, in a similar manner as in Corbo and Strid (2020). The main advantage of modelling the Foreign economy as structural is that shocks that originate in the Foreign economy can be interpreted in terms of the model mechanisms.

The layout of the rest of the document is as follows: the model is presented in Section 2, the calibration of the model is presented in Section 3, and impulse response functions for selected shocks are presented in Section 4. The stationarized and log-linearized model equations are presented in Appendix A, the steady state equations are in Appendix B, derivations of the model equations are in Appendix C, D and E, while the variable and parameter definitions are in Appendix H and Appendix I respectively. Finally, the impulse response functions of the selected model variables to model shocks are reported in the Appendix J.

2 The model

The world economy consists of two economies, Sweden, and the rest of the world, called Foreign.[1] Sweden is a small open economy, which means that Sweden relies heavily on trade with other countries. At the same time, Sweden is a sufficiently small economy relative to the rest of the world that changes in the economic environment or economic decisions in Sweden do not affect Foreign. In contrast, Foreign is a large economy, which means that changes in the economic environment or the economic decisions in Foreign have an impact on Sweden. Households and firms in both economies make decisions based on optimizing behavior and rational, forward-looking expectations. We assume trade in goods and bonds between the two economies, but we abstract from the possibility of labor mobility between countries.

In the Swedish economy, the household sector is composed of two types of households: Ricardian and Non-Ricardian. Both types of households consume and work. The difference between them is that the Ricardian households have access to financial markets, which implies that they can save and borrow. Non-Ricardian households do not have access to financial markets and can neither save nor borrow (which implies that they cannot smooth their consumption over time). Production is carried out by intermediate good firms that rent capital and labor from households. Domestically produced intermediate goods are then combined with imported goods to produce final goods, which are sold either on the domestic market, or on the export market. Separate firms specialize in the business of importing and exporting. Furthermore, Sweden has a detailed fiscal sector with a government that uses several sources of tax revenue to finance government consumption, investment and transfers to households. Figure 1 shows an overview of SELMA; the structure of the Swedish economy and how it connects with the Foreign economy.

Foreign is partly a mirror-image of Sweden. However, as the main focus of the model is the analysis of Sweden, Foreign is modelled with a more sparse structure. In particular, the fiscal sector in Foreign is modelled in much less detail compared to the Swedish economy,[2] and there is only one type of household, the Ricardian household.

Both Sweden and Foreign are affected by two global technology shocks, which determine the long-run path for productivity. They are denoted z_t and γ_t. For Sweden, z_t and γ_t may be interpreted, respectively, as a labor augmenting technological process and a technological process specific to the production of investment goods. z_t^+, which is a function of z_t and γ_t, summarizes the compound effect of technology on the level of production along

[1] For a list of all variables and parameters, see Appendix H and I respectively.

[2] In Foreign, all of the proceeds from taxation are spent on transfers to households and the government runs a balanced budget every period.

the balanced growth path. For the Foreign economy, the similar variable that summarizes the compound effect of technology is denoted by $z^{+}_{F,t}$. In addition to the two global shocks, each of the two economies are affected by a number of country-specific shocks, where some of the shocks are allowed to be correlated.

The remainder of this section describes the problems solved by optimizing agents in the two economies, as well as the policy rules that govern monetary and fiscal policy. A complete list of the equilibrium conditions and the derivations can be found in Appendix A, and Appendices C, D and E respectively.

2.1 The Swedish household sector: Ricardian households

The Swedish household sector consists of a continuum of households with total mass equal to one and indexed by k. They can be divided into two types of representative households, Ricardian households, with mass $(1-s_{nr})$ and Non-Ricardian households with mass s_{nr}. In this section, we describe the Ricardian households. A representative Ricardian household earns income from wages and from the return on its savings, and it decides how much to consume and how to allocate its remaining resources between different kinds of savings. There are four kinds of assets that the household can save in: 1) capital, which is owned by households and rented to firms on a period-by-period basis, 2) private bonds denominated in Swedish currency, 3) private bonds that are denominated in the currency of Foreign, and 4) a portfolio of government bonds denominated in Swedish currency.

A representative household is a large structure with many members who are represented by the unit square $(h,j) \in [0,1] \times [0,1]$, where each member is indexed by h according to their type of labor service they are specialized in and indexed by j according to their degree of disutility of work. We drop the household's index k because all households have the the same optimization problem. The objective of representative large household is to maximize the following expected discounted life time utility:

$$E_0 \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \int_0^{N_{h,t}} j^\eta \, dj dh \right], \tag{1}$$

where ρ_h is the consumption habit formation parameter, ζ_t^c is the consumption preference shock and the composite consumption $\tilde{C}_t$ of household is defined as a constant elasticity of substitution (CES) aggregate:

$$\tilde{C}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} C_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} G_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}$$

where C_t denotes the household's aggregate consumption of private consumption goods which is obtained by integrating over all household members' consumption, $C_t = \int_0^1 \int_0^1 C_{h,j,t} dj dh$. We assume full risk sharing of consumption among household members which implies $C_t = C_{h,j,t}$ for all (h,j). G_t measures government consumption. Note that α_G is a share parameter and $\upsilon_G > 0$, where υ_G measures the elasticity of substitution between private consumption and government consumption. $\upsilon_G \to 0$ implies perfect complementarity, $\upsilon_G \to \infty$ gives perfect substitutability, and $\upsilon_G \to 1$ yields the Cobb-Douglas (CD) case. Following Coenen, Straub, and Trabandt (2013), Bouakez and Rebei (2007), Leeper, Walker, and Yang (2009) and others, we allow government consumption to enter household utility in a non-separable way. This feature has several implications. First, changes in government consumption affect optimal private consumption decisions directly, as opposed to the indirect wealth effect in case of separable government consumption. Second, conditional on the degree of complementarity, a co-movement of private and government consumption may be obtained, which is observed in macro data, see for example the discussion in Galí, López-Salido, and Vallés (2007). Intuitively, examples of government consumption goods that represent complements to private consumption goods are public security provision such as defense or police, and education. The term $\rho_h \tilde{C}_{t-1}$ in the utility function captures an external habit formation, which implies that households dislike to deviate from last period's average consumption.

The term $N_{h,t}$ denotes the employment level for profession h and by integrating disutility of work over j the household utility can be written in the following way:

$$E_0 \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \frac{N_{h,t}^{1+\eta}}{1+\eta} dh \right],$$

The functions $u(\cdot)$ is twice continuously differentiable function and β_t is the subjective discount factor. The term ζ_t^n denotes an economy-wide preference shock to the disutility of labor that evolves stochastically and that causes exogenous shifts in the supply of labor. The term Θ_t^n is an endogenous shifter defined as

$$\Theta_t^n = Z_t^n U_{c,t} \tag{2}$$

where Z_t^n is an approximation for the trend of marginal utility of consumption $U_{c,t}$ and defined by

$$Z_t^n = (Z_{t-1}^n)^{1-\chi_n} (U_{c,t})^{-\chi_n} \tag{3}$$

where $\chi_n \in [0,1]$ and determines the persistency of Z_t^n. The formulation of Θ_t^n implies a "consumption externality" to the labor force participation. When the marginal utility of consumption $U_{c,t}$ is below its trend value Z_t^n, marginal disutility of work goes down for an individual household member through the value of Θ_t^n. This mechanism helps to reduce the short-run "wealth effect" on labor force participation, the magnitude of which is determined by the value of parameter χ_n. The lower the value of χ_n the lower is the "wealth effect" in the short-run.

The budget constraint of the Ricardian household is the following:

$$\underbrace{(1+\tau_t^C)P_t^C C_t}_{\text{Consumption expenditure}} \underbrace{+(1-\tau_t^I)\frac{P_t^I}{\gamma_t}I_t + P_t^K \triangle_t^K}_{\text{Investment expenditure}} + \underbrace{\frac{B_{t+1}^{priv}}{R_t\zeta_t} + B_t^n + \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)}}_{\text{Bond savings}} + T_t = \underbrace{\int_0^1 (1-\tau_t^W) W_{h,t} N_{h,t}\, dh}_{\text{Labor income}} +$$
$$\underbrace{(1-\tau_t^K)\left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t\right) + \iota^K \tau_t^K \delta P_{t-1}^K K_t}_{\text{Capital income}} + \underbrace{B_t^{priv} + \left(\alpha_B + (R_{t-1}^B - 1)\right) B_t + S_t B_t^{FH}}_{\text{Bond income}}$$
$$+ \underbrace{(1-\tau_t^{TR}) TR_t}_{\text{Transfer income}} + \Xi_{B,t} + \Xi_{B^{FH},t} + \Psi_t, \quad (4)$$

P_t^C is the price index of private consumption goods, P_t^I is the price index of investment and P_t^K is the price of capital. R_t is the gross nominal interest rate on private bonds denominated in Swedish currency and $R_{F,t}$ is the gross nominal interest rate of bonds denoted in the currency of Foreign. S_t is the nominal exchange rate, expressed as the price in Swedish currency of one unit of Foreign currency. There are different types of taxes levied on the household: τ_t^C denotes the consumption tax rate, τ_t^W the labor income tax rate and τ_t^K the capital income tax rate. Moreover, τ_t^{TR} denotes the tax rate levied on transfers from the government. We also allow for the possibility of investment tax credit/subsidy τ_t^I.[3]

In the budget constraint, Equation (4), the left-hand side items represent expenditure on private consumption $\left(1+\tau_t^C\right)P_t^C C_t$, investment $\left(1-\tau_t^I\right)\frac{P_t^I}{\gamma_t}I_t$, newly installed capital $P_t^K \triangle_t^K$, domestic private bonds B_{t+1}^{priv}, newly issued debt by the government B_t^n, and private bonds denominated in the currency of Foreign B_{t+1}^{FH}. Following Smets and Wouters (2007), we also include a risk-premium shock ζ_t which affects the household's return on bonds, hence also the Euler equation. $\Xi_{B,t} + \Xi_{B^{FH},t}$ denote lump-sum rebates of financial intermediation costs associated with the risk premium shocks on domestic private bonds and foreign private bonds. The function $\Phi(\cdot)$ represents a premium on Foreign bond holdings, which we will refer to as an external risk premium.[4] Its presence in the budget constraint is motivated below in the discussions of financial assets (Section 2.1.2) and net foreign assets (Section 2.7.4). The term $1/(R_t\zeta_t)$ is the effective price of domestic private bonds, while the effective price of private bonds denominated in the currency of Foreign is $S_t/(R_{F,t}\zeta_t\Phi(\cdot))$. The right-hand-side terms represent labor income net of taxes $(\int_0^1 (1-\tau_t^W)W_{h,t}N_{h,t}dh)$, rental income from the capital stock $\left(1-\tau_t^K\right)(R_t^K u_t K_t)$ and the gross return on bonds carried from the previous period $B_t^{priv} + (\alpha_B + (R_{t-1}^B - 1))B_t + S_t B_t^{FH}$.

The maintenance cost of the stock of capital is $(\frac{P_t^I}{\gamma_t}a(u_t)K_t)$, where $a(u_t)$ is the cost of capital utilization and K_t is the capital stock. The expression $\tau_t^K \frac{P_t^I}{\gamma_t}a(u_t)K_t$ captures the notion that the maintenance cost of capital can be deducted from the capital tax bill. Moreover, $\tau_t^K \delta P_{t-1}^K K_t$ captures the notion that depreciation of capital can be deducted from the capital tax bill at its historical cost. The allowance of tax deduction of depreciation of capital is contingent on the indicator variable $\iota^K \in \{0,1\}$ being set to 1. TR_t and T_t denote lump-sum transfers and taxes, respectively. The last term on the right-hand side, Ψ_t, denotes the sum of profit transfers from firms. Each individual Ricardian household owns an equal share of the domestic firm sector and any profits or losses are returned on a period-by-period basis to the household sector. Since the access to financial markets and the possibility to save is reserved for the Ricardian households, we now describe the average interest rate on government bonds and the capital accumulation equation.

To capture the empirical fact that government bonds have different maturities, which, among other things, leads to an incomplete pass-though of a change in the monetary policy rate to the interest payments for the government in the following period, we follow the approach of Krause and Moyen (2016) and allow the government bonds to have stochastic maturity. The government issues bonds that mature with probability α_B in a given period. Until stochastic maturity, the bond pays a non-state contingent interest rate. The portfolio of government bonds B_{t+1} that the household holds evolves according to

$$B_{t+1} = (1-\alpha_B) B_t + B_t^n \quad (5)$$

[3]Note that the modeling approach of τ_t^C adopted here implies that there is a complete pass-through of changes in the consumption tax rate into the sales price. In other words, the consumption tax rate modeling approach resembles a sales tax as in the U.S.

[4]As will be clear later, the external risk premium does not affect the Foreign household's return on their savings. Hence, $\Phi(\cdot)$ can also be interpreted as a pure exchange-rate shock.

where B_t^n denotes the newly issued debt by the government in period t. Following Krause and Moyen (2016), households are assumed to buy a representative portfolio of government bonds with all possible stochastic maturities. The average interest rate R_t^B on outstanding government debt bought by the household is given by

$$\left(R_t^B - 1\right) B_{t+1} = (1 - \alpha_B)\left(R_{t-1}^B - 1\right) B_t + \left(R_t^{B,n} - 1\right) B_t^n \tag{6}$$

where the interest rate on newly issued government debt is denoted by $R_t^{B,n}$.

2.1.1 Investment and capital services

The stock of capital K_{t+1} owned by the household evolves according to the following accumulation expression:

$$K_{t+1} = (1 - \delta) K_t + \Upsilon_t F(I_t, I_{t-1}) + \triangle_{k,t}^K, \tag{7}$$

where δ is a constant rate of depreciation. The stock of capital in $t+1$ is given by the previous period's stock of capital that survives the depreciation $(1-\delta) K_t$, the stationary investment-specific technology shock Υ_t, the new investment net of adjustment costs regulated by the function $F(I_t, I_{t-1})$, and the amount of capital traded between household k and the other households in Sweden $\triangle_t^K$.

In particular, it is assumed that adjustments in the rate of investment are costly. Hence, the price of one unit of installed capital, P_t^K, may differ from the cost of one unit of investment, which is denoted by $\frac{P_t^I}{\gamma_t}$. The presence of a market where households can trade capital $\triangle_t^K$ allows us to conveniently derive the price P_t^K.[5]

Firms in the intermediate goods sector rent capital services K_t^s from Ricardian households. The amount of capital services rented and used in the intermediate goods production depends on the household's chosen degree of utilization u_t and the household's chosen level of capital K_t. In every period, the individual household observes the going rental rate of capital services, R_t^K, and decides how intensively to use its current stock of capital. A higher degree of utilization u_t implies that more capital services are rented to the firm sector. The cost of a higher utilization rate is higher maintenance costs. In the current version of the model, the households' ability to vary the degree of capital utilization is de-activated, see Section 3.

2.1.2 Financial assets

We assume that there exists a set of contingent claims that allows an individual household member to diversify the component of idiosyncratic risk that is associated with its wage income and employment status, which allows full risk-sharing within the household. However, we also assume that individual members take into account household utility rather than their personal utility while giving thier decisions. This second assumption coming with the first assumption is crucial because under the full consumption risk-sharing being not working (or being unemployed) gives more utility than being employed for an individual member, and thus not internalizing the benefits to the household of members' employment would lead to no participation in the labor market.

Swedish private bonds purchased in period t yield a gross, nominal return of R_t, set by the Riksbank, times an exogenous risk premium ζ_t in the subsequent period, which creates a wedge between the Riksbank policy rate and the return that the household gets. This rate of return is known with certainty at the time of investment. The gross return on Foreign bonds earned by Swedish households, in terms of Foreign currency, is determined by the nominal interest rate in Foreign, $R_{F,t}$, the risk premium ζ_t and by the external risk premium, $\Phi(\cdot)$. The presence of the external risk premium is motivated by two concerns, the first of which is to ensure the existence of a well-defined steady state (see e.g. Schmitt-Grohe and Uribe (2001)). The second concern has to do with model dynamics around the steady state and the empirical failure of the standard uncovered interest parity (UIP) condition. Outside of the steady state, the external risk premium will cause deviations from the standard UIP condition, helping the model to better fit the data, e.g. the behavior of the real exchange rate after a monetary policy shock. We follow Adolfson et al. (2008) and specify the external risk premium as a function of the (aggregate) net foreign asset position of Sweden, of the expected change in the nominal exchange rate and of an exogenous shock, $\widetilde{\phi}_t$.[6]

2.1.3 Wage setting

As in Erceg, Henderson, and Levin (2000), each individual member of the Ricardian household is assumed to supply a differentiated labor service to the intermediate firm sector. The labor market is characterized by monopolistic competition and by staggered nominal wage contracts.

A representative employment agency rents differentiated labor services from Ricardian households and aggregates them into a homogeneous labor service which can be written as $N_t = \left[\int_0^1 (N_{h,t})^{\frac{\varepsilon_{w,t}-1}{\varepsilon_{w,t}}} dh\right]^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}}$, which is

[5] In equilibrium, all Ricardian households will want to hold the same quantity of capital. The market for capital will thus clear at a price at which the individual household wants neither to buy nor to sell any units.

[6] The functional form of $\Phi(\cdot)$ will be discussed further below, in Section (2.7.4).

sold to intermediate firms. The labor type h charges a wage rate $W_{h,t}$ for its differentiated labor service $N_{h,t}$, and the employment agency optimizes the input of different labor services in order to minimize costs. When doing so, it takes the wage rates of differentiated labor services and homogeneous labor service as given. The minimum expenditure required to produce one unit of the homogeneous labor service is given by $W_t = \left[\int_0^1 (W_{h,t})^{(1-\varepsilon_{w,t})} dh\right]^{\frac{1}{1-\varepsilon_{w,t}}}$ and W_t can be interpreted as the aggregate wage index. The agency's demand for labor from the labor type h,

$$N_{h,t} = \left(\frac{W_{h,t}}{W_t}\right)^{-\varepsilon_{w,t}} N_t, \tag{8}$$

is derived from this cost minimization problem. $\varepsilon_{w,t}$ is the wage-elasticity of demand for $N_{h,t}$, $\lambda_t^W = \frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}$ is each labor type's desired wage markup and N_t is the aggregate employment.

The wage setting by the individual household is subject to Calvo-style frictions. At the beginning of each period, labor type h learns if it is allowed to reset its wage in that period or not. The opportunity to reset the wage occurs with constant probability $(1-\xi_w)$. This probability is independent of the number of periods that passed since the last time the household had the possibility to reset its wage.[7] In periods when the wage cannot be reset, it is indexed by a factor $\overline{\Pi}_t^W = \left(\Pi_{t-1}^W\right)^{\chi_w}\left(\Pi_t^{trend}\right)^{1-\chi_w}$, where $\Pi_t^W = \frac{W_t}{W_{t-1}}$ is the aggregate wage inflation in period t and Π_t^{trend} is the inflation trend in the economy. $\chi_w \in [0,1]$ governs the weight on previous period's inflation in the inflation indexation. The higher χ_w is, the higher is the wage inflation inertia. Suppose labor type h has the opportunity to reset its wage in period t. Also recall that it considers households utility rather than its individual utility. It then chooses the optimal wage rate $W_{h,t}^{\text{opt}}$ that maximizes (1), subject to the budget constraint (4), the labor demand schedule (8) and the constraint that the wage rate $W_{h,t+k}$ in any future period $(t+k)$ evolves according to:

$$W_{h,t+k} = \begin{cases} \overline{\Pi}_{t+k}^W W_{h,t+k-1} & \text{with probability } \xi_w, \\ W_{h,t+k}^{\text{opt}} & \text{with probability } (1-\xi_w). \end{cases} \tag{9}$$

We assume that Non-Ricardian households set their wage equal to the average wage of Ricardian households and face identical labor demand. This assumption implies that the group of Ricardian and the group of Non-Ricardian households will have the same average wage rate and supply the same amount of labor.[8]

2.1.4 Labor supply and unemployment

We follow Galí (2011) and Galí, Smets, and Wouters (2012) in modelling labor force participation. Given the assumption that household members take into account the household welfare and their own personal disutility of work, the individual household member (h,j) will find it optimal to participate in the labor market in period t if and only if

$$\Omega_t^c(1-\tau_t^W)\left(\frac{W_{h,t}}{P_t^C}\right) \geq \zeta_t^n \Theta_t^n A_n j^\eta,$$

where Ω_t^c is a modified marginal utility of consumption defined below. Denote the labor supply of the marginal supplier j by $L_{h,t}$. Labor force participation condition is then written as the following:

$$\Omega_t^c(1-\tau_t^W)\left(\frac{W_{h,t}}{P_t^C}\right) = \zeta_t^n \Theta_t^n A_n L_{h,t}^\eta \tag{10}$$

This condition is a unique feature of the Galì approach that enable us to incorporate unemployment into the model in a theoretically coherent way. The condition says that household members are willing to participate to the labor force as long as the consumption utility they receive from their wage income is bigger than or equal to their disutility of work. Aggregate labor supply of the representative household is then given by

$$L_t = \int_0^1 L_{h,t} dh.$$

Having market power enables each labor type (or labor unions) to set its wage with a positive markup over marginal rate of substitution. This results in wages that are higher than in the competitive equilibrium, implying that markets do not clear and that unemployment exists in the model. Unemployment rate now can be written by its standard definition:

$$un_t = \frac{L_t - N_t}{L_t} \tag{11}$$

[7] The opportunity to reset the wage in any given period is also independently distributed across different labor types.

[8] Note that the alternative assumption that both Ricardian and Non-Ricardian households supply their labor services via unions that act as wage setters subject to the demand for labor services would give the same result that wages and labor supply are identical across both groups, see Coenen, Straub, and Trabandt (2013).

2.1.5 First-order conditions

In every period t, the household chooses C_t, I_t, u_t, $\triangle_t^K$, K_{t+1}, B_{t+1}^{priv}, B_{t+1}, B_t^n and B_{t+1}^{FH} in order to maximize Equation (1) subject to (4)-(7). The first-order conditions associated with this problem are presented next. Denote $\Omega_{h,t}^C$ as the marginal utility of consumption including the tax on consumption:

$$\Omega_t^C \equiv \frac{\zeta_t^c u_{C_t}(\tilde{C}_t, \tilde{C}_{t-1})}{1+\tau_t^C} = \frac{U_{c,t}}{1+\tau_t^C}.$$

$\beta_{t+1}^r \equiv \frac{\beta_{t+1}}{\beta_t}$ represents the change in the subjective discount factor between two consecutive periods. θ_t^b, θ_t^S, θ_t^R and θ_t^k are the Lagrange multipliers associated with the budget constraint (4), the equation for the stock of government bonds (5), the equation for the average rate of return on government bonds (6) and the capital accumulation equation (7), respectively.

$$C_t: \quad \theta_t^b P_t^C = \Omega_t^C \tag{12}$$

$$I_t: \quad \theta_t^b \frac{P_t^I}{\gamma_t}\left(1-\tau_t^I\right) = \theta_t^k \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \theta_{t+1}^k \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right], \tag{13}$$

$$u_t: \quad R_t^K K_t = \frac{P_t^I}{\gamma_t} a'(u_t) K_t, \tag{14}$$

$$\triangle_t^K: \quad \theta_t^b P_t^K = \theta_t^k, \tag{15}$$

$$K_{t+1}: \quad \theta_t^k = E_t \beta_{t+1}^r \left[\left(1-\tau_{t+1}^K\right)\theta_{t+1}^b\left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1})\right) + \theta_{t+1}^b \iota^K \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^k (1-\delta)\right], \tag{16}$$

$$B_{t+1}^{priv}: \quad \theta_t^b = E_t \beta_{t+1}^r \theta_{t+1}^b R_t \zeta_t, \tag{17}$$

$$B_{t+1}: \quad E_t\beta_{t+1}^r\theta_{t+1}^b\left(\alpha_B + \left(R_t^B - 1\right)\right) = \theta_t^S - E_t\beta_{t+1}^r\theta_{t+1}^S\left(1-\alpha_B\right) + \left(\theta_t^R - \left(1-\alpha_B\right)E_t\beta_{t+1}^r\theta_{t+1}^R\right)\left(R_t^B - 1\right) \tag{18}$$

$$B_t^n: \quad \theta_t^b \beta_t = \theta_t^S \beta_t + \beta_t \theta_t^R\left(R_t^{B,n} - 1\right) \tag{19}$$

$$R_t^B: \quad \theta_t^R E_t B_{t+1} = E_t \beta_{t+1}^r \theta_{t+1}^b B_{t+1} + E_t \beta_{t+1}^r \theta_{t+1}^R \left(1-\alpha_B\right) B_{t+1} \tag{20}$$

$$B_{t+1}^{FH}: \quad \theta_t^b S_t = E_t\left[\beta_{t+1}^r \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right) R_{F,t}\zeta_t S_{t+1}\theta_{t+1}^b\right]. \tag{21}$$

In periods when there is an opportunity to reset the wage, the household also chooses $W_{h,t}^{opt}$. To simplify notation, let $W_{h,t+k|t} = W_{h,t}^{opt}\overline{\Pi}_t^W\overline{\Pi}_{t+1}^W \ldots \overline{\Pi}_{t+k-1}^W$ denote the wage of labor type h in future period $(t+k)$, given that the household last had the opportunity to reset its wage in period t. The first-order condition of the wage optimization problem may then be written:

$$E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^{k}\beta_{t+i}^r\right) N_{h,t+k|t}\,\theta_{h,t+k}^b\left[\left(1-\tau_{t+k}^W\right)W_{h,t+k|t} - \lambda_{t+k}^W\,\zeta_{t+k}^n A_n \Theta_{t+k}\frac{N_{h,t}^{(\eta)}}{\theta_{h,t+k}^b}\right] = 0, \tag{22}$$

where $\prod_{i=1}^k \beta_{t+i}^r = \beta_{t+1}^r\beta_{t+2}^r \ldots \beta_{t+k}^r$ and $\prod_{i=1}^0 \beta_{t+i}^r \equiv 1$.[9]

[9] For future reference, we also derive the corresponding condition for the case of flexible prices and wages. When the household is free to optimize its wage in every period, the first-order condition becomes:

$$\left(1-\tau_t^W\right)W_{h,t}^{fp} = \lambda_t^W \zeta_t^n A_n \Theta_t^n \frac{(N_{h,t}^{fp})^{(\eta)}}{\theta_{h,t}^{b,fp}}. \tag{23}$$

where $W_{h,t}^{fp}$, $N_{h,t}^{fp}$ and $\theta_{h,t}^{b,fp}$ is the flexible-price equivalent expressions of wages, labor supplies and the Lagrange multiplier for the budget constraint. When wages are flexible, labor type h achieves the desired markup λ_t^W in every period.

2.2 The Swedish household sector: Non-Ricardian households

Non-Ricardian and Ricardian households have identical preferences. The difference between the two types of households is that Non-Ricardian households have no access to capital or bonds markets. In addition, it is assumed, for simplicity, that the wage and employment supplied by Non-Ricardian household equals the average wage and employment supplied by Ricardian households.

Since a Non-Ricardian household has no ability to save nor borrow, its nominal consumption expenditure equals its after-tax wage income plus the transfers it gets from the government. We index Non-Ricardian household with m,but for notational convenience we drop the index. Formally,

$$(1+\tau_t^C)\,P_t^C C_t = (1-\tau_t^W)W_t N_t + \left(1-\tau_t^{TR}\right)TR_t. \tag{24}$$

Note the assumption that the lump-sum taxes are only paid by Ricardian households. Furthermore, note that an increase in government consumption has no direct effect on the consumption decision of the Non-Ricardian household.

2.3 Aggregation of individual household variables

The private and government bonds owned by Ricardian households sum to the following aggregates:

$$B_{t+1}^{priv} = \int_0^{1-s_{nr}} B_{k,t+1}^{priv}dk$$

$$B_{t+1} = \int_0^{1-s_{nr}} B_{k,t+1}dk$$

$$B_t^n = \int_0^{1-s_{nr}} B_{k,t}^n dk$$

$$B_{t+1}^{FH} = \int_0^{1-s_{nr}} B_{k,t+1}^{FH}dk$$

Aggregate consumption and transfers can be expressed as follow:

$$C_t^{agg} = \int_0^{1-s_{nr}} C_{k,t}dk + \int_{1-s_{nr}}^1 C_{m,t}dm$$

$$TR_t^{agg} = \int_0^{1-s_{nr}} TR_{k,t}dk + \int_{1-s_{nr}}^1 TR_{m,t}dm$$

Aggregate private investments, aggregate capital traded between households, the aggregate capital stock and the aggregate capital services respectively sum to:

$$I_{t+1} = \int_0^1 I_{k,t+1}dk$$

$$\triangle_{t+1}^K = \int_0^1 \triangle_{k,t+1}^K dk$$

$$K_{t+1} = \int_0^1 K_{k,t+1}dk$$

$$K_{t+1}^s = \int_0^1 K_{k,t+1}^s dk$$

2.4 The Swedish firm sector

Several different types of firms operate in the Swedish economy. Some of these firms are price setters and others are price takers.

Six types of firms operate in monopolistically competitive markets and they face nominal frictions in their price setting. These are producers of domestic intermediate goods, import firms for non-energy consumer goods, import firms for energy consumer goods, import firms for investment goods, import firms for export goods, and export good producers. The rationale for including three different types of import firms is to be able to better match the macro data on Swedish imports and import prices.

Four types of representative firms operate under perfect competition. These firms take both the prices of their inputs and the prices at which they sell their output as given. Two representative firms produce private final consumption goods and private investment goods, while two other representative firms produce government consumption goods and government investment goods, respectively. The private consumption and investment goods firm use domestically produced intermediate goods as well as imported goods while the public consumption and investment good firms only use domestically produced intermediate inputs.

The optimization problems of the different types of firms are described below. In addition, the firm sector also consists of a number of aggregator firms that aggregate the different varieties of goods that are produced within each of the markets characterized by monopolistic competition. All aggregator firms operate under perfect competition and their problems are not explicitly discussed in the text. Instead, the standard input demand functions and price indices associated with these aggregators are stated as restrictions in the problems of other firms.

2.4.1 Swedish intermediate good producers

A continuum of firms produce domestic intermediate goods, each of which is differentiated from other intermediate goods produced in the sector. The total mass of these firms is unity and they operate in a market characterized by monopolistic competition. Each firm sets its price to minimize the costs of producing the associated output.

A representative aggregator firm buys the different varieties of goods and aggregates it into a homogeneous intermediate good that is sold to the firms producing consumption goods, investment goods and export goods. The demand for the individual variety i, $Y_t(i)$, is a function of the relative price of that variety and of total demand for the homogeneous, intermediate good:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{\frac{\lambda_t}{1-\lambda_t}} Y_t. \tag{25}$$

$P_t(i)$ denotes the price charged by firm i, $P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\lambda_t}} di\right)^{1-\lambda_t}$ is the price associated with the homogeneous, intermediate good and Y_t denotes total demand. λ_t is a time-varying markup over marginal cost that evolves according to an exogenous, stochastic process.[10]

The individual intermediate good firm takes the rental rate of capital services R_t^K, the wage rate $\left(1+\tau_t^{SSC}\right) W_t$ including social security contributions τ_t^{SSC}, and the public capital stock $K_{G,t}$ as given when it decides on an optimal input of production factors: $K_t^s(i)$ and $N_t(i)$. In addition to these two variable costs, firms also incur a fixed cost $z_t^+\phi$ in each period. The cost-minimization problem of firm i is given by

$$\min_{K_t^s(i), L_t(i)} \left\{R_t^K K_t^s(i) + \left(1+\tau_t^{SSC}\right) W_t N_t(i)\right\}$$

s.t.

$$Y_t(i) = \varepsilon_t \left[\tilde{K}_t^s(i)\right]^{\alpha} \left[z_t N_t(i)\right]^{1-\alpha} - z_t^+\phi.$$

where $\tilde{K}_t^s(i)$ denotes a composite capital service input made up by private capital services $K_t^s(i)$ and public capital $K_{G,t}$. We assume the following constant elasticity of substitution (CES) aggregator of private capital services $K_t^s(i)$ and the public capital stock $K_{G,t}$:

$$\tilde{K}_t^s(i) = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(K_t^s(i)\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}} \left(K_{G,t}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K-1}}.$$

Hence, we assume that each intermediate good firm i has access to the same public capital stock. We also assume that public capital grows at the same rate as private capital services along the balanced growth path. The parameter υ_K is the elasticity of substitution between private capital services and the public capital stock,

[10] Note that λ_t may be interpreted as a function of a time varying elasticity of substitution between the different varieties of intermediate goods. One natural interpretation, therefore, of shocks to λ_t is of an exogenous change in the degree of market power enjoyed by the individual firms in this sector.

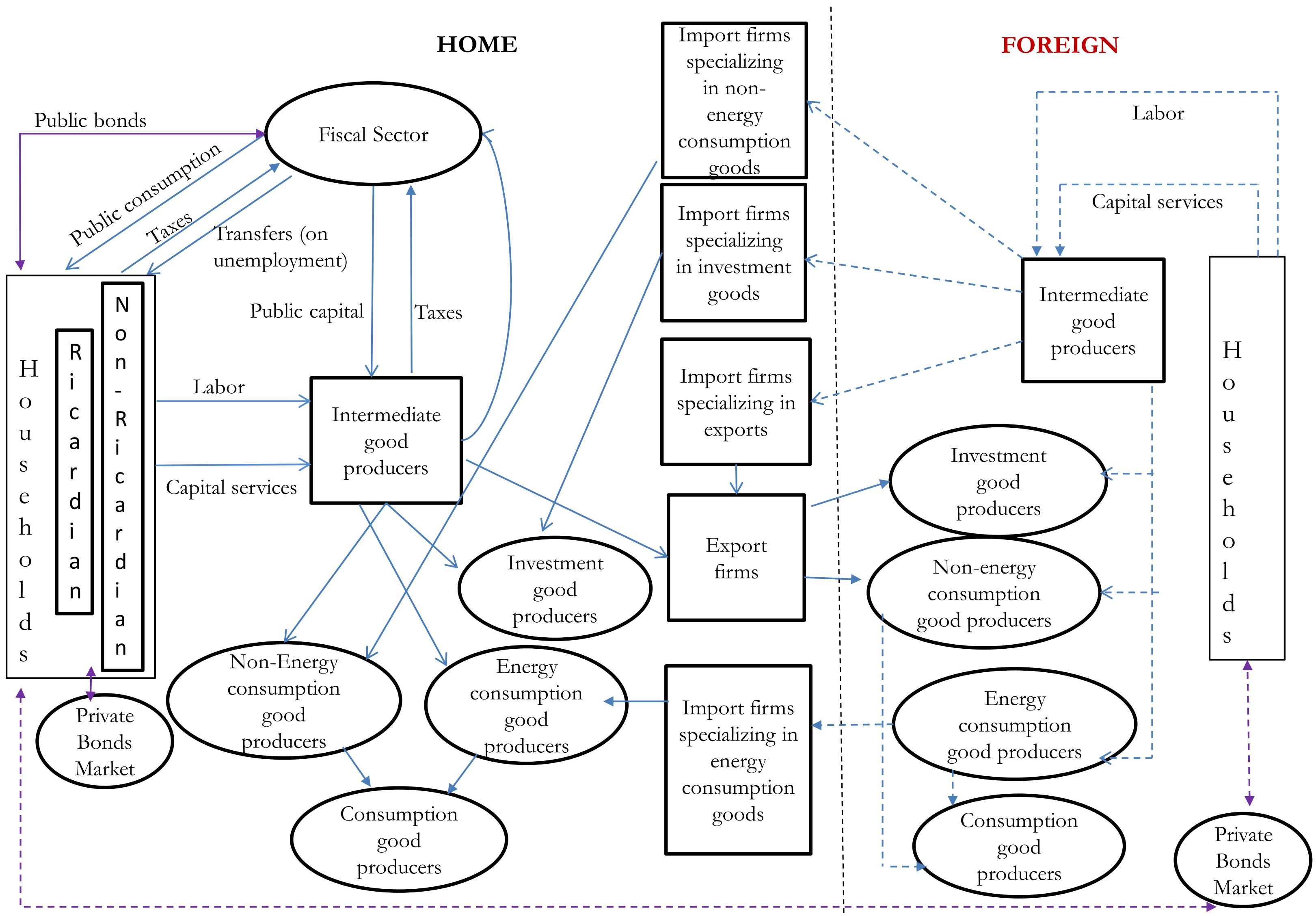


Figure 1: Overview of SELMA

and α_K is a share parameter. ε_t is a stationary stochastic process, with an unconditional mean of unity, that is common to all firms in the Swedish intermediate good sector. The shock captures temporary changes in total factor productivity of the firms. The global stochastic process that governs the labor augmenting technology, z_t, is growth-stationary. Let $\mu_{z,t} = \frac{z_t}{z_{t-1}}$ denote the growth rate of z_t.

The variable z_t^+,which is multiplied by the fixed cost, ensures that the fixed cost grows in proportion to output. It consists of a combination of the labor augmenting technology variable z_t and an investment-specific productivity variable γ_t and is given by

$$z_t^+ = z_t \gamma_t^{\frac{\alpha}{1-\alpha}}. \tag{26}$$

The solution to the cost minimization problem can be expressed in terms of a marginal cost function:

$$MC_t(i) = \frac{\left(\frac{(1+\tau_t^{SSC})W_t}{z_t}\right)^{1-\alpha} \left(R_t^K\right)^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\varepsilon_t \mathit{\Gamma}_{G,t}(i)}. \tag{27}$$

A relationship between the rental rate for capital services and the optimal capital-to-labor ratio:

$$R_t^K = \alpha \varepsilon_t z_t^{1-\alpha} MC_t(i) \left(\frac{\tilde{K}_t^s(i)}{L_t(i)}\right)^{\alpha-1} \left(\mathit{\Gamma}_{G,t}(i)\right)^{\frac{1}{\alpha}}. \tag{28}$$

where

$$\mathit{\Gamma}_{G,t}(i) = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}},$$

The firms face a Calvo style price friction when they set their prices. In every period t, there is a probability $(1-\xi)$ that the individual firm i gets the opportunity to reset its price. With complementary probability ξ the firm does not have this opportunity. In the latter case, the non-reset price $P_{t-1}(i)$ will instead be indexed by $\overline{\Pi}_t$ such that $P_t(i) = \overline{\Pi}_t P_{t-1}(i)$, where $\overline{\Pi}_t = (\Pi_{t-1})^{\chi}\left(\Pi_t^{trend}\right)^{1-\chi}$ is a weighted average of previous period's gross inflation Π_{t-1} and the inflation trend Π_t^{trend}. The inflation trend does in turn follow a stochastic autoregressive process which is specified later. $\chi \in [0,1]$ represents the weight on previous period's inflation in indexation. Suppose firm i has the opportunity to reset its price in period t and let $P_{t+k|t}(i) \equiv P_t^{opt}\overline{\Pi}_{t+1}\cdots\overline{\Pi}_{t+k}$ denote the price that will apply in period $(t+k)$, conditional on the firm not having any opportunity to reset its price between periods t and $(t+k)$. When choosing P_t^{opt}, the firm seeks to maximize the expected, discounted sum of present and future profits, which may be written as

$$E_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left\{P_{t+k|t}(i) Y_{t+k|t}(i) - TC_{t+k|t}\left[Y_{t+k|t}(i)\right]\right\}, \tag{29}$$

where $Y_{t+k|t}(i)$ is the demand in period $(t+k)$ for the output of firm i, conditional on the price $P_{t+k|t}(i)$. $\Lambda_{t,t+k}$ represents the firm's stochastic discount factor and $TC_{t+k|t}[\cdot]$ denotes total cost, as a function of output.[11] The first-order condition associated with this problem may be written as[12]

$$E_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \frac{Y_{t+k|t}}{(\lambda_{t+k}-1)} \left(P_{t+k|t} - \lambda_{t+k} MC_{t+k}\right) = 0. \tag{31}$$

As mentioned above, private consumption goods, investment goods and export goods are assumed to be composites of domestically produced intermediate goods and of imported goods. Before we proceed to describe the firms that are active in the markets for these final goods, we outline the problem of Swedish import firms.

[11] In keeping with the assumption that Ricardian households own the firms, firms discount future profits at the same rate as households discount future income: $\Lambda_{t,t+k} = \frac{\beta_{t+k}\Omega_{t+k}^C P_t^C}{\beta_t \Omega_t^C P_{t+k}^C}$.

[12] All firms that have an opportunity to reset their price in period t will face the same problem. As a consequence, all such firms will choose the same optimal reset price and they will produce the same quantity of output in that period. Therefore, index i is dropped in this equation. In the equilibrium with flexible prices and wages, the corresponding (standard) first-order condition of the firm gives a price equal to the desired markup times the marginal cost:

$$P_t^{\mathrm{fp}} = \lambda_t MC_t^{\mathrm{fp}}. \tag{30}$$

2.4.2 Swedish import firms

Following Corbo and Strid (2020), there are four types of Swedish import firms. One type of firm specializes in the business of importing intermediate goods from Foreign and transforming those imported goods into inputs that are suitable for the production of export goods. A second type of import firm transforms imported goods to inputs suited for the production of private investment goods. The third type of import firm transforms imported goods to inputs suited for the production of non-energy consumption goods, and the fourth specializes in transforming the energy good from Foreign into an input suited for the production of the energy consumption good. We capture the local currency pricing through the import firm's price setting. The import firms face sticky prices, allows for incomplete pass-through from the exchange rate to prices in the importing country. There exists a continuum of individual import firms of each type, and each of these individual firms owns a technology to make one-to-one transformations of the homogeneous Foreign export good into a differentiated import good. The individual heterogeneous import goods are then again transformed into a homogeneous import good by an aggregator firm. Let $n \in \{X, I, \{C, xe\}, \{C, e\}\}$ index the type of import firm, and let $M_t^n(i)$ represent the quantity produced by the individual firm i of type n. The cost to firm i of producing $M_t^{n^{xe}}(i)$ units of the differentiated import good of type $n^{xe} \in \{X, I, \{C, xe\}\}$ is $S_t P_{F,t}\left[M_t^{n^{xe}}(i) + z_t^+ \phi^{M,n^{xe}}\right]$, where $P_{F,t}$ is the price of the homogeneous Foreign intermediate good and $S_t P_{F,t} z_t^+ \phi^{M,n}$ denotes the fixed cost of production. The cost to firm i of producing $M_t^{C,e}(i)$ units of the differentiated energy import good is given by $S_t P_{F,t}^{C,e}\left[M_t^{C,e}(i) + z_t^+ \phi^{M,C,e}\right]$, where $P_{F,t}^{C,e}$ is the price of the Foreign energy good and $P_{F,t}^{C,e} z_t^+ \phi^{M,C,e}$ denotes the fixed cost of production. The price of the differentiated product of the individual import firm i of type n is denoted by $P_t^{M,n}(i)$ and $P_t^{M,n} = \left(\int_0^1 P_t^{M,n}(i)^{\frac{1}{1-\lambda_t^{M,n}}} di\right)^{1-\lambda_t^{M,n}}$ denotes the price of the homogeneous import good of type n. $\lambda_t^{M,n}$ is a time-varying, exogenous markup that is specific to all import firms of type n. The individual firm faces the following demand for its differentiated product:

$$M_t^n(i) = \left[\frac{P_t(i)^{M,n}}{P_t^{M,n}}\right]^{\frac{\lambda_t^{M,n}}{1-\lambda_t^{M,n}}} M_t^n. \tag{32}$$

M_t^n represents the total demand for the homogeneous import good of type n. Like firms in the intermediate good sector, import firms face pricing frictions. With probability $(1 - \xi_{M,n})$, individual firm i will be able to reset its price in period t. The optimal reset price is denoted $P_{t,opt}^{M,n}$. With complementary probability $\xi_{M,n}$, the price from the previous period will instead be indexed according to $P_t^{M,n}(i) = \overline{\Pi}_t^{M,n} P_{t-1}^{M,n}(i)$. The indexing factor $\overline{\Pi}_t^{M,n}$ is defined as $\overline{\Pi}_t^{M,n} = \left(\Pi_{t-1}^{M,n}\right)^{\chi_{m,n}} \left(\Pi_t^{trend}\right)^{1-\chi_{m,n}}$, where $\Pi_{t-1}^{M,n} = \frac{P_{t-1}^{M,n}}{P_{t-2}^{M,n}}$. $\chi_{m,n} \in [0,1]$ represents the weight on previous period's inflation of import goods. Let $P_{t+k|t}^{M,n} = P_{t,opt}^{M,n} \overline{\Pi}_{t+1}^{M,n} \cdots \overline{\Pi}_{t+k}^{M,n}$ denote the price that will apply in period $(t+k)$, conditional on the firm not having any opportunity to reset its price between periods t and $(t+k)$. In periods when the firm does have an opportunity to reset its price, it chooses $P_{t,opt}^{M,n}$ in order to maximize:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \Lambda_{t,t+k} \left\{P_{t+k|t}^{M,n^{xe}}(i)\, M_{t+k|t}^{n^{xe}}(i) - S_t P_{F,t+k} M_{t+k|t}^{n^{xe}}(i) - S_{t+k} P_{F,t+k} z_{t+k}^+ \phi^{M,n^{xe}}\right\} \tag{33}$$

for the non-energy firms and

$$E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \Lambda_{t,t+k} \left\{P_{t+k|t}^{M,C,e}(i)\, M_{t+k|t}^{C,e}(i) - S_t P_{F,t+k}^{C,e}\, M_{t+k|t}^{C,e}(i) - S_{t+k} P_{F,t+k}^{C,e} z_{t+k}^+ \phi^{M,C,e}\right\} \tag{34}$$

for the energy firms. If we write the marginal costs of the firms as

$$\begin{aligned} MC_t^{M,n^{xe}}(i) &= S_t P_{F,t} \\ MC_t^{M,C,e}(i) &= S_t P_{F,t}^{C^e}, \end{aligned} \tag{35}$$

then the first-order condition associated with the firm's maximization problem may be written:[13]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \Lambda_{t,t+k} \frac{M_{t+k|t}^n}{\left(\lambda_{t+k}^{M,n} - 1\right)} \left(P_{t+k|t}^{M,n} - \lambda_{t+k}^{M,n}\, MC_{t+k}^n(i)\right) = 0, \quad n \in \{X, I, \{C, xe\}, \{C, e\}\}. \tag{37}$$

[13]The corresponding first-order condition in the flexible price and wage equilibrium is:

$$P_{t,\text{fp}}^{M,n} = \lambda_t^{M,n}\, MC_t^n(i), \quad n \in \{X, I, C^{xe}, C^e\}. \tag{36}$$

2.4.3 Swedish export firms

Firms in the Swedish export sector use domestically produced intermediate goods and imported goods as inputs in their production of export goods. Export firms act as price takers in the markets for their input goods and as price setters in the market for their output goods. There are infinitely many export good producers, each of which produce a differentiated good that is sold in a market characterized by monopolistic competition. The different export firms share a common production technology and minimize the costs of production by choosing an optimal mix of inputs. Let $D_t^X(i)$ and $M_t^X(i)$ denote, respectively, the quantity of the domestically produced intermediate good and of the imported good used as inputs by individual firm i in the export good sector. The cost minimization problem is given by

$$\min_{D_t^X(i),\, M_t^X(i)} \left\{ P_t\, D_t^X(i) + P_t^{M,X}\, M_t^X(i) \right\}$$

s.t.

$$X_t(i) = \left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x - 1}{\nu_x}} + (1-\psi^X)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x - 1}{\nu_x}} \right]^{\frac{\nu_x}{\nu_x - 1}} - z_t^+ \phi^X.$$

$X_t(i)$ denotes the quantity produced by the individual firm i and ν_x represents the elasticity of substitution between domestically produced and imported inputs in the production of export goods. Furthermore, $\psi^X = \vartheta^X + \frac{1}{1+\omega}\left(1-\vartheta^X\right)$ is the weight of the domestically produced intermediate good in production, where $\vartheta^X \in [0,1]$ may be interpreted as an index of home bias. $z_t^+\phi^X$ is the fixed cost of production. The marginal cost of the export goods is given by[14]

$$MC_t^X = \left[\psi^X \left(P_t\right)^{(1-\nu_x)} + \left(1-\psi^X\right)\left(P_t^{M,X}\right)^{(1-\nu_x)} \right]^{\frac{1}{1-\nu_x}}. \tag{38}$$

A representative aggregator firm buys the different varieties of export goods and aggregates them into a homogeneous export good that is sold to import firms in Foreign. The demand for the individual variety i, $X_t(i)$, is a function of the relative price of that variety and of total demand for Swedish exports: $X_t(i) = \left[\frac{P_t^X(i)}{P_t^X}\right]^{\frac{\lambda_t^X}{1-\lambda_t^X}} X_t$. $P_t^X(i)$ denotes the price charged by firm i, $P_t^X = \left(\int_0^1 P_t^X(i)^{\frac{1}{1-\lambda_t^X}} di\right)^{1-\lambda_t^X}$ is the price of the homogeneous export good and X_t is total demand. λ_t^X denotes the desired markup of Swedish export firms and is governed by an exogenous, stochastic process. The pricing frictions faced by the individual export good producers are of the same type as those faced by firms in the intermediate good sector and the import good sector. The probability that firm i has an opportunity to reset its price in any given period is denoted $(1-\xi_x)$ and the optimal reset price is represented by $P_t^{X,opt}$. The objective function of the export firm may be written

$$E_t \sum_{k=0}^{\infty} (\xi_x)^k \Lambda_{t,t+k} \left\{ P_{t+k|t}^X(i) S_{t+k} X_{t+k|t}(i) - TC_{t+k|t}^X \left[X_{t+k|t}(i)\right] \right\}, \tag{39}$$

where S_t enters the function due to local currency pricing[15], $TC_{t+k|t}^X\left[X_{t+k|t}(i)\right]$ represents total costs and $P_{t+k|t}^X = P_t^{X,opt}\,\overline{\Pi}_{t+1}^X \cdots \overline{\Pi}_{t+k}^X$. $\overline{\Pi}_t^X = \left(\Pi_{t-1}^X\right)^{\chi_x}\left(\Pi_{F,t}^{trend}\right)^{1-\chi_x}$ denotes the indexing factor, $\Pi_{t-1}^X = \frac{P_{t-1}^X}{P_{t-2}^X}$ and $\Pi_{F,t}^{trend}$ is the trend inflation in Foreign. In order for the reset price to be optimal, it must satisfy:[16]

$$E_t \sum_{k=0}^{\infty} (\xi_x)^k \Lambda_{t,t+k} \frac{X_{t+k|t}}{(\lambda_{t+k}^X - 1)} \left(P_{t+k|t}^X S_{t+k} - \lambda_{t+k}^X\, MC_{t+k}^X \right) = 0. \tag{41}$$

[14] For future reference, note that the first-order conditions from this problem may be used to derive input demand equations $D_t^X(i) = \psi^X\left(\frac{MC_t^X}{P_t}\right)^{\nu_x}\left[X_t(i) + z_t^+\phi^X\right]$ and $M_t^X(i) = (1-\psi^X)\left(\frac{MC_t^X}{P_t^{M,X}}\right)^{\nu_x}\left[X_t(i) + z_t^+\phi^X\right]$. $D_t^X(i)$ represents the demand for the domestically produced, intermediate good from the individual export firm i. $M_t^X(i)$ denotes the demand for the imported good from the same firm.

[15] For the export good producer, local currency pricing implies that the export producers price their goods in the currency of Foreign. They are, however, interested in maximizing profits in Swedish currency, which is why the exchange rate enters the equation.

[16] In the equilibrium with flexible prices and wages, the corresponding first-order condition instructs the firm to set its price (times the exchange rate to denote it into Swedish currency) equal to the desired markup times the marginal cost:

$$P_t^{X,\text{fp}} S_t = \lambda_t^X MC_t^{X,\text{fp}}. \tag{40}$$

2.4.4 Swedish investment good producers

After describing the sectors for intermediate goods and for imports and exports, we now turn to the production of private investment goods. The investment good production sector consists of a continuum of investment good firms that operate on a market characterized by perfect competition. This means that they act as price takets, both in the market for their inputs and in the market for their output. The representative investment good producer use domestically produced intermediate goods and imported goods used for investment as inputs in its production of investment goods. Let D_t^I and M_t^I denote, respectively, the quantity of the domestically produced homogenous intermediate good and of the homogenous imported good used as inputs by the representative investment good firm. Furthermore, let $V_t^I = \frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]$ be the output of the investment good firm (meaning that the utilization cost is payed for via investment goods), and P_t^I be the price of investment goods. The maximization problem of the firm is then given by

$$\max_{V_t^I,\, D_t^I,\, M_t^I} \left\{ P_t^I\, V_t^I - P_t\, D_t^I - P_t^{M,I}\, M_t^I \right\}$$

s.t.

$$V_t^I = \left[\left(\psi^I\right)^{\frac{1}{\nu_I}} \left(D_t^I\right)^{\frac{\nu_I - 1}{\nu_I}} + (1-\psi^I)^{\frac{1}{\nu_I}} \left(M_t^I\right)^{\frac{\nu_I - 1}{\nu_I}} \right]^{\frac{\nu_I}{\nu_I - 1}}.$$

$\psi^I = \vartheta^I + \frac{1}{1+\omega}\left(1-\vartheta^I\right)$ is the weight of the domestically produced intermediate good in the production of the investment good, and $\vartheta^v \in [0,1]$ may be interpreted as an index of home bias. The first-order conditions from this problem yield input demand functions $D_t^I = \psi^I \left(\frac{P_t^I}{P_t}\right)^{\nu_I} V_t^I$ and $M_t^I = \left(1-\psi^I\right)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I} V_t^I$. Note that ν_I may be interpreted as the price-elasticity of demand for the two respective inputs. P_t^I is the minimum expenditure needed to produce one unit of each investment good:

$$P_t^I = \left[\psi^I \left(P_t\right)^{1-\nu_I} + \left(1-\psi^I\right)\left(P_t^{M,I}\right)^{1-\nu_I} \right]^{\frac{1}{1-\nu_I}}. \tag{42}$$

Given the assumption of perfect competition, the representative investment good firm make zero profits. Therefore we interpret P_t^I as the appropriate price index for the investment good.

2.4.5 Swedish consumption good producers

In the modelling of private consumption, we follow Corbo and Strid (2020) and let the private consumption goods C_t^{agg} be created by a combination of non-energy consumption goods C_t^{xe} and energy consumption goods C_t^e. These goods are in turn created by combining domestic and imported non-energy goods, $D_t^{C,xe}$ and $M_t^{C,xe}$, and domestic and imported energy goods, $D_t^{C,e}$ and $M_t^{C,e}$, respectively. All firms face perfect competition, which means that they are price takers both regarding their inputs and their outputs. The maximization problem for the representative private consumption good firm is given by

$$\max_{C_t^{agg},\, C_t^{xe},\, C_t^e} \left\{ P_t^C\, C_t^{agg} - P_t^{C,xe}\, C_t^{xe} - P_t^{C,e}\, C_t^e \right\}$$

s.t.

$$C_t^{agg} = \left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}} \left(C_t^{xe}\right)^{\frac{\nu_C - 1}{\nu_C}} + (1-\vartheta^C)^{\frac{1}{\nu_C}} \left(C_t^e\right)^{\frac{\nu_C - 1}{\nu_C}} \right]^{\frac{\nu_C}{\nu_C - 1}}$$

where $P_t^{C,xe}$ is the price of non-energy consumption goods and $P_t^{C,e}$ is the price of energy consumption goods. ϑ^C is the weight of non-energy consumption good in the production function. The first-order conditions from this problem yield input demand functions

$$C_t^{xe} = \vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C} C_t^{agg} \tag{43}$$

$$C_t^e = \left(1-\vartheta^C\right)\left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C} C_t^{agg}. \tag{44}$$

Note that ν_C may be interpreted as the price-elasticity of demand for the two respective inputs. P_t^C is the minimum expenditure needed to produce one unit of each consumption good:

$$P_t^C = \left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(P_t^{C,e}\right)^{1-\nu_C} \right]^{\frac{1}{1-\nu_C}}. \tag{45}$$

The non-energy good producers face the following maximization problem:

$$\max_{C_t^{xe},\, D_t^{C,xe},\, M_t^{C,xe}} \left\{ P_t^{C,xe}\, C_t^{xe} - P_t\, D_t^{C,xe} - P_t^{M,C,xe}\, M_t^{C,xe} \right\}$$

s.t.

$$C_t^{xe} = \left[\left(\vartheta^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + (1-\vartheta^{C,xe})^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}$$

where $\vartheta^{C,xe}$ is the weight of the domestically produced intermediate good in the production of goods. The first-order conditions from this problem yield input demand functions

$$D_t^{C,xe} = \vartheta^{C,xe} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_{C,xe}} C_t^{xe} \tag{46}$$

$$M_t^{C,xe} = \left(1-\vartheta^{C,xe}\right) \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}} C_t^{xe} \tag{47}$$

Note that $\nu_{C,xe}$ may be interpreted as the price-elasticity of demand for the two respective inputs. $P_t^{C,xe}$ is the minimum expenditure needed to produce one unit of each non-energy consumption good:

$$P_t^{C,xe} = \left[\psi^{C,xe} \left(P_t\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C,xe}\right) \left(P_t^{M,C,xe}\right)^{1-\nu_{C,xe}} \right]^{\frac{1}{1-\nu_{C,xe}}} . \tag{48}$$

The energy good producers face the following maximization problem:

$$\max_{C_t^{e},\, D_t^{C,e},\, M_t^{C,e}} \left\{ P_t^{C,e}\, C_t^{e} - P_t^{D,Ce}\, D_t^{C,e} - P_t^{M,C,e}\, M_t^{C,e} \right\}$$

s.t.

$$C_t^{e} = \left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}} \left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + (1-\vartheta^{C,e})^{\frac{1}{\nu_{C,e}}} \left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

where $\vartheta^{C,e}$ is the weight of the domestically produced intermediate good in the production of goods. The first-order conditions from this problem yield input demand functions

$$D_t^{C,e} = \vartheta^{C,e} \left(\frac{P_t^{C,e}}{P_t^{D,C,e}} \right)^{\nu_{C,e}} C_t^{e} \tag{49}$$

$$M_t^{C,e} = \left(1-\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{M,Ce}} \right)^{\nu_{C,e}} C_t^{e} \tag{50}$$

Note that $\nu_{C,e}$ may be interpreted as the price-elasticity of demand for the two respective inputs. $P_t^{C,e}$ is the minimum expenditure needed to produce one unit of each non-energy consumption good:

$$P_t^{C,e} = \left[\vartheta^{C,e} \left(P_t\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right) \left(P_t^{M,C,e}\right)^{1-\nu_{C,e}} \right]^{\frac{1}{1-\nu_{C,e}}} . \tag{51}$$

Note that the price of energy follows a stochastic process which is defined in Section 2.8.

2.4.6 Swedish government consumption and government investment good producers

Two representative firms, a government consumption good producer and a government investment good producer, use only domestically produced inputs in their production of final goods. These representative firms act as price takers, both in the markets for their inputs and in the markets for their respective outputs. There are no pricing frictions in the markets for government consumption and investment goods. Let $D_t^{v^P}$ be the quantity of the domestically produced intermediate goods used as inputs by the representative firm in sector $v^P \in \{G, I^G\}$. Furthermore, let V_t^P, $P \in \{G, I^G\}$ denote the output of such a representative firm. The profit maximization problem of such a representative firm is given by

$$\max_{V_t^P,\, D_t^{v^P}} \left\{ P_t^{v^P} V_t^P - P_t D_t^{v^P} \right\}$$

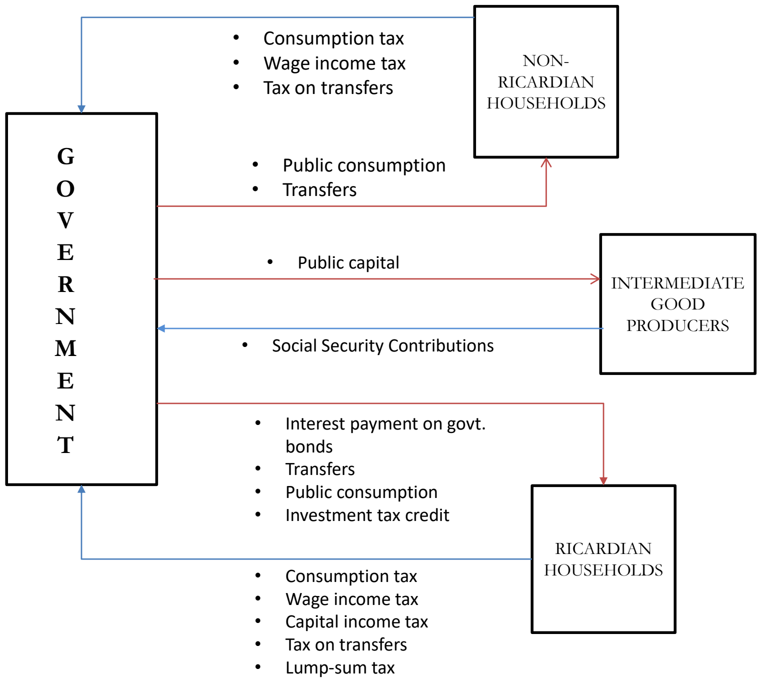
s.t.

$$V_t^P = D_t^{v^P}.$$

which implies that the price of both types of goods is given by

$$P_t^{v^P} = P_t.$$

Figure 2: Fiscal policy block



2.5 Fiscal authority and central bank in Sweden

In Sweden, a fiscal authority controls a large set of fiscal instruments (described in detail below) and the central bank, Riksbank, controls the nominal interest rate on private bonds.The interest rate is set according to a Taylor rule, taking into account the zero or effective lower bound for the nominal interest rate.

2.5.1 The Swedish fiscal authority

The government in Sweden collects taxes levied on household labor income, transfers, private consumption, household capital income, as well as lump-sum taxes. Furthermore, it collects social security contributions from the intermediate good firms. The government uses the tax revenue and issues bonds to finance expenditures. The expenditures consist of government consumption, government investment, lump-sum transfers and an investment tax credit as well as interest payments on government debt. Figure 2 illustrates the fiscal sector and its flows. The government budget constraint is given by

$$\tau_t^C P_t^C C_t^{agg} + (\tau_t^{SSC} + \tau_t^W) W_t N_t + \Upsilon_t^K + B_t^n + T_t = \left(\alpha_B + (R_{t-1}^B - 1)\right) B_t + \tau_t^I \frac{P_t^I}{\gamma_t} I_t + P_t G_t + P_t \frac{I_t^G}{\gamma_t} + (1 - \tau_t^{TR}) TR_t^{agg} \tag{52}$$

where

$$\Upsilon_t^K = \tau_t^K \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t \right) - \iota^K \tau_t^K \delta P_{t-1}^K K_t. \tag{53}$$

$\tau_t^C P_t^C C_t^{agg}$ denotes the aggregate revenue from the tax on private consumption, while $\left(\tau_t^{SSC} + \tau_t^W\right) W_t N_t$ denotes the aggregate revenue from tax on labor income. Υ_t^K denotes the capital income tax revenue, B_t^n denotes the newly issued debt by the government in period t and T_t denotes lump-sum taxes. On the right hand side of the government budget equation, $\left(\alpha_B + \left(R_{t-1}^B - 1\right)\right) B_t$ denotes interest rate payments on previously issued government bonds, where R_t^B is the average interest rate on outstanding government debt. $\tau_t^I \frac{P_t^I}{\gamma_t} I_t$ denotes the expenses due to the investment tax credit. $P_t G_t$ and $\frac{P_t}{\gamma_t} I_t^G$ denote expenses on government consumption and government investment, respectively. Finally $\left(1 - \tau_t^{TR}\right) TR_t^{agg}$ denotes aggregate lump-sum transfers net of taxes.

The government owns and maintains the public capital stock in the economy:

$$K_{G,t+1} = (1 - \delta_G) K_{G,t} + I_t^G$$

where $K_{G,t+1}$ denotes the public capital stock in the next period and I_t^G denotes government investment.[17] [18]

Government surplus We define primary revenues $PREV_t$ as

$$PREV_t = \tau_t^C P_t^C C_t^{agg} + \left(\tau_t^{SSC} + \tau_t^W\right) W_t N_t + \Upsilon_t^K + \tau_t^{TR} TR_t^{agg} + T_t \tag{54}$$

and primary expenditure $PEXP_t$ as

$$PEXP_t = \tau_t^I \frac{P_t^I}{\gamma_t} I_t + P_t G_t + P_t \frac{I_t^G}{\gamma_t} + TR_t^{agg}. \tag{55}$$

Given the primary revenues and the primary expenditure, we use the government budget constraint and define the government surplus $SURP_t$ as[19]

$$SURP_t \equiv \underbrace{\underbrace{PREV_t - PEXP_t}_{\text{primary surplus}} - \underbrace{\left(R_{t-1}^B - 1\right)}_{\text{interest payments}} B_t}_{\text{total surplus (or gov. balance)}} = \alpha_B B_t - B_t^n$$

Hence, the surplus equals the incoming government debt that matures in period t minus the newly issued government debt.

The fiscal instruments Fiscal policy can be conducted using the following different instruments:

$$x_t \in \left\{g_t,\, I_t^G,\, \tau_t^I,\, \tau_t^C,\, \tau_t^W,\, \tau_t^K,\, \tau_t^{TR},\, \tau_t^{SSC}\right\},$$

and tr_t^{agg}. g_t and I_t^G are the government transfers, government consumption and government investment per capita, and τ_t^I, τ_t^C, τ_t^W, τ_t^K, τ_t^{TR}, τ_t^{SSC} are the different tax rates in the economy. tr_t^{agg} is the aggregate transfers in units of domestically produced intermediate goods. The equations for each of the instruments can be divided into two different parts: an AR(1) process and a fiscal feedback rule, so that $x_t = x_t^{AR} + x_t^{Rule}$. The AR(1) part, which exists for all instruments except for the lump-sum tax, can be described by

$$x_t^{AR} = (1-\rho_x)x + \rho_x x_{t-1} + \varepsilon_t^x. \tag{56}$$

The fiscal feedback rule consists of three elements: the deviation of the government debt level as percent of GDP from its target $b_{\bar{y},t} - b_{\bar{y},t}^{Target}$, the deviation of the structural government surplus as percent of steady state GDP from its target $Stsurp_{\bar{y}.t} - Stsurp_{\bar{y},t}^{Target}$, and log deviation of GDP from its steady state level $\hat{y}_t$. On the other hand, the feedback rule for tr_t^{agg}consists of $b_{\bar{y},t} - b_{\bar{y},t}^{Target}$, $Stsurp_{\bar{y}.t} - Stsurp_{\bar{y},t}^{Target}$ and log deviation of unemployment from its steady state level $\hat{un}_t$. In the fiscal rules the surplus target is defined on the structural surplus, that is the cyclically adjusted budget balance.

The structural surplus is defined as the difference between the structural primary revenue, $Stprev_t$ and the structural primary expenditure, $Stpexp_t$, net of the interest payments on the current debt $(R_{t-1}^B - 1)B_t$. The structural primary expenditure is calculated by removing the business-cycle component (i.e. the output gap or unemployment gap reactions of the variables in the fiscal rules) from the actual primary expenditure while the structural primary revenues are calculated by multiplying all tax rates with their respective structural tax bases. Hence the structural surplus is given as[20]:

$$Stsurp_t = Stprev_t - Stpexp_t - (R_{t-1}^B - 1)B_t \tag{57}$$

with

$$Stprev_t = \tau_t^C P^C C^{agg} + \left(\tau_t^{SSC} + \tau_t^W\right) WN + \tau_t^K K \left(R^K - \iota^K \delta \frac{P^K}{\Pi}\right) + \tau_t^{TR} \left(TR_t^{agg} - F_{tr,un} Y \breve{un}_t\right) + T \tag{58}$$

where $P^C C^{agg}$ denotes the steady state consumption tax base while WN is the wage income tax base at the steady state. $K(R^K - \iota^K \delta \frac{P^K}{\Pi})$ and $TR_t^{agg} - F_{tr,y} Y \hat{un}_t$ denote the steady state capital income tax base and structural transfer tax base, respectively. $Stpexp_t$ is defined as:

[17] Note that there is no investment adjustment cost for public capital.

[18] The aggregate investments in the economy can be written as $I_t^{agg} = I_t + I_t^G$.

[19] Government surplus is sometimes referred to as government net lending. Here we do, however, use government surplus to be in line with the literature.

[20] For more detailed information on the structural surplus calculation see Appendix C.7.1.

$$\frac{Stpexp_t}{P_t} = \left(\frac{TR_t^{agg}}{P_t} - F_{tr,un} Y \hat{un}_t\right) + \left(\frac{I_t^G}{\gamma_t} - \mathcal{F}_{IG,y} \frac{I^G}{\gamma Y}(Y_t - Y)\right) + \left(G_t - \mathcal{F}_{g,y} \frac{G}{Y}(Y_t - Y)\right) + \tau_t^I \frac{P^I}{\gamma P_t} I \quad (59)$$

where $\frac{P^I}{\gamma} I$ denotes the steady state level for investment tax base and all other government expenditure terms are adjusted for their respective cyclical component.

A fiscal rule equation has been defined for eight of the instruments. The investment subsidy has not been assigned a rule as there is no such subsidy in Sweden at present. For government consumption and government investment, $x_t^{Rule} \in \{g_t, I_t^G\}$, the rule is given by

$$x_t^{Rule} = \mathcal{F}_{x,b}\left(b_{\bar{y},t} - b_{\bar{y},t}^{Target}\right) + \mathcal{F}_{x,surp}\left(Stsurp_{\bar{y},t} - Stsurp_{\bar{y},t}^{Target}\right) + \mathcal{F}_{x,y}\hat{y}_t. \quad (60)$$

The first two terms on the right-hand-side of the equations are supposed to capture the Swedish fiscal framework, which includes a surplus target and a debt anchor. This kind of feedback for the debt level can be found also in e.g. Coenen, Straub, and Trabandt (2013) and Erceg and Lindé (2013). The third and last part of the equation is supposed to capture automatic stabilizers. For the tax rates, $x_t^{Rule} \in \{\tau_t^C, \tau_t^W, \tau_t^K, \tau_t^{TR}, \tau_t^{SSC}\}$, the rule is given by

$$x_t^{Rule} = \mathcal{F}_{x,b}\left(b_{\bar{y},t} - b_{\bar{y},t}^{Target}\right) + \mathcal{F}_{x,surp}\left(Stsurp_{\bar{y},t} - Stsurp_{\bar{y},t}^{Target}\right). \quad (61)$$

The eighth rule is the transfer rule which is normalized by steady-state GDP, $\bar{y}$:

$$tr_t^{agg,Rule} = \bar{y}F_{tr,b}\left(b_{\bar{y},t} - b_{\bar{y},t}^{Target}\right) + \bar{y}\mathcal{F}_{tr,surp}\left(Stsurp_{\bar{y},t} - Stsurp_{\bar{y},t}^{Target}\right) + \bar{y}\mathcal{F}_{tr,un}\breve{un}_t. \quad (62)$$

There is a mapping between the debt-target and the surplus target. This mapping needs to hold in the steady state, since a certain level of debt in percent of GDP in the long run implies a unique surplus in percent of GDP. The mapping between the debt target and the surplus target is defined as

$$Stsurp_{\bar{y},t}^{Target} = \left(\frac{1}{\mu_{z+}\Pi} - 1\right) b_{\bar{y},t}^{Target}.$$

Debt and surplus target shocks The debt and surplus target are also fiscal policy variables. The fiscal authority might want to deviate from the rules temporarily. This is captured by the debt target shock $\varepsilon_t^{b^{Target}}$.[21] After such a shock, the debt target follows an AR(2) process which is described as

$$b_{\bar{y},t}^{Target} - b_{\bar{y}} = \left(\rho_{1,b^T} + \rho_{2,b^T}\right)\left(b_{\bar{y},t-1}^{Target} - b_{\bar{y}}\right) - \rho_{1,b^T}\rho_{2,b^T}\left(b_{\bar{y},t-2}^{Target} - b_{\bar{y}}\right) + \epsilon_t^{b^{Target}}. \quad (63)$$

Aggregate transfer distribution The share of aggregate transfers that goes to Ricardian and Non-Ricardian households respectively off the steady state is governed by the following equation:

$$\varpi_{dyn}\breve{tr}_t = (1 - \varpi_{dyn})\breve{tr}_t^{nr},$$

where $\breve{tr}_t$ and $\breve{tr}_t^{nr}$ are the deviations in transfers to Ricardians and Non-Ricardians in units of domestically produced intermediate goods. The equation implies that the steady-state distribution of transfers between Ricardian and Non-Ricardian households might differ from the distribution off the steady state.

2.5.2 The Swedish central bank

The Riksbank sets the policy interest rate according to a Taylor rule. We follow Corbo and Strid (2020), and let the interest rate be affected by the deviations of inflation and unemployment from their steady-state levels, and by the changes in inflation and unemployment. The rule is written in deviations from steady state, where $\breve{i}_t$ is defined as the deviation of the policy rate from the neutral interest rate, defined below. $\hat{\Pi}_t^C \equiv ln\left(\frac{\Pi_t^C}{\Pi^C}\right)$ is the deviation of inflation from the Riksbank target rate Π^C which is also the steady state inflation rate. The Riksbank reacts to annual inflation rather than quarterly inflation, where the annual inflation is defined as $\hat{\Pi}_t^{a,C} = \frac{1}{4}\left(\hat{\Pi}_t^C + \hat{\Pi}_{t-1}^C + \hat{\Pi}_{t-2}^C + \hat{\Pi}_{t-3}^C\right)$. $\breve{un}_t$ is the deviation of unemployment rate from its steady state level. Furthermore, there is a lower bound on the interest rate $\underline{\breve{i}}$. If the Taylor rule implies an interest rate level below the lower bound, the interest rate is set to the lower bound. The following two equations govern the interest rate:

[21] Note that the debt target shock can also be used to capture a shock to the surplus target, since the debt target and the surplus target are mirror images in the steady state.

$$\breve{i}_t^{notional} = \rho \breve{i}_{t-1}^{notional} + (1-\rho)\left(r_\pi \hat{\Pi}_{t-1}^{a,C} + r_{un}\breve{un}_{t-1}\right) + r_{\triangle\pi}\left(\hat{\Pi}_t^C - \hat{\Pi}_{t-1}^C\right) + r_{\triangle un}\left(\breve{un}_t - \breve{un}_{t-1}\right) + \epsilon_t^i, \quad (64)$$

$$\breve{i}_t^{ss} = max(\underline{\breve{i}}, \breve{i}_t^{notional} + \breve{i}_t^{nat}) \quad (65)$$

where $\breve{i}_t^{notional}$ denotes the notional nominal policy rate, i.e. the nominal interest rate absent the effective lower bound constraint, $\breve{i}_t^{ss}$ is the deviation of the actual interest rate from its steady state value, and $\breve{i}_t^{nat}$ is the neutral interest rate, both in deviations from their respective steady state values. ϵ_t^i is an exogenous, stochastic shock. The second equation introduces the zero or effective lower bound constraint into the model.

2.5.3 The neutral interest rate

We follow Corbo and Strid (2020) and introduce a neutral interest rate into the model. The neutral rate is introduced for empirical reasons, given the observation that global interest rates have declined over time, at the same time as it is difficult to argue that actual monetary policy have become more and more expansionary. As such, we interpret $\breve{i}_t$ as being the policy rate deviation from the neutral rate rather than the deviation from its steady state level, such that

$$\breve{i}_t = \breve{i}_t^{ss} - \breve{i}_t^{nat} \quad (66)$$

A consequence of this assumption is that the resulting model simulations (except for the simulated policy rate) are not affected by the introduction of a neutral interest rate, except for when the neutral interest rate lies below the lower bound of the interest rate. Furthermore, we assume that the inflation rate in the neutral rate remains constant, so that changes in the neutral rate happens only via changes in the neutral real interest rate. Hence, we can write the neutral interest rate in a similar manner as in Corbo and Strid (2020), as

$$\breve{i}_t^{nat} = r_\mu \hat{\mu}_{z^+,t} - r_\zeta \hat{\zeta}_t + \hat{z}_t^r \quad (67)$$

where $\hat{\mu}_{z^+,t}$ is the log-deviation of growth rate of from its steady-state level, $\hat{\zeta}_t$ is the log deviation of the risk-premium shock from its steady-state level, and $\hat{z}_t^r$ is a shock process introduced to capture factors that are not introduced into the model explicitly, but that can be assumed to change the neutral rate, such as demographic factors. Given how the neutral rate is introduced into the model, we interpret an interest rate that is lower than the neutral rate as expansionary monetary policy, while an interest rate that is higher than the neutral rate as contractionary monetary policy.

2.6 The Foreign economy

We model Sweden as a small open economy. Due to its size relative to Sweden, the Foreign economy instead behaves like a closed economy. From the perspective of the Foreign economy, any transactions between the two countries will be arbitrarily small, compared to the total quantities of goods that are produced and consumed within Foreign. Formally, we assume that the size ω of Foreign tends to infinity, $\omega \to \infty$ implying that the relative size of the Swedish economy in relation to world economy; $\frac{1}{1+\omega}$, tends to zero. Given the size of Foreign, we abstract from modelling the Foreign export and import sectors. The reasons are the following: Firstly, since the exports and imports from Sweden are arbitrarily small compared to aggregate Foreign output, they will not have any effect on the equilibrium allocations and prices in Foreign. Secondly, the modelling of Foreign exports and imports adds an additional layer of complexity, but does not give any additional information to the evolution of Swedish imports and exports that can not be captured by the modelling of the Swedish import and export sectors. Note however, that we still need to model the demand for Swedish exports and supply of Swedish imports. Both are discussed in the market clearing section. The derivation of the export demand is however presented in Section E.

The Foregin firms' optimization problems are to a great extent identical to those in Sweden, up to a scaling factor. There are also, however, important differences between the two economies. Compared to the Swedish economy, the fiscal sector in Foreign is modelled in much less detail. In addition, intermediate goods producers use only private capital as physical capital in their production. Moreover, it is assumed that all households in Foreign are Ricardian households, and that they can only save in bonds denominated in the currency of Foreign. The following sections describe the different agents in Foreign and their decision problems. Because of the similarities with Sweden, the explanations given here are relatively sparse and emphasis is given to areas where the two economies differ.

2.6.1 Foreign households

We model the foreign households slightly different from the households in Sweden. We assume a standard representative household setup which is the most commonly used in DSGE literature, where households (or its members) differentiate from each other only by their labor type but not by their disutility of labor. We also use hours worked as the unit of labor in foreign economy (intensive margin) while we use employment for the Swedish economy (extensive margin). The total mass of households in Foreign is ω. Foreign households' preferences over private consumption and hours worked are identical to the Ricardian households in Sweden. Furthermore, we assume that Foreign households are able to hold assets that yield a risk-free return in terms of Foreign currency, just like the Ricardian households in Sweden are able to hold assets with a risk-free return in terms of Swedish currency. Foreign households are however not allowed to hold bonds in Swedish currency. The problem of the individual household f in Foreign is to choose private consumption $C_{f,t}$, physical capital $K_{f,t+1}$, Investment I_t, capital utilization $u_{f,t}$, the change in capital stock by trading in the market $\triangle^K_{f,t}$, domestic nominal bonds that are denominated in the Foreign currency $B^{FF}_{f,t+1}$ and the nominal wage $W_{f,t}$, in order to maximize

$$E_0 \sum_{t=0}^{\infty} \beta_{F,t} \left[\zeta^c_{F,t} u(C_{f,t}, C_{F,t-1}) - \zeta^n_{F,t} \nu(N_{f,t}) \right], \tag{68}$$

subject to the following budget constraint:

$$P^C_{F,t} C_{f,t} + \frac{P^I_{F,t}}{\gamma_t} I_{f,t} + P^K_{F,t} \triangle^K_{f,t} + \frac{B^{FF}_{f,t+1}}{R_{F,t}\zeta_{F,t}}. = (1-\tau^w_F) W_{f,t} N_{f,t} + R^K_{F,t} u_{f,t} K_{f,t} - \frac{P^I_{F,t}}{\gamma_t} a(u_{f,t}) K_{f,t} + B^{FF}_{f,t} + \Xi_{B^{FF},t} + \Psi_{f,t} + TR_{f,t}, \tag{69}$$

and the capital accumulation process:

$$K_{f,t+1} = (1-\delta_F) K_{f,t} + \Upsilon_{F,t} F(I_{f,t}, I_{f,t-1}) + \triangle^K_{f,t}, \tag{70}$$

$C_{F,t}$ denotes aggregate consumption in Foreign. δ_F is the depreciation rate of the foreign capital and $\Upsilon_{F,t}$ is the stationary investment-specific technology shock. In periods when the household has an opportunity to reset its wage, it also chooses $W^{\text{opt}}_{f,t}$ subject to the following condition:

$$W_{f,t+k} = \begin{cases} \overline{\Pi}^W_{F,t+k-1} W_{f,t+k} & \text{with probability } \xi^F_w \\ W^{\text{opt}}_{f,t+k} & \text{with probability } 1-\xi^F_w \end{cases} \tag{71}$$

for all $k \geq 0$, and taking $N_{f,t+k} = \frac{1}{\omega}\left(\frac{W_{f,t+k}}{W_{F,t+k}}\right)^{-\varepsilon^F_w} N_{F,t}$ as given. Let $W_{f,t+k|t} = W^{opt}_{f,t}\overline{\Pi}^W_{F,t+1}\ldots\overline{\Pi}^W_{F,t+k}$ denote the wage of household f in future period $(t+k)$, given that the household last opportunity to set the wage was in period t. The first-order conditions associated with this problem are presented next. $\Omega^C_{f,t}$ denotes the marginal utility of consumption and $\beta^r_{F,t+1} = \frac{\beta_{F,t+1}}{\beta_{F,t}}$ represents changes in the subjective discount factor between consecutive periods. $\theta^b_{f,t}$ and $\theta^k_{f,t}$denote the Lagrange multipliers associated with the budget constraint (69) and the capital accumulation equation (70), respectively.

$$C_{f,t} : \theta^b_{f,t} P^C_{F,t} = \Omega^C_{f,t}, \tag{72}$$

$$B^{FF}_{f,t+1} : \theta^b_{f,t} P^C_{F,t} = E_t \left[\beta^r_{F,t+1} \theta^b_{f,t+1} P^C_{F,t} R_{F,t} \zeta_{F,t} \right] \tag{73}$$

$$K_{f,t+1} : \theta^k_{f,t} = E_t \beta^r_{F,t+1} \left[\theta^b_{f,t+1} \left(R^K_{F,t+1} u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}} a(u_{f,t+1}) \right) + \theta^k_{f,t+1}(1-\delta_F) \right] \tag{74}$$

$$I_{f,t} : \theta^b_{f,t} \frac{P^I_{F,t}}{\gamma_t} = \theta^k_{f,t} \Upsilon_{F,t} F_1(I_{f,t}, I_{f,t-1}) + E_t \left[\beta^r_{F,t+1} \theta^k_{f,t+1} \Upsilon_{F,t+1} F_2(I_{f,t+1}, I_{f,t}) \right] \tag{75}$$

$$u_{f,t} : R^K_{F,t} K_{f,t} = \frac{P^I_{F,t}}{\gamma_t} a'(u_{f,t}) K_{f,t} \tag{76}$$

$$\triangle^K_{f,t} : \theta^b_{f,t} P^K_{F,t} = \theta^k_{f,t}. \tag{77}$$

The first-order condition associated with $W^{opt}_{f,t}$ is given by

$$E_t \sum_{k=0}^{\infty} \left(\xi^F_w\right)^k \left(\prod_{i=1}^{k} \beta^r_{F,t+i} \right) N_{f,t+k|t}\, \theta^b_{f,t+k} \left[(1-\tau^w_F)\, W_{f,t+k|t} - \lambda^W_F\, \zeta^n_{F,t+k} \frac{\nu'\left(N_{f,t+k|t}\right)}{\theta^b_{f,t+k}} \right] = 0, \tag{78}$$

where $\prod_{i=1}^{k} \beta^r_{F,t+i} = \beta^r_{F,t+1}\beta^r_{F,t+2}\ldots\beta^r_{F,t+k}$ and $\prod_{i=1}^{0} \beta^r_{F,t+i} \equiv 1.$[22]

[22]When the household is free to optimize its wage in every period, as is the case in the equilibrium with flexible prices and wages,

2.6.2 Foreign intermediate good producers

The intermediate good sector in Foreign is consist of a continuum of firms with total mass ω. As in Sweden, a representative aggregator firm buys the different varieties of goods and produces a homogeneous, intermediate good that is sold to firms in other sectors. The demand for the individual variety j, $Y_{F,t}(j)$ is a function of the relative price of that variety and of total demand for the homogeneous, intermediate good: $Y_{F,t}(j) = \frac{1}{\omega}\left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} Y_{F,t}$. $P_{F,t}(j)$ denotes the price charged by firm j, $P_{F,t} = \left(\frac{1}{\omega}\int_0^{\omega} P_{F,t}(j)^{\frac{1}{1-\lambda_{F,t}}} dj\right)^{1-\lambda_{F,t}}$ is the price index associated with the homogeneous, intermediate good and $Y_{F,t}$ denotes total demand. $\lambda_{F,t}$ is a time-varying, stochastic markup. Intermediate good firms in Foreign use labor and capital as inputs in their production. The cost-minimization problem of firm j is:

$$\min_{K_{F,t}(j) L_{F,t}(j)} \left\{ R^K_{F,t} K^s_{F,t}(j) + W_{F,t} N_{F,t}(j) \right\}$$

s.t.

$$Y_{F,t}(j) = \varepsilon_{F,t} \left[K^s_{F,t}(j)\right]^{\alpha_F} \left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z^+_{F,t}\phi_F,$$

where $\varepsilon_{F,t}$ is a stationary stochastic process with an unconditional mean of unity, that is common to all firms in the sector. As in Swedish economy, $z^+_{F,t}$ is a function of the two global stochastic variables z_t and γ_t and is given by

$$z^+_{F,t} = z_t \gamma_t^{\frac{\alpha_F}{1-\alpha_F}}. \tag{79}$$

The cost-minimization problem yields the following expression for nominal marginal cost:

$$MC_{F,t} = \frac{\left(\frac{W_{F,t}}{z_t}\right)^{1-\alpha_F} \left(R^K_{F,t}\right)^{\alpha_F}}{\alpha_F^{\alpha_F} \left(1-\alpha_F\right)^{1-\alpha_F} \varepsilon_{F,t}}. \tag{80}$$

The rental rate of capital services can be written as a function of the marginal cost and the optimal capital-to-labor ratio:

$$R^K_{F,t} = \alpha_F \varepsilon_{F,t} z_t^{1-\alpha_F} MC_{F,t} \left(\frac{K^s_{F,t}}{N_{F,t}}\right)^{\alpha_F - 1}. \tag{81}$$

The price setting problem of intermediate good firms in Foreign is identical to that of the corresponding firms in Sweden. Therefore, we only state the first-order condition associated with that problem and refer the reader to Section 2.4.1 for more details:[23]

$$E_t \sum_{k=0}^{\infty} (\xi^F)^k \Lambda^F_{t,t+k} \frac{Y_{F,t+k|t}}{(\lambda_{F,t+k} - 1)} \left(P_{F,t+k|t} - \lambda_{F,t+k}\, MC_{F,t+k}\right) = 0. \tag{82}$$

2.6.3 Foreign consumption good producers

A representative firm produces consumption goods that are sold to households in Foreign. As in Sweden, the consumption good consists of a combination of non-energy and energy goods. The markets for the inputs and outputs of this representative firm are characterized by perfect competition, flexible prices and zero profits. The production function is similar to that of Sweden, i.e. given by

$$C_{F,t} = \left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_{F.t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{e}_{F.t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}.$$

the first-order condition becomes:

$$\left(1-\tau^F_w\right) W^{\mathrm{fp}}_{f,t} = \lambda^W_F \zeta^n_{F,t} \frac{\nu'\left(N^{\mathrm{fp}}_{f,t}\right)}{\theta^{\mathrm{b,fp}}_{f,t}}.$$

[23]In the equilibrium with flexible prices and wages, the corresponding first-order condition is:

$$P^{\mathrm{fp}}_{F,t} = \lambda_{F,t}\, MC^{\mathrm{fp}}_{F,t}.$$

The respective demand functions for energy and non-energy goods are given by

$$C^{xe}_{F.t} = \vartheta^C_F \left(\frac{P^C_{F,t}}{P^{C,xe}_{F,t}}\right)^{\nu_{F,C}} C_{F,t} \tag{83}$$

$$C^{e}_{F.t} = \left(1-\vartheta^C_F\right) \left(\frac{P^C_{F,t}}{P^{C,e}_{F,t}}\right)^{\nu_{F,C}} C_{F,t} \tag{84}$$

and the price of the Foreign consumption good is given by

$$P^C_{F,t} = \left[\vartheta^C_F \left(P^{C,xe}_{F,t}\right)^{1-\nu_{F,C}} + \left(1-\vartheta^C_F\right)\left(P^{C,e}_{F,t}\right)^{1-\nu_{F,C}}\right]^{\frac{1}{1-\nu_{F,C}}}. \tag{85}$$

Similarly to Sweden, non-energy consumption is produced by combining domestic and imported non-energy goods according to the production function

$$C^{xe}_{F,t} = \left[\left(\vartheta^{C,xe}_F\right)^{\frac{1}{\nu_{F,C,xe}}} \left(D^{C,xe}_{F.t}\right)^{\frac{\nu_{F,C,xe}-1}{\nu_{F,C,xe}}} + \left(1-\vartheta^{C,xe}_F\right)^{\frac{1}{\nu_{F,C,xe}}} \left(M^C_{F.t}\right)^{\frac{\nu_{F,C,xe}-1}{\nu_{F,C,xe}}}\right]^{\frac{\nu_{F,C,xe}}{\nu_{F,C,xe}-1}}.$$

However, due to Foreign being so large compared to Sweden, the share of Swedish imports in the production of Foreign non-energy consumption goes to zero, meaning that $\vartheta^{C,xe}_F \to 1$.This reduces the production function to

$$C^{xe}_{F,t} = D^{C,xe}_{F.t}. \tag{86}$$

Furthermore, due to the assumption of perfect competition, the price of the non-energy good is given by

$$P^{C,xe}_{F,t} = P_{F,t}. \tag{87}$$

Finally, the Foreign energy consumption good is produced by transforming Foreign domestic intermediate goods to energy goods. Their production function is given by

$$C^{e}_{F,t} = D^{C,e}_{F.t}. \tag{88}$$

2.6.4 Fiscal authority and central bank in Foreign

The fiscal authority in Foreign is modelled in a sparse manner. The fiscal authority levies a tax on labor income, and all tax income is returned to households using transfers. The government budget is balanced every period, and there is no government debt. The fiscal authority's budget constraint is given by

$$W_{F,t}N_{F,t}\tau^w_F = TR_{F,t} + G_{F,t}. \tag{89}$$

Concerning monetary policy, it is assumed that the central bank in Foreign sets its policy interest rate according to a Taylor rule, where $\breve{i}_{F,t}$ is the policy rate deviation from the neutral interest rate, following Corbo and Strid (2020). The Taylor rule is similar to Sweden, but reacts to output rather than unemployment, since unemployment is not modelled in Foreign. The Foreign central bank also reacts to annual inflation rather than quarterly inflation, where the annual inflation is defined as $\hat{\Pi}^{a,C}_{F,t} = \frac{1}{4}\left(\hat{\Pi}^C_{F,t} + \hat{\Pi}^C_{F,t-1} + \hat{\Pi}^C_{F,t-2} + \hat{\Pi}^C_{F,t-3}\right)$. As in Sweden, the interest rate in Foreign is restricted by its lower bound $\underline{i_F}$. The interest rate follows the following equations:

$$\breve{i}^{notional}_{F,t} = \rho_F \breve{i}^{notional}_{F,t-1} + (1-\rho_F)\left(r_{F,\pi}\hat{\Pi}^C_{F,t} + r_{F,y}\hat{y}_{F,t}\right) + r_{F,\triangle\pi}\left(\hat{\Pi}^C_{F,t} - \hat{\Pi}^C_{F,t-1}\right) + r_{F,\triangle y}\left(\hat{y}_{F,t} - \hat{y}_{F,t-1}\right) + \epsilon^{i_F}_t, \tag{90}$$

$$\breve{i}^{ss}_{F,t} = max(\underline{i_F}, \breve{i}^{notional}_{F,t} + \breve{i}^{nat}_{F,t}) \tag{91}$$

Just as in Sweden, we define the monetary policy expansion as the difference between the actual rate $\breve{i}^{ss}_{F,t}$ and the the neutral rate $\breve{i}^{nat}_{F,t}$:

$$\breve{i}_{F,t} = \breve{i}^{ss}_{F,t} - \breve{i}^{nat}_{F,t} \tag{92}$$

where the neutral rate is defined as

$$\breve{i}^{nat}_{F,t} = r_{F,\mu}\hat{\mu}_{z+,t} - r_{F,\zeta}\hat{\zeta}_{F,t} + \hat{z}^r_t \tag{93}$$

2.7 Market clearing

In equilibrium, decisions taken by individual households and firms must be consistent with market clearing in the markets for goods, bonds and capital. For most types of goods and assets, the markets need to clear within each country. In principle, the markets for traded goods between the country also needs to clear. For the export goods, this is done by equalizing the supply of export goods, described in Section 2.4.3, and the demand for Swedish exports, which is defined below. The goods that are imported to Sweden from Foreign does however either consist of Foreign domestic goods or Foreign energy goods, which are created by Foreign domestic goods. Since Sweden is so small compared to Foreign, the demand for Foreign domestic goods and energy goods by Swedish firms do not have any effect on the aggregate output in Foreign. Therefore, we abstract from the purchases by Swedish import firms in the Foreign market clearing conditions. The international payments between Sweden and Foreign must however balance, which is achieved via the Balance of Payments equation. The expressions in this section are derived in Appendix E.

2.7.1 Aggregate resources

As a necessary condition for the Swedish market for domestically produced intermediate goods to clear, the sum of output from individual intermediate good producers must equal Swedish final good producers' (i.e. private and government consumption, private and government investment and export good producers) demand for domestically produced intermediate goods. Let Y_t be the amount of domestically produced homogeneous intermediate goods. Also, let $\overset{\leftrightarrow}{P}{}_t^X = \int_0^1 \left(\frac{P_t^X(i)}{P_t^X}\right)^{\frac{\lambda_t^X}{1-\lambda_t^X}} di$ be a measure of price dispersion among firms in the export good sector. Furthermore, let $N_t^D = \int_0^1 N_t(i)di$ denote total demand for labor services from the intermediate good producers. The aggregate resource constraint for Sweden may then be written as

$$Y_t = \psi^{C,xe}\left(\frac{P_t^{C,xe}}{P_t^C}\right)^{\nu_{C,xe}} C_t^{xe} + D_t^{C,e} + \psi^I\left(\frac{P_t^I}{P_t}\right)^{\nu_I}\left[\frac{I_t}{\gamma_t} + a\left(u_t\right)\frac{K_t}{\gamma_t}\right] + \psi^X\left(\frac{MC_t^X}{P_t}\right)^{\nu_x}\left[X_t \overset{\leftrightarrow}{P}{}_t^X + z_t^+\phi^X\right] + G_t + \frac{I_t^G}{\gamma_t} \tag{94}$$

Corresponding definitions are used to write the Foreign aggregate resource constraint. For example, $\overset{\leftrightarrow}{P}{}_{F,t} = \int_0^\omega \frac{1}{\omega}\left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} dj$ is a measure of price dispersion in the Foreign market for intermediate goods, and $N_{F,t} = \int_0^\omega N_{F,t}(j)dj$ denotes the total (or aggregate) demand for labor services in Foreign. Aggregate resources in Foreign are used to satisfy the demand for non-energy consumption goods, energy consumption goods and investment goods. Remember, however, that there is no government consumption and government investment. Therefore, the Foreign aggregate resource constraint contains fewer terms than that of the Swedish economy:

$$\begin{aligned}\varepsilon_{F,t}\left[K_{F,t}^s\right]^{\alpha_F}\left[z_t N_{F,t}\right]^{1-\alpha_F} &= \overset{\leftrightarrow}{P}{}_{F,t}\psi_F^{C.xe}\left(\frac{P_{F,t}^{C,xe}}{P_{F,t}}\right)^{-\nu_{F,C}} C_{F,t}^{xe} + \overset{\leftrightarrow}{P}{}_{F,t}C_{F,t}^{e} \\ &+ \overset{\leftrightarrow}{P}{}_{F,t}\psi_F^I\left(\frac{P_{F,t}^I}{P_{F,t}}\right)^{-\nu_{F,I}}\left[\frac{I_{F,t}}{\gamma_t} + a\left(u_{F,t}\right)\frac{K_{F,t}}{\gamma_t}\right] + G_{F,t} + z_{F,t}^+\omega\phi_F.\end{aligned} \tag{95}$$

For future reference, we also define Swedish and Foreign output (GDP), where GDP is the same as the domestically produced homogeneous input goods Y_t and $Y_{F,t}$, where

$$\overset{\leftrightarrow}{P}{}_t Y_t = \int_0^1 \left(\varepsilon_t\left[K_t^s(i)\right]^\alpha\left[z_t L_t(i)\right]^{1-\alpha} - z_t^+\phi\right) di \tag{96}$$

$$\overset{\leftrightarrow}{P}{}_{F,t} Y_{F,t} = \int_0^\omega \left(\varepsilon_{F,t}\left[K_{F,t}^s(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z_{F,t}^+\phi_F\right) dj \tag{97}$$

In the definition of Swedish and Foreign GDP given by Equation (96) and Equation (97), respectively, capital utilization costs are included, since some of the output goes to paying these costs. These equations implicitly include utilization costs as a part of final demand. This definition is consistent with an interpretation of these costs as a form of investment. Christiano, Trabandt, and Walentin (2011), however, suggest that a second and alternative definition of output be used for the purpose of matching model variables to the data. We adopt their approach and use Y_t^m to denote 'measured output' in Sweden and $Y_{F,t}^m$ in Foreign, which are equal to output less of utilization costs. The capital utilization costs are represented by the term $\psi^I\left(\frac{P_t^I}{P_t}\right)^{-\nu_I} a\left(u_t\right)\frac{K_t}{\gamma_t}$ in Equation (94) and by the term $\psi_F^I\left(\frac{P_{F,t}^I}{P_{F,t}}\right)^{-\nu_{F,I}} a\left(u_{F,t}\right)\frac{K_{F,t}}{\gamma_t}$ in Equation (97). Hence we have

$$Y_t^m = Y_t - \psi^I\left(\frac{P_t^I}{P_t}\right)^{-\nu_I} a\left(u_t\right)\frac{K_t}{\gamma_t} \tag{98}$$

$$Y_{F,t}^{m}=Y_{F,t}-\psi_{F}^{I}\left(\frac{P_{F,t}^{I}}{P_{F,t}}\right)^{-\nu_{F,I}}a\left(u_{F,t}\right)\frac{K_{F,t}}{\gamma_{t}} \tag{99}$$

With this alternative concept of output, capital utilization costs are treated as a proper cost and are not included in gross fixed capital formation.

2.7.2 Market clearing for bonds

There are three different bond markets that have to clear. The first is the market for private bonds denominated in Swedish currency. The market clearing condition is given by

$$B_{t+1}^{priv}=0. \tag{100}$$

The second is the market for Foreign bonds. First define $B_{t+1}^{FH}=\int_0^{1-s_{nr}}B_{k,t+1}^{FH}dk$, the aggregate value of purchases by all Swedish households of such bonds in period t. Since the bonds are traded across the two countries, the clearing condition is given by

$$B_{t+1}^{FH}+\int_0^{\omega}B_{f,t+1}^{FF}df=0. \tag{101}$$

The third bond market is the market for government bonds. In that market, the total amount of newly issued debt by the government B_t^n must equal the total household demand for newly issued government debt. The market clearing condition is given by

$$B_t^n=\int_0^{1-s_{nr}}B_{k,t}^{n}dk. \tag{102}$$

2.7.3 International trade in goods

Let X_t denote aggregate demand for Swedish exports. The consumption and investment good firms in Foreign are using inputs from Sweden in their production. The demand function for Swedish export goods is derived in Appendix E.3.5, and is given by

$$X_t=\left(1-\psi_F^{C,xe}\right)\left(\frac{P_t^X}{P_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}}C_{F,t}^{xe}+\left(1-\psi_F^{I}\right)\left(\frac{P_t^X}{P_{F,t}^{I}}\right)^{-\nu_{F,I}}I_{F,t}. \tag{103}$$

We now move on to the import sector. Since Sweden is arbitrarily small in relation to Foreign, the demand for imports does not affect the supply of goods in Foreign, meaning that Foreign will supply any number of goods that the Swedish households demand for the given price. Define all Swedish non-energy imports M_t^{xe} as

$$M_t^{xe}=\int_0^1 M_t^{C,xe}(i)di+\int_0^1 M_t^{I}(i)di+\int_0^1 M_t^{X}(i)di+z_t^{+}\phi^{M,C,xe}+z_t^{+}\phi^{M,I}+z_t^{+}\phi^{M,X}. \tag{104}$$

Then total Swedish imports are given by

$$M_t=M_t^{xe}+\int_0^1 M_t^{C,e}(i)di+z_t^{+}\phi^{M,C,e}. \tag{105}$$

It is also useful to define total imports of energy goods M_t^e as

$$M_t^e=\int_0^1 M_t^{C,e}(i)di+z_t^{+}\phi^{M,C,e}. \tag{106}$$

2.7.4 Balance of payments and net foreign assets

Two types of international transactions occur between agents in Sweden and Foreign. Firms in the Swedish export and import sectors trade with Foreign firms, and Swedish households buy and sell in the Foreign (international) market for bonds. In the aggregate, the nominal value of these different transactions must balance. For the purpose of defining a balance of payments relationship for Sweden, let us start by adding up the different transactions that occur in the market for international bonds. Note that the value of this aggregate position in Swedish currency is $A_t=\frac{S_tB_{t+1}^{FH}}{R_{F,t}\zeta_t\Phi\left(\tilde{a}_t,s_t,\tilde{\phi}_t\right)}$. A_t may therefore be referred to as the period t nominal aggregate net foreign asset position of Sweden.[24] The value in Swedish currency of the settlement of bonds issued in the previous period is $\int_0^{1-s_{nr}}\left[S_tB_{k,t}^{FH}\right]dk=S_tB_t^{FH}$.

[24]Since all these bonds mature one period after they are issued, A_t will consist of the total value of all outstanding Foreign bonds at the end of period t. Note, however, that Swedish households can both save and borrow in Foreign bonds, implying that B_{t+1}^{FH} may be either a positive or a negative number. If $B_{t+1}^{FH}<0$, the period t aggregate net foreign asset position of Sweden is negative.

$\Phi(\cdot)$ represents an external risk premium on domestic (Swedish) holdings of Foreign bonds, and the choice of a specific functional form for this premium merits some discussion. Note that $s_t = \frac{S_t}{S_{t-1}}$. The value of $\Phi(\cdot)$ is determined by the two aggregate variables $\overline{a}_t = \frac{A_t}{z_t^+ P_t}$ and $E_t\left(\frac{S_{t+1}}{S_t}\frac{S_t}{S_{t-1}}\right) = E_t(s_{t+1}s_t)$, as well as by the (aggregate) shock $\widetilde{\phi}_t$. For notational convenience, we let s_t represent the second argument of the risk premium function and thus write $\Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)$.[25] $\overline{a}_t = \frac{A_t}{z_t^+ P_t} = \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right) z_t^+ P_t}$ denotes the real, stationarized (*per capita*) value of the net foreign asset position. $\Phi(\cdot)$ is assumed to be a negative function of $\overline{a}_t$, with the following interpretation. If $\overline{a}_t < 0$, so that Sweden is a net borrower on the international financial market in period t, then $\Phi(\cdot)$ is more likely to take on a positive value. In this case, $\Phi(\cdot)$ represents a premium that Swedish households will be charged over and above the international risk free, gross interest rate $R_{F,t}$. If $\overline{a}_t > 0$, then Sweden is a net lender and the claims on Foreign bonds owned by Swedish households are more likely to pay a return that is lower than the international rate. See Benigno (2009) for an early application of a similar functional form with this interpretation. The second argument of $\Phi(\cdot)$, $E_t(s_{t+1}s_t) = E_t\left(\frac{S_{t+1}}{S_t}\frac{S_t}{S_{t-1}}\right)$, is due to Adolfson et al. (2008). A positive value of $E_t(s_{t+1}s_t)$ implies a lower value of $\Phi(\cdot)$, *ceteris paribus*. The motivation for including this second argument is purely empirical, as it allows the model to reproduce the observed negative correlation between the risk premium and the expected exchange rate depreciation. Adolfson et al. (2008) offers a possible justification for this specification, namely that domestic investors are more likely to accept a lower expected return on their international bond portfolio if the exchange rate is easier to predict. $\widetilde{\phi}_t$ represents an exogenous shock that will absorb any residual movements in the external risk premium.

The purchase by Swedish households of Foreign bonds create a debit recording in Sweden's balance of payments, the total value of which is $\frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)}$. Part or all of this purchase may be financed by the settlement of international bonds that were acquired in the previous period $S_t B_t^{FH}$. The total or net debit recording arising from financial transactions is therefore:

$$\frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)} - S_t B_t^{FH}.$$

We now turn to the payments that arise from international trade in goods. The value of Sweden's net exports in period t is given by $S_t P_t^X X_t - S_t P_{F,t} M_t^{xe} - S_t P_{F,t}^{C,e} M_t^e$, and represents a credit recording in the current account of Sweden.[26] In equilibrium, the total value of the credit recording from Swedish net exports must be balanced by a debt recording of equal value, arising from the net value of all transactions in the international bond market:

$$S_t P_t^X X_t - S_t P_{F,t} M_t^{xe} - S_t P_{F,t}^{C,e} M_t^e = \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)} - S_t B_t^{FH}. \tag{107}$$

Using the definition of net foreign assets, A_t, to substitute for $S_t B_{t+1}^{FH}$ and $S_{t-1} B_t^{FH}$, this relationship may alternatively be written as:

$$A_t - A_{t-1} = S_t P_t^X X_t - S_t P_{F,t} M_t^{xe} - S_t P_{F,t}^{C,e} M_t^e + \left[\Phi\left(\overline{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right)\zeta_{t-1} R_{F,t-1}\frac{S_t}{S_{t-1}} - 1\right] A_{t-1}. \tag{108}$$

The right-hand-side terms in Equation (108) represent the value of Swedish net exports plus the net return between periods t and $(t-1)$, in Swedish currency, on the net foreign asset position of Sweden. The sum of receipts from net exports and net returns on the international investment position is equal to the change in the international investment position, i.e. the change in Sweden net foreign assets.

2.8 Shock processes

In this section, the shock processes are defined. They are all written in log-linear form, where the variable $\hat{x}_t$ is the log-deviation of the variable x_t from it's steady-state value. There are three types of shocks, global shocks that have an equal impact on both Sweden and Foreign, domestic shocks that affects only Sweden, and Foreign shocks that affect Foreign directly, but leads to spill-over effects on the Swedish economy. Furthermore, following Corbo and Strid (2020), the private bond risk premium shock, ζ_t, the utility of consumption shock ζ_t^c and the investment efficiency shock Υ_t are assumed to be correlated with the equivalent Foreign shock process, so that innovations to the Foreign shock process affects also the Swedish shock process. Finally the Foreign utility of consumption $\zeta_{F,t}^c$ and the Foreign marginal investment efficiency $\Upsilon_{F,t}$ are also allowed to be correlated. The innovations are denoted $\epsilon_{x,t}$ where x is the process in question. In addition, some processes are assumed to also have MA-term..

[25] For an explicit statement of the functional form, see Section (3.1).

[26] Recall from Section (2.4.2) in the main text that the prices of all traded goods are assumed to be set in the currency of the importing country, so called local currency pricing.

2.8.1 Global exogenous shocks

There are three global shock processes, the labor augmenting technology shock, the investment-specific non-stationary technology shock and the shock to the neutral interest rate:

$$\hat{\mu}_{z,t} = \rho_{\mu_z}\, \hat{\mu}_{z,t-1} + \epsilon_{\mu_z,t} \tag{109}$$

$$\hat{\mu}_{\gamma,t} = \rho_{\mu_\gamma}\, \hat{\mu}_{\gamma,t-1} + \epsilon_{\mu_\gamma,t} \tag{110}$$

$$\hat{z}_t^R = \rho_{z^R}\, \hat{z}_{t-1}^R + \epsilon_{z^R,t} + \theta_{z^R}\epsilon_{z^R,t-1} \tag{111}$$

2.8.2 Swedish exogenous shocks

Except for the monetary policy shock and the fiscal shocks, which are defined in previous sections, the Swedish economy shocks are

$$\hat{\beta}_t^r = \rho_\beta \hat{\beta}_{t-1}^r + \epsilon_t^\beta \tag{112}$$

$$\hat{\zeta}_t = corr_\zeta \hat{\zeta}_{F,t} + \rho_\zeta\, \hat{\zeta}_{t-1} + \epsilon_t^\zeta \tag{113}$$

$$\hat{\zeta}_t^c = corr_{\zeta^c} \hat{\zeta}_{F,t}^c + \rho_{\zeta^c}\, \hat{\zeta}_{t-1}^c + \epsilon_t^{\zeta^c} \tag{114}$$

$$\hat{\tilde{\phi}}_t = \rho_{\tilde{\phi}}\, \hat{\tilde{\phi}}_{t-1} + \epsilon_t^{\tilde{\phi}} \tag{115}$$

$$\hat{\zeta}_t^n = \rho_{\zeta^n} \hat{\zeta}_{t-1}^n + \epsilon_t^{\zeta^n} \tag{116}$$

$$\hat{\lambda}_t^W = \rho_{\lambda^W} \hat{\lambda}_{t-1}^W + \epsilon_t^{\lambda^W} \tag{117}$$

$$\hat{\varepsilon}_t = \rho_\varepsilon \hat{\varepsilon}_{t-1} + \epsilon_t \tag{118}$$

$$\hat{\Upsilon}_t = corr_\Upsilon \hat{\Upsilon}_{F,t} + \rho_\Upsilon \hat{\Upsilon}_{t-1} + \epsilon_t^\Upsilon \tag{119}$$

$$\hat{\lambda}_t = \rho_\lambda \hat{\lambda}_{t-1} + \epsilon_t^\lambda \tag{120}$$

$$\hat{\lambda}_t^X = \rho_{\lambda^X} \hat{\lambda}_{t-1}^X + \epsilon_t^{\lambda^X} \tag{121}$$

$$\hat{\lambda}_t^{M,C,xe} = \rho_{\lambda^{M,C,xe}} \hat{\lambda}_{t-1}^{M,C,xe} + \epsilon_t^{\lambda^{M,C,xe}} \tag{122}$$

$$\hat{\lambda}_t^{M,I} = \rho_{\lambda^{M,I}} \hat{\lambda}_{t-1}^{M,I} + \epsilon_t^{\lambda^{M,I}} \tag{123}$$

$$\hat{\lambda}_t^{M,X} = \rho_{\lambda^{M,X}} \hat{\lambda}_{t-1}^{M,X} + \epsilon_t^{\lambda^{M,X}} \tag{124}$$

$$\hat{\lambda}_t^{M,C,e} = \rho_{\lambda^{M,C,xe}} \hat{\lambda}_{t-1}^{M,C,e} + \epsilon_t^{\lambda^{M,C,e}} \tag{125}$$

$$\hat{p}_t^{D,C,e} = \rho_{p^{D,C,e}} \hat{p}_{t-1}^{D,C,e} + \epsilon_t^{p^{D,C,e}} \tag{126}$$

$$\hat{\Pi}_t^{trend} = \rho_{\Pi^{trend}} \hat{\Pi}_{t-1}^{trend} + \epsilon_t^{\Pi^{trend}} \tag{127}$$

2.8.3 Foreign exogenous shocks

The Foreign shocks, except for the monetary policy shock which is defined in a previous section, are

$$\hat{\beta}_{F,t}^r = \rho_{\beta_F} \hat{\beta}_{F,t-1}^r + \epsilon_{F,t}^\beta \tag{128}$$

$$\hat{\zeta}_{F,t} = \rho_{\zeta_F}\, \hat{\zeta}_{F,t-1} + \epsilon_{F,t}^\zeta \tag{129}$$

$$\hat{\zeta}_{F,t}^c = corr_{\zeta_F^c,\Upsilon} \hat{\Upsilon}_{F,t} + \rho_{\zeta_F}\, \hat{\zeta}_{F,t-1}^c + \epsilon_{F,t}^{\zeta^c} \tag{130}$$

$$\hat{\zeta}_{F,t}^n = \rho_{\zeta_F^n}\, \hat{\zeta}_{F,t-1}^n + \epsilon_{F,t}^{\zeta^n} \tag{131}$$

$$\hat{\varepsilon}_{F,t} = \rho_{\varepsilon_F}\, \hat{\varepsilon}_{F,t-1} + \epsilon_{F,t} \tag{132}$$

$$\hat{\Upsilon}_{F,t} = \rho_{\Upsilon_F}\, \hat{\Upsilon}_{F,t-1} + \epsilon_{F,t}^\Upsilon \tag{133}$$

$$\hat{\lambda}_{F,t} = \rho_{\lambda_F} \hat{\lambda}_{F,t-1} + \epsilon_{F,t}^\lambda \tag{134}$$

$$\hat{p}_{F,t}^{C,e} = \rho_{p_F^{D,C,e}} \hat{p}_{F,t-1}^{D,C,e} + \epsilon_{F,t}^{p^{D,C,e}} \tag{135}$$

$$\hat{\Pi}_{F,t}^{trend} = \rho_{\Pi_F^{trend}} \hat{\Pi}_{F,t-1}^{trend} + \epsilon_t^{\Pi_F^{trend}} \tag{136}$$

$$\hat{g}_{F,t} = \rho_{g_F} \hat{g}_{F,t-1} + \epsilon_t^{g_F} \tag{137}$$

3 Calibration

In this section, we report the chosen model parameter values. SELMA is calibrated to match the Swedish economy at a quarterly frequency. Most of the non-fiscal policy parameter values in SELMA are based on the MAJA model presented in Corbo and Strid (2020). The MAJA model is estimated using Swedish data over the period 1995Q2-2018Q4. For certain macro shock parameters there are no corresponding values in MAJA. In those cases, we use the parameter values from the Ramses I model presented in Adolfson et al. (2005), which are estimated using Swedish data over the period 1980Q1-2002Q4. Otherwise we calibrate the parameters to give reasonable dynamics for the Swedish economy. The parameters in Foreign are also taken from MAJA except for the discount factor shock parameter which is taken from Ramses I. Some of the fiscal parameter values in SELMA are based on the model in Coenen, Straub, and Trabandt (2013), which is estimated on euro area data over the period 1985Q1-2010Q2. In the current setup of SELMA, the model user determines which fiscal rule to activate and chooses values for the fiscal policy rule parameters at one's own discretion.

First, the functional forms of the household preferences, the external risk premium and the investment adjustment cost function are presented. Then the values of the great ratios and the steady state values of the model are discussed, and finally, the calibration of the parameters are discussed.

3.1 Functional forms

The utility function is chosen so as to be consistent with a balanced growth path, and is given by

$$u(\tilde{C}_{h,t} - \rho_h \tilde{C}_{t-1}) = \ln(\tilde{C}_{h,t} - \rho_h \tilde{C}_{t-1}). \tag{138}$$

The disutility of labor function for households in Sweden including endogenous shifter and weighting parameter is given by

$$\nu(n_t) = \Theta_t^n A_n \frac{N_t^{1+\eta}}{1+\eta}. \tag{139}$$

The disutility of labor function in foreign economy is standard in the DSGE literature, and the same as in for example Adolfson et al. (2005). It is given by

$$\nu(n_{f,t}) = \frac{N_{f,t}^{1+\eta}}{1+\eta}. \tag{140}$$

The external risk premium function is as in Adolfson et al. (2008) and Corbo and Strid (2020), and is given by

$$\Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right) = e^{-\widetilde{\phi}_a(\overline{a}_t - \overline{a}) - \widetilde{\phi}_s E_t\left[s_{t+1}s_{t-1}\right] + \widetilde{\phi}_t}.$$

The investment adjustment cost function $F(I_t, I_{t-1})$ is taken from Christiano, Eichenbaum, and Evans (2005) and is given by

$$F(I_t, I_{t-1}) = \left[1 - \widetilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.$$

where the functional form of $\widetilde{S}$ is defined as in Adolfson et al. (2005):

$$\widetilde{S}\left(\frac{I_t}{I_{t-1}}\right) = \frac{1}{2}\left[e^{\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - \mu_z + \mu_\gamma\right)} + e^{-\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - \mu_z + \mu_\gamma\right)} - 2\right].$$

3.2 Great ratios and balanced growth path

The steady-state great ratios in SELMA are based on data from Statistic Sweden (SCB), where averages have been taken over the period 2008-2017. Table 1 shows the great ratios. The private investment-to-output ratio $p^I \overline{I}/\overline{y}$ is set to 0.191, and is calculated as the ratio of nominal private investment (including inventories) to nominal market price GDP. For the calculation of the export-to-output ratio $p^X \overline{x}/\overline{y}$ and the import-to-output ratio $p^M \overline{m}/\overline{y}$, we take the average between the nominal exports-to-nominal market price GDP and nominal imports-to-nominal market price GDP to obtain the export and import-to-output ratios. Note that we equalize the export and import-to-output ratio in order to match the assumption of balanced trade in the steady state. This gives the value of 0.433 for the export and import-to-output ratios.

The government consumption-to-output ratio $\overline{g}/\overline{y}$ is calculated as the ratio of nominal government consumption-to-nominal market price GDP, which is 0.258. The government investment to output ratio $\overline{I}^G/\overline{y}$ is calibrated to match the ratio of nominal government investment to nominal market price GDP of 0.044. Private consumption to output ratio $p^C\bar{c}/\bar{y}$, which equals 0.507, is a residual which can be obtained by using the following expression : $p^C\bar{c}/\bar{y} = 1$ - $p^X\bar{x}/\bar{y} + p^M\bar{m}/\bar{y}$-$p^I\bar{I}/\bar{y}$ - $\bar{g}/\bar{y}$ -$\bar{I}^G/\bar{y}$. The aggregate transfers to output ratio $\overline{tr}^{agg}/\bar{y}$ is calibrated to match the ratio of nominal public transfers-to-nominal market price GDP of 0.149.

Table 1 also shows the calibrated values of the parameters pertaining to the Swedish fiscal policy framework. Based on the Swedish fiscal policy framework, the government debt anchor is set at 35 per cent of GDP, and the surplus target is 1/3 of a percent of GDP. Thus, the debt to output ratio target $\bar{b}^{Target}/\bar{y}$ is set to 0.35*4, and the surplus to output ratio target $\overline{surp}^{Target}/\bar{y}$ is set to 1/3*0.01. In the steady state of the model the surplus target is incompatible with a debt target of 35 per cent of GDP, since a surplus every period results in a negative debt in the long run. See *2.5.* for the relationship between the two targets. Therefore the model user has to choose which target should be active in the steady state.

As Table 3 shows, the growth rate of labor augmenting technology μ_z is set to 1.005 which implies that the growth rate of labor augmenting technology is 2 percent per year. Thus, the Swedish economy grows at 2 percent per year along the balanced growth path, which is in line with the Ramses I and II calibration. Note that the composite technological growth rate, μ_{z^+} can be defined as a function of the growth rates of labor augmenting technology, μ_z and investment-specific technology μ_γ: $\mu_{z^+} = \mu_z(\mu_\gamma)^{\frac{\alpha}{1-\alpha}}$. The labor augmenting technological process follows a unit root process, whereas we assume the investment-specific technological process to be stationary, so μ_γ is 1 in the steady state. Finally, the steady state inflation rate is set to 2 percent per year in both Sweden and Foreign.

3.3 Labor market aggregates

Table 2 shows steady state labor market aggregates. The rates of labor force participation and employment are based on their potential values in the NIER database over the period 2008-2017. The rate for labor force participation l is calculated as period average of potential labor force over population aged 15-74, which is 0.716. Similarly, employment rate n is calculated as average of potential employment over population aged 15-74 for the same period, which is 0.668. Unemployment rate is defined by $(l - n)/l$ and we define the steady-state unemployment rate as the average potential unemployment rate over the sample period, which is 0.067.

3.4 Household sector parameters

Table 4 shows the values of the calibrated household parameters. In this table, the values of β and β_F are based on the MAJA model calibration, whereas ρ_h and $\rho_{h,F}$ are based on the MAJA estimation. According to the MAJA estimation, the value of the consumption habit parameter in Sweden ρ_h is 0.74 and in foreign it is 0.64. Household discount factors β and β_F are set to 0.999 which implies that the nominal net interest rate (or the policy rate) is 4.49 percent annually. The inverse Frisch elasticity parameter in the Swedish economy η is estimated to 3.65 and foreign counterpart $\eta_F$is estimated to 6 in MAJA, and we use these values for the calibration. The parameter associated with labor disutility A_n is a function of marginal utility of consumption, real wage, labor supply and wage income tax in steady state, and this can be found in Section I.2.

We follow the Coenen, Straub, and Trabandt (2013) calibration strategy by choosing the value of the share of private consumption in the composite consumption α_G such that the marginal utility of private consumption equals the marginal utility of government consumption. Thus, α_G is calibrated to 0.63. As discussed in Galí, López-Salido, and Vallés (2007) among others, several empirical studies find that private consumption rises after a positive shock to government consumption. Thus, given the size of α_G, we set the elasticity of substitution between private and government consumption υ_G to 0.29 which is taken from the Coenen, Straub, and Trabandt (2013) estimate of υ_G, and this calibrated value produces a positive co-movement between government consumption and private consumption after a positive shock to government consumption spending.

Based on the Coenen, Straub, and Trabandt (2013) calibration strategy, we set the share of aggregate transfers going to Non-Ricardians in steady state, ϖ_{ss} to 0.31 which is the number that ensures a ratio of Non-Ricardian household consumption to Ricardian household consumption in steady state to be 0.8. The share of aggregate transfers going to Non-Ricardians off steady state ϖ_{dyn} is set at 0.75 as we assume that most of the aggregate transfers goes toward Non-Ricardian households as credit constrained households are likely to receive a higher proportion of transfers. ϖ_{dyn} is a policy parameter which can be adjusted by the model user. Finally, the share of Non-Ricardians over total population snr is set at 0.35 in line with the Swedish macro-estimate in Campbell and Mankiw (1991).

3.5 Sticky prices, wage parameters and markups

Table 5 presents the Calvo parameters which measure the fraction of households/firms that keep their wages/prices unchanged each period. Table 6 shows parameters of indexation to previous inflation which capture an inertia in nominal wage/price inflation. Table 7 shows the mark-ups of producers and wage-setters. We set the values of the Calvo probability parameters to match the MAJA estimation.

The values of indexation to previous inflation parameters are obtained from the MAJA. However, most values are calibrated to zero since they are weakly identified and tend to be estimated close to zero. The value of indexation to previous inflation for intermediate goods firms χ is set to match the value of the MAJA estimate of 0.33. The parameter for price indexation of foreign firms in MAJA is estimated to 0.55 and therefore χ_F is set to that value. The value of wage indexation parameter in Sweden χ_W and in the foreign economy $\chi_{F,w}$ are also set to 0 as in MAJA.

We set the steady state markups (intermediate good firms, importing firms) to 1.2, a standard value in the DSGE literature, which is also the same as most of the steady state markups that are reported in the calibration section of MAJA. However, following MAJA, Swedish export goods producing firms' markup is calibrated to a lower value to 1.05, in order to avoid double markup on these goods. For wage markup, recall that η is calibrated to the Riksbank's estimated value, 3.65. Also note that at state state wage markup can be defined as $\lambda^W = (\frac{l}{n})^\eta$. Since all the variables at the right hand side are already calibrated, then wage markup is also calibrated to 1.3, approximately. For foreign economy, we follow MAJA and set the foreign wage markup λ_F^W to 1.6.

3.6 Elasticities of substitution between imported and domestic good parameters, home bias parameters and export demand parameters

Table 8 shows the values of elasticities of substitution between imported intermediate goods and domestically produced intermediate goods for different sectors. The Swedish elasticity of substitution between imported goods and domestically produced intermediate goods for non-energy consumption production $\nu_{C,xe}$ is set to 0.87 which is in line with the MAJA estimation. The elasticities of substitution between imported and domestically produced intermediate goods for investment and export goods productionν_I and ν_x are set to 0.27 and 1.53, respectively, also this in line with the MAJA estimation. The elasticity of substitution between domestic and foreign energy, $\nu_{C,e}$, is set to 0.5 as in Corbo and Strid (2020).

Finally, we need to find the elasticity of substitution between imported and domestic consumption goods in Foreign, $\nu_{F,c}$ which captures the sensitivity of Swedish exports to Foreign import prices. We set $\nu_{F,c}$ to be 0.37 which is based on the MAJA estimation. [27]

The home bias parameters $\vartheta^{C,xe}$, $\vartheta^{C,e}$, ϑ^I and ϑ^X are functions of the elasticities of substitution between imported and domestic goods and the ratio of imports to final goods.[28] The home bias parameters can be found in Section I.2. Lastly, ϑ_F^C captures the Foreign home bias for consumption goods. We set ϑ_F^C to 0.5, so that Equation (B.114) for Swedish exports holds in the steady state: $\overline{x} = \left(1 - \vartheta_F^C\right)\left(p_F^M\right)^{-\nu_{F,c}} \lambda_F^M \overline{y}_F$, and $\vartheta_F^C < 1$ implies that Swedish exports is affected by Foreign output (Foreign consumption).

The parameters that govern the substitution between energy and non-energy in aggregate private consumption are all taken from Corbo and Strid (2020). The elasticities of substitutions in between energy and non-energy consumption in the creation of consumption goods, ν_c and $\nu_{F,c}$, are set to 0.5 in both Sweden and Foreign. The parameter which governs the share of non-energy in total consumption in Sweden and Foreign, ϑ^C and $\vartheta^{C,F}$, are set to match the shares of energy consumption to total private consumption in data reported in Corbo and Strid (2020). These shares are 0.075 in Sweden and 0.09 in Foreign.

The values for the export demand function are taken from Corbo and Strid (2020) and are set to $\omega_C^X = 0.27$ and $\nu_F = 0.37$.

3.7 Capital and investment parameters

Table 9 shows parameters that are associated with capital and investment. We set the capital share in production α to be 0.35, in line with National Institute of Economic Research's (NIER) calculation where the average is taken over the period 2008-2017. The depreciation rate, δ, is calibrated to be 0.0077. This value is solved numerically such that the steady state conditions of the composite capital function and the private capital accumulation function hold.[29] In SELMA, the government provides public capital to intermediate good producers. Hence, the elasticity of substitution between private and public capital υ_K determines the degree of complementary between private and government investment. The parameter α_K represents the weight of private capital in the composite capital function. We follow the Coenen, Straub, and Trabandt (2013) calibration strategy by choosing the value of the share of private capital in the composite capital function α_K such that the marginal product of private capital equals the marginal product of public capital, which implies that α_K is calibrated to 0.83.

Afonso and Alegre (2011) find that an increase in public investment has a crowding-in effect on private investment based on a set of 15 EU countries which includes Sweden. Hence, given the size of α_K, we set υ_K

[27] Since the Foreign economy behaves as a closed economy we do not need to specify value for the elasticity of substitution between imported and domestic inputs in Foreign , $\nu_{F,x}$.

[28] The share of imported energy in the consumption of energy is set to 0.5, following Corbo and Strid (2020).

[29] This method is required due to the inclusion of public capital to the composite capital which is used for the production of intermediate goods.

to 0.25 which is taken from the Coenen, Straub, and Trabandt (2012) calibration of υ_K that produces a positive co-movement between government investment and private investment after a positive shock to government investment under no active fiscal policy rule.[30]

The foreign capital share α_F in production is the same as in Corbo and Strid (2020). The depreciation rate δ_F is calibrated to match the average investment-output ratio as reported in Corbo and Strid (2020).

The investment adjustment cost S'' is set to 8.39 in Sweden and to 3.99 in the foreign economy, which is in line with the MAJA estimation. We set the public capital depreciation rate δ_G to 0.018, which implies a public capital depreciation rate of 7.2 percent per year, this value helps us to have an immediate increase in private investment after a positive government investment shock.[31] In the current calibration of SELMA, the variable capital utilization is deactivated by setting σ_a to a very high number, i.e. 100000.[32]

3.8 Modified UIP condition parameters

Table 10 shows the values of two external risk premium parameters, which are based on those reported by the model estimation of Corbo and Strid (2020). The optimal choice of Swedish bond and Foreign bond holdings by Ricardian households in Sweden pins down the modified uncovered interest rate parity (UIP) condition. In particular, we have two external risk premium parameters in the modified UIP condition. First, $\widetilde{\phi}_a$, which captures the external risk premium parameter associated with net foreign asset position, is set close to 0. It does however need to be set large enough to assure that the economy moves back the its original steady state after a shock, why it is set to 10^{-4}. Second, $\widetilde{\phi}_s$, which represents the external risk premium parameter associated with exchange rate fluctuations, is set to 0.16.

3.9 Monetary policy rule parameters

Table 11 reports the values of the Riksbank's Taylor rule parameters, which are based on the MAJA estimation. In Foreign, the Taylor rule parameter values are the same as in MAJA except for the output response coefficient and the difference in output coefficient which are scaled down values of the coefficients for unemployment rate response and unemployment rate change response in MAJA.

3.10 Tax rates and fiscal rules

Table 12 reports the steady state tax rates. The tax rates are in general calculated by dividing the tax income by the tax base. All data series used are nominal, and are based on yearly data, where averages are taken between 2008-2017. For the labor income tax rate, τ^W, the tax revenue is calculated as the municipal and regional income tax plus the state income tax minus the earned income tax credit.[33] The municipal and regional income tax is however paid on both transfers and wages in Sweden. To estimate how much of the municipal and regional income tax is paid on wages, the tax income has been multiplied by the wage sum and divided by the sum of wages and transfers. The tax base for τ^W is given by the wage sum. For the transfer tax, τ^{TR}, the tax revenue is calculated as the municipal and regional income tax time the transfers divided by the sum of transfers and wages (to get the share of tax revenue paid on transfers). The tax base for τ^{TR} is given by total public transfers. For the capital income tax rate τ^K, we use paid corporate tax as the tax revenue, and use the private sector net surplus excluding the net surplus of small houses as the tax base. For the social security contributions, the tax revenue is given by the paid employee labor taxes plus the pension income contributions paid for employees. The tax base for τ^{SSC} is the wage sum. For the consumption tax, τ^C, the tax value for 2015 from the model IOR is used.[34] The value taken from IOR is the consumption taxes as a share of total consumption that are paid by households. This value is then reformulated into a value added tax. The investment tax credit τ^I is set to zero. To close the government budget constraint, a steady-state value of the lump-sum tax needs to be introduced as well. This is set as a residual so that the government debt is at its target level in steady state. Given our calibration, this implies a lump-sum tax of -1.2 percent of steady-state GDP. The parameters in the fiscal rules are up to the model user to set, except for the coeffient on unemployment in the tranfer rule, which is calibrated to $F_{tr,un} = 0.106$. The coefficient is based on the average value of the elasticity of primary expenditures on unemployment over the period 2009-2018 in Almenberg and Sigonius (2021), who calculate the Swedish automatic stabilizers on both public revenues and expenditures for the years 1998 to 2019.[35]

[30]Note that if we set υ_K to match the Coenen, Straub, and Trabandt (2013) estimate of υ_K (0.84), this will not give us a positive co-movement between public and private investment after a positive shock to government investment under no active fiscal policy rule.

[31]Note that our parameter value is close to Yang, Walker, and Leeper (2010) calibrated value for δ_G which is 0.02.

[32]The capital utilization is de-activated in simulations because to the best of our knowledge, there are no other DSGE models that include capital utilization, public capital and the complementarity between government and private investment in the same model.

[33]The earned income tax credit is calculated as total income tax credit minus the credit for public pensions.

[34]IOR, which is described in Forsfält and Glans (2015), is an input-output model used for forecasts on imports and dis-aggregate production, as well as for analysis of the structure of the Swedish economy.

[35]We use the semi-elasticity for public expenditures, which includes not only unemployment benefits but also costs for active labor market programs (so-called semi automatic stabilizers). These are calculated for the years 2009 to 2018.

3.11 Macro shock parameters

Table 13 reports the values of macroeconomic shock parameters. The values of the Swedish macroeconomic shock persistences and the correlations are based primarily on the MAJA estimation.[36] For the shocks that are not estimated in MAJA, the parameters are based on the RAMSES I estimation. The persistence value for the non-stationary investment-specific technology shock is based on Ramses I estimates since the shock is not active in MAJA. The discount factor shock in MAJA is not active either. Thus, the persistence for the discount factor shock is set to be the same value as the Ramses I estimate of the consumption preference shock persistence.[37] We assume symmetry between Sweden and Foreign, thus the analogous Foreign discount factor parameter has the same value as the Swedish one.

Note that in our model, intermediate good markup shocks follow i.i.d process as in MAJA model, so the shocks do not have a persistent component. Even though there is no persistence component in the intermediate markup shocks, the intermediate good markups in steady state are not zero (see Table 7). The persistence values of Swedish markup shocks to import firms (consumption and investment goods) are taken from the Ramses I estimation. The import firms that specialize in export goods are not modelled in Ramses I model, so we take the average of the two persistence values of Swedish import markup shocks (consumption and investment goods) to obtain the persistence value of Swedish markup shock to import firms that specialize in export goods. The markup shock persistence for the Swedish export firms is also taken from the Ramses I estimation.

The remaining persistence parameters are set to values corresponding the values in Corbo and Strid (2020).

The correlation parameter for the risk premium shocks is also set as reported in Corbo and Strid (2020).[38]

3.12 Average maturity of government bonds

In SELMA, we allow for the government bonds to have a stochastic maturity. The government issues bonds that mature with probability α_B in a given period. Note if α_B is one, then we have a one-period government bond as in the standard DSGE framework. In our current framework, we allow α_B to be less than one, and as a result we have long-term government bonds. We set α_B to match the Swedish National Debt Office data, where the average maturity of Swedish government bonds in 2014 was 4 years. Thus, the probability that government debt will mature each period α_B is set to 0.0625.

4 Impulse response analysis

In this section, we present some impulse response functions from SELMA to show the model's response to selected shocks.[39] Before showing the results from the simulations, we discuss two key equations, the consumption Euler equation and modified UIP condition.[40]

The consumption Euler equation, which is a key equation to determine the Ricardian households' consumption, can be written in log-linearized form as:

$$\hat{\Omega}_t^C = E_t\left[\hat{\zeta}_t + \hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C + \frac{1}{R}\breve{i}_t - \hat{\Pi}_{t+1}^C - \hat{\mu}_{z^+,t+1}\right].$$

A variable with the *hat* notation represents the log-linear approximation of the variable in deviation from its steady-state value (which in turn can be interpreted as percent deviation of the variable from its steady state), and a variable with *breve* notation is interpreted as an absolute deviation of the variable from its steady state. For more details on notations, see Appendix A. $\hat{\Omega}_t^C$ is the marginal utility of consumption, $\hat{\zeta}_t$ is a risk premium shock to private bonds, $\breve{i}_t$ is the monetary policy rate set by the Riksbank, $\hat{\Pi}_{t+1}^C$ is the consumer price inflation (CPIF), and $\hat{\mu}_{z^+,t+1}$ is the compound effects of labor augmenting technological and investment-specific technological processes. $\hat{\beta}_{t+1}^r$ is a time-varying discount factor.

[36]These parameters may however be changed by the model users depending on the investigated scenario.

[37]The reason for choosing the value from RAMSES I instead of MAJA is that in MAJA, the consumption shock persistence is derived by its correlation with foreign consumption, while the persistence on its own lag is zero.

[38]Note that in Corbo and Strid (2020), the correlations between the shock processes are reported rather the parameter values that governs the correlations. Here, the parameter values are reported instead.

[39]The impulse response functions for other selected shocks of the model are plotted in the Appendix J.

[40]For the derivation of the non-linear equations, see Appendix C

Table 1: Calibration: Great ratios

Symbol	Description	Value
$p^I\overline{I}/\overline{y}$	Private investment to output ratio	0.191
$p^C\overline{c}^{agg}/\overline{y}$	Private consumption to output ratio	0.507
$p^X\overline{x}/\overline{y}$	Export to output ratio	0.433
$p^M\overline{m}/\overline{y}$	Import to output ratio	0.433
$\overline{g}/\overline{y}$	Government consumption to output ratio	0.258
$\overline{I}^G/\overline{y}$	Government investment to output ratio	0.044
$\overline{tr}^{agg}/\overline{y}$	Aggregate transfers to output ratio	0.149
$\overline{surp}^{Target}/\overline{y}$	Government surplus to output ratio target	1/3*0.01
$\overline{b}^{Target}/\overline{y}$	Government debt to output ratio target	0.35*4
$p^C_F\overline{c}_F/\overline{y}_F$	Foreign Private Consumption ratio	0.59
$p^I_F\overline{I}_F/\overline{y}_F$	Foreign Private Investment ratio	0.20
$\overline{g}_F/\overline{y}_F$	Foreign government consumption ratio	0.21

Table 2: Calibration: Labor market aggregates

Symbol	Description	Value
l	Labor force participation rate	0.716
n	Employment rate	0.668
un	Unemployment rate	0.067

Table 3: Calibration: Balanced growth path values

Symbol	Description	Value
μ_z	Growth rate of labor augmenting technology	1.005
μ_{z^+}	Composite technological growth rate	1.005
$\mu_{z^+_F}$	Foreign composite technological growth rate	1.005
Π	Gross inflation rate of intermediate goods	1.005
Π^{trend}	Gross inflation trend	1.005
Π^C	Gross inflation rate of consumption goods	1.005
Π^X	Gross inflation rate of export goods	1.005
Π^W	Gross inflation rate of wages	1.010
Π_F	Foreign gross inflation rate of intermediate goods	1.005
Π^C_F	Foreign gross inflation rate of consumption goods	1.005
Π^W_F	Foreign gross inflation rate of wages	1.010
Π^X_F	Foreign gross inflation rate of export goods	1.005

Table 4: Calibration: Household sector parameters

Symbol	Description	Value
β	Household discount factor	0.999
β_F	Foreign household discount factor	0.999
ρ_h	Consumption habit	0.75
$\rho_{h,F}$	Foreign consumption habit	0.64
η	Inverse of Frisch elasticity, labor supply elasticity	3.65
η_F	Foreign inverse of Frisch elasticity, labor supply elasticity	6
α_G	Share of private consumption in the composite consumption	0.63
υ_G	Elasticity of substitution between private and government consumption	0.29
snr	Share of Non-Ricardian over total population	0.35
ϖ_{ss}	Share of aggregate transfers going to Non-Ricardians in steady state	0.31
ϖ_{dyn}	Share of aggregate transfers going to Non-Ricardians off steady state	0.75

Table 5: Calibration: Calvo parameters

Symbol	Description	Value
ξ	Calvo intermediate good prices	0.94
ξ_x	Calvo export prices	0.79
$\xi_{m,C,xe}$	Calvo import prices, import firms specializing in non-energy consumption goods	0.92
$\xi_{m,C,e}$	Calvo import prices, import firms specializing in energy consumption goods	0.10
$\xi_{m,I}$	Calvo import prices, import firms specializing in investment goods	0.79
$\xi_{m,X}$	Calvo import prices, import firms specializing in export goods	0.80
ξ_w	Calvo wages	0.86
ξ^F	Foreign Calvo intermediate good prices	0.92
ξ^F_w	Foreign Calvo wages	0.86

Table 6: Calibration: Inflation indexation

Symbol	Description	Value
χ	Indexation to previous inflation, intermediate goods	0.33
χ_x	Indexation to previous inflation, export goods	0
$\chi_{m,C.xe}$	Indexation to previous inflation, import firms specializing in non-energy consumption goods	0
$\chi_{m,C.e}$	Indexation to previous inflation, import firms specializing in energy consumption goods	0
$\chi_{m,I}$	Indexation to previous inflation, import firms specializing in investment goods	0
$\chi_{m,X}$	Indexation to previous inflation, import firms specializing in export goods	0
χ_w	Indexation to previous wage inflation	0
χ_F	Foreign indexation to previous inflation, intermediate goods	0.55
$\chi_{F,w}$	Foreign indexation to previous wage inflation	0

Table 7: Calibration: Steady state values of markups

Symbol	Description	Value
λ	Intermediate good price markup	1.2
λ^X	Export price markup	1.05
λ^M	Imported good price markup (consumption, investment and export)	1.2
λ^W	Wage markup	1.3
λ_F	Foreign intermediate good price markup	1.2
λ^W_F	Foreign wage markup	1.6

Table 8: Calibration: Elasticities of substitutions in production sector

Symbol	Description	Value
ν_C	Elasticity between non-energy and energy goods in the production of consumption goods	0.5
$\nu_{C,e}$	Elasticity between domestic and imported goods in the production of energy consumption	0.5
$\nu_{C,xe}$	Elasticity between domestic and imported goods in the production of non-energy consumption	0.87
ν_I	Elasticity between domestic and imported goods used for investment goods production	0.27
ν_x	Elasticity between domestic and imported goods used for export goods production	1.53
$\nu_{F,c}$	Elasticity between imported and foreign consumption goods in Foreign	0.37

Table 9: Calibration: Capital and investment parameters

Symbol	Description	Value
α	Capital share in production	0.35
α_K	Share of private capital in composite capital function	0.83
ν_K	Elasticity of substitution between public and private capital	0.25
S''	Investment adjustment cost	8.39
δ	Private capital depreciation rate	0.0077
δ_G	Public capital depreciation rate	0.018
α_F	Foreign capital share in production	0.21
S''_F	Foreign investment adjustment cost	3.99
δ_F	Foreign capital depreciation rate	0.0151

Table 10: Calibration: Modified UIP condition parameters

Symbol	Description	Value
$\widetilde{\phi}_a$	External risk premium associated with net foreign asset position	10^{-4}
$\widetilde{\phi}_s$	External risk premium associated with exchange rate fluctuations	0.16

Table 11: Calibration: Monetary policy rule parameters

Symbol	Description	Value
ρ	Interest rate smoothing coefficient	0.92
r_π	Inflation response coefficient	1.71
r_{un}	Unemployment response coefficient	0.25
$r_{\triangle\pi}$	Difference in inflation response coefficient	0
$r_{\triangle un}$	Difference in unemployment response coefficient	0.17
ρ_F	Foreign interest rate smoothing coefficient	0.93
$r_{F,\pi}$	Foreign inflation response coefficient	1.75
$r_{F,y}$	Foreign output response coefficient	0.068
$r_{F,\triangle\pi}$	Foreign difference in inflation response coefficient	0
$r_{F,\triangle y}$	Foreign difference in output response coefficient	0.137

Table 12: Calibration: Steady state level of tax rates and fiscal rules

Symbol	Description	Value
τ^C	Consumption tax rate	0.171
τ^W	Labor income tax rate	0.247
τ^{SSC}	Social security contribution rate	0.299
τ^K	Capital income tax rate	0.15
τ^{TR}	Tax rate on transfers to households	0.279
τ^I	Investment tax credit	0
τ^W_F	Foreign labor income tax rate	0.247
$F_{tr,un}$	Automatic stabilizer on transfer rule	0.106

Table 13: Calibration: Macro shock parameters

Symbol	Description	Value
ρ_{μ^z}	Persistence, labor augmenting technology shock	0.55
ρ_{γ}	Persistence, investment-specific technology shock	0.75
ρ_{ε}	Persistence, productivity shock	0.87
ρ_{β}	Persistence, discount factor shock	0.935
ρ_{ζ^n}	Persistence, labor disutility shock	0.83
ρ_{ζ^c}	Persistence, consumption preference shock	0
ρ_{ζ}	Persistence, private bond risk premium shock	0.69
$\rho_{\widetilde{\phi}}$	Persistence, external risk premium shock (exchange rate shock)	0.84
ρ_{Υ}	Persistence, stationary investment-specific shock	0
$\rho_{P^{C,e}}$	Persistence, energy price shock	0.91
$\rho_{\lambda^{M,C}}$	Persistence, markup shock to import firms specializing in non-energy consumption goods	0.978
$\rho_{\lambda^{M,I}}$	Persistence, markup shock to import firms specializing in investment goods	0.974
$\rho_{\lambda^{M,X}}$	Persistence, markup shock to import firms specializing in export goods	0.976
ρ_{λ^X}	Persistence, markup shock to exporting good firms	0.894
$\rho_{\Pi^{C,trend}}$	Persistence, inflation trend	0.5
ρ_{ε_F}	Persistence, Foreign productivity shock	0.81
ρ_{β_F}	Persistence, Foreign discount factor shock	0.935
$\rho_{\zeta_F^n}$	Persistence, Foreign labor disutility shock	0.99
$\rho_{\zeta_F^c}$	Persistence, Foreign consumption shock	0.73
ρ_{ζ_F}	Persistence, Foreign private bond risk premium shock	0.91
ρ_{Υ_F}	Persistence, Foreign stationary investment-specific shock	0.66
$\rho_{P_F^{C,e}}$	Persistence, Foreign energy price shock	0.91
$\rho_{\Pi_F^{C,tren}}$	Persistence, Foreign inflation trend	0.95
ρ_{g_F}	Persistence, Foreign government consumption	0.98
$corr_{\Upsilon}$	Parameter governing correlation between Swedish and Foreign investment shock	0.446
$corr_{\zeta}$	Parameter governing correlation between Swedish and Foreign risk premium shock	0.268
$corr_{\zeta^c,\zeta_F^c}$	Parameter governing correlation between Swedish and Foreign consumption preference shock	0.936
$corr_{\zeta_F^c,\Upsilon_F}$	Parameter governing correlation between Foreign consumption preference and investment shock	0.119

Remember that since $\hat{\Omega}_t^C$ is concave, an increase in the marginal utility of consumption means that consumption falls. On the balanced growth path, the marginal utility of consumption will decrease at a constant pace, so that consumption grows at the same rate as the rest of the economy. If, however, the economy is hit by shocks, the underlying economic conditions for the households will change, and the households might change how they choose to consume. Note, however, that households prefer to smooth their consumption. The Ricardian households, who are able to do so, would like the effects of temporary disturbances to affect their current consumption as little as possible, smoothing the burden of the shock over their whole lifespan.[41]

An increase in the monetary policy rate $\breve{i}_t$, will, *ceteris paribus*, increase the Ricardian households' incentives to save, since one extra unit of savings gives a higher level of consumption tomorrow than without a rate increase. As a result, $\hat{\Omega}_t^C$ will increase, meaning that current consumption will decrease. The effective interest rate on private bonds for the household is however given by $\frac{1}{R}\breve{i}_t + \hat{\zeta}_t$. Hence, a shock to the risk premium has a similar direct effect on the household as a change in the nominal interest rate. Now paying attention to the inflation rate, an increase in the inflation rate will, *ceteris paribus*, reduce the real interest rate that the household faces. It will therefore have a similar effect on consumption as a decrease in the nominal interest rate. Turning to the productivity parameter $\hat{\mu}_{z^+,t+1}$, an increase in productivity leads to an immediate consumption increase since the households want to smooth consumption.

Next we discuss the modified UIP condition that determines the exchange rate in the model, which in a log-linearized form is expressed as:

$$\frac{1}{R}\left(\breve{i}_t - \breve{i}_{F,t}\right) = \left(1 - \widetilde{\phi}_s\right) E_t\left[\hat{s}_{t+1}\right] - \widetilde{\phi}_s \hat{s}_t - \widetilde{\phi}_a \breve{a}_t + \hat{\widetilde{\phi}}_t.$$

The Swedish nominal interest rate, $\breve{i}_t$ represents the expected return on Swedish private bonds. Likewise, $\breve{i}_{F,t}$ is the Foreign nominal interest rate. $\hat{s}_t$ is the nominal exchange rate (a higher value means a depreciation of the Swedish krona relative to the Foreign currency). $\breve{a}_t$ is the real value of net foreign asset position of Sweden, which captures Foreign private bonds that are owned by Swedish Ricardian households, and $\hat{\widetilde{\phi}}_t$ is an exchange rate shock, also referred to as the external risk premium shock.

If the Swedish monetary policy rate, $\breve{i}_t$, is higher than the Foreign policy rate, we would expect a depreciation of the exchange rate between today and tomorrow, so that the returns on the two assets equalize. For this to occur, the exchange rate must appreciate today. This is captured by the first term in the right-hand side of the equation.

The next three terms on the right-hand-side of the equation captures an international bond market friction, called the external risk premium. The friction makes the return on Foreign assets held by Swedish Ricardian households decrease with the size of their Foreign bond portfolio. This means that it is more expensive for Swedish Ricardian households to borrow from Foreign households if they already have negative net Foreign assets. Similarly, the return on their Foreign bond portfolio is lower if they have positive net Foreign assets. The friction does also include the shock $\hat{\widetilde{\phi}}_t$.

Now, we continue by describing the model responses to different shocks in the economy. Note that in the IRF-diagrams, the monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized quarter-on-quarter values.

4.1 Fiscal policy rule parameters

In the model simulations described below, we show the outcomes under three different fiscal policy assumptions. In all cases the assume that the automatic stabilizers are in effect. The case *No active fiscal rule* is defined by using the transfer policy rule with $\mathcal{F}_{tr,surp} = 0$, and the response of transfers to changes in the government debt level is set to a very low number, $\mathcal{F}_{tr,b} = -0.0035$. The last part is to ensure the stability of the government debt-to-GDP in the long run.

In the second case, named as *Transfer (tr_t^{agg}) rule - Debt target*, the fiscal rule is defined on aggregate transfers with the following parameterization. The structural surplus gap coefficient $\mathcal{F}_{tr,surp}$, is set to zero while the coefficient on the debt target to is set to $\mathcal{F}_{tr,b} = -0.14$. This parameter value will allow debt to go back to the target level with a reasonable pace.

In the third and the final case, named as *Transfer (tr_t^{agg}) rule - Struct Surp target*, the fiscal rule is also defined on aggregate transfers. In this case aggregate transfers are determined by the feedback rule that respects the structural surplus target. Hence for this case $\mathcal{F}_{tr,surp}$ is set to 5 so that the structural surplus goes back to its target level with a reasonable pace. Finally, $\mathcal{F}_{tr,b}$ is set to zero.

In all fiscal policy rule specifications the AR(1) component parameter ($\rho_{tr^{agg}}$) is set to 0. Since the fiscal rule parameters are choice parameters, the model user may choose other values for the fiscal policy rule parameters compared to our current example. The economic outcome under *No active fiscal rule* is illustrated by the blue solid line in the figures below, whereas the outcomes given *Transfer (tr_t^{agg}) rule - Debt target* and *Transfer*

[41] We assume that Non-Ricardian households do not have access to capital markets. Therefore they consume all of their income in every period.

(tr_t^{agg}) rule - Struct Surp target are illustrated by the red and green dashed lines, respectively. In the graphs below, the monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized quarter-on-quarter values.

In the case of *Transfer (tr_t^{agg}) rule - Struct Surp target* the debt will also converge to its long-run level but more slowly than in the case of *Transfer (tr_t^{agg}) rule - Debt target* due to fact that in the long run closing the debt gap is equivalent to closing the structural surplus gap. This long run equivalence arises due to the dynamic relationship between debt and surplus in the model.

4.2 A monetary policy shock in Sweden

In this section, we describe the model response to a monetary policy shock that increases the annualized quarter-on-quarter Swedish monetary policy rate, $\check{i}_t^{notional}$, by 25 basis points ($\epsilon_t^i = 0.000625$). The economic outcome is illustrated in Figures 3 and 4. First, we describe the outcome given *No active fiscal rule*, which is illustrated by the red line. A positive shock to the policy rate leads to an increase in the returns to saving in private bonds for the Ricardian households in Sweden. Hence, *ceterus paribus*, they would like to decrease their consumption and save more of their income. As a result of decreased demand, the price level and consumption fall.

Furthermore, since it becomes more profitable to save in Swedish bonds than in Foreign bonds, the households would like to sell Foreign bonds and buy Swedish bonds, which leads to an appreciation of the exchange rate. The appreciated exchange rate does in turn lead to lower revenues for the export firms given the same set export price, since the price is set in the currency of Foreign. The firms respond to the decreased revenues by increasing the price of export goods. This leads to a fall in exports.

Private investment decreases following the shock. Due to the lower demand for consumption and exports, output decreases, which reduces the firm's demand for physical capital. Furthermore, the increased interest rate leads to an increased demand for bonds at the expense of other types of savings, in this case, physical capital. Both these channels lead to lower investment.

The demand for the four type of import goods (energy, non-energy, investment and export) are functions of the demand for their respective final good and of the price of the imported intermediate goods relative to the price of the respective final good. As the real exchange rate appreciates, the import firms decrease their prices. For the non-energy consumption good and the investment good firms, the final goods demand effect dominates, and they decrease their output. However, for the energy good and export firms, the price effect dominates. Hence they increase their output. In sum, total imports first decrease, but then increase again above the steady-state level of imports.

Due to the lower demand of domestic intermediate goods, output decreases. This leads to a reduced demand for labor, decreasing employment and nominal wages. However, real wages increase due to lower CPIF inflation. As a result, unemployment increases.[42] The wage income of the Non-Ricardian households decreases due to the reduced employment, which dominates the effect of the real wage on income. Therefore, Non-Ricardian consumption decreases.

Turning to the public sector, the reduced economic activity leads to a decrease in tax income, which in turn reduces the government surplus and increases government debt. The scenario with *Transfer (tr_t^{agg}) rule - Debt target* is illustrated by the green line in Figures 3 and 4. In that scenario, the government decreases the level of transfers as a response to the increased level of debt. The reduced transfers primarily affects Non-Ricardian households, who reduce their consumption demand one-to-one with their reduction in transfer income. Since the demand for consumption goods decreases following the reduced non-ricardian income, the demand for imported consumption goods also decreases, which leads to lower total imports than in the case of no fiscal policy.

The scenario with *Transfer (tr_t^{agg}) rule - Struct Surp target* is depicted with dashed purple line in Figures 3 and 4. In this scenario aggregate transfers are adjusted so that the structural surplus returns to its target level with a reasonable pace. Due to the definition of the structural surplus, it is affected by only discretionary fiscal policies and the debt payments, or permanent disturbances; hence the effects of a temporary monetary policy shock only affects the structural surplus via the debt payments. In order to correct the deviation of structural surplus from its long run target, the aggregate transfers fall, leading to a reduction in the Non-Ricardian households' consumption. However, since the effects of the temporary monetary policy shock on structural surplus is small, following this fiscal rule have limited different effects in comparison to *No active fiscal rule* case.

4.3 A risk premium shock to Swedish private bonds

In this section, we describe the economic outcome after a quarterly increase $\epsilon_t^{\zeta} = 0.0025$, in the internal risk premium ζ_t. The shock can also be interpreted more generally as an increase in the demand for saving in private bonds, or an increase in the demand for debt amortization, rather than an increase in the rate of return on

[42]Employment and output are highly correlated, since the main part of the costs of the production of intermediate goods consists of labor costs.

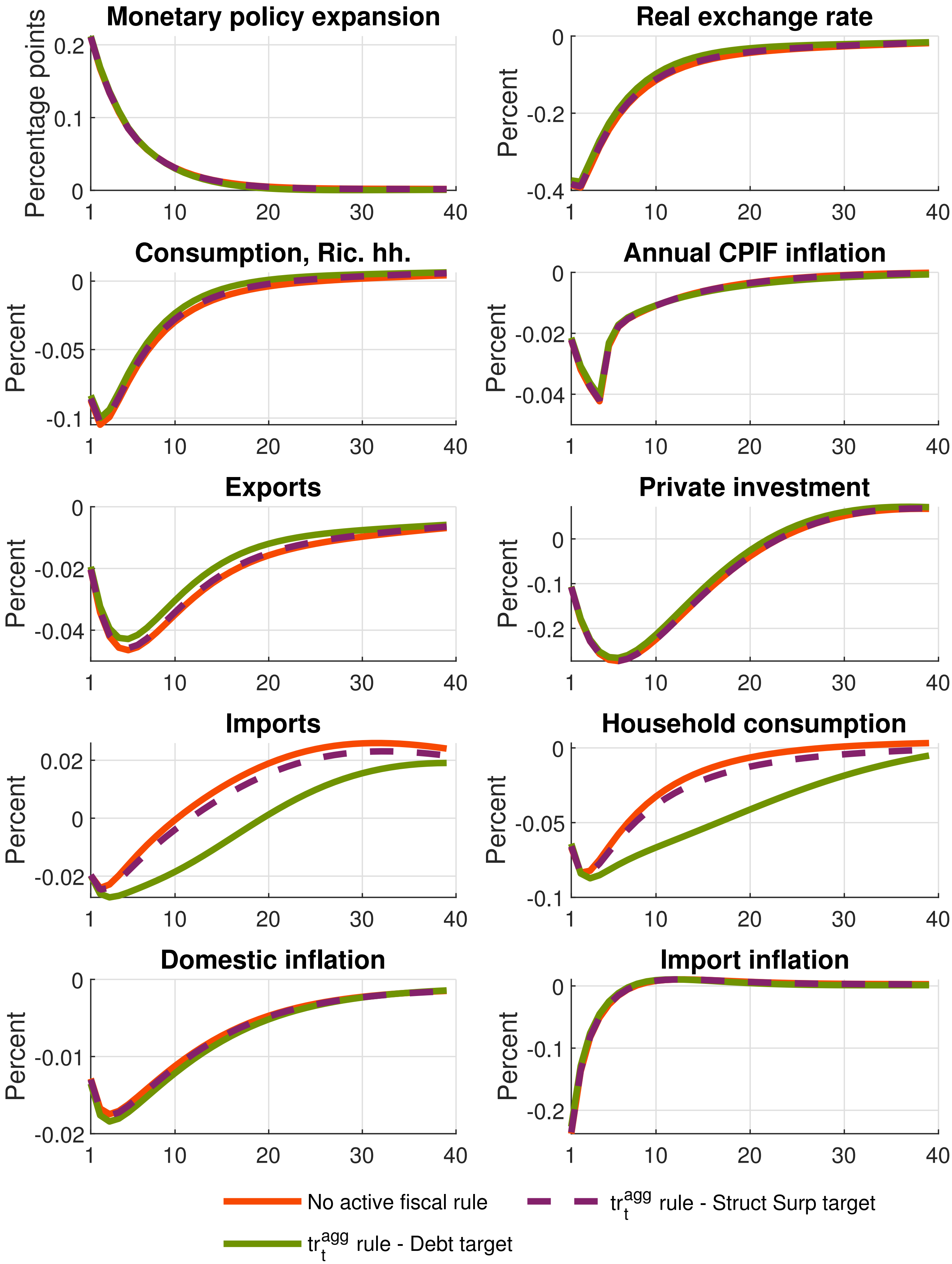


Figure 3: Economic outcome after a shock to the monetary policy rate $\breve{i}_t^{notational}$

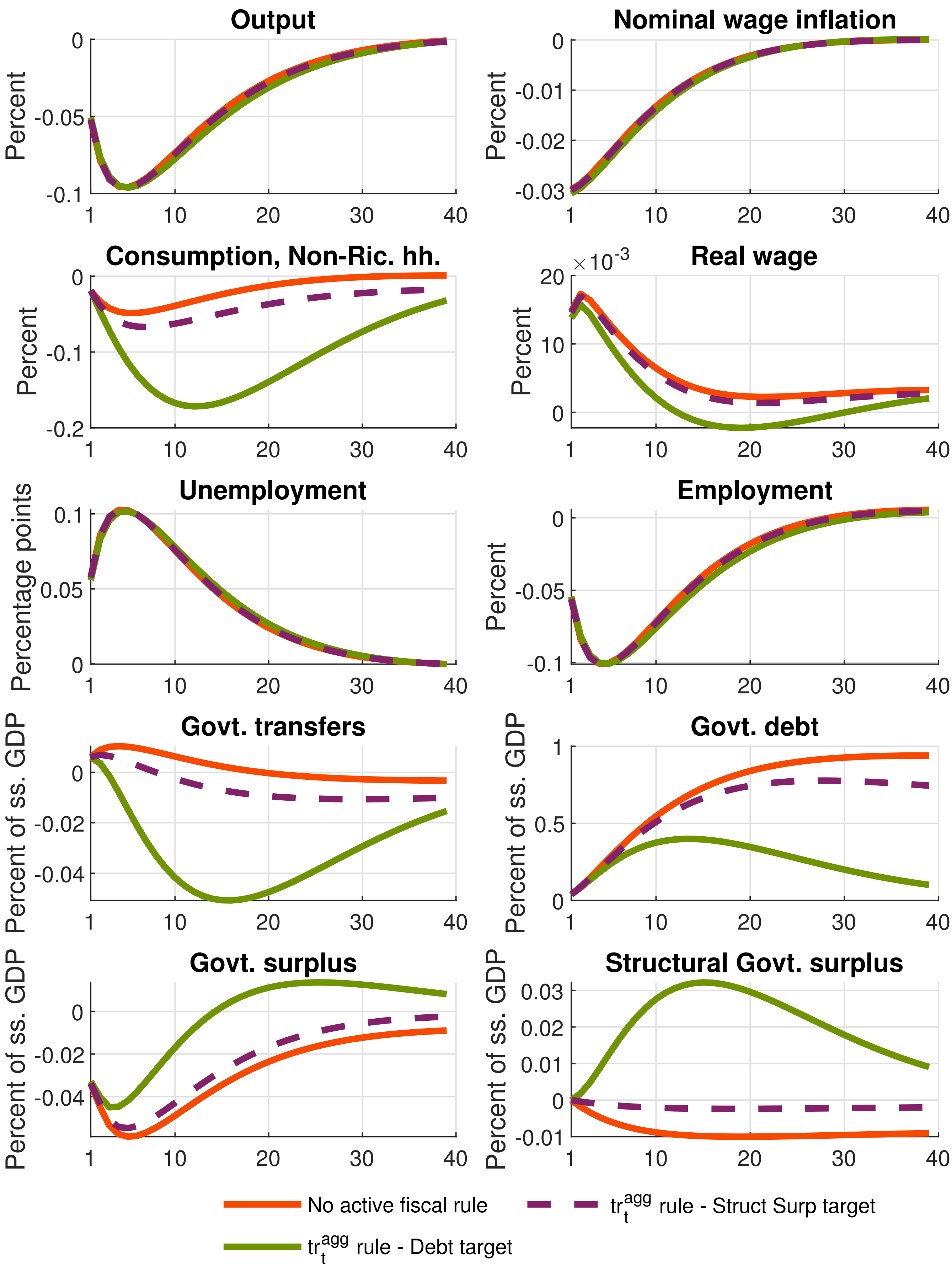


Figure 4: Economic outcome after a shock to the monetary policy rate $\breve{i}_t^{notational}$

bonds. The economic outcome of such a shock is illustrated in the Figures 5 and 6. First, we describe the outcome given *No active fiscal rule*, which is illustrated by the red line. A positive shock to the demand for bond savings leads to a decrease in the demand for consumption of the Ricardian households in Sweden. As a result of the decreased demand, the output of consumption goods falls which leads to a lower demand for domestically produced intermediate goods. This in turn leads to a decreased demand for labor and a higher unemployment rate. The increase in unemployment leads to a small decrease in the monetary policy rate. The decrease is however not large enough to offset the decrease in consumption demand following the risk premium shock.

A lower domestic interest rate together with an unchanged interest rate in the foreign economy leads to a depreciation of the exchange rate. The depreciated exchange rate does in turn lead to higher revenues for the export firms for a given price of exports (which is set in Foreign currency). The firms respond to the increased revenues by decreasing the price of export goods. This leads to a rise in exports. Furthermore, the depreciated exchange rate leads to an increase in import prices. The higher import prices, together with lower demand for Swedish final goods, lead to lower imports. Furthermore, higher import prices lead to an increase in CPIF inflation despite the decrease in domestic prices.

Private investments fall following the shock. The increased return on private bonds leads a re-allocation in the household savings portfolio, decreasing demand for investments. Additionally, the lower demand for domestically produced goods reduces demand for capital which depresses the rental rate of capital and in turn also has a negative effect on investments.

The decreased demand for final goods leads to a lower demand for domestically produced intermediate goods. This does lead to a decreased demand for labor. The decreased demand for labor leads to a decline in wages, both in nominal and in real terms. Furthermore, output and employment decrease and unemployment increases. Due to the lower labor income following the wage and employment decrease, Non-Ricardian households also decrease their consumption.

Turning to the public sector, the decreased economic activity leads to a decrease in tax income, which in turn reduces the government surplus and increases government debt. The scenario with *Transfer (tr_t^{agg}) rule - Debt target* is illustrated by the green line in Figures 5 and 6. In that scenario, the government decreases the level of transfers as a response to the increased level of debt. The transfer decrease primarily affects Non-Ricardian households, who reduce their consumption demand one-to-one with their reduction in transfer income.

The case with *Transfer (tr_t^{agg}) rule - Struct Surp target* is depicted with dashed purple line in Figures 5 and 6. Due to the definition of the structural surplus it is affected by only discretionary fiscal policies and the debt payments, or permanent disturbances; hence the effects of a temporary domestic risk premium shock only affects the structural surplus via the debt payments. In order to correct the deviation of structural surplus from its long run target the aggregate transfers fall compared to the *No active fiscal rule* case, leading to reduction in the Non-Ricardian households' consumption.

4.4 A stationary technology shock

In this section, we describe the economic outcome after a shock of $\epsilon_t = 0.0025$ to the intermediate good producers' technology ε_t. A temporary technology shock increases the productivity of Swedish intermediate good producers; thus, they can produce more output for a given level of inputs. The economic outcome is illustrated in Figures 7 and 8. First, we describe the outcome given the case No active fiscal rule, which is illustrated by the red line.

A positive technology shock generates a temporary increase in total factor productivity, which directly reduces the marginal cost of production. As a consequence, this effect generates downward pressure on domestic inflation. The decrease in domestic inflation generates downward pressure on CPIF inflation. In addition, since firms are able to produce same amount of output with lower input, the demand for labor thus employment decreases and unemployment increases. The increase in unemployment induces the Riksbank to reduce the nominal interest rate despite a modest increase in CPIF inflation, steming from higher import inflation.

The fall in the nominal interest rate in Sweden leads to the return on savings abroad being higher than the return on Swedish bonds. This induces Swedish households to buy Foreign bonds, which leads to an exchange rate depreciation. The exchange rate depreciation increases the markups of the Swedish export firms, who respond by reducing their prices. The opposite holds for the Swedish import firms, increasing the price of Swedish import goods. These changes in prices increases Swedish exports and reduces Swedish imports.

The intermediate good firms substitute capital for labor input in their production, leading to an increase in the demand for capital. The increase in demand for capital, together with a lower interest rate path, leads to higher private investment. The real wage decreases as nominal wage inflation decreases while CPIF inflation increases. Real labor income drops since both employment and real wage decrease. Non-Ricardian households reduce their consumption as their real labor income drops.

A decline in the interest rate path induces Ricardian households to increase their consumption. Overall household consumption decrease in the initial periods, but afterwards the higher Ricardian consumption dominates. The lower wage income reduces the labor income tax revenue and lower household consumption reduces the consumption tax revenue. As a result, the government surplus decreases and the government debt increases.

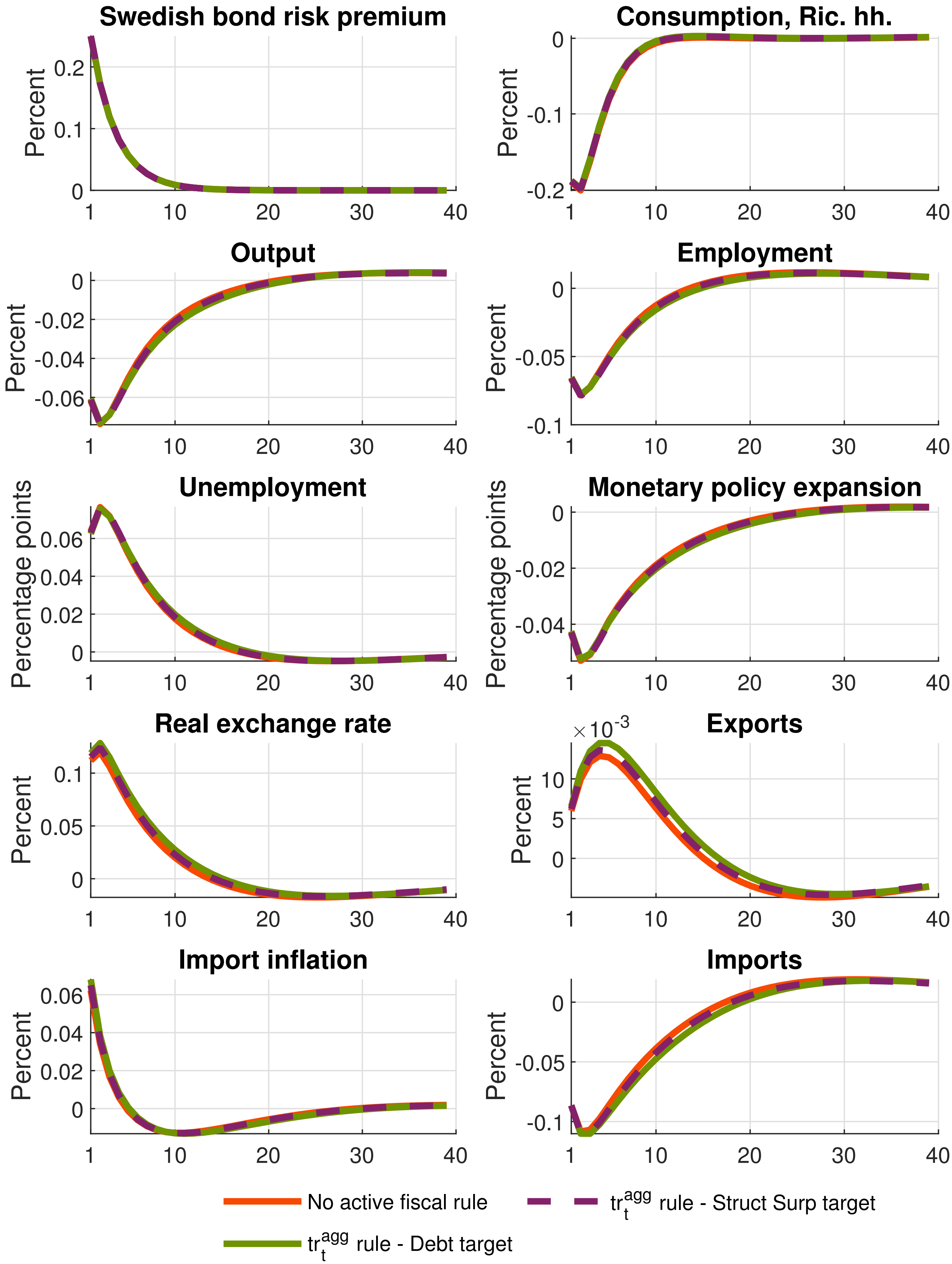


Figure 5: Economic outcome after a risk premium shock to Swedish private bonds ζ_t

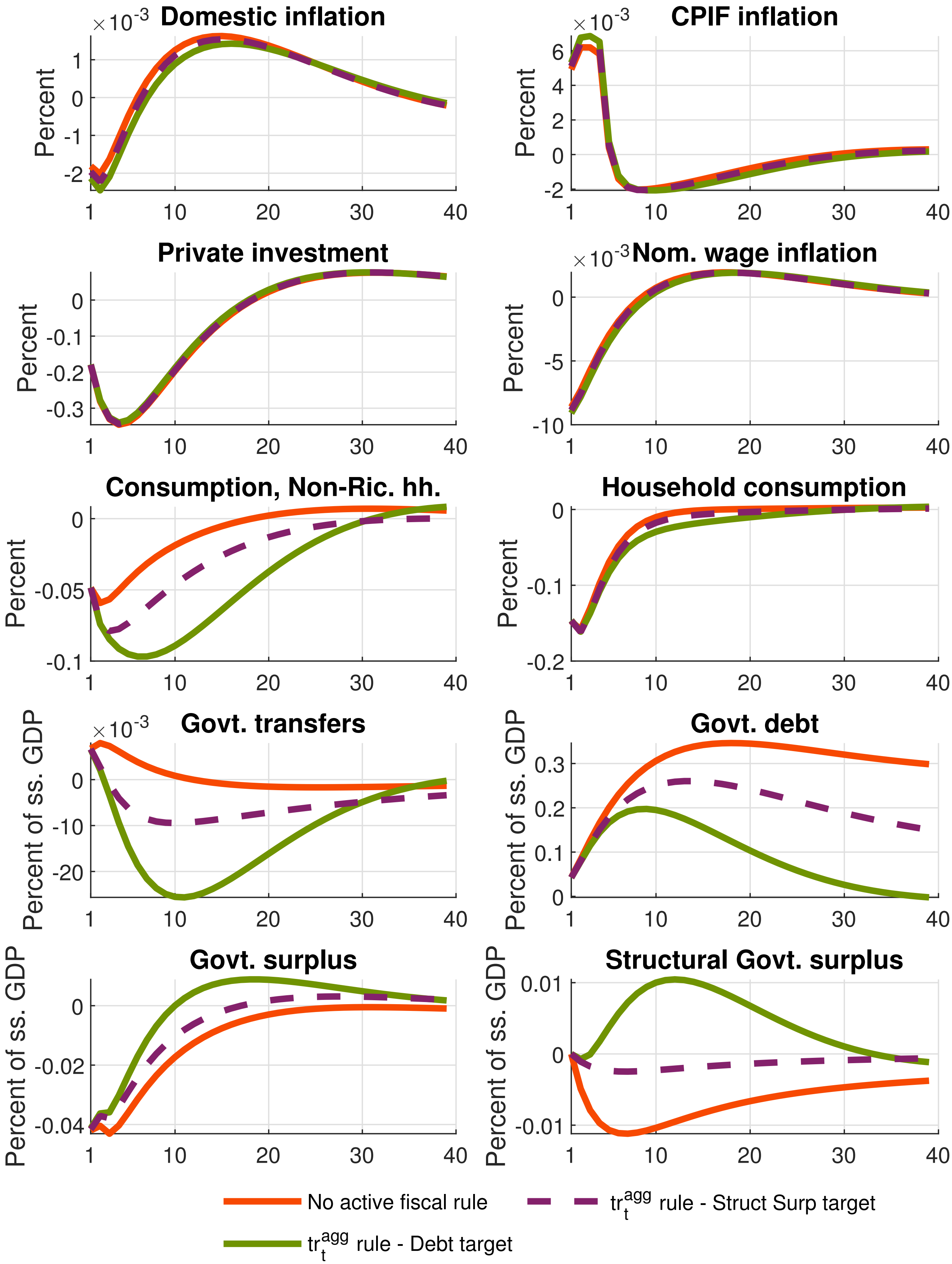


Figure 6: Economic outcome after a risk premium shock to Swedish private bonds ζ_t

The case with *Transfer (tr_t^{agg}) rule - Debt target* is illustrated by the green line in Figures 7 and 8. Particularly, the government responds to the increased government debt by decreasing transfers to the households. The drop in transfers has a negative impact on Non-Ricardian household consumption, which in turn magnifies the decrease in Non-Ricardian household consumption. As a consequence, output also becomes slightly lower. The change in output is however small.

Slightly lower output leads to a small decline in demand for labor and to a small increase in unemployment, and thus leads to a slightly lower monetary policy rate path. The return on Swedish bonds decreases and this leads to slightly higher depreciation and, in turn, to higher import inflation. Higher import inflation causes CPIF inflation to rise and lower demand for labor causes lower wage inflation. Hence, the real wage is lower under *Transfer (tr_t^{agg}) rule - Debt target*. Even if interest rate differentials are small between the different cases fiscal rules, the lower monetary policy path and higher CPIF inflation in the case with the debt target rule induce a lower real interest rate which leads to higher Ricardian household consumption and to higher private investment. Moreover, the lower return on Swedish private bonds induces Ricardian households to re-allocate their resources partly to savings in capital, which further increases the level of private investment relative to the scenario with No active fiscal rule, and partly to private consumption. Hence, Ricardian household consumption and private investment are higher than with No active fiscal rule, whereas Non-Ricardian household consumption and aggregate household consumption are lower.

The case with a *Transfer (tr_t^{agg}) rule - Struct Surp target* is illustrated by the dashed purple line in Figures 7 and 8. Since the changes in the tax income is only temporary it does not affect the structural surplus. Thus, the transfers to the households are mainly driven by unemployment. As a result, there is not a significant difference between *No active fiscal rule* and *Transfer (tr_t^{agg}) rule - Struct Surp target* in any of the variables considered.

4.5 A risk premium shock to Foreign private bonds

In this section, we describe the economic outcome after an increase of $\epsilon_t^{\zeta_F} = 0.0025$, to the Foreign domestic risk premium $\zeta_{F,t}$.This can also be interpreted more generally as negative demand shock in Foreign, or as an increased demand for bond holdings by Foreign households. The economic outcome is illustrated in 9 and 10. First, we describe the outcome *No active fiscal rule*, which is illustrated by the red line.

The shock leads to Foreign households wanting to save more and consume less. The decreased demand leads to a decrease in demand for intermediate goods. As a response to the decreased demand, output and hours worked in Foreign are reduced. This puts a downward pressure on Foreign wages, reducing the costs for the Foreign intermediate good firms. As a response, they reduce their prices, leading to Foreign consumption good firms to reduce their prices. The central bank in Foreign responds to the decline in output and inflation by reducing the monetary policy rate.

Since the foreign risk premium shock is correlated with the risk premium shock to Swedish private bonds, a positive shock to the foreign risk premium leads to an increased demand for bond savings in Sweden as well. Because of increased savings, consumption of the Ricardian households in Sweden and hence output of consumption goods falls. This leads to a lower demand for domestically produced intermediate goods, which in turn leads to a decreased demand for labor and a higher unemployment rate. The latter induces the Swedish central bank to decrease the monetary policy rate. Initially the foreign policy rate is lower than the Swedish rate. However, in the determination of the exchange rate the whole interest rate path is taken into account. After ten quarters the foreign policy rate is higher than the Swedish one and the exchange rate is therefore depreciating.

Demand for Swedish exports falls due to the lower demand from abroad. The decline in Swedish exports leads to an additional decrease in demand for both domestically produced and imported intermediate goods. Due to lower demand in Sweden imports decrease as well.

Responding to the decline in wages following the lower demand, firms reduce their prices. Hence domestic inflation falls. In contrast, imported inflation increases due to the exchange rate depreciation. The effect on domestic inflation does however dominate, leading CPIF inflation to fall.

As described in section 4.3, an increase in the domestic risk premium leads to households requiring a higher return to capital and hence an increased rental cost of capital. This, together with the lower demand for domestically produced goods, reduces the demand for capital and hence private investments fall.

The decreased demand for domestically produced intermediate goods lowers the demand for labor and leads to a decline in wages, both in nominal and in real terms. Due to the lower labor income following the wage and employment decrease, Non-Ricardian households decrease their consumption.

Turning to the public sector, the tax on labor, consisting of both labor taxes levied on households and of social security contributions levied on firms, are the most important income source for the government. Tax income decreases due to the fall in output, leading to an increase in debt. The case with *Transfer (tr_t^{agg}) rule - Debt target* is illustrated by the green line in 9 and 10. In this case, the government decreases the level of transfers as a response to the increased level of debt. The transfer decrease primarily affects Non-Ricardian households, who reduce their consumption demand with their reduction in transfer income. This reduces the primary expenditures of the government (and revenue to some degree since transfer income is taxed), leading to

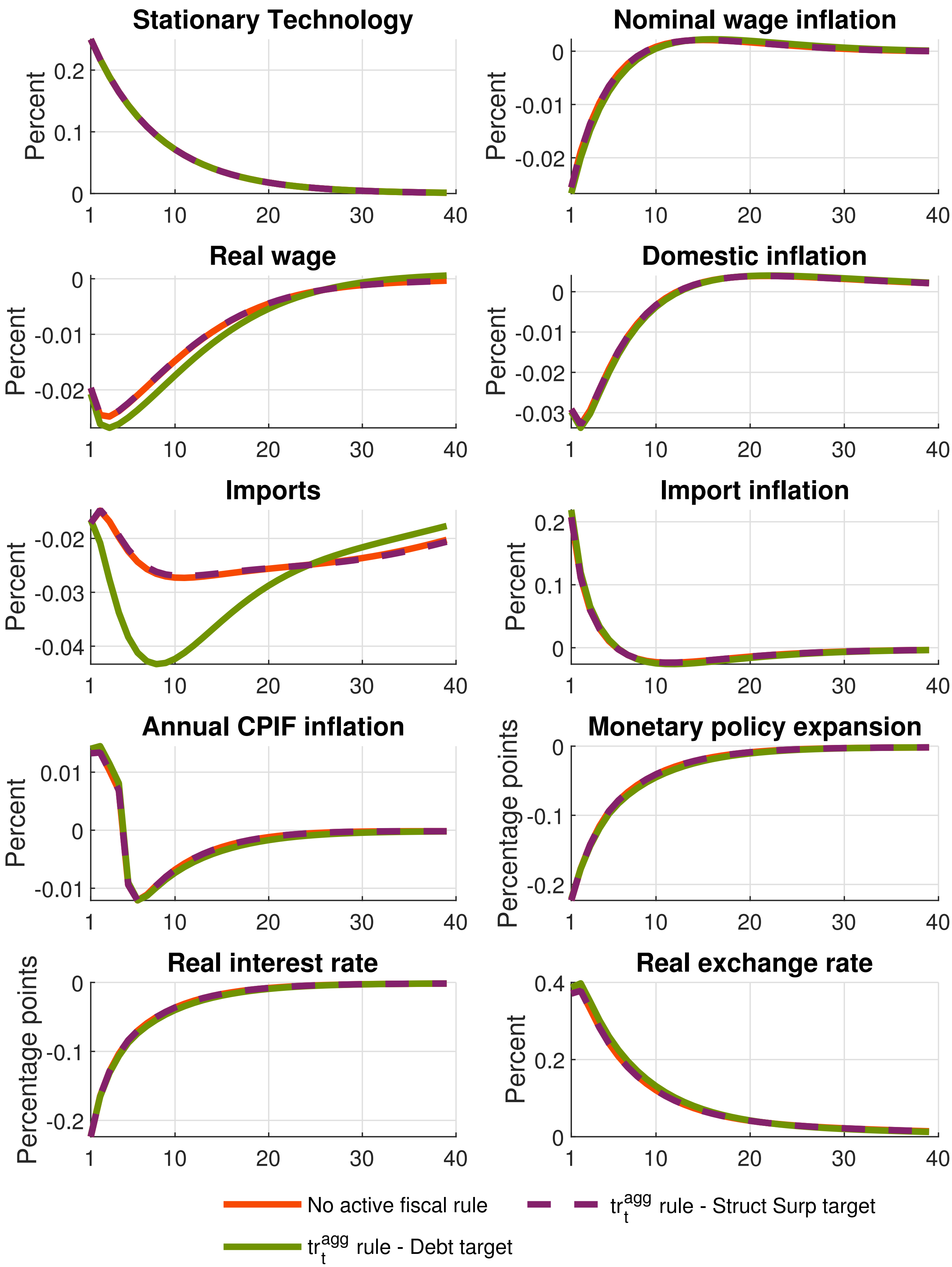


Figure 7: Economic outcome after a shock to productivity ε_t

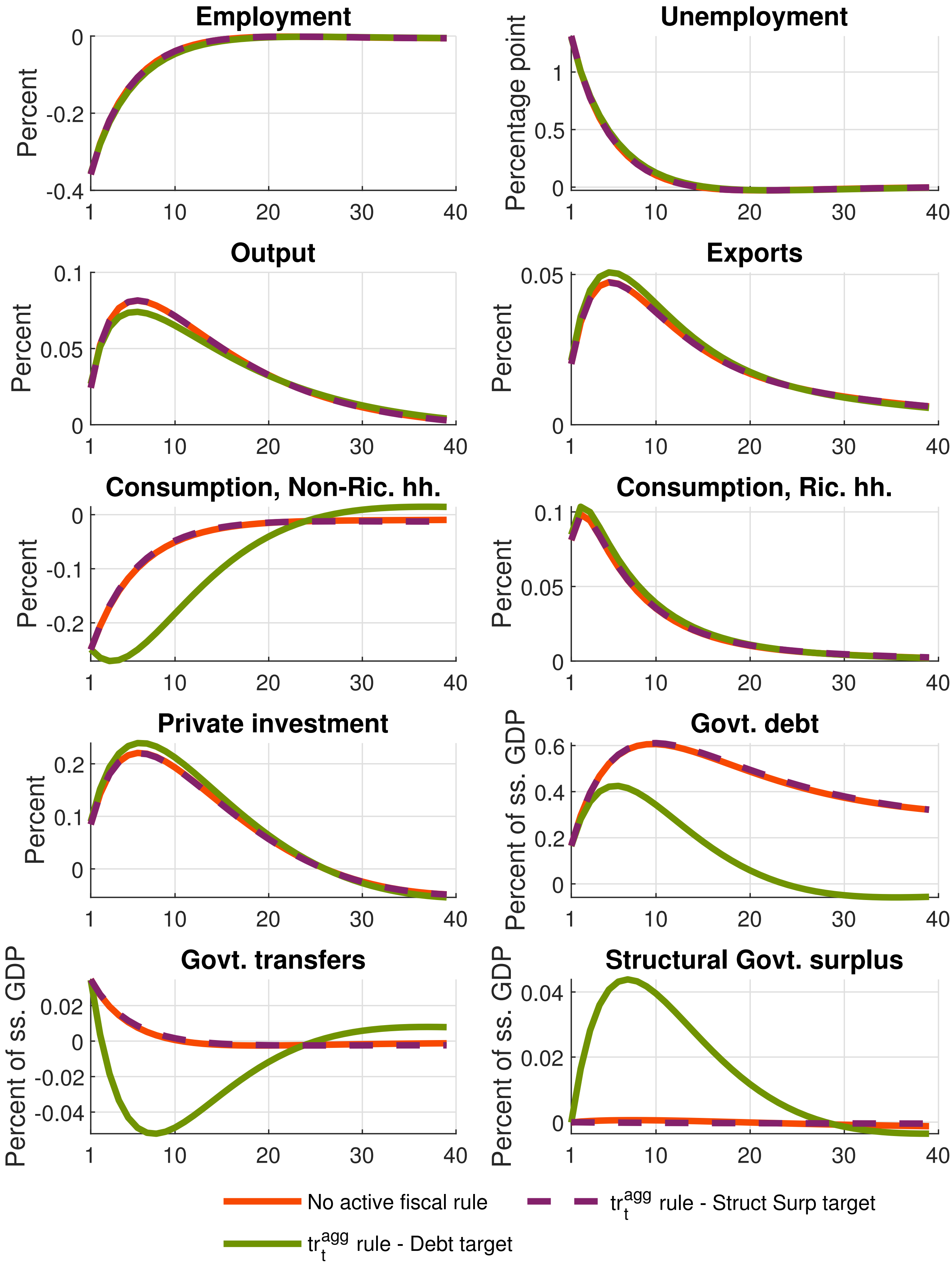


Figure 8: Economic outcome after a shock to productivity ε_t

a higher surplus and a lower debt. Other than for the consumption of Non-Ricardian households, the effect on the non-fiscal variables is small.

The case with *Transfer* (tr_t^{agg}) *rule - Struct Surp target* is illustrated by the dashed purple line in 9 and 10. Since the change in the tax income is only temporary it does not affect the structural surplus. The change in the structural surplus does primarily stem from the increase in debt and the resulting higher debt service cost. The transfers to the households are decreased slightly to return the structural surplus to its target. Since the change in transfers is relatively small, there is only a small decrease in Non-Ricardian consumption, while the other non-fiscal variables are virtually unaffected.

4.6 An external risk premium shock

In this section, we describe the economic outcome after a quarterly increase of 0.25 percent, $\epsilon_t^{\tilde{\tilde{\phi}}_t} = 0.0025$, to the external risk premium $\tilde{\tilde{\phi}}_t$. The positive external risk premium shock makes holding of domestic currency bonds less attractive relative to holding bonds in foreign currency. As a result the exchange rate depreciates. The economic outcome is illustrated in Figures 11 and 12.

First, we describe the outcome given *No active fiscal rule*, which is illustrated by the red line. The depreciation of the Krona leads to a higher marginal cost for the Swedish import firms, leading to a lower markup. To restore the markup, the firms increase their prices, leading to a decrease in Swedish imports. Furthermore, the exchange rate depreciation leads to a higher markup for Swedish export firms. As a response, they reduce their prices, leading to higher exports.

The increased price of imported goods leads to higher costs for the consumption and investment good producers, who therefore increase their prices. The increased CPIF inflation leads to lower Ricardian consumption and private investment. Due to the lower demand for investment and consumption, demand for domestically produced inputs to investment and consumption also decrease. During the first two years however, the increased demand for exports dominates, leading to an increase in output. After two years, output moves below its steady-state level. Due to the increased output, labor demand also increases and is higher than in the steady state during the first two years. This leads employment to be higher and unemployment to be lower than in the steady state during the the same period. After two years however, the effect reverses on both employment and unemployment.

The increased demand for domestic goods leads to an increase in labor demand, putting a slight upwards pressure on nominal wages. The CPIF inflation does however increase more than the nominal wages, which means that real wages fall. This does in turn lead to a lower consumption for Non-Ricardian households. The Non-Ricardian consumption is further dampened by the fact that their transfers decrease due to the output increase.

Turning to the public sector, the increase in output leads to an increase in tax revenue. Simultaneously, the decrease in transfers due to higher output reduces primary expenditure. Therefore, both the government surplus and the government debt are reduced.

The case with *Transfer* (tr_t^{agg}) *rule - Debt target* is illustrated by the green line in Figures 11 and 12. As a response to the decreased debt, following the specified fiscal policy rule, the government increases the transfers to households. Non-Ricardian households increase their consumption one-to-one with any additional transfers they receive. Thus, household consumption increases. The fiscal policy leads to a decreasing government surplus, meaning that the government debt moves towards its steady-state level. The effect on non-fiscal variables other than household consumption is, however, small.

The case with *Transfer* (tr_t^{agg}) *rule - Struct Surp target* is illustrated by the dashed purple line in Figures 11 and 12. Since the changes in the tax income is only temporary it does not affect the structural surplus. The change in the structural surplus does primarily stem from the decrease in debt and the resulting lower debt service cost. The transfers to the households are increased slightly to return the structural surplus to its target. The other fiscal variables are however not affected that much since the increase in transfers are quite small. The same goes through to the non-fiscal variables. Since the change in transfers is quite small, there is only a small increase in Non-Ricardian consumption, while the other non-fiscal variables are virtually unaffected.

4.7 A domestic energy price shock

The increase in the price of domestic energy increases the price of energy consumption, which in turn leads to a higher price of consumption goods, hence higher CPIF-inflation. The increased CPIF-inflation does in turn lead to a monetary policy rate increase.

The higher price of consumption together with the policy rate increase leads to a lower demand for consumption goods, both from Ricardian and Non-Ricardian households. Furthermore, the policy rate increase leads to an exchange rate appreciation, which affects exports negatively the first two years. After two years the nominal exchange rate has however started to depreciate again, and is back above its original levels, leading exports to move above its steady-state level.

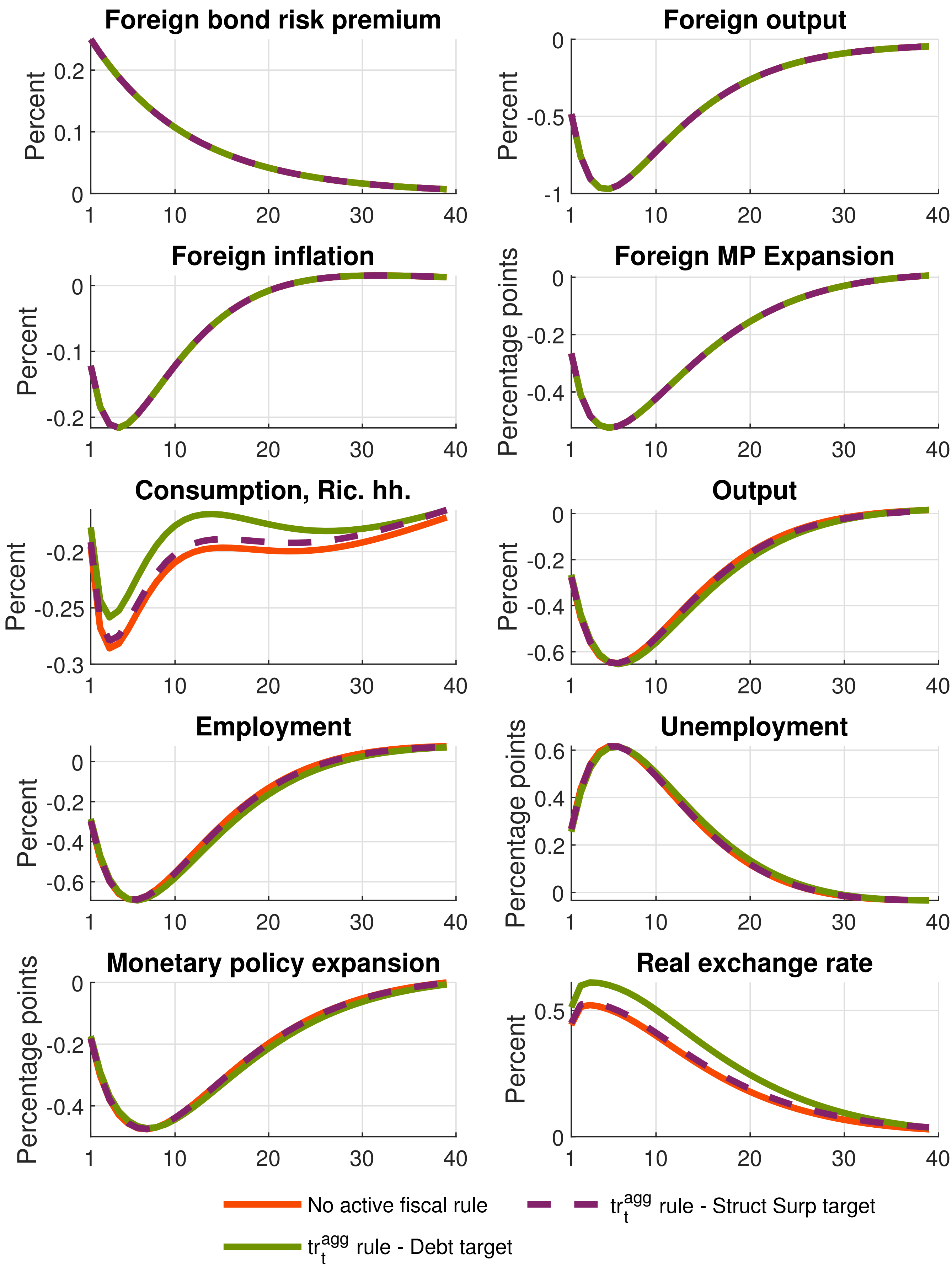


Figure 9: Economic outcome after a shock to Foreign bond risk premium $\zeta_{F,t}$

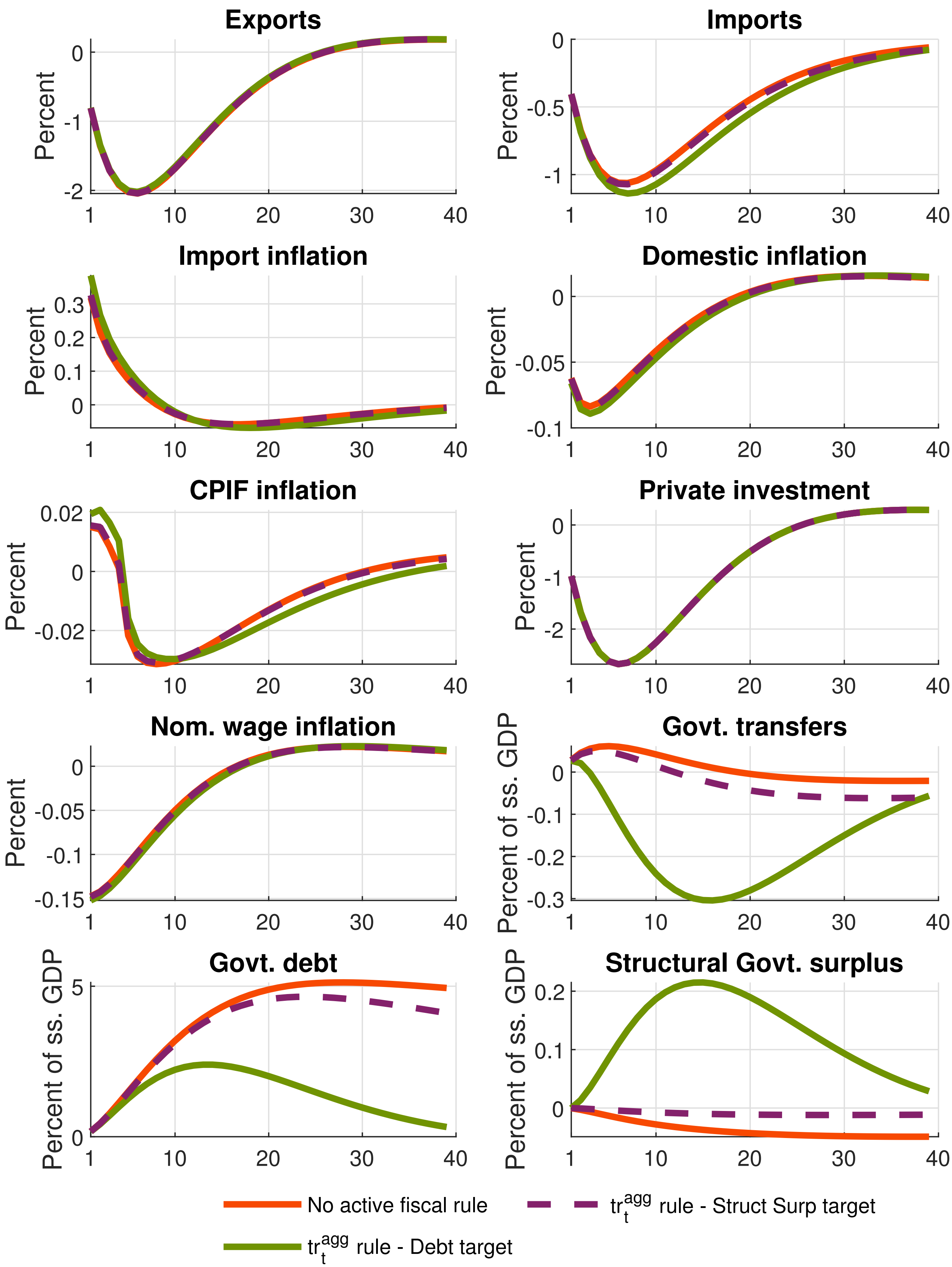


Figure 10: Economic outcome after a shock to Foreign bond risk premium $\zeta_{F,t}$

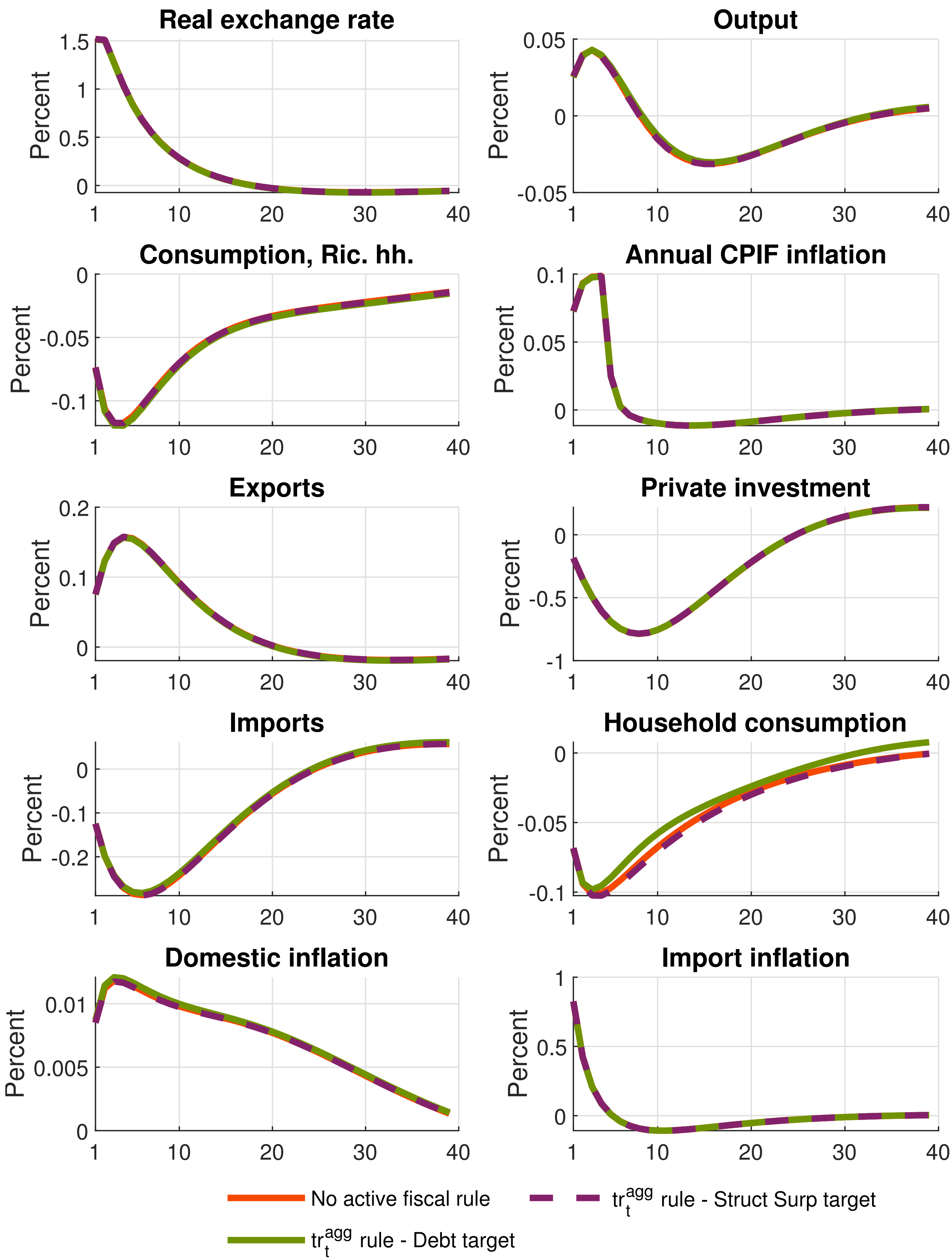


Figure 11: Economic outcome after a shock to the external risk premium $\hat{\tilde{\phi}}_t$

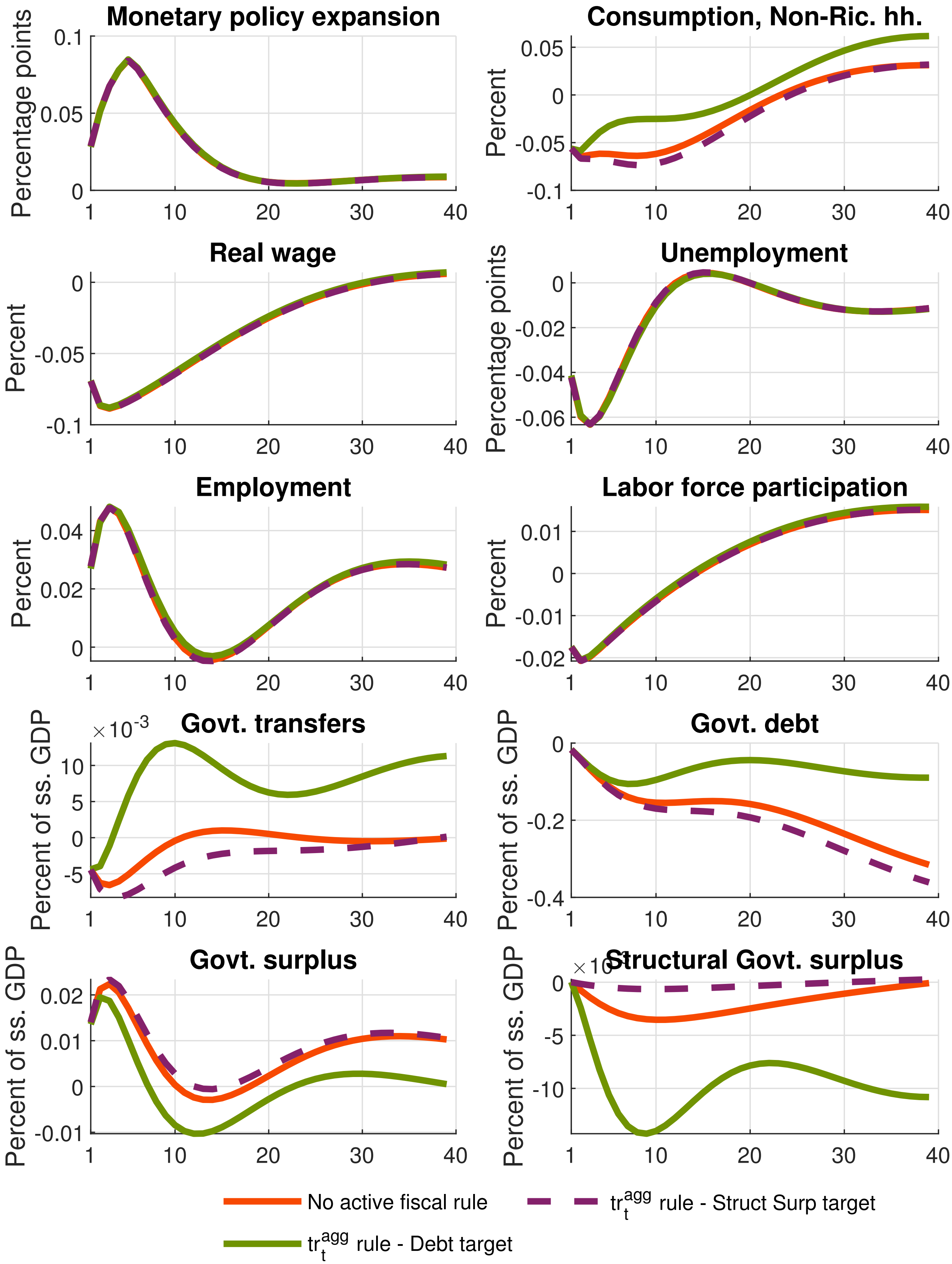


Figure 12: Economic outcome after a shock to the external risk premium $\tilde{\tilde{\phi}}_t$

Investment decreases immediately after the shock, and stays below its steady state level for 9 quarters, due to the increased policy rate and the lower demand for domestic goods following the lower demand for consumtion and export goods. As output starts to increase again and move towards its steady-state level, the demand for capital increases again, leading to increased private investment.

Even though investment and exports after two years are above their respective steady-state levels, output is still below its steady-state level, due to the persistently low consumption demand. The lower output leads to a lower demand for labor, and hence a lower employment, which leads to higher unemployment. The increase in unemployment is however smaller than the decrease in employment. The reason is that the CPIF-inflation leads to a lower real wage, affecting labor force participation negatively.

Turning to the public sector, the lower employment and consumption leads to lower tax revenues. Furthermore, transfers are increased due to the higher unemployment. As a result, the public surplus decreases, and the public debt increases.

The case with *Transfer* (tr_t^{agg}) *rule - Debt target* is illustrated by the green line in Figures 13 and 14. As a response to the increased debt, following the specified fiscal policy rule, the government decreases the transfers to households. Non-Ricardian households decrease their consumption one-to-one with any additional transfers they receive. Thus, household consumption decreases. The conducted fiscal policy increases the government surplus such that the government debt moves towards its steady-state level.

The case with *Transfer* (tr_t^{agg}) *rule - Struct Surp target* is illustrated by the dashed purple line in Figures 13 and 14. Since the changes in the tax income is only temporary it does not affect the structural surplus. The change in the structural surplus does primarily stem from the increased debt and the resulting higher debt service cost. The transfers to the households are decreased slightly to return the structural surplus to its target. The change is however so small. This means that the effect on household consumption is also small. The decreased transfers does however change the path of debt slightly, that becomes slightly smaller than in the *no fiscal policy* case.

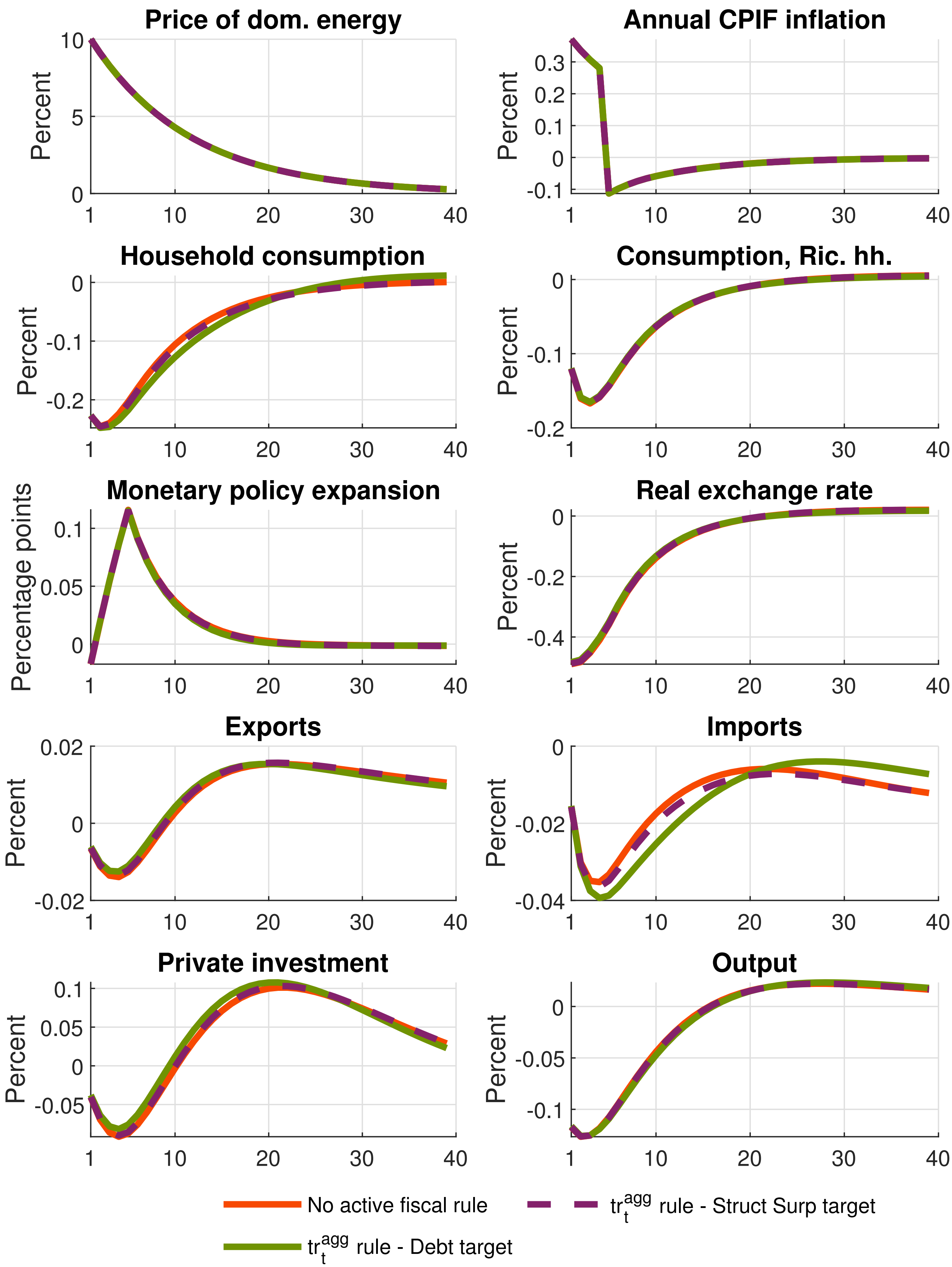


Figure 13: Economic outcome after a shock to the domestic price of energy $\hat{p}_t^{C,D,e}$

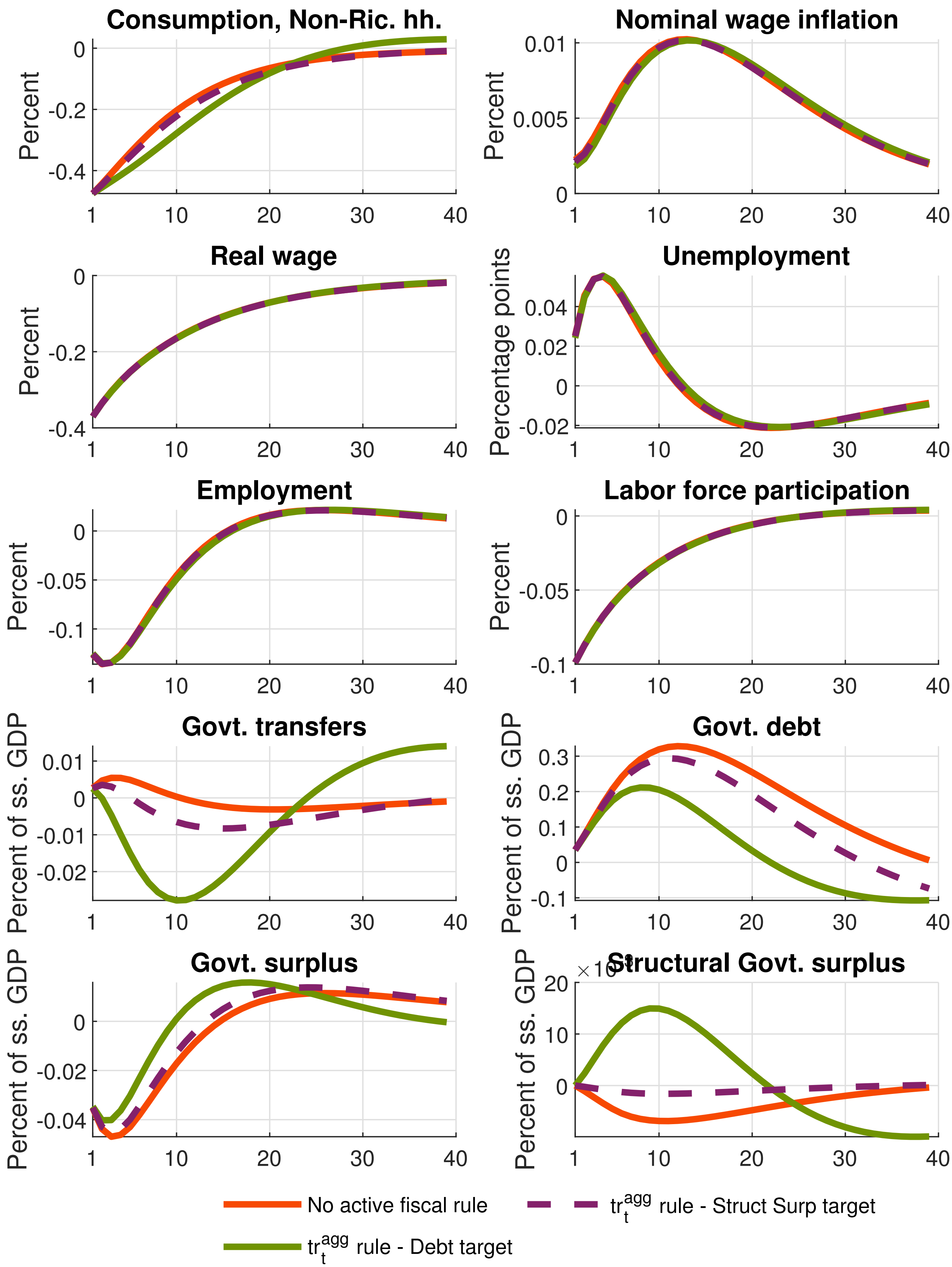


Figure 14: Economic outcome after a shock to the domestic price of energy $\hat{p}_t^{C,D,e}$

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Appendix

A Appendix: Model equations

In this appendix, we present the non-linear model equations and the corresponding log-linearized model equations.

Before we present the model equations, we clarify some notations. Variables that are trending along the balanced growth path have been stationarized. In most cases, we use the *bar* notation to distinguish stationarized variables from the non-stationary variables. For example, $\overline{C}_t^{agg} = \frac{C_t^{agg}}{z_t^+}$ denotes the stationarized level of aggregate household consumption in period t. K_t denotes the non-stationarized level of aggregate capital. Thus, $\overline{K}_t = \frac{K_t}{z_{t-1}\left(\gamma_{t-1}\right)^{\frac{1}{1-\alpha}}}$ denotes the stationarized level of aggregate capital. $K_t^s = u_t K_t$, in turn, denotes the non-stationarized level of aggregate capital services, and $\overline{K}_t^s$ is the stationarized level of aggregate capital services which is defined as $\overline{K}_t^s = u_t \overline{K}_t$. The different indexation variables, such as the gross inflation of intermediate good prices $\overline{\Pi}_t$, constitutes an exception to the *bar* notation. The different gross inflation rates do not need to be stationarized. For these different gross inflation rates, the *bar* instead denotes the corresponding indexation variable.

When applicable, variables that appear in the model equations are expressed in *per capita* terms. For example, $\overline{C}_t^{agg}$ is the stationarized level of aggregate household consumption in the Swedish economy, and $\overline{c}_t^{agg}$ is the stationarized *per capita* level of aggregate household consumption in the Swedish economy. In the Swedish economy, where the size of the population is normalized to unity, the distinction between aggregate and *per capita* terms is trivial. For the Foreign economy, however, it is essential to distinguish, for example, between the stationarized level of aggregate household consumption $\overline{C}_{F,t}$, and the stationarized level of aggregate consumption per inhabitant, which is denoted by $\overline{c}_{F,t} = \frac{C_{F,t}}{z_t^+ \omega}$, where ω is the size of the population in Foreign.[43]

In addition to the non-linear form of model equations, we also present the corresponding log-linearized model equations. In this documentation, a variable with the *hat* notation can be interpreted as a log-linear approximation of the variable around its steady state (percent deviations). Two examples may help to clarify the use of the *hat* notation. The first example is for a variable such as $\overline{c}_t^{agg}$, which has been stationarized. Thus, we have: $\hat{c}_t^{agg} = ln\left(\frac{\overline{c}_t^{agg}}{\overline{c}^{agg}}\right)$, where $\overline{c}^{agg}$ denotes the steady state level of aggregate consumption *per capita* in the Swedish economy. $\hat{c}_t^{agg}$ can be interpreted as a log-linear approximation of the stationarized level of aggregate consumption *per capita* around its steady state level. The second example is for a variable such as Π_t, which does not need to be stationarized. Hence, we have: $\hat{\Pi}_t = ln\left(\frac{\Pi_t}{\Pi}\right)$, where Π denotes the steady state level of gross inflation of intermediate good prices. $\hat{\Pi}_t$ can be interpreted as a log-linear approximation of the gross inflation rate of intermediate goods around its steady state level.

Finally, a variable with *breve* notation is interpreted as an absolute deviation of the variable from its steady state. The first example is for a variable such as the real government debt $\overline{b}_t$, which has been stationarized. Thus, we have: $\breve{b}_t = \overline{b}_t - \overline{b}$, where $\overline{b}$ is the steady state level of the real government debt. $\breve{b}_t$ can be interpreted as an absolute deviation of the stationarized level of the real government debt from its steady state level. The second example for a variable such as $\breve{\tau}_t^C$, is the time-varying consumption tax rate, which does not need to be stationarized. Thus, we have: $\breve{\tau}_t^C = \tau_t^C - \tau^C$, where τ^C is the consumption tax rate in steady state. $\breve{\tau}_t^C$ is interpreted as an absolute deviation of the consumption tax rate from its steady state (percentage point deviations).

Our model equations include both equilibrium conditions and various definitions that are used to solve and simulate the model. In the subsequent sections, we present these equations.

A.1 Sweden: Household sector

Consumption Euler equation:

$$\overline{\Omega}_t^C = R_t \zeta_t E_t \left[\beta_{t+1}^r \frac{1}{\mu_{z^+,t+1}\Pi_{t+1}^C} \overline{\Omega}_{t+1}^C\right] \tag{A.1a}$$

$$\hat{\Omega}_t^C = E_t \left[\hat{\zeta}_t + \hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C + \frac{1}{R}\breve{i}_t - \hat{\Pi}_{t+1}^C - \hat{\mu}_{z^+,t+1}\right] \tag{A.1b}$$

[43] One exception to the *per capita* notation is investment: because we use i_t to denote the net nominal interest rate, $\bar{I}_t$ denotes both the stationarized level of aggregate investment and the stationarized level of aggregate investment per inhabitant in the Swedish economy.

Definition of nominal gross interest rate on private bonds:

$$R_t = 1 + i_t \tag{A.2a}$$

$$\hat{R}_t = \frac{1}{R}\breve{i}_t \tag{A.2b}$$

Lagrange multiplier, marginal utility of consumption equation:

$$\overline{\Omega}_t^C = \frac{\zeta_t^c}{(1+\tau_t^C)\left(\bar{\bar{c}}_t - \rho_h \frac{1}{\mu_{z^+,t}}\bar{\bar{c}}_{t-1}\right)}\left(\alpha_G \frac{\bar{\bar{c}}_t}{\bar{c}_t}\right)^{\frac{1}{\upsilon_G}} \tag{A.3a}$$

$$\hat{\Omega}_t^C = \hat{\zeta}_t^c + \left(1 - \frac{\rho_h}{\mu_{z^+,}}\right)^{-1}\left[-\hat{\bar{c}}_t + \frac{\rho_h}{\mu_{z^+,}}\hat{\bar{c}}_{t-1} - \frac{\rho_h}{\mu_{z^+}}\hat{\mu}_{z^+,t}\right] + \frac{1}{\upsilon_G}\left(\hat{\bar{c}}_t - \hat{c}_t\right) - \frac{1}{1+\tau^C}\breve{\tau}_t^C \tag{A.3b}$$

Marginal utility of consumption equation:

$$\bar{U}_{c,t} = \frac{\zeta_t^c}{\left(\bar{\bar{c}}_t - \rho_h \frac{1}{\mu_{z^+,t}}\bar{\bar{c}}_{t-1}\right)}\left(\alpha_G \frac{\bar{\bar{c}}_t}{\bar{c}_t}\right)^{\frac{1}{\upsilon_G}} \tag{A.4a}$$

$$\hat{U}_{c,t} = \hat{\zeta}_t^c + \left(1 - \frac{\rho_h}{\mu_{z^+,}}\right)^{-1}\left[-\hat{\bar{c}}_t + \frac{\rho_h}{\mu_{z^+,}}\hat{\bar{c}}_{t-1} - \frac{\rho_h}{\mu_{z^+}}\hat{\mu}_{z^+,t}\right] + \frac{1}{\upsilon_G}\left(\hat{\bar{c}}_t - \hat{c}_t\right) \tag{A.4b}$$

Composite consumption function:

$$\bar{\bar{c}}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}}\,\bar{c}_t^{\frac{\upsilon_G-1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}}\,\bar{g}_t^{\frac{\upsilon_G-1}{\upsilon_G}}\right)^{\frac{\upsilon_G}{\upsilon_G-1}} \tag{A.5a}$$

$$\left(\frac{\bar{\bar{c}}}{\bar{c}}\right)^{\frac{\upsilon_G-1}{\upsilon_G}}\hat{\bar{c}}_t = \alpha_G^{\frac{1}{\upsilon_G}}\,\hat{c}_t + (1-\alpha_G)^{\frac{1}{\upsilon_G}}\left(\frac{\bar{g}}{\bar{c}}\right)^{\frac{\upsilon_G-1}{\upsilon_G}}\hat{g}_t \tag{A.5b}$$

Average interest rate on government bonds:

$$\bar{\Omega}_t^R = E_t\beta_{t+1}^r \frac{\bar{\Omega}_{t+1}^C}{\bar{\Omega}_t^C\Pi_{t+1}^C\mu_{z^+,t+1}}\left[1 + \bar{\Omega}_{t+1}^R(1-\alpha_B)\right] \tag{A.6a}$$

$$\hat{\Omega}_t^R = E_t\left[\hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C + \frac{\bar{\Omega}^R(1-\alpha_B)}{1+\bar{\Omega}^R(1-\alpha_B)}\hat{\Omega}_{t+1}^R - \hat{\Omega}_t^C - \hat{\Pi}_{t+1}^C - \hat{\mu}_{z^+,t+1}\right] \tag{A.6b}$$

Euler equation for government bond holdings:

$$1 = E_t\beta_{t+1}^r \frac{\bar{\Omega}_{t+1}^C}{\bar{\Omega}_t^C\Pi_{t+1}^C\mu_{z^+,t+1}}\left[R_t^{B,n} - (1-\alpha_B)\,\bar{\Omega}_{t+1}^R\left(R_{t+1}^{B,n} - R_t^{B,n}\right)\right] \tag{A.7a}$$

$$0 = E_t\hat{\beta}_{t+1}^r + \hat{E_t\Omega}_{t+1}^C - \hat{\Omega}_t^C - \hat{E_t\Pi}_{t+1}^C - \hat{E_t}\mu_{z^+,t+1} + \frac{1}{R^{B,n}}\breve{R}_t^{B,n} - (1-\alpha_B)\,\bar{\Omega}^R\left(E_t\breve{R}_{t+1}^{B,n} - \breve{R}_t^{B,n}\right) \tag{A.7b}$$

Capital utilization decision equation:

$$r_t^K = p_t^I a'(u_t) \tag{A.8a}$$

$$\hat{r}_t^K = \hat{p}_t^I + \sigma_a\hat{u}_t \tag{A.8b}$$

Household purchases of installed capital equation:

$$p_t^K = E_t\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C\Pi_{t+1}}{\overline{\Omega}_t^C\Pi_{t+1}^C\mu_{z^+,t+1}\mu_{\gamma,t+1}}\left[\left(1-\tau_{t+1}^K\right)\left(r_{t+1}^K u_{t+1} - p_{t+1}^I a(u_{t+1})\right) + \iota^K\delta\tau_{t+1}^K\frac{\mu_{\gamma,t+1}}{\Pi_{t+1}}p_t^K + p_{t+1}^K(1-\delta)\right] \tag{A.9a}$$

$$\left(1-\iota^K \frac{1}{H}\tau^K \delta \frac{\mu_\gamma}{\Pi} p^K\right)\left(\hat{p}_t^K - \hat{\Pi}_{t+1} + \hat{\mu}_{\gamma,t+1}\right) =$$

$$E_t \hat{\beta}^r_{t+1} + E_t \hat{\Omega}^C_{t+1} - \hat{\Omega}^C_t - E_t \hat{\Pi}^C_{t+1} - E_t \hat{\mu}_{z^+,t+1} + \frac{1}{H} r^K \left(1-\tau^K\right) E_t \hat{r}^K_{t+1} - \frac{1}{H}\left(r^K - \iota^K \delta \frac{\mu_\gamma}{\Pi} p^K\right) E_t \breve{\tau}^K_{t+1} + \frac{1}{H} p^K (1-\delta) E_t \hat{p}^K_{t+1} \tag{A.9b}$$

Household investment decision equation:

$$p_t^I \left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(\overline{I}_t, \overline{I}_{t-1}, \mu_{z^+,t}, \mu_{\gamma,t}) + E_t \left[\beta^r_{t+1} \frac{\overline{\Omega}^C_{t+1} \Pi_{t+1}}{\overline{\Omega}^C_t \Pi^C_{t+1}} \frac{p^K_{t+1}}{\mu_{z^+,t+1}\mu_{\gamma,t+1}} \Upsilon_{F,t+1} F_2(\overline{I}_{t+1}, \overline{I}_t, \mu_{z^+,t+1}, \mu_{\gamma,t+1})\right] \tag{A.10a}$$

$$(1-\tau^I)\frac{p^I}{p^K \Upsilon} \hat{p}_t^I - \breve{\tau}_t^I = \hat{p}_t^K + \hat{\Upsilon}_t - S''\left(\mu_{z^+}\mu_\gamma\right)^2 E_t \left[\triangle \hat{I}_t + \hat{\mu}_{z^+,t} + \hat{\mu}_{\gamma,t} - \beta \triangle \hat{I}_{t+1} - \beta \hat{\mu}_{z^+,t+1} - \beta \hat{\mu}_{\gamma,t+1}\right] \tag{A.10b}$$

Definition of capital services:

$$\overline{k}_t^s = u_t \bar{k}_t \tag{A.11a}$$

$$\hat{k}_t^s = \hat{u}_t + \hat{k}_t \tag{A.11b}$$

Capital accumulation equation:

$$\overline{k}_{t+1} = (1-\delta)\overline{k}_t \frac{1}{\mu_{z^+,t}\mu_{\gamma,t}} + \Upsilon_t \left[1 - \widetilde{S}\left(\frac{\overline{I}_t \mu_{z^+}\mu_\gamma}{\overline{I}_{t-1}}\right)\right] \overline{I}_t + \overline{\triangle}_t^K \tag{A.12a}$$

$$\hat{k}_{t+1} = \frac{(1-\delta)}{\mu_{z^+}\mu_\gamma}\left(\hat{k}_t - \hat{\mu}_{z^+,t} - \hat{\mu}_{\gamma,t}\right) + \frac{\overline{I}}{\overline{k}} \Upsilon \left(\hat{I}_t + \hat{\Upsilon}_t\right) \tag{A.12b}$$

Optimal wage setting equation:

$$E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^{k} \beta^r_{t+i}\right) n_{t+k|t} \overline{\Omega}^C_{t+k} \frac{1}{(1-\lambda^W_{t+k})} \left[\left(1-\tau^W_{t+k}\right)\overline{w}_{t+k|t} - \lambda^W_{t+k}\zeta^n_{t+k} \frac{\nu'(n_{t+k|t})}{\overline{\Omega}^C_{t+k}}\right] = 0 \tag{A.13a}$$

$$\triangle \hat{w}_t = \beta E_t \left[\triangle \hat{w}_{t+1}\right] - \kappa_W (\hat{\Psi}^W_t - \hat{\lambda}^W_t) + \hat{\overline{\Pi}}^W_t - \hat{\mu}_{z^+,t} - \hat{\Pi}^C_t - \beta E_t \left[\hat{\overline{\Pi}}^W_{t+1} - \hat{\mu}_{z^+,t+1} - \hat{\Pi}^C_{t+1}\right] \tag{A.13b}$$

Labor force participation equation:

$$\overline{\Omega}^c_t (1-\tau^W_t)\overline{w}_t = \zeta^n_t \Theta^n_t A_n l^\eta_t \tag{A.14a}$$

$$\hat{w}_t = \hat{\zeta}^n_t + \hat{\Theta}^n_t + \eta \hat{l}_t - \hat{\Omega}^C_t + \frac{1}{1-\tau^W}\breve{\tau}^W_t \tag{A.14b}$$

Definition of endogenous shifter equation:

$$\Theta^n_t = \bar{Z}^n_t \bar{U}_{c,t} \tag{A.15a}$$

$$\hat{\Theta}^n_t = \hat{Z}^n_t + \hat{U}_{c,t} \tag{A.15b}$$

Trend of wealth effect in endogenous shifter:

$$\bar{Z}^n_t = \left(\frac{\bar{Z}^n_{t-1}}{\mu_{z^+,t}}\right)^{1-\chi_n} (\bar{U}_{c,t})^{-\chi_n} \tag{A.16a}$$

$$\hat{Z}^n_t = (1-\chi_n)\hat{Z}^n_{t-1} - (1-\chi_n)\hat{\mu}_{z^+,t} - \chi_n \hat{U}_{c,t} \tag{A.16b}$$

Unemployment definition:

$$un_t = \frac{L_t - N_t}{L_t} \tag{A.17a}$$

$$\breve{un}_t = \frac{n}{l}\left(\hat{l}_t - \hat{n}_t\right) \tag{A.17b}$$

Real wage markup equation:

$$\overline{\Psi}_t^W = \frac{\left(1-\tau_t^W\right)\overline{w}_t}{\zeta_t^n \frac{\nu'(n_t)}{\overline{\Omega}_t^C}} \tag{A.18a}$$

$$\hat{\Psi}_t^W = \eta\left(\hat{l}_t - \hat{n}_t\right) \tag{A.18b}$$

Definition of wage inflation:

$$\Pi_t^W = \frac{\overline{w}_t}{\overline{w}_{t-1}}\mu_{z^+,t}\Pi_t^C \tag{A.19a}$$

$$\hat{\Pi}_t^W = \triangle\hat{w}_t + \hat{\mu}_{z^+,t} + \hat{\Pi}_t^C \tag{A.19b}$$

Definition of wage inflation indexation:

$$\overline{\Pi}_t^W = (\Pi_{t-1}^W)^{\chi_w}(\Pi_t^{trend})^{1-\chi_w} \tag{A.20a}$$

$$\hat{\overline{\Pi}}_t^W = \chi_w\hat{\Pi}_{t-1}^W + (1-\chi_W)\hat{\Pi}_t^{trend} \tag{A.20b}$$

Real wage relevant to employers:

$$\overline{w}_t^e = \overline{w}_t p_t^C \tag{A.21a}$$

$$\hat{w}_t^e = \hat{w}_t + \hat{p}_t^C \tag{A.21b}$$

Modified uncovered interest rate parity equation:

$$R_t E_t\left[\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C}\right] = R_{F,t}\Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right) E_t\left[\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C} s_{t+1}\right] \tag{A.22a}$$

$$\frac{1}{R}\left(\breve{i}_t - \breve{i}_{F,t}\right) = \left(1-\widetilde{\phi}_s\right)E_t\left[\hat{s}_{t+1}\right] - \widetilde{\phi}_s\hat{s}_t - \widetilde{\phi}_a\breve{a}_t + \hat{\widetilde{\phi}}_t \tag{A.22b}$$

Aggregate consumption:

$$\bar{c}_t^{agg} = (1-s_{nr})\bar{c}_t + s_{nr}\bar{c}_t^{nr} \tag{A.23a}$$

$$\bar{c}^{agg}\hat{c}_t^{agg} = (1-s_{nr})\bar{c}\hat{c}_t + s_{nr}\bar{c}^{nr}\hat{c}_t^{nr} \tag{A.23b}$$

Non-Ricardian budget constraint:

$$(1+\tau_t^C)p_t^C\bar{c}_t^{nr} = (1-\tau_t^W)\bar{w}_t^e n_t + \left(1-\tau_t^{TR}\right)\overline{tr}_t^{nr} \tag{A.24a}$$

$$\left(1+\tau^C\right)p^C\bar{c}^{nr}\left(\hat{c}_t^{nr}+\hat{p}_t^C\right) + p^C\bar{c}^{nr}\breve{\tau}_t^C = (1-\tau^W)\bar{w}^e n\left(\hat{w}_t^e+\hat{n}_t\right) - \bar{w}^e n\breve{\tau}_t^W + (1-\tau^{TR})\breve{tr}_t^{nr} - \overline{tr}^{nr}\breve{\tau}_t^{TR} \tag{A.24b}$$

A.2 Sweden: Firm sector

A.2.1 Sweden: Intermediate good producers

Definition of composite technological growth rate:

$$\mu_{z^+,t} = \mu_{z,t}\left(\mu_{\gamma,t}\right)^{\frac{\alpha}{1-\alpha}} \tag{A.25a}$$

$$\hat{\mu}_{z^+,t} = \hat{\mu}_{z,t} + \frac{\alpha}{1-\alpha}\hat{\mu}_{\gamma,t} \tag{A.25b}$$

Real marginal cost of production for intermediate good producers equation:

$$\overline{mc}_t = \frac{\left(\left(1+\tau_t^{SSC}\right)\overline{w}_t^e\right)^{1-\alpha}(r_t^K)^\alpha}{\varepsilon_t\alpha^\alpha(1-\alpha)^{1-\alpha}\bar{\Gamma}_{G,t}} \tag{A.26a}$$

$$\hat{mc}_t = (1-\alpha)\left(\hat{w}_t^e + \frac{1}{1+\tau^{SSC}}\breve{\tau}_t^{SSC}\right) + \alpha\hat{r}_t^K - \hat{\varepsilon}_t - \hat{\Gamma}_{G,t} \tag{A.26b}$$

Simplifying expression variable Gamma:

$$\bar{\Gamma}_{G,t} = \alpha_K^{\frac{\alpha}{\upsilon_K}}\left(\frac{\bar{\bar{k}}_t^s}{\bar{\bar{k}}_t^s}\right)^{\frac{\alpha}{\upsilon_K}} \tag{A.27a}$$

$$\hat{\Gamma}_{G,t} = -\frac{\alpha}{\upsilon_K}\left(\widehat{\bar{k}^s}_t - \hat{k}_t^s\right) \tag{A.27b}$$

Real rental rate for capital services equation:

$$r_t^K = \alpha\varepsilon_t\left(\frac{\bar{\bar{k}}_t^s}{n_t}\frac{1}{\mu_{z+,t}\mu_{\gamma,t}}\right)^{\alpha-1}\overline{mc}_t\bar{\Gamma}_{G,t}^{\frac{1}{\alpha}} \tag{A.28a}$$

$$\hat{r}_t^K = \hat{\varepsilon}_t + (\alpha-1)\left(\widehat{\bar{k}}_t^s - \hat{n}_t - \hat{\mu}_{z+,t} - \hat{\mu}_{\gamma,t}\right) + \hat{mc}_t + \frac{1}{\alpha}\hat{\Gamma}_{G,t} \tag{A.28b}$$

Composite capital function:

$$\bar{\bar{k}}_t^s = \left(\alpha_K{}^{\frac{1}{\upsilon_K}}\left(\bar{k}_t^s\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}}\left(\bar{k}_{G,t}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K-1}} \tag{A.29a}$$

$$\widehat{\bar{k}}_t^s = \alpha_K{}^{\frac{1}{\upsilon_K}}\left(\frac{\bar{k}^s}{\bar{\bar{k}}^s}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\hat{k}_t^s + (1-\alpha_K)^{\frac{1}{\upsilon_K}}\left(\frac{\bar{k}_G}{\bar{\bar{k}}^s}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\hat{k}_{G,t} \tag{A.29b}$$

Public capital accumulation equation:

$$\bar{k}_{G,t+1} = (1-\delta_G)\bar{k}_{G,t}\frac{1}{\mu_{z+,t}\mu_{\gamma,t}} + \bar{I}_t^G \tag{A.30a}$$

$$\hat{k}_{G,t+1} = \frac{(1-\delta_G)}{\mu_{z+}\mu_\gamma}\left(\hat{k}_{G,t} - \hat{\mu}_{z+,t} - \hat{\mu}_{\gamma,t}\right) + \frac{\bar{I}^G}{\bar{k}_G}\hat{I}_t^G \tag{A.30b}$$

Optimal price of intermediate goods equation:[44]

$$E_t\sum_{k=0}^{\infty}(\xi)^k\left(\prod_{j=1}^{k}\beta_{t+j}^r\right)\frac{\overline{\Omega}_{t+k}^C}{\overline{\Omega}_t^C}\left(\prod_{j=1}^{k}\frac{\Pi_{t+j}}{\Pi_{t+j}^C}\right)\frac{\overline{y}_{t+k|t}}{(\lambda_{t+k}-1)}\left[\left(\prod_{j=1}^{k}\frac{\overline{\Pi}_{t+j}}{\Pi_{t+j}}\right)\frac{p_t^{opt}}{\Pi_t} - \lambda_{t+k}\overline{mc}_{t+k}\right] = 0 \tag{A.31a}$$

$$\hat{\Pi}_t = \beta E_t\left[\hat{\Pi}_{t+1} - \hat{\overline{\Pi}}_{t+1}\right] + \kappa\left(\frac{1}{\kappa}\hat{\lambda}_t + \hat{mc}_t\right) + \hat{\overline{\Pi}}_t \tag{A.31b}$$

Definition of intermediate good price inflation indexation:

$$\overline{\Pi}_t = (\Pi_{t-1})^\chi(\Pi_t^{trend})^{1-\chi} \tag{A.32a}$$

$$\hat{\overline{\Pi}}_t = \chi\hat{\Pi}_{t-1} + (1-\chi)\hat{\Pi}_t^{trend} \tag{A.32b}$$

[44] We scale the markup shock $\hat{\lambda}_t$ by $\frac{1}{\kappa}$.

A.2.2 Sweden: Consumption good producers

Relative price of consumption goods equation:

$$p_t^C = \left[\vartheta^C \left(p_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(p_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}} \tag{A.33a}$$

$$\hat{p_t}^C = \vartheta^C \left(\frac{p^{C,xe}}{p^C}\right)^{1-\nu_C} \hat{p_t}^{C,xe} + \left(1-\vartheta^C\right)\left(\frac{p^{C,e}}{p^C}\right)^{1-\nu_{c,xe}} \hat{p_t}^{C,e} \tag{A.33b}$$

Definition of consumption good price inflation:

$$\Pi_t^C = \frac{p_t^C}{p_{t-1}^C}\Pi_t \tag{A.34a}$$

$$\hat{\Pi}_t^C = \hat{p}_t^C - \hat{p}_{t-1}^C + \hat{\Pi}_t \tag{A.34b}$$

Demand for non-energy consumption goods equation:

$$\bar{c}_t^{xe} = \vartheta^C \left(\frac{p_t^{C,xe}}{p_t^C}\right)^{-\nu_C} \bar{c}_t^{agg} \tag{A.35a}$$

$$\hat{c}_t^{xe} = \nu_C\left(\hat{p}_t^C - \hat{p}_t^{C,xe}\right) + \hat{c}_t^{agg} \tag{A.35b}$$

Demand for energy consumption goods equation:

$$\bar{c}_t^{e} = \left(1-\vartheta^C\right) \left(\frac{p_t^{C,e}}{p_t^C}\right)^{-\nu_C} \bar{c}_t^{agg} \tag{A.36a}$$

$$\hat{c}_t^{e} = \nu_C\left(\hat{p}_t^C - \hat{p}_t^{C,e}\right) + \hat{c}_t^{agg} \tag{A.36b}$$

Relative price of non-energy consumption goods equation:

$$p_t^{C,xe} = \left[\vartheta^{C,xe} + \left(1-\vartheta^{C,xe}\right)\left(p_t^{M,C,xe}\right)^{1-\nu_{c,xe}}\right]^{\frac{1}{1-\nu_{c,xe}}} \tag{A.37a}$$

$$\hat{p_t}^{C,xe} = \left(1-\vartheta^{C,xe}\right)\left(\frac{p^{M,Cxe}}{p^{C,xe}}\right)^{1-\nu_{c,xe}} \hat{p_t}^{M,C,xe} \tag{A.37b}$$

Definition of non-energy consumption good price inflation:

$$\Pi_t^{C,xe} = \frac{p_t^{C,xe}}{p_{t-1}^{C,xe}}\Pi_t \tag{A.38a}$$

$$\hat{\Pi}_t^{C,xe} = \hat{p}_t^{C,xe} - \hat{p}_{t-1}^{C,xe} + \hat{\Pi}_t \tag{A.38b}$$

Relative price of energy consumption goods equation:

$$p_t^{C,e} = \left[\vartheta^{C,e} \left(p_t^{D,C,e}\right)^{1-\nu_{c,e}} + \left(1-\vartheta^{C,e}\right)\left(p_t^{M,C,e}\right)^{1-\nu_{c,e}}\right]^{\frac{1}{1-\nu_{c,e}}} \tag{A.39a}$$

$$\hat{p_t}^{C,e} = \vartheta^{C,e} \left(\frac{p^{D,C,e}}{p^{C,e}}\right)^{1-\nu_{c,e}} \hat{p_t}^{D,C,e} + \left(1-\vartheta^{C,e}\right)\left(\frac{p^{M,C,e}}{p^{C,e}}\right)^{1-\nu_{c,e}} \hat{p_t}^{M,C,e} \tag{A.39b}$$

Definition of energy consumption good price inflation:

$$\Pi_t^{C,e} = \frac{p_t^{C,e}}{p_{t-1}^{C,e}}\Pi_t \tag{A.40a}$$

$$\hat{\Pi}_t^{C,e} = \hat{p}_t^{C,e} - \hat{p}_{t-1}^{C,e} + \hat{\Pi}_t \tag{A.40b}$$

Demand for domestic energy equation:

$$\bar{d}_t^{e} = \vartheta^{C,e} \left(\frac{p_t^{D,C,e}}{p_t^{C,e}}\right)^{-\nu_{C,e}} \bar{c}_t^{e} \tag{A.41a}$$

$$\hat{d}_t^{e} = \nu_{C,e}\left(\hat{p}_t^{C,e} - \hat{p}_t^{D,C,e}\right) + \hat{c}_t^{e} \tag{A.41b}$$

Demand for imported energy equation:

$$\bar{m}_t^{C,e} = \left(1 - \vartheta^{C,e}\right)\left(\frac{p_t^{M,C,e}}{p_t^{C,e}}\right)^{-\nu_{C,e}} \bar{c}_t^{e} \tag{A.42a}$$

$$\hat{c}_t^{e} = \nu_{C,e}\left(\hat{p}_t^{C,e} - \hat{p}_t^{M,C,e}\right) + \hat{c}_t^{e} \tag{A.42b}$$

Definition of domestic energy inflation:

$$\Pi_t^{D,C,e} = \frac{p_t^{D,C,e}}{p_{t-1}^{D,C,e}}\Pi_t \tag{A.43a}$$

$$\hat{\Pi}_t^{D,C,e} = \hat{p}_t^{D,C,e} - \hat{p}_{t-1}^{D,C,e} + \hat{\Pi}_t \tag{A.43b}$$

A.2.3 Sweden: Investment good producers

Relative price of investment goods equation:

$$p_t^{I} = \left[\vartheta^{I} + \left(1 - \vartheta^{I}\right)\left(p_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}} \tag{A.44a}$$

$$\hat{p}_t^{I} = \left(1 - \vartheta^{I}\right)\left(\frac{p^{M,I}}{p^{I}}\right)^{1-\nu_I} \hat{p}_t^{M,I} \tag{A.44b}$$

Definition of investment good price inflation:

$$\Pi_t^{I} = \frac{p_t^{I}}{p_{t-1}^{I}}\Pi_t \tag{A.45a}$$

$$\hat{\Pi}_t^{I} = \hat{p}_t^{I} - \hat{p}_{t-1}^{I} + \hat{\Pi}_t \tag{A.45b}$$

A.2.4 Sweden: Export good producers

Real marginal cost of production for export good producers equation:

$$\overline{mc}_t^{X} = \left[\vartheta^{X} + \left(1 - \vartheta^{X}\right)\left(p_t^{M,X}\right)^{1-\nu_x}\right]^{\frac{1}{1-\nu_x}} \tag{A.46a}$$

$$\hat{mc}_t^{X} = \left(1 - \vartheta^{X}\right)\left(\frac{p^{M,X}}{mc^{X}}\right)^{1-\nu_x} \hat{p}_t^{M,X} \tag{A.46b}$$

Optimal price of export goods equation:[45]

$$E_t \sum_{k=0}^{\infty} (\xi_x)^k \left(\prod_{j=1}^{k} \beta_{t+j}^{r}\right) \frac{\overline{\Omega}_{t+k}^{C}}{\overline{\Omega}_t^{C}} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi_{t+j}^{C}}\right) \frac{\overline{x}_{t+k|t}}{(\lambda_{t+k}^{X} - 1)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}_{t+j}^{X} s_{t+j}}{\Pi_{t+j}}\right) p_t^{X,opt} - \lambda_{t+k}^{X}\overline{mc}_{t+k}^{X}\right] = 0 \tag{A.47a}$$

$$\hat{\Pi}_t^{X} = \beta E_t\left[\hat{\Pi}_{t+1}^{X} - \hat{\overline{\Pi}}_{t+1}^{X}\right] + \kappa_X\left(\frac{1}{\kappa_X}\hat{\lambda}_t^{X} + \hat{mc}_t^{X} - \hat{p}_t^{X}\right) + \hat{\overline{\Pi}}_t^{X} \tag{A.47b}$$

Definition of export good price inflation indexation:

$$\overline{\Pi}_t^{X} = \left(\Pi_{t-1}^{X}\right)^{\chi_x}\left(\Pi_F^{trend}\right)^{1-\chi_x} \tag{A.48a}$$

$$\hat{\overline{\Pi}}_t^{X} = \chi_x \hat{\Pi}_{t-1}^{X} + (1 - \chi_x)\hat{\Pi}_t^{trend} \tag{A.48b}$$

Definition of export good price inflation:

$$\frac{p_t^{X}}{p_{t-1}^{X}} = \frac{\Pi_t^{X} s_t}{\Pi_t} \tag{A.49a}$$

$$\hat{p}_t^{X} = \hat{p}_{t-1}^{X} + \hat{\Pi}_t^{X} - \hat{\Pi}_t + \hat{s}_t \tag{A.49b}$$

[45]We scale the markup shock $\hat{\lambda}_t^X$ by $\frac{1}{\kappa_X}$.

A.2.5 Sweden: Import good producers

Optimal price for import firms specializing in non-energy consumption goods equation:[46]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,C,xe})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^{C,xe}_{t+k}}{\overline{\Omega}^{C}_{t}} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^{C,xe}_{t+j}} \right) \frac{\overline{m}^{C,xe}_{t+k|t}}{\left(\lambda^{M,C}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,C,xe}_{t+j}}{\Pi_{t+j}} \right) p^{M,C,xe}_{t,opt} - \lambda^{M,C,xe}_{t} \overline{mc}^{M,C,xe}_{F,t+k} \right] = 0 \tag{A.50a}$$

$$\hat{\Pi}^{M,C,xe}_t = \beta E_t \left[\hat{\Pi}^{M,C,xe}_{t+1} - \hat{\overline{\Pi}}^{M,C,xe}_{t+1} \right] + \kappa_{M,C,xe} \left(\frac{1}{\kappa_{M,C,xe}} \hat{\lambda}^{M,C,xe}_t + \hat{mc}^{M,C,xe}_{F,t} - \hat{p}^{M,C,xe}_t \right) + \hat{\overline{\Pi}}^{M,C,xe}_t \tag{A.50b}$$

Definition of import price inflation indexation, import firms specializing in non-energy consumption goods:

$$\overline{\Pi}^{M,C,xe}_t = \left(\Pi^{M,C,xe}_{t-1} \right)^{\chi_{m,C,xe}} \left(\Pi^{trend}_t \right)^{1-\chi_{m,C,xe}} \tag{A.51a}$$

$$\hat{\overline{\Pi}}^{M,C,xe}_t = \chi_{m,C,xe} \hat{\Pi}^{M,C,xe}_{t-1} + (1 - \chi_{m,C,xe}) \hat{\Pi}^{trend}_t \tag{A.51b}$$

Definition of import price inflation, import firms specializing in non-energy consumption goods:

$$\frac{p^{M,C,xe}_t}{p^{M,C,xe}_{t-1}} = \frac{\Pi^{M,C,xe}_t}{\Pi_t} \tag{A.52a}$$

$$\hat{p}^{M,C,xe}_t = \hat{p}^{M,C,xe}_{t-1} + \hat{\Pi}^{M,C,xe}_t - \hat{\Pi}_t \tag{A.52b}$$

Optimal price for import firms specializing in investment goods equation:[47]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,I})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^{C}_{t+k}}{\overline{\Omega}^{C}_{t}} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^{C}_{t+j}} \right) \frac{\overline{m}^{I}_{t+k|t}}{\left(\lambda^{M,I}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,I}_{t+j}}{\Pi_{t+j}} \right) p^{M,I}_{t,opt} - \lambda^{M,I}_{t} \overline{mc}^{M,I}_{F,t+k} \right] = 0 \tag{A.53a}$$

$$\hat{\Pi}^{M,I}_t = \beta E_t \left[\hat{\Pi}^{M,I}_{t+1} - \hat{\overline{\Pi}}^{M,I}_{t+1} \right] + \kappa_{M,I} \left(\frac{1}{\kappa_{M,I}} \hat{\lambda}^{M,I}_t + \hat{mc}^{M,I}_{F,t} - \hat{p}^{M,I}_t \right) + \hat{\overline{\Pi}}^{M,I}_t \tag{A.53b}$$

Definition of import price inflation indexation, import firms specializing in investment goods:

$$\overline{\Pi}^{M,I}_t = \left(\Pi^{M,I}_{t-1} \right)^{\chi_{m,I}} \left(\Pi^{trend}_t \right)^{1-\chi_{m,I}} \tag{A.54a}$$

$$\hat{\overline{\Pi}}^{M,I}_t = \chi_{m,I} \hat{\Pi}^{M,I}_{t-1} + (1 - \chi_{m,I}) \hat{\Pi}^{trend}_t \tag{A.54b}$$

Definition of import price inflation, import firms specializing in investment goods:

$$\frac{p^{M,I}_t}{p^{M,I}_{t-1}} = \frac{\Pi^{M,I}_t}{\Pi_t} \tag{A.55a}$$

$$\hat{p}^{M,I}_t = \hat{p}^{M,I}_{t-1} + \hat{\Pi}^{M,I}_t - \hat{\Pi}_t \tag{A.55b}$$

Optimal price for import firms specializing in export goods equation:[48]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,X})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^{C}_{t+k}}{\overline{\Omega}^{C}_{t}} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^{C}_{t+j}} \right) \frac{\overline{m}^{X}_{t+k|t}}{\left(\lambda^{M,X}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,X}_{t+j}}{\Pi_{t+j}} \right) p^{M,X}_{t,opt} - \lambda^{M,X}_{t} \overline{mc}^{M,X}_{F,t+k} \right] = 0 \tag{A.56a}$$

$$\hat{\Pi}^{M,X}_t = \beta E_t \left[\hat{\Pi}^{M,X}_{t+1} - \hat{\overline{\Pi}}^{M,X}_{t+1} \right] + \kappa_{M,X} \left(\frac{1}{\kappa_{M,X}} \hat{\lambda}^{M,X}_t + \hat{mc}^{M,X}_{F,t} - \hat{p}^{M,X}_t \right) + \hat{\overline{\Pi}}^{M,X}_t \tag{A.56b}$$

Definition of import price inflation indexation, import firms specializing in export goods:

$$\overline{\Pi}^{M,X}_t = \left(\Pi^{M,X}_{t-1} \right)^{\chi_{m,X}} \left(\Pi^{trend}_t \right)^{1-\chi_{m,X}} \tag{A.57a}$$

$$\hat{\overline{\Pi}}^{M,X}_t = \chi_{m,X} \hat{\Pi}^{M,X}_{t-1} + (1 - \chi_{m,X}) \hat{\Pi}^{trend}_t \tag{A.57b}$$

[46] We scale the markup shock $\hat{\lambda}^{M,C,xe}_t$ by $\frac{1}{\kappa_{M,C,xe}}$.

[47] We scale the markup shock $\hat{\lambda}^{M,I}_t$ by $\frac{1}{\kappa_{M,I}}$.

[48] We scale the markup shock $\hat{\lambda}^{M,X}_t$ by $\frac{1}{\kappa_{M,X}}$.

Definition of import price inflation, import firms specializing in export goods:

$$\frac{p_t^{M,X}}{p_{t-1}^{M,X}} = \frac{\Pi_t^{M,X}}{\Pi_t} \tag{A.58a}$$

$$\hat{p}_t^{M,X} = \hat{p}_{t-1}^{M,X} + \hat{\Pi}_t^{M,X} - \hat{\Pi}_t \tag{A.58b}$$

Optimal price for import firms specializing in energy consumption goods equation:[49]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,C,e})^k \left(\prod_{j=1}^{k} \beta_{t+j}^r\right) \frac{\overline{\Omega}_{t+k}^C}{\overline{\Omega}_t^C} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi_{t+j}^{C,e}}\right) \frac{\overline{m}_{t+k|t}^{C,e}}{\left(\lambda_{t+k}^{M,C,e} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}_{t+j}^{M,C,e}}{\Pi_{t+j}}\right) p_{t,opt}^{M,C,e} - \lambda_t^{M,C,e} \overline{mc}_{t+k}^{M,C,e}\right] = 0 \tag{A.59a}$$

$$\hat{\Pi}_t^{M,C,e} = \beta E_t \left[\hat{\Pi}_{t+1}^{M,C,e} - \hat{\overline{\Pi}}_{t+1}^{M,C,e}\right] + \kappa_{M,C,e} \left(\frac{1}{\kappa_{M,C,e}} \hat{\lambda}_t^{M,C,e} + \hat{mc}_t^{M,C,e} - \hat{p}_t^{M,C,e}\right) + \hat{\overline{\Pi}}_t^{M,C,e} \tag{A.59b}$$

Definition of import price inflation indexation, import firms specializing in energy consumption goods:

$$\overline{\Pi}_t^{M,C,e} = \left(\Pi_{t-1}^{M,C,e}\right)^{\chi_{m,C,e}} \left(\Pi_t^{trend}\right)^{1-\chi_{m,C,e}} \tag{A.60a}$$

$$\hat{\overline{\Pi}}_t^{M,C,e} = \chi_{m,C,e} \hat{\Pi}_{t-1}^{M,C,e} + (1 - \chi_{m,C,e}) \hat{\Pi}_t^{trend} \tag{A.60b}$$

Definition of import price inflation, import firms specializing in energy consumption goods:

$$\frac{p_t^{M,C,e}}{p_{t-1}^{M,C,e}} = \frac{\Pi_t^{M,C,e}}{\Pi_t} \tag{A.61a}$$

$$\hat{p}_t^{M,C,e} = \hat{p}_{t-1}^{M,C,e} + \hat{\Pi}_t^{M,C,e} - \hat{\Pi}_t \tag{A.61b}$$

Marginal cost of energy importer:

$$\overline{mc}_t^{M,C,e} = p_{F,t}^{C,e} Q_t \frac{p_t^C}{p_{F,t}^C} \tag{A.62a}$$

$$\hat{mc}_t^{M,C,e} = \hat{p}_{F,t}^{C,e} + \hat{Q}_t + \hat{p}_t^C - \hat{p}_{F,t}^C \tag{A.62b}$$

Marginal cost of non-energy importer:

$$\overline{mc}_t^{M,xe} = Q_t \frac{p_t^C}{p_{F,t}^C} \tag{A.63a}$$

$$\hat{mc}_t^{M,xe} = \hat{Q}_t + \hat{p}_t^C - \hat{p}_{F,t}^C \tag{A.63b}$$

Definition of real exchange rate:

$$\frac{Q_t}{Q_{t-1}} = s_t \frac{\Pi_{F,t}^C}{\Pi_t^C} \tag{A.64a}$$

$$\hat{Q}_t - \hat{Q}_{t-1} = \hat{s}_t + \hat{\Pi}_{F,t}^C - \hat{\Pi}_t^C \tag{A.64b}$$

A.3 Swedish monetary policy rule

Monetary policy rule:

$$\breve{i}_t^{notional} = \rho \breve{i}_{t-1}^{notional} + (1-\rho)\left(r_\pi \hat{\Pi}_{t-1}^{a,C} + r_{un} \breve{un}_{t-1}\right) + r_{\triangle\pi}\left(\hat{\Pi}_t^C - \hat{\Pi}_{t-1}^C\right) + r_{\triangle un}\left(\breve{un}_t - \breve{un}_{t-1}\right) + \epsilon_t^i, \tag{A.65}$$

$$\hat{\Pi}_t^{a,C} = \frac{1}{4}\left(\hat{\Pi}_t^C + \hat{\Pi}_{t-1}^C + \hat{\Pi}_{t-2}^C + \hat{\Pi}_{t-3}^C\right)$$

[49]We scale the markup shock $\hat{\lambda}_t^{M,C,e}$ by $\frac{1}{\kappa_{M,C,e}}$.

Nominal interest rate with and without the zero lower bound:

$$\breve{i}_t^{ss} = \max\left\{\breve{\underline{i}}, \breve{i}_t^{notional} + \breve{i}_t^{nat}\right\} \tag{A.66}$$

Real interest rate:

$$\breve{r}_t = \breve{i}_t - \hat{\Pi}_{t+1}^c \tag{A.67}$$

Monetary policy expansion, definition:

$$\breve{i}_t = \breve{i}_t^{ss} - \breve{i}_t^{nat} \tag{A.68}$$

The neutral interest rate:

$$\breve{i}_t^{nat} = r_\mu \hat{\mu}_{z+,t} - r_\zeta \hat{\zeta}_t + \hat{z}_t^r \tag{A.69}$$

A.4 Swedish fiscal authority

Government budget constraint:

$$\tau_t^C p_t^C \bar{c}_t^{agg} + (\tau_t^{SSC} + \tau_t^W) p_t^C \bar{w}_t n_t + \bar{\Upsilon}_t^K + \bar{b}_t^n + \bar{t}_t = \left(\alpha_B + (R_{t-1}^B - 1)\right) \frac{\bar{b}_t}{\mu_{z+,t}\Pi_t} + \bar{g}_t + \tau_t^I p_t^I \bar{I}_t + \bar{I}_t^G + (1-\tau_t^{TR}) \bar{tr}_t^{agg} \tag{A.70a}$$

$$\begin{aligned} p^C \bar{c}^{agg} \breve{\tau}_t^C + \tau^C p^C \bar{c}^{agg}\left(\hat{p}_t^C + \hat{c}_t^{agg}\right) + p^C \bar{w} n \breve{\tau}_t^W + p^C \bar{w} n \breve{\tau}_t^{SSC} + (\tau^{SSC} + \tau^W) p^C \bar{w} n \left(\hat{p}_t^C + \hat{w}_t + \hat{n}_t\right) + \breve{\Upsilon}_t^K + \breve{b}_t^n + \breve{t}_t \\ = (\alpha_B + R^B - 1)\left(\frac{1}{\mu_{z+}\Pi}\breve{b}_t - \frac{\bar{b}}{\mu_{z+}\Pi}\left(\hat{\mu}_{z+,t} + \hat{\Pi}_t\right)\right) + \frac{\bar{b}}{\mu_{z+}\Pi}\breve{R}_{t-1}^B + \bar{I}^G \hat{I}_t^G + \bar{g}\hat{g}_t + (1-\tau^{TR})\breve{tr}_t^{agg} \\ - \bar{tr}^{agg}\breve{\tau}_t^{TR} + p^I \bar{I} \breve{\tau}_t^I + \tau^I p^I \bar{I}\left(\hat{p}_t^I + \hat{I}_t\right) \end{aligned} \tag{A.70b}$$

Law of motion for aggregate total government debt stock:

$$\bar{b}_{t+1} = (1-\alpha_B)\,\bar{b}_t \frac{1}{\mu_{z+,t}\Pi_t} + \bar{b}_t^n \tag{A.71a}$$

$$\breve{b}_{t+1} = \frac{1-\alpha_B}{\mu_{z+}\Pi}\breve{b}_t - \frac{(1-\alpha_B)\,\bar{b}}{\mu_{z+}\Pi}\left(\hat{\mu}_{z+,t} + \hat{\Pi}_t\right) + \breve{b}_t^n \tag{A.71b}$$

Definition of average interest rate on all outstanding government debt:

$$\left(R_t^B - 1\right)\bar{b}_{t+1} = (1-\alpha_B)\left(R_{t-1}^B - 1\right)\bar{b}_t \frac{1}{\mu_{z+,t}\Pi_t} + \left(R_t^{B,n} - 1\right)\bar{b}_t^n \tag{A.72a}$$

$$\bar{b}\breve{R}_t^B + \left(R^B - 1\right)\breve{b}_{t+1} = \frac{(1-\alpha_B)\left(R^B - 1\right)}{\mu_{z+}\Pi}\left[\frac{\bar{b}}{R^B - 1}\breve{R}_{t-1}^B + \breve{b}_t - \bar{b}\left(\hat{\mu}_{z+,t} + \hat{\Pi}_t\right)\right] + \bar{b}^n \breve{R}_t^{B,n} + \left(R^{B,n} - 1\right)\breve{b}_t^n \tag{A.72b}$$

Capital income tax revenues:

$$\bar{\Upsilon}_t^K = \frac{\bar{k}_t}{\mu_{z+,t}\mu_{\gamma,t}}\tau_t^K\left(r_t^K u_t - p_t^I a(u_t) - \iota^K \delta \frac{\mu_{\gamma,t} p_{t-1}^K}{\Pi_t}\right) \tag{A.73a}$$

$$\begin{aligned} \breve{\Upsilon}_t^K = \frac{\tau^K \bar{k}}{\mu_{z+}\mu_\gamma}\left[\left(r^K - \iota^K \delta \frac{p^K \mu_\gamma}{\Pi}\right)\left(\hat{k}_t - \hat{\mu}_{z+,t}\right) + r^K\left(\hat{r}_t^K - \hat{\mu}_{\gamma,t}\right) + \iota^K \delta \frac{p^K \mu_\gamma}{\Pi}\left(\hat{\Pi}_t - \hat{p}_{t-1}^K\right)\right] \\ + \frac{\bar{k}}{\mu_{z+}\mu_\gamma}\left(r^K - \iota^K \delta \frac{p^K \mu_\gamma}{\Pi}\right)\breve{\tau}_t{}^K \end{aligned} \tag{A.73b}$$

Aggregate transfers:

$$\overline{tr}_t^{agg} = (1 - s_{nr})\overline{tr}_t + s_{nr}\overline{tr}_t^{nr} \tag{A.74a}$$

$$\breve{tr}_t^{agg} = (1 - s_{nr})\breve{tr}_t + s_{nr}\breve{tr}_t^{nr} \tag{A.74b}$$

Transfer allocation:

$$\varpi_{dyn}\left(\overline{tr}_t - \overline{tr}\right) = (1 - \varpi_{dyn})(\overline{tr}_t^{nr} - \overline{tr}^{nr}) \tag{A.75a}$$

$$\varpi_{dyn}\breve{tr}_t = (1 - \varpi_{dyn})\breve{tr}_t^{nr} \tag{A.75b}$$

Government surplus:

$$\overline{surp}_t = \alpha_B \frac{\bar{b}_t}{\mu_{z+t}\Pi_t} - \bar{b}_t^n \tag{A.76a}$$

$$su\breve{r}p_t = \frac{\alpha_B}{\mu_{z+}\Pi}\left(\breve{b}_t - \bar{b}\left(\hat{\mu}_{z+t} + \hat{\Pi}_t\right)\right) - \breve{b}_t^n \tag{A.76b}$$

Fiscal policy rule for aggregate transfers:

$$\begin{aligned}\breve{tr}_t^{agg} =& \rho_{tr^{agg}}\breve{tr}_{t-1}^{agg} \\ &+ \bar{y}\left(\mathcal{F}_{tr,b}\left(\breve{b}_{\bar{y},t} - \breve{b}_{\bar{y},t}^{Target}\right) + \mathcal{F}_{tr,surp}\left(Sts\breve{u}rp_{\bar{y},t} - St\breve{s}urp_{\bar{y},t}^{Target}\right) + \mathcal{F}_{tr,un}\breve{un}_t\right) \\ &+ \epsilon_t^{tr^{agg}}\end{aligned} \tag{A.77}$$

Fiscal policy rule for government consumption:

$$\begin{aligned}\hat{g}_t =& \rho_g \hat{g}_{t-1} \\ &+ \mathcal{F}_{g,b}\left(\breve{b}_{\bar{y},t} - \breve{b}_{\bar{y},t}^{Target}\right) + \mathcal{F}_{g,surp}\left(St\breve{s}urp_{\bar{y},t} - St\breve{s}urp_{\bar{y},t}^{Target}\right) + \mathcal{F}_{g,y}\hat{y}_t \\ &+ \epsilon_t^g\end{aligned} \tag{A.78}$$

Fiscal policy rule for government investment:

$$\begin{aligned}\hat{I}_t^G =& \rho_{IG}\hat{I}_{t-1}^G \\ &+ \mathcal{F}_{IG,b}\left(\breve{b}_{\bar{y},t} - \breve{b}_{\bar{y},t}^{Target}\right) + \mathcal{F}_{IG,surp}\left(St\breve{s}urp_{\bar{y},t} - St\breve{s}urp_{\bar{y},t}^{Target}\right) + \mathcal{F}_{IG,y}\hat{y}_t \\ &+ \epsilon_t^{IG}\end{aligned} \tag{A.79}$$

Fiscal policy rule for consumption tax, labor tax, social security contribution, capital tax and transfer tax:

$$\begin{aligned}\breve{\tau}_t^x =& \rho_{\tau^x}\breve{\tau}_{t-1}^x \\ &+ \mathcal{F}_{\tau^x,b}\left(\breve{b}_{\bar{y},t} - \breve{b}_{\bar{y},t}^{Target}\right) + \mathcal{F}_{\tau^x,surp}\left(St\breve{s}urp_{\bar{y},t} - St\breve{s}urp_{\bar{y},t}^{Target}\right) \\ &+ \epsilon_t^{\tau^x}, \quad x \in \{C, W, SSC, K, TR\}\end{aligned} \tag{A.80}$$

Investment tax credits:

$$\tau_t^I = \rho_{\tau^I}\tau_{t-1}^I + \epsilon_t^{\tau^I} \tag{A.81}$$

Debt target equation:

$$\breve{b}_{\bar{y},t}^{Target} = \left(\rho_{1,b^T} + \rho_{2,b^T}\right)\breve{b}_{\bar{y},t-1}^{Target} - \rho_{1,b^T}\rho_{2,b^T}\left(\breve{b}_{\bar{y},t-2}^{Target}\right) + \epsilon_t^{b^{Target}} \tag{A.82}$$

Structural government surplus:

$$\overline{Stsurp}_t = \overline{Stprev}_t - \overline{Stpexp}_t - \frac{R^B_{t-1} - 1}{\Pi_t \mu_{z^+,t}} \bar{b}_t \tag{A.83a}$$

$$St\breve{s}urp_t = St\breve{p}rev_t - St\breve{p}exp_t - \left(\frac{R^B - 1}{\Pi \mu_{z^+}} \breve{b}_t + \bar{b} \left(\frac{1}{R^B - 1} \breve{R}^B_{t-1} - \hat{\mu}_{z^+,t} - \hat{\pi}_t \right) \right) \tag{A.83b}$$

Structural primary expenditures:

$$\overline{Stpexp}_t = ((\bar{tr}^{agg}_t - \bar{y}\mathcal{F}_{tr,un}\breve{un}_t)) + \left(\bar{I}^G - \mathcal{F}_{IG,y} \bar{I}^G \frac{(\bar{y}_t - \bar{y})}{\bar{y}} \right) + \left(\bar{g} - \mathcal{F}_{g,y} \bar{g} \frac{(\bar{y}_t - \bar{y})}{\bar{y}} \right) + \tau^I_t p^I \bar{I} \tag{A.84a}$$

$$St\breve{p}exp_t = \left(\breve{tr}^{agg}_t - \bar{y}\mathcal{F}_{tr,un}\breve{un}_t \right) + \bar{I}^G \left(\hat{I}^G_t - \mathcal{F}_{IG,y}\hat{y}_t \right) + \bar{g}\left(\hat{g}_t - \mathcal{F}_{g,y}\hat{y}_t \right) + p^I \bar{I} \breve{\tau}^I_t \tag{A.84b}$$

Structural primary revenues:

$$\overline{Stprev}_t = \tau^C_t p^C \bar{c}^{agg} + (\tau^{SSC}_t + \tau^W_t)\overline{w}n + \tau^K_t \frac{\overline{k}_t}{\mu_{z^+}\mu_{\gamma,}} \left(r^K_t - \iota^K \delta \frac{\mu_{\gamma,t} p^K}{\Pi} \right) + \tau^{TR}_t \left(\overline{tr}^{agg}_t - \bar{y}\mathcal{F}_{tr,un}\breve{un}_t \right) + \bar{t} \tag{A.85a}$$

$$\begin{aligned} St\breve{p}rev_t =& \breve{\tau}^C_t p^C \bar{c}^{agg} + (\breve{\tau}^{SSC}_t + \breve{\tau}^W_t)\overline{w}n + \breve{\tau}^K_t \frac{\overline{k}}{\mu_{z^+}\mu_\gamma} \left(r^K - \iota^K \delta \frac{p^K \mu_\gamma}{\Pi} \right) \\ &+ \breve{\tau}^{TR}_t (\breve{tr}^{agg}_t - \mathcal{F}_{tr,un}\bar{y}\breve{un}_t) + \breve{t}_t \end{aligned} \tag{A.85b}$$

Relation between debt target and surplus target:

$$Stsurp^{Target}_{\bar{y},t} = \left(\frac{1}{\mu_{z^+}\mathit{\Pi}} - 1 \right) b^{Target}_{\bar{y},t} \tag{A.86a}$$

$$St\breve{s}urp^{Target}_{\bar{y},t} = \left(\frac{1}{\mu_{z^+}\mathit{\Pi}} - 1 \right) \breve{b}^{Target}_{\bar{y},t} \tag{A.86b}$$

A.5 Auxiliary variables

There are some variables which do not affect the simulations of the model, but which are used for the purpose of illustration and comparison with data. These are called auxiliary variables, and are stated below.

Aggregate investment:

$$\bar{I}^{agg}_t = \bar{I}_t + \bar{I}^G_t \tag{A.87a}$$

$$\hat{I}_t = \frac{\bar{I}}{\bar{I} + \bar{I}^G} \hat{I}_t + \frac{\bar{I}^G}{\bar{I} + \bar{I}^G} \hat{I}^G_t \tag{A.87b}$$

Price of aggregate investment:

$$p^{Iagg}_t = \frac{\bar{I}_t}{\bar{I}^{agg}_t} p^I_t + \frac{\bar{I}^G_t}{\bar{I}^{agg}_t} \tag{A.88a}$$

$$\hat{p}^{Iagg}_t = \frac{\bar{I}}{\bar{I}^{agg}} \frac{p^I}{p^{Iagg}} (\hat{p}^I_t + \hat{I}_t) + \frac{\bar{I}}{\bar{I}^{agg}} \frac{1}{p^{Iagg}} \hat{I}^G_t - \hat{I}^{agg}_t \tag{A.88b}$$

Aggregate investment inflation:

$$p^{Iagg}_t = p^{Iagg}_{t-1} \frac{\Pi^{Iagg}_t}{\Pi_t} \tag{A.89a}$$

$$\hat{p}^{Iagg}_t = \hat{p}^{Iagg}_{t-1} + \hat{\Pi}^{Iagg}_t - \hat{\Pi}_t \tag{A.89b}$$

Aggregate import prices:

$$p_t^M \overline{m}_t^D = \overline{m}_t^{C,xe} p_t^{MC,xe} + \overline{m}_t^I p_t^{MI} + \overline{m}_t^X p_t^{MX} + \overline{m}_t^{C,e} p_t^{MC,e} \tag{A.90a}$$

$$\hat{p}_t^M = \frac{\overline{m}^{C,xe}}{\overline{m}^D} \hat{p}_t^{MC,xe} + \frac{\overline{m}^I}{\overline{m}^D} \hat{p}_t^{MI} + \frac{\overline{m}^X}{\overline{m}^D} \hat{p}_t^{MX} + \frac{\overline{m}^{C,e}}{\overline{m}^D} \hat{p}_t^{MC,e} \tag{A.90b}$$

Aggregate import inflation:

$$\frac{p_t^M}{p_{t-1}^M} = \frac{\Pi_t^M}{\Pi_t} \tag{A.91a}$$

$$\hat{p}_t^M = \hat{p}_{t-1}^M + \hat{\Pi}_t^M - \hat{\Pi}_t \tag{A.91b}$$

Consumption tax revenues:

$$\overline{Rev_t}^{\tau^C} = \tau_t^C p_t^C \bar{c}_t^{agg} \tag{A.92a}$$

$$\breve{Rev}_t^{\tau^C} = p^C \bar{c}^{agg} \breve{\tau}_t^C + \tau^C p^C \bar{c}^{agg} \left(\hat{p}_t^C + \hat{c}_t{}^{agg} \right) \tag{A.92b}$$

Labor tax revenues:

$$\overline{Rev_t}^{\tau^W} = \tau_t^W p_t^C \bar{w}_t n_t \tag{A.93a}$$

$$\breve{Rev}_t^{\tau^W} = p^C \bar{w} n \breve{\tau} + \tau^W p^C \bar{w} n \left(\hat{p}_t^C + \hat{w}_t + \hat{n}_t \right) \tag{A.93b}$$

Social security contribution revenues:

$$\overline{Rev_t}^{\tau^{SSC}} = \tau_t^{SSC} p_t^C \bar{w}_t n_t \tag{A.94a}$$

$$\breve{Rev}_t^{\tau^{SSC}} = p^C \bar{w} n \breve{\tau} + \tau^{SSC} p^C \bar{w} n \left(\hat{p}_t^C + \hat{w}_t + \hat{n}_t \right) \tag{A.94b}$$

Transfer tax revenues:

$$\overline{Rev_t}^{\tau^{TR}} = \tau_t^{TR} \bar{tr}_t^{agg} \tag{A.95a}$$

$$\breve{Rev}_t^{\tau^{TR}} = \tau^{TR} \breve{tr}_t^{agg} + \bar{tr}^{agg} \breve{\tau}_t^{TR} \tag{A.95b}$$

Primary revenues:

$$\overline{PRev}_t = \overline{Rev_t}^{\tau^C} + \overline{Rev_t}^{\tau^W} + \overline{Rev_t}^{\tau^{SSC}} + \overline{Rev_t}^{\tau^{TR}} + \bar{\Upsilon}_t^K \tag{A.96a}$$

$$P\breve{rev}_t = R\breve{e}v_t^{\tau^C} + R\breve{e}v_t^{\tau^W} + R\breve{e}v_t^{\tau^{SSC}} + R\breve{e}v_t^{\tau^{TR}} + \breve{\Upsilon}_t^K \tag{A.96b}$$

Investment tax credit expenditures:

$$\overline{Exp_t}^{\tau^I} = \tau_t^I p_t^I \bar{I}_t \tag{A.97a}$$

$$\breve{Exp}_t^{\tau^I} = p^I \bar{I} \breve{\tau}_t^I + \tau^I p^I \bar{I} \left(\hat{p}_t^I + \hat{I}_t \right) \tag{A.97b}$$

Primary expenditure:

$$\overline{Pexp}_t = \tau_t^I p_t^I \bar{I}_t + \overline{g}_t + \bar{I}_t^G + \bar{tr}_t^{agg} \tag{A.98a}$$

$$P\breve{e}xp_t = p^I \bar{I} \breve{\tau}_t^I + \tau^I p^I \bar{I} \left(\hat{p}_t^I + \hat{I}_t \right) + \bar{g}\hat{g}_t + \bar{I}_G \hat{I}_{G,t} + \breve{tr}_t^{agg} \tag{A.98b}$$

Primary surplus:

$$\overline{Psurp}_t = \overline{PRev}_t - \overline{Pexp}_t \tag{A.99a}$$
$$P\breve{surp}_t = P\breve{rev}_t - P\breve{exp}_t \tag{A.99b}$$

Aggregate transfers, percent of GDP:

$$tr^{agg}oy_t = \frac{\bar{tr}_t^{agg}}{\overline{y}_t^m} \tag{A.100a}$$
$$tr^{a\breve{g}g}oy_t = \frac{1}{\overline{y}}\breve{tr}_t^{agg} - \frac{\bar{tr}^{agg}}{\overline{y}}\hat{y}_t^m \tag{A.100b}$$

Government debt to GDP:

$$boy_t = \frac{\bar{b}_t}{\overline{y}_t^m} \tag{A.101a}$$
$$b\breve{o}y_t = \frac{1}{\overline{y}}\breve{b}_t - \frac{\overline{b}}{\overline{y}}\hat{y}_t^m \tag{A.101b}$$

Surplus to GDP:

$$surpoy_t = \frac{\overline{surp_t}}{\overline{y}_t^m} \tag{A.102a}$$
$$sur\breve{p}oy_t = \frac{1}{\overline{y}}s\breve{u}rp_t - \frac{\overline{surp}}{\overline{y}}\hat{y}_t^m \tag{A.102b}$$

Net exports:

$$\overline{nx}_t = \bar{x}_t - \bar{m}_t \tag{A.103a}$$
$$n\breve{x}_t = \bar{x}\hat{x}_t - \bar{m}\hat{m}_t \tag{A.103b}$$

A.6 Foreign: Household sector

Foreign consumption Euler equation:

$$\overline{\Omega}_{F,t}^C = R_{F,t}\zeta_{F,t}E_t\left[\beta_{F,t+1}^r \frac{\overline{\Omega}_{F,t+1}^C}{\mu_{z_F^+,t+1}\Pi_{F,t+1}^C}\right] \tag{A.104a}$$
$$\hat{\Omega}_{F,t}^C = E_t\left[\hat{\zeta}_{F,t} + \hat{\beta}_{F,t+1}^r + \hat{\Omega}_{F,t+1}^C + \frac{1}{R_F}\breve{i}_{F,t} - \hat{\Pi}_{F,t+1}^C - \hat{\mu}_{z_F^+,t+1}\right] \tag{A.104b}$$

Foreign marginal utility of consumption equation:

$$\overline{\Omega}_{F,t}^C = \frac{\zeta_{F,t}^c}{\left(\overline{c}_{F,t} - \rho_{h,F}\frac{1}{\mu_{z_F^+,t}}\overline{c}_{F,t-1}\right)} \tag{A.105a}$$
$$\hat{\Omega}_{F,t}^C = \hat{\zeta}_{F,t}^c\left(1 - \frac{\rho_{h,F}}{\mu_{z_F^+}}\right)^{-1}\left[-\hat{c}_{F,t} + \frac{\rho_{h,F}}{\mu_{z_F^+}}\left(\hat{c}_{F,t-1} - \hat{\mu}_{z_F^+,t}\right)\right] \tag{A.105b}$$

Foreign capital utilization decision equation:

$$r_{F,t}^K = p_{F,t}^I a'(u_{F,t}) \tag{A.106a}$$
$$\hat{r}_{F,t}^K = \hat{p}_{F,t}^I + \sigma_a\hat{u}_{F,t} \tag{A.106b}$$

Foreign household purchases of installed capital equation:

$$p_{F,t}^K = E_t\beta_{F,t+1}^r \frac{\overline{\Omega}_{F,t+1}^C}{\overline{\Omega}_{F,t}^C}\frac{\Pi_{F,t+1}}{\Pi_{F,t+1}^C}\frac{1}{\mu_{z_F^+,t+1}\mu_{\gamma,t+1}}\left[r_{F,t+1}^K u_{F,t+1} - p_{F,t+1}^I a(u_{F,t+1}) + p_{F,t+1}^K(1-\delta_F)\right] \tag{A.107a}$$

$$\left(\hat{p}^K_{F,t} - \hat{\Pi}_{F,t+1} + \hat{\mu}_{\gamma,t+1}\right) =$$

$$E_t\hat{\beta}^r_{F,t+1} + E_t\hat{\Omega}^C_{F,t+1} - \hat{\Omega}^C_{F,t} - E_t\hat{\Pi}^C_{F,t+1} - E_t\hat{\mu}_{z^+_F,t+1} + \frac{1}{H_F} r^K_F E_t \hat{r}^K_{F,t+1} + \frac{1}{H_F} p^K_F (1-\delta_F) E_t \hat{p}^K_{F,t+1} \quad \text{(A.107b)}$$

Foreign household investment decision equation:

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t} F_1(\overline{I}_{F,t}, \overline{I}_{F,t-1}, \mu_{z^+_F,t}, \mu_{\gamma,t}) + E_t\left[\beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\overline{\Omega}^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{p^K_{F,t+1}}{\mu_{z^+_F,t+1}\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(\overline{I}_{F,t+1}, \overline{I}_{F,t}, \mu_{z^+_F,t+1}\mu_{\gamma,t+1})\right] \quad \text{(A.108a)}$$

$$\frac{p^I_F}{p^K_F \Upsilon_F}\hat{p}^I_{F,t} = \hat{p}^K_{F,t} + \hat{\Upsilon}_{F,t} - S''\left(\mu_{z^+_F}\mu_\gamma\right)^2 E_t\left[\triangle\hat{I}_{F,t} + \hat{\mu}_{z^+_F,t} + \hat{\mu}_{\gamma,t} - \beta_F\triangle\hat{I}_{F,t+1} - \beta_F\hat{\mu}_{z^+_F,t+1} - \beta_F\hat{\mu}_{\gamma,t+1}\right] \quad \text{(A.108b)}$$

Foreign definition of capital services:

$$\overline{k}^s_{F,t} = u_{F,t}\bar{k}_{F,t} \quad \text{(A.109a)}$$

$$\hat{k}^s_{F,t} = \hat{u}_{F,t} + \hat{k}_{F,t} \quad \text{(A.109b)}$$

Foreign capital accumulation equation:

$$\overline{k}_{F,t+1} = (1-\delta)\overline{k}_{F,t}\frac{1}{\mu_{z^+_F,t}\mu_{\gamma,t}} + \Upsilon_{F,t}\left[1 - \widetilde{S}\left(\frac{\overline{I}_{F,t}\mu_{z^+_F}\mu_\gamma}{\overline{I}_{F,t-1}}\right)\right]\overline{I}_{F,} + \overline{\triangle}^K_{F,t} \quad \text{(A.110a)}$$

$$\hat{k}_{F,t+1} = \frac{(1-\delta_F)}{\mu_{z^+_F}\mu_\gamma}\left(\hat{k}_{F,t} - \hat{\mu}_{z^+_F,t} - \hat{\mu}_{\gamma,t}\right) + \frac{\overline{I}_F}{\overline{k}_F}\Upsilon_F\left(\hat{I}_{F,t} + \Upsilon_{F,t}\right) \quad \text{(A.110b)}$$

Foreign optimal wage setting equation:

$$E_t\sum_{k=0}^{\infty}\left(\xi^F_w\right)^k\left(\prod_{j=1}^{k}\beta^r_{F,t+j}\right)n_{F,t+k|t}\overline{\Omega}^C_{F,t+k}\left[\left(1-\tau^w_F\right)\overline{w}_{F,t+k|t} - \lambda^W_F\zeta^n_{F,t+k}\frac{\nu'(n_{F,t+k|t})}{\overline{\Omega}^C_{F,t+k}}\right] = 0 \quad \text{(A.111a)}$$

$$\triangle\hat{w}_{F,t} = \beta_F E_t\left[\triangle\hat{w}_{F,t+1}\right] - \kappa_{F,W}\hat{\Psi}^W_{F,t} + \hat{\overline{\Pi}}^W_{F,t} - \hat{\mu}_{z^+_F,t} - \hat{\Pi}^C_{F,t} - \beta_F E_t[\hat{\overline{\Pi}}^W_{F,t+1} - \hat{\mu}_{z^+_F,t+1} - \hat{\Pi}^C_{F,t+1}] \quad \text{(A.111b)}$$

Foreign real wage markup equation:

$$\overline{\Psi}^W_{F,t} = \frac{\left(1-\tau^w_F\right)\overline{w}_{F,t}}{\zeta^n_{F,t}\frac{\nu'_F(n_{F,t})}{\overline{\Omega}^C_{F,t}}} \quad \text{(A.112a)}$$

$$\hat{\Psi}^W_{F,t} = \hat{w}_{F,t} - \hat{\zeta}^n_{F,t} - \eta_F\hat{n}_{F,t} + \hat{\Omega}^C_{F,t} \quad \text{(A.112b)}$$

Definition of Foreign wage inflation:

$$\Pi^W_{F,t} = \frac{\overline{w}_{F,t}}{\overline{w}_{F,t-1}}\mu_{z^+_F,t}\Pi^C_{F,t} \quad \text{(A.113a)}$$

$$\hat{\Pi}^W_{F,t} = \triangle\hat{w}_{F,t} + \hat{\mu}_{z^+_F,t} + \hat{\Pi}^C_{F,t} \quad \text{(A.113b)}$$

Definition of Foreign wage inflation indexation:

$$\overline{\Pi}^W_{F,t} = (\Pi^W_{F,t-1})^{\chi_{F,w}}(\Pi^{trend}_{F,t})^{1-\chi_{F,w}} \quad \text{(A.114a)}$$

$$\hat{\overline{\Pi}}^W_{F,t} = \chi_{F,w}\hat{\Pi}^W_{F,t-1} + \left(1-\chi_{F,w}\right)\hat{\Pi}^{trend}_{F,t} \quad \text{(A.114b)}$$

Real wage relevant to Foreign employers:

$$\overline{w}^e_{F,t} = \overline{w}_{F,t}p^C_{F,t} \quad \text{(A.115a)}$$

$$\hat{w}^e_{F,t} = \hat{w}_{F,t} + \hat{p}^C_{F,t} \quad \text{(A.115b)}$$

A.7 Foreign: Firm sector

A.7.1 Foreign: Intermediate good producers

Definition of Foreign composite technological growth rate

$$\mu_{z_F^+,t} = \mu_{z,t} \left(\mu_{\gamma,t}\right)^{\frac{\alpha_F}{1-\alpha_F}} \tag{A.116a}$$

$$\hat{\mu}_{z_F^+,t} = \hat{\mu}_{z,t} + \frac{\alpha_F}{1-\alpha_F}\hat{\mu}_{\gamma,t} \tag{A.116b}$$

Real marginal cost of production for Foreign intermediate good producers equation:

$$\overline{mc}_{F,t} = \frac{\left(\overline{w}^e_{F,t}\right)^{1-\alpha_F} \left(r^K_{F,t}\right)^{\alpha_F}}{\varepsilon_{F,t}\alpha_F^{\alpha_F}\left(1-\alpha_F\right)^{1-\alpha_F}} \tag{A.117a}$$

$$\hat{mc}_{F,t} = \left(1-\alpha_F\right)\hat{w}^e_{F,t} + \alpha_F \hat{r}^K_{F,t} - \hat{\varepsilon}_{F,t} \tag{A.117b}$$

Real rental rate for capital services equation:

$$r^K_{F,t} = \alpha_F \varepsilon_{F,t} \left(\frac{\bar{k}^s_{F,t}}{n_{F,t}} \frac{1}{\mu_{z_F^+,t}\mu_{\gamma,t}}\right)^{\alpha_F - 1} \overline{mc}_{F,t} \tag{A.118a}$$

$$\hat{r}^K_{F,t} = \hat{\varepsilon}_{F,t} + \left(\alpha_F - 1\right)\left(\hat{k}^s_{F,t} - \hat{n}_{F,t} - \hat{\mu}_{z_F^+,t} - \hat{\mu}_{\gamma,t}\right) + \hat{mc}_{F,t} \tag{A.118b}$$

Optimal price of Foreign intermediate goods equation:[50]

$$E_t \sum_{k=0}^{\infty} \left(\xi^F\right)^k \left(\prod_{j=1}^{k} \beta^r_{F,t+j}\right) \frac{\overline{\Omega}^C_{F,t+k}}{\overline{\Omega}^C_{F,t}} \left(\prod_{j=1}^{k} \frac{\Pi_{F,t+j}}{\Pi^C_{F,t+j}}\right) \frac{\overline{y}_{F,t+k|t}}{\left(\lambda_{F,t+k} - 1\right)}$$
$$\left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}_{F,t+j}}{\Pi_{F,t+j}}\right) \frac{p^{opt}_{F,t}}{\Pi_{F,t}} - \lambda_{F,t+k}\overline{mc}_{F,t+k}\right] = 0 \tag{A.119a}$$

$$\hat{\Pi}_{F,t} = \beta_F E_t \left[\hat{\Pi}_{F,t+1} - \hat{\overline{\Pi}}_{F,t+1}\right] + \kappa_F \left(\frac{1}{\kappa_F}\hat{\lambda}_{F,t} + \hat{mc}_{F,t}\right) + \hat{\overline{\Pi}}_{F,t} \tag{A.119b}$$

Definition of Foreign intermediate good price inflation indexation:

$$\overline{\Pi}_{F,t} = \left(\Pi_{F,t-1}\right)^{\chi_F} \left(\Pi^{trend}_{F,t}\right)^{1-\chi_F} \tag{A.120a}$$

$$\hat{\overline{\Pi}}_{F,t} = \chi_F \hat{\Pi}_{F,t-1} + \left(1-\chi_F\right)\hat{\Pi}^{trend}_{F,t} \tag{A.120b}$$

A.7.2 Foreign: Consumption good producers

Relative price of Foreign consumption goods equation:

$$p^C_{F,t} = \left[\vartheta^C_F \left(p^{C,xe}_{F,t}\right)^{1-\nu_{F,C}} + \left(1-\vartheta^C_F\right)\left(p^{C,e}_{F,t}\right)^{1-\nu_{F,C}}\right]^{\frac{1}{1-\nu_{F,C}}} \tag{A.121a}$$

$$\hat{p}^C_{F,t} = \vartheta^C_F \left(\frac{p^{C,xe}_F}{p^C_F}\right)^{1-\nu_{F,C}} \hat{p}^{C,xe}_{F,t} + \left(1-\vartheta^C_F\right)\left(\frac{p^{C,e}_F}{p^C_F}\right)^{1-\nu_{F,C}} \hat{p}^{C,e}_{F,t} \tag{A.121b}$$

Definition of Foreign consumption good price inflation:

$$\Pi^C_{F,t} = \frac{p^C_{F,t}}{p^C_{F,t-1}}\Pi_{F,t} \tag{A.122a}$$

$$\hat{\Pi}^C_{F,t} = \hat{p}^C_{F,t} - \hat{p}^C_{F,t-1} + \hat{\Pi}_{F,t} \tag{A.122b}$$

[50]We scale the markup shock $\hat{\lambda}_{F,t}$ by $\frac{1}{\kappa_F}$.

Demand for non-energy consumption goods equation:

$$\bar{c}^{xe}_{F,t} = \vartheta^C_F \left(\frac{p^{C,xe}_{F,t}}{p^C_{F,t}} \right)^{-\nu_{F,C}} \bar{c}_{F,t} \tag{A.123a}$$

$$\hat{c}^{xe}_{F,t} = \nu_{F,C} \left(\hat{p}^C_{F,t} - \hat{p}^{C,xe}_{F,t} \right) + \hat{c}_{F,t} \tag{A.123b}$$

Demand for energy consumption goods equation:

$$\bar{c}^{e}_{F,t} = \left(1 - \vartheta^C_F \right) \left(\frac{p^{C,e}_{F,t}}{p^C_{F,t}} \right)^{-\nu_{F,C}} \bar{c}_{F,t} \tag{A.124a}$$

$$\hat{c}^{e}_{F,t} = \nu_{F,C} \left(\hat{p}^C_{F,t} - \hat{p}^{C,e}_{F,t} \right) + \hat{c}_{F,t} \tag{A.124b}$$

Relative price of non-energy consumption good:

$$p^{C,xe}_{F,t} = 1 \tag{A.125a}$$

$$\hat{p}^{C,xe}_{F,t} = 0 \tag{A.125b}$$

Definition of Foreign non-energy consumption good price inflation:

$$\Pi^{C,xe}_{F,t} = \Pi_{F,t} \tag{A.126a}$$

$$\hat{\Pi}^{C,xe}_{F,t} = \hat{\Pi}_{F,t} \tag{A.126b}$$

Definition of Foreign energy consumption good price inflation:

$$\Pi^{C,e}_{F,t} = \frac{p^{C,e}_{F,t}}{p^{C,e}_{F,t-1}} \Pi_{F,t} \tag{A.127a}$$

$$\hat{\Pi}^{C,e}_{F,t} = \hat{p}^{C,e}_{F,t} - \hat{p}^{C,e}_{F,t-1} + \hat{\Pi}_{F,t} \tag{A.127b}$$

A.7.3 Foreign: Investment good producers

Relative price of Foreign investment:

$$p^{I}_{F,t} = 1 \tag{A.128a}$$

$$\hat{p}^{I}_{F,t} = 0 \tag{A.128b}$$

Foreign investment inflation:

$$\Pi^{I}_{F,t} = \frac{p^{I}_{F,t}}{p^{I}_{F,t-1}} \Pi_{F,t} \tag{A.129a}$$

$$\hat{\Pi}^{I}_{F,t} = \hat{p}^{I}_{F,t} - \hat{p}^{I}_{F,t-1} + \hat{\Pi}^{I}_{F,t-1} \tag{A.129b}$$

A.7.4 Price of Swedish exports in terms of Foreign intermediate goods

Definition of relative price of Swedish export goods in terms of Foreign intermediate goods:

$$\widetilde{p}^{X}_{t} = \frac{p^X_t \, p^C_{F,t}}{Q_t \, p^C_t} \tag{A.130a}$$

$$\hat{\widetilde{p}}^{X}_{t} = \hat{p}^X_t + \hat{p}^C_{F,t} - \hat{Q}_t - \hat{p}^C_t \tag{A.130b}$$

A.8 Foreign monetary policy rule

Foreign monetary policy rule:

$$\breve{i}_{F,t}^{notational} = \rho_F \breve{i}_{F,t-1}^{notational} + (1-\rho_F)\left(r_{F,\pi}\hat{\Pi}_{F,t-1}^{a,C} + r_{F,y}\hat{y}_{F,t-1}\right) + r_{F,\triangle\pi}\left(\hat{\Pi}_{F,t}^{C} - \hat{\Pi}_{F,t-1}^{C}\right) + r_{F,\triangle y}\left(\hat{y}_{F,t} - \hat{y}_{F,t-1}\right) + \epsilon_t^{i_F}, \tag{A.131}$$

$$\hat{\Pi}_{F,t}^{a,C} = \frac{1}{4}\left(\hat{\Pi}_{F,t}^{C} + \hat{\Pi}_{F,t-1}^{C} + \hat{\Pi}_{F,t-2}^{C} + \hat{\Pi}_{F,t-3}^{C}\right)$$

Foreign nominal interest rate with and without the zero lower bound:

$$\breve{i}_{F,t}^{ss} = max(\underline{i_F}, \breve{i}_{F,t}^{notional} + \breve{i}_{F,t}^{nat}) \tag{A.132}$$

Definition of monetary policy expansion

$$\breve{i}_{F,t} = \breve{i}_{F,t}^{ss} - \breve{i}_{F,t}^{nat} \tag{A.133}$$

Foreign nominal interest rate with and without the zero lower bound:

$$\breve{i}_{F,t}^{nat} = r_{F,\mu}\hat{\mu}_{z_F^+,t} - r_{F,\zeta}\hat{\zeta}_{F,t} + \hat{z}_t^r \tag{A.134}$$

Foreign real interest rate:

$$\breve{r}_{F,t} = \breve{i}_t^F - \hat{\Pi}_{F,t+1}^c \tag{A.135}$$

A.9 Market clearing

A.9.1 Swedish aggregate resource constraint

$$\bar{y}_t = \vartheta^{C,xe}\left(p_t^{C,xe}\right)^{\nu_{c,xe}}\bar{c}_t^{xe} + \bar{d}_t^{C,e} + \vartheta^I\left(p_t^I\right)^{\nu_I}\left[\bar{I}_t + a(u_t)\frac{\bar{k}_t}{\mu_{z_F^+,t}\mu_{\gamma,t}}\right]$$
$$+ \vartheta^X\left(\overline{mc}_t^X\right)^{\nu_x}\left[\bar{x}_t \overleftrightarrow{P}_t^X + \phi^X\right] + \bar{g}_t + \bar{I}_t^G \tag{A.136a}$$

$$\hat{y}_t = \vartheta^{C,xe}(p^{C,xe})^{\nu_c}\frac{\bar{c}^{xe}}{\bar{y}}\left(\nu_{c,xe}\hat{p}_t^{C,xe} + \hat{c}_t^{xe}\right) + \frac{\bar{d}^{C,e}}{\bar{y}}\hat{d}_t^e + \vartheta^I(p^I)^{\nu_I}\frac{\bar{I}}{\bar{y}}\left(\nu_I\hat{p}_t^I + \hat{I}_t + \frac{a'\bar{k}}{\mu_{z_F^+}\mu_\gamma\bar{I}}\hat{u}_t\right)$$
$$+ \vartheta^X\left(\overline{mc}^X\right)^{\nu_x}\frac{(\bar{x}+\phi^X)}{\bar{y}}\left(\nu_x\hat{mc}_t^X + \frac{\bar{x}}{(\bar{x}+\phi^X)}\hat{x}_t\right) + \frac{\bar{g}}{\bar{y}}\hat{g}_t + \frac{\bar{I}^G}{\bar{y}}\hat{I}_t^G \tag{A.136b}$$

A.9.2 Foreign aggregate resource constraint

$$\bar{y}_{F,t} = \bar{c}_{F,t}^{xe} + \bar{c}_{F,t}^{e} + \bar{I}_{F,t} + a(u_{F,t})\bar{k}_{F,t}\frac{1}{\mu_{z_F^+,t}\mu_{\gamma,t}} + \bar{g}_t \tag{A.137a}$$

$$\hat{y}_{F,t} = \frac{\bar{c}^{xe}}{\bar{y}_F}\hat{c}_t^{xe} + \frac{\bar{c}^e}{\bar{y}_F}\hat{c}_t^e + \frac{\bar{I}_F}{\bar{y}_F}\left(\hat{I}_{F,t} + \frac{a'\bar{k}_F}{\mu_{z_F^+}\mu_\gamma\bar{I}_F}\hat{u}_{F,t}\right) + \frac{\bar{g}_F}{\bar{y}_F}\hat{g}_{F,t} \tag{A.137b}$$

A.9.3 Balance of payments

$$\bar{a}_t = p_t^X\bar{x}_t - \bar{mc}_t^{M,xe}\overline{m}_t^{xe} - \bar{mc}_t^{M,C,e}\overline{m}_t^e + \Phi\left(\bar{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right)R_{F,t-1}\zeta_{t-1}\,s_t\,\bar{a}_{t-1}\frac{1}{\mu_{z_F^+,t}\Pi_t} \tag{A.138a}$$

$$\breve{a}_t = p^X\bar{x}\left(\hat{p}_t^X + \hat{x}_t\right) - \bar{mc}^{M,xe}\overline{m}^{xe}\left(\hat{mc}_t^{M,xe} + \hat{m}_t^{xe}\right) - \bar{mc}^{M,C,e}\overline{m}^e\left(\hat{mc}_t^{M,C,e} + \hat{m}_t^e\right)$$
$$+ \frac{\bar{a}}{\beta}\left[-\widetilde{\phi}_a\breve{a}_{t-1} - \widetilde{\phi}_s(\hat{s}_t + \hat{s}_{t-1}) + \hat{\widetilde{\phi}}_{t-1} + \frac{1}{R_F}\breve{i}_{F,t-1} + \hat{\zeta}_{t-1} + \hat{s}_t - \hat{\mu}_{z_F^+,t} - \hat{\Pi}_t\right] + \frac{1}{\beta}\breve{a}_{t-1} \tag{A.138b}$$

A.9.4 Swedish exports

$$\bar{x}_t = \left(1-\vartheta_F^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe} + \left(1-\vartheta_F^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \bar{I}_{F,t} \tag{A.139a}$$

$$\hat{x}_t = -\nu_F \hat{\widetilde{p}}_t^X + \omega_C^X \hat{c}_{F,t}^{xe} + \left(1-\omega_C^X\right)\hat{I}_{F,t} \tag{A.139b}$$

A.9.5 Swedish imports for non-energy consumption

$$\overleftrightarrow{P}_t^{M,C,xe}\overline{m}_t^{C,xe} = \left(1-\vartheta^{C,xe}\right)\left[\vartheta^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1}+1-\vartheta^{C,xe}\right]^{\frac{\nu_{C,xe}}{1-\nu_{C,xe}}}\overline{c}_t^{xe} \tag{A.140a}$$

$$\hat{m}_t^{C,xe} = (1-\vartheta^{C,xe})\left(\frac{p^{C,xe}}{p^{M,C,xe}}\right)^{\nu_{C,xe}} \frac{\overline{c}^{xe}}{\overline{m}^{C,xe}}\left[\hat{c}_t^{xe} - \nu_{C,xe}\vartheta^{C,xe}\left(p^{C,xe}\right)^{\nu_{C,xe}-1}\hat{p}_t^{M,C,xe}\right] \tag{A.140b}$$

A.9.6 Swedish imports for investment

$$\overleftrightarrow{P}_t^{M,I}\overline{m}_t^I = (1-\vartheta^I)\left[\vartheta^I\left(p_t^{M,I}\right)^{\nu_I-1}+1-\vartheta^I\right]^{\frac{\nu_I}{1-\nu_I}}\left[\overline{I}_t + a(u_t)\overline{k}_t\frac{1}{\mu_{z^+,t},\mu_{\gamma,t}}\right] \tag{A.141a}$$

$$\hat{m}_t^I = (1-\vartheta^I)\left(\frac{p^I}{p^{M,I}}\right)^{\nu_I}\frac{\overline{I}}{\overline{m}^I}\left[\hat{I}_t + \left(\frac{a'}{\mu_{z^+}\mu_\gamma-1+\delta}\right)\hat{u}_t - \nu_I\vartheta^I\left(p^I\right)^{\nu_I-1}\hat{p}_t^{M,I}\right] \tag{A.141b}$$

A.9.7 Swedish imports for export

$$\overleftrightarrow{P}_t^{M,X}\overline{m}_t^X = \left(1-\vartheta^X\right)\left[\vartheta^X\left(p_t^{M,X}\right)^{\nu_x-1}+1-\vartheta^X\right]^{\frac{\nu_x}{1-\nu_x}}\left[\overline{x}_t\overleftrightarrow{P}_t^X+\phi^X\right] \tag{A.142a}$$

$$\hat{m}_t^X = (1-\vartheta^X)\left(\frac{\overline{mc}^X}{p^{M,X}}\right)^{\nu_x}\frac{\overline{x}}{\overline{m}^X}\left[\hat{x}_t - \nu_x\vartheta^X\lambda^X\left(\overline{mc}^X\right)^{\nu_x-1}\hat{p}_t^{M,X}\right] \tag{A.142b}$$

A.9.8 Imports of non-energy goods including fixed costs

$$\overline{m}_t^{xe} = \overleftrightarrow{P}_t^{M,C,xe}\overline{m}_t^{C,xe} + \overleftrightarrow{P}_t^{M,I}\overline{m}_t^I + \overleftrightarrow{P}_t^{M,X}\overline{m}_t^X + \phi^{M,C,xe} + \phi^{M,I} + \phi^{M,X} \tag{A.143a}$$

$$\hat{m}_t^{xe} = \frac{\overline{m}^{C,xe}}{\overline{m}^{xe}}\hat{m}_t^{C,xe} + \frac{\overline{m}^I}{\overline{m}^{xe}}\hat{m}_t^I + \frac{\overline{m}^X}{\overline{m}^{xe}}\hat{m}_t^X \tag{A.143b}$$

A.9.9 Imports of non-energy goods excluding fixed costs

$$\overline{m}_t^{D,xe} = \overleftrightarrow{P}_t^{M,C,xe}\overline{m}_t^{C,xe} + \overleftrightarrow{P}_t^{M,I}\overline{m}_t^I + \overleftrightarrow{P}_t^{M,X}\overline{m}_t^X \tag{A.144a}$$

$$\hat{m}_t^{D,xe} = \frac{\overline{m}^{C,xe}}{\overline{m}^{D,xe}}\hat{m}_t^{C,xe} + \frac{\overline{m}^I}{\overline{m}^{D,xe}}\hat{m}_t^I + \frac{\overline{m}^X}{\overline{m}^{D,xe}}\hat{m}_t^X \tag{A.144b}$$

A.9.10 Imports of energy goods including fixed cost

$$\overline{m}_t^e = \overleftrightarrow{P}_t^{M,C,e}\overline{m}_t^{C,e} + \phi^{M,C,e} \tag{A.145a}$$

$$\hat{m}_t^e = \frac{\overline{m}^{C,e}}{\overline{m}^e}\hat{m}_t^{C,e} \tag{A.145b}$$

A.9.11 Aggregate imports excluding fixed costs

$$\overline{m}_t^D = \overline{m}_t^{D,xe} + \overleftrightarrow{P}_t^{M,C,e}\overline{m}_t^{C,e} \tag{A.146a}$$

$$\hat{m}_t^D = \frac{\overline{m}^{D,xe}}{\overline{m}^D}\hat{m}_t^{D,xe} + \frac{\overline{m}^{C,e}}{\overline{m}^D}\hat{m}_t^{C,e} \tag{A.146b}$$

A.9.12 Aggregate imports including fixed costs

$$\overline{m}_t = \overline{m}_t^{xe} + \overleftrightarrow{P}_t^{M,C,e}\overline{m}_t^{C,e} + \phi^{M,C,e} \tag{A.147a}$$

$$\hat{m}_t = \frac{\overline{m}^{xe}}{\overline{m}}\hat{m}_t^{xe} + \frac{\overline{m}^{C,e}}{\overline{m}}\hat{m}_t^{C,e} \tag{A.147b}$$

A.9.13 Swedish aggregate output

$$\overline{y}_t \overleftrightarrow{P}_t = \left(\varepsilon_t \left[\frac{\bar{\tilde{k}}_t^s}{\mu_{z+,t}\mu_{\gamma,t}}\right]^\alpha n_t^{1-\alpha}\right) - \phi \tag{A.148a}$$

$$\hat{y}_t = \frac{\lambda}{F}\left(\hat{\varepsilon}_t + \alpha\left(\hat{\tilde{k}}_t^s - \hat{\mu}_{z+,t} - \hat{\mu}_{\gamma,t}\right) + (1-\alpha)\hat{n}_t\right) \tag{A.148b}$$

A.9.14 Measured Swedish aggregate output

$$\overline{y}_t^m = \overline{y}_t - \vartheta^I (p_t^I)^{\nu_I} a(u_t)\frac{\overline{k}_t}{\mu_{z+,t}\mu_{\gamma,t}} \tag{A.149a}$$

$$\hat{y}_t^m = \hat{y}_t - \frac{\vartheta^I (p^I)^{\nu_I}}{\overline{y}}\left(\frac{r^K}{p^I}\frac{\overline{k}}{\mu_{z+}\mu_\gamma}\right)\hat{u}_t \tag{A.149b}$$

A.9.15 Foreign aggregate output

$$\overline{y}_{F,t} \overleftrightarrow{P}_{F,t} = \left(\varepsilon_{F,t}\left[\frac{\bar{\tilde{k}}_{F,t}^s}{\mu_{z_F^+,t}\mu_{\gamma,t}}\right]^{\alpha_F} n_{F,t}^{1-\alpha_F}\right) - \phi_F \tag{A.150a}$$

$$\hat{y}_t = \lambda_F\left(\hat{\varepsilon}_t + \alpha_F\left(\hat{\tilde{k}}_{F,t}^s - \hat{\mu}_{z_F^+,t} - \hat{\mu}_{\gamma,t}\right) + (1-\alpha_F)\hat{n}_{F,t}\right) \tag{A.150b}$$

A.9.16 Measured Foreign aggregate output

$$\overline{y}_{F,t}^m = \overline{y}_{F,t} - a(u_{F,t})\frac{\overline{k}_{F,t}}{\mu_{z_F^+,t}\mu_{\gamma,t}} \tag{A.151a}$$

$$\hat{y}_{F,t}^m = \hat{y}_{F,t} - \frac{1}{\overline{y}_F}\left(\frac{r_F^K}{p_F^I}\frac{\bar{k}_F}{\mu_{z_F^+}\mu_\gamma}\right)\hat{u}_{F,t} \tag{A.151b}$$

A.10 Stochastic exogenous shocks

A.10.1 Global exogenous shocks

Labor augmenting technology shock:

$$\hat{\mu}_{z,t} = \rho_{\mu_z} \hat{\mu}_{z,t-1} + \epsilon_{\mu_z,t} \tag{A.152}$$

Investment-specific technology shock:

$$\hat{\mu}_{\gamma,t} = \rho_{\mu_\gamma} \hat{\mu}_{\gamma,t-1} + \epsilon_{\mu_\gamma,t} \tag{A.153}$$

Neutral rate shock:

$$\hat{z}_t^R = \rho_{z^R} \hat{z}_{t-1}^R + \epsilon_{z^R,t} + \theta_{z^R} \epsilon_{z^R,t-1} \tag{A.154}$$

A.10.2 Swedish exogenous shocks

Discount factor shock:

$$\hat{\beta}_t^r = \rho_\beta \hat{\beta}_{t-1}^r + \epsilon_t^\beta \tag{A.155}$$

Monetary policy shock

$$\epsilon_{i,t} \tag{A.156}$$

Private bond risk premium shock:

$$\hat{\zeta}_t = corr_\zeta \hat{\zeta}_{F,t} + \rho_\zeta \hat{\zeta}_{t-1} + \epsilon_t^\zeta \tag{A.157}$$

Consumption preference shock:

$$\hat{\zeta}_t^c = corr_{\zeta^c} \hat{\zeta}_{F,t}^c + \rho_{\zeta^c} \hat{\zeta}_{t-1}^c + \epsilon_t^{\zeta^c} \tag{A.158}$$

Exchange rate shock (external risk premium shock):

$$\hat{\tilde{\phi}}_t = \rho_{\tilde{\phi}} \hat{\tilde{\phi}}_{t-1} + \epsilon_t^{\tilde{\phi}} \tag{A.159}$$

Labor disutility shock:

$$\hat{\zeta}_t^n = \rho_{\zeta^n} \hat{\zeta}_{t-1}^n + \epsilon_t^{\zeta^n} \tag{A.160}$$

Wage markup shock:

$$\hat{\lambda}_t^W = \rho_{\lambda^W} \hat{\lambda}_{t-1}^W + \epsilon_t^{\lambda^W} \tag{A.161}$$

Productivity shock (stationary technology shock):

$$\hat{\varepsilon}_t = \rho_\varepsilon \hat{\varepsilon}_{t-1} + \epsilon_t \tag{A.162}$$

Stationary investment-specific shock:

$$\hat{\Upsilon}_t = corr_\Upsilon \hat{\Upsilon}_{F,t} + \rho_\Upsilon \hat{\Upsilon}_{t-1} + \epsilon_t^\Upsilon \tag{A.163}$$

Intermediate good price markup shock:

$$\hat{\lambda}_t = \rho_\lambda \hat{\lambda}_{t-1} + \epsilon_t^\lambda \tag{A.164}$$

Export price markup shock:

$$\hat{\lambda}_t^X = \rho_{\lambda^X} \hat{\lambda}_{t-1}^X + \epsilon_t^{\lambda^X} \tag{A.165}$$

Markup shock to import firms specializing in non-energy consumption goods:

$$\hat{\lambda}_t^{M,C,xe} = \rho_{\lambda^{M,C,xe}} \hat{\lambda}_{t-1}^{M,C,xe} + \epsilon_t^{\lambda^{M,C,xe}} \tag{A.166}$$

Markup shock to import firms specializing in investment goods:

$$\hat{\lambda}_t^{M,I} = \rho_{\lambda^{M,I}} \hat{\lambda}_{t-1}^{M,I} + \epsilon_t^{\lambda^{M,I}} \tag{A.167}$$

Markup shock to import firms specializing in export goods:

$$\hat{\lambda}_t^{M,X} = \rho_{\lambda^{M,X}} \hat{\lambda}_{t-1}^{M,X} + \epsilon_t^{\lambda^{M,X}} \tag{A.168}$$

Markup shock to import firms specializing in energy consumption goods:

$$\hat{\lambda}_t^{M,C,e} = \rho_{\lambda^{M,C,xe}} \hat{\lambda}_{t-1}^{M,C,e} + \epsilon_t^{\lambda^{M,C,e}} \tag{A.169}$$

Domestic energy price shock:

$$\hat{p}_t^{D,C,e} = \rho_{p^{D,C,e}} \hat{p}_{t-1}^{D,C,e} + \epsilon_t^{p^{D,C,e}} \tag{A.170}$$

Inflation trend shock:

$$\hat{\Pi}_t^{trend} = \rho_{\Pi^{trend}} \hat{\Pi}_{t-1}^{trend} + \epsilon_t^{\Pi^{trend}} \tag{A.171}$$

A.10.3 Foreign exogenous shocks

Discount factor shock:

$$\hat{\beta}_{F,t}^r = \rho_{\beta_F} \hat{\beta}_{F,t-1}^r + \epsilon_{F,t}^{\beta} \tag{A.172}$$

Monetary policy shock

$$\epsilon_{i,t}^F \tag{A.173}$$

Private bond risk premium shock:

$$\hat{\zeta}_{F,t} = \rho_{\zeta_F}\, \hat{\zeta}_{F,t-1} + \epsilon_{F,t}^{\zeta} \tag{A.174}$$

Consumption preference shock:

$$\hat{\zeta}_{F,t}^c = corr_{\zeta_F^c,\Upsilon_F}\, \hat{\Upsilon}_{F,t} + \rho_{\zeta_F^c}\, \hat{\zeta}_{F,t-1}^c + \epsilon_{F,t}^{\zeta^c} \tag{A.175}$$

Labor disutility shock:

$$\hat{\zeta}_{F,t}^n = \rho_{\zeta_F^n}\, \hat{\zeta}_{F,t-1}^n + \epsilon_{F,t}^{\zeta^n} \tag{A.176}$$

Productivity shock (stationary technology shock):

$$\hat{\varepsilon}_{F,t} = \rho_{\varepsilon_F}\, \hat{\varepsilon}_{F,t-1} + \epsilon_{F,t} \tag{A.177}$$

Stationary investment-specific shock:

$$\hat{\Upsilon}_{F,t} = \rho_{\Upsilon_F}\, \hat{\Upsilon}_{F,t-1} + \epsilon_{F,t}^{\Upsilon} \tag{A.178}$$

Intermediate good price markup shock:

$$\hat{\lambda}_{F,t} = \rho_{\lambda_F} \hat{\lambda}_{F,t-1} + \epsilon_{F,t}^{\lambda} \tag{A.179}$$

Foreign domestic energy price shock:

$$\hat{p}_{F,t}^{C,e} = \rho_{p_F^{D,C,e}} \hat{p}_{F,t-1}^{D,C,e} + \epsilon_{F,t}^{p^{D,C,e}} \tag{A.180}$$

Foreign inflation trend shock:

$$\hat{\Pi}_{F,t}^{C,trend} = \rho_{\Pi_F^{C,trend}} \hat{\Pi}_{F,t-1}^{C,trend} + \epsilon_t^{\Pi_F^{C,trend}} \tag{A.181}$$

Foreign government consumption shock:

$$\hat{g}_{F,t} = \rho_{g_F} \hat{g}_{F,t-1} + \epsilon_t^{g_F} \tag{A.182}$$

B Appendix: Steady state

B.1 The Swedish economy

B.1.1 Sweden: Household sector

Consumption Euler equation:

$$R = \frac{\mu_{z+}\Pi^C}{\beta} \tag{B.1}$$

Definition of nominal gross interest rate on private bonds:

$$R = 1 + i \tag{B.2}$$

Lagrange multiplier, Marginal utility of consumption equation:

$$\overline{\Omega}^C = \frac{(\alpha_G)^{\frac{1}{\upsilon_G}}}{(1+\tau^C)\,\bar{c}\left(1-\frac{\rho_h}{\mu_{z+}}\right)}\left(\frac{\bar{\bar{c}}}{\bar{c}}\right)^{\frac{1}{\upsilon_G}-1} \tag{B.3}$$

Marginal utility of consumption equation:

$$\bar{U}_c = \frac{(\alpha_G)^{\frac{1}{\upsilon_G}}}{\bar{c}\left(1-\frac{\rho_h}{\mu_{z+}}\right)}\left(\frac{\bar{\bar{c}}}{\bar{c}}\right)^{\frac{1}{\upsilon_G}-1} \tag{B.4}$$

Composite consumption function:

$$\frac{\bar{\bar{c}}}{\bar{c}} = \left(\alpha_G^{\frac{1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}}\left(\frac{\bar{g}}{\bar{c}}\right)^{\frac{\upsilon_G-1}{\upsilon_G}}\right)^{\frac{\upsilon_G}{\upsilon_G-1}} \tag{B.5}$$

Average interest rate on government bonds:

$$\bar{\Omega}^R = \frac{\beta}{\Pi^C\mu_{z+} - \beta(1-\alpha_B)} \tag{B.6}$$

Euler equation for government bond holdings:

$$R^{B,n} = \frac{\Pi^C\mu_{z+}}{\beta} \tag{B.7}$$

Capital utilization decision equation:

$$r^K = p^I a' \tag{B.8}$$

Household purchases of installed capital equation:

$$p^K = \frac{\beta\left(1-\tau^K\right)r^K}{\mu_{z+}\mu_\gamma - \beta(1-\delta) - \beta\tau^K\delta\frac{\mu_\gamma}{\Pi}} \tag{B.9}$$

Household investment decision equation:

$$p^I\left(1-\tau^I\right) = p^K \tag{B.10}$$

Definition of capital services:

$$\bar{k}^s = \overline{k} \tag{B.11}$$

Capital accumulation equation:

$$1 = (1-\delta)\frac{1}{\mu_{z+}\mu_\gamma} + \frac{\overline{I}}{\bar{\bar{k}}} \tag{B.12}$$

Optimal wage setting equation:

$$\left(1-\tau_{w}\right)\overline{w}=\lambda^{W}\zeta^{n}\frac{\nu'(n)}{\overline{\Omega}^{C}} \tag{B.13}$$

Labor force participation equation:

$$\overline{\Omega}^{C}\left(1-\tau_{w}\right)\overline{w}=\zeta^{n}\Theta^{n}A_{n}l^{\eta} \tag{B.14}$$

Definition of endogenous shifter:

$$\Theta^{n}=\bar{Z}^{n}\bar{U}_{c} \tag{B.15}$$

Trend of wealth effect in endogenous shifter:

$$\bar{Z}^{n}=(\mu_{z^{+}})^{(\frac{\chi_{n}-1}{\chi_{n}})}(\bar{U}_{c})^{-1} \tag{B.16}$$

Unemployment rate definition:

$$un=\frac{l-n}{l} \tag{B.17}$$

Real wage markup equation:

$$\overline{\Psi}^{W}=(\tfrac{l}{n})^{\eta} \tag{B.18}$$

Definition of wage inflation:

$$\Pi^{W}=\mu_{z^{+}}\Pi^{C} \tag{B.19}$$

Definition of wage inflation indexation:

$$\overline{\Pi}^{W}=\Pi^{W} \tag{B.20}$$

Real wage relevant to employers:

$$\overline{w}^{e}=\overline{w}p^{C} \tag{B.21}$$

Modified uncovered interest rate parity equation:

$$R=sR_{F} \tag{B.22}$$

Aggregate consumption:

$$\bar{c}^{agg}=(1-s_{nr})\bar{c}+s_{nr}\bar{c}^{nr} \tag{B.23}$$

Non-Ricardian budget constraint:

$$(1+\tau^{C})p^{C}\bar{c}^{nr}=\left(1-\tau^{W}\right)\bar{w}^{e}n+\left(1-\tau^{TR}\right)\overline{tr}^{nr} \tag{B.24}$$

B.1.2 Sweden: Firm sector

Definition of composite technological growth rate:

$$\mu_{z^{+}}=\mu_{z}\left(\mu_{\gamma}\right)^{\frac{\alpha}{1-\alpha}} \tag{B.25}$$

Real marginal cost of production for intermediate good producers equation:

$$\overline{mc}=\frac{\left(\left(1+\tau^{SSC}\right)\overline{w}^{e}\right)^{1-\alpha}(r^{K})^{\alpha}}{\varepsilon\alpha^{\alpha}(1-\alpha)^{1-\alpha}\bar{\Gamma}_{G}} \tag{B.26}$$

Simplifying expression variable Gamma:

$$\bar{\Gamma}_{G,t} = \alpha_K^{\frac{\alpha}{\upsilon_K}} \left(\frac{\bar{\tilde{k}}_t^s}{\bar{\bar{k}}_t^s} \right)^{\frac{\alpha}{\upsilon_K}} \tag{B.27}$$

Real rental rate for capital services equation:

$$r^K = \alpha\varepsilon \left(\frac{\bar{\tilde{k}}^s}{n} \frac{1}{\mu_{z+}\mu_\gamma} \right)^{\alpha-1} \overline{mc} \bar{\Gamma}_{G,t}^{\frac{1}{\alpha}} \tag{B.28}$$

Composite capital function:

$$\bar{\tilde{k}}^s = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(\bar{k}^s\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}} \left(\bar{k}_G\right)^{\frac{\upsilon_K-1}{\upsilon_K}} \right)^{\frac{\upsilon_K}{\upsilon_K-1}} \tag{B.29}$$

Public capital accumulation equation:

$$1 = (1-\delta_G) \frac{1}{\mu_{z+}\mu_\gamma} + \frac{\overline{I}^G}{\bar{k}_G} \tag{B.30}$$

Optimal price of intermediate goods equation:

$$\overline{mc} = \frac{1}{\lambda} \tag{B.31}$$

Definition of intermediate good price inflation indexation:

$$\overline{\Pi} = \Pi^C \tag{B.32}$$

Relative price of consumption goods equation:

$$p^C = \left[\vartheta^C \left(p^{C,xe}\right)^{1-\nu_c} + \left(1-\vartheta^C\right) \left(p^{C,e}\right)^{1-\nu_c} \right]^{\frac{1}{1-\nu_c}} \tag{B.33}$$

Definition of consumption good price inflation:

$$\Pi^C = \Pi \tag{B.34}$$

Demand for non-energy consumption goods equation:

$$\bar{c}^{xe} = \vartheta^C \left(\frac{p^{C,xe}}{p^C} \right)^{-\nu_C} \bar{c}^{agg} \tag{B.35}$$

Demand for energy consumption goods equation:

$$\bar{c}^e = \left(1-\vartheta^C\right) \left(\frac{p^{C,e}}{p^C} \right)^{-\nu_C} \bar{c}^{agg} \tag{B.36}$$

Relative price of consumption goods equation:

$$p^{C,xe} = \left[\vartheta^{C,xe} + \left(1-\vartheta^{C,xe}\right) \left(p^{M,C,xe}\right)^{1-\nu_{c,xe}} \right]^{\frac{1}{1-\nu_{c,xe}}} \tag{B.37}$$

Definition of non-energy consumption good price inflation:

$$\Pi^{C,xe} = \Pi \tag{B.38}$$

Relative price of energy consumption goods equation:

$$p^{C,e} = \left[\vartheta^{C,e}\left(p^{D,C,e}\right)^{1-\nu_{c,e}} + \left(1-\vartheta^{C,e}\right)\left(p^{M,C,e}\right)^{1-\nu_{c,e}}\right]^{\frac{1}{1-\nu_{c,e}}} \tag{B.39}$$

Definition of consumption good price inflation:

$$\Pi^{C,e} = \Pi \tag{B.40}$$

Demand for non-energy consumption goods equation:

$$\bar{d}^{e} = \vartheta^{C,e}\left(\frac{p^{D,C,e}}{p^{C,e}}\right)^{-\nu_{C,e}} \bar{c}^{e} \tag{B.41}$$

Import demand for energy consumption goods equation:

$$\bar{m}^{C,e} = \left(1-\vartheta^{C,e}\right)\left(\frac{p^{M,C,e}}{p^{C,e}}\right)^{-\nu_{C,e}} \bar{c}^{e} \tag{B.42}$$

Definition of consumption good price inflation:

$$\Pi^{D,C,e} = \Pi \tag{B.43}$$

Relative price of investment goods equation:

$$p^{I} = \left[\vartheta^{I} + \left(1-\vartheta^{I}\right)\left(p^{M,I}\right)^{1-\nu_{I}}\right]^{\frac{1}{1-\nu_{I}}} \tag{B.44}$$

Definition of investment good price inflation:

$$\Pi^{I} = \Pi \tag{B.45}$$

Real marginal cost for export good producers equation:

$$\overline{mc}^{X} = \left[\vartheta^{X} + \left(1-\vartheta^{X}\right)\left(p^{M,X}\right)^{1-\nu_{x}}\right]^{\frac{1}{1-\nu_{x}}} \tag{B.46}$$

Optimal price of export goods equation:

$$\overline{mc}^{X} = \frac{p^{X}}{\lambda^{X}} \tag{B.47}$$

Definition of export good price inflation indexation:

$$\overline{\Pi}^{X} = \Pi^{C} \tag{B.48}$$

Definition of export good price inflation:

$$\Pi^{X} = \Pi \tag{B.49}$$

Optimal price for import firms specializing in non-energy consumption goods equation:

$$p^{M,C,xe} = \lambda^{M}\overline{mc}^{M,xe} \tag{B.50}$$

Definition of import price inflation indexation, import firms specializing in non-energy consumption goods:

$$\overline{\Pi}^{M,C,xe} = \Pi^{C,xe} \tag{B.51}$$

Definition of import price inflation, import firms specializing in non-energy consumption goods:

$$\Pi^{M,C,xe} = \Pi \tag{B.52}$$

Optimal price for import firms specializing in investment goods equation:

$$p^{M,I} = \lambda^M \overline{mc}^{M,xe} \tag{B.53}$$

Definition of import price inflation indexation, import firms specializing in investment goods:

$$\overline{\Pi}^{M,I} = \Pi^C \tag{B.54}$$

Definition of import price inflation, import firms specializing in investment goods:

$$\Pi^{M,I} = \Pi \tag{B.55}$$

Optimal price for import firms specializing in export goods equation:

$$p^{M,X} = \lambda^M \overline{mc}^{M,xe} \tag{B.56}$$

Definition of import price inflation indexation, import firms specializing in export goods:

$$\overline{\Pi}^{M,X} = \Pi^C \tag{B.57}$$

Definition of import price inflation, import firms specializing in export goods:

$$\Pi^{M,X} = \Pi \tag{B.58}$$

Optimal price for import firms specializing in energy consumption goods equation:

$$p^{M,C,e} = \lambda^M \overline{mc}^{M,C,e} \tag{B.59}$$

Definition of import price inflation indexation, import firms specializing in energy consumption goods:

$$\overline{\Pi}^{M,C,e} = \Pi^{C,e} \tag{B.60}$$

Definition of import price inflation, import firms specializing in energy consumption goods:

$$\Pi^{M,C,e} = \Pi \tag{B.61}$$

Marginal cost of energy importer:

$$\overline{mc}^{M,C,e} = p_F^e Q \frac{p^C}{p_F^C} \tag{B.62}$$

Marginal cost of non-energy importer:

$$\overline{mc}^{M,xe} = Q \frac{p^C}{p_F^C} \tag{B.63}$$

Definition of real exchange rate:

$$s = 1 \tag{B.64}$$

B.1.3 Swedish monetary policy rule

Monetary policy rule:

$$i = R - 1 \tag{B.65}$$

B.1.4 Swedish fiscal authority equations

Government budget constraint:

$$\tau^C p^C \bar{c}^{agg} + (\tau^{SSC} + \tau^W) p^C \bar{w} n + \bar{\Upsilon}^K + \bar{b}^n + \bar{t} = \left(\alpha_B + (R^B - 1)\right) \bar{b} \frac{1}{\mu_{z+} \mathit{\Pi}} + \bar{g} + \tau^I p^I \bar{I} + \bar{I}^G + (1 - \tau^{TR}) \bar{tr}^{agg}$$

Law of motion for aggregate total government debt stock:

$$\bar{b}^n = \left(1 - \frac{1 - \alpha_B}{\mu_{z+} \mathit{\Pi}}\right) \bar{b}$$

Definition of average interest rate on all outstanding government debt:

$$R^B = R^{B,n}$$

Capital income tax revenues:

$$\bar{\Upsilon}^K = \frac{\bar{k}}{\mu_{z+} \mu_\gamma} \tau^K \left(r^K - \delta \frac{\mu_\gamma p^K}{\Pi}\right)$$

Aggregate transfers:

$$\bar{tr}^{agg} = (1 - s_{nr}) \bar{tr} + s_{nr} \bar{tr}^{nr}$$

Transfer allocation:

$$\varpi_{ss} \bar{tr} = (1 - \varpi_{ss}) \bar{tr}^{nr}$$

Government surplus:

$$\overline{surp} = \alpha_B \frac{\bar{b}}{\mu_{z+} \Pi} - \bar{b}^n \tag{B.66}$$

B.1.5 Auxiliary variables

Aggregate investment:

$$\bar{I}^{agg} = \bar{I} + \bar{I}^G \tag{B.67}$$

Price of aggregate investment:

$$p^{Iagg} = \frac{\bar{I}}{\bar{I}^{agg}} p^I + \frac{\bar{I}^G}{\bar{I}^{agg}} \tag{B.68}$$

Aggregate investment inflation:

$$\Pi^{Iagg} = \Pi \tag{B.69}$$

Aggregate import prices:

$$p^M = \frac{\overline{m}^{C,xe}}{\overline{m}^D} p^{MC,xe} + \frac{\overline{m}^I}{\overline{m}^D} p^{MI} + \frac{\overline{m}^X}{\overline{m}^D} p^{MX} + \frac{\overline{m}^{C,e}}{\overline{m}^D} p^{MC,e} \tag{B.70}$$

Aggregate import inflation:

$$\Pi^M = \Pi \tag{B.71}$$

Consumption tax revenues:

$$\overline{Rev}^{\tau^C} = \tau^C p^C \bar{c}^{agg} \tag{B.72}$$

Labor tax revenues:

$$\overline{Rev}^{\tau^W} = \tau^W p^C \bar{w} n \tag{B.73}$$

Social security contribution revenues:

$$\overline{Rev}^{\tau^{SSC}} = \tau^{SSC} p^C \bar{w} n \tag{B.74}$$

Transfer tax revenues:

$$\overline{Rev}^{\tau^{TR}} = \tau^{TR} \bar{tr}^{agg} \tag{B.75}$$

Primary revenues:

$$\overline{Prev} = \overline{Rev}^{\tau^C} + \overline{Rev}^{\tau^W} + \overline{Rev}^{\tau^{SSC}} + \overline{Rev}^{\tau^{TR}} + \bar{\Upsilon}^K \tag{B.76}$$

Investment tax credit expenditures:

$$\overline{Exp}^{\tau^I} = \tau^I p^I \bar{I} \tag{B.77}$$

Primary Expenditure:

$$\overline{Pexp} = \tau^I p^I \bar{I} + \overline{g} + \bar{I}^G + \bar{tr}^{agg} \tag{B.78}$$

Primary surplus:

$$\overline{Psurp} = \overline{Prev} - \overline{Pexp} \tag{B.79}$$

Aggregate transfers, percent of GDP:

$$tr^{agg}oy = \frac{\bar{tr}^{agg}}{\overline{y}^m} \tag{B.80}$$

Government debt to GDP:

$$boy = \frac{\bar{b}}{\overline{y}^m} \tag{B.81}$$

Surplus to GDP:

$$surpoy = \frac{\overline{surp}}{\overline{y}^m} \tag{B.82}$$

Net exports:

$$\overline{nx} = \bar{x} - \bar{m} \tag{B.83}$$

B.2 Foreign economy

B.2.1 Foreign: Household sector

Foreign consumption Euler equation:

$$R_F = \frac{\mu_{z_F^+} \Pi_F^C}{\beta_F} \tag{B.84}$$

Foreign marginal utility of consumption equation:

$$\overline{\Omega}_F^C = \frac{1}{\bar{c}_F (1 - \frac{\rho_{h,F}}{\mu_{z_F^+}})} \tag{B.85}$$

Foreign capital utilization decision equation:

$$r_F^K = p_F^I {a'}_F \tag{B.86}$$

Foreign household purchases of installed capital equation:

$$p_F^K = \frac{\beta_F r_F^K}{\mu_{z_F^+} \mu_\gamma - \beta_F (1 - \delta_F)} \tag{B.87}$$

Foreign household investment decision equation:

$$p_F^I = p_F^K \tag{B.88}$$

Foreign definition of capital services:

$$\bar{k}_F^s = \bar{k}_F \tag{B.89}$$

Foreign capital accumulation equation:

$$1 = (1 - \delta_F) \frac{1}{\mu_{z_F^+} \mu_\gamma} + \frac{\bar{I}_F}{\bar{k}_F} \tag{B.90}$$

Foreign optimal wage setting equation:

$$(1 - \tau_F^w)\, \overline{w}_F = \lambda_F^W \zeta_F^n \frac{\nu'(n_F)}{\overline{\Omega}_F^C} \tag{B.91}$$

Foreign real wage markup equation:

$$\overline{\Psi}_F^W = \lambda_F^W \tag{B.92}$$

Definition of Foreign wage inflation:

$$\Pi_F^W = \mu_{z_F^+} \Pi_F^C \tag{B.93}$$

Definition of Foreign wage inflation indexation equation:

$$\overline{\Pi}_F^W = \Pi_F^W \tag{B.94}$$

Real wage relevant to Foreign employers:

$$\overline{w}_F^e = \overline{w}_F p_F^C \tag{B.95}$$

B.2.2 Foreign: Firm sector

Definition of Foreign composite technological growth rate:

$$\mu_{z_F^+} = \mu_z \left(\mu_\gamma\right)^{\frac{\alpha_F}{1-\alpha_F}} \tag{B.96}$$

Real marginal cost of production for Foreign intermediate good producers equation:

$$\overline{mc}_F = \frac{\left(\overline{w}_F^e\right)^{1-\alpha_F}\left(r_F^K\right)^{\alpha_F}}{\varepsilon_F \alpha_F^{\alpha_F}\left(1-\alpha_F\right)^{1-\alpha_F}} \tag{B.97}$$

Foreign rental rate for capital services equation:

$$r_F^K = \alpha_F \varepsilon_F \left(\frac{\bar{k}_F^s}{n_F}\frac{1}{\mu_{z_F^+}\mu_\gamma}\right)^{\alpha_F - 1} \overline{mc}_F \tag{B.98}$$

Optimal price of Foreign intermediate goods equation:

$$\overline{mc}_F = \frac{1}{\lambda_F} \tag{B.99}$$

Foreign Intermediate good inflation indexation:

$$\overline{\Pi}_F = \Pi_F^C \tag{B.100}$$

Relative price of Foreign consumption goods equation:

$$p_F^C = \left[\vartheta_F^C \left(p_F^{C,xe}\right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right)\left(p_F^{C,e}\right)^{1-\nu_{F,C}}\right]^{\frac{1}{1-\nu_{F,C}}} \tag{B.101}$$

Definition of Foreign consumption good price inflation:

$$\Pi_F^C = \Pi_F \tag{B.102}$$

Demand for non-energy consumption:

$$\bar{c}_F^{xe} = \vartheta_F^C \left(\frac{p_F^{C,xe}}{p_F^C}\right)^{-\nu_{F,C}} \bar{c}_F \tag{B.103}$$

Demand for energy consumption:

$$\bar{c}_F^{e} = \left(1-\vartheta_F^C\right) \left(\frac{p_F^{C,e}}{p_F^C}\right)^{-\nu_{F,C}} \bar{c}_F \tag{B.104}$$

Relative price of non-energy consumption good:

$$p_F^{C,xe} = 1 \tag{B.105}$$

Definition of Foreign non-energy consumption good price inflation:

$$\Pi_F^{C,xe} = \Pi_F \tag{B.106}$$

Definition of Foreign energy consumption good price inflation:

$$\Pi_F^{C,e} = \Pi_F \tag{B.107}$$

Definition of Foreign investment good price inflation:

$$\Pi_F^I = \Pi_F \tag{B.108}$$

Definition of relative price of Swedish export goods in terms of Foreign intermediate goods:

$$\widetilde{p}^X = \frac{p^X p_F^C}{Q p^C} \tag{B.109}$$

B.2.3 Foreign monetary policy rule

Foreign monetary policy rule:

$$i_F = R_F - 1 \tag{B.110}$$

B.3 Market clearing

B.3.1 Swedish aggregate resource constraint

$$\begin{aligned}\bar{y} &= \vartheta^{C,xe}(p^{C,xe})^{\nu_{c,xe}}\bar{c}^{xe} + \bar{d}^{C,e} + \vartheta^{I}(p^{I})^{\nu_I}\bar{I} \\ &+ \vartheta^{X}(\overline{mc}^{X})^{\nu_x}\left(\bar{x} + \phi^{X}\right) + \bar{g} + \bar{I}^{G}\end{aligned} \tag{B.111}$$

B.3.2 Foreign aggregate resource constraint

$$\bar{y}_F = \bar{c}_F^{xe} + \bar{c}_F^{e} + \bar{I}_F \tag{B.112}$$

B.3.3 Balance of payments

$$\bar{a} = \frac{\beta}{(1-\beta)}(\bar{mc}^{M,xe}\overline{m}^{xe} - \bar{mc}^{M,C,e}\overline{m}^{e} - p^{X}\bar{x}) \tag{B.113}$$

B.3.4 Swedish exports

$$\bar{x} = \left(1 - \vartheta_F^{C,xe}\right)\left(\frac{\widetilde{p}^{X}}{p_F^{C,xe}}\right)^{-\nu_{F,C,xe}}\bar{c}_F^{xe} + \left(1 - \vartheta_F^{I}\right)\left(\frac{\widetilde{p}^{X}}{p_F^{I}}\right)^{-\nu_{F,I}}\bar{I}_F \tag{B.114}$$

B.3.5 Swedish imports for consumption

$$\overline{m}^{C,xe} = (1 - \vartheta^{C,xe})\left[\vartheta^{C,xe}(p^{M,C,xe})^{\nu_{c,xe}-1} + 1 - \vartheta^{C,xe}\right]^{\frac{\nu_{c,xe}}{1-\nu_{c,xe}}}\bar{c}^{xe} \tag{B.115}$$

B.3.6 Swedish imports for investment

$$\overline{m}^{I} = (1 - \vartheta^{I})\left[\vartheta^{I}(p^{M,I})^{\nu_I-1} + 1 - \vartheta^{I}\right]^{\frac{\nu_I}{1-\nu_I}}\bar{I} \tag{B.116}$$

B.3.7 Swedish imports for export

$$\overline{m}^{X} = (1 - \vartheta^{X})\left[\vartheta^{X}(p^{M,X})^{\nu_x-1} + 1 - \vartheta^{X}\right]^{\frac{\nu_x}{1-\nu_x}}(\bar{x} + \phi^{X}) \tag{B.117}$$

B.3.8 Import of non-energy goods including fixed costs

$$\overline{m}^{xe} = \overline{m}^{C,xe} + \overline{m}^{I} + \overline{m}^{X} + \phi^{M,C,xe} + \phi^{M,I} + \phi^{M,X} \tag{B.118}$$

B.3.9 Import of non-energy goods excluding fixed costs

$$\overline{m}^{D,xe} = \overline{m}^{C,xe} + \overline{m}^{I} + \overline{m}^{X} \tag{B.119}$$

B.3.10 Import of energy goods including fixed costs

$$\overline{m}^{e} = \overline{m}^{C,e} + \phi^{M,e} \tag{B.120}$$

B.3.11 Aggregate imports excluding fixed costs

$$\overline{m}^{D} = \overline{m}^{D,xe} + \overline{m}^{C,e} \tag{B.121}$$

B.3.12 Aggregate imports including fixed costs

$$\overline{m} = \overline{m}^{xe} + \overline{m}^{C,e} + \phi^{M,e} \tag{B.122}$$

B.3.13 Swedish aggregate output

$$\overline{y} = \varepsilon \left[\frac{\bar{\tilde{k}}^{s}}{\mu_{z+}\mu_{\gamma}} \right]^{\alpha} n^{1-\alpha} - \phi \tag{B.123}$$

B.3.14 Measured Swedish aggregate output

$$\overline{y}^{m} = \overline{y} \tag{B.124}$$

B.3.15 Foreign aggregate output

$$\overline{y}_{F} = \varepsilon \left[\frac{\bar{\tilde{k}}^{s}_{F}}{\mu_{z^{+}_{F}}\mu_{\gamma}} \right]^{\alpha} n_{F}^{1-\alpha} - \phi_{F} \tag{B.125}$$

B.3.16 Measured Foreign aggregate output

$$\overline{y}^{m}_{F} = \overline{y}_{F} \tag{B.126}$$

C Technical appendix: The Swedish economy

In this technical appendix, we derive the key equilibrium conditions and model equations for the Swedish economy.

C.1 Household sector

There are two types of households, Ricardian households and Non-Ricardian households. The problem of Ricardian household is described in Section C.1.1 and the problem of Non-Ricardian household is described in Section C.1.10.

C.1.1 Ricardian household

There is a continuum of household members who are represented by the unit square $(h, j) \in [0, 1] \times [0, 1]$, where each member is indexed by h according to their type of labor service they are specialized in and indexed by j according to their degree of disutility of work. The utility function of househol memeber (h, j) is defined as:

$$E_0^{h,j} \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_{h,j,t} - \rho_h \tilde{C}_{t-1}) - 1(h,j) \zeta_t^n \Theta_t^n A_n j^{\eta} \right]. \tag{C.1}$$

where β is the household's factor, $\tilde{C}_{h,j,t}$ is composite consumption of household member (h, j), $1(h, j)$ is an indicator that is equal to one if the household member works and zero otherwise. ζ_t^c is the consumption preference shock, ρ_h is the consumption habit formation parameter. We assume external habit formation and in line with that $\tilde{C}_{t-1}$ is aggregate consumption. ζ_t^n is a labor disutility preference shock,$\Theta_t^n$is the endogenous shifter, $A_n$is a parameter that determines the weight of disutility of work.

Under symmetric equilibrium and full consumption risk sharing $\tilde{C}_{h,j,t} = \tilde{C}_t$ for all (h, j) and integrating over all household members' utilities gives

$$E_0 \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \int_0^{N_{h,t}} j^\eta \, dj dh \right], \tag{C.2}$$

$$= E_0 \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \frac{N_{h,t}^{1+\eta}}{1+\eta} dh \right]. \tag{C.3}$$

The composite consumption $\tilde{C}_t$ is consist of C_t and public consumption G_t. The weight on private consumption is α_G and υ_G is the elasticity of substitution between private and public consumption. The composite consumption function is given by:

$$\tilde{C}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} C_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1 - \alpha_G)^{\frac{1}{\upsilon_G}} G_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}. \tag{C.4}$$

The Ricardian household chooses private consumption C_t, investment I_t, capacity utilization u_t, capital K_{t+1}, transacted capital between households $\triangle_t^K$, domestic nominal private bonds B_{t+1}^{priv}, domestic nominal government bonds B_{t+1}, newly issued domestic nominal government bonds B_t^n and Foreign nominal private bonds B_{t+1}^{FH}. The aggregate nominal wage W_t is described later. The household's budget constraint is given by:

$$\begin{aligned}
&(1+\tau_t^C) P_t^C C_t + (1-\tau_t^I) \frac{P_t^I}{\gamma_t} I_t + P_t^K \triangle_t^K + \frac{B_{t+1}^{priv}}{R_t \zeta_t} + B_t^n + \frac{S_t B_{t+1}^{FH}}{R_{F,t} \zeta_t \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)} + T_t = \\
&(1-\tau_t^W) \int_0^1 W_{h,t} N_{h,t} dh + (1-\tau_t^K) \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t \right) + \iota^K \tau_t^K \delta P_{t-1}^K K_t \\
&+ B_t^{priv} + \left(\alpha_B + (R_{t-1}^B - 1) \right) B_t + S_t B_t^{FH} + (1-\tau_t^{TR}) TR_t + \Xi_{B,t} + \Xi_{B^{FH},t} + \Psi_t.
\end{aligned} \tag{C.5}$$

Now, we explain the budget constraint. The right-hand side of the budget constraint represents the Ricardian household's income sources. $(1-\tau_t^W) \int_0^1 W_{h,t} N_{h,t} dh$ captures the after-tax labor income, where τ_t^W is the labor income tax rate. $\left(1-\tau_t^K\right) R_t^K u_t K_t$ represents the return from renting capital services to intermediate good firms and taxes on the return from renting capital services to intermediate good firms. The term $\iota^K \tau_t^K \delta P_{t-1}^K K_t$ captures the notion that the capital depreciation can be exempted from taxation, and ι^K is an indicator variable, where $\iota^K \in \{0, 1\}$. If ι^K is set to 1, the capital depreciation can be exempted from taxation. $\left(1-\tau_t^K\right) \frac{P_t^I}{\gamma_t} a(u_t) K_t$ captures that the maintenance cost of capital can be deducted from the capital tax bill. The stock of private bonds from the previous period is B_t^{priv}. $S_t B_t^{FH}$ is the return from owning Foreign bonds and the return is affected by nominal exchange rate S_t. $\left(1-\tau_t^{TR}\right) TR_t$ represents transfers from the government and τ_t^{TR} is the tax rate on transfers. Ψ_t is a lump-sum profit from owning Swedish firms. The Ricardian household owns a representative portfolio of government bonds B_t. The government issues bonds that mature with a probability α_B in a given period. Until stochastic maturity, the government pays a non-state contingent interest rate R_{t-1}^B on the government bonds. $\Xi_{B,t}$ and $\Xi_{B^{BF},t}$, t are financial intermediation premia associated with Swedish and Foreign bonds that are rebated in form of lump-sum payments.

Now, we explain the left-hand side of the budget constraint which represents the Ricardian household's expenditures. This term $\left(1+\tau_t^C\right) P_t^C C_t$ captures the consumption expenditure, where τ_t^C is the consumption tax rate and P_t^C is the price of consumption goods. The Ricardian household uses some of her\his income for purchasing investment goods which are captured by the following term $\left(1-\tau_t^I\right) \frac{P_t^I}{\gamma_t} I_t$, where τ_t^I represents the investment tax credit, and P_t^I is the price of investment goods subjected to investment-specific technological process γ_t. The Ricardian household can trade capital in the capital market which is captured by the following term $P_t^K \triangle_t^K$, where P_t^K is the price of capital. The Ricardian household buys Swedish private bonds B_{t+1}^{priv} and the effective price of Swedish private bonds is $\frac{1}{R_t \zeta_t}$, where R_t is the nominal gross interest rate and ζ_t is a risk premium shock to private bonds. The Ricardian household can also invest in newly issued government bonds B_t^n. Finally, the Ricardian household can buy Foreign bonds B_{t+1}^{FH} and the effective price of Foreign bonds is $\frac{S_t}{R_{F,t} \zeta_t \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)}$. $R_{F,t}$ is Foreign nominal gross interest rate and $\Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)$ is the external risk premium term. For the exact functional form of $\Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)$, please see Section 3. Finally, the Ricardian household pays the lump-sum taxes T_t.

Now, we present law of motion equations. First, the stock of government bonds that the Ricardian household holds evolves as:

$$B_{t+1} = (1-\alpha_B)\, B_t + B_t^n, \tag{C.6}$$

where B_t^n denotes the newly issued debt by the government in period t. Following Krause and Moyen, 2016, Ricardian households are assumed to buy a representative portfolio of government bonds with all possible stochastic maturities.

Second, the average interest rate R_t^B on outstanding government debt bought by Ricardian household h is given by:

$$\left(R_t^B - 1\right) B_{t+1} = (1-\alpha_B)\left(R_{t-1}^B - 1\right) B_t + \left(R_t^{B,n} - 1\right) B_t^n \tag{C.7}$$

where the interest rate on newly issued government debt is denoted by $R_t^{B,n}$.

Finally, the capital accumulation equation for private capital is given by:

$$K_{t+1} = (1-\delta)K_t + \Upsilon_t F(I_t, I_{t-1}) + \triangle_t^K. \tag{C.8}$$

C.1.2 Ricardian household's first-order conditions

Ricardian household chooses C_t, I_t, u_t, $\triangle_t^K$, K_{t+1}, B_{t+1}^{priv}, B_{t+1}, B_t^n, and B_{t+1}^{FH} to maximize its expected utility (C.1) subject to the composite consumption equation (C.4), the budget constraint (C.5), the capital accumulation equation (C.8), the government bond equation (C.6) and the average interest rate on long-term government debt equation (C.7) .

We derive the FOC:s by setting up the Lagrangian $\mathscr{L}_t$. We denote θ_t^b as the Lagrange multiplier associated with the budget constraint (C.5), θ_t^k as the Lagrange multiplier associated with the capital accumulation equation (C.8), θ_t^S as the Lagrange multiplier associated with the stock of long-term government bond accumulation equation (C.6), and θ_t^R as the Lagrange multiplier associated with the average interest rate on outstanding government debt equation (C.7). The Lagrangian for the household's optimization problem is expressed as:

$$\begin{aligned}
\mathscr{L}_t = {} & E_0 \sum_{t=0}^{\infty} \beta_t \Big\{ \left[\zeta_t^c u(\tilde{C}_t, \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \frac{N_{h,t}^{1+\eta}}{1+\eta} dh \right] \\
& + \theta_t^b \left[(1-\tau_t^W) \int_0^1 W_{h,t} N_{h,t} dh + \left(1-\tau_t^K\right)\left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t\right)\right] \\
& + \theta_t^b \left[\iota^K \tau_t^K \delta P_{t-1}^K K_t + B_t^{priv} + \left(\alpha_B + \left(R_{t-1}^B - 1\right)\right) B_t + S_t B_t^{FH} + \left(1-\tau_t^{TR}\right) TR_t + \Xi_{B,t} + \Xi_{B^{FH},t} + \Psi_t \right] \\
& - \theta_t^b \left[\left(1+\tau_t^C\right) P_t^C C_t + \left(1-\tau_t^I\right) \frac{P_t^I}{\gamma_t} I_t + P_t^K \triangle_t^K + \frac{B_{t+1}^{priv}}{R_t \zeta_t} + B_{h,t}^n + \frac{S_t B_{t+1}^{FH}}{R_{F,t} \zeta_t \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)} + T_t \right] \\
& + \theta_t^S \left[(1-\alpha_B) B_t + B_t^n - B_{t+1} \right] \\
& + \theta_t^R \left[(1-\alpha_B)\left(R_{t-1}^B - 1\right) B_t + \left(R_t^{B,n} - 1\right) B_t^n - \left(R_t^B - 1\right) B_{t+1} \right] \\
& + \theta_t^k \left[(1-\delta) K_t + \Upsilon_t F(I_t, I_{t-1}) + \triangle_t^K - K_{t+1} \right] \Big\}.
\end{aligned} \tag{C.9}$$

When one is solving this optimization problem, one has to keep in mind that the utility function is a function of C_t via the following composite consumption function:

$$\tilde{C}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} C_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} G_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}.$$

First, we derive the FOC for C_t. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to C_t, and we obtain the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial C_t} = \beta_t \left[\zeta_t^c u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \theta_t^b P_t^C \left(1+\tau_t^C\right)\right] = 0.$$

Rearranging the first order condition above equation, we have the following equation:

$$\theta_t^b P_t^C \left(1 + \tau_t^C\right) = \zeta_t^c u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1}). \tag{C.10}$$

We define Ω_t^C as the marginal utility of consumption:

$$\Omega_t^C = \frac{\zeta_t^c u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1})}{1 + \tau_t^C} = \frac{U_{c,t}}{1+\tau_t^C}. \tag{C.11}$$

Note that the definition of Ω_t^C includes consumption taxes to simplify the derivations below.

We use Equation (C.11) to rewrite Equation (C.10) as

$$\theta_t^b P_t^C = \Omega_t^C. \tag{C.12}$$

Equation (C.12), which is the same as Equation (12) in Section 2.1.5, represents the FOC for C_t.

Second, we derive the FOC with respect to I_t. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to I_t, and we have the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial I_t} = \beta_t \left[-\theta_t^b \frac{P_t^I}{\gamma_t}\left(1 - \tau_t^I\right) + \theta_t^k\ F_1(I_t, I_{t-1})\right] + E_t\left[\beta_{t+1}\theta_{t+1}^k\ F_2(I_{t+1}, I_t)\right] = 0.$$

Rearranging the above equation and using the following definition: $\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$, we have the following FOC for I_t:

$$\theta_{h,t}^b \frac{P_t^I}{\gamma_t}\left(1 - \tau_t^I\right) = \theta_t^k \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \theta_{t+1}^k \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right]. \tag{C.13}$$

Equation (C.13), which is the same as Equation (13) in Section 2.1.5, captures the FOC for I_t.

Third, we derive the FOC for u_t. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to u_t, and we have the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial u_t} = \left(1 - \tau_t^K\right)\beta_t\theta_t^b\ R_t^K K_t - \left(1 - \tau_t^K\right)\beta_t\theta_t^b\ \frac{P_t^I}{\gamma_t} a'(u_t) K_t = 0.$$

Rewriting the above equation, we obtain the following FOC for u_t:

$$R_t^K K_t = \frac{P_t^I}{\gamma_t} a'(u_t) K_t. \tag{C.14}$$

Equation (C.14), which is the same as Equation (14) in Section 2.1.5, represents the FOC for u_t.

Fourth, we find the FOC for Δ_t^K. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to Δ_t^K, and we obtain the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial \Delta_t^K} = -\theta_t^b \beta_t P_t^K + \theta_t^k \beta_t = 0.$$

We rewrite the above equation, and we have the following FOC for Δ_t^K:

$$\theta_t^b P_t^K = \theta_t^k. \tag{C.15}$$

Equation (C.15), which is the same as Equation (15) in Section 2.1.5, represents the FOC for Δ_t^K.

Fifth, we find the FOC for K_{t+1}. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to K_{t+1}, and we have the following equation:

$$\begin{aligned}\frac{\partial \mathscr{L}_t}{\partial K_{t+1}} &= -\beta_t\theta_t^k + E_t\beta_{t+1}\left[\left(1 - \tau_{t+1}^K\right)\left(\theta_{t+1}^b R_{t+1}^K u_{t+1} - \theta_{t+1}^b \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1})\right)\right] \\ &\quad + E_t\beta_{t+1}\left[\iota^K \theta_{t+1}^b \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^k (1 - \delta)\right] = 0.\end{aligned}$$

Rearranging the above equation and using the following definition: $\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$, we obtain the following FOC for K_{t+1}:

$$\theta_t^k = E_t\beta_{t+1}^r \left[\left(1-\tau_{t+1}^K\right)\theta_{t+1}^b \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1})\right) + \theta_{t+1}^b \iota^K \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^k (1-\delta)\right]. \tag{C.16}$$

Equation (C.16), which is the same as Equation (16) in Section 2.1.5, represents the FOC for K_{t+1}.

Sixth, we derive the FOC for B_{t+1}^{priv}. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to B_{t+1}^{priv}, and this gives us the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial B_{t+1}^{priv}} = -\beta_t \theta_t^b \frac{1}{R_t \zeta_t} + E_t \beta_{t+1} \theta_{t+1}^b = 0.$$

We rearrange the above equation, and then we use the following definition: $\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$ and multiply both sides by P_t^C. Hence, we have the following FOC for B_{t+1}^{priv}:

$$\theta_t^b P_t^C = E_t \beta_{t+1}^r \theta_{t+1}^b P_t^C R_t \zeta_t. \tag{C.17}$$

Equation (C.17), which is the same as Equation (17) in Section 2.1.5, captures the FOC for B_{t+1}^{priv}.

Seventh, we take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to B_{t+1}. We have the following FOC for B_{t+1}:

$$\begin{aligned}\frac{\partial \mathscr{L}_t}{\partial B_{h,t+1}} = {} & E_t\beta_{t+1}\theta_{t+1}^b \left(\alpha_B + \left(R_t^B - 1\right)\right) + E_t\beta_{t+1}\theta_{t+1}^S \left(1-\alpha_B\right) - \beta_t \theta_t^S \\ & + \beta_{t+1}\theta_{t+1}^R \left(1-\alpha_B\right)\left(R_t^B - 1\right) - \beta_t\theta_t^R \left(R_t^B - 1\right) = 0.\end{aligned}$$

We rearrange the above equation, and then we use the following definition: $\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$. We have the following equation:

$$\begin{aligned}\frac{\partial \mathscr{L}_t}{\partial B_{t+1}} = {} & E_t\beta_{t+1}^r\theta_{t+1}^b \left(\alpha_B + \left(R_t^B - 1\right)\right) + E_t\beta_{t+1}^r\theta_{t+1}^S \left(1-\alpha_B\right) - \theta_t^S \\ & + E_t\left(\beta_{t+1}^r\theta_{t+1}^R \left(1-\alpha_B\right) - \theta_t^R\right)\left(R_t^B - 1\right) = 0.\end{aligned} \tag{C.18}$$

The above equation can be rewritten as follows:

$$E_t\beta_{t+1}^r\theta_{t+1}^b \left(\alpha_B + \left(R_t^B - 1\right)\right) = \theta_t^S - E_t\beta_{t+1}^r\theta_{t+1}^S \left(1-\alpha_B\right) + \left(\theta_t^R - \left(1-\alpha_B\right)E_t\beta_{t+1}^r\theta_{t+1}^R\right)\left(R_t^B - 1\right). \tag{C.19}$$

Equation (C.19), which is the same as Equation (18) in Section 2.1.5, which captures the FOC of government bond holdings.

Eighth, we take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to B_t^n. We have the following FOC for B_t^n:

$$\frac{\partial \mathscr{L}_t}{\partial B_t^n} = -\theta_t^b \beta_t + \theta_t^S \beta_t + \beta_t \theta_t^R \left(R_t^{B,n} - 1\right) = 0. \tag{C.20}$$

Rearranging the above equation, we have the following FOC for B_t^n:

$$\theta_t^b \beta_t = \theta_t^S \beta_t + \beta_t \theta_t^R \left(R_t^{B,n} - 1\right). \tag{C.21}$$

Equation (C.21), which is the same as Equation (19) in Section 2.1.5, captures the FOC of newly issued government bonds.

Ninth, we take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to R_t^B. We have the following FOC for R_t^B:

$$\frac{\partial \mathscr{L}_t}{\partial R_t^B} = E_t\beta_{t+1}\theta_{t+1}^b B_{t+1} + E_t\beta_{t+1}\theta_{t+1}^R \left(1-\alpha_B\right) B_{t+1} - \beta_t\theta_t^R B_{t+1} = 0. \tag{C.22}$$

We use the following definition: $\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$, and the above equation can be rewritten as follows:

$$\theta_t^R B_{t+1} = E_t\beta_{t+1}^r\theta_{t+1}^b B_{t+1} + E_t\beta_{t+1}^r\theta_{t+1}^R \left(1-\alpha_B\right) B_{t+1}. \tag{C.23}$$

Equation (C.23), which is the same as Equation (20) in Section 2.1.5, captures the FOC for average interest rate on outstanding government debt (or the price of government bonds that the household is willing to pay).

Finally, we find the FOC for B_{t+1}^{FH}. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to B_{t+1}^{FH}, and we have the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial B_{t+1}^{FH}} = -\frac{\beta_t \theta_t^b S_t}{\Phi(\overline{a}_t, s_t, \widetilde{\phi}_t) R_{F,t}} + E_t \left[\beta_{t+1} \theta_{t+1}^b S_{t+1} \right] = 0.$$

Rearranging the above equation and using the following definition: $\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$, we have the following FOC for $B_{h,t+1}^{FH}$:

$$\theta_t^b S_t = E_t \left[\beta_{t+1}^r \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t) R_{F,t} \zeta_t S_{t+1} \theta_{t+1}^b \right]. \tag{C.24}$$

Equation (C.24), which is the same as Equation (21) in Section 2.1.5, captures the FOC for B_{t+1}^{FH}.

C.1.3 Consumption Euler equation

In this section, we derive the stationarized version of consumption Euler equation (A.1a).

We use equation (C.12), which shows $\theta_t^b P_t^C = \Omega_t^C$ and the following definitions: $p_t^C = \frac{P_t^C}{P_t}$, and $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$. Thus, we can rewrite Equation (C.17) as follows:

$$\theta_t^b P_t^C = E_t \left[\beta_{t+1}^r \theta_{t+1}^b P_t^C \frac{P_{t+1}^C}{P_{t+1}^C} R_t \zeta_t \right],$$

$$\Omega_t^C = E_t \left[\beta_{t+1}^r \frac{R_t \zeta_t}{\Pi_{t+1}^C} \Omega_{t+1}^C \right].$$

Given our assumptions about the possibility of households to diversify the idiosyncratic risk component associated with their wage income, all households in Sweden will choose the same level of consumption in every period (see Section 2.1.2 in the main text). We may drop the subscript h from the above equation. We have the following non-stationarized version of the consumption Euler equation:

$$\Omega_t^C = E_t \left[\beta_{t+1}^r \frac{R_t \zeta_t}{\Pi_{t+1}^C} \Omega_{t+1}^C \right]. \tag{C.25}$$

We now stationarize the consumption Euler equation. In particular, we stationarize Equation (C.25) by using the following definitions: $\mu_{z^+,t+1} = \frac{z_{t+1}^+}{z_t^+}$, $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$. Equation (C.25) becomes:

$$z_t^+ \Omega_t^C = E_t \left[\beta_{t+1}^r z_{t+1}^+ \frac{z_t^+}{z_{t+1}^+} \frac{R_t \zeta_t}{\Pi_{t+1}^C} \Omega_{t+1}^C \right],$$

and we obtain the following stationarized version of consumption Euler equation:

$$\overline{\Omega}_t^C = R_t \zeta_t E_t \left[\beta_{t+1}^r \frac{1}{\mu_{z^+,t+1} \Pi_{t+1}^C} \overline{\Omega}_{t+1}^C \right]. \tag{C.26}$$

Equation (C.26), which represents the stationarized version of consumption Euler equation, is the same as Equation (A.1a).

C.1.4 Marginal utility of consumption

In this section, first we explicitly define the functional form of the household utility function. Second, we derive the stationarized version of marginal utility of consumption equation (A.3a).

Recall from Section 3.1, we have the following functional form for the utility function:

$$u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) = ln(\tilde{C}_t - \rho_h \tilde{C}_{t-1}).$$

Recall, Equation (C.11), which shows the definition of marginal utility of consumption including the consumption tax, is expressed as:

$$\Omega_t^C = \frac{\zeta_t^c u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1})}{1+\tau_t^C}.$$

Recall, the composite consumption function is expressed as:

$$\tilde{C}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} C_{h,t}^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} G_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}.$$

Using the above utility functional form and the above composite consumption function and taking the first derivative of the utility function with respect to C_t, we can obtain the following marginal utility of consumption equation $U_{c,t}$:

$$U_{c,t} = u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) = \frac{\zeta_t^c}{\tilde{C}_t - \rho_h \tilde{C}_{t-1}} \left(\alpha_G \frac{\tilde{C}_t}{C_t} \right)^{\frac{1}{\upsilon_G}}. \tag{C.27}$$

Using Equation (C.27), we can rewrite Equation (C.11) as:

$$\Omega_t^C = U_{c,t} \frac{1}{1+\tau_t^C} = \frac{\zeta_t^c}{\tilde{C}_t - \rho_h \tilde{C}_{t-1}} \left(\alpha_G \frac{\tilde{C}_t}{C_t} \right)^{\frac{1}{\upsilon_G}} \frac{1}{1+\tau_t^C}. \tag{C.28}$$

We stationarize Equation (C.28) by using the following definitions: $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$, $\overline{C}_t = \frac{C_t}{z_t^+}$ and $\overline{\overline{C}}_t = \frac{\tilde{C}_t}{z_t^+}$. Equation (C.28) becomes:

$$z_t^+ \Omega_t^C = \frac{\zeta_t^c}{(1+\tau_t^C)\left(\frac{1}{z_t^+} \tilde{C}_t - \rho_h \frac{1}{z_t^+} \frac{z_{t-1}^+}{z_{t-1}^+} \tilde{C}_{t-1} \right)} \left(\alpha_G \frac{z_t^+ \tilde{C}_t}{z_t^+ C_t} \right)^{\frac{1}{\upsilon_G}},$$

and we obtain the following equation:

$$\overline{\Omega}_t^C = \frac{\zeta_t^c}{(1+\tau_t^C)\left(\overline{\overline{C}}_t - \rho_h \frac{1}{\mu_{z^+,t}} \overline{\overline{C}}_{t-1} \right)} \left(\alpha_G \frac{\overline{\overline{C}}_t}{\overline{C}_t} \right)^{\frac{1}{\upsilon_G}}.$$

We define $\bar{G}_t$ as $\frac{G_t}{z_t^+}$, and the composite consumption function can be written in stationarized form as follows:

$$\overline{\overline{C}}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} \bar{C}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} \bar{G}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}. \tag{C.29}$$

In the Swedish economy, where the size of the population is normalized to unity, the distinction between aggregate and *per capita* variables are trivial. Nonetheless, we express the above equation in *per capita* terms, so we denote $\bar{c}_t$ as the stationarized aggregate consumption of Ricardian households *per capita* terms, and $\bar{\bar{c}}_t$ as the stationarized composite consumption in *per capita* terms. Hence, the stationarized version of marginal utility of consumption equation can be written as:

$$\overline{\Omega}_t^C = \frac{\zeta_t^c}{(1+\tau_t^C)\left(\bar{\bar{c}}_t - \rho_h \frac{1}{\mu_{z^+,t}} \bar{\bar{c}}_{t-1} \right)} \left(\alpha_G \frac{\bar{\bar{c}}_t}{\bar{c}_t} \right)^{\frac{1}{\upsilon_G}}. \tag{C.30}$$

Equation (C.30), which represents the stationarized version of marginal utility of consumption equation, is the same as Equation (A.3a).

We can rewrite Equation (C.29) in *per capita* terms. We denote $\bar{g}_t$ as the stationarized government consumption in *per capita* terms. Thus, the stationarized composite consumption equation in *per capita* terms can be expressed as:

$$\bar{\bar{c}}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} \bar{c}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} \bar{g}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}.$$

The above equation is the same as Equation (A.5a).

C.1.5 Capital utilization and household purchases of installed capital

This section derives the capital utilization decision equation (A.8a) and the household purchases of installed capital equation (A.9a) respectively.

First, we derive the capital utilization decision equation. Recall, Equation (C.14), which shows the FOC for $u_{h,t}$, is written as:

$$R_t^K K_t = \frac{P_t^I}{\gamma_t} a'(u_t) K_t.$$

Using the following definitions: $r_t^K = \frac{\gamma_t R_t^K}{P_t}$, and $p_t^I = \frac{P_t^I}{P_t}$, the above equation can be rewritten as follows:

$$\frac{\gamma_t R_t^K}{P_t} = \frac{P_t^I}{P_t} a'(u_t),$$

$$r_t^K = p_t^I a'(u_t).$$

Note that the optimal rate of utilization is a function of the two aggregate relative prices r_t^K and p_t^I. All households in Sweden will then choose the same utilization rate, and the subscript h may be dropped from the above equation. Thus, we have the following capital utilization decision equation:

$$r_t^K = p_t^I a'(u_t). \tag{C.31}$$

Equation (C.31), which captures the capital utilization decision, is the same as Equation (A.8a).

Next, we derive the household purchases of installed capital equation (A.9a). Recall, Equation (C.16), which represents the FOC for $K_{h,t+1}$, is expressed as:

$$\theta_t^k = E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \theta_{t+1}^b \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) \right) + \theta_{t+1}^b \iota^K \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^k (1-\delta) \right].$$

Using Equation (C.15) that shows $\theta_t^b P_t^K = \theta_t^k$, we can rewrite the above equation as:

$$\theta_t^b P_t^K = E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \theta_{t+1}^b \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) \right) + \theta_{t+1}^b \iota^K \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^b P_{t+1}^K (1-\delta) \right].$$

We use Equation (C.12) that shows $\theta_t^b P_t^C = \Omega_t^C$ and use the following definition: $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$. Thus, we can rewrite the above equation as follows:

$$\begin{aligned} P_t^C \theta_t^b P_t^K = {} & E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \theta_{t+1}^b P_{t+1}^C \frac{1}{\Pi_{t+1}^C} \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) + \iota^K \frac{\tau_{t+1}^K}{\left(1 - \tau_{t+1}^K\right)} \delta P_t^K \right) \right] \\ & + E_t \beta_{t+1}^r \left[\theta_{t+1}^b P_{t+1}^C \frac{1}{\Pi_{t+1}^C} P_{t+1}^K (1-\delta) \right], \end{aligned}$$

and

$$\Omega_t^C P_t^K = E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \Omega_{t+1}^C \frac{1}{\Pi_{t+1}^C} \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) + \iota^K \frac{\tau_{t+1}^K}{\left(1 - \tau_{t+1}^K\right)} \delta P_t^K \right) + \Omega_{t+1}^C \frac{1}{\Pi_{t+1}^C} P_{t+1}^K (1-\delta) \right].$$

We multiply both sides of the above equation by $\frac{\gamma_t}{P_t}$, and then we rewrite the above equation as follows:

$$\begin{aligned} \frac{\gamma_t P_t^K}{P_t} = {} & E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} \frac{\gamma_t}{P_t} \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) + \iota^K \frac{\tau_{t+1}^K}{\left(1 - \tau_{t+1}^K\right)} \delta P_t^K \right) \right] \\ & + E_t \beta_{t+1}^r \left[\frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} \frac{\gamma_t}{P_t} P_{t+1}^K (1-\delta) \right]. \end{aligned}$$

In order to stationarize the above equation, we use the following definitions: $r_{t+1}^K = \frac{\gamma_{t+1} R_{t+1}^K}{P_{t+1}}$, $p_{t+1}^I = \frac{P_{t+1}^I}{P_{t+1}}$, $\mu_{\gamma,t+1} = \frac{\gamma_{t+1}}{\gamma_t}$, $p_t^K = \frac{\gamma_t P_t^K}{P_t}$, and $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. Thus, we have the following equation for the household purchases of installed capital:

$$\frac{\gamma_t P_t^K}{P_t} = E_t \beta_{t+1}^r \left[\left(1-\tau_{t+1}^K\right) \frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} \frac{\gamma_t}{P_t} \frac{P_{t+1}}{\gamma_{t+1}} \left(r_{t+1}^K u_{t+1} - p_{t+1}^I a(u_{t+1}) + \iota^K \frac{\tau_{t+1}^K}{\left(1-\tau_{t+1}^K\right)} \delta \frac{\mu_{\gamma,t+1}}{\Pi_{t+1}} p_t^K \right) \right]$$
$$+ E_t \beta_{t+1}^r \left[\frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} \frac{\gamma_t}{P_t} P_{t+1}^K (1-\delta) \right].$$

We use the following definition: $p_t^K = \frac{\gamma_t P_t^K}{P_t}$, and the above equation can be written as follows:

$$p_t^K = E_t \beta_{t+1}^r \frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{\Pi_{t+1}}{\Pi_{t+1}^C} \frac{1}{\mu_{\gamma,t+1}} \left[\left(1-\tau_{t+1}^K\right) \left(r_{t+1}^K u_{t+1} - p_{t+1}^I a(u_{t+1}) \right) + \iota^K \tau_{t+1}^K \delta \frac{\mu_{\gamma,t+1}}{\Pi_{t+1}} p_t^K + p_{t+1}^K (1-\delta) \right]. \tag{C.32}$$

Using the following definitions: $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$ and $\mu_{z^+,t+1} = \frac{z_{t+1}^+}{z_t^+}$, Equation (C.32) can be written as:

$$p_t^K = E_t \beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\overline{\Omega}_t^C} \frac{\Pi_{t+1}}{\Pi_{t+1}^C} \frac{1}{\mu_{z^+,t+1} \mu_{\gamma,t+1}} \left[\left(1-\tau_{t+1}^K\right) \left(r_{t+1}^K u_{t+1} - p_{t+1}^I a(u_{t+1}) \right) + \iota^K \delta \tau_{t+1}^K \frac{\mu_{\gamma,t+1}}{\Pi_{t+1}} p_t^K + p_{t+1}^K (1-\delta) \right]. \tag{C.33}$$

Equation (C.33) is the same as Equation (A.9a), which shows the stationarized version of the household purchase of installed capital.

C.1.6 Investment decision

This section derives the household investment decision equation (A.10a). Recall that we have Equation (C.13) that shows the following FOC for $I_{h,t}$:

$$\theta_t^b \frac{P_t^I}{\gamma_t} \left(1-\tau_t^I\right) = \theta_t^k \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta_{t+1}^r \theta_{t+1}^k \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

The above equation can be expressed as:

$$P_t^I \left(1-\tau_t^I\right) = \frac{\gamma_t \theta_t^k}{\theta_t^b} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta_{t+1}^r \frac{\gamma_t \theta_{t+1}^k}{\theta_t^b} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

We use Equation (C.15), which shows $\theta_t^b P_t^K = \theta_t^k$. We can rewrite the above equation as follows:

$$P_t^I \left(1-\tau_t^I\right) = \frac{\gamma_t \theta_t^b P_t^K}{\theta_t^b} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta_{t+1}^r \frac{\gamma_t \theta_{t+1}^b P_{t+1}^K}{\theta_t^b} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right],$$
$$P_t^I \left(1-\tau_t^I\right) = \gamma_t P_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta_{t+1}^r \frac{\gamma_t \theta_{t+1}^b P_{t+1}^K}{\theta_t^b} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

We use the following definition: $p_t^I = P_t^I / P_t$, and the above equation becomes:

$$p_t^I \left(1-\tau_t^I\right) = \frac{\gamma_t P_t^K}{P_t} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta_{t+1}^r \frac{\gamma_t \theta_{t+1}^b P_{t+1}^K}{P_t \theta_t^b} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

We multiply the second term on the right hand side of the above equation by $\frac{P_{t+1}\gamma_{t+1}}{P_{t+1}\gamma_{t+1}}$. We use the following definitions: $\mu_{\gamma,t+1} = \frac{\gamma_{t+1}}{\gamma_t}$ and $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. The above equation can then be rewritten as follows:

$$p_t^I \left(1-\tau_t^I\right) = \frac{\gamma_t P_t^K}{P_t} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \frac{\gamma_{t+1} P_{t+1}^K}{P_{t+1}} \frac{P_{t+1}}{P_t} \frac{\gamma_t}{\gamma_{t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right],$$
$$p_t^I \left(1-\tau_t^I\right) = \frac{\gamma_t P_t^K}{P_t} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \frac{\gamma_{t+1} P_{t+1}^K}{P_{t+1}} \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

Using the following definition: $p_t^K = \frac{\gamma_t P_t^K}{P_t}$, this gives us the following equation:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right].$$

Using the following definitions: $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$ and $\Omega_{h,t}^C = \theta_{h,t}^b P_t^C$, we can rewrite the above equation as follows:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\theta_{t+1}^b P_{t+1}^C}{\theta_t^b P_t^C} \frac{P_t^C}{P_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right],$$

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\theta_{t+1}^b P_{t+1}^C}{\theta_t^b P_t^C} \frac{1}{\Pi_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right],$$

and we can obtain the following equation:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right].$$

Hence, we have the following equation for the household investment decision:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right]. \quad \text{(C.34)}$$

Now, we continue the effort to stationarize Equation (C.34). Using the following definitions: $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$ and $\mu_{z^+,t+1} = \frac{z_{t+1}^+}{z_t^+}$, Equation (C.34) can be written as follows:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{z_{t+1}^+\Omega_{t+1}^C}{z_t^+\Omega_t^C} \frac{z_t^+}{z_{t+1}^+} \frac{1}{\Pi_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{t+1}^\gamma} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right],$$

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\overline{\Omega}_t^C} \frac{\Pi_{t+1}}{\Pi_{t+1}^C} \frac{p_{t+1}^K}{\mu_{z^+,t+1}\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right]. \quad \text{(C.35)}$$

Furthermore, we need to express $F_1(I_t, I_{t-1})$ and $F_2(I_{t+1}, I_t)$ as functions of stationary variables. Recall from Section 3.1, we have the following investment adjustment cost function $F(I_t, I_{t-1})$:

$$F(I_t, I_{t-1}) = \left[1 - \widetilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.$$

We take the first derivative of $F(I_t, I_{t-1})$ with respect to I_t, and we can find $F_1(I_t, I_{t-1})$. We then take the first derivative of $F(I_{t+1}, I_t)$ with respect to I_t, and we can find $F_2(I_{t+1}, I_t)$. We have the following results:

$$F_1(I_t, I_{t-1}) = -\widetilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \left[1 - \widetilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right], \quad \text{(C.36)}$$

and

$$F_2(I_{t+1}, I_t) = \widetilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2. \quad \text{(C.37)}$$

We express Equation (C.36) and Equation (C.37) by applying the following definition: $\overline{I}_t = \frac{I_t}{z_t^+\gamma_t}$. Using this definition, together with $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$ and $\mu_{\gamma,t} = \frac{\gamma_t^+}{\gamma_{t-1}^+}$, the ratio $\frac{I_t}{I_{t-1}}$ can be written as: $\mu_{z^+,t}\,\mu_{\gamma,t}\,\frac{\overline{I}_t}{\overline{I}_{t-1}}$. We use the notation $F_1(\overline{I}_t, \overline{I}_{t-1}, \mu_{z^+,t}, \mu_{\gamma,t})$ to express $F_1(I_t, I_{t-1})$ as a function of the stationary variables $\overline{I}_t$, $\overline{I}_{t-1}$, $\mu_{z^+,t}$ and $\mu_{\gamma,t}$. Moreover, $F_2(\overline{I}_{t+1}, \overline{I}_t, \mu_{z^+,t+1}, \mu_{\gamma,t+1})$ represents $F_2(I_{t+1}, I_t)$ expressed as a function of stationary variables. Hence, Equation (C.36) and Equation (C.37) become:

$$F_1(\overline{I}_t, \overline{I}_{t-1}, \mu_{z^+,t}, \mu_{\gamma,t}) = -\widetilde{S}'\left(\frac{\mu_{z^+,t}\ \mu_{\gamma,t}\overline{I}_t}{\overline{I}_{t-1}}\right)\frac{\mu_{z^+,t}\ \mu_{\gamma,t}\overline{I}_t}{\overline{I}_{t-1}} + \left[1 - \widetilde{S}\left(\frac{\mu_{z^+,t}}{\overline{I}_{t-1}}\right)\right], \quad \text{(C.38)}$$

and

$$F_2(\overline{I}_{t+1}, \overline{I}_t, \mu_{z^+,t+1}, \mu_{\gamma,t+1}) = \widetilde{S}'\left(\frac{\mu_{z^+,t+1}\ \mu_{\gamma,t+1}\overline{I}_{t+1}}{\overline{I}_t}\right)\left(\frac{\mu_{z^+,t+1}\ \mu_{\gamma,t+1}\overline{I}_{t+1}}{\overline{I}_t}\right)^2. \quad \text{(C.39)}$$

With these notations, we can rewrite Equation (C.35) as:

$$p_t^I\left(1-\tau_t^I\right)=p_t^K\Upsilon_tF_1(\overline{I}_t,\overline{I}_{t-1},\mu_{z^+,t},\mu_{\gamma,t})+E_t\left[\beta_{t+1}^r\frac{\overline{\Omega}_{t+1}^C}{\overline{\Omega}_t^C}\frac{\Pi_{t+1}}{\Pi_{t+1}^C}\frac{p_{t+1}^K}{\mu_{z^+,t+1}\mu_{\gamma,t+1}}\Upsilon_{t+1}F_2(\overline{I}_{t+1},\overline{I}_t,\mu_{z^+,t+1},\mu_{\gamma,t+1})\right].\tag{C.40}$$

Equation (C.40), which represents the stationarized version of the household investment decision equation, is the same as Equation (A.10a).

C.1.7 Modified uncovered interest rate parity

This section derives the stationarized version of uncovered interest rate parity equation (A.22a).

Recall, Equation (C.24), which shows the FOC for B_{t+1}^{FH}, is written as:

$$\theta_t^bS_t=E_t\left[\beta_{t+1}^r\Phi(\overline{a}_t,s_t,\widetilde{\phi}_t)R_{F,t}\zeta_tS_{t+1}\theta_{t+1}^b\right].$$

Using the following definitions: $\theta_t^bP_t^C=\Omega_t^C$ and $\Pi_{t+1}^C=\frac{P_{t+1}^C}{P_t^C}$, the above equation can be written as follows.

$$\theta_t^bP_t^C=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\frac{P_t^C}{P_{t+1}^C}\theta_{t+1}^bP_{t+1}^CR_{F,t}\zeta_t\frac{S_{t+1}}{S_t}\right],\tag{C.41}$$

$$\Omega_t^C=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\frac{1}{\Pi_{t+1}^C}\Omega_{t+1}^CR_{F,t}\zeta_t\frac{S_{t+1}}{S_t}\right].$$

Using the following definition: $s_{t+1}=\frac{S_{t+1}}{S_t}$, the above equation can be expressed as:

$$\Omega_t^C=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\frac{1}{\Pi_{t+1}^C}\Omega_{t+1}^CR_{F,t}\zeta_ts_{t+1}\right].\tag{C.42}$$

Recall, we have the following non-stationarized version of consumption Euler equation (C.25), which is expressed as:

$$\Omega_t^C=E_t\left[\beta_{t+1}^r\frac{R_t\zeta_t}{\Pi_{t+1}^C}\Omega_{t+1}^C\right].$$

Using the above non-stationarized version of consumption Euler equation and Equation (C.42), we can obtain the following non-stationarized version of the uncovered interest parity equation:

$$E_t\left[\beta_{t+1}^r\frac{R_t\zeta_t}{\Pi_{t+1}^C}\Omega_{t+1}^C\right]=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\zeta_t\frac{R_{F,t}}{\Pi_{t+1}^C}\Omega_{t+1}^Cs_{t+1}\right].$$

Now, we stationarize the above equation. Using the following definitions: $\overline{\Omega}_{t+1}^C=z_{t+1}^+\Omega_{t+1}^C$ and $\mu_{z^+,t+1}=\frac{z_{t+1}^+}{z_t^+}$, the above equation becomes:

$$E_t\left[\beta_{t+1}^r\frac{R_t\zeta_t}{\Pi_{t+1}^C}z_t^+\frac{z_{t+1}^+}{z_{t+1}^+}\Omega_{t+1}^C\right]=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\frac{R_{F,t}\zeta_t}{\Pi_{t+1}^C}z_t^+\frac{z_{t+1}^+}{z_{t+1}^+}\Omega_{t+1}^Cs_{t+1}\right].$$

We have the following stationarized version of the uncovered interest rate parity equation:

$$R_tE_t\left[\beta_{t+1}^r\frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C}\right]=R_{F,t}\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)E_t\left[\beta_{t+1}^r\frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C}s_{t+1}\right].\tag{C.43}$$

Equation (C.43) is the same as Equation (A.22a), which shows the stationarized version of the modified uncovered interest rate parity equation.

C.1.8 Average interest rate on government bonds and Euler equation for government bonds

In this section, we derive the optimal condition for average interest rate on government bonds, Equation (A.6a) and Euler equation for government bond holdings, Equation (A.7a).

The FOC for average interest rate on outstanding government debt, Equation (C.23) can be written as:

$$\frac{\theta_t^R}{\theta_t^b} = E_t \beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \left[1 + \frac{\theta_{t+1}^R}{\theta_{t+1}^b} (1 - \alpha_B)\right]. \tag{C.44}$$

The FOC of newly issued government bonds, Equation (C.21) can be written as:

$$\frac{\theta_t^S}{\theta_t^b} = 1 - \frac{\theta_t^R}{\theta_t^b} \left(R_t^{B,n} - 1\right). \tag{C.45}$$

The FOC of government bond holdings, Equation (C.19) can be written as:

$$\frac{\theta_t^S}{\theta_t^b} + \frac{\theta_t^R}{\theta_t^b} \left(R_t^B - 1\right) = E_t \beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \left[\left(\alpha_B + \left(R_t^B - 1\right)\right) + \frac{\theta_{t+1}^S}{\theta_{t+1}^b} (1 - \alpha_B) + \frac{\theta_{t+1}^R}{\theta_{t+1}^b} (1 - \alpha_B) \left(R_t^B - 1\right)\right]. \tag{C.46}$$

Using Equation (C.45), we can rewrite Equation (C.46) as:

$$1 + \frac{\theta_t^R}{\theta_t^b} \left(R_t^B - R_t^{B,n}\right) = E_t \beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \left[R_t^B - (1 - \alpha_B) \frac{\theta_{t+1}^R}{\theta_{t+1}^b} \left(R_{t+1}^{B,n} - R_t^B\right)\right].$$

Using Equation (C.44), we can rewrite the above equation as:

$$1 = E_t \beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \left[R_t^{B,n} - (1 - \alpha_B) \frac{\theta_{t+1}^R}{\theta_{t+1}^b} \left(R_{t+1}^{B,n} - R_t^{B,n}\right)\right]. \tag{C.47}$$

We use the following definitions: $\theta_t^b P_t^C = \Omega_t^C$, $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$, $\bar{\Omega}_t^R = \frac{\theta_t^R}{\theta_t^b}$, and $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$. Thus, we can rewrite Equation (C.44) as:

$$\bar{\Omega}_t^R = E_t \beta_{t+1}^r \frac{\bar{\Omega}_{t+1}^C}{\bar{\Omega}_t^C \Pi_{t+1}^C \mu_{z^+,t+1}} \left[1 + \bar{\Omega}_{t+1}^R (1 - \alpha_B)\right]. \tag{C.48}$$

Equation (C.48), which represents the stationarized version of the optimal condition for average interest rate on government bonds, is the same as Equation (A.6a).

We use the following definitions: $\theta_t^b P_t^C = \Omega_t^C$, $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$, $\bar{\Omega}_t^R = \frac{\theta_t^R}{\theta_t^b}$, and $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$ as well as we drop the subscript h. Thus, we can rewrite Equation (C.47) as:

$$1 = E_t \beta_{t+1}^r \frac{\bar{\Omega}_{t+1}^C}{\bar{\Omega}_t^C \Pi_{t+1}^C \mu_{z^+,t+1}} \left[R_t^{B,n} - (1 - \alpha_B) \bar{\Omega}_{t+1}^R \left(R_{t+1}^{B,n} - R_t^{B,n}\right)\right]. \tag{C.49}$$

Equation (C.49), which represents the stationarized version of Euler equation for government bond holdings, is the same as Equation (A.7a).

C.1.9 Wage setting

This section derives Equation (A.13a), which represents the stationarized version of the optimal wage setting equation. Ricardian household member labor type h choose the optimal wage rate $W_{h,t}^{opt}$ that maximizes the expected utility of household (C.1) rather than its individual utility, subject to the household budget constraint (C.5), the labor demand schedule (C.50), and the Calvo wage contract (C.51). In each period, the individual labor type resets its wage with probability $(1 - \xi_w)$. With probability ξ_w, the household member cannot reset its wage, in which case the wage rate evolves according to: $W_{h,t+k|t} = W_{h,t}^{opt} \overline{\Pi}_{t1}^W \overline{\Pi}_{t+2}^W \ldots \overline{\Pi}_{t+k}^W$. Note $\overline{\Pi}_t^W = \left(\Pi_{t-1}^W\right)^{\chi_w} \left(\Pi_t^{C,trend}\right)^{1-\chi_w}$.

The demand for labor is given by

$$N_{h,t+k|t} = \left(\frac{W_{h,t+k|t}}{W_{t+k}}\right)^{-\varepsilon_{w,t}} N_{t+k} \tag{C.50}$$

and the Calvo wage contract is given by

$$W_{h,t+k} = \begin{cases} \overline{\Pi}^{W}_{t+k} W_{h,t+k-1} & \text{with probability } \xi_w, \\ W^{\text{opt}}_{h,t+k} & \text{with probability } (1-\xi_w). \end{cases} \tag{C.51}$$

We let θ_t^b denote the Lagrange multiplier associated with the budget constraint (C.5). To solve the optimization problem, we set up the following Lagrangian:

$$\begin{aligned}
&\mathscr{L}_t^W = \beta_t \left[\zeta_t^c u(\tilde{C}_t, \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \frac{N_{h,t}^{1+\eta}}{1+\eta} dh\right] \\
&+ \theta_t^b \Bigg[(1-\tau_t^W) \int_0^1 W_{h,t} N_{h,t} dh + (1-\tau_t^K)\left(R_t^K u_t K_{h,t} - \frac{P_t^I}{\gamma_t} a(u_t) K_t\right) + \iota^K \tau_t^K \delta P_{t-1}^K K_t + \\
&+ B_t^{priv} + \left(\alpha_B + \left(R_{t-1}^B - 1\right)\right) B_t + S_t B_t^{FH} + (1-\tau_t^{TR}) TR_t + \Xi_{B,t} + \Xi_{B^{FH},t} + \Psi_t\Bigg] \\
&- \theta_t^b \left[(1+\tau_t^C) P_t^C C_t + (1-\tau_t^I) \frac{P_t^I}{\gamma_t} I_t + P_t^K \triangle_t^K + \frac{B_{t+1}^{priv}}{R_t \zeta_t} + B_t^n + \frac{S_t B_{t+1}^{FH}}{R_{F,t} \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)} + T_t\right] \\
&+ \xi_w \beta_{t+1} \left[\zeta_{t+1}^c u(\tilde{C}_{t+1}, \tilde{C}_t) - \zeta_{t+1}^n \Theta_{t+1}^n A_n \int_0^1 \frac{N_{h,t+1}^{1+\eta}}{1+\eta} dh\right] \\
&+ \theta_{t+1}^b \Bigg[(1-\tau_{t+1}^W) \int_0^1 W_{h,t+1} N_{h,t+1} dh + \left(1-\tau_{t+1}^K\right)\left(R_{t+1}^K u_{t+1} K_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) K_{t+1}\right) + \iota^K \tau_{t+1}^K \delta P_t^K K_{t+1} + \\
&+ B_{t+1}^{priv} + \left(\alpha_B + \left(R_t^B - 1\right)\right) B_{t+1} + S_{t+1} B_{t+1}^{FH} + (1-\tau_{t+1}^{TR}) TR_{t+1} + \Xi_{B,t+1} + \Xi_{B^{FH},t+1} + \Psi_{t+1}\Bigg] \\
&- \theta_{t+1}^b \left[(1+\tau_{t+1}^C) P_{t+1}^C C_{t+1} + (1-\tau_{t+1}^I) \frac{P_{t+1}^I}{\gamma_{t+1}} I_{t+1} + P_{t+1}^K \triangle_{t+1}^K + \frac{B_{t+2}^{priv}}{R_{t+1} \zeta_{t+1}} + \frac{S_{t+1} B_{t+2}^{FH}}{R_{F,t+1} \Phi(\overline{a}_{t+1}, s_{t+1}, \widetilde{\phi}_{t+1})} + T_{t+1}\right] \\
&+ (\xi_w)^2 \beta_{t+2} \left[\zeta_{t+2}^c u(\tilde{C}_{t+2}, \tilde{C}_{t+1}) - \zeta_{t+2}^n \Theta_{t+2}^n A_n \int_0^1 \frac{N_{h,t+2}^{1+\eta}}{1+\eta} dh\right] \\
&+ \theta_{t+2}^b \Bigg[(1-\tau_{t+2}^W) \int_0^1 W_{h,t+2} N_{h,t+2} dh + \left(1-\tau_{t+2}^K\right)\left(R_{t+2}^K u_{t+2} K_{h,t+2} - \frac{P_{t+2}^I}{\gamma_{t+2}} a(u_{t+2}) K_{t+2}\right) + \iota^K \tau_{t+2}^K \delta P_{t+1}^K K_{t+2} + \\
&+ B_{t+2}^{priv} + \left(\alpha_B + \left(R_{t+1}^B - 1\right)\right) B_{t+2} + S_{t+2} B_{t+2}^{FH} + (1-\tau_{t+2}^{TR}) TR_{t+2} + \Xi_{B,t+2} + \Xi_{B^{FH},t+2} + \Psi_{t+2}\Bigg] \\
&- \theta_{t+2}^b \left[(1+\tau_{t+2}^C) P_{t+2}^C C_{t+2} + (1-\tau_{t+2}^I) \frac{P_{t+2}^I}{\gamma_{t+2}} I_{t+2} + P_{t+2}^K \triangle_{t+2}^K + \frac{B_{t+3}^{priv}}{R_{t+2} \zeta_{t+2}} + \frac{S_{t+2} B_{t+3}^{FH}}{R_{F,t+2} \Phi(\overline{a}_{t+2}, s_{t+2}, \widetilde{\phi}_{t+2})} + T_{t+2}\right] \\
&+ \dots
\end{aligned} \tag{C.52}$$

We take the first derivative of $\mathscr{L}_t^W$ with respect to $W_{h,t}^{opt}$, and we obtain the following equation:

$$\begin{aligned}
\frac{\partial \mathscr{L}_{h,t}^W}{\partial W_{h,t}^{opt}} &= \beta_t E_t[-\zeta_t^n \nu'(N_{h,t|t}) \frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}} + \theta_t^b (1-\tau_t^W) \left(N_{h,t|t} + W_{h,t}^{opt} \frac{\partial N_{h,t|t}}{\partial W_{h,t|t}^{opt}} \right)] \\
&+ \xi_w \; E_t \beta_{t+1}[-\zeta_{t+1}^n \nu'(N_{h,t+1|t}) \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}} \\
&+ \theta_{t+1}^b (1-\tau_{t+1}^W) \left(N_{h,t+1|t} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}} + W_{h,t+1|t} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}} \right)] \\
&+ (\xi_w)^2 E_t \beta_{t+2}[-\zeta_{t+2}^n \nu'(N_{h,t+2|t}) \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}} \\
&+ \theta_{t+2}^b (1-\tau_{t+2}^W) \left(N_{h,t+2|t} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}} + W_{h,t+2|t} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}} \right)] + \ldots = 0.
\end{aligned} \tag{C.53}$$

We use the following definition: $\beta_{t+k}^r = \frac{\beta_{t+k}}{\beta_{t+k-1}}$, and then we rearrange the above equation. Note that $\frac{\beta_{t+2}}{\beta_t} = \beta_{t+1}^r \beta_{t+2}^r$. We have the following equation:

$$\begin{aligned}
0 &= \theta_t^b E_t[W_{h,t}^{opt} \frac{\partial N_{h,t}}{\partial W_{h,t}^{opt}} (1-\tau_t^W) \left(\frac{N_{h,t|t}}{W_{h,t}^{opt}} \left(\frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}} \right)^{-1} + 1 \right) - \zeta_t^n \frac{\nu'(N_{h,t|t})}{\theta_t^b} \frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}}] \\
&+ \xi_w E_t \beta_{t+1}^r \theta_{t+1}^b [W_{h,t+1|t} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}} (1-\tau_{t+1}^W) \left(\frac{N_{h,t+1|t}}{W_{h,t+1|t}} \left(\frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \right)^{-1} + 1 \right) \\
&- \zeta_{t+1}^n \frac{\nu'(N_{h,t+1|t})}{\theta_{t+1}^b} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}}] \\
&+ (\xi_w)^2 E_t \beta_{t+1}^r \beta_{t+2}^r \theta_{t+2}^b [W_{h,t+2|t} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}} (1-\tau_{t+2}^W) \left(\frac{N_{h,t+2|t}}{W_{h,t+2|t}} \left(\frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \right)^{-1} + 1 \right) \\
&- \zeta_{t+2}^n \frac{\nu'(N_{h,t+2|t})}{\theta_{t+2}^b} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}}] + \ldots
\end{aligned} \tag{C.54}$$

Recall, we have the following definition: $W_{h,t+k|t} = W_{h,t}^{opt} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W \ldots \overline{\Pi}_{t+k}^W$. Thus, the partial derivative of $W_{h,t+k|t}$ with respect to $W_{h,t}^{opt}$ is:

$$\frac{\partial W_{h,t+k|t}}{\partial W_{h,t}^{opt}} = \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W \ldots \overline{\Pi}_{t+k}^W. \tag{C.55}$$

Using Equation (C.55), Equation (C.54) can be written as:

$$\begin{aligned}
0 &= \theta_t^b E_t[W_{h,t}^{opt} \frac{\partial N_{h,t}}{\partial W_{h,t}^{opt}} (1-\tau_t^W) \left(\frac{N_{h,t|t}}{W_{h,t}^{opt}} \left(\frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}} \right)^{-1} + 1 \right) - \zeta_t^n \frac{\nu'(N_{h,t|t})}{\theta_t^b} \frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}}] \\
&+ \xi_w E_t \beta_{t+1}^r \theta_{t+1}^b [W_{h,t+1|t} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \overline{\Pi}_{t+1}^W (1-\tau_{t+1}^W) \left(\frac{N_{h,t+1|t}}{W_{h,t+1|t}} \left(\frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \right)^{-1} + 1 \right) \\
&- \zeta_{t+1}^n \frac{\nu'(N_{h,t+1|t})}{\theta_{t+1}^b} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \overline{\Pi}_{t+1}^W] \\
&+ (\xi_w)^2 E_t \beta_{t+1}^r \beta_{t+2}^r \theta_{t+2}^b [W_{h,t+2|t} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W (1-\tau_{t+2}^W) \left(\frac{N_{h,t+2|t}}{W_{h,t+2|t}} \left(\frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \right)^{-1} + 1 \right) \\
&- \zeta_{t+2}^n \frac{\nu'(N_{h,t+2|t})}{\theta_{t+2}^b} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W] + \ldots
\end{aligned} \tag{C.56}$$

Using the labor demand schedule, which is captured by Equation (C.50), we can find the following wage-elasticity of labor demand:

$$-\frac{\partial N_{h,t+k|t}}{\partial W_{h,t+k|t}} \frac{W_{h,t+k|t}}{N_{h,t+k|t}} = \varepsilon_{w,t}. \tag{C.57}$$

Using the following definition: $\varepsilon_{w,t} = \frac{\lambda_t^W}{\lambda_t^W - 1}$ and the result from Equation (C.57), we have the following equation:

$$\frac{\partial N_{h,t+k|t}}{\partial W_{h,t+k|t}} \frac{W_{h,t+k|t}}{N_{h,t+k|t}} = \frac{\lambda_t^W}{1-\lambda_t^W}. \tag{C.58}$$

Using the result from Equation (C.58), the derivative of $N_{h,t+k|t}$ with respect to $W_{h,t+k|t}$ is:

$$\frac{\partial N_{h,t+k|t}}{\partial W_{h,t+k|t}} = \frac{\lambda_t^W}{1-\lambda_t^W}\frac{N_{h,t+k|t}}{W_{h,t+k|t}}. \tag{C.59}$$

We use Equation (C.58) and Equation (C.59); hence, Equation (C.56) can be expressed as:

$$\begin{aligned}
0 &= \theta_t^b E_t[\frac{\lambda_t^W}{(1-\lambda_t^W)}\frac{N_{h,t|t}}{W_{h,t}^{opt}}\left[(1-\tau_t^W)W_{h,t}^{opt}\frac{1}{\lambda_t^W} - \zeta_t^n\frac{\nu'(N_{h,t|t})}{\theta_t^b}\right]] \\
&+ \xi_w E_t\ \beta_{t+1}^r\theta_{t+1}^b[\frac{\lambda_{t+1}^W}{(1-\lambda_{t+1}^W)}\frac{N_{h,t+1|t}}{W_{h,t+1|t}}\overline{\Pi}_{t+1}^W\left[(1-\tau_{t+1}^W)W_{h,t+1|t}\frac{1}{\lambda_{t+1}^W} - \zeta_{t+1}^n\frac{\nu'(N_{h,t+1|t})}{\theta_{t+1}^b}\right]] \\
&+ (\xi_w)^2 E_t\ \beta_{t+1}^r\beta_{t+2}^r\theta_{t+2}^b[\frac{\lambda_{t+2}^W}{(1-\lambda_{t+2}^W)}\frac{N_{h,t+2|t}}{W_{h,t+2|t}}\overline{\Pi}_{t+1}^W\overline{\Pi}_{t+2}^W\left[(1-\tau_{t+2}^W)W_{h,t+2|t}\frac{1}{\lambda_{t+2}^W} - \zeta_{t+2}^n\frac{\nu'(N_{h,t+2|t})}{\theta_{t+2}^b}\right]] + \ldots
\end{aligned}$$

We use the following definition: $W_{h,t+k|t} = W_{h,t}^{opt}\overline{\Pi}_{t+1}^W\overline{\Pi}_{t+2}^W\ldots\overline{\Pi}_{t+k}^W$, and we multiply both sides of the above equation by $W_{h,t}^{opt}$. We have following equation:

$$\begin{aligned}
0 &= \theta_t^b\frac{1}{1-\lambda_t^W}E_t[N_{h,t|t}\left[(1-\tau_t^W)W_{h,t}^{opt} - \lambda_t^W\zeta_t^n\frac{\nu'(N_{h,t|t})}{\theta_t^b}\right]] \\
&+ \xi_w E_t\ \beta_{t+1}^r\theta_{t+1}^b\frac{1}{1-\lambda_{t+1}^W}[N_{h,t+1|t}\left[(1-\tau_{t+1}^W)W_{h,t+1|t} - \lambda_{t+1}^W\zeta_{t+1}^n\frac{\nu'(N_{h,t+1|t})}{\theta_{t+1}^b}\right]] \\
&+ (\xi_w)^2 E_t\ \beta_{t+1}^r\beta_{t+2}^r\theta_{t+2}^b\frac{1}{1-\lambda_{t+2}^W}[N_{h,t+2|t}\left[(1-\tau_{t+2}^W)W_{h,t+2|t} - \lambda_{t+2}^W\zeta_{t+2}^n\frac{\nu'(N_{h,t+2|t})}{\theta_{t+2}^b}\right]] + \ldots
\end{aligned}$$

Using the following definitions: $\prod_{i=1}^k \beta_{t+i}^r = \beta_{t+1}^r\beta_{t+2}^r\ldots\beta_{t+k}^r$ and $\prod_{i=1}^0 \beta_{t+i}^r = 1$, the above equation can be written as:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)N_{h,t+k|t}\theta_{t+k}^b\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)W_{h,t+k|t} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(N_{h,t+k|t})}{\theta_{t+k}^b}\right] = 0. \tag{C.60}$$

Equation (C.60) is the FOC for $W_{h,t}^{opt}$, which is the optimal wage decision by a household member with labor type h. Equation (C.60) is the same as Equation (22) in Section 2.1.5.

Since all labor types in the household face the same optimization problem, we can drop the subscript h from the above equation. Thus, the optimal wage setting condition can be rewritten as:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}\theta_{t+k}^b\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)W_{t+k|t} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{\theta_{t+k}^b}\right] = 0. \tag{C.61}$$

Using the following definition: $\theta_{t+k}^b\ P_{t+k}^C = \Omega_{t+k}^C$, the above equation can be written as follows:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}\theta_{t+k}^b P_{t+k}^C\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)\frac{W_{t+k|t}}{P_{t+k}^C} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{\theta_{t+k}^b P_{t+k}^C}\right] = 0,$$

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}\Omega_{t+k}^C\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)\frac{W_{t+k|t}}{P_{t+k}^C} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{\Omega_{t+k}^C}\right] = 0. \tag{C.62}$$

We continue the stationarization of Equation (C.62) by using the following definitions: $\overline{w}_{t+k|t} = \frac{W_{t+k|t}}{z_{t+k}^+ P_{t+k}^C}$ and $\overline{\Omega}_{t+k}^C = z_{t+k}^+\Omega_{t+k}^C$. The above equation can be expressed as:

$$E_t\sum_{k=1}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}z_{t+k}^+\Omega_{t+k}^C\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)\frac{W_{t+k|t}}{z_{t+k}^+P_{t+k}^C} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{z_{t+k}^+\Omega_{t+k}^C}\right] = 0,$$

and we have the following stationarized version of the optimal wage setting equation:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}\overline{\Omega}_{t+k}^C\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)\overline{w}_{t+k|t} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{\overline{\Omega}_{t+k}^C}\right] = 0. \tag{C.63}$$

Equation (C.63) is the same as Equation (A.13a), which shows the stationarized version of the optimal wage setting equation. We assume that Non-Ricardian households set their wage rate equal to the average wage rate of Ricardian households and face identical labor demand, this assumption implies that Ricardian and Non-Ricardian households will have the same wage rate and supply the same amount of labor.

C.1.10 Non-Ricardian Household

We assume that Non-Ricardian households has the same wage, employment and labor supply as Ricardian households. Hence, we have the following results:

$$\begin{aligned} W_{m,t} &= W_t \\ n_{m,t} &= n_t \\ l_{m,t} &= l_t \end{aligned}$$

Since Non-Ricardian households are not able to save, each Non-Ricardian household m sets her nominal consumption expenditure equal to after-tax disposable wage income plus transfers. We have the following the nominal consumption expenditure for Non-Ricardian household m:

$$\left(1+\tau_t^C\right) P_t^C C_{m,t} = \left(1-\tau_t^W\right) W_{m,t} N_{m,t} + \left(1-\tau_t^{TR}\right) TR_{m,t}.$$

We can drop subscript m from the above equation since we assume all Non-Ricardian households face the same budget constraint and will choose the same level of consumption. We denote C_t^{nr} as aggregate Non-Ricardian household consumption, and TR_t^{nr} as aggregate transfers to Non-Ricardian households. The above equation can be written as:

$$\left(1+\tau_t^C\right) P_t^C C_t^{nr} = \left(1-\tau_t^W\right) W_t N_t + \left(1-\tau_t^{TR}\right) TR_t^{nr}.$$

We can express the above equation in *per capita* terms. Especially, we define $\overline{c}_t^{nr}$ as the stationarized aggregate Non-Ricardian household consumption in *per capita* terms, $\overline{tr}_t^{nr}$ is transfers to Non-Ricardian households in *per capita* terms, and n_t is aggregate employment per capita (employment rate). We use the following definitions to stationarize the above equation: $\overline{c}_t^{nr} = \frac{c_t^{nr}}{z_t^+}$, $\overline{tr}_t^{nr} = \frac{tr_t^{nr}}{P_t z_t^+}$, $p_t^C = \frac{P_t^C}{P_t}$, $\overline{w}_t^e = \frac{W_t}{z_t^+ P_t}$. The above equation can be rewritten as:

$$\left(1+\tau_t^C\right) p_t^C \overline{c}_t^{nr} = \left(1-\tau_t^W\right) \overline{w}_t^e n_t + \left(1-\tau_t^{TR}\right) \overline{tr}_t^{nr}. \tag{C.64}$$

Equation (C.64) is the same as Equation (A.24a), which captures the stationarized aggregate Non-Ricardian household consumption.

C.1.11 Aggregation of households

Recall, snr is a share of Non-Ricardian households over total population, and we denote C_t^{agg} as aggregate household consumption. Aggregate private consumption C_t^{agg} is a sum of aggregate Ricardian household consumption and aggregate Non-Ricardian household consumption, which is written as:

$$C_t^{agg} = \int_0^{1-s_{nr}} C_{k,t} dh + \int_{1-s_{nr}}^1 C_{m,t} dm.$$

C_t is aggregate Ricardian household consumption and C_t^{nr} as aggregate Non-Ricardian household consumption. Aggregate private consumption can be written as:

$$C_t^{agg} = (1-s_{nr}) C_t + s_{nr} C_t^{nr}.$$

We can express the above equation in *per capita* terms. Especially, we define $\overline{c}_t^{nr}$ as the stationarized aggregate Non-Ricardian household consumption in *per capita* terms, $\overline{c}_t$ as the stationarized aggregate Ricardian household consumption in *per capita* terms, and $\overline{c}_t^{agg}$ as the stationarized aggregate household consumption in *per capita* terms. As in Section C.1.10, we can stationarize the above equation by using the following definitions: $\overline{c}_t^{agg} = \frac{c_t^{agg}}{z_t^+}$, $\overline{c}_t = \frac{c_t}{z_t^+}$, and $\overline{c}_t^{nr} = \frac{c_t^{nr}}{z_t^+}$. Thus, we have the following equation:

$$\overline{c}_t^{agg} = (1-s_{nr}) \overline{c}_t + s_{nr} \overline{c}_t^{nr}. \tag{C.65}$$

Equation (C.65) is the same as Equation (A.23a), which captures the stationarized aggregate private consumption equation.

Aggregate transfers TR_t is a sum of transfers to Ricardian and Non-Ricardian households:

$$TR_t^{agg} = \int_0^{1-s_{nr}} TR_{k,t}dk + \int_{1-s_{nr}}^{1} TR_{m,t}dm.$$

We denote TR_t^{agg} as aggregate transfers, TR_t^{nr} as aggregate transfers to Non-Ricardian households, and TR_t as aggregate transfers to Ricardian households. Thus, aggregate transfer equation TR_t^{agg} can be expressed as:

$$TR_t^{agg} = (1 - s_{nr})TR_t + s_{nr}TR_t^{nr}.$$

We can express the above equation in *per capita* terms. Especially, $\overline{tr}_t^{nr}$ is transfers to Non-Ricardian households in *per capita* terms, $\overline{tr}_t$ is transfers to Ricardian households in *per capita* terms, and $\overline{tr}_t^{agg}$ is aggregate transfers in *per capita* terms. We stationarize the above equation by using the following definitions: $\overline{tr}_t = \frac{tr_t}{P_t z_t^+}$, $\overline{tr}_t^{nr} = \frac{tr_t^{nr}}{P_t z_t^+}$ and $\overline{tr}_t^{agg} = \frac{tr_t^{agg}}{P_t z_t^+}$. Hence, we have the following equation:

$$\overline{tr}_t^{agg} = (1 - s_{nr})\overline{tr}_t + s_{nr}\overline{tr}_t^{nr}. \tag{C.66}$$

Equation (C.66) is the same as Equation (A.74a), which is the stationarized version of aggregate transfer equation.

The stationarized version of aggregate transfer distribution off steady state equation is given by:

$$\varpi_{dyn}\left(\overline{tr}_t - \overline{tr}\right) = (1 - \varpi_{dyn})(\overline{tr}_t^{nr} - \overline{tr}^{nr}). \tag{C.67}$$

Equation (C.67) is the same as Equation (A.75a).

Similarly, the transfer distribution in steady state equation is expressed as:

$$\varpi_{ss}\overline{tr} = (1 - \varpi_{ss})\overline{tr}^{nr}.$$

C.2 Intermediate good producers

In this section, first we derive the stationarized version of the real marginal cost of production for intermediate good producers, Equation (A.26a). Second, we derive the stationarized version of the real rental rate for capital services, Equation (A.28a). There is a continuum of intermediate good producers of mass one, and i denotes the individual firm in the Swedish economy. Now, we present the optimization problem of intermediate good producers in the Swedish economy.

Firm i chooses capital services $K_t^s(i)$ and labor input $N_t(i)$ to minimize the following cost function:

$$TC_t(i) = R_t^K K_t^s(i) + \left(1 + \tau_t^{SSC}\right) W_t N_t(i) \tag{C.68}$$

subject to the production constraint:

$$Y_t(i) = \varepsilon_t \left[\tilde{K}_t^s(i)\right]^{\alpha} \left[z_t N_t(i)\right]^{1-\alpha} - z_t^+ \phi. \tag{C.69}$$

where τ_t^{SSC} denotes the social security – or payroll – tax paid by firms.

$\tilde{K}_t^s(i)$ denotes a composite capital input made up by private capital services $K_t^s(i)$ and public capital $K_{G,t}$. We assume the following constant elasticity of substitution (CES) aggregator of private capital services $K_t^s(i)$ and public capital stock $K_{G,t}(i)$:

$$\tilde{K}_t^s(i) = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(K_t^s(i)\right)^{\frac{\upsilon_K - 1}{\upsilon_K}} + (1 - \alpha_K)^{\frac{1}{\upsilon_K}} \left(K_{G,t}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K - 1}}.$$

Hence, we assume that each intermediate-good firm i has access to the same public capital stock. We also assume that public capital grows at the same speed as private capital services along the balanced growth path. The parameter υ_K is the elasticity of substitution between private capital services and the public capital stock, and α_K is a share parameter. For $\alpha_K = 1$ we obtain the standard production function without public capital stock. For $\upsilon_K \to 1$ the production function converges to a Cobb-Douglas specification.

We denote $\theta_t(i)$ as the Lagrange multiplier associated with the production constraint (C.69). To solve the optimization problem, we set up the following Lagrangian $\mathscr{L}_t(i)$:

$$\mathscr{L}_t(i) = R_t^K K_t^s(i) + \left(1+\tau_t^{SSC}\right) W_t N_t(i) - \theta_t(i)\left[\varepsilon_t\left[\tilde{K}_t^s(i)\right]^\alpha [z_t N_t(i)]^{1-\alpha} - z_t^+\phi - Y_t(i)\right], \tag{C.70}$$

where

$$\tilde{K}_t^s(i) = \left(\alpha_K{}^{\frac{1}{\upsilon_K}}\left(K_t^s(i)\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}}\left(K_{G,t}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K-1}}.$$

We take the partial derivative of $\mathscr{L}_t(i)$ with respect to $K_t^s(i)$ and $N_t(i)$ respectively, and we can find the FOC for $K_t^s(i)$ and $N_t(i)$.

The FOC for $K_t(i)$ is:

$$R_t^K - \alpha\theta_t(i)\varepsilon_t\frac{\tilde{K}_t^s(i)^\alpha}{K_t^s(i)}[z_t N_t(i)]^{1-\alpha}\alpha_K{}^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K-1}{\upsilon_K}} = 0. \tag{C.71}$$

The FOC for $N_t(i)$ is:

$$\left(1+\tau_t^{SSC}\right)W_t - \theta_t(i)(1-\alpha)\varepsilon_t\left[\tilde{K}_t^s(i)\right]^\alpha z_t^{1-\alpha}[N_t(i)]^{-\alpha} = 0. \tag{C.72}$$

Using Equation (C.71) and Equation (C.72), we obtain the following capital-labor input efficiency condition:

$$K_t^s(i) = \frac{\alpha}{1-\alpha}\frac{\left(1+\tau_t^{SSC}\right)W_t}{R_t^K}N_t(i)\alpha_K{}^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}^s{}_t(i)}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}. \tag{C.73}$$

Note that Equation (C.69) can be written as:

$$\left[Y_t(i) + z_t^+\phi\right] = \varepsilon_t\left[\tilde{K}_t^s(i)\right]^\alpha[z_t N_t(i)]^{1-\alpha}. \tag{C.74}$$

Now, we find the total cost of production equation. We substitute Equation (C.71) and Equation (C.72) into Equation (C.68), and we have the following equation:

$$TC_t(i) = \theta_t(i)\left[\alpha\varepsilon_t\tilde{K}_t^s(i)^\alpha[z_t N_t(i)]^{1-\alpha}\alpha_K{}^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + (1-\alpha)\varepsilon_t\left[\tilde{K}_t{}^s(i)\right]^\alpha z_t^{1-\alpha}[N_t(i)]^{1-\alpha}\right]. \tag{C.75}$$

Using Equation (C.74), we can rewrite Equation (C.75) as follows:

$$TC_t(i) = \theta_t(i)\left[(1-\alpha) + \alpha\alpha_K{}^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right]\left(Y_t(i) + z_t^+\phi\right). \tag{C.76}$$

We use Equation (C.76), and we take the partial derivative of $TC_t(i)$ with respect to $Y_t(i)$. Hence, the lagrangian multiplier, $\theta_t(i)$, can be defined as the marginal cost of production $MC_t(i)$:

$$\frac{\partial TC_t(i)}{\partial Y_t(i)} = MC_t(i) = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}\varepsilon_t}\left(\frac{(1+\tau_t^{SSC})W_t}{z_t}\right)^\alpha\left(\frac{K_t^s(i)}{\alpha_K\tilde{K}_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}}. \tag{C.77}$$

There are three equations that summarize optimal input decisions by firms: 1) the capital-labor input efficiency condition, 2) the optimal capital inputs in terms of marginal cost and 3) the composite capital equation. First, we find the capital-labor input efficiency equation. In particular, we rewrite Equation (C.73) as follows:

$$\frac{K_t^s(i)}{L_t(i)} = \frac{\alpha}{1-\alpha}\frac{\left(1+\tau_t^{SSC}\right)W_t}{R_t^K}\alpha_K{}^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}. \tag{C.78}$$

Equation (C.78) is the capital-labor input efficiency condition.

Second, we find the equation for the optimal capital input in terms of marginal cost. Using Equation (C.77) and Equation (C.78), Equation (C.71). can be written as

$$R_t^K = \alpha MC_t(i) \left[z_t\right]^{1-\alpha} \varepsilon_t \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} \left(\frac{\left(1+\tau_t^{SSC}\right) W_t}{R_t^K}\right)^{\alpha-1} \Gamma_{G,t} \tag{C.79}$$

$$\Gamma_{G,t} = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}},$$

$$R_t^K = \alpha MC_t(i) \left[z_t\right]^{1-\alpha} \varepsilon_t \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} \left(\frac{\left(1+\tau_t^{SSC}\right) W_t}{R_t^K}\right)^{\alpha-1} \Gamma_{G,t}$$

The above equation is the optimal capital input in terms of marginal cost.

Finally, we have the following composite capital function:

$$\tilde{K}_t^s(i) = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(K_t^s(i)\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}} \left(K_{G,t}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K-1}}.$$

We can simplify Equation (C.79) by letting: $\Gamma_{G,t}(i) = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}}$. Thus, Equation (C.79) can be rewritten as:

$$MC_t(i) = \frac{R_t^K}{\alpha z_t^{1-\alpha} \varepsilon_t \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} ((1+\tau_t^{SSC}) W_t)^{\alpha-1} (R_t^K)^{1-\alpha} \Gamma_{G,t}}.$$

The above equation can be expressed as:

$$MC_t(i) = \frac{\left(\frac{(1+\tau_t^{SSC}) W_t}{z_t}\right)^{1-\alpha} \left(R_t^K\right)^{\alpha}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} \varepsilon_t \Gamma_{G,t}(i)}. \tag{C.80}$$

Equation (C.80), which is the same as Equation (27) in Section 2.4.1, is the nominal marginal cost of production for the intermediate good firm i.

Firms hiring from homogeneous labor and private markets, i.e. face the same wage and rental rates. In the standard model without public capital, this implies that marginal costs are identical across firms. With added public capital, the expression $\Gamma_{G,t}(i)$ in principle would make marginal costs different across firms. For simplicity, we assume that each firm uses a constant private to public capital ratio in its production. This means that the amount of private capital services operated is proportional and constant in relation to the amount of public capital used. For example, the number of plants operated by a firm requires the same number of roads to get to the plants. With this assumption, marginal costs are identical across firms, and thus we can drop the subscript i. Equation (C.80) can be written as:

$$MC_t = \frac{\left(\frac{(1+\tau_t^{SSC}) W_t}{z_t}\right)^{1-\alpha} \left(R_t^K\right)^{\alpha}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} \varepsilon_t \Gamma_{G,t}}, \tag{C.81}$$

where

$$\Gamma_{G,t} = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}},$$

Equation (C.81) is the non-stationarized version of the nominal marginal cost of production for intermediate good firm.

Next, we derive the non-stationarized version of the rental rate for capital services. Using Equation (C.71) and that the lagrange multiplier $\theta_t(i)$ equals the marginal cost $MC_t(i)$, we obtain the following equation:

$$R_t^K = \alpha \varepsilon_t z_t^{1-\alpha} MC_t(i) \left(\frac{\tilde{K}_t^s(i)}{N_t(i)}\right)^{\alpha-1} \left(\Gamma_{G,t}(i)\right)^{\frac{1}{\alpha}}, \tag{C.82}$$

where

$$\Gamma_{G,t} = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}},$$

Equation (C.82), which captures the nominal rental rate for capital services, is the same as Equation (28) in Section 2.4.1.

We assume identical capital labor ratios, identical marginal costs and identical private to public capital ratios. This means that we can drop the subscript i and rewrite equation (C.82) as:

$$R_t^K = \alpha \varepsilon_t z_t^{1-\alpha} MC_t \left(\frac{\tilde{K}_t^s}{N_t} \right)^{\alpha-1} (\varGamma_{G,t})^{\frac{1}{\alpha}} . \tag{C.83}$$

Equation (C.83), captures the non-stationarized version of rental rate for capital services.

Now, we find the stationarized version of the marginal cost of production for intermediate good producers. We stationarize Equation (C.81) by applying the following definitions: $r_t^K = \frac{\gamma_t R_t^K}{P_t}$, $\overline{w}_t^e = \frac{W_t}{z_t^+ P_t}$, $z_t^+ = z_t (\gamma_t)^{\frac{\alpha}{1-\alpha}}$, and $\overline{mc}_t = \frac{MC_t}{P_t}$. Stationarizing $\varGamma_{G,t}$ is trivial as private and public capital services have the same growth rate along a balanced growth path. Equation (C.81) can be written as follows:

$$\frac{MC_t}{P_t} = \frac{\left(\frac{(1+\tau_t^{SSC}) W_t}{z_t} \right)^{1-\alpha} \left(\frac{1}{P_t} \right)^{1-\alpha} \left(\frac{1}{P_t} \right)^{\alpha} \left(R_t^K \right)^{\alpha} \frac{\gamma_t^{\alpha}}{\gamma_t^{\alpha}}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} \varepsilon_t \varGamma_{G,t}},$$

$$\overline{mc}_t = \frac{\left(\frac{(1+\tau_t^{SSC}) W_t}{z_t (\gamma_t)^{\alpha/(1-\alpha)} P_t} \right)^{1-\alpha} \left(\frac{\gamma_t R_t^K}{P_t} \right)^{\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} \varepsilon_t \bar{\varGamma}_{G,t}},$$

$$\overline{mc}_t = \frac{\left(\frac{(1+\tau_t^{SSC}) W_t}{z_t^+ P_t} \right)^{1-\alpha} \left(\frac{\gamma_t R_t^K}{P_t} \right)^{\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} \varepsilon_t \bar{\varGamma}_{G,t}}.$$

Thus, the stationarized version of the marginal cost of production for intermediate good producers (the real marginal cost of production) equation can be expressed as:

$$\overline{mc}_t = \frac{\left(\left(1+\tau_t^{SSC}\right) \overline{w}_t^e \right)^{1-\alpha} (r_t^K)^{\alpha}}{\varepsilon_t \alpha^{\alpha} (1-\alpha)^{1-\alpha} \bar{\varGamma}_{G,t}}. \tag{C.84}$$

Equation (C.84), which represents the real marginal cost of production for intermediate good producers, is the same as Equation (A.26a) in Section A.2.

Lastly, we find the stationarized version of the rental rate for capital services. We stationarize Equation (C.83) by applying the following definitions: $r_t^K = \frac{\gamma_t R_t^K}{P_t}$, $z_t^+ \gamma_t = z_t \gamma_t^{1/1-\alpha}$, $\bar{K}_t^s = \frac{K_t^s}{z_{t-1}^+ \gamma_{t-1}}$, and $\overline{mc}_t = \frac{MC_t}{P_t}$. We follow the similar steps when deriving the stationarized version of the marginal cost for intermediate good producers equation. Hence, Equation (C.83) can be written as:

$$r_t^K = \alpha \varepsilon_t \left(\frac{\bar{\tilde{K}}_t^s}{N_t} \frac{1}{\mu_{z^+,t} \mu_{\gamma,t}} \right)^{\alpha-1} \overline{mc}_t \, (\varGamma_{G,t})^{\frac{1}{\alpha}}$$

Furthermore, we can rewrite the above equation in terms of *per capita*, so we denote $\bar{\tilde{k}}_t^s$ as stationarized capital services *per capita*, and n_t as aggregate labor input *per capita*. Hence, we can rewrite the above equation as:

$$r_t^K = \alpha \varepsilon_t \left(\frac{\bar{\tilde{k}}_t^s}{n_t} \frac{1}{\mu_{z^+,t} \mu_{\gamma,t}} \right)^{\alpha-1} \overline{mc}_t \, (\varGamma_{G,t})^{\frac{1}{\alpha}} \tag{C.85}$$

Equation (C.85), which is the real rental rate for capital services equation, is the same as Equation (A.28a) in Section A.2.

Finally, the equation for composite capital in stationary form is written as:

$$\bar{\tilde{k}}_t^s = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(\bar{k}_t^s \right)^{\frac{\upsilon_K - 1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}} \left(\bar{k}_{G,t} \right)^{\frac{\upsilon_K - 1}{\upsilon_K}} \right)^{\frac{\upsilon_K}{\upsilon_K - 1}} \tag{C.86}$$

Equation (C.86) is the same as Equation (A.29a) in Section A.2.

Note that, the law of motion for public capital is given by:

$$K_{G,t+1} = (1-\delta_G)K_{G,t} + I_t^G.$$

We stationarize the law of motion for public capital by dividing the non-stationarized function by $\gamma_t z_t^+$, and we have the following equation:

$$\frac{K_{G,t+1}}{\gamma_t z_t^+} = (1-\delta_G)\frac{K_{G,t}}{\gamma_t z_t^+} + \frac{I_t^G}{\gamma_t z_t^+}$$

The above equation can be written in *per capita* terms:

$$\bar{k}_{G,t+1} = (1-\delta_G)\bar{k}_{G,t}\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}} + \bar{I}_t^G. \tag{C.87}$$

Equation (C.87) is the same as Equation (A.30a) in Section A.2.

C.2.1 Optimal price of intermediate goods

In this section, we derive the stationarized version of the optimal price of intermediate goods equation (A.31a). In this section, firm i chooses the optimal price $P_t^{opt}(i)$ that maximizes its profit, subject to its demand schedule and the Calvo price contract. In each period, the individual firm i resets its price with probability $(1-\xi)$. With probability ξ, the firm cannot reset its price, and then it faces the following price evolution: $P_{t+k|t}(i) = P_t^{opt}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}$. We define the stochastic discount factor as $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega_{t+k}^C}{\Omega_t^C}\frac{P_t^C}{P_{t+k}^C}$.

Firm i chooses the optimal price of intermediate goods $P_t^{opt}(i)$ to maximize the following profit function:

$$\underset{P_t^{opt}(i)}{max}\; E_t\sum_{k=0}^{\infty}(\xi)^k\Lambda_{t,t+k}\left\{P_{t+k|t}(i)Y_{t+k|t}(i) - TC_{t+k|t}\left[Y_{t+k|t}(i)\right]\right\}\upsilon_K \tag{C.88}$$

subject to the demand function:

$$Y_{t+k|t}(i) = \left(\frac{P_{t+k|t}(i)}{P_{t+k}}\right)^{\frac{\lambda_{t+k}}{1-\lambda_{t+k}}} Y_{t+k}, \tag{C.89}$$

and the Calvo price setting contract:

$$P_{t+k}(i) = \begin{cases}\overline{\Pi}_{t+k}P_{t+k-1}(i) & \text{with probability } \xi \\ P_{t+k}^{opt}(i) & \text{with probability } (1-\xi).\end{cases} \tag{C.90}$$

The FOC for $P_t^{opt}(i)$ is:

$$\begin{aligned}
&E_t\{Y_{t|t}(i) + P_t^{opt}(i)\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)} - MC_t(i)\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)} \\
&\quad + \xi\Lambda_{t,t+1}\left[\frac{\partial P_{t+1|t}(i)}{\partial P_t^{opt}(i)}Y_{t+1|t}(i) + P_{t+1|t}(i)\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\frac{\partial P_{t+1|t}(i)}{\partial P_t^{opt}(i)} - MC_{t+1}(i)\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\frac{\partial P_{t+1|t}(i)}{\partial P_t^{opt}(i)}\right] \\
&\quad + (\xi)^2\Lambda_{t,t+2} \\
&\quad \left[\frac{\partial P_{t+2|t}(i)}{\partial P_t^{opt}(i)}Y_{t+2|t}(i) + P_{t+2|t}(i)\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\frac{\partial P_{t+2|t}(i)}{\partial P_t^{opt}(i)} - MC_{t+2}(i)\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\frac{\partial P_{t+2|t}(i)}{\partial P_t^{opt}(i)}\right] \\
&\quad + \ldots\} = 0.
\end{aligned} \tag{C.91}$$

Recall, we have the following definition: $P_{t+k|t}(i) = P_t^{opt}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}$. Hence, the partial derivative of $P_{t+k|t}(i)$ with respect to $P_t^{opt}(i)$ is:

$$\frac{\partial P_{t+k|t}(i)}{\partial P_t^{opt}(i)} = \overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}. \tag{C.92}$$

Using Equation (C.92), Equation (C.91) can be rewritten as:

$$
\begin{aligned}
&E_t\{Y_{t|t}(i) + P_t^{opt}(i)\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)} - MC_t(i)\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)} \\
&+ \xi \Lambda_{t,t+1}\left[\overline{\Pi}_{t+1}Y_{t+1|t}(i) + P_{t+1|t}(i)\overline{\Pi}_{t+1}\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)} - MC_{t+1}(i)\overline{\Pi}_{t+1}\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\right] \\
&+ (\xi)^2 \Lambda_{t,t+2} \\
&\left[\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}Y_{t+2|t}(i) + P_{t+2|t}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)} - MC_{t+2}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\right] \\
&+ \ldots\} = 0.
\end{aligned}
$$

Based on Equation (C.81), we have the following result: $MC_{t+k}(i) = MC_{t+k}$. We rearrange the above equation, and we obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)}\left[P_t^{opt}(i)\left(\frac{Y_{t|t}(i)}{P_t^{opt}(i)}\left(\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)}\right)^{-1} + 1\right) - MC_t\right] \\
&+ \xi \Lambda_{t,t+1}\overline{\Pi}_t\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\left[P_{t+1|t}(i)\left(\frac{Y_{t+1|t}(i)}{P_{t+1|t}(i)}\left(\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\right)^{-1} + 1\right) - MC_{t+1}\right] \\
&+ (\xi)^2 \Lambda_{t,t+2}\overline{\Pi}_t\overline{\Pi}_{t+1}\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\left[P_{t+2|t}(i)\left(\frac{Y_{t+2|t}(i)}{P_{t+2|t}(i)}\left(\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\right)^{-1} + 1\right) - MC_{t+2}\right] + \ldots\} = 0.
\end{aligned} \tag{C.93}
$$

Given the demand schedule for intermediate goods, which is captured by Equation (C.89), we can find the following price elasticity of demand for intermediate goods:

$$
-\frac{\partial Y_{t+k|t}(i)}{\partial P_{t+k|t}(i)}\frac{P_{t+k|t}(i)}{Y_{t+k|t}(i)} = \frac{\lambda_{t+k}}{\lambda_{t+k} - 1}. \tag{C.94}
$$

Using the result from Equation (C.94), the derivative of $Y_{t+k|t}(i)$ with respect to $P_{t+k|t}(i)$ is:

$$
\frac{\partial Y_{t+k|t}(i)}{\partial P_{t+k|t}(i)} = \frac{\lambda_{t+k}}{1 - \lambda_{t+k}}\frac{Y_{t+k|t}(i)}{P_{t+k|t}(i)}. \tag{C.95}
$$

Using Equation (C.94) and Equation (C.95), we can rewrite Equation (C.93) as follows:

$$
\begin{aligned}
&E_t\{\frac{Y_{t|t}(i)}{P_t^{opt}(i)}\frac{\lambda_t}{1 - \lambda_t}\left[P_t^{opt}(i)\frac{1}{\lambda_t} - MC_t\right] \\
&+ \xi \Lambda_{t,t+1}\overline{\Pi}_{t+1}\frac{Y_{t+1|t}(i)}{P_{t+1|t}(i)}\frac{\lambda_{t+1}}{1 - \lambda_{t+1}}\left[P_{t+1|t}(i)\frac{1}{\lambda_{t+1}} - MC_{t+1}\right] \\
&+ (\xi)^2 \Lambda_{t,t+2}\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\frac{Y_{t+2|t}(i)}{P_{t+2|t}(i)}\frac{\lambda_{t+2}}{1 - \lambda_{t+2}}\left[P_{t+2|t}(i)\frac{1}{\lambda_{t+2}} - MC_{t+2}\right] + \ldots\} = 0.
\end{aligned}
$$

We use the following definition: $P_{t+k|t}(i) = P_t^{opt}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}$. We multiply both sides of the above equation by $P_t^{opt}(i)$ and -1. We can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{Y_{t|t}(i)}{\lambda_t - 1}\left[P_t^{opt}(i) - \lambda_t MC_t\right] \\
&+ \xi \Lambda_{t,t+1}\frac{Y_{t+1|t}(i)}{\lambda_{t+1} - 1}\left[P_{t+1|t}(i) - \lambda_{t+1}MC_{t+1}\right] \\
&+ (\xi)^2 \Lambda_{t,t+2}\frac{Y_{t+2|t}(i)}{\lambda_{t+2} - 1}\left[P_{t+2|t}(i) - \lambda_{t+2}MC_{t+2}\right] + \ldots\} = 0.
\end{aligned}
$$

We can drop the subscript i from the above equation because when firms can reset their price, they will set the same price. As a result, all firms will choose the same quantity of output. We rewrite the above equation, and the optimal price of intermediate goods equation can be expressed as:

$$
E_t\sum_{k=0}^{\infty}(\xi)^k \Lambda_{t,t+k}\frac{Y_{t+k|t}}{(\lambda_{t+k} - 1)}\left[P_{t+k|t} - \lambda_{t+k}MC_{t+k}\right] = 0. \tag{C.96}
$$

Equation (C.96), which is the non-stationarized version of the optimal price of intermediate goods equation, is the same as Equation (31) in Section 2.4.1.

Now, we would like to derive the stationarized version of the optimal price of intermediate goods equation. We use the following definition: $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega^C_{t+k}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+k}}$, and then we expand Equation (C.96). Hence, we have the following equation:

$$\begin{aligned}
&E_t\{\frac{Y_{t|t}}{(\lambda_t-1)}\left[P^{opt}_t-\lambda_t MC_t\right]\\
&+\xi\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{Y_{t+1|t}}{(\lambda_{t+1}-1)}\left[P_{t+1|t}-\lambda_{t+1}MC_{t+1}\right]\\
&+(\xi)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{Y_{t+2|t}}{(\lambda_{t+2}-1)}\left[P_{t+2|t}-\lambda_{t+2}MC_{t+2}\right]+\ldots\}=0.
\end{aligned}$$

We multiply the third term of the above equation by $\frac{P^C_{t+1}}{P^C_{t+1}}$, and we obtain the following equation:

$$\begin{aligned}
&E_t\{\frac{Y_{t|t}}{(\lambda_t-1)}\left[P^{opt}_t-\lambda_t MC_t\right]\\
&+\xi\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{Y_{t+1|t}}{(\lambda_{t+1}-1)}\left[P_{t+1|t}-\lambda_{t+1}MC_{t+1}\right]\\
&+(\xi)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{Y_{t+2|t}}{(\lambda_{t+2}-1)}\left[P_{t+2|t}-\lambda_{t+2}MC_{t+2}\right]+\ldots\}=0.
\end{aligned}$$

We use the following definition: $P_{t+k|t}=P^{opt}_t\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}$. We multiply the optimal firm price P^{opt}_t by $\frac{1}{P_{t-1}}\frac{P_{t-1}}{P_t}$, multiply the marginal utility of consumption Ω^C_{t+k} by z^+_{t+k}, and divide the output of firm $Y_{t+k|t}$ by z^+_{t+k}. We multiply the nominal marginal cost MC_t by $\frac{1}{P}_t$, multiply MC_{t+1} by $\frac{P_{t+1}}{P_t}\frac{1}{P_{t+1}}$, and multiply MC_{t+2} by $\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\frac{1}{P_{t+2}}$. Thus, the above equation can be rewritten as:

$$\begin{aligned}
&E_t\{\frac{Y_{t|t}}{(\lambda_t-1)\,z^+_t}\left[\frac{P^{opt}_t}{P_{t-1}}\frac{P_{t-1}}{P_t}-\lambda_t\frac{MC_t}{P_t}\right]\\
&+\xi\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{Y_{t+1|t}}{(\lambda_{t+1}-1)\,z^+_{t+1}}\left[\frac{\overline{\Pi}_{t+1}P^{opt}_t}{P_t}\frac{P_{t-1}}{P_{t-1}}-\lambda_{t+1}\frac{P_{t+1}}{P_t}\frac{MC_{t+1}}{P_{t+1}}\right]\\
&+(\xi)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{Y_{t+2|t}}{(\lambda_{t+2}-1)\,z^+_{t+2}}\left[\frac{\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}P^{opt}_tP_{t-1}}{P_{t-1}P_t}-\lambda_{t+2}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\frac{MC_{t+2}}{P_{t+2}}\right]\\
&+\ldots\}=0.
\end{aligned}$$

Using the following definitions: $p^{opt}_t=\frac{P^{opt}_t}{P_{t-1}}$, $\Pi_{t+k}=\frac{P_{t+k}}{P_{t+k-1}}$, $\Pi^C_{t+k}=\frac{P^C_{t+k}}{P^C_{t+k-1}}$, and $\overline{mc}_{t+k}=\frac{MC_{t+k}}{P_{t+k}}$, we can obtain the following equation:

$$\begin{aligned}
&E_t\{\frac{Y_{t|t}}{(\lambda_t-1)\,z^+_t}\left[\frac{p^{opt}_t}{\Pi_t}-\lambda_t\overline{mc}_t\right]\\
&+\xi\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_t z^+_t}\frac{1}{\Pi^C_{t+1}}\frac{Y_{t+1|t}}{(\lambda_{t+1}-1)\,z^+_{t+1}}\left[\frac{\overline{\Pi}_{t+1}p^{opt}_t}{\Pi_t}-\lambda_{t+1}\Pi_{t+1}\overline{mc}_{t+1}\right]\\
&+(\xi)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_t z^+_t}\frac{1}{\Pi^C_{t+2}\Pi^C_{t+1}}\frac{Y_{t+2|t}}{(\lambda_{t+2}-1)\,z^+_{t+2}}\left[\frac{\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}p^{opt}_t}{\Pi_t}-\lambda_{t+2}\Pi_{t+2}\Pi_{t+1}\overline{mc}_{t+2}\right]\\
&+\ldots\}=0.
\end{aligned}$$

We express the above equation in terms of *per capita*, so we denote $y_{t+k|t}$ as output *per capita*, and we rearrange the above equation. Thus, we have the following equation:

$$
\begin{aligned}
E_t\{ & \frac{y_{t|t}}{(\lambda_t-1)\, z_t^+}\left[\frac{p_t^{opt}}{\Pi_t}-\lambda_t\overline{mc}_t\right] \\
& +\xi\frac{\beta_{t+1}}{\beta_t}\frac{\Omega_{t+1}^C z_{t+1}^+}{\Omega_t^C z_t^+}\frac{\Pi_{t+1}}{\Pi_{t+1}^C}\frac{y_{t+1|t}}{(\lambda_{t+1}-1)\, z_{t+1}^+}\left[\frac{\overline{\Pi}_{t+1}p_t^{opt}}{\Pi_{t+1}\Pi_t}-\lambda_{t+1}\overline{mc}_{t+1}\right] \\
& +(\xi)^2\,\frac{\beta_{t+2}}{\beta_t}\frac{\Omega_{t+2}^C z_{t+2}^+}{\Omega_t^C z_t^+}\frac{\Pi_{t+2}\Pi_{t+1}}{\Pi_{t+2}^C\Pi_{t+1}^C}\frac{y_{t+2|t}}{(\lambda_{t+2}-1)\, z_{t+2}^+}\left[\frac{\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}p_t^{opt}}{\Pi_{t+2}\Pi_{t+1}\Pi_t}-\lambda_{t+2}\overline{mc}_{t+2}\right] \\
& +\ldots\}=0.
\end{aligned}
$$

Using the following definitions: $\beta_{t+j}^r=\frac{\beta_{t+j}}{\beta_{t+j-1}}$, $\overline{\Omega}_{t+k}^C=\Omega_{t+k}^C z_{t+k}^+$, and $\overline{y}_{t+k|t}=\frac{y_{t+k}}{z_{t+k}^+}$, we rewrite the above equation. We have the following stationarized version of the optimal price of intermediate goods equation under the sticky price assumption:

$$
E_t\sum_{k=0}^{\infty}(\xi)^k\left(\prod_{j=1}^{k}\beta_{t+j}^r\right)\frac{\overline{\Omega}_{t+k}^C}{\overline{\Omega}_t^C}\left(\prod_{j=1}^{k}\frac{\Pi_{t+j}}{\Pi_{t+j}^C}\right)\frac{\overline{y}_{t+k|t}}{(\lambda_{t+k}-1)}\left[\left(\prod_{j=1}^{k}\frac{\overline{\Pi}_{t+j}}{\Pi_{t+j}}\right)\frac{p_t^{opt}}{\Pi_t}-\lambda_{t+k}\overline{mc}_{t+k}\right]=0. \tag{C.97}
$$

Equation (C.97), which is the stationarized version of the optimal price of intermediate goods, is the same as Equation (A.31a).

C.3 Private consumption good producers

C.3.1 Consumption good producers

This section presents the optimization problem of the consumption good producers in the Swedish economy and derives the demand functions of non-energy and energy consumption, and derives the relative price of the consumption goods equation.

The optimization problem of the representative non-energy consumption good producer can be defined as follows:

$$
\max_{C_t^{agg},C_t^{xe},C_t^{e}} P_t^C C_t^{agg}-P_t^{C,xe}C_t^{xe}-P_t^{C,e}C_t^{e}
$$

subject to the CES aggregate consumption good function

$$
C_t^{agg}=\left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}}+\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{e}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}}. \tag{C.98}
$$

By substituting the CES aggregate consumption good equation (C.98) into the above profit function, we can rewrite the profit function as:

$$
P_t^C\left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}}+\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{e}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}}-P_t^{C,xe}C_t^{xe}-P_t^{C,e}C_t^{e}.
$$

Taking the derivatives of C_t^{xe} and C_t^{e} respectively gives us the two following first-order-conditions:

$$
\begin{aligned}
\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}-1}P_t^C\left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}}+\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{e}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}-1}-P_t^{C,xe}=0 \\
\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{e}\right)^{\frac{\nu_C-1}{\nu_C}-1}P_t^C\left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}}+\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{e}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}-1}-P_t^{C,e}=0
\end{aligned}
$$

If we substitute the expression in square brackets by the production function, and put the prices of non-energy and energy respectively on the right-hand-side of the equation, the two equations can be written as

$$
\begin{aligned}
P_t^{C,xe}&=\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{-\frac{1}{\nu_C}}P_t^C\left(C_t^{agg}\right)^{\frac{1}{\nu_C}} \\
P_t^{C,e}&=\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{e}\right)^{-\frac{1}{\nu_C}}P_t^C\left(C_t^{agg}\right)^{\frac{1}{\nu_C}}.
\end{aligned}
$$

Rearrange and multiply through with ν_C in the two equations above to get the demand function for the non-energy and energy consumption goods:

$$C_t^{xe} = \vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C} C_t^{agg} \tag{C.99}$$

$$C_t^{e} = \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C} C_t^{agg} \tag{C.100}$$

which are the same equations that are presented in Equation (43) and Equation (44). Next, we create the price index. This is done by substituting the two demand functions into the production function:

$$\begin{aligned}
C_t^{agg} &= \left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}} \left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}} + \left(1-\vartheta^C\right)^{\frac{1}{\nu_C}} \left(C_t^{e}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}} \\
C_t^{agg} &= \left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}} \left(\vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C} C_t^{agg}\right)^{\frac{\nu_C-1}{\nu_C}} + \left(1-\vartheta^C\right)^{\frac{1}{\nu_C}} \left(\left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C} C_t^{agg}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}} \\
C_t^{agg} &= \left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}+\frac{\nu_C-1}{\nu_C}} \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C-1} \left(C_t^{agg}\right)^{\frac{\nu_C-1}{\nu_C}} + \left(1-\vartheta^C\right)^{\frac{1}{\nu_c}+\frac{\nu_C-1}{\nu_C}} \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C-1} \left(C_t^{agg}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}} \\
C_t^{agg} &= C_t^{agg}\left[\left(\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C-1} + \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C-1}\right]^{\frac{\nu_C}{\nu_C-1}} \\
1 &= \left[\left(\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C-1} + \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C-1}\right]^{\frac{\nu_C}{\nu_C-1}} \\
1 = \left(P_t^C\right)^{(\nu_C-1)\frac{\nu_C}{\nu_C-1}} &\left[\vartheta^C \left(\frac{1}{P_t^{C,xe}}\right)^{\nu_C-1} + \left(1-\vartheta^C\right) \left(\frac{1}{P_t^{C,e}}\right)^{\nu_C-1}\right]^{\frac{\nu_C}{\nu_C-1}} \\
\left(P_t^C\right)^{-\nu_C} &= \left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right) \left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{\nu_C}{\nu_C-1}} \\
P_t^C &= \left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right) \left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}}
\end{aligned}$$

which is the same function as is presented in Equation (45). Now, we are going to stationarize all equations and write the equations in per capita form (using the fact the the population is normalized to 1 for convenience). We use the definitions $p_t^C = P_t^C/P_t$, $p_t^{C,xe} = P_t^{C,xe}/P_t$, $p_t^{C,e} = P_t^{C,e}/P_t$, $\bar{c}_t^{agg} = C_t^{agg}/z_t^+$, $\bar{c}_t^{xe} = C_t^{xe}/z_t^+$, $\bar{c}_t^{e} = C_t^{e}/z_t^+$. The non-energy consumption demand function can be written as

$$\begin{aligned}
C_t^{xe} &= \vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C} C_t^{agg} \\
\frac{C_t^{xe}}{z_t^+} &= \vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\frac{P_t}{P_t}\right)^{\nu_C} \frac{C_t^{agg}}{z_t^+} \\
\bar{c}_t^{xe} &= \vartheta^C \left(\frac{p_t^C}{p_t^{C,xe}}\right)^{\nu_C} \bar{c}_t^{agg}
\end{aligned} \tag{C.101}$$

Equation (C.101), which captures the demand for non-energy consumption goods, is the same as Equation (A.35a).

Next, we stationarize the demand for energy goods:

$$\begin{aligned}
C_t^{e} &= \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C} C_t^{agg} \\
\frac{C_t^{e}}{z_t^+} &= \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\frac{P_t}{P_t}\right)^{\nu_C} \frac{C_t^{agg}}{z_t^+} \\
\bar{c}_t^{e} &= \left(1-\vartheta^C\right) \left(\frac{p_t^C}{p_t^{C,e}}\right)^{\nu_C} \bar{c}_t^{agg}
\end{aligned} \tag{C.102}$$

Equation (C.102), which captures the demand for energy consumption goods, is the same as Equation (A.36a).

Finally, we stationarize the price index:

$$
\begin{aligned}
P_t^C &= \left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}} \\
\frac{P_t^C}{P_t} &= \frac{1}{P_t}\left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}} \\
\frac{P_t^C}{P_t} &= \left[\left(\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(P_t^{C,e}\right)^{1-\nu_C}\right) P_t^{\nu_C-1}\right]^{\frac{1}{1-\nu_C}} \\
p_t^C &= \left[\left(\vartheta^C \left(\frac{P_t^{C,xe}}{P_t}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(\frac{P_t^{C,e}}{P_t}\right)^{1-\nu_C}\right)\right]^{\frac{1}{1-\nu_C}} \\
p_t^C &= \left[\left(\vartheta^C \left(p_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(p_t^{C,e}\right)^{1-\nu_C}\right)\right]^{\frac{1}{1-\nu_C}}. \qquad \text{(C.103)}
\end{aligned}
$$

Equation (C.103), which captures the demand for energy consumption goods, is the same as Equation (REF).

C.3.2 Non-energy consumption good producers

This section presents the optimization problem of non-energy consumption good producers in the Swedish economy and derives the relative price of the non-energy consumption goods equation.

The optimization problem of the representative non-energy consumption good producer can be defined as follows:

$$
\underset{D_t^{C,xe},M_t^{C,xe}}{max} P_t^{C,xe} C_t^{xe} - P_t D_t^{C,xe} - P_t^{M,C,xe} M_t^{C,xe}
$$

subject to the CES aggregate consumption good function

$$
C_t^{xe} = \left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_c}} \left(D_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}. \qquad \text{(C.104)}
$$

By substituting the CES aggregate consumption good equation (D.49) into the above profit function, we can rewrite the profit function as:

$$
P_t^{C,xe}\left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}} - P_t D_t^{C,xe} - P_t^{M,C,xe} M_t^{C,xe}.
$$

First, we derive the demand function for the domestically produced intermediate goods used as inputs by the representative firm in the consumption good sector. The FOC for the domestically produced intermediate goods $D_t^{C,xe}$ is:

$$
\frac{\nu_{C,xe}}{\nu_{C,xe}-1} P_t^{C,xe}\left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^{C}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}-1} \frac{\nu_{C,xe}-1}{\nu_{C,xe}} \left(D_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}-1} \left(\psi^{C,xe}\right)^{\frac{1}{\nu}}
$$

We can rewrite the above FOC for $D_t^{C,xe}$ as:

$$
P_t^{C,xe}\left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^{C}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}-1} \left(D_t^{C,xe}\right)^{\frac{-1}{\nu_{C,xe}}} \left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} - P_t = 0.
$$

Note that the CES aggregate consumption good equation (D.49) can be written as:

$$
\left(C_t^{xe}\right)^{\frac{1}{\nu_{C,xe}}} = \left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^{C}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{1}{\nu_{C,xe}-1}}. \qquad \text{(C.105)}
$$

Using Equation (C.105), we can rewrite the FOC for $D_t^{C,xe}$ as:

$$
P_t^{C,xe} \left(C_t^{xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^{C,xe}\right)^{\frac{-1}{\nu_{C,xe}}} \left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} - P_t = 0.
$$

The demand function for the domestically produced intermediate goods can be written as:

$$D_t^{C,xe} = \psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_{C,xe}} C_t^{xe}. \tag{C.106}$$

Next, we find the demand function for imported goods used as inputs by the representative firm in the consumption good sector. The FOC for the imported good M_t^C is:

$$P_t^{C,xe} \left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^C\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} \right]^{\frac{1}{\nu_{C,xe}-1}} \left(M_t^{C,xe}\right)^{\frac{-1}{\nu_{C,xe}}} \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} - P_t^{M,C,xe} = 0.$$

Using Equation (C.105), the FOC for $M_t^{C,xe}$ can be rewritten as:

$$P_t^{C,xe} \left(C_t^{xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{-1}{\nu_{C,xe}}} \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} - P_t^{M,C,xe} = 0.$$

The demand function for the imported goods can be expressed as:

$$M_t^{C,xe} = \left(1-\psi^{C,xe}\right) \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}} C_t^{xe}. \tag{C.107}$$

Substituting the above demand functions (C.106) and (C.107) into the CES aggregate consumption equation (D.49), and we have the following equation:

$$C_t^{xe} = \left[\psi^{C,xe} \left(C_t^{xe}\right)^{\frac{\nu_{C,xe}-1}{C,xe}} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(C_t^{xe}\right)^{\frac{\nu_{C,xe}-1}{C,xe}} \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}} \tag{C.108}$$

$$\begin{aligned}
C_t^{xe} &= C_t^{xe} \left[\psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}, \\
1 &= \left(P_t^{C,xe}\right)^{\nu_{C,xe}} \left[\psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}, \\
\left(P_t^{C,xe}\right)^{\nu_{C,xe}} &= \left[\psi^{C,xe} \left(\frac{1}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(\frac{1}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}, \\
P_t^{C,x} &= \left[\psi^{C,xe} \left(\frac{1}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(\frac{1}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{1}{\nu_{C,xe}-1}}.
\end{aligned}$$

The above equation can be rewritten as:

$$P_t^{C,xe} = \left[\psi^{C,xe} \left(P_t\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C,xe}\right) \left(P_t^{M,C,xe}\right)^{1-\nu_{C,xe}} \right]^{\frac{1}{1-\nu_{C,xe}}}. \tag{C.109}$$

Equation (C.109) captures the aggregate consumption price index. Equation (C.109) is the same as Equation (48) in Section 2.4.5.

Using the following definitions: $p_t^{C,xe} = P_t^{C,xe}/P_t$ and $p_t^{M,C,xe} = P_t^{M,C,xe}/P_t$, Equation (C.109) becomes:

$$\begin{aligned}
\frac{P_t^{C,xe}}{P_t} &= \left[\left(P_t\right)^{\nu_{C,xe}-1} \left(\psi^{C,xe} \left(P_t\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C,xe}\right) \left(P_t^{M,C,xe}\right)^{1-\nu_{C,xe}} \right) \right]^{\frac{1}{1-\nu_{C,xe}}}, \\
\frac{P_t^{C,xe}}{P_t} &= \left[\psi^{C,xe} + \left(1-\psi^{C,xe}\right) \left(\frac{P_t^{M,C,xe}}{P_t} \right)^{1-\nu_{C,xe}} \right]^{\frac{1}{1-\nu_{C,xe}}}.
\end{aligned}$$

Thus, the relative price of consumption goods equation is expressed as:

$$p_t^{C,xe} = \left[\psi^{C,xe} + \left(1-\psi^{C,xe}\right) \left(p_t^{M,C,xe}\right)^{1-\nu_{C,xe}} \right]^{\frac{1}{1-\nu_{C,xe}}}. \tag{C.110}$$

Note that: $\psi^{C,xe} = \vartheta^{C,xe} + \frac{1}{1+\omega}(1-\vartheta^{C,xe})$, where $\frac{1}{1+\omega}$ is the relative size of the Swedish economy, and $\vartheta^{C,xe} \in [0,1]$ is a measure of home bias in the production of consumption goods in Sweden. Since the size of the Foreign economy ω is infinitely larger than the Swedish economy, we have: $\psi^{C,xe} = \vartheta^{C,xe}$. Equation (C.110) becomes:

$$p_t^{C,xe} = \left[\vartheta^{C,xe} + \left(1-\vartheta^{C,xe}\right)\left(p_t^{M,C,xe}\right)^{1-\nu_{C,xe}}\right]^{\frac{1}{1-\nu_{C,xe}}}. \tag{C.111}$$

Equation (C.111), which captures the relative price of consumption goods, is the same as Equation (A.37a).

C.3.3 Energy good producers

This section presents the optimization problem of the energy consumption good producers in the Swedish economy, derives the demand functions of domestic and imported energy consumption, and derives the relative price of the energy consumption goods equation.

The optimization problem of the representative non-energy consumption good producer can be defined as follows:

$$\max_{C_t^e, D_t^{C,e}, M_t^{C,e}} P_t^C C_t^e - P_t^{D,C,e} D_t^{C,e} - P_t^{M,C,e} M_t^{C,e}$$

subject to the CES aggregate consumption good function

$$C_t^e = \left[\left(\psi^{C,e}\right)^{\frac{1}{\nu_{c,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\psi^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}. \tag{C.112}$$

First note, that the definition of $\psi^{C,e} = \vartheta^{C,e} + \frac{1}{1+\omega}(1-\vartheta^{C,xe})$ and since $\omega \to \infty$, $\psi^{C,e} = \vartheta^{C,e}$. By substituting the CES aggregate consumption good equation (C.112) into the above profit function, we can rewrite the profit function as:

$$P_t^{C,e}\left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{c,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} - P_t^{D,C,e}D_t^{C,e} - P_t^{M,C,e}M_t^{C,e}.$$

Taking the derivatives of $D_t^{C,e}$ and $M_t^{C,e}$ respectively gives us the two following first-order-conditions:

$$\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}-1} P_t^{C,e}\left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}-1} - P_t^{D,C,e} = 0$$

$$\left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}-1} P_t^{C,e}\left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}-1} - P_t^{M,C,e} = 0$$

If we substitute the expression in square brackets by the production function, and put the prices of non-energy and energy respectively on the right-hand-side of the equation, the two equations can be written as

$$P_t^{D,C,e} = \left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(D_t^{C,e}\right)^{-\frac{1}{\nu_{C,e}}} P_t^{C,e}\left(C_t^e\right)^{\frac{1}{\nu_{C,e}}}$$
$$P_t^{M,C,e} = \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{-\frac{1}{\nu_{C,e}}} P_t^{C,e}\left(C_t^e\right)^{\frac{1}{\nu_{C,e}}}.$$

Rearrange and multiply through with $\nu_{C,e}$ in the two equations above to get the demand function for the non-energy and energy consumption goods:

$$D_t^{C,e} = \vartheta^{C,e}\left(\frac{P_t^{C,e}}{P_t^{D,C,e}}\right)^{\nu_{C,e}} C_t^e \tag{C.113}$$

$$M_t^{C,e} = \left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{M,C,e}}\right)^{\nu_{C,e}} C_t^e \tag{C.114}$$

which are the same equations that are presented in section 2.4.5. Next, we create the price index. This is done by substituting the two demand functions into the production function:

$$C_t^e = \left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{c,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

$$C_t^e = \left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{c,e}}}\left(\vartheta^{C,e}\left(\frac{P_t^{C,e}}{P_t^{D,C,e}}\right)^{\nu_{C,e}} C_t^e\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(\left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{M,C,e}}\right)^{\nu_{C,e}} C_t^e\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

$$C_t^e = \left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}+\frac{\nu_{C,e}-1}{\nu_{C,e}}}\left(\frac{P_t^{C,e}}{P_t^{D,C,e}}\right)^{\nu_{C,e}-1}\left(C_t^e\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}+\frac{\nu_{C,e}-1}{\nu_{C,e}}}\left(\frac{P_t^{C,e}}{P_t^{M,C,e}}\right)^{\nu_{C,e}-1}\left(C_t^e\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

$$C_t^e = C_t^e\left[\left(\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{D,C,e}}\right)^{\nu_{C,e}-1} + \left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{M,C,e}}\right)^{\nu_{C,e}-1}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

$$1 = \left[\left(\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{D,C,e}}\right)^{\nu_{C,e}-1} + \left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{M,C,e}}\right)^{\nu_{C,e}-1}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

$$1 = \left(P_t^{C,e}\right)^{(\nu_C-1)\frac{\nu_C}{\nu_C-1}}\left[\left(\vartheta^{C,e}\right)\left(\frac{1}{P_t^{D,C,e}}\right)^{\nu_{C,e}-1} + \left(1-\vartheta^{C,e}\right)\left(\frac{1}{P_t^{M,C,e}}\right)^{\nu_{C,e}-1}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

$$\left(P_t^{C,e}\right)^{-\nu_C} = \left[\vartheta^{C,e}\left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(P_t^{M,C,e}\right)^{1-\nu_{C,e}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

$$P_t^{C,e} = \left[\vartheta^{C,e}\left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(P_t^{M,C,e}\right)^{1-\nu_{C,e}}\right]^{\frac{1}{1-\nu_{C,e}}}$$

which is the same function as is presented in section 2.4.5. Now, we are going to stationarize all equations and write the equations in per capita form (using the fact the the population is normalized to 1 for convenience). We use the definitions $p_t^{C,e} = P_t^{C,e}/P_t$, $p_t^{D,C,e} = P_t^{D,C,e}/P_t$, $p_t^{M,C,e} = P_t^{M,C,e}/P_t$, $\bar{c}_t^e = C_t^e/z_t^+$, $\bar{d}_t^{C,e} = D_t^{C,e}/z_t^+$, $\bar{m}_t^{C,e} = M_t^{C,e}/z_t^+$. The non-energy consumption demand function can be written as

$$\begin{aligned} D_t^{C,e} &= \vartheta^{C,e}\left(\frac{P_t^{C,e}}{P_t^{D,C,e}}\right)^{\nu_{C,e}} C_t^e \\ \frac{D_t^{C,e}}{z_t^+} &= \vartheta^{C,e}\left(\frac{P_t^{C,e}}{P_t^{D,C,e}}\frac{P_t}{P_t}\right)^{\nu_{C,e}} \frac{C_t^e}{z_t^+} \\ \bar{d}_t^{C,e} &= \vartheta^{C,e}\left(\frac{p_t^{C,e}}{p_t^{D,C,e}}\right)^{\nu_{C,e}} \bar{c}_t^e \end{aligned} \tag{C.115}$$

Equation (C.115), which captures the demand for non-energy consumption goods, is the same as Equation (A.41a).

Next, we stationarize the demand for energy goods:

$$\begin{aligned} M_t^{C,e} &= \left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{M,C,e}}\right)^{\nu_{C,e}} C_t^e \\ \frac{M_t^{C,e}}{z_t^+} &= \left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{M,C,e}}\frac{P_t}{P_t}\right)^{\nu_{C,e}} \frac{C_t^e}{z_t^+} \\ \bar{m}_t^{C,e} &= \left(1-\vartheta^{C,e}\right)\left(\frac{p_t^{C,e}}{p_t^{M,C,e}}\right)^{\nu_{Ce}} \bar{c}_t^e \end{aligned} \tag{C.116}$$

Equation (C.116), which captures the demand for energy consumption goods, is the same as Equation (A.42a).

Finally, we stationarize the price index:

$$P_t^{C,e} = \left[\vartheta^{C,e}\left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(P_t^{M,C,e}\right)^{1-\nu_{C,e}}\right]^{\frac{1}{1-\nu_{C,e}}}$$

$$\frac{P_t^{C,e}}{P_t} = \frac{1}{P_t}\left[\vartheta^{C,e}\left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(P_t^{M,C,e}\right)^{1-\nu_{C,e}}\right]^{\frac{1}{1-\nu_{C,e}}}$$

$$\frac{P_t^{C,e}}{P_t} = \left[\left(\vartheta^{C,e}\left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(P_t^{M,C,e}\right)^{1-\nu_{C,e}}\right)(P_t)^{\nu_{C,e}-1}\right]^{\frac{1}{1-\nu_{C,e}}}$$

$$p_t^{C,e} = \left[\left(\vartheta^{C,e}\left(\frac{P_t^{D,C,e}}{P_t}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{M,C,e}}{P_t}\right)^{1-\nu_{C,e}}\right)\right]^{\frac{1}{1-\nu_{C,e}}}$$

$$p_t^{C,e} = \left[\left(\vartheta^{C,e}\left(p_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(p_t^{M,C,e}\right)^{1-\nu_{C,e}}\right)\right]^{\frac{1}{1-\nu_{C,e}}}. \tag{C.117}$$

Equation (C.117), which captures the demand for energy consumption goods, is the same as Equation (A.39a).

C.4 Private investment good producers

This section presents the optimization problem of the investment good producers, and derives the relative price of investment goods equation (A.44a). We define V_t^I to be the output of a representative investment firm. We define V_t^I as $\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]$.

A profit function of the representative investment good producer is defined as:

$$P_t^I V_t^I - P_t D_t^I - P_t^{M,I} M_t^I.$$

The investment good producer faces the following CES investment function:

$$V_t^I = \left[\left(\psi^I\right)^{\frac{1}{\nu_I}}\left(D_t^I\right)^{\frac{\nu_I-1}{\nu_I}} + \left(1-\psi^I\right)^{\frac{1}{\nu_I}}\left(M_t^I\right)^{\frac{\nu_I-1}{\nu_I}}\right]^{\frac{\nu_I}{\nu_I-1}}. \tag{C.118}$$

The optimization problem can be defined as follows:

$$\max_{D_t^I, M_t^I} P_t^I V_t^I - P_t D_t^I - P_t^{M,I} M_t^I$$

subject to

$$V_t^I = \left[\left(\psi^I\right)^{\frac{1}{\nu_I}}\left(D_t^I\right)^{\frac{\nu_I-1}{\nu_I}} + \left(1-\psi^I\right)^{\frac{1}{\nu_I}}\left(M_t^I\right)^{\frac{\nu_I-1}{\nu_I}}\right]^{\frac{\nu_I}{\nu_I-1}}.$$

We follow the similar steps as described in Section C.3 to find the demand functions for the domestically produced intermediate goods and for the imported goods used in the production of investment goods. The demand function for the domestically produced intermediate goods used in the production of investment goods D_t^I is:

$$D_t^I = \psi^I\left(\frac{P_t^I}{P_t}\right)^{\nu_I} V_t^I.$$

This demand function is the same the demand function that is presented in Section 2.4.4.

Using the following definition: $V_t^I = \frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]$, the above demand function for the domestically produced intermediate goods used in the production of investment goods can be rewritten as:

$$D_t^I = \psi^I\left(\frac{P_t^I}{P_t}\right)^{\nu_I}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{C.119}$$

The demand function for the imported goods used in the production of investment goods is:

$$M_t^I = (1-\psi^I)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I} V_t^I.$$

This demand function is the same the demand function that is presented in Section 2.4.4.

Using the following definition: $V_t^I = \frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]$, the above demand function for the imported goods used in the production of investment goods can be rewritten as:

$$M_t^I = (1-\psi^I)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I} \frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{C.120}$$

Substituting the above demand functions (C.119) and (C.120) into the CES investment equation (C.118), this gives us the following aggregate investment price index equation:

$$P_t^I = \left[\psi^I \left(P_t\right)^{1-\nu_I} + \left(1-\psi^I\right)\left(P_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{C.121}$$

Equation (C.121) captures the aggregate investment price index. Equation (C.121) is the same as Equation (42) in Section 2.4.4.

Equation (C.121) can be rewritten as:

$$\frac{P_t^I}{P_t} = \left[\psi^I + \left(1-\psi^I\right)\left(\frac{P_t^{M,I}}{P_t}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{C.122}$$

Using the following definitions: $p_t^I = P_t^I/P_t$ and $p_t^{M,I} = P_t^{M,I}/P_t$. The relative price of investment goods equation can be expressed as:

$$p_t^I = \left[\psi^I + \left(1-\psi^I\right)\left(p_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{C.123}$$

Note that: $\psi^I = \vartheta^I + \frac{1}{1+\omega}(1-\vartheta^I)$. $\frac{1}{1+\omega}$ is the relative size of the Swedish economy, and $\vartheta^I \in [0,1]$ is a measure of home bias in the production of investment goods in the Swedish economy. Since the size of the Foreign economy ω is infinitely larger than the Swedish economy, we have: $\psi^I = \vartheta^I$. Thus, the relative price of investment goods equation can be rewritten as:

$$p_t^I = \left[\vartheta^I + \left(1-\vartheta^I\right)\left(p_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{C.124}$$

Equation (C.124), which represents the relative price of investment goods equation, is the same as Equation (A.44a).

C.5 Export good producers

This section presents the optimization problem of export good producers in the Swedish economy. First, we derive the real marginal cost of production for export good producers, Equation (A.46a). Second, we derive the optimal price for export good producers, Equation (A.47a). There is a continuum of export good producers. Each firm i produces export goods $X_t(i)$ by using domestically produced intermediate goods $D_t^X(i)$ and imported goods $M_t^X(i)$ as inputs.

The export good firm i faces the following cost minimization problem:

$$\min_{D_t^X(i),\, M_t^X(i)} \quad TC_t^X(i) = P_t D_t^X(i) + P_t^{M,X} M_t^X(i), \tag{C.125}$$

subject to the following production function:

$$X_t(i) = \left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}} - z_t^+\phi^X. \tag{C.126}$$

We let $\theta_t^X(i)$ to be the Lagrange multiplier associated with the production constraint for export goods (C.126). To solve the optimization problem, we set up the following Lagrangian $\mathscr{L}_t^X(i)$:

$$\begin{aligned}\mathscr{L}_t^X(i) &= \left[P_t D_t^X(i) + P_t^{M,X} M_t^X(i)\right] \\ &+ \theta_t^X(i)\left\{X_t(i) - \left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}} + z_t^+\phi^X\right\}.\end{aligned}$$

We take the partial derivative of $\mathscr{L}_t^X(i)$ with respect to $D_t^X(i)$ and $M_t^X(i)$ respectively, and we can find the FOC for $D_t^X(i)$ and $M_t^X(i)$.

The FOC for $D_t^X(i)$ is:

$$P_t = \theta_t^X(i)\left(\psi^X\right)^{\frac{1}{\nu_x}}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}} D_t^X(i)^{\frac{-1}{\nu_x}}. \tag{C.127}$$

The FOC for $M_t^X(i)$ is:

$$P_t^{M,X} = \theta_t^X(i)\left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}} M_t^X(i)^{\frac{-1}{\nu_x}}. \tag{C.128}$$

We rewrite Equation (C.126) as:

$$\left[X_t(i) + z_t^+\phi^X\right] = \left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}}. \tag{C.129}$$

We use (C.127) and Equation (C.128) to find the following total cost function:

$$\begin{aligned} TC_t^X(i) = {} & \theta_t^X(i)\left(\psi^X\right)^{\frac{1}{\nu_x}}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}} D_t^X(i)^{\frac{-1}{\nu_x}} D_t^X(i) \\ & + \theta_t^X(i)\left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}} M_t^X(i)^{\frac{-1}{\nu_x}} M_t^X(i), \end{aligned}$$

$$\begin{aligned} & TC_t^X(i) = \\ & \theta_t^X(i)\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}}. \end{aligned} \tag{C.130}$$

Using Equation (C.129), Equation (C.130) can be rewritten as follows:

$$\begin{aligned} TC_t^X(i) &= \theta_t^X(i)\left[X_t(i) + z_t^+\phi^X\right]^{\frac{\nu_x-1}{\nu_x}}\left[X_t(i) + z_t^+\phi^X\right]^{\frac{1}{\nu_x}}, \\ TC_t^X(i) &= \theta_t^X(i)\left[X_t(i) + z_t^+\phi^X\right]. \end{aligned} \tag{C.131}$$

We take the derivative of the function of $TC_t^X(i)$ with respect to $X_t(i)$, and we have the following definition:

$$MC_t^X(i) = \frac{\partial TC_t^X(i)}{\partial X_t(i)} = \theta_t^X(i). \tag{C.132}$$

Equation (C.132) implies that $\theta_t^X(i) = MC_t^X(i)$. We can now establish that $\theta_t^X(i)$ is the nominal marginal cost of production for export good producers $MC_t^X(i)$.

We use the result from Equation (C.132), and then we rearrange Equation (C.127) and Equation (C.128). We have the following equations:

$$D_t^X(i) = \psi^X\left(\frac{MC_t^X(i)}{P_t}\right)^{\nu_x}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}}\left(D_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left(M_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}}, \tag{C.133}$$

$$M_t^X(i) = \left(1-\psi^X\right)\left(\frac{MC_t^X(i)}{P_t^{M,X}}\right)^{\nu_x}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}}\left(D_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left(M_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}}. \tag{C.134}$$

We substitute Equation (C.133) and Equation (C.134) into Equation (C.125), and Equation (C.125) can be expressed as:

$$TC_t^X(i) =$$
$$\left[\left(\psi^X\right)^{\frac{1}{\nu_x}}\left(D_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}}+\left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left(M_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}}$$
$$\left[\psi^X\left(MC_t^X(i)\right)^{\nu_x}(P_t)^{(1-\nu_x)}+\left(1-\psi^X\right)\left(MC_t^X(i)\right)^{\nu_x}\left(P_t^{M,X}\right)^{(1-\nu_x)}\right].$$

Using Equation (C.129), the above equation can be simplified to the following equation:

$$TC_t^X(i) = \left[X_t(i)+z_t^+\phi^X\right]\left[\psi^X\left(MC_t^X(i)\right)^{\nu_x}(P_t)^{(1-\nu_x)}+\left(1-\psi^X\right)\left(MC_t^X(i)\right)^{\nu_x}\left(P_t^{M,X}\right)^{(1-\nu_x)}\right]. \quad (C.135)$$

We use Equation (C.135) and Equation (C.132), and we take the partial derivative of $TC_t^X(i)$ with respect to $X_t(i)$. We have the following equation:

$$MC_t^X(i) = \frac{\partial TC_t^X(i)}{\partial X_t(i)} = \left[\psi^X\left(MC_t^X(i)\right)^{\nu_x}(P_t)^{(1-\nu_x)}+\left(1-\psi^X\right)\left(MC_t^X(i)\right)^{\nu_x}\left(P_t^{M,X}\right)^{(1-\nu_x)}\right]. \quad (C.136)$$

We rearrange the above equation, and we can obtain the nominal marginal cost of production for Swedish exports, which depends on the domestic intermediate good and imported good prices. The nominal marginal cost of Swedish export goods can be expressed as:

$$MC_t^X = \left[\psi^X(P_t)^{(1-\nu_x)}+\left(1-\psi^X\right)\left(P_t^{M,X}\right)^{(1-\nu_x)}\right]^{\frac{1}{1-\nu_x}}. \quad (C.137)$$

Equation (C.137) is the same as Equation (38) in Section 2.4.3. The marginal cost of firm i is independent to the firm-specific variables, so there is no subscript$i$ in the RHS of Equation (C.137). As a consequence, we have the following result: $MC_t^X(i) = MC_t^X$.

Using the following definition: $p_t^{M,X} = \frac{P_t^{M,X}}{P_t}$ and $\overline{mc}_t = \frac{MC_t^X}{P_t}$, Equation (C.137) can be rewritten as:

$$\overline{mc}_t^X = \left[\psi^X+\left(1-\psi^X\right)\left(p_t^{M,X}\right)^{1-\nu_x}\right]^{\frac{1}{1-\nu_x}}. \quad (C.138)$$

Note that: $\psi^X = \vartheta^X + \frac{1}{1+\omega}(1-\vartheta^X)$, where $\frac{1}{1+\omega}$ is the relative size of the Swedish economy, and $\vartheta^X \in [0,1]$ is a measure of home bias in the production of export goods in Sweden. We thus have $\psi^X \to \vartheta^X$ as $\omega \to \infty$, and Equation (C.138) becomes:

$$\overline{mc}_t^X = \left[\vartheta^X+\left(1-\vartheta^X\right)\left(p_t^{M,X}\right)^{1-\nu_x}\right]^{\frac{1}{1-\nu_x}}. \quad (C.139)$$

Equation (C.139), which represents the real marginal cost of production for export good producers, is the same as Equation (A.46a).

Furthermore, we can find the demand function for domestically produced intermediate goods that are used in the production of export goods by firm i and the demand function for imported goods that are used in the production of export goods by firm i. We apply Equation (C.129) and the result that the marginal cost across firms are equal: $MC_t^X(i) = MC_t^X$. Thus, we can rewrite Equation (C.133) and Equation (C.134) as follows.

The demand function for domestically produced intermediate goods that are used in the production of export goods $D_t^X(i)$ is:

$$D_t^X(i) = \psi^X\left(\frac{MC_t^X}{P_t}\right)^{\nu_x}\left[X_t(i)+z_t^+\phi^X\right]. \quad (C.140)$$

The demand function for imported goods that are used in the production of export goods $M_t^X(i)$ is:

$$M_t^X(i) = \left(1-\psi^X\right)\left(\frac{MC_t^X}{P_t^{M,X}}\right)^{\nu_x}\left[X_t(i)+z_t^+\phi^X\right]. \quad (C.141)$$

These demand functions are the same as the demand functions in the footnote of Section 2.4.3.

Now, we derive the optimal price of export goods equation (A.47a). Export firm i chooses the optimal price of export goods $P_t^{X,opt}(i)$ that maximizes its profit, subject to its demand schedule for export goods and the Calvo price contract. In each period, the individual firm resets its price with probability $(1-\xi_x)$. With probability ξ_x, the firm cannot reset its price, and then it faces the following price evolution: $P^X_{t+k|t}(i) = P^{X,opt}_t(i)\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}\ldots\overline{\Pi}^X_{t+k}$. Recall that the prices of Swedish export goods are set in the currency of Foreign, so called local currency pricing. We define the stochastic discount factor as $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega^C_{t+k}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+k}}$.

The profit function of firm i is written as:

$$\max_{P^{X,opt}_t(i)} E_t\sum_{k=0}^{\infty}(\xi_x)^k\Lambda_{t,t+k}\left\{P^X_{t+k|t}(i)S_{t+k}X_{t+k|t}(i) - TC^X_{t+k|t}\left[X_{t+k|t}(i)\right]\right\}, \tag{C.142}$$

subject to the demand function:

$$X_{t+k|t}(i) = \left(\frac{P^X_{t+k|t}(i)}{P^X_{t+k}}\right)^{\frac{\lambda^X_{t+k}}{1-\lambda^X_{t+k}}} X_{t+k}, \tag{C.143}$$

and the Calvo price setting contract:

$$P^X_{t+k}(i) = \begin{cases}\overline{\Pi}^X_{t+k}P^X_{t+k-1}(i) & \text{with probability } \xi_x\\ P^{X,opt}_{t+k}(i) & \text{with probability } (1-\xi_x).\end{cases} \tag{C.144}$$

The FOC of $P^{X,opt}_t(i)$ is:

$$\begin{aligned}
&E_t\{S_tX_{t|t}(i) + S_tP^{X,opt}_t(i)\frac{\partial X_{t|t}(i)}{\partial P^{X,opt}_t(i)} - MC^X_t(i)\frac{\partial X_{t|t}(i)}{\partial P^{X,opt}_t(i)}\\
&\quad + \xi_x\Lambda_{t,t+1}\left[S_{t+1}\frac{\partial P^X_{t+1|t}(i)}{\partial P^{X,opt}_t(i)}X_{t+1|t}(i) + S_{t+1}P^X_{t+1|t}(i)\frac{\partial X_{t+1|t}(i)}{\partial P^X_{t+1|t}(i)}\frac{\partial P^X_{t+1|t}(i)}{\partial P^{X,opt}_t(i)} - MC^X_{t+1}(i)\frac{\partial X_{t+1|t}(i)}{\partial P^X_{t+1|t}(i)}\frac{\partial P^X_{t+1|t}(i)}{\partial P^{X,opt}_t(i)}\right]\\
&\quad + (\xi_x)^2\Lambda_{t,t+2}\\
&\quad\left[S_{t+2}\frac{\partial P^X_{t+2|t}(i)}{\partial P^{X,opt}_t(i)}X_{t+2|t}(i) + S_{t+2}P^X_{t+2|t}(i)\frac{\partial X_{t+2|t}(i)}{\partial P^X_{t+2|t}(i)}\frac{\partial P^X_{t+2|t}(i)}{\partial P^{X,opt}_t(i)} - MC^X_{t+2}(i)\frac{\partial X_{t+2|t}(i)}{\partial P^X_{t+2|t}(i)}\frac{\partial P^X_{t+2|t}(i)}{\partial P^{X,opt}_t(i)}\right]\\
&\quad + \ldots\} = 0.
\end{aligned} \tag{C.145}$$

Recall, we have the following definition: $P^X_{t+k|t}(i) = P^{X,opt}_t(i)\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}\ldots\overline{\Pi}^X_{t+k}$. Hence, the partial derivative of $P^X_{t+k|t}(i)$ with respect to $P^{X,opt}_t(i)$ is:

$$\frac{\partial P^X_{t+k|t}(i)}{\partial P^{X,opt}_t(i)} = \overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}\ldots\overline{\Pi}^X_{t+k}. \tag{C.146}$$

Using Equation (C.146), Equation (C.145) can be rewritten as:

$$\begin{aligned}
&E_t\{S_tX_{t|t}(i) + S_tP^{X,opt}_t(i)\frac{\partial X_{t|t}(i)}{\partial P^{X,opt}_t(i)} - MC^X_t(i)\frac{\partial X_{t|t}(i)}{\partial P^{X,opt}_t(i)}\\
&\quad + \xi_x\Lambda_{t,t+1}\left[S_{t+1}\overline{\Pi}^X_{t+1}X_{t+1|t}(i) + S_{t+1}P^X_{t+1|t}(i)\overline{\Pi}^X_{t+1}\frac{\partial X^X_{t+1|t}(i)}{\partial P^X_{t+1|t}(i)} - MC^X_{t+1}(i)\overline{\Pi}^X_{t+1}\frac{\partial X_{t+1|t}(i)}{\partial P^X_{t+1|t}(i)}\right]\\
&\quad + (\xi_x)^2\Lambda_{t,t+2}\\
&\quad\left[S_{t+2}\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}X_{t+2|t}(i) + S_{t+2}P^X_{t+2|t}(i)\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}\frac{\partial X_{t+2|t}(i)}{\partial P^X_{t+2|t}(i)} - MC^X_{t+2}(i)\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}\frac{\partial X_{t+2|t}(i)}{\partial P^X_{t+2|t}(i)}\right]\\
&\quad + \ldots\} = 0.
\end{aligned}$$

From Equation (C.137), we have the following result: $MC^X_{t+k}(i) = MC^X_{t+k}$. We rearrange the above equation, and we can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{\partial X_{t|t}(i)}{\partial P_t^{X,opt}(i)}\left[S_t P_t^{X,opt}(i)\left(\frac{X_{t|t}(i)}{P_t^{X,opt}(i)}\left(\frac{\partial X_{t|t}(i)}{\partial P_t^{X,opt}(i)}\right)^{-1}+1\right)-MC_t^X\right]\\
&+\xi_x\Lambda_{t,t+1}\overline{\Pi}_{t+1}^X\frac{\partial X_{t+1|t}(i)}{\partial P_{t+1|t}^X(i)}\left[S_{t+1}P_{t+1|t}^X(i)\left(\frac{X_{t+1|t}(i)}{P_{t+1|t}^X(i)}\left(\frac{\partial X_{t+1|t}(i)}{\partial P_{t+1|t}^X(i)}\right)^{-1}+1\right)-MC_{t+1}^X\right]\\
&+(\xi_x)^2\Lambda_{t,t+2}\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X\frac{\partial X_{t+2|t}(i)}{\partial P_{t+2|t}^X(i)}\left[S_{t+2}P_{t+2|t}^X(i)\left(\frac{X_{t+2|t}(i)}{P_{t+2|t}^X(i)}\left(\frac{\partial X_{t+2|t}(i)}{\partial P_{t+2|t}^X(i)}\right)^{-1}+1\right)-MC_{t+2}^X\right]+\ldots\}=0.
\end{aligned}
\tag{C.147}
$$

Given the demand schedule for export goods, which is captured by Equation (C.143), we can find the following price elasticity of demand for export goods:

$$
-\frac{\partial X_{t+k|t}(i)}{\partial P_{t+k|t}^X(i)}\frac{P_{t+k|t}^X(i)}{X_{t+k|t}(i)}=\frac{\lambda_{t+k}^X}{\lambda_{t+k}^X-1}.
\tag{C.148}
$$

Using the result from Equation (C.148), the derivative of $X_{t+k|t}(i)$ with respect to $P_{t+k|t}^X(i)$ is:

$$
\frac{\partial X_{t+k|t}(i)}{\partial P_{t+k|t}^X(i)}=\frac{\lambda_{t+k}^X}{1-\lambda_{t+k}^X}\frac{X_{t+k|t}(i)}{P_{t+k|t}^X(i)}.
\tag{C.149}
$$

Using Equation (C.148) and Equation (C.149), we can rewrite Equation (C.147) as follows:

$$
\begin{aligned}
&E_t\{\frac{X_{t|t}(i)}{P_t^{X,opt}(i)}\frac{\lambda_t^X}{1-\lambda_t^X}\left[S_t P_t^{X,opt}(i)\frac{1}{\lambda_t^X}-MC_t^X\right]\\
&+\xi_x\Lambda_{t,t+1}\overline{\Pi}_{t+1}^X\frac{X_{t+1|t}(i)}{P_{t+1|t}^X(i)}\frac{\lambda_{t+1}^X}{1-\lambda_{t+1}^X}\left[S_{t+1}P_{t+1|t}^X(i)\frac{1}{\lambda_{t+1}^X}-MC_{t+1}^X\right]\\
&+(\xi_x)^2\Lambda_{t,t+2}\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X\frac{X_{t+2|t}(i)}{P_{t+2|t}^X(i)}\frac{\lambda_{t+2}^X}{1-\lambda_{t+2}^X}\left[S_{t+2}P_{t+2|t}^X(i)\frac{1}{\lambda_{t+2}^X}-MC_{t+2}^X\right]+\ldots\}=0.
\end{aligned}
$$

We use the following definition: $P_{t+k|t}^X(i)=P_t^{X,opt}(i)\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X\ldots\overline{\Pi}_{t+k}^X$. We multiply both sides of the above equation by $P_t^{X,opt}(i)$ and -1. We can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{X_{t|t}(i)}{\lambda_t^X-1}\left[S_t P_t^{X,opt}(i)-\lambda_t^X MC_t^X\right]\\
&+\xi_x\Lambda_{t,t+1}\frac{X_{t+1|t}(i)}{\lambda_{t+1}^X-1}\left[S_{t+1}P_{t+1|t}^X(i)-\lambda_{t+1}^X MC_{t+1}^X\right]\\
&+(\xi_x)^2\Lambda_{t,t+2}\frac{X_{t+2|t}(i)}{\lambda_{t+2}^X-1}\left[S_{t+2}P_{t+2|t}^X(i)-\lambda_{t+2}^X MC_{t+2}^X\right]+\ldots\}=0.
\end{aligned}
$$

We can drop the subscript i from the above equation because when firms can reset their price, they will set the same price. As a result, all firms that reset their price in period t will choose the same quantity of output. We rewrite the above equation, and the optimal price of export goods equation can be expressed as:

$$
E_t\sum_{k=0}^{\infty}(\xi_x)^k\Lambda_{t,t+k}\frac{X_{t+k|t}}{(\lambda_{t+k}^X-1)}\left[S_{t+k}P_{t+k|t}^X-\lambda_{t+k}^X MC_{t+k}^X\right]=0.
\tag{C.150}
$$

Equation (C.150), which is the non-stationarized version of the optimal price of export goods, is the same as Equation (41) in Section 2.4.3.

The above equation can be rewritten in terms of *per capita* quantities. To this end, we let $x_{t+k|t}$ denote the per capita quantity of output of export firms that reset their price in period t. The optimal price for export goods equation can be written as:

$$
E_t\sum_{k=0}^{\infty}(\xi_x)^k\Lambda_{t,t+k}\frac{x_{t+k|t}}{(\lambda_{t+k}^X-1)}\left[S_{t+k}P_{t+k|t}^X-\lambda_{t+k}^X MC_{t+k}^X\right]=0.
\tag{C.151}
$$

Now, we stationarize Equation (C.151). We use the following definition: $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega^C_{t+k}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+k}}$, and we expand Equation (C.151). Thus, we have the following equation:

$$
\begin{aligned}
&E_t\{\frac{x_{t|t}}{(\lambda^X_t-1)}\left[S_tP^{X,opt}_t-\lambda^X_tMC^X_t\right]\\
&+\xi_x\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{x_{t+1|t}}{(\lambda^X_{t+1}-1)}\left[S_{t+1}P^X_{t+1|t}-\lambda^X_{t+1}MC^X_{t+1}\right]\\
&+(\xi_x)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{x_{t+2|t}}{(\lambda^X_{t+2}-1)}\left[S_{t+2}P^X_{t+2|t}-\lambda^X_{t+2}MC^X_{t+2}\right]+\ldots\}=0.
\end{aligned}
$$

We expand further and multiply through by $\frac{1}{P_t}$ to get:

$$
\begin{aligned}
&E_t\{\frac{x_{t|t}}{(\lambda^X_t-1)}\frac{P_t}{P_t}\left[\frac{S_tP^{X,opt}_t}{P_t}-\frac{\lambda^X_tMC^X_t}{P_t}\right]\\
&+\xi_x\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{x_{t+1|t}}{(\lambda^X_{t+1}-1)}\frac{P_{t+1}}{P_t}\left[\frac{S_{t+1}P^X_{t+1|t}}{P_{t+1}}-\frac{\lambda^X_{t+1}MC^X_{t+1}}{P_{t+1}}\right]\\
&+(\xi_x)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{x_{t+2|t}}{(\lambda^X_{t+2}-1)}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\left[\frac{S_{t+2}P^X_{t+2|t}}{P_{t+2}}-\frac{\lambda^X_{t+2}MC^X_{t+2}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We use the following definition: $P^X_{t+k|t} = P^{X,opt}_t\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}\ldots\overline{\Pi}^X_{t+k}$. We multiply the marginal utility of consumption Ω^C_{t+k} by z^+_{t+k}, and divide export goods $x_{t+k|t}$ by z^+_{t+k} and expand further to get:

$$
\begin{aligned}
&E_t\{\frac{x_{t|t}}{(\lambda^X_t-1)\,z^+_t}\left[\frac{S_tP^{X,opt}_t}{P_t}-\lambda^X_t\frac{MC^X_t}{P_t}\right]\\
&+\xi_x\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_tz^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{x_{t+1|t}}{\left(\lambda^X_{t+1}-1\right)z^+_{t+1}}\frac{P_{t+1}}{P_t}\left[\frac{S_{t+1}\overline{\Pi}^X_{t+1}P^{X,opt}_t}{P_{t+1}}-\lambda^X_{t+1}\frac{MC^X_{t+1}}{P_{t+1}}\right]\\
&+(\xi_x)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_tz^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{x_{t+2|t}}{\left(\lambda^X_{t+2}-1\right)z^+_{t+2}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\\
&\left[\frac{S_{t+2}\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}P^{X,opt}_t}{P_{t+2}}-\lambda^X_{t+2}\frac{MC^X_{t+2}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We multiply the second term of the above equation by $\frac{S_t}{S_t}$, and $\frac{P_t}{P_t}$, whereas we multiply the third term of the above equation by $\frac{S_{t+1}S_t}{S_{t+1}S_t}$ and $\frac{P_tP_{t+1}}{P_tP_{t+1}}$. We have the following equation:

$$
\begin{aligned}
&E_t\{\frac{x_{t|t}}{(\lambda^X_t-1)\,z^+_t}\left[\frac{S_tP^{X,opt}_t}{P_t}-\lambda^X_t\frac{MC^X_t}{P_t}\right]\\
&+\xi_x\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_tz^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{x_{t+1|t}}{\left(\lambda^X_{t+1}-1\right)z^+_{t+1}}\frac{P_{t+1}}{P_t}\left[\frac{P_t}{P_t}\frac{S_t}{S_t}\frac{S_{t+1}\overline{\Pi}^X_{t+1}P^{X,opt}_t}{P_{t+1}}-\lambda^X_{t+1}\frac{MC^X_{t+1}}{P_{t+1}}\right]\\
&+(\xi_x)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_tz^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{x_{t+2|t}}{\left(\lambda^X_{t+2}-1\right)z^+_{t+2}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\\
&\left[\frac{S_{t+1}S_t}{S_{t+1}S_t}\frac{P_tP_{t+1}}{P_tP_{t+1}}\frac{S_{t+2}\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}P^{X,opt}_t}{P_{t+2}}-\lambda^X_{t+2}\frac{MC^X_{t+2}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We use the following definitions: $p^{X,opt}_t = \frac{S_tP^{X,opt}_t}{P_t}$, $\overline{mc}^X_{t+k} = \frac{MC^X_{t+k}}{P_{t+k}}$, $s_{t+k} = \frac{S_{t+k}}{S_{t+k-1}}$, $\Pi_{t+k} = \frac{P_{t+k}}{P_{t+k-1}}$, and

$\Pi^C_{t+k} = \frac{P^C_{t+k}}{P^C_{t+k-1}}$. We can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{x_{t|t}}{(\lambda^X_t - 1)\, z^+_t}\left[p^{X,opt}_t - \lambda^X_t \overline{mc}^X_t\right] \\
&+ \xi_x \frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1} z^+_{t+1}}{\Omega^C_t z^+_t}\frac{\Pi_{t+1}}{\Pi^C_{t+1}}\frac{x_{t+1|t}}{(\lambda^X_{t+1} - 1)\, z^+_{t+1}}\left[\frac{\overline{\Pi}^X_{t+1} s_{t+1} p^{X,opt}_t}{\Pi_{t+1}} - \lambda^X_{t+1}\overline{mc}^X_{t+1}\right] \\
&+ (\xi_x)^2 \frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2} z^+_{t+2}}{\Omega^C_t z^+_t}\frac{\Pi_{t+2}\Pi_{t+1}}{\Pi^C_{t+2}\Pi^C_{t+1}}\frac{x_{t+2|t}}{(\lambda^X_{t+2} - 1)\, z^+_{t+2}} \\
&\left[\frac{\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2} s_{t+1} s_{t+2} p^{X,opt}_t}{\Pi_{t+2}\Pi_{t+1}} - \lambda^X_{t+2}\overline{mc}^X_{t+2}\right] + \ldots\} = 0.
\end{aligned}
$$

Using the following definitions: $\beta^r_{t+j} = \frac{\beta_{t+j}}{\beta_{t+j-1}}$, $\overline{\Omega}^C_{t+k} = \Omega^C_{t+k} z^+_{t+k}$ and $\overline{x}_{t+k} = \frac{x_{t+k}}{z^+_{t+k}}$, the stationarized version of the optimal price of export goods equation can be written as:

$$
E_t \sum_{k=0}^{\infty} (\xi_x)^k \left(\prod_{j=1}^{k}\beta^r_{t+j}\right)\frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t}\left(\prod_{j=1}^{k}\frac{\Pi_{t+j}}{\Pi^C_{t+j}}\right)\frac{\overline{x}_{t+k|t}}{(\lambda^X_{t+k} - 1)}\left[\left(\prod_{j=1}^{k}\frac{\overline{\Pi}^X_{t+j} s_{t+j}}{\Pi_{t+j}}\right)p^{X,opt}_t - \lambda^X_{t+k}\overline{mc}^X_{t+k}\right] = 0. \quad \text{(C.152)}
$$

Equation (C.152), which is the stationarized version of the optimal price of export goods, is the same as Equation (A.47a).

C.6 Import good producers

This section presents the optimization problem of import good producers in the Swedish economy and derives optimal prices. There are four different types of import firms in the Swedish economy. Each type operates in a separate and monopolistically competitive market. The first type of import firm is denoted by index Cxe and provides imported inputs to the non-energy consumption good producers. The second type of import firm is denoted by index I and provides imported goods to investment good producers. The third type of import firm is denoted by index X and provides imported goods to export firms. The fourth type of firm is denoted by index Ce and provides imported goods to the energy consumption producers. We derive the optimal price for import firms specializing in consumption goods, Equation (A.50a), investment goods, Equation (A.53a), export goods Equation (A.56a) and energy consumption goods, Equation (REF).

Let $n\epsilon\{Cxe, I, X, Ce\}$ be the index for different types of import firm, and let $M^n_t(i)$ represents the quantity produced by the individual firm i of type n. The individual import firm i of type n chooses the optimal price of imported goods $P^{M,n}_{t,opt}(i)$ that maximizes its profit, subject to its demand schedule and the Calvo sticky price friction. In each period, the individual firm i resets its price with probability $(1-\xi_{m,n})$. With probability $\xi_{m,n}$, the firm cannot reset its price, and then it faces the following price evolution: $P^{M,n}_{t+k|t}(i) = P^{M,n}_{t,opt}(i)\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\ldots\overline{\Pi}^{M,n}_{t+k}$. We define the stochastic discount factor as $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega^C_{t+k}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+k}}$.

The firms have to purchace the import goods from abroad, which differs between the three non-energy good firms, and the energy good firms. The non-energy good firms purchace Foreign domestic intermediate goods for the price $S_t P_{F,t}$ and transforms the goods to import good suitable for the respective input good purchaser. The energy good firms purchace foreign energy goods for the price $S_t P^{C,e}_{F,t}$ and transform them into goods suitable for the Swedish energy producer. This means that we can define the marginal cost for the different types of firms as

$$
\begin{aligned}
MC^{M,n}_t =& S_t P_{F,t} = \frac{Q_t P^C_t}{p^C_{F,t}}, n \in \{Cxe, I, X\} \\
MC^{M,n}_t =& S_t P^{C,e}_{F,t} = \frac{Q_t P^C_t}{p^C_{F,t}} p^{C,e}_{F,t}, n \in \{Ce\}
\end{aligned}
$$

using the definition that $Q_t = S_t P^C_{F,t}/P^C_t$, $p^C_{F,t} = P^C_{F,t}/P_{F,t}$ and $p^{C,e}_{F,t} = P^{C,e}_{F,t}/P_{F,t}$.
The optimization problem of import good producers under sticky prices can then be defined as follows:

$$
\max_{P^{M,n}_{t,opt}(i)} E_t \sum_{k=0}^{\infty}(\xi_{m,n})^k \Lambda_{t,t+k}\left\{P^{M,n}_{t+k|t}(i) M^n_{t+k|t}(i) - MC^{M,n}_{F,t+k} M^n_{t+k|t}(i) - MC^{M,n}_{t+k} z^+_{t+k}\phi^{M,n}\right\} \quad \text{(C.153)}
$$

subject to the demand function:

$$M^n_{t+k|t}(i) = \left(\frac{P^{M,n}_{t+k|t}(i)}{P^{M,n}_{t+k}}\right)^{\frac{\lambda^{M,n}_{t+k}}{1-\lambda^{M,n}_{t+k}}} M^n_{t+k}, \tag{C.154}$$

and the Calvo price setting contract:

$$P^{M,n}_{t+k}(i) = \begin{cases} \overline{\Pi}^{M,n}_{t+k} P^{M,n}_{t+k-1}(i) & \text{with probability } \xi_{m,n} \\ P^{M,n}_{t+k,opt}(i) & \text{with probability } (1-\xi_{m,n}). \end{cases} \tag{C.155}$$

The FOC of $P^{M,n}_{t,opt}(i)$ is:

$$\begin{aligned}
&E_t\{M^n_{t|t}(i) + P^{M,n}_{t,opt}(i)\frac{\partial M^n_{t|t}(i)}{\partial P^{M,n}_{t,opt}(i)} - MC^{M,n}_{t+k}\frac{\partial M^n_{t|t}(i)}{\partial P^{M,n}_{t,opt}(i)} \\
&+ \xi_{m,n}\Lambda_{t,t+1}\left[\frac{\partial P^{M,n}_{t+1|t}(i)}{\partial P^{M,n}_{t,opt}(i)}M^n_{t+1|t}(i) + P^{M,n}_{t+1|t}(i)\frac{\partial M^n_{t+1|t}(i)}{\partial P^{M,n}_{t+1|t}(i)}\frac{\partial P^{M,n}_{t+1|t}(i)}{\partial P^{M,n}_{t,opt}(i)} - MC^{M,n}_{t+1}\frac{\partial M^n_{t+1|t}(i)}{\partial P^{M,n}_{t+1|t}(i)}\frac{\partial P^{M,n}_{t+1|t}(i)}{\partial P^{M,n}_{t,opt}(i)}\right] \\
&+ (\xi_{m,n})^2\Lambda_{t,t+2} \\
&\left[\frac{\partial P^{M,n}_{t+2|t}(i)}{\partial P^{M,n}_{t,opt}(i)}M^n_{t+2|t}(i) + P^{M,n}_{t+2|t}(i)\frac{\partial M^n_{t+2|t}(i)}{\partial P^{M,n}_{t+2|t}(i)}\frac{\partial P^{M,n}_{t+2|t}(i)}{\partial P^{M,n}_{t,opt}(i)} - MC^{M,n}_{t+2}\frac{\partial M^n_{t+2|t}(i)}{\partial P^{M,n}_{t+2|t}(i)}\frac{\partial P^{M,n}_{t+2|t}(i)}{\partial P^{M,n}_{t,opt}(i)}\right] \\
&+ \ldots\} = 0.
\end{aligned} \tag{C.156}$$

Recall, we have the following definition: $P^{M,n}_{t+k|t}(i) = P^{M,n}_{t,opt}(i)\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\ldots\overline{\Pi}^{M,n}_{t+k}$. Hence, the partial derivative of $P^{M,n}_{t+k|t}(i)$ with respect to $P^{M,n}_{t,opt}(i)$ is:

$$\frac{\partial P^{M,n}_{t+k|t}(i)}{\partial P^{M,n}_{t,opt}(i)} = \overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\ldots\overline{\Pi}^{M,n}_{t+k}. \tag{C.157}$$

Using Equation (C.157), Equation (C.156) can be rewritten as:

$$\begin{aligned}
&E_t\{M^n_{t|t}(i) + P^{M,n}_{t,opt}(i)\frac{\partial M^n_{t|t}(i)}{\partial P^{M,n}_{t,opt}(i)} - MC^{M,n}_{t}\frac{\partial M^n_{t|t}(i)}{\partial P^{M,n}_{t,opt}(i)} \\
&+ \xi_{m,n}\Lambda_{t,t+1}\left[\overline{\Pi}^{M,n}_{t+1}M^n_{t+1|t}(i) + P^{M,n}_{t+1|t}(i)\overline{\Pi}^{M,n}_{t+1}\frac{\partial M^{M,n}_{t+1|t}(i)}{\partial P^{M,n}_{t+1|t}(i)} - MC^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+1}\frac{\partial M^n_{t+1|t}(i)}{\partial P^{M,n}_{t+1|t}(i)}\right] \\
&+ (\xi_{m,n})^2\Lambda_{t,t+2} \\
&\left[\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}M^n_{t+2|t}(i) + P^{M,n}_{t+2|t}(i)\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\frac{\partial M^n_{t+2|t}(i)}{\partial P^{M,n}_{t+2|t}(i)} - MC^{M,n}_{t+2}\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\frac{\partial M^n_{t+2|t}(i)}{\partial P^{M,n}_{t+2|t}(i)}\right] \\
&+ \ldots\} = 0.
\end{aligned}$$

We rearrange the above equation, and we can obtain the following equation:

$$\begin{aligned}
&E_t\{\frac{\partial M^n_{t|t}(i)}{\partial P^{M,n}_{t,opt}(i)}\left[P^{M,n}_{t,opt}(i)\left(\frac{M^n_{t|t}(i)}{P^{M,n}_{t,opt}(i)}\left(\frac{\partial M^n_{t|t}(i)}{\partial P^{M,n}_{t,opt}(i)}\right)^{-1} + 1\right) - MC^{M,n}_{t}\right] \\
&+ \xi_{m,n}\Lambda_{t,t+1}\overline{\Pi}^{M,n}_{t+1}\frac{\partial M^n_{t+1|t}(i)}{\partial P^{M,n}_{t+1|t}(i)}\left[P^{M,n}_{t+1|t}(i)\left(\frac{M^n_{t+1|t}(i)}{P^{M,n}_{t+1|t}(i)}\left(\frac{\partial M^n_{t+1|t}(i)}{\partial P^{M,n}_{t+1|t}(i)}\right)^{-1} + 1\right) - MC^{M,n}_{t+1}\right] \\
&+ (\xi_{m,n})^2\Lambda_{t,t+2}\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\frac{\partial M^n_{t+2|t}(i)}{\partial P^{M,n}_{t+2|t}(i)}\left[P^{M,n}_{t+2|t}(i)\left(\frac{M^n_{t+2|t}(i)}{P^{M,n}_{t+2|t}(i)}\left(\frac{\partial M^n_{t+2|t}(i)}{\partial P^{M,n}_{t+2|t}(i)}\right)^{-1} + 1\right) - MC^{M,n}_{t+2}\right] + \ldots\} = 0.
\end{aligned} \tag{C.158}$$

Given the demand schedule for imported goods, which is captured by Equation (C.154), we can find the following price elasticity of demand for imported goods:

$$-\frac{\partial M^n_{t+k|t}(i)}{\partial P^{M,n}_{t+k|t}(i)}\frac{P^{M,n}_{t+k|t}(i)}{M^n_{t+k|t}(i)}=\frac{\lambda^{M,n}_{t+k}}{\lambda^{M,n}_{t+k}-1}. \tag{C.159}$$

Using the result from Equation (C.159), the derivative of $M^n_{t+k|t}(i)$ with respect to $P^{M,n}_{t+k|t}(i)$ is:

$$\frac{\partial M^n_{t+k|t}(i)}{\partial P^{M,n}_{t+k|t}(i)}=\frac{\lambda^{M,n}_{t+k}}{1-\lambda^{M,n}_{t+k}}\frac{M^n_{t+k|t}(i)}{P^{M,n}_{t+k|t}(i)}. \tag{C.160}$$

Using Equation (C.159) and Equation (C.160), we can rewrite Equation (C.158) as follows:

$$\begin{aligned}
&E_t\{\frac{M^n_t(i)}{P^{M,n}_{t,opt}(i)}\frac{\lambda^{M,n}_t}{1-\lambda^{M,n}_t}\left[P^{M,n}_{t,opt}(i)\frac{1}{\lambda^{M,n}_t}-MC^{M,n}_{t+k}\right]\\
&+\xi_{m,n}\Lambda_{t,t+1}\overline{\Pi}^{M,n}_{t+1}\frac{M^n_{t+1|t}(i)}{P^{M,n}_{t+1|t}(i)}\frac{\lambda^{M,n}_{t+1}}{1-\lambda^{M,n}_{t+1}}\left[P^{M,n}_{t+1|t}(i)\frac{1}{\lambda^{M,n}_{t+1}}-MC^{M,n}_{t+1}\right]\\
&+(\xi_{m,n})^2\Lambda_{t,t+2}\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\frac{M^n_{t+2|t}(i)}{P^{M,n}_{t+2|t}(i)}\frac{\lambda^{M,n}_{t+2}}{1-\lambda^{M,n}_{t+2}}\left[P^{M,n}_{t+2|t}(i)\frac{1}{\lambda^{M,n}_{t+2}}-MC^{M,n}_{t+2}\right]+\ldots\}=0.
\end{aligned}$$

We use the following definition: $P^{M,n}_{t+k|t}(i)=P^{M,n}_{t,opt}(i)\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\ldots\overline{\Pi}^{M,n}_{t+k}$. We multiply both sides of the above equation by $P^{M,n}_{t,opt}(i)$ and -1. We can obtain the following equation:

$$\begin{aligned}
&E_t\{\frac{M^n_{t|t}(i)}{\lambda^{M,n}_t-1}\left[P^{M,n}_{t,opt}(i)-\lambda^{M,n}_tMC^{M,n}_t\right]\\
&+\xi_{m,n}\Lambda_{t,t+1}\frac{M^n_{t+1|t}(i)}{\lambda^{M,n}_{t+1}-1}\left[P^{M,n}_{t+1|t}(i)-\lambda^{M,n}_{t+1}MC^{M,n}_{t+1}\right]\\
&+(\xi_{m,n})^2\Lambda_{t,t+2}\frac{M^n_{t+2|t}(i)}{\lambda^{M,n}_{t+2}-1}\left[P^{M,n}_{t+2|t}(i)-\lambda^{M,n}_{t+2}MC^{M,n}_{t+2}\right]+\ldots\}=0.
\end{aligned}$$

We can drop the subscript i from the above equation because when firms can reset their price, they will set the same price. As a result, all firms that reset their price in period t will choose the same quantity of output. We rewrite the above equation, and the optimal price of imported goods equation can be expressed as:

$$E_t\sum_{k=0}^{\infty}(\xi_{m,n})^k\Lambda_{t,t+k}\frac{M^n_{t+k|t}}{\left(\lambda^{M,n}_{t+k}-1\right)}\left[P^{M,n}_{t+k|t}-\lambda^{M,n}_{t+k}MC^{M,n}_{t+k}\right]=0. \tag{C.161}$$

Equation (C.161), which is the non-stationarized version of the optimal price of imported goods, is the same as Equation (37) in Section 2.4.2.

We can rewrite the above equation in *per capita* terms, so we let m^n_t denote *per capita* output of import firms of type n who reset their price in period t. The optimal price of imported goods equation can be written as:

$$E_t\sum_{k=0}^{\infty}(\xi_{m,n})^k\Lambda_{t,t+k}\frac{m^n_{t+k|t}}{\left(\lambda^{M,n}_{t+k}-1\right)}\left[P^{M,n}_{t+k|t}-\lambda^{M,n}_{t+k}MC^{M,n}_{t+k}\right]=0. \tag{C.162}$$

Now, we stationarize Equation (C.162). We use the following definition: $\Lambda_{t,t+k}=\frac{\beta_{t+k}}{\beta_t}\frac{\Omega^C_{t+k}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+k}}$, and we expand Equation (C.162). Thus, we have the following equation:

$$\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{(\lambda^{M,n}_t-1)}\left[P^{M,n}_{t,opt}-\lambda^{M,n}_tMC^{M,n}_t\right]\\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{m^n_{t+1|t}}{(\lambda^{M,n}_{t+1}-1)}\left[P^{M,n}_{t+1|t}-\lambda^{M,n}_{t+1}MC^{M,n}_{t+1}\right]\\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{m^n_{t+2|t}}{(\lambda^{M,n}_{t+2}-1)}\left[P^{M,n}_{t+2|t}-\lambda^{M,n}_{t+2}MC^{M,n}_{t+2}\right]+\ldots\}=0.
\end{aligned}$$

We multiply all terms of the above equation by $\frac{1}{P_t}$ and expand. We can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{(\lambda^{M,n}_t-1)}\left[\frac{P^{M,n}_{t,opt}}{P_t}-\frac{\lambda^{M,n}_t MC^{M,n}_t}{P_t}\right]\\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{m^n_{t+1|t}}{(\lambda^{M,n}_{t+1}-1)}\frac{P_{t+1}}{P_t}\left[\frac{P^{M,n}_{t+1|t}}{P_{t+1}}-\frac{\lambda^{M,n}_{t+1}MC^{M,n}_{t+2}}{P_{t+1}}\right]\\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{m^n_{t+2|t}}{(\lambda^{M,n}_{t+2}-1)}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\left[\frac{P^{M,n}_{t+2|t}}{P_{t+2}}-\frac{\lambda^{M,n}_{t+2}MC^{M,n}_{t+3}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We use the following definition: $P^{M,n}_{t+k|t}=P^{M,n}_{t,opt}\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\ldots\overline{\Pi}^{M,n}_{t+k}$. We multiply the marginal utility of consumption Ω^C_{t+k} by z^+_{t+k}, and divide imported goods $m^n_{t+k|t}$ by z^+_{t+k}, and expand further to get:

$$
\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{\left(\lambda^{M,n}_t-1\right)z^+_t}\left[\frac{P^{M,n}_{t,opt}}{P_t}-\lambda^{M,n}_t\frac{MC^{M,n}_t}{P_t}\right]\\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{m^n_{t+1|t}}{\left(\lambda^{M,n}_{t+1}-1\right)z^+_{t+1}}\frac{P_{t+1}}{P_t}\left[\frac{\overline{\Pi}^{M,n}_{t+1}P^{M,n}_{t,opt}}{P_{t+1}}-\lambda^{M,n}_{t+1}\frac{MC^{M,n}_{t+1}}{P_{t+1}}\right]\\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{m^n_{t+2|t}}{\left(\lambda^{M,n}_{t+2}-1\right)z^+_{t+2}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\\
&\left[\frac{\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}P^{M,n}_{t,opt}}{P_{t+2}}-\lambda^{M,n}_{t+2}\frac{MC^{M,n}_{t+2}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We multiply the second term of the above equation by $\frac{P_t}{P_t}$, whereas we multiply the third term of the above equation by $\frac{P_tP_{t+1}}{P_tP_{t+1}}$. We have the following equation:

$$
\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{\left(\lambda^{M,n}_t-1\right)z^+_t}\left[\frac{P^{M,n}_{t,opt}}{P_t}-\lambda^{M,n}_t\frac{MC^{M,n}_t}{P_t}\right]\\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{m^n_{t+1|t}}{\left(\lambda^{M,n}_{t+1}-1\right)z^+_{t+1}}\frac{P_{t+1}}{P_t}\left[\frac{P_t}{P_t}\frac{\overline{\Pi}^{M,n}_{t+1}P^{M,n}_{t,opt}}{P_{t+1}}-\lambda^{M,n}_{t+1}\frac{MC^{M,n}_{t+1}}{P_{t+1}}\right]\\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{m^n_{t+2|t}}{\left(\lambda^{M,n}_{t+2}-1\right)z^+_{t+2}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\\
&\left[\frac{P_tP_{t+1}}{P_tP_{t+1}}\frac{\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}P^{M,n}_{t,opt}}{P_{t+2}}-\lambda^{M,n}_{t+2}\frac{MC^{M,n}_{t+2}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We use the following definitions: $p^{M,n}_{t,opt}=\frac{P^{M,n}_{t,opt}}{P_t}$, $\overline{mc}^{M,n}_{F,t+k}=\frac{MC^{M,n}_{t+k}}{P_{t+k}}$, $\Pi_{t+k}=\frac{P_{t+k}}{P_{t+k-1}}$, and $\Pi^C_{t+k}=\frac{P^C_{t+k}}{P^C_{t+k-1}}$. We can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{\left(\lambda^{M,n}_t-1\right)z^+_t}\left[p^{M,n}_{t,opt}-\lambda^{M,n}_t\overline{mc}^{M,n}_{F,t}\right]\\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_t z^+_t}\frac{\Pi_{t+1}}{\Pi^C_{t+1}}\frac{m^n_{t+1|t}}{\left(\lambda^{M,n}_{t+1}-1\right)z^+_{t+1}}\left[\frac{\overline{\Pi}^{M,n}_{t+1}p^{M,n}_{t,opt}}{\Pi_{t+1}}-\lambda^{M,n}_{t+1}\overline{mc}^{M,n}_{F,t+1}\right]\\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_t z^+_t}\frac{\Pi_{t+2}\Pi_{t+1}}{\Pi^C_{t+2}\Pi^C_{t+1}}\frac{m^n_{t+2|t}}{\left(\lambda^{M,n}_{t+2}-1\right)z^+_{t+2}}\\
&\left[\frac{\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}p^{M,n}_{t,opt}}{\Pi_{t+2}\Pi_{t+1}}-\lambda^{M,n}_{t+2}\overline{mc}^{M,n}_{F,t+2}\right]+\ldots\}=0.
\end{aligned}
$$

Using the following definitions: $\beta^r_{t+j}=\frac{\beta_{t+j}}{\beta_{t+j-1}}$, $\overline{\Omega}^C_{t+k}=\Omega^C_{t+k}z^+_{t+k}$ and $\overline{m}^n_{t+k}=\frac{m^n_{t+k}}{z^+_t}$, the stationarized version of optimal price of imported goods equation can be written as:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^C_{t+j}} \right) \frac{\overline{m}^n_{t+k|t}}{\left(\lambda^{M,n}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,n}_{t+j}}{\Pi_{t+j}} \right) p^{M,n}_{t,opt} - \lambda^{M,n}_{t+k} \overline{mc}^{M,n}_{F,t+k} \right] = 0. \quad \text{(C.163)}$$

Equation (C.163), which is the stationarized version of optimal price of imported goods. Recall that, $n \epsilon \{Cxe, I, X, Ce\}$ represents different types of import firms in the Swedish economy.

Replacing the index n with C, we have the following optimal price for import firms specializing in consumption goods:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,C})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^C_{t+j}} \right) \frac{\overline{m}^C_{t+k|t}}{\left(\lambda^{M,C}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,C}_{t+j}}{\Pi_{t+j}} \right) p^{M,C}_{t,opt} - \lambda^{M,C}_{t+k} \overline{mc}^{M,n}_{F,t+k} \right] = 0. \quad \text{(C.164)}$$

Equation (C.164), which is the stationarized version of optimal price for import firms specializing in consumption goods is the same as Equation (A.50a).

Replacing the index n with I, we have the following optimal price for import firms specializing in investment goods:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,I})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^C_{t+j}} \right) \frac{\overline{m}^I_{t+k|t}}{\left(\lambda^{M,I}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,I}_{t+j}}{\Pi_{t+j}} \right) p^{M,I}_{t,opt} - \lambda^{M,I}_{t+k} \overline{mc}^{M,n}_{F,t+k} \right] = 0. \quad \text{(C.165)}$$

Equation (C.166), which is the stationarized version of optimal price for import firms specializing in investment goods is the same as Equation (A.53a).

Replacing the index n with X, we have the following optimal price for import firms specializing in export goods:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,X})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^C_{t+j}} \right) \frac{\overline{m}^X_{t+k|t}}{\left(\lambda^{M,X}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,X}_{t+j}}{\Pi_{t+j}} \right) p^{M,X}_{t,opt} - \lambda^{M,X}_{t+k} \overline{mc}^{M,n}_{F,t+k} \right] = 0. \quad \text{(C.166)}$$

Equation (C.166) which is the stationarized version of optimal price for import firms specializing in export goods is the same as Equation (A.56a).
Finally, note that the stationarized marginal costs of the firms can be written as

$$\overline{mc}^{M,n}_t = \frac{S_t P_{F,t}}{P_t} = \frac{Q_t p^C_t}{p^C_{F,t}}, n \in \{Cxe, I, X\} \quad \text{(C.167)}$$

$$\overline{mc}^{M,n}_t = \frac{S_t P^{C,e}_{F,t}}{P_t} = \frac{Q_t p^C_t}{p^C_{F,t}} p^{C,e}_{F,t}, n \in \{Ce\} \quad \text{(C.168)}$$

Equation (C.168), which is the stationarized marginal cost for the energy importer, is the same as Equation (A.62a).

C.7 Fiscal authority

Given the fiscal authority budget constraint described in section 2.5, we can derive the stationarized version of the government budget constraint. The government budget constraint is given by:

$$\tau^C_t P^C_t C^{agg}_t + (\tau^{SSC}_t + \tau^W_t) W_t N_t + \Upsilon^K_t + B^n_t + T_t = \left(\alpha_B + (R^B_{t-1} - 1)\right) B_t + \tau^I_t \frac{P^I_t}{\gamma_t} I_t + P_t G_t + P_t \frac{I^G_t}{\gamma_t} + (1 - \tau^{TR}_t) TR^{agg}_t. \quad \text{(C.169)}$$

We stationarize the above equation by dividing the above equation by $z^+_t P_t$:

$$\tau^C_t \frac{P^C_t C^{agg}_t}{z^+_t P_t} + (\tau^{SSC}_t + \tau^W_t) \frac{W_t N_t}{z^+_t P_t} + \frac{\Upsilon^K_t}{z^+_t P_t} + \frac{B^n_t}{z^+_t P_t} + \frac{T_t}{z^+_t P_t} = \left(\alpha_B + (R^B_{t-1} - 1)\right) \frac{B_t}{z^+_t P_t} + \tau^I_t \frac{P^I_t}{z^+_t P_t \gamma_t} I_t + \frac{G_t}{z^+_t} + \frac{I^G_t}{\gamma_t z^+_t} + (1 - \tau^{TR}_t) \frac{TR^{agg}_t}{z^+_t P_t}.$$

In the Swedish economy, where the size of the population is normalized to unity, the distinction between aggregate and *per capita* variables are trivial. Nonetheless, we can express the above equation in terms of *per capita*. We denote $\bar{c}_t^{agg}$ as the stationarized aggregate consumption in *per capita*, $\bar{g}_t$ is the stationarized government consumption in *per capita*, $\bar{t}_t$ is lump-sum tax in *per capita*, and $\overline{tr}_t^{agg}$ is aggregate transfers in *per capita*. We use the following definitions: $\bar{c}_t^{agg} = \frac{c_t^{agg}}{z_t^+}$, $\overline{w}_t = \frac{W_t}{z_t^+ P_t^C}$, $p_t^C = \frac{P_t^C}{P_t}$, $\bar{t}_t = \frac{T_t}{z_t^+ P_t}$, $\overline{\Upsilon}_t^K = \frac{\Upsilon_t^K}{z_t^+ P_t}$, $\bar{b}_t^n = \frac{B_t^n}{z_t^+ P_t}$, $\bar{b}_t = \frac{B_t}{z_{t-1}^+ P_{t-1}}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, $\mu_{\gamma,t} = \frac{\gamma_t^+}{\gamma_{t-1}^+}$, $p_t^I = \frac{P_t^I}{P_t}$, $\bar{I}_t = \frac{I_t}{z_t^+ \gamma_t}$, $\bar{g}_t = \frac{g_t}{z_t^+}$, $\bar{I}_t^G = \frac{I_t^G}{\gamma_t z_t^+}$, and $\overline{tr}_t^{agg} = \frac{tr_t^{agg}}{z_t^+ P_t}$.[51]

The above government budget constraint can be written as:

$$\tau_t^C p_t^C \bar{c}_t^{agg} + (\tau_t^{SSC} + \tau_t^W) p_t^C \bar{w}_t n_t + \bar{\Upsilon}_t^K + \bar{b}_t^n + \bar{t}_t = \Big(\alpha_B + (R_{t-1}^B - 1)\Big) \frac{\bar{b}_t}{\mu_{z^+,t} \Pi_t} + \bar{g}_t + \tau_t^I p_t^I \bar{I}_t + \bar{I}_t^G + (1 - \tau_t^{TR}) \bar{tr}_t^{agg}. \tag{C.170}$$

Note that equation (C.170) is the same as equation (A.70a) in Section A.4.

Capital income tax revenues are given by:

$$\Upsilon_t^K = \tau_t^K \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t \right) - \iota^K \tau_t^K \delta P_{t-1}^K K_t. \tag{C.171}$$

We stationarize the above equation by dividing the above equation by $z_t^+ P_t$:

$$\frac{\Upsilon_t^K}{z_t^+ P_t} = \tau_t^K \frac{1}{z_t^+ P_t} \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t z_t^+ P_t} a(u_t) K_t \right) - \iota^K \tau_t^K \delta P_{t-1}^K K_t \frac{1}{z_t^+ P_t}.$$

Using the following definitions: $\overline{\Upsilon}_t^K = \frac{\Upsilon_t^K}{z_t^+ P_t}$, $\overline{K}_t = \frac{K_t}{z_{t-1}(\gamma_{t-1})^{\frac{1}{1-\alpha}}}$, $r_{t+1}^K = \frac{\gamma_{t+1} R_{t+1}^K}{P_{t+1}}$, $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, $\mu_{\gamma,t} = \frac{\gamma_t^+}{\gamma_{t-1}^+}$, $p_t^I = \frac{P_t^I}{P_t}$, $p_t^K = \frac{\gamma_t P_t^K}{P_t}$, the above equation becomes:

$$\bar{\Upsilon}_t^K = \tau_t^K \left(\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}} r_t^K u_t \overline{K}_t - \frac{1}{\mu_{z^+,t}\mu_{\gamma,t}} p_t^I a(u_t) \overline{K}_t \right) - \iota^K \tau_t^K \delta \frac{p_{t-1}^K}{\mu_{z^+,t}\Pi_t} \overline{K}_t.$$

We express the above equation in *per capita* terms as follows:

$$\bar{\Upsilon}_t^K = \tau_t^K \left(\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}} r_t^K u_t \bar{k}_t - \frac{1}{\mu_{z^+,t}\mu_{\gamma,t}} p_t^I a(u_t) \bar{k}_t \right) - \iota^K \tau_t^K \delta \frac{p_{t-1}^K}{\mu_{z^+,t}\Pi_t} \bar{k}_t,$$

$$\bar{\Upsilon}_t^K = \frac{\bar{k}_t}{\mu_{z^+,t}\mu_{\gamma,t}} \tau_t^K \left(r_t^K u_t - p_t^I a(u_t) - \iota^K \delta \frac{\mu_{\gamma,t} p_{t-1}^K}{\Pi_t} \right). \tag{C.172}$$

Equation (C.172) is the same as Equation (A.73a) in Section A.4.

From Section 2.5, the government surplus is given by:

$$SURP_t = \alpha_B B_t - B_t^n.$$

We stationarize the above equation by dividing the above equation by $z_t^+ P_t$:

$$\frac{SURP_t}{P_t z_t^+} = \alpha_B \frac{B_t}{P_t z_t^+} - \frac{B_t^n}{P_t z_t^+}.$$

Using the following definitions: $\overline{surp}_t = \frac{SURP_t}{P_t z_t^+}$, $\bar{b}_t = \frac{B_t}{z_{t-1}^+ P_{t-1}}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, and $\bar{b}_t^n = \frac{B_t^n}{z_t^+ P_t}$, the above equation can be written as:

$$\overline{surp}_t = \alpha_B \frac{\bar{b}_t}{\mu_{z^+t} \Pi_t} - \bar{b}_t^n. \tag{C.173}$$

Note that equation (C.173) is the same as equation (A.76a) in Section A.4.

[51] Two exceptions to the *per capita* notation are private and government investment: because we use i_t to denote the net nominal interest rate, $\bar{I}_t$ denotes both the stationarized level of aggregate investment and the stationarized level of aggregate investment per inhabitant in the Swedish economy. $\bar{I}_t^G$ is the stationarized level of government investment and the stationarized level of government investment *per capita*.

C.7.1 The structural surplus

The structural surplus is defined as the difference between the structural primary revenue, $Stprev_t$ and the structural primary expenditure, $Stpexp_t$, net of the interest payments on the current debt:

$$Stsurp_t = Stprev_t - Stpexp_t - (R^B_{t-1} - 1)B_t \tag{C.174}$$

where the structural primary expenditure is given by the cyclically adjusted government expenditure:

$$\frac{Stpexp_t}{P_t} = \left(\frac{TR^{agg}_t}{P_t} - F_{tr,un}Y\hat{un}_t\right) + \left(\frac{I^G_t}{\gamma_t} - \mathcal{F}_{IG,y}\frac{I^G(Y_t - Y)}{Y}\right) + \left(G_t - \mathcal{F}_{g,y}\frac{G(Y_t - Y)}{Y}\right) + \tau^I_t\frac{P^I}{\gamma P_t}I \tag{C.175}$$

and the structural primary revenues are given by the structural tax bases times the tax rates:

$$Stprev_t = \tau^C_t P^C C^{agg} + \left(\tau^{SSC}_t + \tau^W_t\right)WN + \tau^K_t K\left(R^K - \iota^K\delta\frac{P^K}{\Pi}\right) + \tau^{TR}_t(TR^{agg}_t - F_{tr,un}P_t\hat{un}_t) + T \tag{C.176}$$

where the variables without any time notation are the steady-state values of the stationarized per-capita equivalents of the variables.

The structural surplus and its right-hand-side variables can be stationarized by dividing all variables with $P_t z^+_t$ except for B_t, which is stationarized via $\bar{b}_t = B_t/(P_{t-1}z^+_{t-1})$, which can be written as $\bar{b} = B_t/(P_{t-1}z^+_{t-1}) = B_t\Pi_t\mu_{z^+,t}/(P_t z^+_t)$. Hence we can write the stationarized structural surplus as

$$\overline{Stsurp}_t = \overline{Stprev}_t - \overline{Stpexp}_t - \frac{(R^B_{t-1} - 1)}{\Pi_t\mu_{z^+,t}}\bar{b}_t \tag{C.177}$$

Equation C.177 is equivalent to equation A.83a in Section A.4.

The equation for structural primary expenditures (equation C.176) is stationarized by dividing through the equation by $P_t z^+_t$ to get

$$\frac{Stpexp_t}{P_t z^+_t} = \left(\frac{TR^{agg}_t}{P_t z^+_t} - \frac{F_{tr,un}Y\hat{un}_t}{z^+_t}\right) + \left(\frac{I^G_t}{\gamma_t z^+_t} - \mathcal{F}_{IG,y}\frac{I^G(Y_t - Y)}{Yz^+_t}\right) + \left(\frac{G_t}{z^+_t} - \mathcal{F}_{g,y}\frac{G(Y_t - Y)}{Yz^+_t}\right) + \tau^I_t\frac{P^I}{\gamma P_t z^+_t}I$$

Define $\bar{g}_t = \frac{g_t}{z^+_t}$, $\bar{I}^G_t = \frac{I^G_t}{\gamma_t z^+_t}$, and $\overline{tr}^{agg}_t = \frac{tr^{agg}_t}{z^+_t P_t}$

$$\overline{Stpexp}_t = (\bar{tr}^{agg}_t - \mathcal{F}_{tr,un}\bar{y}\hat{un}_t) + \left(\bar{I}^G - \mathcal{F}_{IG,y}\bar{I}^G\frac{(\bar{y}_t - \bar{y})}{\bar{y}}\right) + \left(\bar{g} - \mathcal{F}_{g,y}\bar{g}\frac{(\bar{y}_t - \bar{y})}{\bar{y}}\right) + \tau^I_t p^I\bar{I} \tag{C.178}$$

Equation C.178 is equivalent to equation A.84a in Section A.4.

The equation for structural primary revenues (equation C.175) is stationarized by dividing through the equation by $P_t z^+_t$ to get:

$$\frac{Stprev_t}{P_t z^+_t} = \frac{\tau^C_t P^C C^{agg}}{P_t z^+_t} + \frac{\left(\tau^{SSC}_t + \tau^W_t\right)WN}{P_t z^+_t} + \frac{\tau^K_t K(R^K - \iota^K\delta\frac{P^K}{\Pi})}{P_t z^+_t} + \frac{\tau^{TR}_t(TR^{agg}_t - F_{tr,un}P_t\hat{un}_t)}{P_t z^+_t} + \frac{T}{P_t z^+_t}$$

by using the definitions $\bar{c}^{agg}_t = \frac{c^{agg}_t}{z^+_t}$, $p^C_t = \frac{P^C_t}{P_t}$, $\overline{w}_t = \frac{W_t}{z^+_t P^C_t}$, $\bar{t}_t = \frac{t_t}{z^+_t P_t}$, $\overline{\Upsilon}^K_t = \frac{\Upsilon^K_t}{z^+_t P_t}$, $\overline{tr}^{agg}_t = \frac{tr^{agg}_t}{z^+_t P_t}$, $r^K_{t+1} = \frac{\gamma_{t+1}R^K_{t+1}}{P_{t+1}}$, $\mu_{z^+,t} = \frac{z^+_t}{z^+_{t-1}}$, $\mu_{\gamma,t} = \frac{\gamma^+_t}{\gamma^+_{t-1}}$, $p^K_t = \frac{\gamma_t P^K_t}{P_t}$,

$$\overline{Stprev}_t = \tau^C_t p^C\bar{c}^{agg} + (\tau^{SSC}_t + \tau^W_t)\overline{w}n + \tau^K_t\frac{\overline{k}}{\mu_{z^+}\mu_\gamma}\left(r^K - \iota^K\delta\frac{\mu_\gamma p^K}{\Pi}\right) + \tau^{TR}_t(\overline{tr}^{agg}_t - \mathcal{F}_{tr,un}\bar{y}\hat{un}_t) + \bar{t} \tag{C.179}$$

Equation C.179 is equivalent to equation A.85a in Section A.4.

D Technical appendix: Foreign economy

In this technical appendix, first we present the optimization problems for households and firms in the Foreign economy. Second, we present the key equilibrium conditions and model equations for the Foreign economy. We denote ω as the size of Foreign economy. We denote the subscript f as the individual household in the Foreign economy, and we denote the subscript j as the individual firm in the Foreign economy. We use the subscript F to denote the aggregate variables and the economy-wide average variables for the Foreign economy.

D.1 Foreign: Household sector

In this section, first we present the optimization problem of households in the Foreign economy. Second, we present the first-order conditions (FOCs) for households in the Foreign economy. The optimization problem of households in the Foreign economy are similar to those of the Swedish economy. However, households in the Foreign economy do not have an access to the market for bonds denominated in the Swedish currency. Moreover, there are no Non-Ricardian consumers and no fiscal sector. Households in the Foreign economy can only buy bonds that are denominated in the currency of Foreign. We let $\theta^b_{f,t}$ denote the Lagrangian multiplier associated with the budget constraint and $\theta^k_{f,t}$ denote Lagrangian multiplier associated with capital accumulation equation for the household f .

The utility function of individual household f is defined as:

$$E_0 \sum_{t=0}^{\infty} \beta_{F,t} \left[u(C_{f,t} - \rho_{F,h} C_{F,t-1}) - \zeta^n_{F,t} \nu(N_{f,t}) \right]. \tag{D.1}$$

where $\rho_{F,h}$ is the consumption habit formation parameter.
The individual household f chooses consumption $C_{f,t}$, physical capital $K_{f,t+1}$, investment $I_{f,t}$, capital utilization $u_{f,t}$, the change in capital stock by trading in the market $\triangle^K_{f,t}$, domestic nominal bonds that are denominated in the Foreign currency $B^{FF}_{f,t+1}$ and the nominal wage $W_{f,t}$ to maximize its expected utility subject to,

the budget constraint:

$$\begin{aligned} P^C_{F,t} C_{f,t} + \frac{P^I_{F,t}}{\gamma_t} I_{f,t} + P^K_{F,t} \triangle^K_{f,t} + \frac{B^{FF}_{f,t+1}}{R_{F,t}\zeta_{F,t}} &= (1-\tau^w_F) W_{f,t} N_{f,t} + R^K_{F,t} u_{f,t} K_{f,t} - \frac{P^I_{F,t}}{\gamma_t} a(u_{f,t}) K_{f,t} \\ &+ B^{FF}_{f,t} + \Xi_{B^{FF},t} + \Psi_{f,t} + TR_{f,t}, \end{aligned} \tag{D.2}$$

the labor demand schedule:

$$N_{f,t} = \frac{1}{\omega} \left(\frac{W_{f,t}}{W_{F,t}} \right)^{-\varepsilon^F_w} N_{F,t}, \tag{D.3}$$

the capital accumulation:

$$K_{f,t+1} = (1-\delta) K_{f,t} + \Upsilon_{F,t} F(I_{f,t}, I_{f,t-1}) + \triangle^K_{f,t}, \tag{D.4}$$

the Calvo wage contract:

$$W_{f,t+k} = \begin{cases} \overline{\Pi}^W_{F,t+k} W_{f,t+k-1} & \text{with probability } \xi^F_w \\ W^{\text{opt}}_{f,t+k} & \text{with probability } (1-\xi^F_w). \end{cases} \tag{D.5}$$

We make the use of the following definition: $\beta^r_{F,t+1} = \frac{\beta_{F,t+1}}{\beta_{F,t}}$, and we follow the similar steps to those in Section C.1.2, we can obtain the FOC for $C_{f,t}$, $B^{FF}_{f,t+1}$, $K_{f,t+1}$, $I_{f,t}$, $u_{f,t}$ and $\triangle^K_{f,t}$.

The FOC for $C_{f,t}$ is:

$$\theta^b_{f,t} P^C_{F,t} = \Omega^C_{f,t}. \tag{D.6}$$

Equation (D.6) is the same as Equation (72) in Section 2.6.1.

The FOC for $B^{FF}_{f,t+1}$ is:

$$\theta^b_{f,t} P^C_{F,t} = E_t \left[\beta^r_{F,t+1} \theta^b_{f,t+1} P^C_{F,t} R_{F,t} \zeta_{F,t} \right]. \tag{D.7}$$

Equation (D.7) is the same as Equation (73) in Section 2.6.1.

The FOC for $K_{f,t+1}$ is:

$$\theta^k_{f,t} = E_t \beta^r_{F,t+1} \left[\theta^b_{f,t+1} \left(R^K_{F,t+1} u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}} a(u_{f,t+1}) \right) + \theta^k_{f,t+1}(1-\delta_F) \right]. \tag{D.8}$$

Equation (D.8) is the same as Equation (74) in Section 2.6.1.

The FOC for $I_{f,t}$ is:

$$\theta^b_{f,t}\frac{P^I_{F,t}}{\gamma_{F,t}} = \theta^k_{f,t}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\theta^k_{f,t+1}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right]. \tag{D.9}$$

Equation (D.9) is the same as Equation (75) in Section 2.6.1.

The FOC for $u_{f,t}$ is:

$$R^K_{F,t}K_{f,t} = \frac{P^I_{F,t}}{\gamma_t}a'(u_{f,t})K_{f,t}. \tag{D.10}$$

Equation (D.10) is the same as Equation (76) in Section 2.6.1.

The FOC for $\triangle^K_{f,t}$. is:

$$\theta^b_{f,t}P^K_{F,t} = \theta^k_{f,t}. \tag{D.11}$$

Equation (D.11) is the same as Equation (77) in Section 2.6.1.

D.1.1 Foreign: Consumption Euler equation

This section presents the stationarized version of consumption Euler equation. We use Equation (D.6) and Equation (D.7), and apply the following definitions: $p^C_{F,t} = \frac{P^C_{F,t}}{P_{F,t}}$, and $\Pi^C_{F,t+1} = \frac{P^C_{F,t+1}}{P^C_{F,t}}$ to find the non-stationarized version of consumption Euler equation for the Foreign economy.

Following the similar steps that are used to derive the non-stationarized version of consumption Euler equation for the Swedish economy in Section C.1.3, we can obtain the following non-stationarized version of consumption Euler equation for the Foreign economy:

$$\Omega^C_{F,t} = E_t\left[\beta^r_{F,t+1}\frac{R_{F,t}\zeta_{F,t}}{\Pi^C_{F,t+1}}\Omega^C_{F,t+1}\right]. \tag{D.12}$$

We use the following definitions: $\mu_{z^+_F,t+1} = \frac{z^+_{F,t+1}}{z^+_{F,t}}$, $\overline{\Omega}^C_{F,t} = z^+_{F,t}\Omega^C_{F,t}$. To find the stationarized version of consumption Euler equation for the Foreign economy. Following the similar steps as in Section C.1.3, we can find the stationarized version of consumption Euler equation for the Foreign economy. Thus, Equation (D.12) becomes:

$$\overline{\Omega}^C_{F,t} = R_{F,t}\zeta_{F,t}E_t\left[\beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\mu_{z^+_F,t+1}\Pi^C_{F,t+1}}\right]. \tag{D.13}$$

Equation (D.13), which represents the stationarized version of consumption Euler equation for the Foreign economy, is the same as Equation (A.104a).

D.1.2 Foreign: Marginal utility of consumption

In this section, we present the stationarized version of marginal utility of consumption for the Foreign economy equation.

The Foreign utility function is the same as the Swedish utility function, and the functional form of the Foreign utility function is:

$$u(C_{f,t} - \rho_{F,h}C_{F,t-1}) = ln(C_{f,t} - \rho_{F,h}C_{F,t-1}).$$

Note that we abstract from government consumption in the foreign economy and thereby we can abstract from potential non-separability between foreign private consumption and government consumption.
We use the above utility functional form and follow the similar steps that are used to derive the non-stationarized version of the Swedish marginal utility of consumption equation in Section C.1.4. Thus, we can obtain the following non-stationarized version of the Foreign marginal utility of consumption:

$$\Omega^C_{F,t} = \frac{1}{(C_{F,t} - \rho_{F,h}C_{F,t-1})}. \tag{D.14}$$

Equation (D.14) can be written in *per capita* terms by applying the following definition: $c_{F,t} = \frac{C_{F,t}}{\omega}$. We define

$c_{F,t}$ as Foreign consumption *per capita* and ω is the size of the Foreign economy. We stationarize Equation (D.14) by using the following definitions: $\mu_{z^+_F,t+1} = \frac{z^+_{F,t+1}}{z^+_{F,t}}$, $\overline{\Omega}^C_{F,t} = z^+_{F,t}\Omega^C_{F,t}$, and $\overline{c}_{F,t} = \frac{c_{F,t}}{z^+_{F,t}}$. Equation (D.14) becomes:

$$\overline{\Omega}^C_{F,t} = \frac{1}{\left(\overline{c}_{F,t} - \rho_{F,h}\frac{1}{\mu_{z^+_F,t}}\overline{c}_{F,t-1}\right)}. \tag{D.15}$$

Equation (D.15), which represents the stationarized version of the Foreign marginal utility of consumption, is the same as Equation (A.105a).

D.1.3 Foreign: Capital utilization and household purchase of installed capital

This section derives stationarized capital utilization decision equation and the household purchases of installed capital equation respectively.

First, we derive the capital utilization decision equation. Recall, Equation (D.10), which shows the FOC for $u_{f,t}$, is written as:

$$R^K_{F,t}K_{f,t} = \frac{P^I_{F,t}}{\gamma_t}a'(u_{f,t})K_{f,t}.$$

Using the following definitions: $r^K_{F,t} = \frac{\gamma_t R^K_{F,t}}{P_{F,t}}$, and $p^I_{F,t} = \frac{P^I_{F,t}}{P_{F,t}}$, the above equation can be rewritten as follows:

$$\frac{\gamma_t R^K_{F,t}}{P_{F,t}} = \frac{P^I_{F,t}}{P_{F,t}}a'(u_{f,t}),$$

$$r^K_{F,t} = p^I_{F,t}a'(u_{f,t}).$$

Note that the optimal rate of utilization is a function of the two aggregate relative prices $r^K_{F,t}$ and $p^I_{F,t}$. All households in Foreign will then choose the same utilization rate, and the subscript f may be dropped from the above equation. Thus, we have the following capital utilization decision equation:

$$r^K_{F,t} = p^I_{F,t}a'(u_{F,t}). \tag{D.16}$$

Equation (D.16), which captures the capital utilization decision, is the same as Equation (A.106a).

Next, we derive the household purchases of installed capital equation (A.107a). Recall, Equation (D.8), which represents the FOC for $K_{f,t+1}$, is expressed as:

$$\theta^k_{f,t} = E_t\beta^r_{F,t+1}\left[\theta^b_{f,t+1}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right) + \theta^k_{f,t+1}(1-\delta_F)\right].$$

Using Equation (D.11) that shows $\theta^b_{f,t}P^K_{F,t} = \theta^k_{f,t}$, we can rewrite the above equation as:

$$\theta^b_{f,t}P^K_{F,t} = E_t\beta^r_{F,t+1}\left[\theta^b_{f,t+1}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right) + \theta^b_{f,t+1}P^K_{F,t+1}(1-\delta_F)\right].$$

We use Equation (D.6) that shows $\theta^b_{f,t}P^C_{F,t} = \Omega^C_{f,t}$ and use the following definition: $\Pi^C_{F,t+1} = \frac{P^C_{F,t+1}}{P^C_{F,t}}$. Thus, we can rewrite the above equation as follows:

$$\begin{aligned}P^C_{F,t}\theta^b_{f,t}P^K_{F,t} = {} & E_t\beta^r_{F,t+1}\left[\theta^b_{f,t+1}P^C_{F,t+1}\frac{1}{\Pi^C_{F,t+1}}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right)\right] \\ & + E_t\beta^r_{F,t+1}\left[\theta^b_{f,t+1}P^C_{F,t+1}\frac{1}{\Pi^C_{F,t+1}}P^K_{F,t+1}(1-\delta_F)\right],\end{aligned}$$

and

$$\Omega^C_{f,t}P^K_{F,t} = E_t\beta^r_{F,t+1}\left[\Omega^C_{f,t+1}\frac{1}{\Pi^C_{F,t+1}}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right) + \Omega^C_{f,t+1}\frac{1}{\Pi^C_{F,t+1}}P^K_{F,t+1}(1-\delta_F)\right].$$

We multiply both sides of the above equation by $\frac{\gamma_t}{P_{F,t}}$, and then we rewrite the above equation as follows:

$$\begin{aligned}\frac{\gamma_t P^K_{F,t}}{P_{F,t}} &= E_t \beta^r_{F,t+1}\left[\frac{\Omega^C_{f,t+1}}{\Omega^C_{f,t}}\frac{1}{\Pi^C_{F,t+1}}\frac{\gamma_t}{P_{F,t}}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right)\right] \\ &+ E_t \beta^r_{F,t+1}\left[\frac{\Omega^C_{f,t+1}}{\Omega^C_{f,t}}\frac{1}{\Pi^C_{F,t+1}}\frac{\gamma_t}{P_{F,t}}P^K_{F,t+1}(1-\delta_F)\right].\end{aligned}$$

In order to stationarize the above equation, we use the following definitions: $r^K_{F,t+1} = \frac{\gamma_{t+1}R^K_{t+1}}{P_{F,t+1}}$, $p^I_{F,t+1} = \frac{P^I_{F,t+1}}{P_{F,t+1}}$, $\mu_{\gamma,t+1} = \frac{\gamma_{t+1}}{\gamma_t}$, $p^K_{F,t} = \frac{\gamma_t P^K_{F,t}}{P_{F,t}}$, and $\Pi_{F,t+1} = \frac{P_{F,t+1}}{P_{F,t}}$. Also, since all households choose the same level of consumption and the same utilization rate, the subscript f may be dropped from the above equation. Thus, we have the following equation for the household purchases of installed capital:

$$\begin{aligned}\frac{\gamma_t P^K_{F,t}}{P_{F,t}} &= E_t \beta^r_{F,t+1}\left[\frac{\Omega^C_{F,t+1}}{\Omega^C_{F,t}}\frac{1}{\Pi^C_{F,t+1}}\frac{\gamma_t}{P_{F,t}}\frac{P_{F,t+1}}{\gamma_{t+1}}\left(r^K_{F,t+1}u_{F,t+1} - p^I_{F,t+1}a(u_{F,t+1})\right)\right] \\ &+ E_t \beta^r_{F,t+1}\left[\frac{\Omega^C_{F,t+1}}{\Omega^C_{F,t}}\frac{1}{\Pi^C_{F,t+1}}\frac{\gamma_t}{P_{F,t}}P^K_{F,t+1}(1-\delta_F)\right].\end{aligned}$$

We use the following definition: $p^K_{F,t} = \frac{\gamma_t P^K_{F,t}}{P_{F,t}}$, and the above equation can be written as follows:

$$p^K_{F,t} = E_t \beta^r_{F,t+1}\frac{\Omega^C_{F,t+1}}{\Omega^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{1}{\mu_{\gamma,t+1}}\left[r^K_{F,t+1}u_{F,t+1} - p^I_{F,t+1}a(u_{F,t+1}) + p^K_{F,t+1}(1-\delta_F)\right]. \tag{D.17}$$

Using the following definitions: $\overline{\Omega}^C_{F,t} = z^+_{F,t}\Omega^C_{F,t}$ and $\mu_{z^+_F,t+1} = \frac{z^+_{F,t+1}}{z^+_{F,t}}$, Equation (D.17) can be written as:

$$p^K_{F,t} = E_t \beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\overline{\Omega}^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{1}{\mu_{z^+_F,t+1}\mu_{\gamma,t+1}}\left[r^K_{F,t+1}u_{F,t+1} - p^I_{F,t+1}a(u_{F,t+1}) + p^K_{F,t+1}(1-\delta_F)\right]. \tag{D.18}$$

Equation (D.18) is the same as Equation (A.107a), which shows the stationarized version of the household purchase of installed capital.

D.1.4 Foreign: Investment

This section derives the household investment decision equation (A.108a). Recall that we have Equation (D.9) that shows the following FOC for $I_{f,t}$:

$$\theta^b_{f,t}\ \frac{P^I_{F,t}}{\gamma_{F,t}} = \theta^k_{f,t}\ \Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\theta^k_{f,t+1}\ \Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

The above equation can be expressed as:

$$P^I_{F,t} = \frac{\gamma_t\theta^k_{f,t}}{\theta^b_{f,t}}\ \Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\gamma_t\theta^k_{f,t+1}}{\theta^b_{f,t}}\ \Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

We use Equation (D.11), which shows $\theta^b_{f,t}P^K_{F,t} = \theta^k_{f,t}$. We can rewrite the above equation as follows:

$$\begin{aligned}P^I_{F,t} &= \frac{\gamma_t\theta^b_{f,t}P^K_{F,t}}{\theta^b_{f,t}}\ \Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\gamma_t\theta^b_{f,t+1}P^K_{F,t+1}}{\theta^b_{f,t}}\ \Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right], \\ P^I_{F,t} &= \gamma_t P^K_{F,t}\ \Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\gamma_t\theta^b_{f,t+1}P^K_{F,t+1}}{\theta^b_{f,t}}\ \Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].\end{aligned}$$

We use the following definition: $p^I_{F,t} = P^I_{F,t}/P_{F,t}$, and the above equation becomes:

$$p^I_{F,t} = \frac{\gamma_t P^K_{F,t}}{P_{F,t}}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\gamma_t\theta^b_{f,t+1}P^K_{F,t+1}}{P_{F,t}\theta^b_{f,t}}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

We multiply the second term on the right hand side of the above equation by $\frac{P_{F,t+1}\gamma_{t+1}}{P_{F,t+1}\gamma_{t+1}}$. We use the following definitions: $\mu_{\gamma,t+1} = \frac{\gamma_{t+1}}{\gamma_t}$ and $\Pi_{F,t+1} = \frac{P_{F,t+1}}{P_{F,t}}$. The above equation can then be rewritten as follows:

$$p^I_{F,t} = \frac{\gamma_t P^K_{F,t}}{P_{F,t}}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}}{\theta^b_{f,t}}\frac{\gamma_{t+1}P^K_{F,t+1}}{P_{F,t+1}}\frac{P_{F,t+1}}{P_{F,t}}\frac{\gamma_t}{\gamma_{t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right],$$

$$p^I_{F,t} = \frac{\gamma_t P^K_{F,t}}{P_{F,t}}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}}{\theta^b_{f,t}}\frac{\gamma_{t+1}P^K_{F,t+1}}{P_{F,t+1}}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

Using the following definition: $p^K_{F,t} = \frac{\gamma_{F,t}P^K_{F,t}}{P_{F,t}}$, this gives us the following equation:

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}}{\theta^b_{f,t}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

Using the following definitions: $\Pi^C_{F,t+1} = \frac{P^C_{F,t+1}}{P^C_{F,t}}$ and $\Omega^C_{f,t} = \theta^b_{f,t}P^C_{F,t}$, we can rewrite the above equation as follows:

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}P^C_{F,t+1}}{\theta^b_{f,t}P^C_{F,t}}\frac{P^C_{F,t}}{P^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right]$$

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}P^C_{F,t+1}}{\theta^b_{f,t}P^C_{F,t}}\frac{1}{\Pi^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right],$$

and we can obtain the following equation:

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\Omega^C_{f,t+1}}{\Omega^C_{f,t}}\frac{1}{\Pi^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

Since all households choose the same level of investment we replace the subscript f with F and write the equation in aggregate variables. Hence, we have the following equation for the household investment decision:

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(I_{F,t}, I_{F,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\Omega^C_{F,t+1}}{\Omega^C_{F,t}}\frac{1}{\Pi^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{F,t+1}, I_{F,t})\right]. \quad \text{(D.19)}$$

Now, we continue the effort to stationarize Equation (D.19). Using the following definitions: $\overline{\Omega}^C_{F,t} = z^+_{F,t}\Omega^C_{F,t}$ and $\mu_{z^+_F,t+1} = \frac{z^+_{F,t+1}}{z^+_{F,t}}$, Equation (D.19) can be written as follows:

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(I_{F,t}, I_{F,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{z^+_{F,t+1}\Omega^C_{F,t+1}}{z^+_{F,t}\Omega^C_{F,t}}\frac{z^+_{F,t}}{z^+_{F,t+1}}\frac{1}{\Pi^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{F,t+1}, I_{F,t})\right],$$

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(I_{F,t}, I_{F,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\overline{\Omega}^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{p^K_{F,t+1}}{\mu_{z^+_F,t+1}\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{F,t+1}, I_{F,t})\right]. \quad \text{(D.20)}$$

Furthermore, we need to express $F_1(I_{F,t}, I_{F,t-1})$ and $F_2(I_{F,t+1}, I_{F,t})$ as functions of stationary variables. Recall from Section 3.1 , we have the following investment adjustment cost function $F(I_{F,t}, I_{F,t-1})$:

$$F(I_{F,t}, I_{F,t-1}) = \left[1 - \widetilde{S}\left(\frac{I_{F,t}}{I_{F,t-1}}\right)\right]I_{F,t}.$$

We take the first derivative of $F(I_{F,t}, I_{F,t-1})$ with respect to $I_{F,t}$, and we can find $F_1(I_{F,t}, I_{F,t-1})$. We then take the first derivative of $F(I_{F,t+1}, I_{F,t})$ with respect to $I_{F,t}$, and we can find $F_2(I_{F,t+1}, I_{F,t})$. We have the following results:

$$F_1(I_{F,t}, I_{F,t-1}) = -\widetilde{S}'\left(\frac{I_{F,t}}{I_{F,t-1}}\right)\frac{I_{F,t}}{I_{F,t-1}} + \left[1 - \widetilde{S}\left(\frac{I_{F,t}}{I_{F,t-1}}\right)\right], \quad \text{(D.21)}$$

and

$$F_2(I_{F,t+1}, I_{F,t}) = \widetilde{S}'\left(\frac{I_{F,t+1}}{I_{F,t}}\right)\left(\frac{I_{F,t+1}}{I_{F,t}}\right)^2. \quad \text{(D.22)}$$

We express Equation (D.21) and Equation (D.22) by applying the following definition: $\overline{I}_{F,t} = \frac{I_{F,t}}{z^+_{F,t}\gamma_t}$. Using this definition, together with $\mu_{z^+_F,t} = \frac{z^+_{F,t}}{z^+_{F,t-1}}$ and $\mu_{\gamma,t} = \frac{\gamma^+_t}{\gamma^+_{t-1}}$, the ratio $\frac{I_{F,t}}{I_{F,t-1}}$ can be written as: $\mu_{z^+_F,t}\,\mu_{\gamma,t}\,\frac{\overline{I}_{F,t}}{\overline{I}_{F,t-1}}$. We use the notation $F_1(\overline{I}_{F,t},\overline{I}_{F,t-1},\mu_{z^+,t},\mu_{\gamma,t})$ to express $F_1(I_{F,t},I_{F,t-1})$ as a function of the stationary variables $\overline{I}_{F,t}$, $\overline{I}_{F,t-1}$, $\mu^F_{z^+_F,t}$ and $\mu^F_{\gamma,t}$. Moreover, $F_2(\overline{I}_{F,t+1},\overline{I}_{F,t},\mu_{z^+_F,t+1},\mu_{\gamma,t+1})$ represents $F_2(I_{F,t+1},I_{F,t})$ expressed as a function of stationary variables. Hence, Equation (D.21) and Equation (D.22) become:

$$F_1(\overline{I}_{F,t},\overline{I}_{F,t-1},\mu_{z^+_F,t},\mu_{\gamma,t}) = -\widetilde{S}'\left(\frac{\mu_{z^+_F,t}\,\mu_{\gamma,t}\overline{I}_t}{\overline{I}_{F,t-1}}\right)\frac{\mu_{z^+_F,t}\,\mu_{\gamma,t}\overline{I}_{F,t}}{\overline{I}_{F,t-1}} + \left[1-\widetilde{S}\left(\frac{\mu_{z^+_F,t}}{\overline{I}_{F,t-1}}\right)\right],$$

and

$$F_2(\overline{I}_{F,t+1},\overline{I}_{F,t},\mu_{z^+_F,t+1},\mu_{\gamma,t+1}) = \widetilde{S}'\left(\frac{\mu_{z^+_F,t+1}\,\mu_{\gamma,t+1}\overline{I}_{F,t+1}}{\overline{I}_{F,t}}\right)\left(\frac{\mu_{z^+_F,t+1}\,\mu_{\gamma,t+1}\overline{I}_{F,t+1}}{\overline{I}_{F,t}}\right)^2.$$

With these notations, we can rewrite Equation (D.20) as:

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(\overline{I}_{F,t},\overline{I}_{F,t-1},\mu_{z^+_F,t},\mu_{\gamma,t}) + E_t\left[\beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\overline{\Omega}^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{p^K_{F,t+1}}{\mu_{z^+_F,t+1}\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(\overline{I}_{F,t+1},\overline{I}_{F,t},\mu_{z^+_F,t+1},\mu_{\gamma,t+1})\right] \tag{D.23}$$

Equation (D.23), which represents the stationarized version of the household investment decision equation, is the same as Equation (A.108a).

D.1.5 Foreign: Wage setting

This section presents the stationarized version of the optimal wage setting equation. In this section, the household f chooses the optimal wage rate $W^{opt}_{f,t}$ that maximizes the expected utility (D.1), subject to the budget constraint (D.2), the labor demand schedule (D.3), and the Calvo wage contract (D.5). In each period, the individual household resets its wage with probability $(1-\xi^F_w)$. With probability ξ^F_w, the household cannot reset its wage, in which case the wage rate evolves according to: $W_{f,t+k|t} = W^{opt}_{f,t}\overline{\Pi}^W_{F,t+1}\overline{\Pi}^W_{F,t+2}\ldots\overline{\Pi}^W_{F,t+k}$. Note that $\overline{\Pi}^W_{F,t} = (\Pi^W_{F,t-1})^{\chi_{F,w}}\left(\Pi^{C,trend}_{F,t}\right)^{1-\chi_{F,w}}$.

We apply the following definitions: $\varepsilon^F_w = \frac{\lambda^W_F}{\lambda^W_F-1}$ and $\frac{\partial N_{f,t+k|t}}{\partial W_{f,t+k|t}}\frac{W_{f,t+k|t}}{N_{f,t+k|t}} = \frac{\lambda^W_F}{1-\lambda^W_F}$. We also use the following definition: $W_{f,t+k|t} = W^{opt}_{f,t}\overline{\Pi}^W_{F,t+1}\overline{\Pi}^W_{F,t+2}\ldots\overline{\Pi}^W_{F,t+k}$. We follow the same steps to those in Section C.1.9; hence, we can find the following non-stationarized version of the optimal wage setting equation for the Foreign economy:

$$E_t\sum_{k=0}^{\infty}\left(\xi^F_w\right)^k\left(\prod_{i=1}^{k}\beta^r_{F,t+i}\right)N_{f,t+k|t}\,\theta^b_{f,t+k}\left[(1-\tau^w_F)\,W_{f,t+k|t} - \lambda^W_F\,\zeta^n_{F,t+k}\frac{\upsilon'\left(N_{f,t+k|t}\right)}{\theta^b_{f,t+k}}\right] = 0. \tag{D.24}$$

Equation (D.24) is the same as Equation (78) in Section 2.6.1.

Equation (D.24) can be expressed in terms of *per capita* quantities by applying the following definition: $n_{F,t} = \frac{N_{F,t}}{\omega}$. We define $n_{F,t}$ as Foreign aggregate hours *per capita* and ω is the size of the Foreign economy. We stationarize Equation (D.24) by using the following definitions: $\overline{w}_{F,t+k|t} = \frac{W_{F,t+k|t}}{z^+_{F,t+k}P^C_{F,t+k}}$ and $\overline{\Omega}^C_{F,t+k} = z^+_{F,t+k}\Omega^C_{F,t+k}$.

We follow the similar steps as in Section C.1.9, and we can obtain the following stationarized version of the optimal wage setting equation for the Foreign economy:

$$E_t\sum_{k=0}^{\infty}\left(\xi^F_w\right)^k\left(\prod_{j=1}^{k}\beta^r_{F,t+j}\right)n_{F,t+k|t}\overline{\Omega}^C_{F,t+k}\left[(1-\tau^w_F)\,\overline{w}_{F,t+k|t} - \lambda^W_F\,\zeta^n_{F,t+k}\frac{\nu'(n_{F,t+k|t})}{\overline{\Omega}^C_{F,t+k}}\right] = 0. \tag{D.25}$$

Equation (D.25), which represents the stationarized version of the optimal wage setting for the Foreign economy, is the same as Equation (A.111a).

D.2 Foreign: Intermediate good producers

In this section, we present the stationarized version of the expression for the real marginal cost of production of Foreign intermediate good producers. Intermediate good producers in the Foreign economy use labor and private capital as inputs, so different from the Swedish economy they do not use public capital. As in Swedish economy, $z^+_{F,t}$ combines a global labor augmenting technological process z_t and a technological process specific to the production of investment goods γ_t. There is a continuum of intermediate good producers of mass ω. The individual firm in the Foreign economy is denoted by j. Firm j uses capital services $K_{f,t}(j)$ and labor $L_{F,t}(j)$ to minimize the following cost function:

$$TC_t(j) = R^K_{F,t} K^s_{F,t}(j) + W_{F,t} N_{F,t}(j) \tag{D.26}$$

subject to the production constraint:

$$Y_{F,t}(j) = \varepsilon_{F,t} \left[K^s_{F,t}(j)\right]^{\alpha_F} \left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z^+_{F,t}\phi_F. \tag{D.27}$$

We denote $\theta_{F,t}(j)$ as the Lagrange multiplier associated with the production constraint D.27. To solve the optimization problem, we set up the following Lagrangian $\mathscr{L}_{F,t}(j)$:

$$\mathscr{L}_t(i) = R^K_{F,t} K^s_{F,t}(j) + W_{F,t} N_{F,t}(j) - \theta_{F,t}(j)\left[\varepsilon_{F,t}\left[K^s_{F,t}(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z^+_{F,t}\phi_F - Y_{F,t}(j)\right].$$

We take the partial derivative of $\mathscr{L}_{F,t}(j)$ with respect to $K^s_{F,t}(j)$ and $N_{F,t}(j)$ respectively, and we can find the FOCs.

The FOC for $K_{F,t}(j)$ is:

$$R^K_{F,t} - \alpha_F \theta_{F,t}(j)\varepsilon_{F,t}\left[K^s_{F,t}(j)\right]^{\alpha_F - 1}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} = 0. \tag{D.28}$$

The FOC for $N_{F,t}(j)$ is:

$$W_{F,t} - \theta_{F,t}(j)\left(1-\alpha_F\right)\varepsilon_{F,t}\left[K^s_{F,t}(j)\right]^{\alpha_F} z_t^{1-\alpha}\left[N_{F,t}(j)\right]^{-\alpha_F} = 0. \tag{D.29}$$

Using Equation (D.28) and Equation (D.29), we obtain the following capital-labor input efficiency condition:

$$K^s_{F,t}(j) = \frac{\alpha_F}{1-\alpha_F}\frac{W_{F,t}}{R^K_{F,t}} N_{F,t}(j).$$

Note that Equation (D.27) can be written as:

$$\left[Y_{F,t}(j) + z^+_t \phi^F\right] = \varepsilon_{F,t}\left[K^s_{F,t}(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F}. \tag{D.30}$$

Now, we find the total cost of production equation. We substitute Equation (D.28) and Equation (D.29) into Equation (D.26) , and we have the following equation:

$$TC_{F,t}(j) = \theta_{F,t}(j)\left[\alpha_F \varepsilon_{F,t} K^s_{F,t}(j)^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} + \left(1-\alpha_F\right)\varepsilon_{F,t}\left[K^s_{F,t}(j)\right]^{\alpha_F} z_t^{1-\alpha_F}\left[N_{F,t}(j)\right]^{1-\alpha_F}\right]. \tag{D.31}$$

Using Equation (D.30), we can rewrite Equation (D.31) as follows:

$$TC_{F,t}(j) = \theta_{F,t}(j)\left[\left(1-\alpha_F\right) + \alpha_F\right]\left(Y_{F,t}(j) + z^+_{F,t}\phi^F\right). \tag{D.32}$$

We use Equation (D.32), and we take the partial derivative of $TC_{F,t}(j)$ with respect to $Y_{F,t}(j)$. Hence, the lagrangian multiplier, $\theta_{F,t}(j)$, can be defined as the marginal cost of production $MC_{F,t}(j)$:

$$\frac{\partial TC_{F,t}(j)}{\partial Y_{F,t}(j)} = MC_{F,t}(j) = \theta_{F,t}(j).$$

Combining the two first-order conditions and solving for $N_{F,t}(j)$ yields

$$N_{F,t}(j) = \frac{(1-\alpha_F) R^K_{F,t} K^S_{F,t}(j)}{\alpha_F W^K_{F,t}}. \tag{D.33}$$

Substituting this expression back into the first-order condition with respect to for $N_{F,t}(j)$ gives

$$\theta_{F,t}(j) = \frac{W_{F,t}}{(1-\alpha_F)\varepsilon_{F,t}\left[K^S_{F,t}(j)\right]^{\alpha_F} z_{F,t}^{1-\alpha_F}}\left[N_{F,t}(j)\right]^{\alpha_F} = \frac{\left[R^K_{F,t}\right]^{\alpha_F} W_{F,t}^{1-\alpha_F}}{\alpha_F^{\alpha_F}(1-\alpha_F)^{1-\alpha_F}\varepsilon_{F,t} z_{F,t}^{1-\alpha_F}},$$

and hence

$$\frac{\partial TC_{F,t}(j)}{\partial Y_{F,t}(j)} = MC_{F,t}(j) = \theta_{F,t}(j) = \frac{W_{F,t}^{1-\alpha_F}\left(R^K_{F,t}\right)^{\alpha_F}}{\alpha_F^{\alpha_F}(1-\alpha_F)^{1-\alpha_F}\varepsilon_{F,t} z_t^{1-\alpha_F}} \tag{D.34}$$

Equation (D.34), which is the same as Equation (80) in Section 2.6.2, is the nominal marginal cost of production for the intermediate good firm j.
There are two equations that summarize optimal input decisions by firms: 1) the capital-labor input efficiency condition and 2) the optimal capital inputs in terms of marginal cost First, we find the capital-labor input efficiency equation. In particular, we rewrite Equation (D.33) as follows:

$$\frac{K^s_{F,t}(j)}{N_{F,t}(j)} = \frac{\alpha_F}{1-\alpha_F}\frac{W_{F,t}}{R^K_{F,t}}. \tag{D.35}$$

Equation (D.35) is the capital-labor input efficiency condition.
Firms hiring from homogeneous labor and private markets, i.e. face the same wage and rental rates. This implies that marginal costs are identical across firms. Equation (D.34) can be written as:

$$MC_{F,t} = \frac{\left(\frac{W_{F,t}}{z_t}\right)^{1-\alpha_F}\left(R^K_{F,t}\right)^{\alpha_F}}{\alpha_F^{\alpha_F}\left(1-\alpha_F\right)^{1-\alpha_F}\varepsilon_{F,t}}. \tag{D.36}$$

Equation (D.36) is the non-stationarized version of the nominal marginal cost of production for intermediate good firm.

Next, we derive the non-stationarized version of the rental rate for capital services. Using Equation (D.35) and Equation (D.36), we obtain the following equation:

$$R^K_{F,t} = \alpha_F\varepsilon_{F,t} MC_{F,t}(j)\left[K^s_{F,t}(j)\right]^{\alpha_F-1}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F}. \tag{D.37}$$

Since we have identical capital labor ratios and identical marginal costs, we can drop the subscript j and rewrite Equation (D.37) as:

$$R^K_{F,t} = \alpha_F\varepsilon_{F,t} z_t^{1-\alpha_F} MC_{F,t}\left(\frac{K^s_{F,t}}{N_{F,t}}\right)^{\alpha_F-1}. \tag{D.38}$$

Equation (D.38) is the same as Equation (81) in Section 2.6.2 and captures the non-stationarized version of rental rate for capital services.

Now, we find the stationarized version of the marginal cost of production for intermediate good producers. We stationarize Equation (D.36) by applying the following definitions: $r^K_{F,t} = \frac{\gamma_t R^K_{F,t}}{P_{F,t}}$, $\overline{w}^e_{F,t} = \frac{W_{F,t}}{z^+_{F,t}P_{F,t}}$, $z^+_{F,t} = z_t\left(\gamma_t\right)^{\frac{\alpha_F}{1-\alpha_F}}$, and $\overline{mc}_{F,t} = \frac{MC_{F,t}}{P_{F,t}}$. Equation (D.36) can be written as follows:

$$\frac{MC_{F,t}}{P_{F,t}} = \frac{\left(\frac{W_{F,t}}{z_t}\right)^{1-\alpha_F}\left(\frac{1}{P_{F,t}}\right)^{1-\alpha_F}\left(\frac{1}{P_{F,t}}\right)^{\alpha_F}\left(R^K_{F,t}\right)^{\alpha_F}\frac{\gamma_t^{\alpha_F}}{\gamma_t^{\alpha_F}}}{\alpha_F^{\alpha_F}\left(1-\alpha_F\right)^{1-\alpha_F}\varepsilon_{F,t}},$$

$$\overline{mc}_{F,t} = \frac{\left(\frac{W_{F,t}}{z_t(\gamma_t)^{\alpha_F/(1-\alpha_F)}\,P_{F,t}}\right)^{1-\alpha_F}\left(\frac{\gamma_t R^K_{F,t}}{P_{F,t}}\right)^{\alpha_F}}{\alpha_F^{\alpha_F}\left(1-\alpha_F\right)^{1-\alpha_F}\varepsilon_{F,t}},$$

$$\overline{mc}_{F,t} = \frac{\left(\frac{W_{F,t}}{z^+_{F,t}P_{F,t}}\right)^{1-\alpha_F}\left(\frac{\gamma_t R^K_{F,t}}{P_{F,t}}\right)^{\alpha_F}}{\alpha_F^{\alpha_F}\left(1-\alpha_F\right)^{1-\alpha_F}\varepsilon_{F,t}}.$$

Thus, the stationarized version of the marginal cost of production for intermediate good producers (the real marginal cost of production) equation can be expressed as:

$$\overline{mc}_{F,t} = \frac{\left(\overline{w}^e_{F,t}\right)^{1-\alpha_F}\left(r^K_{F,t}\right)^{\alpha_F}}{\varepsilon_{F,t}\alpha_F^{\alpha_F}(1-\alpha_F)^{1-\alpha_F}}. \tag{D.39}$$

Equation (D.39), which represents the real marginal cost of production for intermediate good producers, is the same as Equation (A.117a) in Section A.

Lastly, we find the stationarized version of the rental rate for capital services. We stationarize Equation (D.38) by applying the following definitions: $r^K_{F,t} = \frac{\gamma_t R^K_{F,t}}{P_{F,t}}$, $z^+_{F,t}\gamma_t = z_t\gamma_t^{1/(1-\alpha_F)}$, $\bar{K}^s_{F,t} = \frac{K^s_{F,t}}{z^+_{F,t-1}\gamma_{t-1}}$, and $\overline{mc}_{F,t} = \frac{MC_{F,t}}{P_{F,t}}$. We follow the similar steps when deriving the stationarized version of the marginal cost for intermediate good producers equation. Hence, Equation (D.38) can be written as:

$$r^K_{F,t} = \alpha_F \varepsilon_{F,t} \left(\frac{\bar{K}^s_{F,t}}{N_{F,t}} \frac{1}{\mu_{z^+_F,t}\mu_{\gamma,t}} \right)^{\alpha_F - 1} \overline{mc}_{F,t}$$

Furthermore, we can rewrite the above equation in terms of *per capita*, so we denote $\bar{k}^s_{F,t}$ as stationarized capital services *per capita*, and $n_{F,t}$ as aggregate labor input *per capita* by using the following definitions: $\bar{k}^s_{F,t} = \frac{\bar{K}^s_{F,t}}{\omega}$ and $n_{F,t} = \frac{N_{F,t}}{\omega}$. Hence, we can rewrite the above equation as:

$$r^K_{F,t} = \alpha_F \varepsilon_{F,t} \left(\frac{\bar{k}^s_{F,t}}{n_{F,t}} \frac{1}{\mu_{z^+_F,t}\mu_{\gamma,t}} \right)^{\alpha_F - 1} \overline{mc}_{F,t} \tag{D.40}$$

Equation (D.40), which is the real rental rate for capital services equation, is the same as Equation (A.118a) in Section A.

D.2.1 Foreign: Optimal price of intermediate goods

In this section, we present the stationarized version of the expression for the optimal price of intermediate goods for Foreign intermediate good producers. The firm j chooses the optimal price $P^{opt}_{F,t}(j)$ that maximizes its profit, subject to its demand schedule and the Calvo price contract. In each period, the individual firm j resets its price with probability $(1-\xi^F)$. With probability ξ^F, the firm cannot reset its price, and then it faces the following price evolution: $P_{F,t+k|t}(j) = P^{opt}_{F,t}(j)\overline{\Pi}_{F,t+1}\overline{\Pi}_{F,t+2}\ldots\overline{\Pi}_{F,t+k}$. We define the stochastic discount factor as $\Lambda^F_{t,t+k} = \frac{\beta_{F,t+k}}{\beta_{F,t}} \frac{\Omega^C_{F,t+k}}{\Omega^C_{F,t}} \frac{P^C_{F,t}}{P^C_{F,t+k}}$.

Firm j chooses the optimal price of intermediate goods $P^{opt}_{F,t}(j)$ to maximize the following profit function:

$$\max_{P^{opt}_{F,t}(j)} E_t \sum_{k=0}^{\infty} (\xi^F)^k \Lambda^F_{t,t+k} \left\{ P_{F,t+k|t}(j) Y_{F,t+k|t}(j) - TC_{F,t+k|t}\left[Y_{F,t+k|t}(j)\right] \right\}$$

subject to the demand function:

$$Y_{F,t+k|t}(j) = \frac{1}{\omega} \left(\frac{P_{F,t+k|t}(j)}{P_{F,t+k}} \right)^{\frac{\lambda_{F,t+k}}{1-\lambda_{F,t+k}}} Y_{F,t+k},$$

and the Calvo price setting contract:

$$P_{F,t+k}(j) = \begin{cases} \overline{\Pi}_{F,t+k} P_{F,t+k-1}(j) & \text{with probability } \xi^F \\ P^{opt}_{F,t+k}(j) & \text{with probability } (1-\xi^F). \end{cases}$$

We apply the following definitions: $-\frac{\partial Y_{F,t+k|t}(j)}{\partial P_{F,t+k|t}(j)} \frac{P_{F,t+k|t}(j)}{Y_{F,t+k|t}(j)} = \frac{\lambda_{F,t+k}}{\lambda_{F,t+k}-1}$, and $P_{F,t+k|t}(j) = P^{opt}_{F,t}(j)\overline{\Pi}_{F,t+1}\overline{\Pi}_{F,t+2}\ldots\overline{\Pi}_{F,t+k}$, and we follow the same steps to those in Section C.2.1. Hence, we can find the following non-stationarized version of the optimal price of intermediate goods equation for the Foreign economy:

$$E_t \sum_{k=0}^{\infty} (\xi^F)^k \Lambda^F_{t,t+k} \frac{Y_{F,t+k|t}}{(\lambda_{F,t+k}-1)} \left(P_{F,t+k|t} - \lambda_{F,t+k}\, MC_{F,t+k} \right) = 0. \tag{D.41}$$

Equation (D.41), which is the non-stationarized version of the optimal price of intermediate goods equation, is the same as Equation (82) in Section 2.6.2.

Equation (D.41) can be written in terms of *per capita* quantities by using the following definition: $y_{F,t} = \frac{Y_{F,t}}{\omega}$. We define $y_{F,t}$ as Foreign aggregate output *per capita* and ω is the size of the Foreign economy. We stationarize Equation (D.41) by using the following definitions: $p^{opt}_{F,t} = \frac{P^{opt}_{F,t}}{P_{F,t-1}}$, $\Pi_{F,t+k} = \frac{P_{F,t+k}}{P_{F,t+k-1}}$, $\Pi^C_{F,t+k} = \frac{P^C_{F,t+k}}{P^C_{F,t+k-1}}$, and $\overline{mc}_{F,t+k} = \frac{MC_{F,t+k}}{P_{F,t+k}}$. We also use the following definitions: $\bar{y}_{F,t} = \frac{y_{F,t}}{z^+_{F,t}}$ and $\overline{\Omega}^C_{F,t+k} = z^+_{F,t+k}\Omega^C_{F,t+k}$ when we stationarize Equation (D.41).

We follow the similar steps as in Section C.2.1, and we can obtain the following stationarized version of the optimal price for intermediate good producers in the Foreign economy:

$$E_t \sum_{k=0}^{\infty} \left(\xi^F\right)^k \left(\prod_{j=1}^{k} \beta^r_{F,t+j}\right) \frac{\overline{\Omega}^C_{F,t+k}}{\overline{\Omega}^C_{F,t}} \left(\prod_{j=1}^{k} \frac{\Pi_{F,t+j}}{\Pi^C_{F,t+j}}\right) \frac{\overline{y}_{F,t+k|t}}{(\lambda_{F,t+k}-1)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}_{F,t+j}}{\Pi_{F,t+j}}\right) \frac{p^{opt}_{F,t}}{\Pi_{F,t}} - \lambda_{F,t+k} \overline{mc}_{F,t+k}\right] = 0. \tag{D.42}$$

Equation (D.42), which captures the stationarized version of the optimal price for Foreign intermediate good producers, is the same as Equation (A.119a).

D.3 Foreign: Consumption good producers

This section presents the optimization problem of the consumption good producers in the Foreign economy and derives the demand functions of non-energy and energy consumption and derives the relative price of the consumption goods equation.

The optimization problem of the representative consumption good producer can be defined as follows:

$$\max_{C_{F,t}, C^{xe}_{F,t}, C^{e}_{F,t}} P^C_{F,t} C_{F,t} - P^{C,xe}_{F,t} C^{xe}_{F,t} - P^{C,e}_{F,t} C^{e}_{F,t}$$

subject to the CES aggregate consumption good function

$$C_{F,t} = \left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_t\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{e}_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}. \tag{D.43}$$

By substituting the CES aggregate consumption good equation (D.43) into the above profit function, we can rewrite the profit function as:

$$P^C_{F,t}\left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_t\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{e}_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}} - P^{C,xe}_{F,t} C^{xe}_{F,t} - P^{C,e}_{F,t} C^{e}_{F,t}.$$

Taking the derivatives of $C^{xe}_{F,t}$ and $C^{e}_{F,t}$ respectively gives us the two following first-order-conditions:

$$\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}-1} P^C_{F,t}\left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_t\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{e}_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}-1} - P^{C,xe}_{F,t} = 0$$

$$\left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{e}_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}-1} P^C_{F,t}\left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_t\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{e}_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}-1} - P^{C,e}_{F,t} = 0$$

If we substitute the expression in square brackets by the production function, and put the prices of non-energy and energy respectively on the right-hand-side of the equation, the two equations can be written as

$$\begin{aligned} P^{C,xe}_{F,t} &= \left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_{F,t}\right)^{-\frac{1}{\nu_{F,C}}} P^C_{F,t} \left(C_{F,t}\right)^{\frac{1}{\nu_{F,C}}} \\ P^{C,e}_{F,t} &= \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{e}_{F,t}\right)^{-\frac{1}{\nu_{F,C}}} P^C_{F,t} \left(C_{F,t}\right)^{\frac{1}{\nu_{F,C}}}. \end{aligned}$$

Rearrange and multiply through with $\nu_{F,C}$ in the two equations above to get the demand function for the non-energy and energy consumption goods:

$$C^{xe}_{F,t} = \vartheta^C_F \left(\frac{P^C_{F,t}}{P^{C,xe}_{F,t}}\right)^{\nu_C} C_{F,t} \tag{D.44}$$

$$C^{e}_{F,t} = \left(1-\vartheta^C_F\right) \left(\frac{P^C_{F,t}}{P^{C,e}_{F,t}}\right)^{\nu_C} C_{F,t} \tag{D.45}$$

which are the same equations that are presented in Equation (83) and Equation (84). Next, we create the price index. This is done by substituting the two demand functions into the production function:

$$C_{F,t} = \left[\left(\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}} \left(C_t^{xe}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}} \left(C_{F,t}^{e}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$C_{F,t} = \left[\left(\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}} \left(\vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_C} C_{F,t} \right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}} \left(\left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_C} C_{F,t} \right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$C_{F,t} = \left[\left(\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}+\frac{\nu_{F,C}-1}{\nu_{F,C}}} \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}-1} \left(C_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}+\frac{\nu_{F,C}-1}{\nu_{F,C}}} \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}-1} \left(C_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} \right]$$

$$C_{F,t} = \left[\vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}-1} + \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}-1} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$1 = \left[\vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}-1} + \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}-1} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$1 = \left(P_{F,t}^C\right)^{\left(\nu_{F,C}-1\right)\frac{\nu_{F,C}}{\nu_{F,C}-1}} \left[\vartheta_F^C \left(\frac{1}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}-1} + \left(1-\vartheta_F^C\right) \left(\frac{1}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}-1} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$\left(P_{F,t}^C\right)^{-\nu_{F,C}} = \left[\vartheta_F^C \left(P_{F,t}^{C,xe}\right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(P_{F,t}^{C,e}\right)^{1-\nu_{F,C}} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$P_{F,t}^C = \left[\vartheta_F^C \left(P_{F,t}^{C,xe}\right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(P_{F,t}^{C,e}\right)^{1-\nu_{F,C}} \right]^{\frac{1}{1-\nu_{F,C}}}$$

which is the same function as is presented in . We use the definitions $p_{F,t}^C = P_{F,t}^C/P_{F,t}$, $p_{F,t}^{C,xe} = P_{F,t}^{C,xe}/P_{F,t}$, $p_{F,t}^{C,e} = P_{F,t}^{C,e}/P_{F,t}$, $\bar{c}_{F,t} = C_{F,t}/z_{F,t}^+$, $\bar{c}_{F,t}^{xe} = C_{F,t}^{xe}/z_{F,t}^+$, $\bar{c}_{F,t}^{e} = C_{F,t}^{e}/z_{F,t}^+$. The non-energy consumption demand function can be written as

$$\begin{aligned} C_{F,t}^{xe} &= \vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}} C_{F,t} \\ \frac{C_{F,t}^{xe}}{z_t^+} &= \vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \frac{P_{F,t}}{P_{F,t}} \right)^{\nu_{F,C}} \frac{C_{F,t}}{z_t^+} \\ \bar{c}_{F,t}^{xe} &= \vartheta_F^C \left(\frac{p_{F,t}^C}{p_{F,t}^{C,xe}} \right)^{\nu_{F,C}} \bar{c}_{F,t} \end{aligned} \tag{D.46}$$

Equation (D.46), which captures the demand for non-energy consumption goods, is the same as Equation (A.123a).

Next, we stationarize the demand for energy goods:

$$\begin{aligned} C_{F,t}^{e} &= \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}} C_{F,t} \\ \frac{C_{F,t}^{e}}{z_t^+} &= \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \frac{P_{F,t}}{P_{F,t}} \right)^{\nu_{F,C}} \frac{C_{F,t}}{z_t^+} \\ \bar{c}_{F,t}^{e} &= \left(1-\vartheta_F^C\right) \left(\frac{p_{F,t}^C}{p_{F,t}^{C,e}} \right)^{\nu_{F,C}} \bar{c}_{F,t} \end{aligned} \tag{D.47}$$

Equation (D.47), which captures the demand for energy consumption goods, is the same as Equation (A.124a).

Finally, we stationarize the price index:

$$P_{F,t}^{C} = \left[\vartheta_F^C \left(P_{F,t}^{C,xe} \right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(P_{F,t}^{C,e} \right)^{1-\nu_{F,C}} \right]^{\frac{1}{1-\nu_{F,C}}}$$

$$\frac{P_{F,t}^{C}}{P_{F,t}} = \frac{1}{P_{F,t}} \left[\vartheta_F^C \left(P_{F,t}^{C,xe} \right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(P_{F,t}^{C,e} \right)^{1-\nu_{F,C}} \right]^{\frac{1}{1-\nu_{F,C}}}$$

$$\frac{P_{F,t}^{C}}{P_{F,t}} = \left[\left(\vartheta_F^C \left(P_{F,t}^{C,xe} \right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(P_{F,t}^{C,e} \right)^{1-\nu_{F,C}} \right) P_{F,t}^{\nu_{F,C}-1} \right]^{\frac{1}{1-\nu_{F,C}}}$$

$$p_{F,t}^{C} = \left[\left(\vartheta_F^C \left(\frac{P_{F,t}^{C,xe}}{P_{F,t}} \right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^{C,e}}{P_{F,t}} \right)^{1-\nu_{F,C}} \right) \right]^{\frac{1}{1-\nu_{F,C}}}$$

$$p_{F,t}^{C} = \left[\left(\vartheta_F^C \left(p_{F,t}^{C,xe} \right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(p_{F,t}^{C,e} \right)^{1-\nu_{F,C}} \right) \right]^{\frac{1}{1-\nu_{F,C}}}. \tag{D.48}$$

Equation (D.48), which captures the demand for energy consumption goods, is the same as Equation (A.121a).

D.3.1 Foreign: Non-energy consumption good producers

This section presents the optimization problem of non-energy consumption good producers in the Foreign economy and derives the relative price of the non-energy consumption goods equation (A.37a).

The optimization problem of the representative non-energy consumption good producer can be defined as follows:

$$\max_{D_{F,t}^{C,xe}, M_{F,t}^{C,xe}} P_{F,t}^{C,xe} C_{F,t}^{xe} - P_{F,t} D_{F,t}^{C,xe} - P_{F,t}^{M,C,xe} M_{F,t}^{C,xe}$$

subject to the CES aggregate consumption good function

$$C_{F,t}^{xe} = \left[\left(\psi_F^{C,xe} \right)^{\frac{1}{\nu_{F,C.xe}}} \left(D_{F,t}^{C,xe} \right)^{\frac{\nu_{F,C,xe}-1}{\nu_{F,C,xe}}} + \left(1-\psi_F^{C,xe}\right)^{\frac{1}{\nu_{F,C,xe}}} \left(M_{F,t}^{C,xe} \right)^{\frac{\nu_{F,C,xe}-1}{\nu_{F,C,xe}}} \right]^{\frac{\nu_{F,C,xe}}{\nu_{F,C,xe}-1}}. \tag{D.49}$$

Following the same procedures as in Section (C.3), we can obtain the demand function for intermediate goods used in the production of non-energy consumption goods in the Foreign economy, $D_{F,t}^{C,xe}$, and the demand function for imported goods used in the production of non-energy consumption goods in the Foreign economy, $M_{F,t}^{C,xe}$. The demand for intermediate goods used in the production of non-energy consumption goods in Forein economy, $D_{F,t}^{C,xe}$ is:

$$D_{F,t}^{C,xe} = \psi_F^{C,xe} \left(\frac{P_{F,t}^{C,xe}}{P_{F,t}} \right)^{\nu_{F,C,xe}} C_{F,t}^{xe}. \tag{D.50}$$

The demand for import goods used in the production of non-energy consumption goods in Forein economy, $M_{F,t}^{C,xe}$ is:

$$M_{F,t}^{C,xe} = (1-\psi_F^{C,xe}) \left(\frac{P_{F,t}^{C,xe}}{P_t^X} \right)^{\nu_{F,C,xe}} C_{F,t}^{xe}.$$

Note that $\psi_F^{C,xe} = \vartheta_F^{C,xe} + \frac{\omega}{1+\omega}\left(1-\vartheta_F^{C,xe}\right)$, and that, Foreign being infinitely large compared to Sweden. This means that $\psi_F^{C,xe} \to 1$, and that the share of imports become arbitrtarily small. Hence, the production function reduces to:

$$C_{F,t}^{xe} = D_{F,t}^{C,xe} \tag{D.51}$$

and the profit function then reduces to

$$P_{F,t}^{C,xe} D_{F,t}^{C,xe} - P_{F,t} D_{F,t}^{C,xe}.$$

This means that we get the following relationship between the price of Foreign domestic goods and Forein non-energy goods:

$$P_t^{C,xe} = P_{F,t} \tag{D.52}$$

We stationarize this equation by using the definition $p_t^{C.xe} = P_t^{C,xe}/P_{F,t}$ which simply means that

$$p_t^{C.xe} = 1 \tag{D.53}$$

Equation (D.53), which captures the relative price of consumption goods, is the same as Equation (A.125a).

D.4 Foreign: Investment good producers

This section presents optimization problem of investment good producers and derives the relative price of investment goods equation for the Foreign economy. Note that if the Swedish economy is infinitely small relative to the Foreign economy, then the investment goods and the intermedite goods will have the same price. Then the intermediate goods and investment goods will essentially be the same. We define $V_{F,t}^{I}$ to be the output of a representative investment firm. We define $V_{F,t}^{I}$ as $\frac{1}{\gamma_{F,t}}\left[I_{F,t}+a(u_t)K_{F,t}\right]$.

The representative investment good producer maximizes the following profit function:

$$\max_{D_{F,t}^{I},M_{F,t}^{I}} P_{F,t}^{I}V_{F,t}^{I}-P_{F,t}D_{F,t}^{I}-P_{F,t}^{M}M_{F,t}^{I},$$

subject to the following CES aggregate investment good function:

$$V_{F,t}^{I}=\left[\left(\psi_F^{I}\right)^{\frac{1}{\nu_{F,c}}}\left(D_{F,t}^{I}\right)^{\frac{\nu_{F,I}-1}{\nu_{F,I}}}+\left(1-\psi_F^{I}\right)^{\frac{1}{\nu_{F,I}}}\left(M_{F,t}^{I}\right)^{\frac{\nu_{F,I}-1}{\nu_{F,I}}}\right]^{\frac{\nu_{F,I}}{\nu_{F,I}-1}}. \tag{D.54}$$

Following the same procedures as in Section (D.3), we can obtain the demand function for intermediate goods used in the production of investment goods in the Foreign economy, $D_{F,t}^{I}$, and the demand function for imported goods used in the production of investment goods in the Foreign economy, $M_{F,t}^{I}$.
The demand for intermediate goods used in the production of investment goods in Forein economy, $D_{F,t}^{I}$ is:

$$D_{F,t}^{I}=\psi_F^{I}\left(\frac{P_{F,t}^{I}}{P_{F,t}}\right)^{\nu_I}V_{F,t}^{I}. \tag{D.55}$$

The demand for import goods used in the production of investment goods in Foreign economy, $M_{F,t}^{I}$ is:

$$M_{F,t}^{I}=(1-\psi_F^{I})\left(\frac{P_{F,t}^{I}}{P_t^{X}}\right)^{\nu_I}V_{F,t}^{I}.$$

The assumption that Sweden is small economy implies that Foreign imports have a negligible share in the production of foreign investment goods. Thus, $\psi_F^{I}\to 1$ and the demand for the domestically produced intermediate good for investment good production is:

$$D_{F,t}^{I}=V_{F,t}^{I} \tag{D.56}$$

and the price index for foreign investment goods is:

$$P_{F,t}^{I}=P_{F,t}.$$

Using definition of relative foreign investment price $p_{F,t}^{I}=\frac{P_{F,t}^{I}}{P_{F,t}}$, we have

$$p_{F,t}^{I}=1. \tag{D.57}$$

E Technical appendix: Market clearing

This section shows how to derive the Swedish aggregate resource constraint, the expressions for Swedish exports and imports, the Foreign aggregate resource constraint, as well as the balance of payments equation. These equilibrium conditions are stated and discussed in Section (2.7) in the main text. In addition, this section also includes a discussion of the value of the different fixed cost of production that exist in the Swedish and Foreign firm sectors.

Several of the equilibrium conditions to be discussed here are stated in two versions: one version that applies to the general case when the size of the Foreign economy, ω, can take on any non-negative value; and a second version that applies to the limiting case when ω tends to infinity. In all derivations that apply to the second case, we assume that all relative prices and all stationarized real quantities, expressed in *per capita* terms, take on non-negative, finite values in the limit as $\omega\to\infty$. We thus assume, for example, that the stationarized (*per capita*) level of Swedish exports converges to a non-negative, finite number as $\omega\to\infty$, i.e. that $\lim_{\omega\to\infty}\left(\frac{X_t}{z_t^{+}}\right)=\lim_{\omega\to\infty}\left(\overline{x}_t\right)$ is a non-negative, finite number. Concerning these derivations and the associated notation, a note of caution is warranted. In order to keep the notation relatively simple, we do not stringently distinguish between, on the hand, relative prices and stationarized (*per capita*) quantities that apply to any equilibrium where ω take on any positive, finite value and, on the hand, the corresponding relative prices and stationarized (*per capita*) quantities that apply to the limiting equilibrium. In the end, our focus is on the limiting equilibrium that obtains when $\omega \to \infty$ and when we state equilibrium conditions in terms of relative prices and stationarized (*per capita*) quantities, the final aim is always to make statements about this limiting equilibrium.

E.1 Swedish aggregate resource constraint

This subsection shows how to derive the aggregate resource constraint of the Swedish economy. In the first part of the subsection, different market clearing conditions are combined to derive the non-stationarized version of the Swedish aggregate resource constraint. This is Equation (94) in the main text. In a second part of the section, we derive a stationarized version of the constraint that applies to the limiting case when $\omega \to \infty$ and that can be used to solve the model with numerical methods. The result is Equation (A.136a).

E.1.1 Market clearing in Sweden

Y_t represents total demand for the homogeneous intermediate good, which in turn is the sum of demand from consumption and investment good producers, from export good producers and from the government. We denote these different demand components $D_t^{C,xe}$,$D_t^{C,e}$, D_t^I, D_t^X and G_t and $D_t^{I^G}$, respectively, and thus write:

$$Y_t = D_t^{C,xe} + D_t^{C,e} + D_t^I + D_t^X + G_t + D_t^{I^G} \tag{E.1}$$

Based on Equation (C.140), we have: $D_t^X(i) = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t(i) + z_t^+ \phi^X\right]$. $D_t^X(i)$ represents demand from the individual export good producer i and where $X_t(i)$ denotes production of the same firm. Let $X_t^P = \int_0^1 X_t(i)di$ denote total production of Swedish export goods. Thus, total demand for the homogeneous, intermediate good from Swedish export good producers can be written as:

$$\begin{aligned} D_t^X &= \int_0^1 D_t^X(i)di = \int_0^1 \left\{\psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t(i) + z_t^+ \phi^X\right]\right\} di, \\ &= \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[\int_0^1 X_t(i)di + z_t^+ \phi^X\right] = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t^P + z_t^+ \phi^X\right]. \end{aligned} \tag{E.2}$$

The market for differentiated Swedish export goods clears when production of each individual export good firm i equals demand for the export goods produced by the same firm. For the individual firm i, this implies $X_t(i) = \left[\frac{P_t^X(i)}{P_t^X}\right]^{\frac{\lambda_t^X}{1-\lambda_t^X}} X_t$, where X_t denotes total demand for the homogeneous Swedish export good (see Section 2.4.3 in the main text). Aggregating over all firms in the export good sector, and defining $\overleftrightarrow{P}_t^X = \int_0^1 \left[\frac{P_t^X(i)}{P_t^X}\right]^{\frac{\lambda_t^X}{1-\lambda_t^X}} di$ as a measure of price dispersion in the export good sector, we have:

$$X_t^P = \int_0^1 X_t(i)di = \int_0^1 \left\{\left[\frac{P_t^X(i)}{P_t^X}\right]^{\frac{\lambda_t^X}{1-\lambda_t^X}} X_t\right\} di = X_t \overleftrightarrow{P}_t^X. \tag{E.3}$$

Substituting $X_t \overleftrightarrow{P}_t^X$ into Equation (E.2), we have the following equation:

$$D_t^X = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t \overleftrightarrow{P}_t^X + z_t^+ \phi^X\right].$$

Note we have the following demand functions: Equation (C.106), $D_t^{C,xe} = \psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t}\right)^{\nu_{C,xe}} C_t^{xe}$; Equation (C.119), $D_t^I = \psi^I \left(\frac{P_t^I}{P_t}\right)^{\nu_I} \left[\frac{I_t}{\gamma_t} + a(u_t)\frac{K_t}{\gamma_t}\right]$; $D_t^X = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t \overleftrightarrow{P}_t^X + z_t^+ \phi^X\right]$ and $D_t^{I^G} = \frac{I_t^G}{\gamma_t}$.

Substituting D_t^C, D_t^I, D_t^X and $D_t^{I^G}$ into Equation (E.1), we have the following equation:

$$Y_t = \psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t}\right)^{\nu_{C,xe}} C_t^{agg} + D_t^{C,e} + \psi^I \left(\frac{P_t^I}{P_t}\right)^{\nu_I} \left[\frac{I_t}{\gamma_t} + a(u_t)\frac{K_t}{\gamma_t}\right] + \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t \overleftrightarrow{P}_t^X + z_t^+ \phi^X\right] + G_t + \frac{I_t^G}{\gamma_t}. \tag{E.4}$$

Equation (E.4) is the same as Equation (94) in the main text.

E.1.2 Stationarizing the Swedish aggregate resource constraint

In the stationarization of the Swedish aggregate resource constraint, we make use of the following definitions of relative prices and of the real marginal cost of export good producers: $p_t^{C,xe} = \frac{P_t^{C,xe}}{P_t}$, $p_t^I = \frac{P_t^I}{P_t}$, $\overline{mc}_t^X = \frac{MC_t^X}{P_t}$. Furthermore, real variables are stationarized as follows: $\overline{C}_t^{xe} = \frac{C_t^{xe}}{z_t^+}$, $\overline{D}_t^e = \frac{D_t^e}{z_t^+}$, $\overline{I}_t = \frac{I_t}{z_t \gamma_t^{\frac{1}{1-\alpha}}}$, $\overline{K}_t = \frac{K_t}{z_{t-1}(\gamma_{t-1})^{\frac{1}{1-\alpha}}}$, $\overline{X}_t = \frac{X_t}{z_t^+}$, and $\overline{G}_t = \frac{G_t}{z_t^+}$. With these definitions, Equation (E.4) can be rewritten as follows:

$$\frac{Y_t}{z_t^+} = \psi^{C,xe}\left(\frac{P_t^{C,xe}}{P_t}\right)^{\nu_{C,xe}} \frac{C_t^{xe}}{z_t^+} + \frac{D_t^e}{z_t^+} + \psi^I \left(\frac{P_t^I}{P_t}\right)^{\nu_I}\left[\frac{I_t}{z_t^+\gamma_t} + a(u_t)\frac{K_t}{z_t^+\gamma_t}\right] + \psi^X\left(\frac{MC_t^X}{P_t}\right)^{\nu_x}\left[\frac{\overleftrightarrow{P}_t^X X_t}{z_t^+} + \phi^X\right] + \frac{G_t}{z_t^+} + \frac{I_t^G}{z_t^+\gamma_t}.$$

Expressing the above equation in *stationarized per capita* terms, we have the following equation:

$$\bar{y}_t = \psi^{C,xe}\left(p_t^{C,xe}\right)^{\nu_{C,xe}} \bar{c}_t^{xe} + \bar{d}_t^{C,e} + \psi^I\left(p_t^I\right)^{\nu_I}\left[\overline{I}_t + a(u_t)\,\bar{k}_t \frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}\right] + \psi^X\left(\overline{mc}_t^X\right)^{\nu_x}\left[\bar{x}_t \overleftrightarrow{P}_t^X + \phi^X\right] + \bar{g}_t + \overline{I}_t^G \tag{E.5}$$

The last step to take, in order to arrive at Equation (A.136a), is to consider the implications of letting the size of the Foreign economy, ω, tend to infinity. Consider $\psi^C = \vartheta^C + \frac{1}{1+\omega}\left(1 - \vartheta^C\right)$, where $\vartheta^C \in [0,1]$. Note that $\lim_{\omega\to\infty}\psi^C = \vartheta^C$. By analogous arguments, we have $\lim_{\omega\to\infty}\psi^I = \vartheta^I$ and $\lim_{\omega\to\infty}\psi^X = \vartheta^X$. Substituting for ψ^C, ψ^I and ψ^X in Equation (E.5), we have:

$$\bar{y}_t = \vartheta^{C,xe}\left(p_t^{C,xe}\right)^{\nu_{C,xe}} \bar{c}_t^{xe} + \bar{d}_t^{C,e} + \vartheta^I\left(p_t^I\right)^{\nu_I}\left[\overline{I}_t + a(u_t)\,\bar{k}_t \frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}\right] + \vartheta^X\left(\overline{mc}_t^X\right)^{\nu_x}\left[\bar{x}_t \overleftrightarrow{P}_t^X + \phi^X\right] + \bar{g}_t + \overline{I}_t^G. \tag{E.6}$$

Equation (E.6) is the same as Equation (A.136a).

E.2 Fixed costs

In the Swedish economy, $z_t^+\phi$, $z_t^+\phi^X$ and $z_t^+\phi^{M,n}$ for $n \in \{Cxe, I, X, Ce\}$ represent real, fixed costs associated with the production, respectively, of intermediate goods, export goods and the three different types of import goods. $z_t^+\phi_F$, $z_t^+\phi_F^M$ and $z_t^+\phi_F^X$ represent corresponding fixed costs in the Foreign economy. For all of these different fixed costs, it is assumed that their value is such that along the balanced growth path, *ex post* profits are zero. In this subsection, we discuss the implication of this assumption for the value of ϕ, the stationarized fixed cost associated with the production of intermediate goods in Sweden. ϕ is chosen as an example and it should be noted that the same reasoning that applies to the value of ϕ also applies to the values of the other fixed costs mentioned here.

$Y_t(i)$ denotes the supply of good i from firm i in the Swedish intermediate good sector. $P_t(i)$ and $TC_t(i)$ represents, respectively, the price charged by firm i and the total cost of production of the same firm. Profits in period t may thus be written $P_t(i)Y_t(i) - TC_t(i)$, and real profits are

$$Y_t(i) - \frac{1}{P_t(i)}TC_t(i). \tag{E.7}$$

We assume that along a balanced growth path, enough time has elapsed that all firms charge the same price. This price will be the firms' desired price, i.e. the one that maximizes profits. We now focus on an equilibrium associated with such a balanced growth path, and we thus drop the subscript i from $P_t(i)$. Note that it follows, if profits are maximized, that $P_t = \lambda MC_t$, where λ is the (steady state) value of the desired markup and where MC_t denotes the nominal, marginal cost. Using equations (C.76) and (C.77) from Section (C.2), total costs may be written: $TC_t(i) = MC_t\left[(1-\alpha) + \alpha\alpha_K^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}}\right]\left[Y_t(i) + z_t^+\phi\right]$. Use this expression to substitute for $TC_t(i)$ in (E.7):

$$Y_t(i) - \frac{1}{P_t}MC_t F_t(i)\left[Y_t(i) + z_t^+\phi\right]. \tag{E.8}$$

where

$$F_t(i) = \left[(1-\alpha) + \alpha\alpha_K^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}}\right]$$

Using $P_t = \lambda MC_t$ to substitute for $\frac{MC_t}{P_t}$ in the above equation, we have the following equation:

$$Y_t(i) - \frac{F_t(i)}{\lambda}\left[Y_t(i) + z_t^+\phi\right] = \left(1 - \frac{F_t(i)}{\lambda}\right)Y_t(i) - \frac{F_t(i)}{\lambda}z_t^+\phi. \tag{E.9}$$

Impose zero profits and rearrange:

$$\left(\frac{\lambda - F_t(i)}{\lambda}\right)Y_t(i) - \frac{F_t(i)}{\lambda}z_t^+\phi = 0, \tag{E.10}$$

$$\phi = \left(\frac{\lambda}{F_t(i)} - 1\right)\frac{Y_t(i)}{z_t^+}. \tag{E.11}$$

Aggregate over all firms, using the definition $Y_t^P = \int_0^1 Y_t(i)di$ for total (aggregate) production of intermediate goods:

$$\int_0^1 \phi di = \phi = \int_0^1 \left(\frac{\lambda}{F_t(i)} - 1\right)\frac{Y_t(i)}{z_t^+}di = \left(\frac{\lambda}{F_t} - 1\right)\frac{Y_t^P}{z_t^+}. \tag{E.12}$$

From the previous subsection we have $Y_t^P = Y_t \overleftrightarrow{P}_t$, where $\overleftrightarrow{P}_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{\lambda_t}{1-\lambda_t}} di$. Given, however, that we consider the case (along a balanced growth path) where all intermediate good firms charge the same price P_t, we have $Y_t^P = Y_t$ and thus $\phi = \left(\frac{\lambda}{F_t} - 1\right)\frac{Y_t}{z_t^+}$. Using our notation for stationarized variables expressed in *per capita* terms, this may be stated: $\phi = \left(\frac{\lambda}{F_t} - 1\right)\overline{y}_t$. This expression indicates that $\overline{y}_t$ is a constant, and this is indeed the case along a balanced growth path, where Swedish output (GDP) grows at the constant rate $\mu_{z^+} = \frac{z_t^+}{z_{t-1}^+}$. We may therefore write $\phi = \left(\frac{\lambda}{F} - 1\right)\overline{y}$. By analogous reasoning, the following results may be obtained: $\phi^X = \left(\lambda^X - 1\right)\overline{x}$, $\phi^{M,n} = \left(\lambda^{M,n} - 1\right)\overline{m}^n$ for $n \in \{\{C, xe\}, I, X, \{C, e\}\}$, $\phi_F = \left(\lambda_F - 1\right)\overline{y}_F$, [52]

E.3 Imports and exports

This section contains derivations of the expressions for Swedish imports and exports. Because one country's imports is the other countries exports, this is also a treatment of Foreign exports and imports. The first part of this section focuses on Swedish imports, while a second part contains the derivations of an expression for Swedish exports.

E.3.1 Swedish imports of consumption goods

The total demand for the homogeneous imported intermediate good used in the production of non-energy consumption goods is denoted $M_t^{P,C,xe}$. This must equal the production of these goods minus the fixed costs:

$$M_t^{P,Cxe} = \int_0^1 \left[M_t^{C,xe}(i)\right] di \tag{E.13}$$

From Section 2.4.2 we have $M_t^{C,xe}(i) = \left[\frac{P_t^{M,C,xe}(i)}{P_t^{M,C,xe}}\right]^{\frac{\lambda_t^{M,C,xe}}{1-\lambda_t^{M,Cxe}}} M_t^{C,xe}$, where $P_t^{M,C,xe}(i)$ is the price charged by firm i and where $P_t^{M,C,xe}$ is the price index of the homogeneous Swedish import good. Now let $\overleftrightarrow{P}_t^{M,C,xe} = \int_0^1 \left(\frac{P_t^{M,C,xe}(i)}{P_t^{M,C,xe}}\right)^{\frac{\lambda_t^{M,C,xe}}{1-\lambda_t^{M,C,xe}}} di$ be a measure of price dispersion in the sector for Swedish import goods. Thus, the total demand for the homogeneous imported intermediate good used in the production of consumption goods can be written as:

$$M_t^{P,Cxe} = \int_0^1 M_t^{C,xe}(i)di = \int_0^1 \left[\frac{P_t(i)^{M,C,xe}}{P_t^{M,C,xe}}\right]^{\frac{\lambda_t^{M,C,xe}}{1-\lambda_t^{M,C,xe}}} M_t^{C,xe}di = \overleftrightarrow{P}_t^{M,C,xe} M_t^{C,xe}. \tag{E.14}$$

Recall from Equation (C.107), we have the following demand function for imported consumption goods:

[52] There are four different types of import firms in the Swedish economy, as described in Section (2.4.2) in the main text. For each of these three types, there exists an exogenous markup $\lambda_t^{M,n}$, that fluctuates stochastically around its long-run (unconditional) mean. Three markup shocks in the Swedish import sector are assumed to share the same unconditional mean. Also, note that $\overline{m}^n$ refers to the total demand for Swedish import good of type n, and that $\overline{m}_F^D$ denotes total demand (from Foreign consumption good producers and Foreign export firms) of the homogenous Foreign import good.

$$M_t^{Cxe} = \left(1-\psi^{C,xe}\right)\left(\frac{P_t^{C,xe}}{P_t^{M,Cxe}}\right)^{\nu_{C,xe}} C_t^{xe}.$$

Substituting the above demand function for imported consumption goods into Equation (E.14), we have the following equation:

$$\overleftrightarrow{P}_t^{M,C,xe} M_t^{C,xe} = \overleftrightarrow{P}_t^{M,C,xe}\left(1-\psi^{C,xe}\right)\left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}}\right)^{\nu_{C,xe}} C_t^{xe}. \tag{E.15}$$

Using the following definition $p_t^{M,C,xe} = \frac{P_t^{M,C,xe}}{P_t}$ and Equation (C.109) which captures the consumption good price index, we can obtain the following equation for price ratio:

$$\frac{P_t^{C,xe}}{P_t^{M,C,xe}} = \frac{1}{P_t^{M,C,xe}}\left[\psi^{C,xe}\left(P_t\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C}\right)\left(P_t^{M,C,xe}\right)^{1-\nu_{C,xe}}\right]^{\frac{1}{1-\nu_{C,xe}}},$$

$$\frac{P_t^{C,xe}}{P_t^{M,C,xe}} = \left[\psi^{C,xe}\left(\frac{P_t}{P_t^{M,C,xe}}\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C,xe}\right)\left(\frac{P_t^{M,C,xe}}{P_t^{M,C,xe}}\right)^{1-\nu_{C,xe}}\right]^{\frac{1}{1-\nu_{C,xe}}},$$

$$\frac{P_t^{C,xe}}{P_t^{M,C,xe}} = \left[\psi^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1 - \psi^{C,xe}\right]^{\frac{1}{1-\nu_{C,xe}}}. \tag{E.16}$$

Using Equation (E.16), Equation (E.15) can be written as follows:

$$\overleftrightarrow{P}_t^{M,C,xe} M_t^{C,xe} = \left(1-\psi^{C,xe}\right)\left[\psi^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1 - \psi^{C,xe}\right]^{\frac{\nu_{C,xe}}{1-\nu_{C,xe}}} C_t^{xe} \tag{E.17}$$

We stationarize this expression by dividing both sides by z_t^+ and express the above equation in *per capita* terms. Thus, we have the following equation:

$$\overleftrightarrow{P}_t^{M,C,xe}\overline{m}_t^{C,xe} = \left(1-\psi^{C,xe}\right)\left[\psi^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1 - \psi^{C,xe}\right]^{\frac{\nu_{C,xe}}{1-\nu_{C,xe}}} \overline{c}_t^{xe}. \tag{E.18}$$

Next, consider the limit as ω tends to infinity. Recall from Section (E.1.2) that $\lim_{\omega\to\infty}\psi^C = \vartheta^C$. Then we can rewrite the above equation as:

$$\overleftrightarrow{P}_t^{M,C.xe}\overline{m}_t^{C,xe} = \left(1-\vartheta^{C,xe}\right)\left[\vartheta^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1 - \vartheta^{C,xe}\right]^{\frac{\nu_{C,xe}}{1-\nu_{C,xe}}} \overline{c}_t^{xe}. \tag{E.19}$$

Note that Equation (E.19) is the same as Equation (A.140a) in Section A.9.

E.3.2 Swedish imports of investment goods

The total demand for the homogeneous imported intermediate good used in the production of investment goods is denoted $M_t^{P,I}$. This must equal the production of these goods minus the fixed costs:

$$M_t^{P,I} = \int_0^1 \left[M_t^I(i)\right] di \tag{E.20}$$

From Section 2.4.2 we have $M_t^I(i) = \left[\frac{P_t^{M,I}(i)}{P_t^{M,I}}\right]^{\frac{\lambda_t^{M,I}}{1-\lambda_t^{M,I}}} M_t^I$, where $P_t^{M,I}(i)$ is the price charged by firm i and where $P_t^{M,I}$ is the price index of the homogeneous Swedish import good. Now let $\overleftrightarrow{P}_t^{M,I} = \int_0^1\left(\frac{P_t^{M,I}(i)}{P_t^{M,I}}\right)^{\frac{\lambda_t^{M,I}}{1-\lambda_t^{M,I}}} di$ be a measure of price dispersion in the sector for Swedish import goods. Thus, the total demand for the homogeneous imported intermediate good used in the production of investment goods can be written as:

$$M_t^{P,I} = \int_0^1 M_t^I(i)di = \int_0^1\left[\frac{P_t(i)^{M,I}}{P_t^{M,I}}\right]^{\frac{\lambda_t^{M,I}}{1-\lambda_t^{M,I}}} M_t^I di = \overleftrightarrow{P}_t^{M,I} M_t^I. \tag{E.21}$$

Recall from Equation(C.120), we have the following demand function for imported investment goods:

$$M_t^I = (1-\psi^I)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{E.22}$$

Substituting the above demand function into Equation (E.21), we have the following equation:

$$\overleftrightarrow{P}_t^{M,I} M_t^I = \overleftrightarrow{P}_t^{M,I}\left(1-\psi^I\right)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{E.23}$$

Using the definition $p_t^{M,I} = \frac{P_t^{M,I}}{P_t}$ and Equation (C.121) which shows the investment good price index, we can obtain the following price ratio:

$$\frac{P_t^I}{P_t^{M,I}} = \frac{1}{P_t^{M,I}}\left[\psi^I\left(P_t\right)^{1-\nu_I} + \left(1-\psi^I\right)\left(P_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}},$$

$$\frac{P_t^I}{P_t^{M,I}} = \left[\psi^I\left(\frac{P_t}{P_t^{M,I}}\right)^{1-\nu_I} + \left(1-\psi^I\right)\left(\frac{P_t^{M,I}}{P_t^{M,I}}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}},$$

$$\frac{P_t^I}{P_t^{M,I}} = \left[\psi^I\left(p_t^{M,I}\right)^{\nu_I-1} + 1-\psi^I\right]^{\frac{1}{1-\nu_I}}.$$

We substitute the above price ratio equation into Equation (E.23). Thus, we have the following equation:

$$\overleftrightarrow{P}_t^{M,I} M_t^I = \left(1-\psi^I\right)\left[\psi^I\left(p_t^{M,I}\right)^{\nu_I-1} + 1-\psi^I\right]^{\frac{\nu_I}{1-\nu_I}}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{E.24}$$

We stationarize the above expression by dividing both sides by z_t^+ and then express the above equation in *per capita* terms. Thus, we have the following equation:

$$\overleftrightarrow{P}_t^{M,I}\overline{m}_t^I = \left(1-\psi^I\right)\left[\psi^I\left(p_t^{M,I}\right)^{\nu_I-1} + 1-\psi^I\right]^{\frac{\nu_I}{1-\nu_I}}\left[\bar{I}_t + a(u_t)\bar{k}_t\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}\right] \tag{E.25}$$

Next, consider the limit as ω tends to infinity. Recall from Section (E.1.2) that $\lim_{\omega\to\infty}\psi^I = \vartheta^I$. Then we can write as:

$$\overleftrightarrow{P}_t^{M,I}\overline{m}_t^I = \left(1-\vartheta^I\right)\left[\vartheta^I\left(p_t^{M,I}\right)^{\nu_I-1} + 1-\vartheta^I\right]^{\frac{\nu_I}{1-\nu_I}}\left[\bar{I}_t + a(u_t)\bar{k}_t\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}\right] \tag{E.26}$$

Note that Equation (E.26) is the same as Equation (A.141a) in Section A.9.

E.3.3 Swedish imports of export goods

The total demand for the homogeneous imported intermediate good used in the production of export goods is denoted $M_t^{P,X}$. This must equal the production of these goods minus the fixed costs:

$$M_t^{P,X} = \int_0^1 \left[M_t^X(i)\right] di \tag{E.27}$$

From Section 2.4.2 we have $M_t^X(i) = \left[\frac{P_t^{M,X}(i)}{P_t^{M,X}}\right]^{\frac{\lambda_t^{M,X}}{1-\lambda_t^{M,X}}} M_t^X$, where $P_t^{M,X}(i)$ is the price charged by firm i and where $P_t^{M,X}$ is the price index of the homogeneous Swedish import good. Now let $\overleftrightarrow{P}_t^{M,X} = \int_0^1\left(\frac{P_t^X(i)}{P_t^{M,X}}\right)^{\frac{\lambda_t^{M,X}}{1-\lambda_t^{M,X}}} di$ be a measure of price dispersion in the sector for Swedish import goods. Thus, the total demand for the homogeneous imported intermediate good used in the production of export goods can be written as:

$$M_t^{P,X} = \int_0^1 M_t^X(i)di = \int_0^1\left[\frac{P_t(i)^{M,X}}{P_t^{M,X}}\right]^{\frac{\lambda_t^{M,I}}{1-\lambda_t^{M,I}}} M_t^X di = \overleftrightarrow{P}_t^{M,X} M_t^X. \tag{E.28}$$

Recall from Equation (C.141), we have the following demand function for imported goods for export production:

$$M_t^X(i) = \left(1-\psi^X\right)\left(\frac{MC_t^X}{P_t^{M,X}}\right)^{\nu_x}\left[X_t(i) + z_t^+\phi^X\right]. \tag{E.29}$$

Substituting the above demand function into Equation (E.28), we have the following equation:

$$\overleftrightarrow{P}_t^{M,X} M_t^X = \overleftrightarrow{P}_t^{M,X} \left(1-\psi^X\right)\left(\frac{MC_t^X}{P_t^{M,X}}\right)^{\nu_X} \int_0^1 \left[X_t(i) + z_t^+ \phi^X\right] di. \tag{E.30}$$

Using the definition $p_t^{M,X} = \frac{P_t^{M,X}}{P_t}$ and Equation (C.137) which captures nominal marginal cost of export production , we can obtain the following marginal cost of export production ratio:

$$\begin{aligned}
\frac{MC_t^X}{P_t^{M,X}} &= \frac{1}{P_t^{M,X}}\left[\psi^X \left(P_t\right)^{(1-\nu_x)} + \left(1-\psi^X\right)\left(P_t^{M,X}\right)^{(1-\nu_x)}\right]^{\frac{1}{1-\nu_x}}, \\
\frac{MC_t^X}{P_t^{M,X}} &= \left[\psi^X \left(\frac{P_t}{P_t^{M,X}}\right)^{1-\nu_x} + \left(1-\psi^X\right)\left(\frac{P_t^{M,X}}{P_t^{M,X}}\right)^{1-\nu_x}\right]^{\frac{1}{1-\nu_x}}, \\
\frac{MC_t^X}{P_t^{M,X}} &= \left[\psi^X \left(p_t^{M,X}\right)^{\nu_x-1} + 1-\psi^X\right]^{\frac{1}{1-\nu_x}}.
\end{aligned}$$

Defining the price dispersion of export goods as $\overleftrightarrow{P}_t^X$, and using the above marginal cost of export production ratio, we can rewrite Equation (E.30) as follows:

$$\overleftrightarrow{P}_t^{M,X} M_t^X = \left(1-\psi^X\right)\left[\psi^X \left(p_t^{M,X}\right)^{\nu_x-1} + 1-\psi^X\right]^{\frac{1}{1-\nu_x}} \left[\overleftrightarrow{P}_t^X X_t + z_t^+ \phi^X\right]. \tag{E.31}$$

We stationarize this expression by dividing both sides by z_t^+ and express the above equation in *per capita* terms. Thus, we have the following equation:

$$\overleftrightarrow{P}_t^{M,X} \overline{m}_t^X = \left(1-\psi^X\right)\left[\psi^X \left(p_t^{M,X}\right)^{\nu_x-1} + 1-\psi^X\right]^{\frac{1}{1-\nu_x}} \left[\overleftrightarrow{P}_t^X \overline{x}_t + \phi^X\right]. \tag{E.32}$$

Next, consider the limit as ω tends to infinity. Recall from Section (E.1.2) that $\lim_{\omega\to\infty}\psi^X = \vartheta^X$. Then we can write as follows:

$$\overleftrightarrow{P}_t^{M,X} \overline{m}_t^X = \left(1-\vartheta^X\right)\left[\psi^X \left(p_t^{M,X}\right)^{\nu_x-1} + 1-\psi^X\right]^{\frac{1}{1-\nu_x}} \left[\overleftrightarrow{P}_t^X \overline{x}_t + \phi^X\right] \tag{E.33}$$

Note that Equation (E.33) is the same as Equation (A.142a) in Section A.9.

E.3.4 Total Swedish non-energy imports

Total demand for Swedish imports from Foreign is given by

$$M_t^{xe} = \overleftrightarrow{P}_t^{M,C,xe} M_t^{C,xe} + \overleftrightarrow{P}_t^{M,I} M_t^I + \overleftrightarrow{P}_t^{M,X} M_t^X + z_t^+\phi^{M,C,xe} + z_t^+\phi^{M,I} + z_t^+\phi^{M,X}. \tag{E.34}$$

We stationarize the above expression by dividing both sides by z_t^+ and express the above equation in *per capita* terms.

$$\overline{m}_t^{xe} = \overleftrightarrow{P}_t^{M,C} \overline{m}_t^C + \overleftrightarrow{P}_t^{M,I} \overline{m}_t^I + \overleftrightarrow{P}_t^{M,X} \overline{m}_t^X + \phi^{C,xe} + \phi^{M,I} + \phi^{M,X} \tag{E.35}$$

Note that Equation (E.35) is the same is Equation (A.143a) in Section A.9.
It is also useful to have an equation of total import demand $\overline{m}_t^{D,xe}$, that is, the amount of import goods which are used as intermediate goods in the other sectors in the economy. This expression is given by removing the fixed cost:

$$\overline{m}_t^{D,xe} = \overleftrightarrow{P}_t^{M,C} \overline{m}_t^C + \overleftrightarrow{P}_t^{M,I} \overline{m}_t^I + \overleftrightarrow{P}_t^{M,X} \overline{m}_t^X. \tag{E.36}$$

Note that Equation (E.36) is the same is Equation (A.144a) in Section A.9.

E.3.5 Swedish exports

We turn now to the discussion of Swedish exports, and the demand for Swedish export goods in Foreign.

Since we allow the two economies, Sweden and Foreign, to potentially grow at different paces via $z_t^+$and $z_{F,t}^+$, we also need to make some additional assumptions about the weights of Swedish export goods in the production of Foreign, to assure that Swedish exports grow at the same rate as output on the balanced growth path. More specifically, we need to let the weights $\psi_{F,t}^I$ and $\psi_{F,t}^C$ vary over time. We abstract from the time varying weights in the main text to make it more easy for the reader to follow, since $\lim_{\omega\to\infty}\psi_{F,t}^I = 1$ and $\lim_{\omega\to\infty}\psi_{F,t}^{C,xe} = 1$. The demand for Swedish exports that goes to Foreign investment is given by

$$X_t^I = \left(1 - \psi_{F,t}^I\right)\left(\frac{P_t^X}{P_{F,t}^I}\right)^{-\nu_{F,I}} I_{F,t} \tag{E.37}$$

where ψ_F^I is the share of Swedish exports in Foreign investment good production. P_t^X is the price of Swedish export goods in Foreign currency, $P_{F,t}^I$ is the price of the Foreign investment good and $I_{F,t}$ is Foreign investment.

Similarly, the demand for Swedish exports that goes to Foreign consumption is given by

$$X_t^C = \left(1 - \psi_{F,t}^{C,xe}\right)\left(\frac{P_t^X}{P_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} C_{F,t}^{xe} \tag{E.38}$$

where $\psi_{F,t}^{C,xe}$ is the share of Swedish exports in Foreign non-energy consumption good production. P_t^X is the price of Swedish export goods in Foreign currency, $P_{F,t}^{C,xe}$ is the price of the Foreign non-energy good and $C_{F,t}^{xe}$ is Foreign non-energy consumption.

This means that total demand for exports is given by

$$X_t = \left(1 - \psi_{F,t}^{C,xe}\right)\left(\frac{P_t^X}{P_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} C_{F,t}^{xe} + \left(1 - \psi_{F,t}^I\right)\left(\frac{P_t^X}{P_{F,t}^I}\right)^{-\nu_{F,I}} I_{F,t} \tag{E.39}$$

We stationarize this equation by dividing through with $z_t^+$and divide the prices with $P_{F,t}$, using the definitions $\widetilde{p}_t^X = \frac{P_t^X}{P_{F,t}}$, $p_{F,t}^{C,xe} = \frac{P_{F,t}^{C,xe}}{P_{F,t}}$ and $p_{F,t}^I = \frac{P_{F,t}^I}{P_{F,t}}$ to get

$$\frac{X_t}{z_t^+} = \left(1 - \psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \frac{C_{F,t}^{xe}}{z_t^+} + \left(1 - \psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \frac{I_{F,t}}{z_t^+}$$

$$\frac{X_t}{z_t^+} = \left(1 - \psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \frac{C_{F,t}^{xe}}{z_{F,t}^+}\frac{z_{F,t}^+}{z_t^+} + \left(1 - \psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \frac{I_{F,t}}{z_{F,t}^+}\frac{z_{F,t}^+}{z_t^+} \tag{E.40}$$

$$\frac{X_t}{z_t^+} = \frac{\omega}{\omega}\left(1 - \psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \frac{C_{F,t}^{xe}}{z_{F,t}^+}\frac{z_{F,t}^+}{z_t^+} + \frac{\omega}{\omega}\left(1 - \psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \frac{I_{F,t}}{z_{F,t}^+}\frac{z_{F,t}^+}{z_t^+} \tag{E.41}$$

$$\frac{X_t}{z_t^+} = \frac{\omega}{\omega}\left(1 - \psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \frac{C_{F,t}^{xe}}{z_{F,t}^+}\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} + \frac{\omega}{\omega}\left(1 - \psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \frac{I_{F,t}}{z_{F,t}^+}\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} \tag{E.42}$$

Write it in per-capita form (using the fact that the size of the population in Sweden is 1 and the size of the population in Foreign is ω):

$$\bar{x}_t = \omega\left(1 - \psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe}\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} + \omega\left(1 - \psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \bar{I}_{F,t}\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} \tag{E.43}$$

Now, we let $\psi_{F,t}^{C,xe} = \tilde{\vartheta}_{F,t}^{C,xe} + \frac{\omega}{1+\omega}(1 - \tilde{\vartheta}_{F,t}^{C,xe})$ and $\psi_{F,t}^I = \tilde{\vartheta}_{F,t}^I + \frac{\omega}{1+\omega}(1 - \tilde{\vartheta}_{F,t}^I)$ where $\tilde{\vartheta}_{F,t}^{C,xe}$ and $\tilde{\vartheta}_{F,t}^I$ denotes the home-bias in the production functions of Foreign exports and investment. Using these expressions we can write

$$\begin{aligned}\bar{x}_t =& \gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)}\left[\omega\left(1 - \tilde{\vartheta}_{F,t}^{C,xe} - \frac{\omega}{1+\omega}(1 - \tilde{\vartheta}_{F,t}^{C,xe})\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe}\right] \\ &+ \gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)}\left[\omega\left(1 - \tilde{\vartheta}_{F,t}^I - \frac{\omega}{1+\omega}(1 - \tilde{\vartheta}_{F,t}^I)\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \bar{I}_{F,t}\right]\end{aligned} \tag{E.44}$$

or

$$\bar{x}_t = \gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} \left[\left(\frac{\omega(1+\omega)-\omega^2}{1+\omega}(1-\tilde{\vartheta}_{F,t}^{C,xe}) \right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}} \right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe} + \left(\frac{\omega(1+\omega)-\omega^2}{1+\omega}(1-\tilde{\vartheta}_{F,t}^{I}) \right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{I}} \right)^{-\nu_{F,I}} \bar{I}_{F,t} \right]$$

Note that since $\omega \to \infty$,we can use l'hôspital's rule, which means that

$$lim_{\omega\to\infty} \frac{\omega(1+\omega)-\omega^2}{1+\omega} = \frac{1+2\omega-2\omega}{1} = 1$$

Hence, we can write the above expression as

$$\bar{x}_t = \gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} \left[\left(1-\tilde{\vartheta}_{F,t}^{C,xe}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}} \right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe} + \left(1-\tilde{\vartheta}_{F,t}^{I}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{I}} \right)^{-\nu_{F,I}} \bar{I}_{F,t} \right].$$

Now, we make the two following restrictive assumptions above the home bias processes:

$$\begin{aligned} \tilde{\vartheta}_{F,t}^{C,xe} &= 1-(1-\vartheta_{F,t}^{C,xe})\gamma_t^{-\alpha_F/(1-\alpha_F)+\alpha/(1-\alpha)} \\ \tilde{\vartheta}_{F,t}^{I} &= 1-(1-\vartheta_{F,t}^{I})\gamma_t^{-\alpha_F/(1-\alpha_F)+\alpha/(1-\alpha)} \end{aligned}$$

These imply that if the global economy grows faster than the Swedish economy due to the investment technology process, then the Swedish market share of the global economy will shrink over time and vice versa. Inserting these two equations into the demand for export gives us the following stationarized export demand function:

$$\bar{x}_t = \left(1-\vartheta_F^{C,xe}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}} \right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe} + \left(1-\vartheta_F^{I}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{I}} \right)^{-\nu_{F,I}} \bar{I}_{F,t}. \tag{E.45}$$

Equation (E.45) is the same as Equation (A.139a) in Appendix A.

E.4 Swedish aggregate output

E.4.1 Swedish aggregate output

Swedish aggregate output is given by:

$$\overleftrightarrow{P}_t Y_t = \int_0^1 \left(\varepsilon_t \left[\tilde{K}_t^s(i)\right]^\alpha [z_t N_t(i)]^{1-\alpha} - z_t^+ \phi \right) di. \tag{E.46}$$

We stationarize the above equation by dividing the equation with z_t^+. Note that $z_t^+ = z_t \gamma_t^{\frac{\alpha}{1-\alpha}}$, and the definition of stationarized capital services is given by: $\bar{\tilde{k}}_t^s(i) = \tilde{K}_t^s(i)/(z_{t-1}\gamma_{t-1}^{\frac{1}{1-\alpha}}) = \tilde{K}_t^s(i)/\left[(z_t\gamma_t^{\frac{1}{1-\alpha}})/(\mu_{z,t}\mu_{\gamma,t}^{\frac{1}{1-\alpha}})\right]$. This means that if we divide both sides of the Swedish aggregate output expression with z_t^+ , we have the following equation:

$$\begin{aligned} \frac{Y_t}{z_t^+} \overleftrightarrow{P}_t &= \int_0^1 \left(\varepsilon_t \left[\frac{\tilde{K}_t^s(i)}{(z_t\gamma_t^{\frac{1}{1-\alpha}})/(\mu_{z,t}\mu_{\gamma,t}^{\frac{1}{1-\alpha}})} \right]^\alpha \left[\frac{N_t(i)}{z_t^+} \right]^{1-\alpha} \right) di - \phi, \\ \overline{Y}_t \overleftrightarrow{P}_t &= \int_0^1 \left(\varepsilon_t \left[\frac{\bar{\tilde{K}}_t^s(i)}{\mu_{z^+,t}\mu_{\gamma,t}} \right]^\alpha L_t(i)^{1-\alpha} \right) di - \phi \end{aligned}$$

We can remove the subscript i since it is shown above that the firms in Sweden choose the same capital stock. We rewrite the above equation in terms of stationarized variables in per-capita:

$$\overline{y}_t \overleftrightarrow{P}_t = \left(\varepsilon_t \left[\frac{\bar{\tilde{k}}_t^s}{\mu_{z^+,t}\mu_{\gamma,t}} \right]^\alpha n_t^{1-\alpha} \right) - \phi \tag{E.47}$$

Note that Equation (E.47) is the same as Equation (A.148a) in Section A.

E.4.2 Measured Swedish aggregate output

The measured Swedish aggregate input is given by

$$Y_t^m = Y_t - \psi^I \left(\frac{P_t^I}{P_t}\right)^{\nu_I} a(u_t) \frac{K_t}{\gamma_t}. \tag{E.48}$$

We follow Section E.4.1. In particular, we divide both sides of the above equation by z_t^+ and express the equation in per-capita terms. This yields:

$$\overline{y}_t^m = \overline{y}_t - \vartheta^I (p_t^I)^{\nu_I} a(u_t) \frac{\overline{k}_t}{\mu_{z^+,t}\mu_{\gamma,t}}. \tag{E.49}$$

Note that since $\omega \to \infty$ and $\psi^I = \vartheta^I + \frac{1}{1+\omega}(1-\vartheta^I)$,we get that $\psi^I = \vartheta^I$. Note that Equation (E.49) is the same as Equation (A.149a) in Section A.

E.5 Foreign aggregate resource constraint

The derivation of the aggregate resource constraint for Foreign proceeds in much the same way as the corresponding derivations for Sweden, which were laid out in Section (E.1) above. In the first part of this section, we derive the non-stationary version of the Foreign aggregate resource constraint, which corresponds to Equation (95) in the main text. In the second part of the section, a stationarized version is derived (Equation A.137a).

E.5.1 Market clearing in Foreign

The market for intermediate goods in Foreign clears when the production of each individual firm j, $Y_{F,t}(j)$, equals the demand for the output of the same firm: $Y_{F,t}(j) = \frac{1}{\omega}\left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} Y_{F,t}$. $Y_{F,t}$ represents aggregate demand for the homogeneous, intermediate good in Foreign, and $P_{F,t}$ is the associated aggregate price index. $P_{F,t}(j)$ denotes the price charged by the individual firm j and $\lambda_{F,t}$ is a time varying markup. After having defined $Y_{F,t}^P = \int_0^\omega Y_{F,t}(j)dj$ as aggregate production of intermediate goods and $\overleftrightarrow{P}_{F,t} = \int_0^\omega \frac{1}{\omega}\left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} dj$ as a measure of price dispersion, we proceed by aggregating over all firms in the sector:

$$Y_{F,t}^P = \int_0^\omega \left\{\frac{1}{\omega}\left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} Y_{F,t}\right\} dj = Y_{F,t}\int_0^\omega \frac{1}{\omega}\left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} dj = Y_{F,t}\overleftrightarrow{P}_{F,t}. \tag{E.50}$$

Recall from Section (2.6.2) that $Y_{F,t}(j) = \varepsilon_{F,t}\left[\tilde{K}_{F,t}^s(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z_{F,t}^+\phi_F$. Once again aggregating over all firms in the sector, write:

$$Y_{F,t}^P = \int_0^\omega \left(\varepsilon_{F,t}\left[\tilde{K}_{F,t}^s(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z_{F,t}^+\phi_F\right) dj = Y_{F,t}\overleftrightarrow{P}_{F,t}. \tag{E.51}$$

Turn now to the demand side of the economy. In Foreign, demand for the homogeneous intermediate good comes from non-energy consumption good producers, energy consumption good producers and investment good producers:

$$Y_{F,t} = D_{F,t}^{C,xe} + D_{F,t}^{C,e} + D_{F,t}^I + G_{F,t} \tag{E.52}$$

From Equation (D.51), we have $D_{F,t}^{C,xe} = C_{F,t}^{xe}$ represents demand from non-energy foreign consumption good production. Similarly, demand for intermediate goods for foreign energy consumption production is $D_{F,t}^{C,e} = C_{F,t}^e$ and from Equation (D.56), we have $D_{F,t}^I = V_{F,t}^I$ represents demand from foreign investment good production where $V_{F,t}^I = \frac{1}{\gamma_{F,t}}\left[I_{F,t} + a(u_t)K_{F,t}\right]$.

Continue by substituting for $D_{F,t}^{C,xe}$ and $D_{F,t}^I$ in Equation (E.52). We have:

$$Y_{F,t} = C_{F,t}^{xe} + C_{F,t}^e + V_{F,t}^I + G_{F,t}. \tag{E.53}$$

Now use Equation (E.53) to substitute for $Y_{F,t}$ in (E.50), and then combine this expression with (E.51) to get:

$$Y_{F,t}^P = \int_0^\omega \varepsilon_{F,t}\left[\tilde{K}_{F,t}^s(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} dj - z_{F,t}^+\omega\phi_F =$$

$$\overleftrightarrow{P}_{F,t}C_{F,t}^{xe} + \overleftrightarrow{P}_{F,t}C_{F,t}^e + \overleftrightarrow{P}_{F,t}\frac{1}{\gamma_{F,t}}\left[I_{F,t} + a(u_t)K_{F,t}\right] + \overleftrightarrow{P}_{F,t}G_{F,t}.$$

Rearrange slightly to get the aggregate resource constraint for Foreign, the same as Equation (95) in the main text:[53]

$$\varepsilon_{F,t}\left[K_{F,t}^{s}\right]^{\alpha_F}\left[z_t N_{F,t}\right]^{1-\alpha_F}=\overleftrightarrow{P}_{F,t}C_{F,t}^{xe}+\overleftrightarrow{P}_{F,t}C_{F,t}^{e}$$

$$+\overleftrightarrow{P}_{F,t}\frac{1}{\gamma_{F,t}}\left[I_{F,t}+a(u_t)K_{F,t}\right]+\overleftrightarrow{P}_{F,t}G_{F,t}+z_{F,t}^{+}\omega\phi_F. \tag{E.54}$$

E.5.2 Stationarizing and simplifying the Foreign aggregate resource constraint

The following definitions are needed to stationarize Equation (E.54), and to express it in per capita values: $\bar{y}_{F,t}=\frac{Y_{F,t}}{z_{F,t}^{+}\omega}$, $\bar{c}_{F,t}^{xe}=\frac{C_{F,t}^{xe}}{z_{F,t}^{+}\omega}$, $\bar{c}_{F,t}^{e}=\frac{C_{F,t}^{e}}{z_{F,t}^{+}\omega}$, $\bar{g}_{F,t}=\frac{G_{F,t}}{z_{F,t}^{+}\omega}$, and $\overline{I}_{F,t}=\frac{I_{F,t}}{\omega z_t\gamma_t^{\frac{1}{1-\alpha_F}}}$. Also, let $\bar{k}_{F,t}^{s}=\frac{K_{F,t}^{s}}{z_{t-1}\gamma_{t-1}^{\frac{1}{1-\alpha_F}}\omega}$ and $n_{F,t}=\frac{N_{f,t}}{\omega}$. Moreover, use $Y_{F,t}^{P}=Y_{F,t}\overleftrightarrow{P}_{F,t}$ Using these definitions, and then dividing through by $z_{F,t}^{+}\omega$, we get:

$$\bar{y}_{F,t}=\bar{c}_{F,t}^{xe}+\bar{c}_{F,t}^{e}+\overline{I}_{F,t}+a\left(u_{F,t}\right)\bar{k}_{F,t}\frac{1}{\mu_{z_F^{+},t}\mu_{\gamma,t}}+\bar{g}_{F,t} \tag{E.55}$$

E.6 Balance of payments and net foreign assets

Section (2.7.4) in the main text established the balance of payments identity of Sweden (Equation 107), which we reproduce here for convenience:

$$S_tP_t^{X}X_t-S_tP_{Ft}M_t^{xe}-S_tP_{Ft}^{C,e}M_t^{e}=\frac{S_tB_{t+1}^{FH}}{R_{F,t}\zeta_t\Phi(\overline{a}_t,s_t,\widetilde{\phi}_t)}-S_tB_t^{FH}. \tag{E.56}$$

Note that the prices of imported goods are the same as the marginal costs of the import firms, $S_tP_{Ft}^{C,e}=MC_t^{M,C,e}$ and $S_tP_{Ft}=MC_t^{M,xe}$. Therefore, we can write the expression as the following:

$$S_tP_t^{X}X_t-MC_t^{M,xe}M_t^{xe}-MC_t^{M,C,e}M_t^{e}=\frac{S_tB_{t+1}^{FH}}{R_{F,t}\zeta_t\Phi(\overline{a}_t,s_t,\widetilde{\phi}_t)}-\frac{S_t}{S_{t-1}}S_{t-1}B_t^{FH}.$$

Now substitute A_t for $\frac{S_tB_{t+1}^{FH}}{R_{F,t}\zeta_t\Phi(\overline{a}_t,s_t,\widetilde{\phi}_t)}$, as well as A_{t-1} for $\frac{S_{t-1}B_t^{FH}}{R_{F,t-1}\zeta_{t-1}\Phi(\overline{a}_{t-1},s_{t-1},\widetilde{\phi}_{t-1})}$. Then subtract A_{t-1} from both sides to get:

$$A_t-A_{t-1}=S_tP_t^{X}X_t-MC_t^{M,xe}M_t^{xe}-MC_t^{M,C,e}M_t^{e}+\Phi\left(\overline{a}_{t-1},s_{t-1},\widetilde{\phi}_{t-1}\right)R_{F,t-1}\zeta_{t-1}\frac{S_t}{S_{t-1}}A_{t-1}-A_{t-1}.$$

Simplify the last term to arrive at:

$$A_t-A_{t-1}=S_tP_t^{X}X_t-MC_t^{M,xe}M_t^{xe}-MC_t^{M,C,e}M_t^{e}+\left[\Phi\left(\overline{a}_{t-1},s_{t-1},\widetilde{\phi}_{t-1}\right)R_{F,t-1}\zeta_{t-1}\frac{S_t}{S_{t-1}}-1\right]A_{t-1},$$

which is the same as Equation (108) in the main text.
Let $\overline{a}_t=\frac{A_t}{z_t^{+}P_t}$ denote the stationarized net foreign asset position of the Swedish economy, and use the following definitions to rewrite the above condition: $\widetilde{p}_{F,t}^{X}=\frac{P_{F,t}^{X}}{P_t}$, $\bar{mc}_t^{M,C,xe}=\frac{MC_t^{M,xe}}{P_t}$, $\overline{m}_t^{xe}=\frac{M_t}{z_t^{+}}$, $\bar{mc}_t^{M,C,e}=\frac{MC_t^{M,C,e}}{P_t}$, $\bar{m}_t^{e}=\frac{M_t^{e}}{z_t^{+}}$, $p_t^{X}=\frac{S_tP_t^{X}}{P_t}$, $\overline{x}_t=\frac{X_t}{z_t^{+}}$, $s_t=\frac{S_t}{S_{t-1}}$, $\mu_{z^+,t}=\frac{z_t^{+}}{z_{t-1}^{+}}$ and $\Pi_t=\frac{P_t}{P_{t-1}}$. The above equation can be written as:

$$\bar{mc}_t^{M,C,xe}\,\overline{m}_t^{xe}+\bar{mc}_t^{M,C,e}\bar{m}_t^{e}+\overline{a}_t=p_t^{X}\,\overline{x}_t+\Phi\left(\overline{a}_{t-1},s_{t-1},\widetilde{\phi}_{t-1}\right)R_{F,t-1}\zeta_{t-1}\,s_t\,\overline{a}_{t-1}\frac{1}{\mu_{z^+,t}\Pi_t}.$$

[53]Note that $\psi_F^{C.xe}\left(\frac{P_{F,t}}{P_{F,t}^{C,xe}}\right)^{-\nu_{F,C}}C_{F,t}^{xe}=C_{F,t}^{xe}$ and $\psi_F^{I}\left(\frac{P_{F,t}}{P_{F,t}^{I}}\right)^{-\nu_{F,I}}\left[\frac{I_{F,t}}{\gamma_t}+a\left(u_{F,t}\right)\frac{K_{F,t}}{\gamma_t}\right]=\frac{1}{\gamma_{F,t}}\left[I_{F,t}+a(u_t)K_{F,t}\right]$.

Note that the demand function for individual goods imported energy goods are given by $\bar{m}_t^{C,e}(i) = \left(\frac{p_t^{M,C,e}(i)}{p_t^{M,C,e}}\right)^{\frac{\lambda_t^{M,C,e}}{1-\lambda_t^{M,C,e}}} \bar{m}_t^{C,e}$. Denote $\overset{\leftrightarrow}{p}_t^{M,C,e} = \int_0^1 \left(\frac{p_t^{M,C,e}(i)}{p_t^{M,C,e}}\right)^{\frac{\lambda_t^{M,C,e}}{1-\lambda_t^{M,C,e}}} di$ and rearrange slightly to get an expression that is identical to Equation (A.138a) in Appendix A:

$$\overline{a}_t = p_t^X\, \overline{x}_t - \bar{m}c_t^{M,xe}\, \overline{m}_t^{xe} - \bar{m}c_t^{M,C,e}\bar{m}_t^e + \Phi\left(\overline{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right) R_{F,t-1}\zeta_{t-1}\, s_t\, \overline{a}_{t-1}\frac{1}{\mu_{z^+,t}\,\Pi_t}. \tag{E.57}$$

E.7 Total energy imports

Total energy imports is given by

$$M_t^e = \int_0^1 M_t^{C,e}(i)di + z_t^+\phi^{M,C,e} \tag{E.58}$$

Note that the demand function for individual goods imported energy goods are given by $M_t^{C,e}(i) = \left(\frac{P_t^{M,C,e}(i)}{P_t^{M,C,e}}\right)^{\frac{\lambda_t^{M,C,e}}{1-\lambda_t^{M,C,e}}} P_t^{C,e}$. Denote $\overset{\leftrightarrow}{P}_t^{M,C,e} = \int_0^1 \left(\frac{p_t^{M,C,e}(i)}{p_t^{M,C,e}}\right)^{\frac{\lambda_t^{M,C,e}}{1-\lambda_t^{M,C,e}}} di$ to write the import function as

$$M_t^e = \overset{\leftrightarrow}{P}_t^{M,C,e} M_t^{C,e} + z_t^+\phi^{M,C,e} \tag{E.59}$$

To stationarize, divide through with z_t^+ to get

$$\bar{m}_t^e = \overset{\leftrightarrow}{P}_t^{M,C,e}\bar{m}_t^{C,e} + \phi^{M,C,e} \tag{E.60}$$

which is the same equation as Equation (A.147a) in Appendix A.

F Appendix: Log-linearization

F.1 Log-linearization method

Suppose, we have the following function: $F(X_t, Z_t)$. We take the natural log of the function $F(X_t, Z_t)$, and then we take a first-order Taylor approximation of the function $F(X_t, Z_t)$ around the steady state X and the steady state Z respectively.

A log-linear approximation of $F(X_t, Z_t)$ around the steady state X and the steady state Z is:

$$ln\left[F(X_t, Z_t)\right] \approx ln\left[F(X, Z)\right] + \frac{F_X(X,Z)}{F(X,Z)}X\left[\frac{X_t - X}{X}\right] + \frac{F_Z(X,Z)}{F(X,Z)}Z\left[\frac{Z_t - Z}{Z}\right].$$

Note that: $\hat{X}_t = ln(X_t) - ln(X) \approx \frac{X_t - X}{X}$. We can interpret $\hat{X}_t$ as a log-linear approximation of the variable X_t around its steady state value X.

F.2 Example of log-linearization method

In this section, we demonstrate how to implement a first-order log-linear approximation. For an illustrative purpose, we will take a first-order log-linear approximation of the stationarized version of consumption Euler equation (C.26).

Recall, the stationarized version of consumption Euler equation is written as:

$$\overline{\Omega}_t^C = R_t\zeta_t E_t\left[\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C}\right].$$

Note that $\beta^r = \beta$ and $\zeta = 1$. Thus, the steady state of consumption Euler equation can be expressed as:

$$\begin{aligned}\overline{\Omega}^C &= R\beta \frac{1}{\mu_{z+}\Pi^C}\overline{\Omega}^C \\ R &= \frac{\mu_{z+}\Pi^C}{\beta}.\end{aligned} \tag{F.1}$$

The first-order log-linear approximation of the LHS of Equation (C.26) is:

$$ln\overline{\Omega}_t^C \approx ln\overline{\Omega}^C + \frac{1}{\overline{\Omega}^C}\overline{\Omega}^C\left(\frac{\overline{\Omega}_t^C - \overline{\Omega}^C}{\overline{\Omega}^C}\right) = ln\overline{\Omega}^C + \widehat{\Omega}_t^C. \tag{F.2}$$

We have the following definitions:

$$\begin{aligned}\hat{\zeta}_t &= \frac{\zeta_t - \zeta}{\zeta} \\ \zeta &= 1 \\ \breve{i}_t &= R_t - R.\end{aligned}$$

We use the above definitions, and the first-order log-linear approximation of the RHS of Equation (C.26) can be written as follows:

$$\begin{aligned}ln\overline{\Omega}^C &+ \frac{1}{\overline{\Omega}^C}\beta\overline{\Omega}^C\frac{1}{\mu_{z+}\Pi^C}(R_t - R) \\ &+ \frac{1}{\overline{\Omega}^C}\beta\overline{\Omega}^C\frac{R\zeta}{\mu_{z+}\Pi^C}\left(\frac{\zeta_t - \zeta}{\zeta}\right) \\ &+ \frac{1}{\overline{\Omega}^C}R\beta\overline{\Omega}^C\frac{1}{\mu_{z+}\Pi^C}\left(\frac{\beta_{t+1}^r - \beta}{\beta}\right) \\ &+ \frac{1}{\overline{\Omega}^C}R\beta\overline{\Omega}^C\frac{1}{\mu_{z+}\Pi^C}\left(\frac{\overline{\Omega}_{t+1}^C - \overline{\Omega}^C}{\overline{\Omega}^C}\right) \\ &- \frac{1}{\overline{\Omega}^C}R\beta\overline{\Omega}^C\frac{1}{\Pi^C}\left(\frac{1}{\mu_{z+}}\right)^2\mu_{z+}\left(\frac{\mu_{z+,t+1} - \mu_{z+}}{\mu_{z+}}\right) \\ &- \frac{1}{\overline{\Omega}^C}R\beta\overline{\Omega}^C\frac{1}{\mu_{z+}}\left(\frac{1}{\Pi^C}\right)^2\Pi^C\left(\frac{\Pi_{t+1}^C - \Pi^C}{\Pi^C}\right).\end{aligned} \tag{F.3}$$

Using Equation (F.1) and the above definitions, Equation (F.3) can be written as:

$$ln\overline{\Omega}^C + \hat{\zeta}_t + \frac{1}{R}(R_t - R) + \hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C. \tag{F.4}$$

Combing Equation (F.2) and Equation (F.4), this gives us the following log-linearized version of consumption of Euler equation:

$$\hat{\Omega}_t^C = E_t\left[\hat{\zeta}_t + \hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C + \frac{1}{R}\breve{i}_t - \hat{\Pi}_{t+1}^C - \hat{\mu}_{z^+,t+1}\right]. \tag{F.5}$$

Equation (F.5) is the same as Equation (A.1b).

G Appendix: Derivation of log-linear wage equation

This section derives the log-linearized version of the optimal wage condition, Equation (A.13b). First, we present the key equations that will be used to derive the final log-linearized version of the optimal wage equation. The key equations are the real wage markup, aggregate wage index and labor demand equation. Second, we log-linearize Equation (C.63) which is the nonlinear version of the optimal wage equation and then use the key equations to obtain the final log-linearized version of the optimal wage equation.

G.1 Real wage markup

In this section, we present Equation (A.18b) that captures the log-linearized version of the real wage markup.

Recall, Equation (A.18a), which shows the non-linearized version of the real wage markup, is expressed as:

$$\overline{\Psi}_t^W = \frac{\left(1-\tau_t^W\right)\overline{w}_t}{\zeta_t^n \frac{\nu'(n_t)}{\overline{\Omega}_t^C}}. \tag{G.1}$$

Recall from Section 3.1, the labor disutility function is specified as:

$$\nu(N_{h,t}) = \Theta_t^n A_n \frac{N_{h,t}^{1+\eta}}{1+\eta}. \tag{G.2}$$

We can drop the subscript h since all household members choose the same optimal wage when they have chance to update their wage. As a result those labor types will have the same employment in this model. We can also rewrite the labor disutility function in terms of per *capita*, so n_t is denoted as employment per *capita* (employment rate). Thus, we have the following labor disutility function:

$$\nu(n_t) = \Theta_t^n A_n \frac{n_t^{1+\eta}}{1+\eta}. \tag{G.3}$$

The first derivative of $\nu(n_t)$ with respect to n_t is:

$$\nu'(n_t) = \Theta_t^n A_n n_t^{\eta}. \tag{G.4}$$

The second derivative of $\nu(n_t)$ with respect to n_t is:

$$\nu''(n_t) = \eta\Theta_t^n A_n n_t^{\eta-1}. \tag{G.5}$$

We can rewrite Equation (G.1) as follows:

$$\overline{\Psi}_t^W = \frac{\left(1-\tau_t^W\right)\overline{w}_t}{\zeta_t^n \frac{\Theta_t^n A_n n_t^{\eta}}{\overline{\Omega}_t^C}}. \tag{G.6}$$

We apply the log-linearization method from Section F.1 to Equation (G.6), and we can obtain the following log-linearized version of the real wage markup:

$$\hat{\Psi}_t^W = \hat{w}_t - \frac{1}{1-\tau^W}\breve{\tau}_t^W - \hat{\zeta}_t^n - \eta\hat{n}_t + \hat{\Omega}_t^C. \tag{G.7}$$

Furthermore, the labor force participation condition, Equation (A.14a) can be written as:

$$\hat{w}_t = \hat{\zeta}_t^n + \hat{\Theta}_t^n + \eta\hat{l}_t - \hat{\Omega}_t^C + \frac{1}{1-\tau^W}\breve{\tau}_t^W \tag{G.8}$$

When we combine Equation (G.8) and Equation (G.7) we obtain the following relationship between the wage markup and unemployment:

$$\hat{\Psi}_t^W = \eta\hat{un}_t \tag{G.9}$$

Equation (G.9) is the same as Equation (A.18b).

G.2 Aggregate wage index

In this section, we present the log-linearized version of the aggregate wage index. Recall from the main text in Section 2.1.3, the aggregate wage index in non-linear terms is specified as follows:

$$W_t = \left[\int_0^1 W_{h,t}^{\left(1-\varepsilon_{w,t}\right)} dh\right]^{\frac{1}{1-\varepsilon_{w,t}}}. \tag{G.10}$$

Recall from Section 2.1.3, we have the following Calvo wage contract:

$$W_{h,t} = \begin{cases} \overline{\Pi}_t^W W_{h,t-1} & \text{with probability } \xi_w \\ W_{h,t}^{\text{opt}} & \text{with probability } (1-\xi_w). \end{cases} \tag{G.11}$$

We apply the above Calvo wage contract and the following definition: $\varepsilon_{w,t} = \frac{\lambda_t^W}{\lambda_t^W - 1}$. Thus, we can rewrite Equation (G.10) as follows:

$$W_t = \left[\int_0^1 W_{h,t}^{\frac{1}{1-\lambda_t^W}} dh \right]^{1-\lambda_t^W}, \tag{G.12}$$

$$W_t^{\frac{1}{1-\lambda_t^W}} = \int_0^1 (W_{h,t})^{\frac{1}{1-\lambda_t^W}} dh, \tag{G.13}$$

$$W_t^{\frac{1}{1-\lambda_t^W}} = \int_0^{\xi_w} \left(\overline{\Pi}_t^W W_{h,t-1} \right)^{\frac{1}{1-\lambda_t^W}} dh + \int_{\xi_w}^1 \left(W_{h,t}^{opt} \right)^{\frac{1}{1-\lambda_t^W}} dh, \tag{G.14}$$

$$W_t^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W \right)^{\frac{1}{1-\lambda_t^W}} \int_0^{\xi_w} (W_{h,t-1})^{\frac{1}{1-\lambda_t^W}} dh + \int_{\xi_w}^1 \left(W_{h,t}^{opt} \right)^{\frac{1}{1-\lambda_t^W}} dh. \tag{G.15}$$

Now, we evaluate the integrals in Equation (G.15). First, we evaluate the first term of the RHS of Equation (G.15). Recall that the opportunity to reset the wage in any given period is governed by a random variable, and that this variable is identically and independently distributed across individual labor types and across different time periods. From this assumption, it follows that the subset of labor types that do not have the opportunity to reset their wage in period t will constitute a representative sample of all labor types. The wages posted by those same labor types in period $(t-1)$ will, by the same argument, constitute a representative sample of all the individual wages that were posted in that period. By the law of large numbers, the term $\int_0^{\xi_w} (W_{h,t-1})^{\frac{1}{1-\lambda_t^W}} dh$ may therefore be evaluated as follows:

$$\int_0^{\xi_w} (W_{h,t-1})^{\frac{1}{1-\lambda_t^W}} dh = \xi_w W_{t-1}^{\frac{1}{1-\lambda_t^W}}, \tag{G.16}$$

where $W_{t-1} = \left[\int_0^1 W_{h,t-1}^{\frac{1}{1-\lambda_t^W}} dh \right]^{1-\lambda_t^W}$. Using Equation (G.16), Equation (G.15) can be written as:

$$W_t^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W \right)^{\frac{1}{1-\lambda_t^W}} \xi_w (W_{t-1})^{\frac{1}{1-\lambda_t^W}} + \int_{\xi_w}^1 \left(W_{h,t}^{opt} \right)^{\frac{1}{1-\lambda_t^W}} dh. \tag{G.17}$$

Now, we evaluate the second term of the RHS of Equation (G.15). All labor types that get the opportunity to reset their wage in period t will face the same maximization problem. This follows from our assumption concerning the existence of contingent claims that allow individual household members to diversify the risk associated with the nominal wage friction (see Section 2.1.2 in the main text). As a consequence, all labor types that have the opportunity to reset their wage in period t will choose the same optimal wage, and we may therefore write $W_{h,t}^{opt} = W_t^{opt}$. Hence, Equation (G.17) can be written as follows:

$$W_t^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W \right)^{\frac{1}{1-\lambda_t^W}} \xi_w (W_{t-1})^{\frac{1}{1-\lambda_t^W}} + \left(W_t^{opt} \right)^{\frac{1}{1-\lambda_t^W}} \int_{\xi_w}^1 dh. \tag{G.18}$$

Using the following result: $\int_{\xi_w}^1 dh = (1-\xi_w)$, Equation (G.18) can be expressed as:

$$W_t^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W \right)^{\frac{1}{1-\lambda_t^W}} \xi_w (W_{t-1})^{\frac{1}{1-\lambda_t^W}} + (1-\xi_w) \left(W_t^{opt} \right)^{\frac{1}{1-\lambda_t^W}}. \tag{G.19}$$

We stationarize the above equation. Using the following definitions: $\overline{w}_t = \frac{W_t}{z_t^+ P_t^C}$, $\overline{w}_t^{opt} = \frac{W_t^{opt}}{z_t^+ P_t^C}$, $\Pi_t^C = \frac{P_t^C}{P_{t-1}^C}$ and $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, Equation (G.19) becomes:

$$\left(\frac{W_t}{z_t^+ P_t^C} \right)^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W \right)^{\frac{1}{1-\lambda_t^W}} \xi_w \left(\frac{1}{\mu_{z^+,t} \Pi_t^C} \frac{W_{t-1}}{z_{t-1}^+ P_{t-1}^C} \right)^{\frac{1}{1-\lambda_t^W}} + (1-\xi_w) \left(\frac{W_t^{opt}}{z_t^+ P_t^C} \right)^{\frac{1}{1-\lambda_t^W}}.$$

The above equation can be simplified to the following equation:

$$(\overline{w}_t)^{\frac{1}{1-\lambda_t^W}} = \xi_w \left(\frac{\overline{\Pi}_t^W}{\mu_{z^+,t} \Pi_t^C} \right)^{\frac{1}{1-\lambda_t^W}} (\overline{w}_{t-1})^{\frac{1}{1-\lambda_t^W}} + (1-\xi_w) \left(\overline{w}_t^{opt} \right)^{\frac{1}{1-\lambda_t^W}}. \tag{G.20}$$

Equation (G.20) captures the stationarized version of the aggregate wage index.

We apply the log-linearization method from Section F.1 to Equation (G.20), and we can obtain the following log-linearized version of the aggregate wage index:

$$\hat{w}_t = \xi_w \left(\hat{\overline{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C + \hat{w}_{t-1} \right) + (1-\xi_w)\hat{w}_t^{opt}. \tag{G.21}$$

Equation (G.21) captures the log-linearized version the aggregate wage index.

G.3 Labor demand

In this section, we present the log-linearized version of labor demand equation.

Recall from Section C.1.9, we have the following labor demand schedule:

$$N_{h,t+k|t} = \left(\frac{W_{h,t+k|t}}{W_{t+k}} \right)^{-\varepsilon_{w,t+k}} N_{t+k}.$$

Recall, $N_{h,t+k|t}$ denotes demand for labor type h, whereas N_{t+k} is aggregate labor demand. First, we can drop the subscript h from the labor demand schedule since all labor types choose the same optimal wage as a result the same amount of labor supply. Second, we stationarize the above demand schedule by using the following definitions: $\overline{w}_{t+k|t} = \frac{W_{t+k|t}}{z_{t+k}^+ P_{t+k}^C}$ and $\overline{w}_{t+k} = \frac{W_{t+k}}{z_{t+k}^+ P_{t+k}^C}$. We also rewrite the above equation in per *capita* terms. The stationarized version of the labor demand schedule is:

$$n_{t+k|t} = \left(\frac{\overline{w}_{t+k|t}}{\overline{w}_{t+k}} \right)^{-\varepsilon_{w,t+k}} n_{t+k}. \tag{G.22}$$

Recall from Section C.1.9, we have the following definition:

$$W_{t+k|t} = W_t^{opt} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W \dots \overline{\Pi}_{t+k}^W. \tag{G.23}$$

We stationarize Equation (G.23) by applying the following definitions: $\overline{w}_{t+k} = \frac{W_{t+k}}{z_{t+k}^+ P_{t+k}^C}$, $\overline{w}_t^{opt} = \frac{W_t^{opt}}{z_t^+ P_t^C}$, $\Pi_{t+k}^C = \frac{P_{t+k}^C}{P_{t+k-1}^C}$, and $\mu_{z^+,t+k} = \frac{z_{t+k}^+}{z_{t+k-1}^+}$. Thus, Equation (G.23) becomes:

$$\overline{w}_{t+k|t} = \frac{\overline{w}_t^{opt} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W \dots \overline{\Pi}_{t+k}^W}{\mu_{z^+,t+1}\mu_{z^+,t+2} \dots \mu_{z^+,t+k} \Pi_{t+1}^C \Pi_{t+2}^C \dots \Pi_{t+k}^C}. \tag{G.24}$$

Recall, Equation (A.19a), which shows the definition of wage inflation, is expressed as:

$$\Pi_t^W = \frac{\overline{w}_t}{\overline{w}_{t-1}} \mu_{z^+,t} \Pi_t^C.$$

Note that along the balanced growth path, the definition of wage inflation is:

$$\Pi_t^W = \overline{\Pi}^W = \mu_{z^+} \Pi^C. \tag{G.25}$$

Along the balanced growth path, Equation (G.24) can be expressed as:

$$\overline{w} = \left(\frac{\overline{\Pi}^W}{\mu_{z^+} \Pi^C} \right)^k \overline{w}^{opt}. \tag{G.26}$$

We apply the log-linearization method from Section F.1 to Equation (G.22) and also take into account Equation

(G.24), (G.25) and (G.26) when log-linearizing Equation (G.22). Hence, we can obtain the following log-linearized version of the labor demand equation:

$$\begin{aligned}
&\hat{n}_{t+k|t} = \varepsilon_w \hat{w}_{t+k} + \hat{n}_{t+k} - \varepsilon_w \hat{w}_t^{opt} \\
&- \varepsilon_w \hat{\overline{\Pi}}_{t+1}^W - \varepsilon_w \hat{\overline{\Pi}}_{t+2}^W - \ldots - \varepsilon_w \hat{\overline{\Pi}}_{t+k}^W \\
&+ \varepsilon_w \hat{\mu}_{z^+,t+1} + \varepsilon_w \hat{\mu}_{z^+,t+2} + \ldots + \varepsilon_w \hat{\mu}_{z^+,t+k} \\
&+ \varepsilon_w \hat{\Pi}_{t+1}^C + \varepsilon_w \hat{\Pi}_{t+2}^C + \ldots + \varepsilon_w \hat{\Pi}_{t+k}^C \\
&+ \frac{\lambda^W}{(1-\lambda^W)^2} log(\frac{\overline{w}^{opt}}{\overline{w}}) \hat{\lambda}_{t+1}^W + \frac{\lambda^W}{(1-\lambda^W)^2} log \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C} \right)^{-2} \hat{\lambda}_{t+2}^W \ldots + \frac{\lambda^W}{(1-\lambda^W)^2} log \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C} \right)^{-k} \hat{\lambda}_{t+k}^W
\end{aligned} \tag{G.27}$$

Equation (G.27) is the log-linearized version of labor demand equation.

G.4 Optimal wage equation

In this section, we derive the log-linearized version of the optimal wage equation, Equation (A.13b). The first step is to log-linearize Equation (C.63), which captures the nonlinear version of the optimal wage condition. In the second step, we use the following key equations: Equation (G.7), Equation (G.21) and Equation (G.27) to obtain the final log-linearized version of the optimal wage condition, Equation (A.13b).

Recall, we have the nonlinear version of the optimal wage equation:

$$E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^{k} \beta_{t+i}^r \right) n_{t+k|t} \overline{\Omega}_{t+k}^C \frac{1}{1-\lambda_{t+k}^W} \left[\left(1 - \tau_{t+k}^W\right) \overline{w}_{t+k|t} - \lambda_{t+k}^W \zeta_{t+k}^n \frac{\nu'(n_{t+k|t})}{\overline{\Omega}_{t+k}^C} \right] = 0 \tag{G.28}$$

Equation (G.28) is the same as Equation (C.63) which shows the stationarized version of the optimal wage setting equation.

We rewrite Equation (G.28) as follows:

$$E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^{k} \beta_{t+i}^r \right) n_{t+k|t} \overline{\Omega}_{t+k}^C \frac{1}{1-\lambda_{t+k}^W} \left[\left(1 - \tau_{t+k}^W\right) \overline{w}_{t+k|t} \right] = E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^{k} \beta_{t+i}^r \right) n_{t+k|t} \overline{\Omega}_{t+k}^C \frac{\lambda_{t+k}^W}{1-\lambda_{t+k}^W} \zeta_{t+k}^n \frac{\nu'(n_{t+k|t})}{\overline{\Omega}_{t+k}^C}. \tag{G.29}$$

We expand Equation (G.29), and we can obtain the following equation:

$$\begin{aligned}
&n_{t|t} \overline{\Omega}_t^C \frac{1}{1-\lambda_t^W} \left(1 - \tau_t^W\right) \overline{w}_t^{opt} + E_t \left[\xi_w \beta_{t+1}^r n_{t+1|t} \overline{\Omega}_{t+1}^C \frac{1}{1-\lambda_{t+1}^W} \left(1 - \tau_{t+1}^W\right) \overline{w}_{t+1|t} \right] \\
&+ E_t \left[(\xi_w)^2 \beta_{t+1}^r \beta_{t+2}^r n_{t+2|t} \overline{\Omega}_{t+2}^C \frac{1}{1-\lambda_{t+2}^W} \left(1 - \tau_{t+2}^W\right) \overline{w}_{t+2|t} + \ldots \right] \\
&= \\
&n_{t|t} \overline{\Omega}_t^C \frac{\lambda_t^W}{1-\lambda_t^W} \zeta_t^n \frac{\nu'(n_{t|t})}{\overline{\Omega}_t^C} \\
&+ E_t \left[\xi_w \beta_{t+1}^r n_{t+1|t} \overline{\Omega}_{t+1}^C \frac{\lambda_{t+1}^W}{1-\lambda_{t+1}^W} \zeta_{t+1}^n \frac{\nu'(n_{t+1|t})}{\overline{\Omega}_{t+1}^C} \right] \\
&+ E_t \left[(\xi_w)^2 \beta_{t+1}^r \beta_{t+2}^r n_{t+2|t} \overline{\Omega}_{t+2}^C \frac{\lambda_{t+2}^W}{1-\lambda_{t+2}^W} \zeta_{t+2}^n \frac{\nu'(n_{t+2|t})}{\overline{\Omega}_{t+2}^C} + \ldots \right].
\end{aligned} \tag{G.30}$$

Recall from Section G.3, we have the following definition:

$$\overline{w}_{t+k|t} = \frac{\overline{w}_t^{opt} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W \ldots \overline{\Pi}_{t+k}^W}{\mu_{z^+,t+1} \mu_{z^+,t+2} \ldots \mu_{z^+,t+k} \Pi_{t+1}^C \Pi_{t+2}^C \ldots \Pi_{t+k}^C}. \tag{G.31}$$

Recall from Section G.3, along the balanced growth path, the definition of wage inflation is written as:

$$\overline{\Pi}^W = \mu_{z^+} \Pi^C. \tag{G.32}$$

Along the balanced growth path, Equation (G.31) can be expressed as:

$$\overline{w} = \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C} \right)^k \overline{w}^{opt}. \tag{G.33}$$

When log-linearizing Equation (G.30), we take account of Equation (G.31), (G.32) and (G.33). We also let $H_1 = \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\overline{w}$. Note that variables such as $n_{t+k|t}$, β_t^r, and $\overline{\Omega}_{t+k}^C$ are that common to both sides of Equation (G.30) will cancel out after we have log-linearized. We therefore ignore them when log-linearizing, but we still have to log-linearize $\frac{1}{\Omega_{t+k}^C}$ as this particular variable only appears in the RHS of Equation (G.30). Note that in equilibrium, we have $\beta^r = \beta$ and define $\breve{\tau}_t^W$ as $\tau_t^W - \tau^W$.

Now, we log-linearize the LHS of Equation (G.30) and we can obtain the following equation:

$$\begin{aligned}
&\ln H_1 + \frac{1}{H_1} n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\overline{w}\hat{w}_t^{opt} \\
&+ \frac{1}{H_1}\ \xi_w \beta n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\hat{w}_t^{opt} \\
&+ \frac{1}{H_1}\ \left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{w}_t^{opt} + \ldots \\
&+ \frac{1}{H_1}\ \xi_w \beta n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\hat{\overline{\Pi}}_t^W \\
&+ \frac{1}{H_1}\ \left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}E\left[\hat{\overline{\Pi}}_{t+1}^W\right] + \ldots \\
&+ \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\overline{\Pi}}_{t+2}^W + \ldots\right] \\
&- \frac{1}{H_1}\ E_t\left[\xi_w \beta n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\hat{\mu}_{z^+,t+1}\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\mu}_{z^+,t+1} - \ldots\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\mu}_{z^+,t+2} - \ldots\right] \\
&- \frac{1}{H_1}\ E_t\left[\xi_w \beta n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\hat{\Pi}_{t+1}^C\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\Pi}_{t+1}^C - \ldots\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\Pi}_{t+2}^C - \ldots\right] \\
&- \frac{1}{H_1}\ n\overline{\Omega}^C \frac{1}{1-\lambda^W}\overline{w}\breve{\tau}_t^W - \frac{1}{H_1}E_t\left[\xi_w \beta n\overline{\Omega}^C\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\breve{\tau}_{t+1}^W\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\breve{\tau}_{t+2}^W - \ldots\right] \\
&+ \frac{1}{H_1}\ n\overline{\Omega}^C \frac{1}{(1-\lambda^W)^2}\overline{w}\left(1-\tau^W\right)\lambda^W\hat{\lambda}_t^W + \frac{1}{H_1}E_t\left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{(1-\lambda^W)^2}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\bar{w}\hat{\lambda}_{t+1}^W\right] \\
&+ \frac{1}{H_1}E_t\left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{(1-\lambda^W)^2}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\bar{w}\hat{\lambda}_{t+2}^W - \ldots\right].
\end{aligned} \tag{G.34}$$

We let $H_2 = \frac{\lambda^W}{1-\lambda^W}\zeta^n\frac{\nu'(n)}{\overline{\Omega}^C}$. Also recall Equation G.4 below:

$$\nu'(n_t) = \Theta_t^n A_n n_t^\eta, \tag{G.35}$$

and denote first derivative of $\nu'(n_t)$ with respect to Θ_t^n as $\nu'_{\Theta^n}(n_t)$, which is $\nu'_{\Theta^n}(n_t) = \frac{\partial\nu'(n_t)}{\partial\Theta_t^n} = A_n n_t^\eta = \frac{\nu'(n_t)}{\Theta_t^n}$. In steady state $\nu'_{\Theta^n}(n) = \frac{\nu'(n)}{\Theta^n}$ and we use it below while we log-linearize the RHS of Equation (G.30), and we have the following equation:

$$
\begin{aligned}
&\ln H_2 + \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\frac{\nu'(n)}{\overline{\Omega}^C}\zeta^n\hat{\zeta}^n_t \\
&+ \frac{1}{H_2}\ E_t\left[\xi_w\beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\frac{\nu'(n)}{\overline{\Omega}^C}\zeta^n\hat{\zeta}^n_{t+1}\right] \\
&+ \frac{1}{H_2}\ E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\frac{\nu'(n)}{\overline{\Omega}^C}\zeta^n\hat{\zeta}^n_{t+2} + \dots\right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\frac{1}{\overline{\Omega}^C}\nu^{''}(n)n\hat{n}_{t|t} \\
&+ \frac{1}{H_2}\ E_t\left[\xi_w\beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\frac{1}{\overline{\Omega}^C}\nu^{''}(n)n\hat{n}_{t+1|t}\right] \\
&+ \frac{1}{H_2}\ E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\frac{1}{\overline{\Omega}^C}\nu^{''}(n)n\hat{n}_{t+2|t} + \dots\right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\frac{1}{\overline{\Omega}^C}\frac{\nu'(n)}{\Theta^n}\Theta^n\hat{\Theta}^n_t \\
&+ \frac{1}{H_2}\ E_t\left[\xi_w\beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\frac{1}{\overline{\Omega}^C}\frac{\nu'(n)}{\Theta^n}\Theta^n\Theta^{\hat{n}}_{t+1}\right] \\
&+ \frac{1}{H_2}\ E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\frac{1}{\overline{\Omega}^C}\frac{\nu'(n)}{\Theta^n}\Theta^n\Theta^{\hat{n}}_{t+2} + \dots\right] \\
&- \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\nu'(n)\left(\frac{1}{\overline{\Omega}^C}\right)^2\overline{\Omega}^C\hat{\Omega}^C_t \\
&- \frac{1}{H_2}\ E_t\left[\xi_w\beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\nu'(n)\left(\frac{1}{\overline{\Omega}^C}\right)^2\overline{\Omega}^C\hat{\Omega}^C_{t+1}\right] \\
&- \frac{1}{H_2}\ E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W}\zeta^n\nu'(n)\left(\frac{1}{\overline{\Omega}^C}\right)^2\overline{\Omega}^C\hat{\Omega}^C_{t+2} + \dots\right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C\zeta^n\frac{\nu'(n)}{\overline{\Omega}^C}\frac{1}{(1-\lambda^W)^2}\lambda^W\hat{\lambda}^W_t \\
&+ \frac{1}{H_2}\ E_t\left[\xi_w\beta n\overline{\Omega}^C\zeta^n\frac{\nu'(n)}{\overline{\Omega}^C}\frac{\lambda^W}{(1-\lambda^W)^2}\hat{\lambda}^W_{t+1}\right] \\
&+ \frac{1}{H_2}\ E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C\zeta^n\frac{\nu'(n)}{\overline{\Omega}^C}\frac{\lambda^W}{(1-\lambda^W)^2}\hat{\lambda}^W_{t+2} + \dots\right].
\end{aligned}
\tag{G.36}
$$

Note that in steady state, Equation (G.5) is: $\nu''(n) = \eta\Theta^n A_n n^{\eta-1}$. We use the following results: $\nu''(n) = \eta\Theta^n A_n n^{\eta-1}$ and $\nu'(n) = \Theta^n A_n n^\eta$. Thus, $\nu''(n)n$ can be defined as:

$$
\nu''(n)n = \eta\Theta^n A_n n^\eta = \eta\nu'(n). \tag{G.37}
$$

Using Equation (G.37), Equation (G.36) can be rewritten as follows:

$$
\begin{aligned}
&\ln H_2 + \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}_t^n \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}_{t+1}^n \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2\, n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}_{t+2}^n + \ldots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \eta \nu'(n) \hat{n}_{t|t} \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \eta \nu'(n) \hat{n}_{t+1|t} \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2\, n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \eta \nu'(n) \hat{n}_{t+2|t} + \ldots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \hat{\Theta^n}_{t|t} \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \hat{\Theta^n}_{t+1|t} \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2\, n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \hat{\Theta^n}_{t+2|t} + \ldots \right] \\
&- \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}_t^C \\
&- \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}_{t+1}^C \right] \\
&- \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2\, n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}_{t+2}^C + \ldots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{1}{(1-\lambda^W)^2} \lambda^W \hat{\lambda}_t^W \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{\lambda^W}{(1-\lambda^W)^2} \hat{\lambda}_{t+1}^W \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2\, n\overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{\lambda^W}{(1-\lambda^W)^2} \hat{\lambda}_{t+2}^W + \ldots \right].
\end{aligned}
\tag{G.38}
$$

We use the following result: $H_1 = H_2$, and multiply the terms with $\check{\tau}_{t+k}^W$ by $\frac{(1-\tau^W)}{(1-\tau^W)}$. We then combine Equation

(G.34) and (G.38). Thus, we have the following equation:

$$
\begin{aligned}
& n\overline{\Omega}^C \left(1-\tau^W\right) \overline{w}\hat{w}_t^{opt} \\
& +\xi_w \beta n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right) \overline{w}\hat{w}_t^{opt} \\
& +\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \overline{w}\hat{w}_t^{opt} + \ldots \\
& +E_t \left[\xi_w \beta n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right) \overline{w}\hat{\overline{\Pi}}_{t+1}^W\right] \\
& +E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \overline{w}\hat{\overline{\Pi}}_{t+1}^W\right] + \ldots \\
& +E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \overline{w}\hat{\overline{\Pi}}_{t+2}^W + \ldots\right] \\
& -E_t \left[\xi_w \beta n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right) \overline{w}\hat{\mu}_{z^+,t+1}\right] \\
& -E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \overline{w}\hat{\mu}_{z^+,t+1} - \ldots\right] \\
& -E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \overline{w}\hat{\mu}_{z^+,t+2} - \ldots\right] \\
& -E_t \left[\xi_w \beta n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right) \overline{w}\hat{\Pi}_{t+1}^C\right] \\
& -E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \overline{w}\hat{\Pi}_{t+1}^C - \ldots\right] \\
& -E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \overline{w}\hat{\Pi}_{t+2}^C - \ldots\right] \\
& -n\overline{\Omega}^C \left(1-\tau^W\right) \overline{w} \frac{1}{\left(1-\tau^W\right)} \breve{\tau}_t^W \\
& -E_t \left[\xi_w \beta n\overline{\Omega}^C \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right) \left(1-\tau^W\right) \overline{w} \frac{1}{\left(1-\tau^W\right)} \breve{\tau}_{t+1}^W\right] \\
& -E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \left(1-\tau^W\right) \overline{w} \frac{1}{\left(1-\tau^W\right)} \breve{\tau}_{t+2}^W - \ldots\right] \\
& +n\overline{\Omega}^C \frac{1}{\left(1-\lambda^W\right)} \overline{w} \left(1-\tau^W\right) \lambda^W \hat{\lambda}_t^W \\
& +E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{\left(1-\lambda^W\right)} \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right) \bar{w}\hat{\lambda}_{t+1}^W\right] \\
& +E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega} \frac{\lambda^W}{\left(1-\lambda^W\right)} \left(1-\tau^W\right) \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^2 \bar{w}\hat{\lambda}_{t+2}^W + \ldots\right]. \\
= \; & n\overline{\Omega}^C \hat{\zeta}_t^n \\
& +E_t \left[\xi_w \beta n\overline{\Omega}^C \hat{\zeta}_{t+1}^n\right] \\
& +E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \hat{\zeta}_{t+2}^n + \ldots\right] \\
& +n\overline{\Omega}^C \eta\hat{n}_{t|t} \\
& +E_t \left[\xi_w \beta n\overline{\Omega}^C \eta\hat{n}_{t+1|t}\right] \\
& +E_t \left[\left(\xi_w \beta\right)^2 n\overline{\Omega}^C \eta\hat{n}_{t+2|t} + \ldots\right]
\end{aligned} \tag{G.39}
$$

$$
\begin{aligned}
&+\ n\overline{\Omega}^C \hat{\Theta}^n{}_{t|t} \\
&+\ E_t\left[\xi_w \beta n \overline{\Omega}^C \hat{\Theta}^n{}_{t+1|t}\right] \\
&+\ E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C \hat{\Theta}^n{}_{t+2|t} + \ldots\right] \\
&- n\overline{\Omega}^C \hat{\Omega}^C_t \\
&- E_t\left[\xi_w \beta n \overline{\Omega}^C \hat{\Omega}^C_{t+1}\right] \\
&- E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C \hat{\Omega}^C_{t+2} + \ldots\right] \\
&+ n\overline{\Omega}^C \frac{1}{(1-\lambda^W)} \hat{\lambda}^W_t \\
&+ E_t\left[\xi_w\beta n\overline{\Omega}^C \frac{1}{(1-\lambda^W)} \hat{\lambda}^W_{t+1}\right] \\
&+ E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C \frac{1}{(1-\lambda^W)} \hat{\lambda}^W_{t+2} + \ldots\right].
\end{aligned}
$$

We have the following summation formula for an infinite geometric series:

$$
b + bz + bz^2 + \ldots + bz^{n-1} + \ldots = \frac{b}{1-z}.
$$

We assume that $|z| < 1$.
We make use of following definitions $(1-\tau^W)\overline{w} = \lambda^W \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C}$ and $\frac{\overline{\Pi}^W}{\mu_{z^+,t}\Pi^C} = 1$. We also gather all $\hat{\lambda}^W_{t+k}$ terms on the right hand side by using the following for all $t+k$:

$$
(\xi_w\beta)^k\, n\overline{\Omega}^C \frac{\nu'(n)}{\overline{\Omega}^C} \frac{\lambda^W}{(1-\lambda^W)} \hat{\lambda}^W_{t+k} - (\xi_w\beta)^k\, n\overline{\Omega} \frac{\lambda^W}{(1-\lambda^W)} \left(1-\tau^W\right) \overline{w} \hat{\lambda}^W_{t+k} = (1-\lambda^W)\,(\xi_w\beta)^k\, n\overline{\Omega}^C \frac{\nu'(n)}{\overline{\Omega}^C} \frac{\lambda^W}{(1-\lambda^W)} \hat{\lambda}^W_{t+k}
$$

We apply the summation formula for an infinite geometric series to Equation (G.39), and then we simplify Equation (G.39). This gives us the following equation:

$$
\begin{aligned}
&\frac{\hat{w}^{opt}_t}{(1-\xi_w\beta)} + \frac{\xi_w\beta}{(1-\xi_w\beta)} \hat{\overline{\Pi}}^W_{t+1} + \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)} E_t \hat{\overline{\Pi}}^W_{t+2} + \ldots \\
&- \frac{\xi_w\beta}{(1-\xi_w\beta)} E_t \hat{\mu}_{z^+,t+1} - \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)} E_t \hat{\mu}_{z^+,t+2} - \ldots \\
&- \frac{\xi_w\beta}{(1-\xi_w\beta)} E_t \hat{\Pi}^C_{t+1} - \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)} E_t \hat{\Pi}^C_{t+2} - \ldots \\
&- \frac{1}{(1-\tau^W)} \breve{\tau}^W_t - \xi_w\beta \frac{1}{(1-\tau^W)} E_t \breve{\tau}^W_{t+1} - (\xi_w\beta)^2 \frac{1}{(1-\tau^W)} E_t \breve{\tau}^W_{t+2} - \ldots \\
&= E_t \sum_{k=0}^{\infty} (\xi_w\beta)^k \left[\hat{\zeta}^n_{t+k} + \eta \hat{n}_{t+k|t} + \hat{\Theta}^n_{t+k} - \hat{\Omega}^C_{t+k} + \hat{\lambda}^W_{t+k}\right].
\end{aligned}
\tag{G.40}
$$

Substituting the labor demand equation (G.27) into Equation (G.40), Equation (G.40) becomes

$$
\begin{aligned}
&\frac{\hat{w}_t^{opt}}{(1-\xi_w\beta)} + \frac{\xi_w\beta}{(1-\xi_w\beta)} E_t\hat{\overline{\Pi}}_{t+1}^W + \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)} E_t\hat{\overline{\Pi}}_{t+2}^W + \ldots \\
&- \frac{\xi_w\beta}{(1-\xi_w\beta)} E_t\hat{\mu}_{z^+,t+1} - \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)} E_t\hat{\mu}_{z^+,t+2} - \ldots \\
&- \frac{\xi_w\beta}{(1-\xi_w\beta)} E_t\hat{\Pi}_{t+1}^C - \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)} E_t\hat{\Pi}_{t+2}^C - \ldots \\
&= E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}_{t+k}^n + \hat{\Theta}_{t+k}^n - \hat{\Omega}_{t+k}^C + \hat{\lambda}_{t+k}^W + \frac{1}{(1-\tau^W)}\breve{\tau}_{t+k}^W\right] \\
&+ \eta\left[\varepsilon_w\hat{w}_t + \hat{n}_t - \varepsilon_w\hat{w}_t^{opt}\right] \\
&+ (\xi_w\beta)\eta E_t\left[\varepsilon_w\hat{w}_{t+1} + \hat{n}_{t+1} - \varepsilon_w\hat{w}_t^{opt} - \varepsilon_w\hat{\overline{\Pi}}_{t+1}^W + \varepsilon_w\hat{\mu}_{z^+,t+1} + \varepsilon_w\hat{\Pi}_{t+1}^C\right] \\
&+ (\xi_w\beta)^2\eta E_t\left[\varepsilon_w\hat{w}_{t+2} + \hat{n}_{t+2} - \varepsilon_w\hat{w}_t^{opt} - \varepsilon_w\hat{\overline{\Pi}}_{t+1}^W - \varepsilon_w\hat{\overline{\Pi}}_{t+2}^W\right] \\
&+ (\xi_w\beta)^2\eta E_t\left[\varepsilon_w\hat{\mu}_{z^+,t+1} + \varepsilon_w\hat{\mu}_{z^+,t+2} + \varepsilon_w\hat{\Pi}_{t+1}^C + \varepsilon_w\hat{\Pi}_{t+2}^C\right] + \ldots \\
&+ (\xi_w\beta)\frac{\lambda^W}{(1-\lambda^W)^2}log(\frac{\bar{w}^{opt}}{\bar{w}})\hat{\lambda}_{t+1}^W + (\xi_w\beta)^2\frac{\lambda^W}{(1-\lambda^W)^2}log\left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^{-2}\hat{\lambda}_{t+2}^W\ldots + (\xi_w\beta)^k\frac{\lambda^W}{(1-\lambda^W)^2}log\left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^{-k}\hat{\lambda}_{t+k}^W.
\end{aligned}
$$

Since the all the terms of last row of the above equation cancels out in the steps below, we continue without that part of the equation. With this simplification, the above equation can be written as:

$$
\begin{aligned}
&\frac{\hat{w}_t^{opt}}{(1-\xi_w\beta)} + \frac{\xi_w\beta}{(1-\xi_w\beta)} E_t\left[\hat{\overline{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right] \\
&+ \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)} E_t\left[\hat{\overline{\Pi}}_{t+2}^W - \hat{\mu}_{z^+,t+2} - \hat{\Pi}_{t+2}^C\right] + \ldots \\
&= E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}_{t+k}^n + \hat{\Theta}_{t+k}^n - \hat{\Omega}_{t+k}^C + \hat{\lambda}_{t+k}^W + \frac{1}{(1-\tau^W)}\breve{\tau}_{t+k}^W\right] \\
&+ E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\,\eta\;E_t\left[\varepsilon_w\hat{w}_{t+k} + \hat{n}_{t+k} - \varepsilon_w\hat{w}_t^{opt}\right] \\
&- (\xi_w\beta)\,\eta\varepsilon_w\;E_t\left[\hat{\overline{\Pi}}_{t+1}^W + (\xi_w\beta)\hat{\overline{\Pi}}_{t+1}^W + (\xi_w\beta)^2\hat{\overline{\Pi}}_{t+1}^W + \ldots\right] \\
&+ (\xi_w\beta)\,\eta\varepsilon_w\;E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C\right] \\
&+ (\xi_w\beta)\,\eta\varepsilon_w\;E_t\left[(\xi_w\beta)\left(\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C\right) + (\xi_w\beta)^2\left(\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C\right) + \ldots\right] \\
&- (\xi_w\beta)^2\,\eta\varepsilon_w\;E_t\left[\hat{\overline{\Pi}}_{t+2}^W + (\xi_w\beta)\hat{\overline{\Pi}}_{t+2}^W + (\xi_w\beta)^2\hat{\overline{\Pi}}_{t+2}^W + \ldots\right] \\
&+ (\xi_w\beta)^2\,\eta\varepsilon_w\;E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}_{t+2}^C\right] \\
&+ (\xi_w\beta)^2\,\eta\varepsilon_w\;E_t\left[(\xi_w\beta)\left(\hat{\mu}_{z^+,t+2} + \hat{\Pi}_{t+2}^C\right) + (\xi_w\beta)^2\left(\hat{\mu}_{z^+,t+2} + \hat{\Pi}_{t+2}^C\right) + \ldots\right] \\
&+ \ldots .
\end{aligned}
\tag{G.41}
$$

We apply the above summation formula for an infinite geometric series to Equation (G.41), and we have the following equation:

$$
\begin{aligned}
&\frac{\hat{w}_t^{opt}}{(1-\xi_w\beta)} + \frac{\xi_w\beta}{(1-\xi_w\beta)}E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right] \\
&+ \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}E_t\left[\hat{\bar{\Pi}}_{t+2}^W - \hat{\mu}_{z^+,t+2} - \hat{\Pi}_{t+2}^C\right] + \ldots \\
&= \\
&E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}_{t+k}^n + \hat{\Theta}_{t+k}^n - \hat{\Omega}_{t+k}^C + \hat{\lambda}_{t+k}^W + \frac{1}{(1-\tau^W)}\check{\tau}_{t+k}^W + \eta\varepsilon_w\hat{w}_{t+k} + \eta\hat{n}_{t+k}\right] \\
&- \eta\varepsilon_w\frac{\hat{w}_t^{opt}}{(1-\xi_w\beta)} - \frac{\xi_w\beta}{(1-\xi_w\beta)}\eta\varepsilon_w E_t\left[\hat{\bar{\Pi}}_{t+1}^W\right] \\
&- \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}\eta\varepsilon_w E_t\left[\hat{\bar{\Pi}}_{t+2}^W\right] - \ldots \\
&+ \frac{\xi_w\beta}{(1-\xi_w\beta)}\eta\varepsilon_w E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C\right] + \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}\eta\varepsilon_w E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}_{t+2}^C\right] + \ldots
\end{aligned}
\tag{G.42}
$$

We multiply both sides of Equation (G.42) by $(1-\xi_w\beta)$ and rearrange the equation. Hence, Equation (G.42) can be rewritten as:

$$
\begin{aligned}
&(1+\eta\varepsilon_w)\hat{w}_t^{opt} = \\
&(1-\xi_w\beta)E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}_{t+k}^n + \hat{\Theta}_{t+k}^n - \hat{\Omega}_{t+k}^C + \hat{\lambda}_{t+k}^W + \frac{1}{(1-\tau^W)}\check{\tau}_{t+k}^W + \eta\varepsilon_w\hat{w}_{t+k} + \eta\hat{n}_{t+k}\right] \\
&-(1+\eta\varepsilon_w)(\xi_w\beta)\hat{\bar{\Pi}}_{t+1}^W - (1+\eta\varepsilon_w)(\xi_w\beta)^2E_t\left[\hat{\bar{\Pi}}_{t+2}^W\right] - \ldots \\
&+(1+\eta\varepsilon_w)(\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C\right] + (1+\eta\varepsilon_w)(\xi_w\beta)^2E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}_{t+2}^C\right] + \ldots
\end{aligned}
\tag{G.43}
$$

Dividing both sides of Equation (G.43) by $(1+\eta\varepsilon_w)$, we have the following equation:

$$
\begin{aligned}
&\hat{w}_t^{opt} = \\
&\frac{(1-\xi_w\beta)}{(1+\eta\varepsilon_w)}E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}_{t+k}^n + \hat{\Theta}_{t+k}^n - \hat{\Omega}_{t+k}^C + \hat{\lambda}_{t+k}^W + \frac{1}{(1-\tau^W)}\check{\tau}_{t+k}^W + \eta\varepsilon_w\hat{w}_{t+k} + \eta\hat{n}_{t+k}\right] \\
&-(\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W\right] - (\xi_w\beta)^2E_t\left[\hat{\bar{\Pi}}_{t+2}^W\right] - \ldots \\
&+(\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C\right] + (\xi_w\beta)^2E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}_{t+2}^C\right] + \ldots
\end{aligned}
\tag{G.44}
$$

Now, we iterate Equation (G.44) one period forward and multiply both sides of the equation by $(\xi_w\beta)$. Thus, we have the following equation:

$$
\begin{aligned}
&(\xi_w\beta)\hat{w}_{t+1}^{opt} = \\
&\frac{(1-\xi_w\beta)}{(1+\eta\varepsilon_w)}E_t\sum_{k=1}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}_{t+k}^n + \hat{\Theta}_{t+k}^n - \hat{\Omega}_{t+k}^C + \hat{\lambda}_{t+k}^W + \frac{1}{(1-\tau^W)}\check{\tau}_{t+k}^W + \eta\varepsilon_w\hat{w}_{t+k} + \eta\hat{n}_{t+k}\right] \\
&-(\xi_w\beta)^2E_t\hat{\bar{\Pi}}_{t+2}^W - (\xi_w\beta)^3E_t\left[\hat{\bar{\Pi}}_{t+3}^W\right] - \ldots \\
&+(\xi_w\beta)^2E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}_{t+2}^C\right] + (\xi_w\beta)^3E_t\left[\hat{\mu}_{z^+,t+3} + \hat{\Pi}_{t+3}^C\right] + \ldots
\end{aligned}
\tag{G.45}
$$

Subtracting Equation (G.45) from (G.44), we have the following equation:

$$
\begin{aligned}
&\hat{w}_t^{opt} - (\xi_w\beta)E_t\hat{w}_{t+1}^{opt} = \\
&\frac{(1-\xi_w\beta)}{(1+\eta\varepsilon_w)}\left[\hat{\zeta}_t^n + \hat{\Theta}_t^n - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\check{\tau}_t^W + \eta\varepsilon_w\hat{w}_t + \eta\hat{n}_t\right] \\
&+(\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C - \hat{\bar{\Pi}}_{t+1}^W\right].
\end{aligned}
$$

The above equation can be written as:

$$\begin{aligned}
\hat{w}_t^{opt} &= (\xi_w\beta)E_t\hat{w}_{t+1}^{opt} \\
&+ \frac{(1-\xi_w\beta)}{(1+\eta\varepsilon_w)}\left[\hat{\zeta}_t^n + \hat{\Theta}_t^n - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\breve{\tau}_t^W + \eta\varepsilon_w\hat{w}_t + \eta\hat{n}_t\right] \\
&+ (\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C - \hat{\bar{\Pi}}_{t+1}^W\right].
\end{aligned} \tag{G.46}$$

Recall from Equation (G.21) in Section G.2, we have the following log-linearized version of the aggregate wage index, which is expressed as:

$$\hat{w}_t = \xi_w\left(\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C + \hat{w}_{t-1}\right) + (1-\xi_w)\hat{w}_t^{opt}.$$

We rewrite the above aggregate wage index as:

$$\hat{w}_t^{opt} = \frac{1}{(1-\xi_w)}\hat{w}_t - \frac{\xi_w}{(1-\xi_w)}\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C + \hat{w}_{t-1}\right]. \tag{G.47}$$

We iterate Equation (G.47) one period forward and multiply the equation by $(\xi_w\beta)$, and we have the following equation:

$$(\xi_w\beta)E_t\hat{w}_{t+1}^{opt} = \frac{(\xi_w\beta)}{(1-\xi_w)}E_t\hat{w}_{t+1} - \frac{(\xi_w\beta)\xi_w}{(1-\xi_w)}E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C + \hat{w}_t\right]. \tag{G.48}$$

Substituting Equation (G.47) and (G.48) into Equation (G.46), Equation (G.46) becomes:

$$\begin{aligned}
\hat{w}_t &= \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C + \hat{w}_{t-1}\right] \\
&+ \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[\hat{\zeta}_t^n + \hat{\Theta}_t^n - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\breve{\tau}_t^W + \eta\varepsilon_w\hat{w}_t + \eta\hat{n}_t\right] \\
&+ (1-\xi_w)(\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C - \hat{\bar{\Pi}}_{t+1}^W\right] + (\xi_w\beta)E_t\left[\hat{w}_{t+1}\right] \\
&- (\xi_w)^2\beta\ E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C + \hat{w}_t\right].
\end{aligned}$$

We add $\hat{w}_t - \hat{w}_t$ to the second term of the RHS of the above equation, and then we rearrange the above equation. Hence, we have the following equation:

$$\begin{aligned}
\hat{w}_t &= \xi_w\hat{w}_{t-1} + (\xi_w\beta)E_t\left[\hat{w}_{t+1}\right] - \beta(\xi_w)^2\hat{w}_t \\
&+ \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[\hat{w}_t - \hat{w}_t + \hat{\zeta}_t^n + \hat{\Theta}_t^n - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\breve{\tau}_t^W + \eta\varepsilon_w\hat{w}_t + \eta\hat{n}_t\right] \\
&+ \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] \\
&+ \left((\xi_w)^2\beta + \xi_w\beta - (\xi_w)^2\beta\right)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C - \hat{\bar{\Pi}}_{t+1}^W\right].
\end{aligned} \tag{G.49}$$

We add $\xi_w\hat{w}_t - \xi_w\hat{w}_t$ to the LHS and $\beta\xi_w\hat{w}_t - \beta\xi_w\hat{w}_t$ to the RHS of Equation (G.49). Thus, we have the following equation:

$$\begin{aligned}
\hat{w}_t + (\xi_w\hat{w}_t - \xi_w\hat{w}_t) - \xi_w\hat{w}_{t-1} &= (\xi_w\beta)E_t\left[\hat{w}_{t+1}\right] + (\beta\xi_w\hat{w}_t - \beta\xi_w\hat{w}_t) - \beta(\xi_w)^2\hat{w}_t \\
&+ \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[-\hat{w}_t + \hat{\zeta}_t^n + \eta\hat{n}_t - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\breve{\tau}_t^W + (1+\eta\varepsilon_w)\hat{w}_t\right] \\
&+ \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] - (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right].
\end{aligned}$$

Note that: $\triangle\hat{w}_t = (\hat{w}_t - \hat{w}_{t-1})$ and the above equation can be written as:

$$\begin{aligned}
\xi_w\triangle\hat{w}_t &= (\xi_w\beta)E_t\left[\triangle\hat{w}_{t+1}\right] + (1-\xi_w)(\xi_w\beta)\hat{w}_t - (1-\xi_w)\hat{w}_t \\
&- \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[\hat{w}_t - \hat{\zeta}_t^n - \hat{\Theta}_t^n - \eta\hat{n}_t + \hat{\Omega}_t^C - \hat{\lambda}_t^W - \frac{1}{(1-\tau^W)}\breve{\tau}_t^W - (1+\eta\varepsilon_w)\hat{w}_t\right] \\
&+ \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] - (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right].
\end{aligned} \tag{G.50}$$

Recall from Equation (G.7) in Section G.1, we have the following log-linearized version of the real wage markup equation:

$$\hat{\Psi}_t^W = \hat{w}_t - \frac{1}{(1-\tau^W)}\breve{\tau}_t^W - \hat{\zeta}_t^n - \eta\hat{n}_t + \hat{\Omega}_t^C.$$

Using the above real wage markup equation, Equation (G.50) can be written as:

$$\begin{aligned}\xi_w \triangle\hat{w}_t &= (\xi_w\beta)E_t\left[\triangle\hat{w}_{t+1}\right] + (1-\xi_w)(\xi_w\beta)\hat{w}_t - (1-\xi_w)\hat{w}_t \\ &- \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[\hat{\Psi}_t^W - \hat{\lambda}_t^W - (1+\eta\varepsilon_w)\hat{w}_t\right] + \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] \\ &- (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right].\end{aligned} \tag{G.51}$$

Equation (G.51) can be simplified as follows:

$$\begin{aligned}\xi_w \triangle\hat{w}_t &= (\xi_w\beta)E_t\left[\triangle\hat{w}_{t+1}\right] + (1-\xi_w)\hat{w}_t - (1-\xi_w)\hat{w}_t - \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}(\hat{\Psi}_t^W - \hat{\lambda}_t^W) \\ &+ \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] - (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right],\end{aligned}$$

$$\begin{aligned}\triangle\hat{w}_t &= \beta E_t\left[\triangle\hat{w}_{t+1}\right] - \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)\xi_w}(\hat{\Psi}_t^W - \hat{\lambda}_t^W) \\ &+ \hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C - \beta E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right].\end{aligned} \tag{G.52}$$

We let $\kappa_W = \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)\xi_w}$. Thus, Equation (G.52) can be written as:

$$\triangle\hat{w}_t = \beta E_t\left[\triangle\hat{w}_{t+1}\right] - \kappa_W(\hat{\Psi}_t^W - \hat{\lambda}_t^W) + \hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C - \beta E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right]. \tag{G.53}$$

Equation (G.53), which represents the log-linearized version of the optimal wage equation, is the same as Equation (A.13b).

H Appendix: List of variables, relative prices and definitions

H.1 List of global variables

Table 14: Global variables

Symbol	**Description**
z_t	State of labor augmenting technology
γ_t	State of investment-specific technology
$z_t^+ = z_t\left(\gamma_t\right)^{\frac{\alpha}{1-\alpha}}$	Composite state of technology
$z_{Ft}^+ = z_t\left(\gamma_t\right)^{\frac{\alpha_F}{1-\alpha_F}}$	Composite state of technology
$\mu_{z,t} = \frac{z_t}{z_{t-1}}$	Growth rate of labor augmenting technology
$\mu_{\gamma,t} = \frac{\gamma_t}{\gamma_{t-1}}$	Growth rate of investment-specific technology
$\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$	Composite technological growth rate
$\mu_{z_F^+,t} = \frac{z_{Ft}^+}{z_{Ft-1}^+}$	Composite technological growth rate

As mentioned earlier in Section 2, the long-run path for productivity is affected by two global, stochastic processes, z_t and γ_t. z_t^+ and z_{Ft}^+ may be interpreted as the compound effect of the labor augmenting technological process z_t and the investment-specific technological process γ_t.

Using the following definitions from this section: $z_t^+ = z_t \left(\gamma_t\right)^{\frac{\alpha}{1-\alpha}}$, $\mu_{\gamma,t} = \frac{\gamma_t}{\gamma_{t-1}}$ and $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, we can rewrite the definition of $\mu_{z^+,t}$ as:

$$\mu_{z^+,t} = \frac{z_t \left(\gamma_t\right)^{\frac{\alpha}{1-\alpha}}}{z_{t-1} \left(\gamma_{t-1}\right)^{\frac{\alpha}{1-\alpha}}}.$$

We rewrite the above expression, and we have the following alternative expression for the definition of $\mu_{z^+,t}$:

$$\mu_{z^+,t} = \mu_{z,t}(\mu_{\gamma,t})^{\frac{\alpha}{1-\alpha}}.$$

H.2 List of Swedish variables

In this section, we present the list of variables that are specific to the Swedish economy, with a focus on aggregate variables.

Table 15: Swedish variables

Symbol	Description
	Continues on next page
C_t^{agg}	Aggregate household consumption
C_t	Aggregate Ricardian consumption
C_t^{nr}	Aggregate Non-Ricardian consumption
G_t	Government consumption
$\widetilde{C}_t$	Aggregate composite consumption
B_{t+1}^{priv}	Domestic nominal private bonds held by Ricardian households in the Swedish economy
B_{t+1}^{FH}	Foreign nominal bonds held by Ricardian households in the Swedish economy
B_t^n	Newly issued domestic nominal government bonds held by Ricardian households in the Swedish economy
B_{t+1}	Domestic nominal government bonds held by Ricardian households in the Swedish economy
R_t^B	Average interest rate on outstanding government debt
$R_t^{B,n}$	Interest rate on newly issued government debt
Ω_t^C	Average marginal utility of consumption
β_t	Discount factor
$\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$	Discount factor ratio
R_t	Nominal gross interest rate
i_t	Nominal net interest rate
R_t^K	Nominal rental rate of capital services
$r_t^K = \frac{\gamma_t R_t^K}{P_t}$	Real rental rate of capital services
P_t^I	Price of private investment
u_t	Average rate of capital utilization
I_t	Aggregate private investment
I_t^G	Government investment
Ψ_t	Lump-sum profits to Ricardian households
P_t^K	Price of capital
K_t^s	Aggregate capital services
K_t	Aggregate capital
$\triangle_t^K$	Traded capital
$K_{G,t}$	Public capital
$\widetilde{K}_t^s$	Composite capital services
N_t	Aggregate labor labor demand
L_t	Aggregate labor force participation
un_t	Unemployment rate
Θ_t^n	Endogenous shifter of disutility or work
Z_t^n	Trend for marginal utility of consumption
W_t	Nominal wage index
W_t^{opt}	Optimal nominal wage
$\overline{w}_t = \frac{W_t}{z_t^+ P_t^C}$	Stationarized real wage
$\overline{w}_t^e = \frac{W_t}{z_t^+ P_t}$	Stationarized real wage relevant to employers
Π_t^W	Gross rate of aggregate wage inflation
$\overline{\Pi}_t^W$	Wage indexation factor
Ψ_t^W	Real wage markup
θ_t^b	Lagrange multiplier associated with the Ricardian household budget constraint
θ_t^k	Lagrange multiplier associated with the capital accumulation equation

Symbol	Description
θ_t^R	Lagrange multiplier associated with the average rate of return on government bonds
θ_t^{S}	Lagrange multiplier associated with the government bond equation
$\bar{\Omega}_t^R$	$\theta_t^{\mathrm{R}}/\theta_t^b$
$A_t = S_t B_{t+1}^{FH}$	Net foreign assets of the Swedish economy
$\overline{a}_t = \frac{A_t}{z_t^+ P_t}$	Stationarized net foreign assets of the Swedish economy
$\Lambda_{t,t+1}$	Stochastic discount factor
P_t^{opt}	Optimal price of intermediate goods
$P_{t,opt}^{M,n}$	Optimal price of imported goods of type n used as inputs in the production of final good $n \in \{C, I, X\}$
$P_t^{X,opt}$	Optimal price of export goods
P_t	Price of intermediate goods
P_t^C	Price of consumption goods
C_t^{xe}	Consumption of non-energy goods
C_t^e	Consumption of energy goods
$P_t^{C,xe}$	Price of non-energy consumption goods
$P_t^{C,e}$	Price of energy consumption goods
$P_t^{M,n}$	Price of imported goods of type n used as inputs in the production of final good $n \in \{Cxe, I, X, Ce\}$
P_t^X	Price of export goods
$P_t^{C,D,e}$	Price of domestically produced energy goods
Π_t	Gross inflation rate of intermediate goods
Π_t^{trend}	Inflation trend
Π_t^C	Gross inflation rate of consumption goods
$\Pi_t^{C,xe}$	Gross inflation rate of non-energy consumption goods
$\Pi_t^{C,e}$	Gross inflation rate of energy consumption goods
$\Pi_t^{M,n}$	Gross inflation of imported goods of type $n \in \{Cxe, I, X, Ce\}$
Π_t^X	Gross inflation rate of export goods
$\overline{\Pi}_t$	Indexation factor, intermediate good prices
$\overline{\Pi}_t^X$	Indexation factor, export good prices
$\overline{\Pi}_t^{M,n}$	Indexation factor, prices of import goods of type $n \in \{Cxe, I, X, Ce\}$
$D_t^{C,xe}$	Quantity of domestically produced intermediate goods used by consumption good producers
$M_t^{C,xe}$	Quantity of imported goods used by consumption good producers
D_t^I	Quantity of domestically produced intermediate goods used by investment good producers
M_t^I	Quantity of imported goods used by investment good producers
D_t^X	Quantity of domestically produced intermediate goods used by export good producers
M_t^X	Quantity of imported goods used by export good producers
$D_t^{C,e}$	Quantity of domestic goods used by energy good producers
$M_t^{C,e}$	Quantity of imported goods used by energy good producers
$M_t^{D,e}$	Swedish energy import goods excluding fixed costs (Total energy imports excluding fixed costs)
M_t	Swedish import goods taking into account fixed costs (Total imports with fixed costs)
M_t^D	Swedish import goods excluding fixed costs (Total imports excluding fixed costs)
M_t^e	Swedish imports of energy goods including fixed costs
X_t	Swedish exports
TC_t	Total cost of producing intermediate goods
TC_t^X	Total cost of producing export goods
MC_t	Nominal marginal cost of intermediate good firms
MC_t^X	Nominal marginal cost for export good firms
$\overline{mc}_t$	Real marginal cost for intermediate good firms

Table 15 – continued from previous page	
Symbol	**Description**
$\overline{mc}_t^X$	Real marginal cost for export good firms
MC_t^n	Nominal marginal cost of import good firms, $n \in \{\{C, xe\}, I, X, \{C, e\}\}$
$\overline{mc}_t^n$	Real marginal cost of import good firms, $n \in \{\{C, xe\}, I, X, \{C, e\}\}$
Y_t	Aggregate output
Y_t^m	Measured aggregate output
S_t	Nominal exchange rate: the Swedish currency price of a unit of Foreign currency
$s_t = \frac{S_t}{S_{t-1}}$	Rate of change in nominal exchange rate
$Q_t = \frac{S_t P_{F,t}^C}{P_t^C}$	Real exchange rate
$\overleftrightarrow{P}_t$	Intermediate good price dispersion
$\overleftrightarrow{P}_t^X$	Export price dispersion
$\overleftrightarrow{P}_t^{M,n}$	Import price dispersion of type n used as inputs in the production of final good $n \in \{\{C, xe\}, I, X, \{C, e\}\}$
λ_t	Intermediate good price markup
λ_t^X	Export price markup
$\lambda_t^{M,C,xe}$	Import price markup, import firms specializing in non-energy consumption goods
$\lambda_t^{M,I}$	Import price markup, import firms specializing in investment goods
$\lambda_t^{M,X}$	Import price markup, import firms specializing in export goods
$\lambda_t^{M,C,e}$	Import price markup, import firms specializing in energy consumption goods
ζ_t^n	Labor disutility shock
ζ_t	Private bond risk premium shock
$\widetilde{\phi}_t$	External risk premium shock (exchange rate shock)
ε_t	Productivity shock (stationary technology shock)
$\epsilon_{i,t}$	Monetary policy shock
τ_t^C	Consumption tax rate
τ_t^W	Labor income tax rate
τ_t^{SSC}	Social security contribution tax rate
τ_t^K	Capital income tax rate
τ_t^{TR}	Transfer tax rate
τ_t^I	Investment tax credit
TR_t^{agg}	Government transfers
TR_t	Government transfers to Ricardian households
TR_t^{nr}	Government transfers to Non-Ricardian households
T_t	Lump-sum tax on Ricardian households
B_t	Government debt
$SURP_t$	Government budget surplus

H.3 List of Swedish relative prices

In this section, we present the list of Swedish relative prices.

Table 16: Swedish relative prices

Symbol	Description
$p_t^{opt} = \frac{P_t^{opt}}{P_{t-1}}$	Relative optimal price of intermediate goods
$p_t^C = \frac{P_t^C}{P_t}$	Relative price of consumption goods
$p_t^{C,xe} = \frac{P_t^{C,xe}}{P_t}$	Relative price of non-energy consumption goods
$p_t^{C,e} = \frac{P_t^{C,e}}{P_t}$	Relative price of energy consumption goods
$p_t^{C,D,e} = \frac{P_t^{C,D,e}}{P_t}$	Relative price of domestic energy goods
$p_t^I = \frac{P_t^I}{P_t}$	Relative price of investment goods
$p_t^K = \frac{\gamma_t P_t^K}{P_t}$	Relative price of capital
$p_t^X = \frac{S_t P_t^X}{P_t}$	Relative price of export goods
$p_t^{X,opt} = \frac{S_t P_t^{X,opt}}{P_t}$	Relative optimal price of export goods
$p_t^{M,n} = \frac{P_t^{M,n}}{P_t}$	Relative price of import goods of type $n \in \{\{C,xe\}, I, X, \{C,e\}\}$
$p_{t,opt}^{M,n} = \frac{P_{t,opt}^{M,n}}{P_t}$	Relative optimal price of import goods of type $n \in \{\{C,xe\}, I, X, \{C,e\}\}$

Note that: the relative price of Swedish export goods in terms of Foreign intermediate goods $\frac{P_t^X}{P_{F,t}}$ can be expressed as $\frac{p_t^X p_{F,t}^C}{Q_t p_t^C}$.

H.4 List of Foreign variables

In this section, we present the list of variables that are specific to the Foreign economy, with a focus on the aggregate variables. We use the subscript F to denote the aggregate variables and the economy-wide average variables for the Foreign economy.

Table 17: Foreign variables

Symbol	Description
	Continued on next page
$C_{F,t}$	Aggregate household consumption
$C_{F,t}^{xe}$	Aggregate non-energy consumption
$C_{F,t}^{e}$	Aggregate energy consumption
B_{t+1}^{FF}	Domestic nominal bonds held by households in the Foreign economy
$\Omega_{F,t}^{C}$	Average marginal utility of consumption
$\beta_{F,t}$	Discount factor
$\beta_{F,t}^{r} = \frac{\beta_{F,t+1}}{\beta_{F,t}}$	Discount factor ratio
$R_{F,t}$	Nominal gross interest rate
$i_{F,t}$	Nominal net interest rate
$\Psi_{F,t}$	Lump-sum transfers from firms to households
$TR_{F,t}$	Lump-sum transfers from government to households
$N_{F,t}$	Aggregate labor supply
$L_{F,t}$	Aggregate labor demand
$W_{F,t}$	Aggregate nominal wage index
$\theta_{F,t}^{b}$	Lagrange multiplier associated with the household budget constraint
$\overline{w}_{F,t} = \frac{W_{F,t}}{z_t^+ P_{F,t}^C}$	Stationarized real wage
$\overline{w}_{F,t}^{e} = \frac{W_{F,t}}{z_t^+ P_{F,t}}$	Stationarized real wage relevant to employers
$\Pi_{F,t}^{W}$	Gross rate of aggregate wage inflation
$\overline{\Pi}_{F,t}^{W}$	Wage indexation factor
$\Psi_{F,t}^{W}$	Real wage markup
$\Lambda_{t,t+1}^{F}$	Stochastic discount factor
$P_{F,t}^{opt}$	Optimal price of intermediate goods
$P_{F,t}$	Price of intermediate goods
$P_{F,t}^{C}$	Price of consumption goods
$P_{F,t}^{C,xe}$	Price of non-energy consumption goods
$P_{F,t}^{C,e}$	Price of energy consumption goods
$\Pi_{F,t}$	Gross inflation rate of intermediate goods
$\Pi_{F,t}^{C}$	Gross inflation rate of consumption goods
$\Pi_{F,t}^{trend}$	Inflation trend
$\Pi_{F,t}^{C,xe}$	Gross inflation rate of non-energy consumption goods
$\Pi_{F,t}^{C,e}$	Gross inflation rate of energy consumption goods
$\overline{\Pi}_{F,t}$	Indexation factor, intermediate good prices
$TC_{F,t}$	Total cost of producing intermediate goods
$MC_{F,t}$	Nominal marginal cost for intermediate good firms
$\overline{mc}_{F,t} = \frac{MC_{F,t}}{P_{F,t}}$	Real marginal cost of intermediate good firms
$Y_{F,t}$	Aggregate output
$G_{F,t}$	Government Consumption
$\lambda_{F,t}$	Intermediate price markup
$\zeta_{F,t}$	Private bond risk premium shock
$\zeta_{F,t}^{n}$	Labor disutility shock
$\varepsilon_{F,t}$	Productivity shock
$\epsilon_{i,t}^{F}$	Monetary policy shock

H.5 List of Foreign relative prices

In this section, we present the list of Foreign relative prices.

Table 18: Foreign relative prices

Symbol	Description
Continued on next page	
$p_{F,t}^{opt} = \frac{P_{F,t}^{opt}}{P_{F,t-1}}$	Relative optimal price of intermediate goods
$p_{F,t}^{C} = \frac{P_{F,t}^{C}}{P_{F,t}}$	Relative price of consumption goods
$p_{F,t}^{C,xe} = \frac{P_{F,t}^{C,xe}}{P_{F,t}}$	Relative price of non-energy consumption goods
$p_{F,t}^{C,e} = \frac{P_{F,t}^{C,e}}{P_{F,t}}$	Relative price of energy consumption goods

I Appendix: Model parameters and functional forms

I.1 Model parameters

In this section, we present the list of parameters that are used in the model equations that are listed in Appendix A.

Table 19: Model parameters

Symbol	Description
ω	Size of Foreign economy relative to the Swedish economy
μ_z	Gross growth rate of labor augmenting technology
μ_γ	Gross growth rate of investment-specific technology
μ_{z^+}	Composite technological growth rate
β	Discount factor
β_F	Foreign discount factor
ρ_h	Consumption habit
$\rho_{F,h}$	Foreign consumption habit
α_G	Share of private consumption in the composite consumption
υ_G	Elasticity of substitution between private and public consumption
snr	Share of Non-Ricardian households over total population
ϖ_{ss}	Share of aggregate transfers going to Non-Ricardians in steady state
ϖ_{dyn}	Share of aggregate transfers going to Non-Ricardians off steady state
S''	Investment adjustment cost
χ	Indexation to previous inflation, intermediate goods
χ_F	Foreign indexation to previous inflation, intermediate goods
$\chi_{m,C,xe}$	Indexation to previous inflation, import firms specializing in non-energy consumption goods
$\chi_{m,C,e}$	Indexation to previous inflation, import firms specializing in energy consumption goods
$\chi_{m,I}$	Indexation to previous inflation, import firms specializing in investment goods
$\chi_{m,X}$	Indexation to previous inflation, import firms specializing in export goods
$\chi_{F,m}$	Foreign indexation to previous inflation, imported goods
χ_x	Indexation to previous inflation, export goods

Continued on next page

Table 19 – continued from previous page	
Symbol	**Description**
$\chi_{F,x}$	Foreign indexation to previous inflation, export goods
χ_w	Indexation to previous wage inflation
$\chi_{F,w}$	Foreign indexation to previous wage inflation
Π^C	Gross inflation target
Π_F^C	Foreign gross inflation target
$\widetilde{\phi}_a$	External risk premium parameter associated with net foreign asset
$\widetilde{\phi}_s$	External risk premium parameter associated with exchange rate
λ^W	Wage markup
λ_F^W	Foreign wage markup
λ	Intermediate good price markup
λ_F	Foreign intermediate good price markup
$\lambda^{M,C,xe}$	Import price markup, import firms specializing in non-energy consumption goods
$\lambda^{M,I}$	Import price markup, import firms specializing in investment goods
$\lambda^{M,X}$	Import price markup, import firms specializing in export goods
$\lambda^{M,C,e}$	Import price markup, import firms specializing in energy consumption goods
λ_F^M	Foreign import price markup
λ^X	Export price markup
λ_F^X	Foreign export price markup
τ_F^w	Tax on labor in Foreign
ν_C	Elasticity of substitution between non-energy and energy goods used for consumption goods production
$\nu_{C,xe}$	Elasticity of substitution between domestic and imported goods used for non-energy consumption goods production
$\nu_{C,e}$	Elasticity of substitution between domestic and imported goods used for energy consumption goods production
ν_I	Elasticity of substitution between domestic and imported goods used for investment goods production
ν_x	Elasticity of substitution between domestic and imported goods used for export goods production
$\nu_{F,C}$	Elasticity of substitution between imported and foreign consumption goods in Foreign
ν_K	Elasticity of substitution between private and public capital
α_K	Share of private capital in composite capital
ϑ^C	Weight of non-energy in the production of consumption goods
$\vartheta^{C,xe}$	Home bias in the production of non-energy consumption goods
$\vartheta^{C,e}$	Home bias in the production of energy consumption goods
ϑ^I	Home bias in the production of investment goods
ϑ^X	Home bias in the production of export goods
ϑ_F^C	Foreign home bias in the production of consumption
A_n	Labor disutility
$A_{F,n}$	Foreign labor disutility
A_F	Foreign production parameter
η	Inverse of Frisch elasticity
χ_n	Parameter associated with persistency of trend component of endogenous shifter in labor disutility
η_F	Foreign inverse of Frisch elasticity
α	Capital share in production
δ	Private capital depreciation rate
δ_G	Public capital depreciation rate
σ_a	Capital utilization cost, $\sigma_a = a''/a'$
a'	Parameter associated with capital utilization cost
a''	Parameter associated with capital utilization cost
ι^K	Indicator parameter for tax deduction of depreciation of capital
ξ	Calvo domestic prices
ξ_x	Calvo export prices
	Continued on next page

Table 19 – continued from previous page	
Symbol	**Description**
$\xi_{m,C,xe}$	Calvo import prices, import firms specializing in non-energy consumption goods
$\xi_{m,C,e}$	Calvo import prices, import firms specializing in energy consumption goods
$\xi_{m,I}$	Calvo import prices, import firms specializing in investment goods
$\xi_{m,X}$	Calvo import prices, import firms specializing in export goods
ξ_w	Calvo wages
ξ^F	Foreign Calvo domestic prices
ξ_x^F	Foreign Calvo export prices
ξ_m^F	Foreign Calvo import prices
ξ_w^F	Foreign Calvo wages
ω_C^X	Weight on consumption in investment demand
ν_F	Price elasticity of export demand
ρ	Interest rate smoothing, Taylor rule
r_π	Inflation response, Taylor rule
r_{un}	Unemployment response, Taylor rule
$r_{\triangle\pi}$	Difference in inflation response, Taylor rule
$r_{\triangle un}$	Difference in unemployment response, Taylor rule
ρ_F	Foreign interest rate smoothing, Taylor rule
$r_{F,\pi}$	Foreign inflation response, Taylor rule
$r_{F,y}$	Foreign output response, Taylor rule
$r_{F,\triangle\pi}$	Foreign difference in inflation response, Taylor rule
$r_{F,\triangle y}$	Foreign difference in output response, Taylor rule
ρ_ζ	Persistence, private bond risk premium shock
ρ_{μ^z}	Persistence, labor augmenting technology shock
ρ_γ	Persistence, investment-specific technology shock
ρ_{ζ^c}	Persistence, consumption shock
$\rho_{\zeta_F^c}$	Persistence, Foreign consumption shock
$\rho_{\tilde{\phi}}$	Persistence, exchange rate shock (external risk premium shock)
ρ_{ζ^n}	Persistence, labor disutility preference shock
$\rho_{p^{D,C,e}}$	Persistence, domestic energy price
$\rho_{p_F^{D,C,e}}$	Persistence, Foreign energy price
ρ_ε	Persistence, productivity shock
ρ_{ε_F}	Persistence, Foreign productivity shock
$\rho_{\zeta_F^n}$	Persistence, Foreign labor disutility preference shock
ρ_{ζ_F}	Persistence, Foreign private bond risk premium shock
ρ_{I^G}	Persistence, government investment shock
ρ_g	Persistence, government consumption shock
ρ_{τ^C}	Persistence, consumption tax shock
$\rho_{\tau^{SSC}}$	Persistence, social security contribution shock
ρ_{τ^W}	Persistence, labor income tax shock
ρ_{τ^K}	Persistence, capital income tax shock
ρ_{τ^I}	Persistence, investment tax credit shock
$\rho_{\tau^{TR}}$	Persistence, transfer tax shock
$\rho_{tr^{agg}}$	Persistence, aggregate transfer shock
ρ_{1,b^T}	Persistence, debt target shock AR(1)
ρ_{2,b^T}	Persistence, debt target shock AR(2)
ρ_{λ^W}	Persistence, wage markup shock to intermediate good producers
ρ_λ	Persistence, markup shock to intermediate good producers
ρ_{λ_F}	Persistence, markup shock to Foreign intermediate good producers
$\rho_{\lambda^{M,C}}$	Persistence, markup shock to import firms specializing in consumption goods
$\rho_{\lambda^{M,I}}$	Persistence, markup shock to import firms specializing in investment goods
$\rho_{\lambda^{M,X}}$	Persistence, markup shock to import firms specializing in export goods
ρ_{λ^X}	Persistence, markup shock to exporting good firms
$\rho_{\Pi^{C,trend}}$	Persistence, inflation trend shock
	Continued on next page

Table 19 – continued from previous page	
Symbol	**Description**
$corr_{\zeta}$	Parameter governing correlation, Swedish risk premium
$corr_{\Upsilon}$	Parameter governing correlation, Swedish investment efficiency
$corr_{\zeta^c}$	Parameter governing correlation, Swedish consumption shock
$corr_{\zeta_F^c,\Upsilon_F}$	Parameter governing correlation between consumption and investment in Foreign
α_B	Probability of debt maturing in every period (i.e. average maturity)
υ^{be}	Budget elasticity
$\mathcal{F}_{tr,b}$	Debt gap coefficient in aggregate transfer policy rule
$\mathcal{F}_{tr,surp}$	Surplus gap coefficient in aggregate transfer policy rule
$\mathcal{F}_{tr,y}$	Output gap coefficient in aggregate transfer policy rule
$\mathcal{F}_{g,b}$	Debt gap coefficient in government consumption policy rule
$\mathcal{F}_{g,surp}$	Surplus gap coefficient in government consumption policy rule
$\mathcal{F}_{g,y}$	Output gap coefficient in government consumption policy rule

I.2 Auxiliary parameters

In this section, we present the list of auxiliary parameters that are used in our model equations, which are shown in Appendix A. The auxiliary model parameters are functions of structural parameters, which are calibrated and can be found in Section I.1.

Table 20: Auxiliary model parameters

Symbol	Description
$\varepsilon_w^F = \frac{\lambda_F^W}{\lambda_F^W - 1}$	Foreign wage-elasticity of labor demand
$\kappa_W = \frac{(1-\xi_w)(1-\xi_w\beta)}{\xi_w(1+\eta\varepsilon_w)}$	Slope of wage Phillips curve
$\kappa_{F,W} = \frac{\left(1-\xi_w^F\right)\left(1-\xi_w^F\beta_F\right)}{\xi_w\left(1+\eta_F\varepsilon_w^F\right)}$	Slope of Foreign wage Phillips curve
$\kappa = \frac{(1-\xi\beta)(1-\xi)}{\xi}$	Slope of Phillips curve, intermediate goods
$\kappa_F = \frac{\left(1-\xi^F\beta_F\right)\left(1-\xi^F\right)}{\xi^F}$	Slope of Foreign Phillips curve, intermediate goods
$\kappa_X = \frac{(1-\xi_x)(1-\xi_x\beta)}{\xi_x}$	Slope of Phillips curve, export goods
$\kappa_{F,X} = \frac{\left(1-\xi_x^F\right)\left(1-\xi_x^F\beta\right)}{\xi_x^F}$	Slope of Foreign Phillips curve, export goods
$\kappa_{M,C,xe} = \frac{(1-\xi_{m,C,xe})(1-\beta\xi_{m,C,xe})}{\xi_{m,C,xe}}$	Slope of Phillips curve, import firms specializing in non-energy consumption goods
$\kappa_{M,C,e} = \frac{(1-\xi_{m,C,e})(1-\beta\xi_{m,C,e})}{\xi_{m,C,e}}$	Slope of Phillips curve, import firms specializing in energy consumption goods
$\kappa_{M,I} = \frac{(1-\xi_{m,I})(1-\beta\xi_{m,I})}{\xi_{m,I}}$	Slope of Phillips curve, import firms specializing in investment goods
$\kappa_{M,X} = \frac{(1-\xi_{m,X})(1-\beta\xi_{m,X})}{\xi_{m,X}}$	Slope of Phillips curve, import firms specializing in export goods
$\kappa_{F,M} = \frac{(1-\xi_m^F)(1-\beta_F\xi_m^F)}{\xi_m^F}$	Slope of Foreign Phillips curve, imported goods
$\vartheta^{C,xe} = \frac{\left(1-\frac{\overline{m}^{D,Cxe}}{\overline{c}}\right)}{\left(\frac{\overline{m}^{D,Cxe}}{\overline{c}}\left[\left(p^M\right)^{\nu_c-1}-1\right]\right)+1}$	Home bias for non-energy consumption goods
$\vartheta^{C,e} = \frac{\left(1-\frac{\overline{m}^{D,Ce}}{\overline{c}}\right)}{\left(\frac{\overline{m}^{D,Ce}}{\overline{c}}\left[\left(p^M\right)^{\nu_c-1}-1\right]\right)+1}$	Home bias for energy consumption goods
$\vartheta^{I} = \frac{\left(1-\frac{\overline{m}^{D,I}}{\overline{I}}\right)}{\left(\frac{\overline{m}^{D,I}}{\overline{I}}\left[\left(p^M\right)^{\nu_I-1}-1\right]\right)+1}$	Home bias for investment goods
$\vartheta^{X} = \frac{\left(1-\frac{\overline{m}^{D,X}}{\overline{x}}\right)}{\left(\frac{\overline{m}^{D,X}}{\overline{x}}\left[\left(p^M\right)^{\nu_x-1}-1\right]\right)+1}$	Home bias for export goods
$\psi^{C,xe} = \vartheta^{C,xe} + \frac{1}{1+\omega}(1-\vartheta^{C,xe})$	Weight of the domestically produced intermediate goods in the production of non-energy consumption goods

Symbol	Description
$\psi^{C,e} = \vartheta^{C,e} + \frac{1}{1+\omega}(1-\vartheta^{C,e})$	Weight of the domestically produced intermediate goods in the production of energy consumption goods
$\psi^{X} = \vartheta^{X} + \frac{1}{1+\omega}(1-\vartheta^{X})$	Weight of the domestically produced intermediate goods in the production of export goods
$\psi^{I} = \vartheta^{I} + \frac{1}{1+\omega}(1-\vartheta^{I})$	Weight of the domestically produced intermediate goods in the production of investment goods
$\psi_F^{C,xe} = 1 - \frac{1}{1+\omega}(1-\vartheta_F^{C,xe})$	Weight of the domestically produced intermediate goods in the production of non-energy consumption goods, Foreign
$\psi_F^{X} = 1 - \frac{1}{1+\omega}(1-\vartheta_F^{X})$	Weight of the domestically produced intermediate goods in the production of export goods, Foreign
$\phi = (\lambda - 1)\overline{y}$	Fixed cost for intermediate good producers
$\phi^X = (\lambda^X - 1)\overline{x}$	Fixed cost for export good producers
$\phi^{M,C,xe} = (\lambda^{M,C,xe} - 1)\overline{m}^{C,xe}$	Fixed cost for import firms specializing in non-energy consumption goods
$\phi^{M,C,e} = (\lambda^{M,C,xe} - 1)\overline{m}^{C,e}$	Fixed cost for import firms specializing in energy consumption goods
$\phi^{M,I} = (\lambda^{M,I} - 1)\overline{m}^{I}$	Fixed cost for import firms specializing in investment goods
$\phi^{M,X} = (\lambda^{M,X} - 1)\overline{m}^{X}$	Fixed cost for import firms specializing in export goods
$\phi^{Mxe} = \phi^{M,C} + \phi^{M,I} + \phi^{M,X}$	Total fixed cost of the imported good sector
$\phi_F = (\lambda_F - 1)\overline{y}_F$	Fixed cost for Foreign intermediate good producers
$\phi_F^X = (\lambda_F^X - 1)\overline{x}_F$	Fixed cost for Foreign export good producers
$\phi_F^M = (\lambda_F^M - 1)\overline{m}_F$	Fixed cost for Foreign import good producers
$a' = \frac{r^K}{p^I}$	Parameter associated with capital utilization cost
$a'' = a'\sigma_a$	Parameter associated with capital utilization cost
$A_n = \frac{(\overline{\Omega}^C(1-\tau^W)\overline{w})}{(\lambda^W \zeta^n \Theta n^\eta)}$	Parameter associated with labor disutility function
$\rho_{\Pi^{C,trend}}$	
$A_{F,n} = \frac{(\overline{\Omega}_F^C(1-\tau_F^W)\overline{w}_F)}{(\lambda_F^W \zeta_F^n l_F^\eta)}$	Parameter associated with Foreign labor disutility function
$H = \left(1-\tau^K\right) r^K + \left(1 - \delta + \iota^K \tau^K \delta \frac{\mu_\gamma}{\Pi}\right) p^K$	Parameter associated with household purchases of installed capital equation
$A_F = \frac{\overline{y}_F}{(1/\lambda_F l_F)}$	Parameter associated with Foreign intermediate good production function

J Appendix: Impulse Response Functions

In this section the impulse response functions (IRF) of the main variables to some selected shocks of the model are reported. Since the model has a balanced growth path, the impulse responses are measured as deviations from the balanced growth path steady state.

In the model simulations, we show the outcomes under three different specification for fiscal policy. The rule in general form is defined as:

$$\breve{tr}_t^{agg} = \rho_{tr}\breve{tr}_{t-1}^{agg} + F_{tr,b}\bar{y}\left(b_{\bar{y},t} - b_{\bar{y},t}^{Target}\right) + F_{tr,surp}\bar{y}\left(StSurp_{\bar{y},t} - StSurp_{\bar{y},t}^{Target}\right) + F_{tr,y}\bar{y}\hat{y}_t.$$

where $\bar{y}$ is the steady-state GDP, $b_{\bar{y}.t} - b_{\bar{y},t}^{Target}$ is the deviation of the government debt level as a percent of GDP from its target, $StSurp_{\bar{y}.t} - StSurp_{\bar{y},t}^{Target}$ is the deviation of the structural surplus level as a percent of GDP from its target, and $\hat{y}_t$ is the log deviation of GDP from its steady state level.

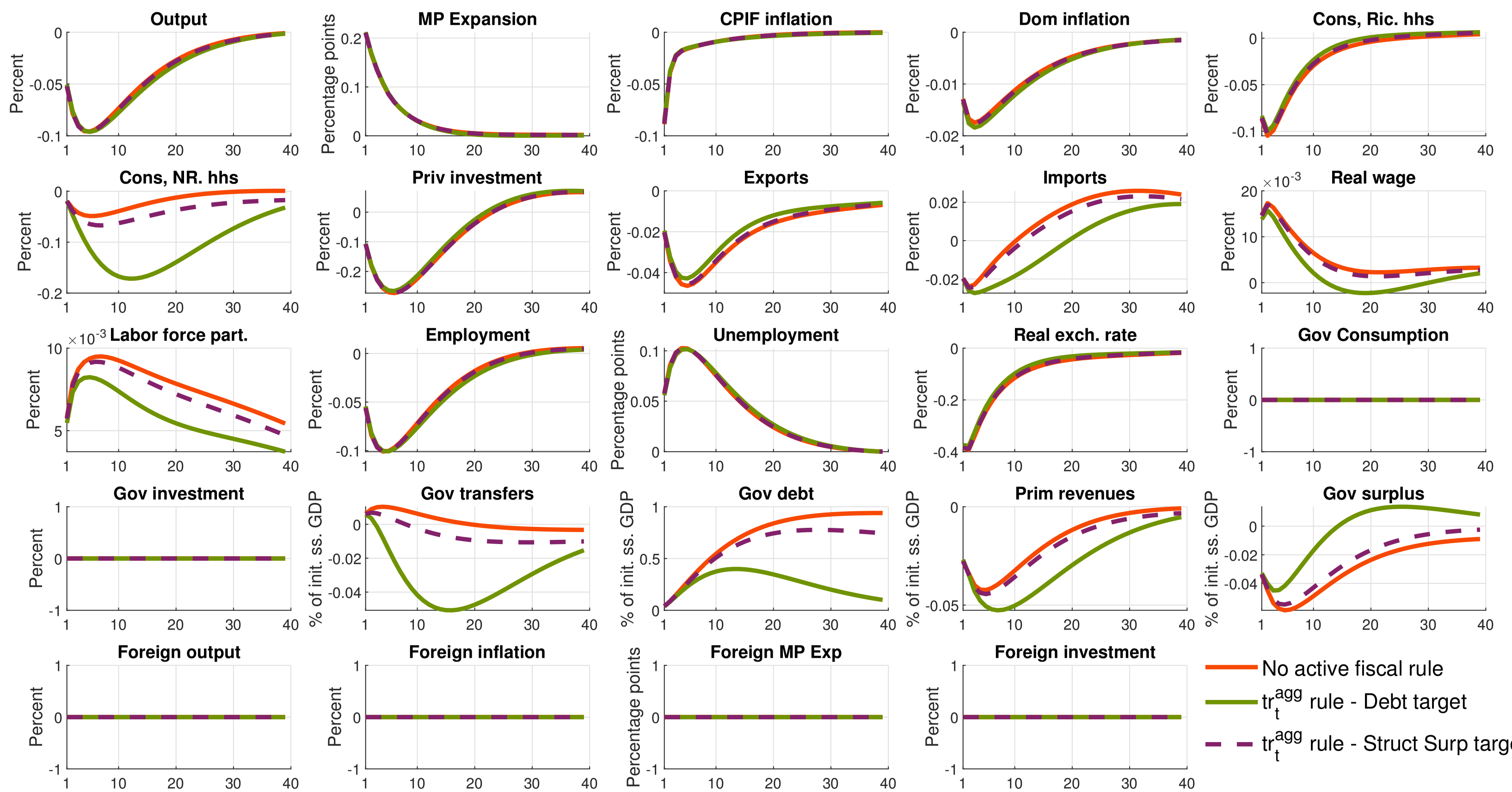
In all cases we assume that semi-automatic stabilizers are in effect and hence we set $\mathcal{F}_{tr,un} = 0.3795$ following Flodén (2009) for each fiscal rule specifations. The case *No active fiscal fule* is defined by using the transfer policy rule with $\mathcal{F}_{tr,surp} = 0$, and the response of transfers to changes in the government debt level is set to a very low number, $\mathcal{F}_{tr,b} = -0.0035$. The last part is to ensure stability of the government debt-to-GDP in the long run.

In the second case, named as *Transfer (tr_t^{agg}) rule - Debt target*, the fiscal rule is defined on aggregate transfers with the following parameterization. The structural surplus gap coefficient $\mathcal{F}_{tr,surp}$, is set to zero while the coefficient on the debt target to is set to $\mathcal{F}_{tr,b} = -0.14$. This parameter value will allow debt to go back to the target level with a reasonable pace.

In the third and the final case, named as *Transfer (tr_t^{agg}) rule - Struct Surp target*, the fiscal rule is also defined on aggregate transfers. This time aggregate transfers are determined by the feedback rule that respects the structural surplus target. Hence for this case is $\mathcal{F}_{tr,surp}$, is set to 5 so that the structural surplus goes back to its target level with a reasonable pace and $\mathcal{F}_{tr,b}$ is set to zero.

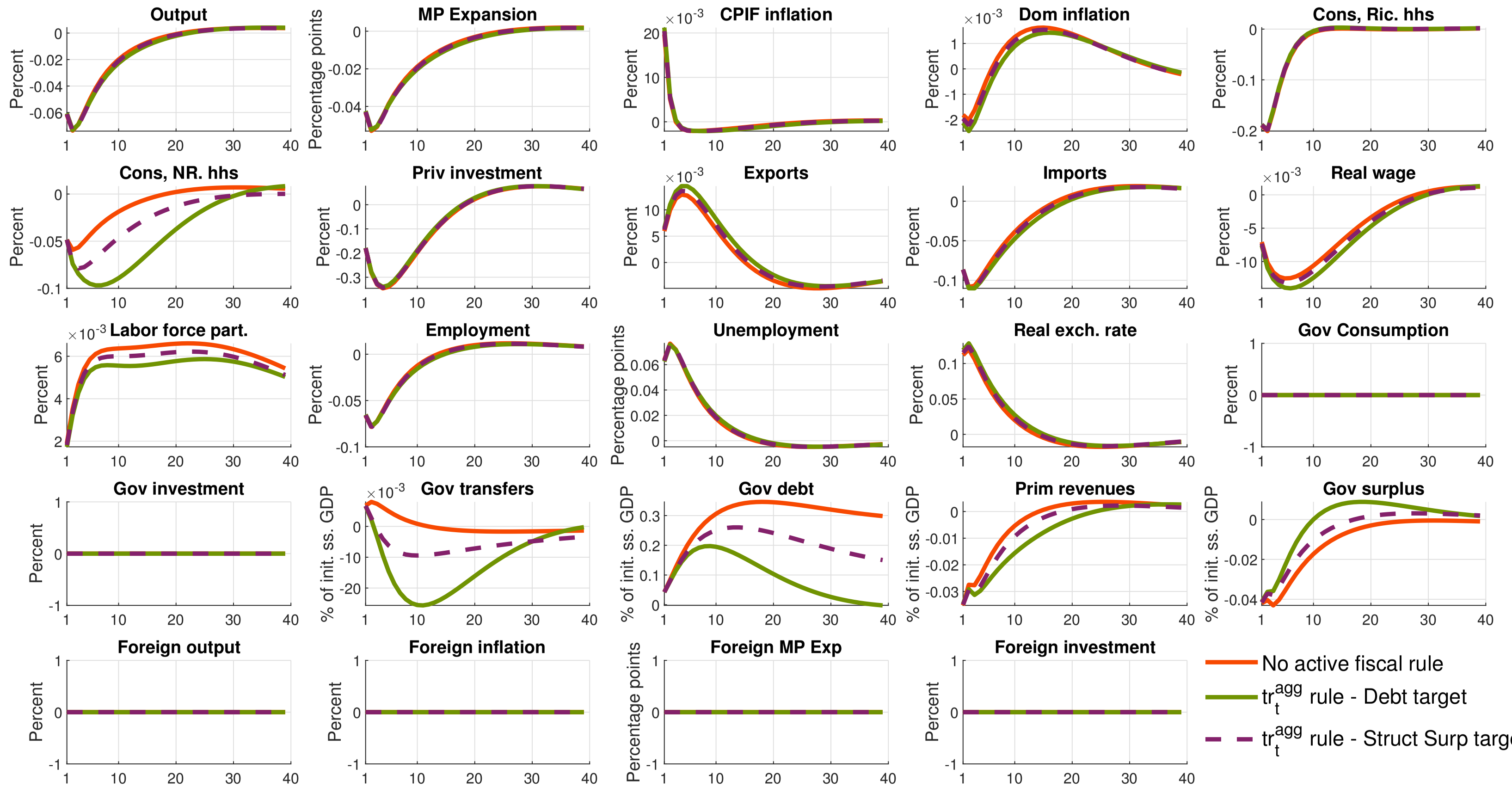
In all fiscal policy rule specifications the AR(1) component parameter ($\rho_{tr^{agg}}$) is set to 0. The IRFs under *No active fiscal fule* is illustrated by the blue solid line in the figures below, whereas the outcomes given *Transfer (tr_t^{agg}) rule - Debt target* and *Transfer (tr_t^{agg}) rule - Struct Surp target* are illustrated by the red and green dashed lines,respectively.

In the graphs below, the monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized quarter-on-quarter values.



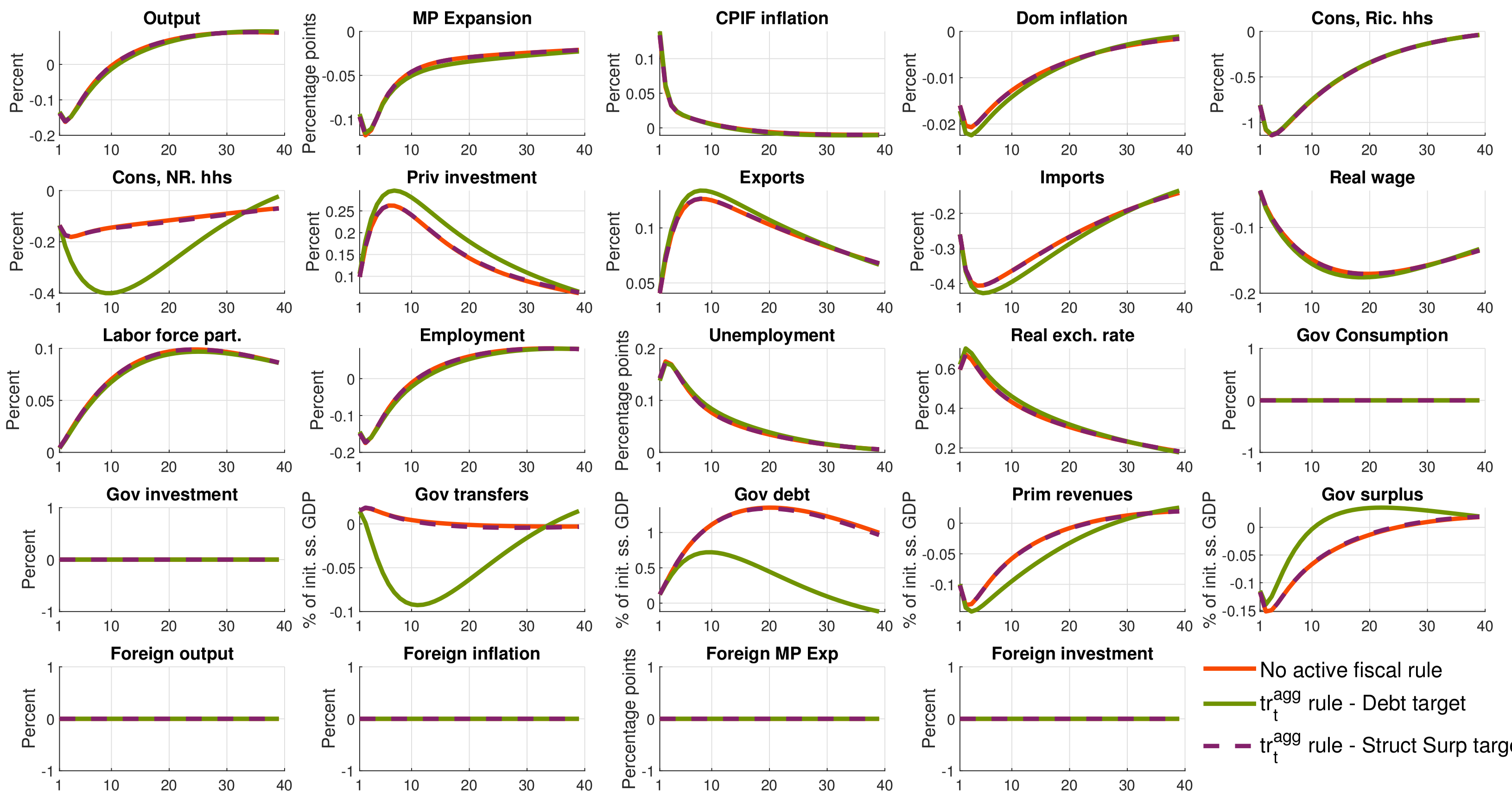
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 15: Monetary Policy Shock



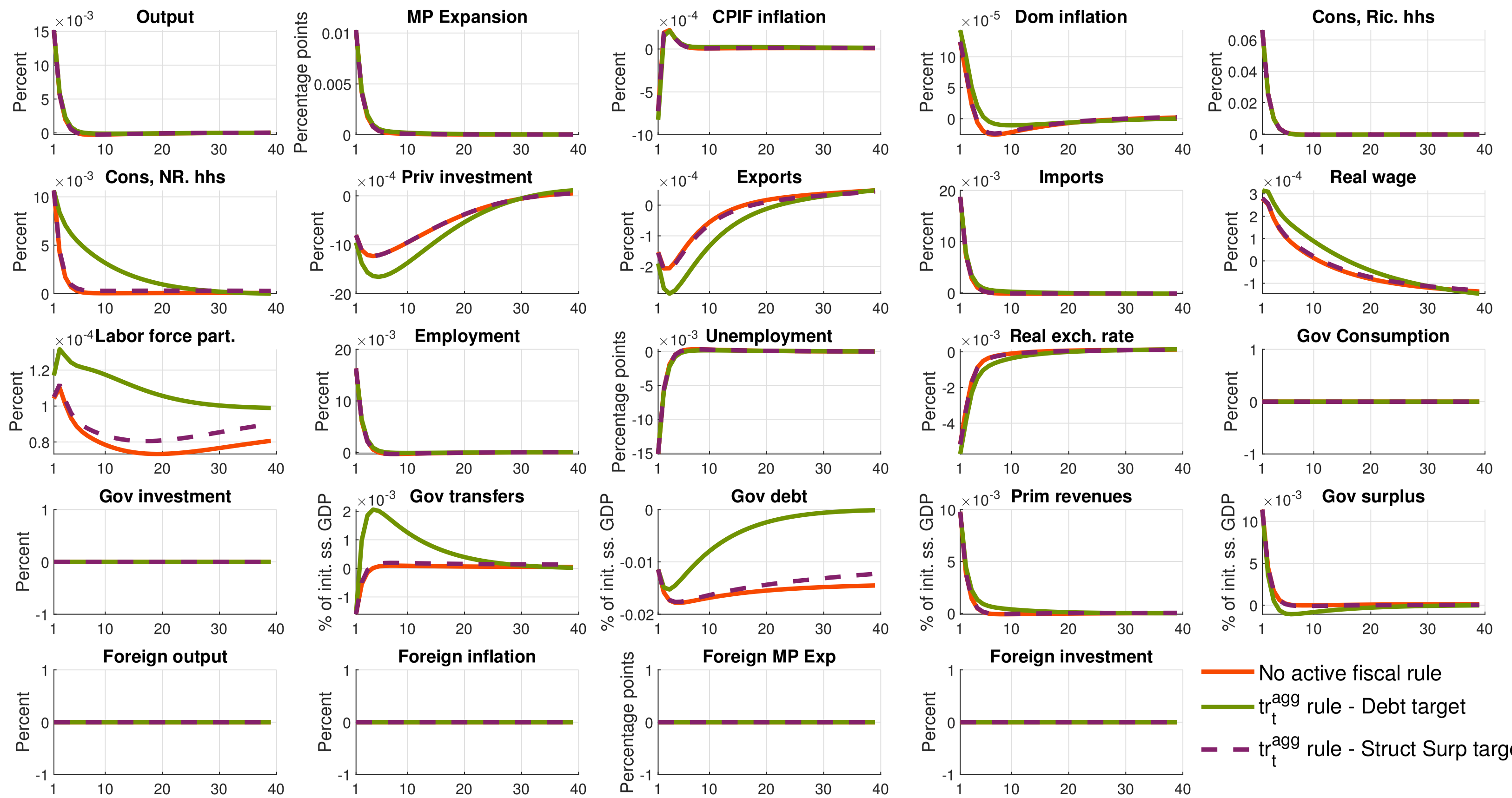
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 16: Risk Premium Shock



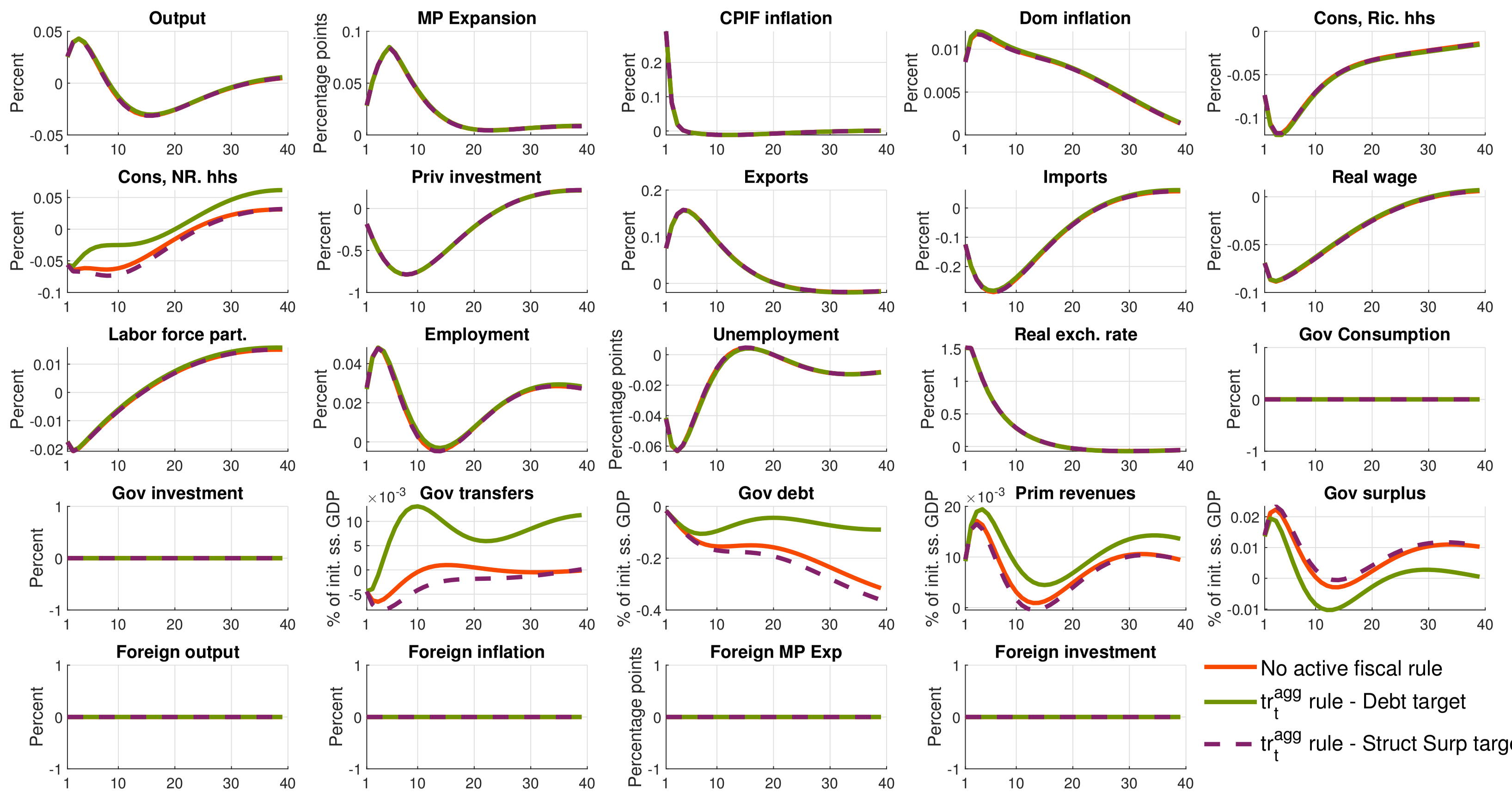
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 17: Discount Factor Shock



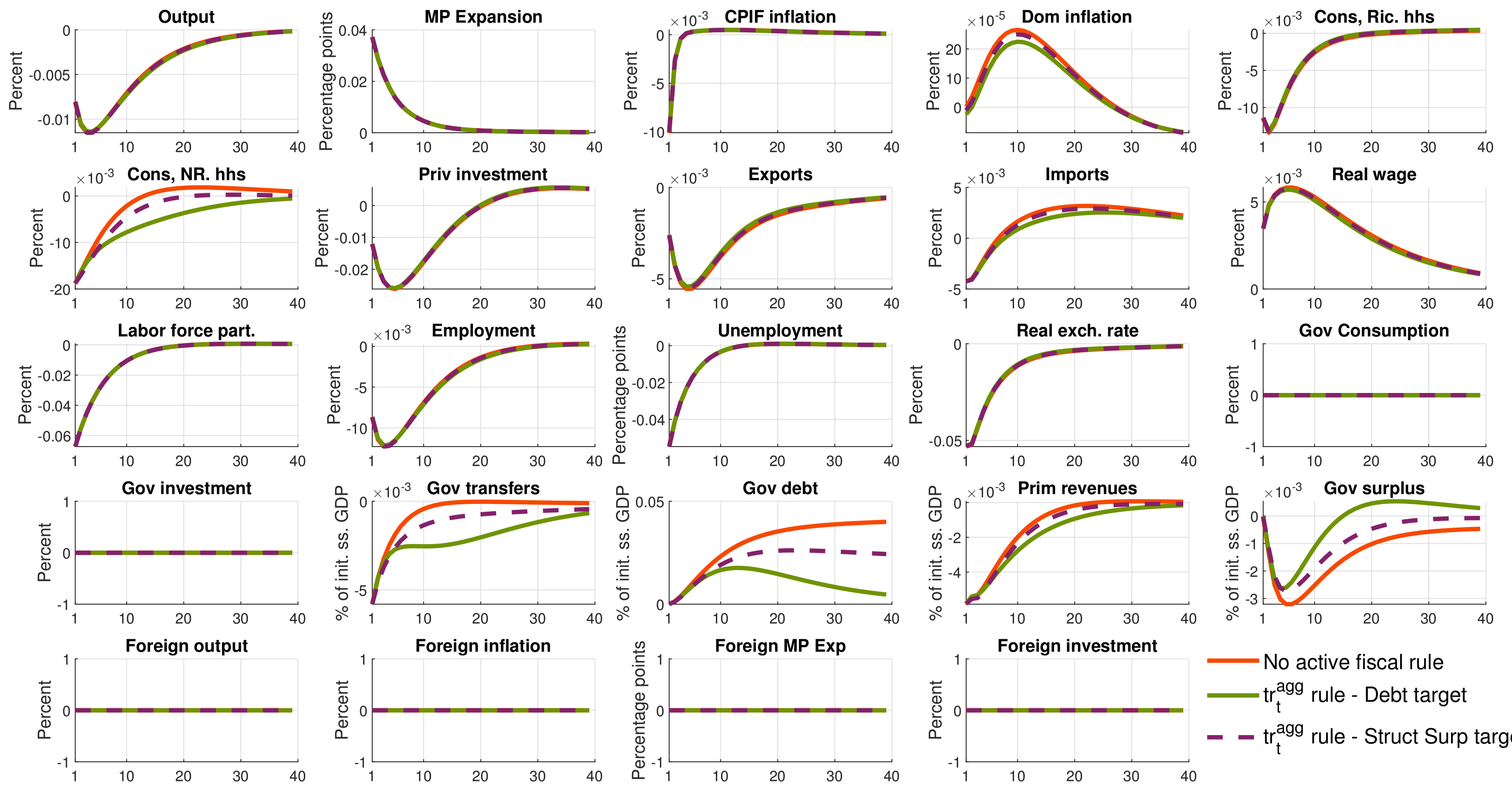
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 18: Consumption Preference Shock



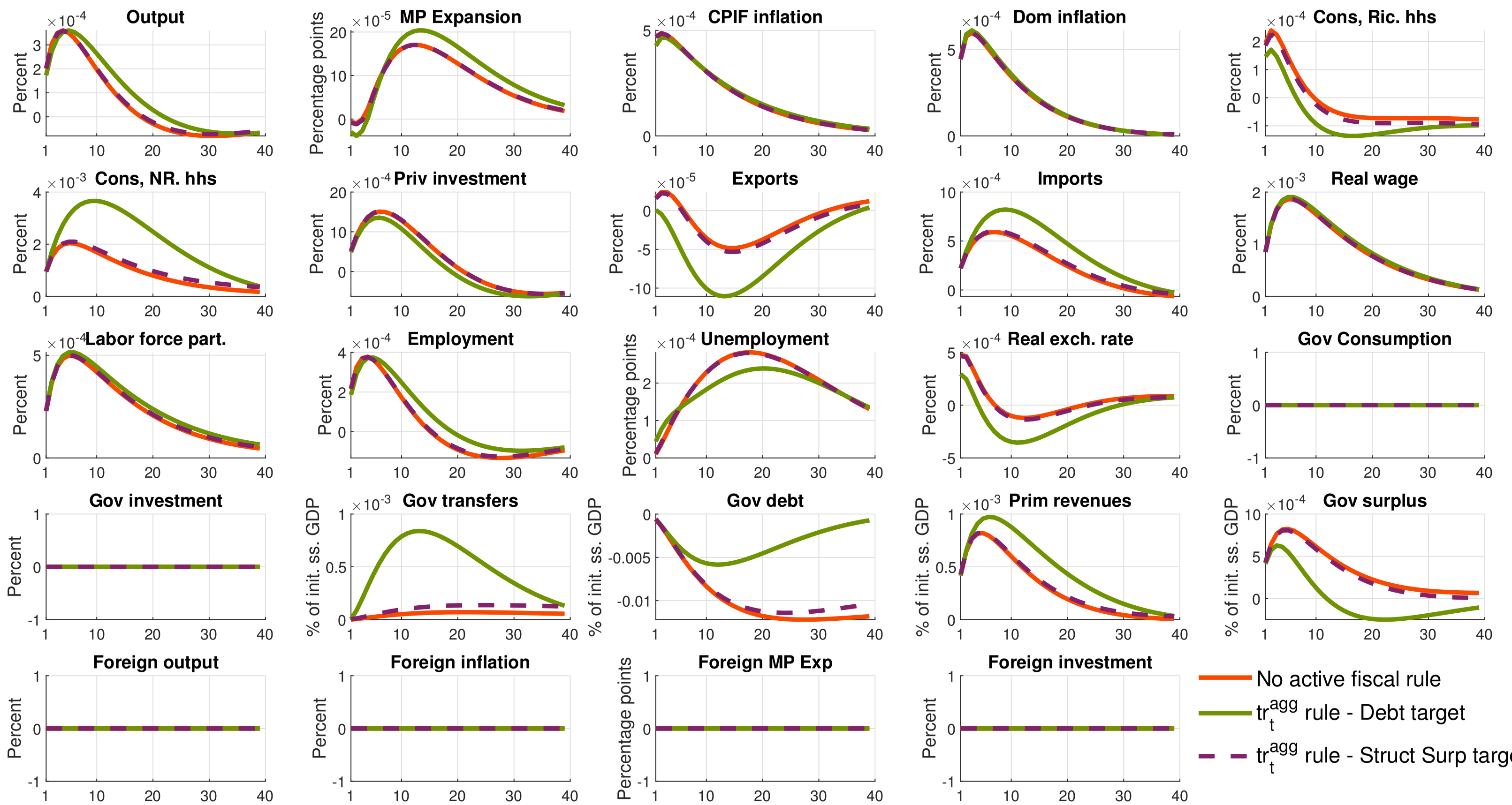
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 19: UIP Risk Premium Shock



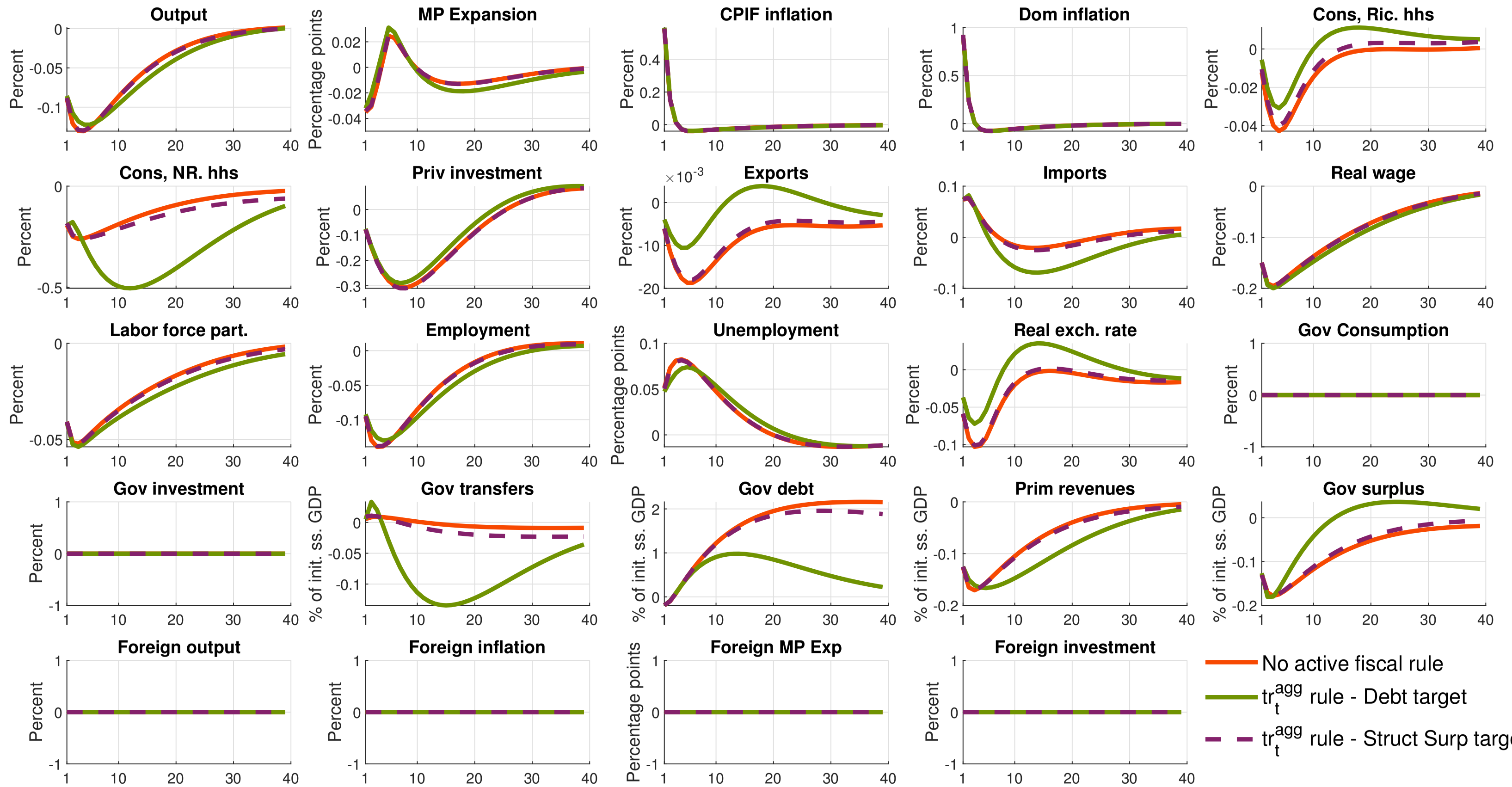
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 20: Labor Preference Shock



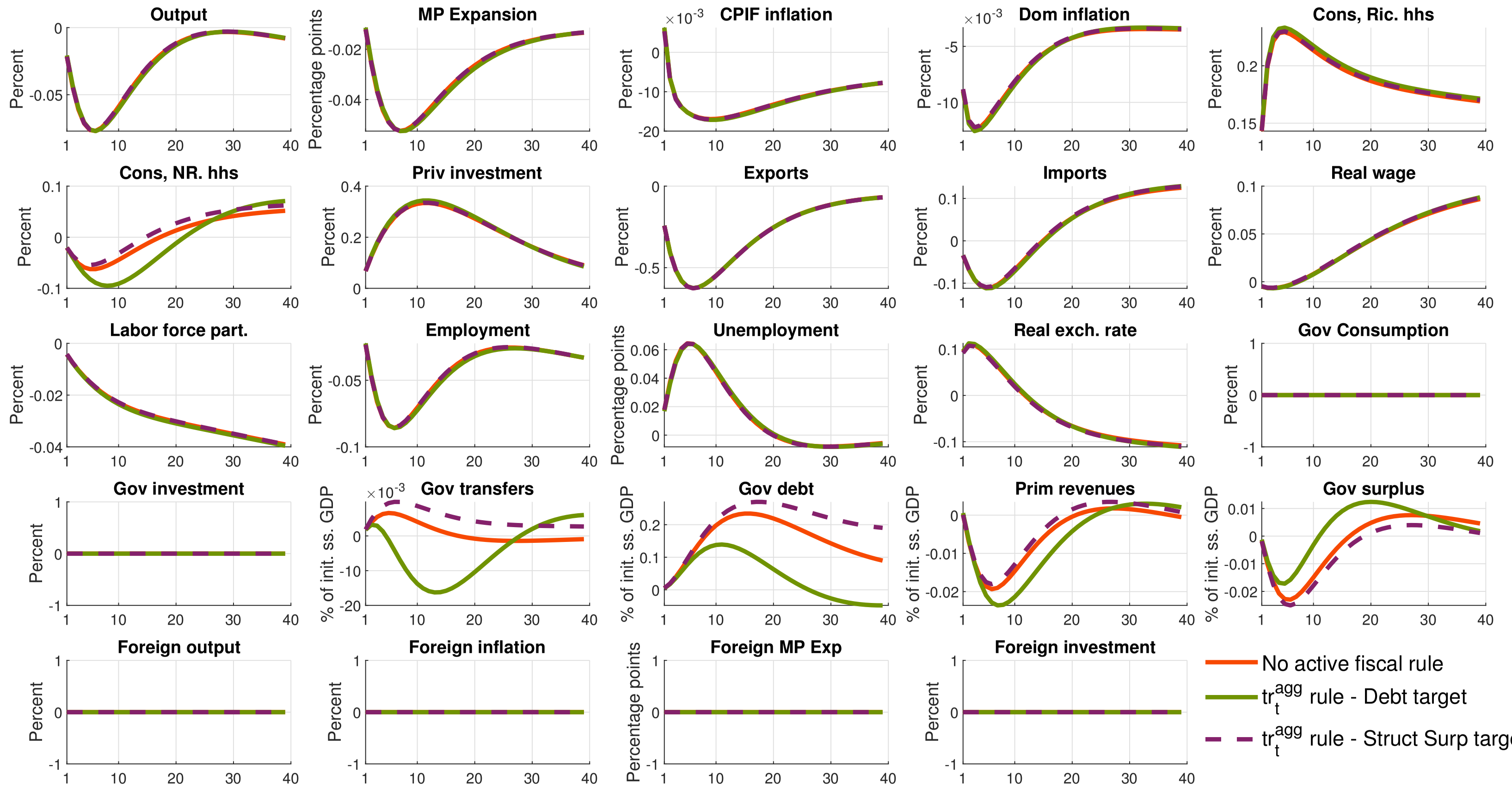
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 21: Wage Markup Shock



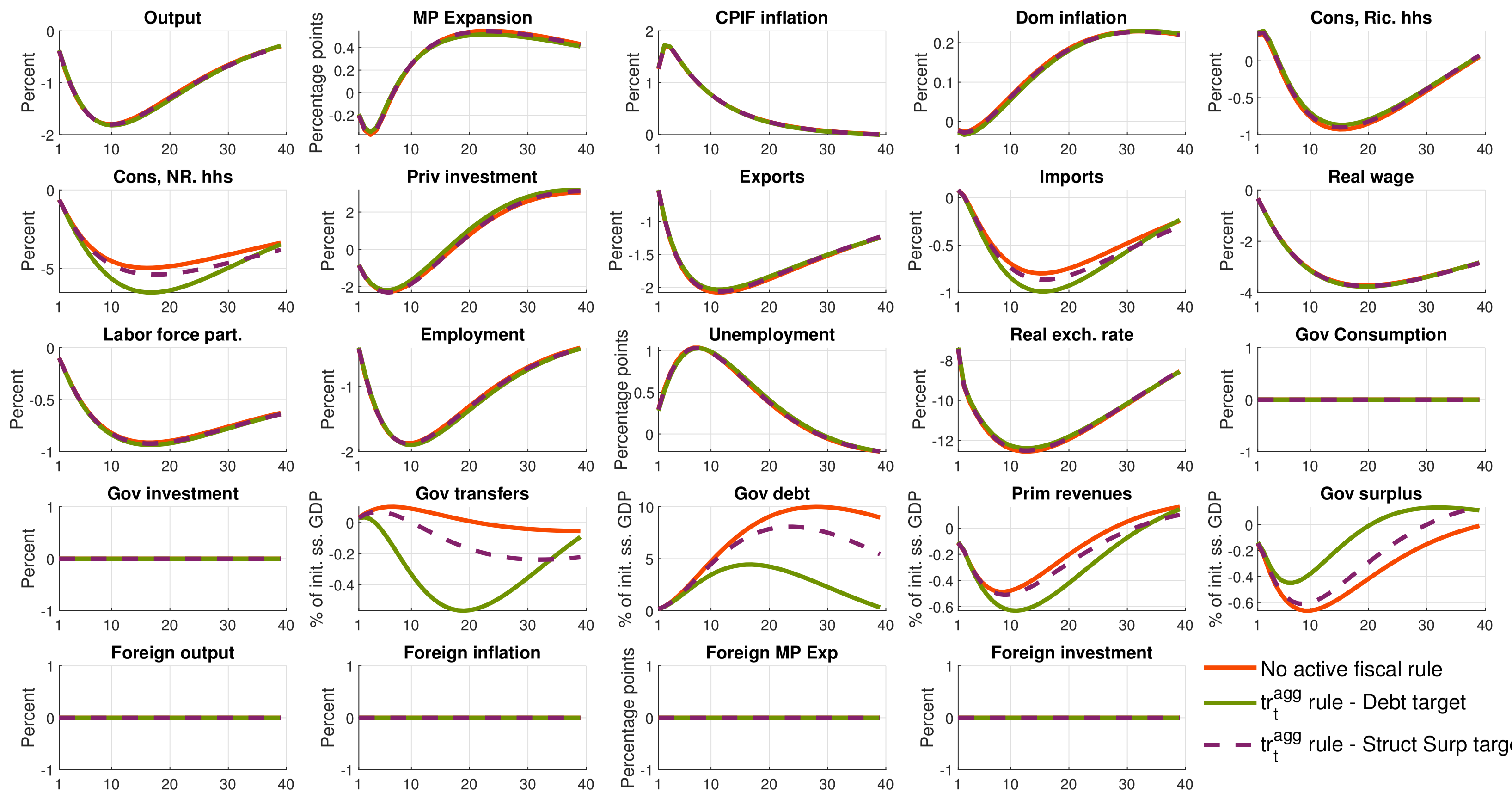
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 22: Intermediate Goods Price Markup Shock



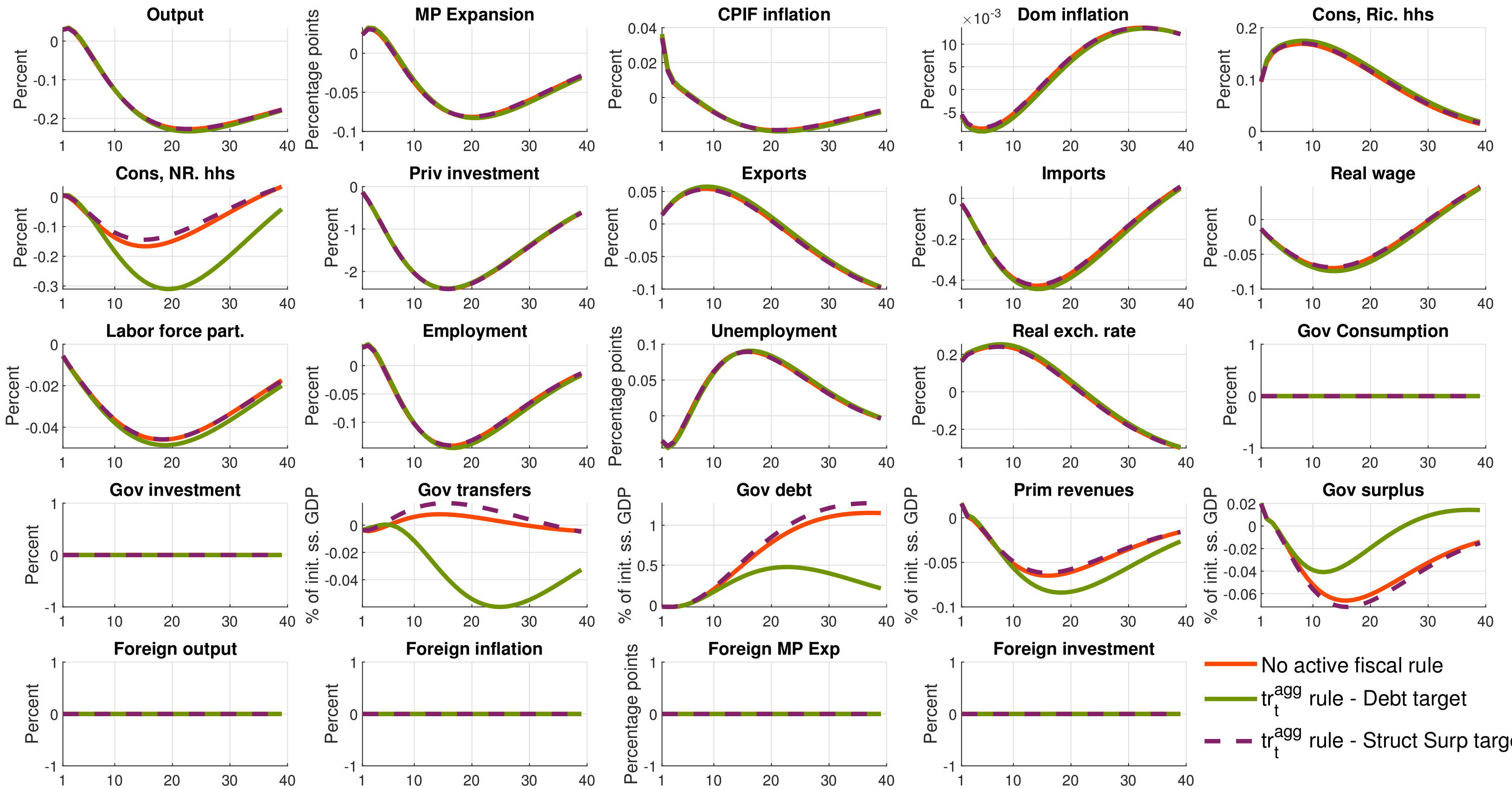
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 23: Export Goods Price Markup Shock



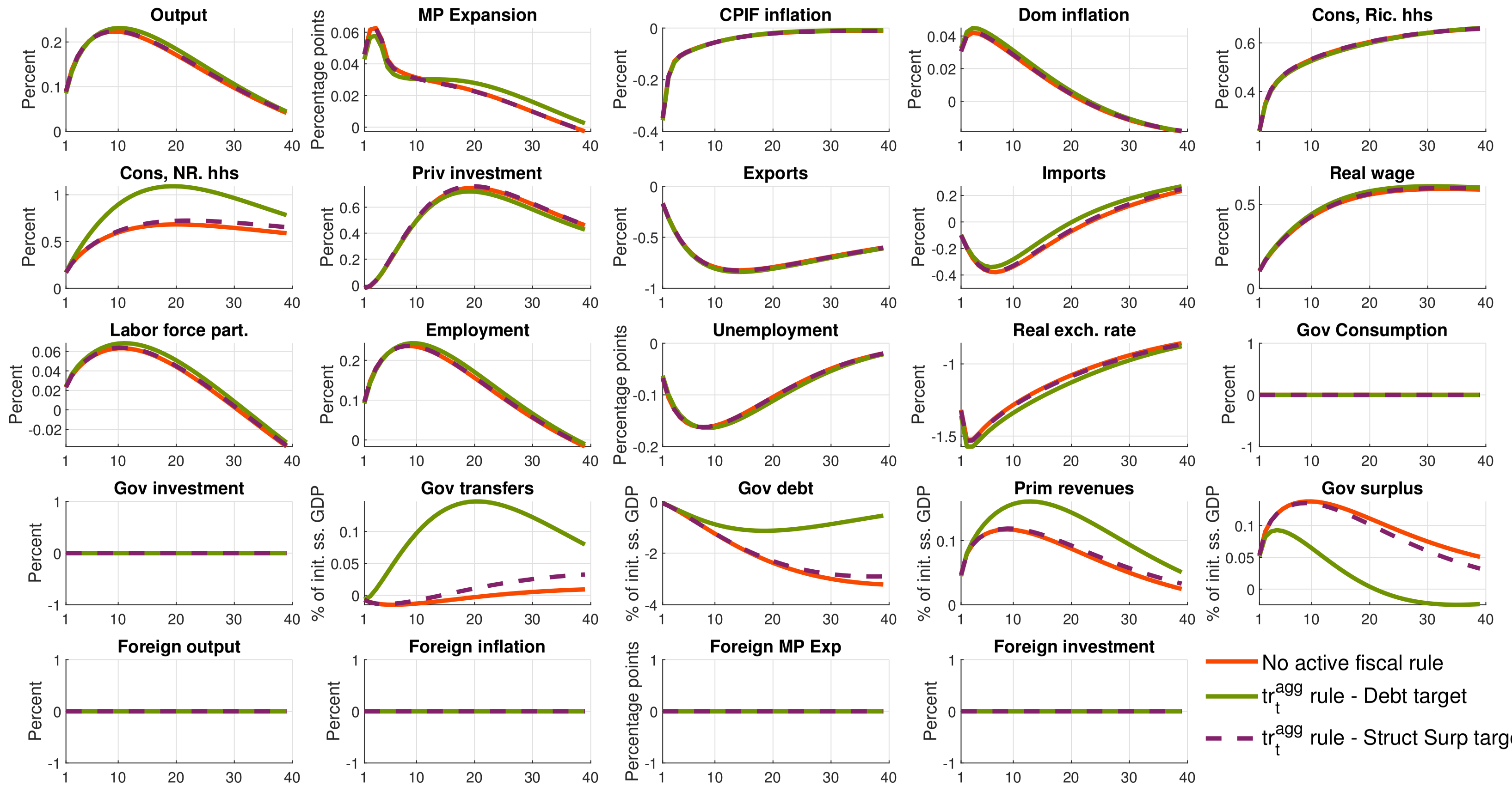
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 24: Import to Non-Energy Consumption Goods Price Markup Shock



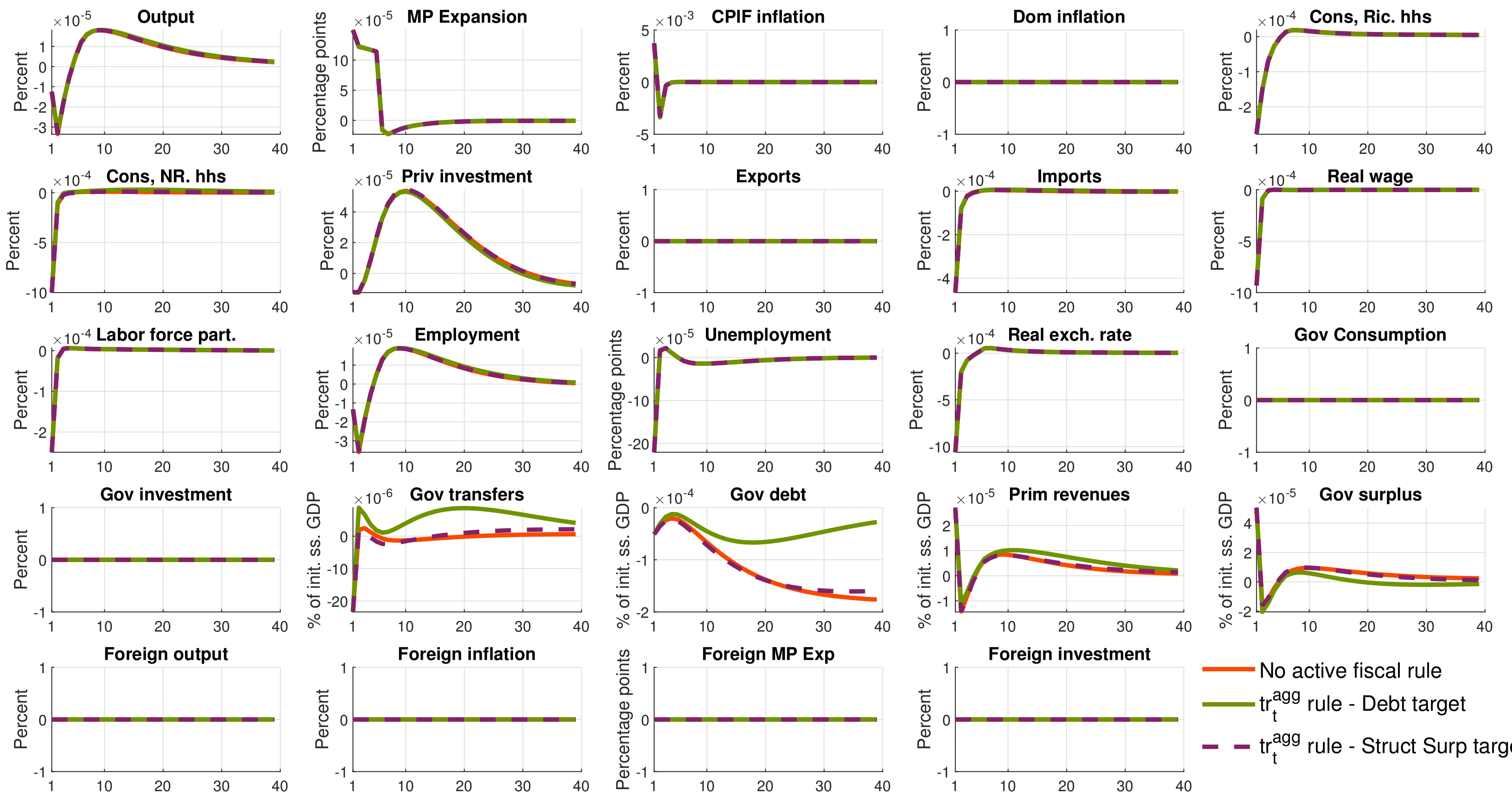
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 25: Import to Investment Goods Price Markup Shock



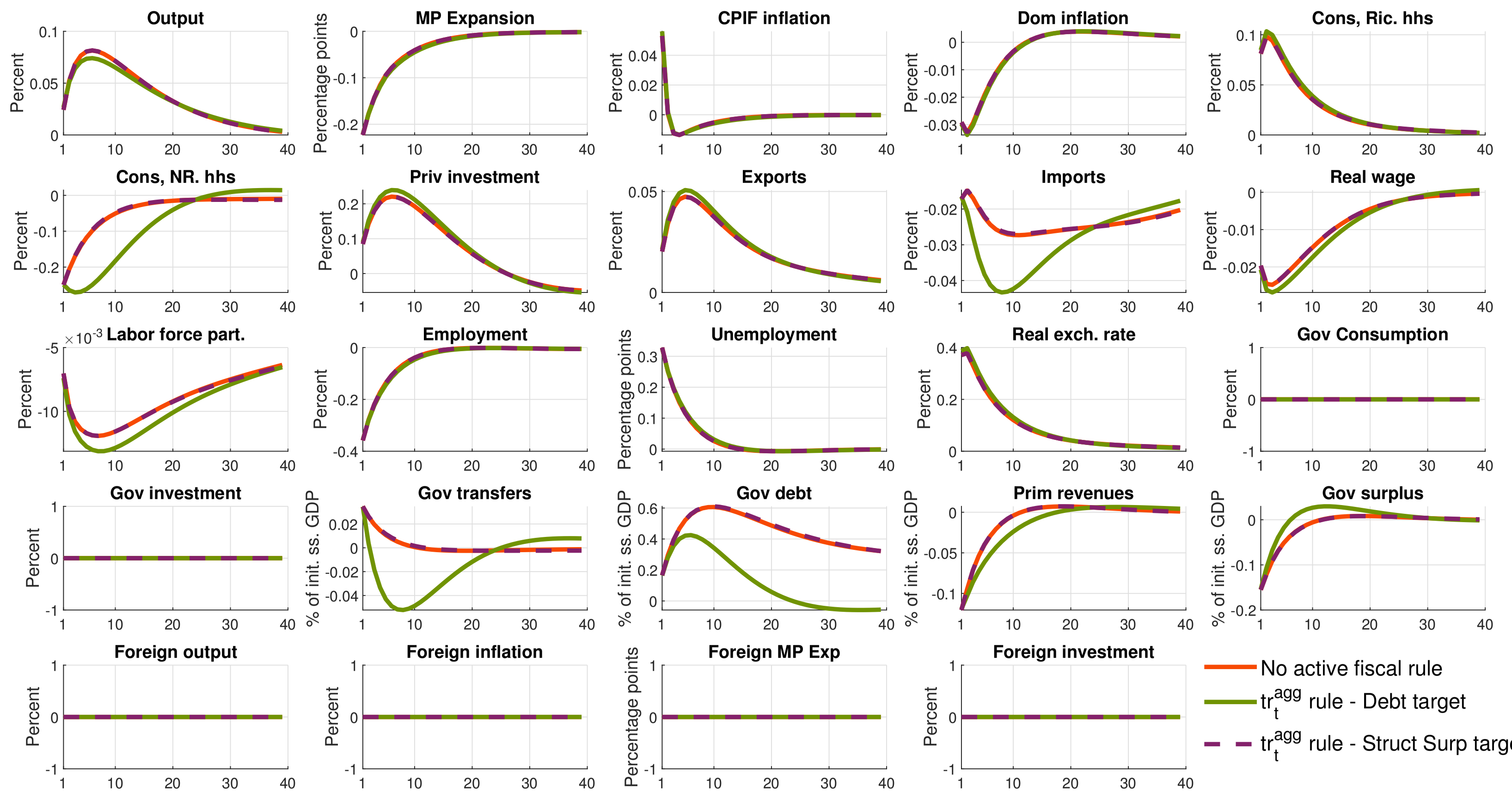
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 26: Import to Export Goods Price Markup Shock



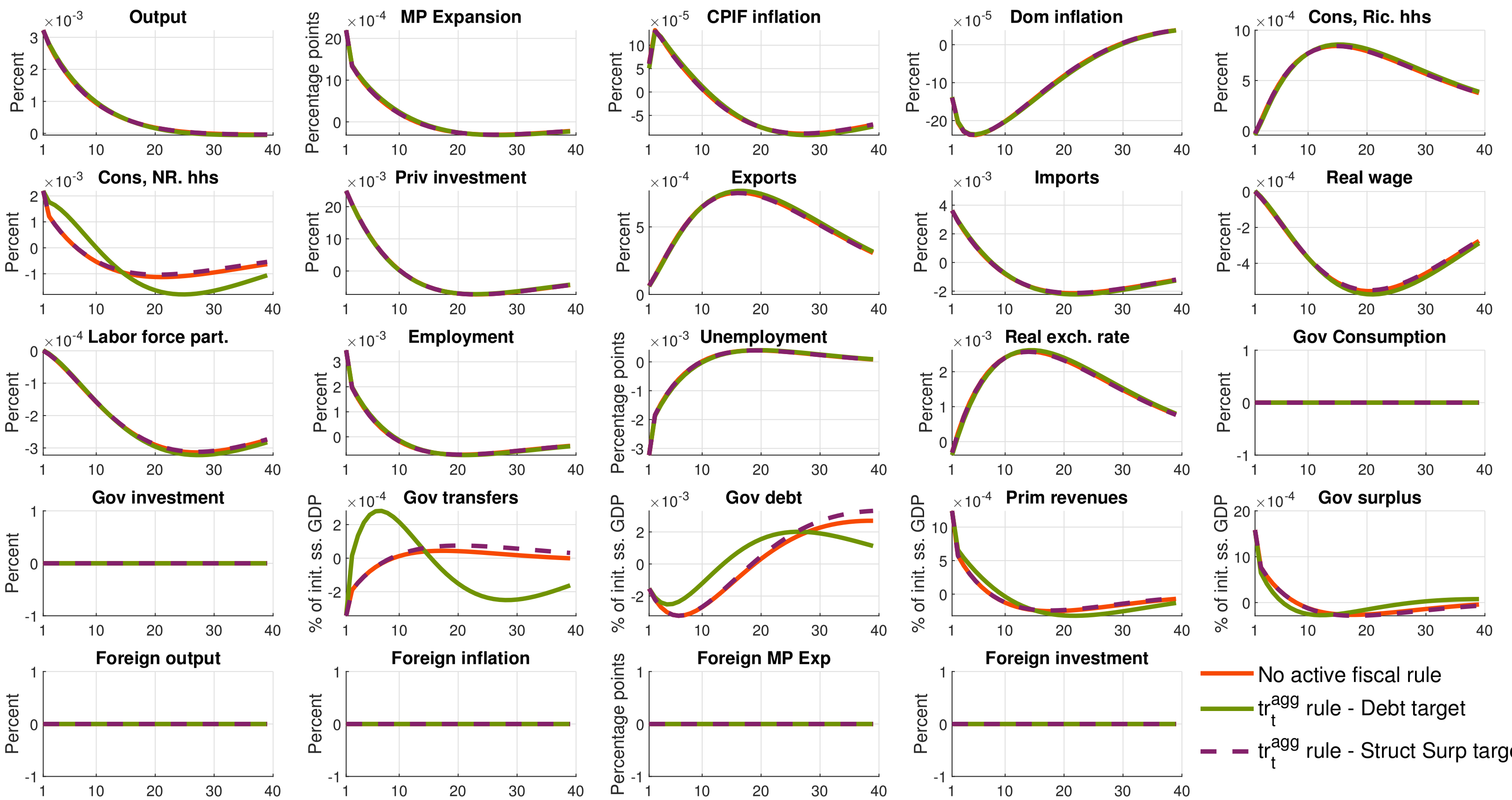
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 27: Import to Energy Consumption Goods Price Markup Shock



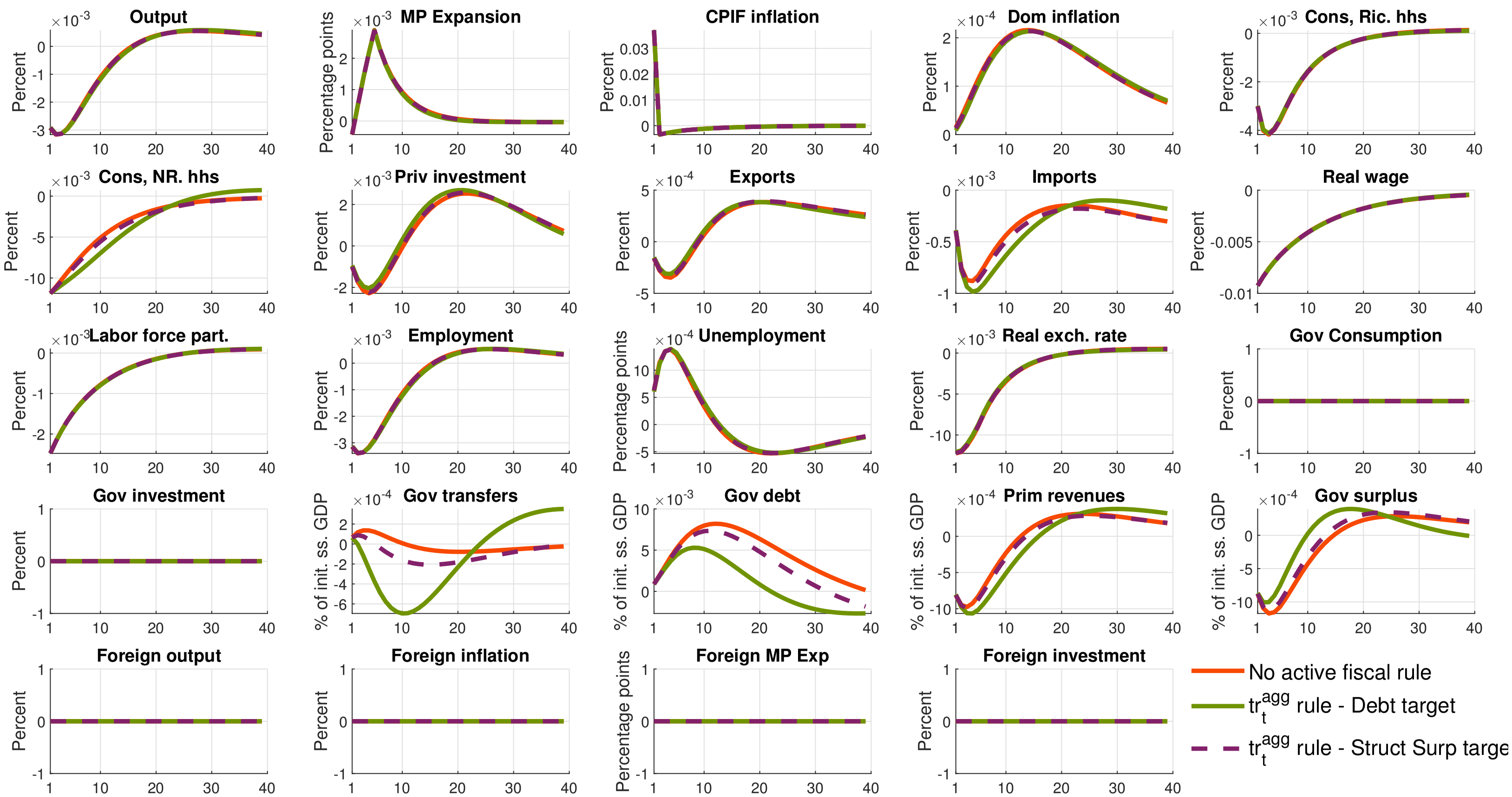
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 28: Stationary Technology Shock



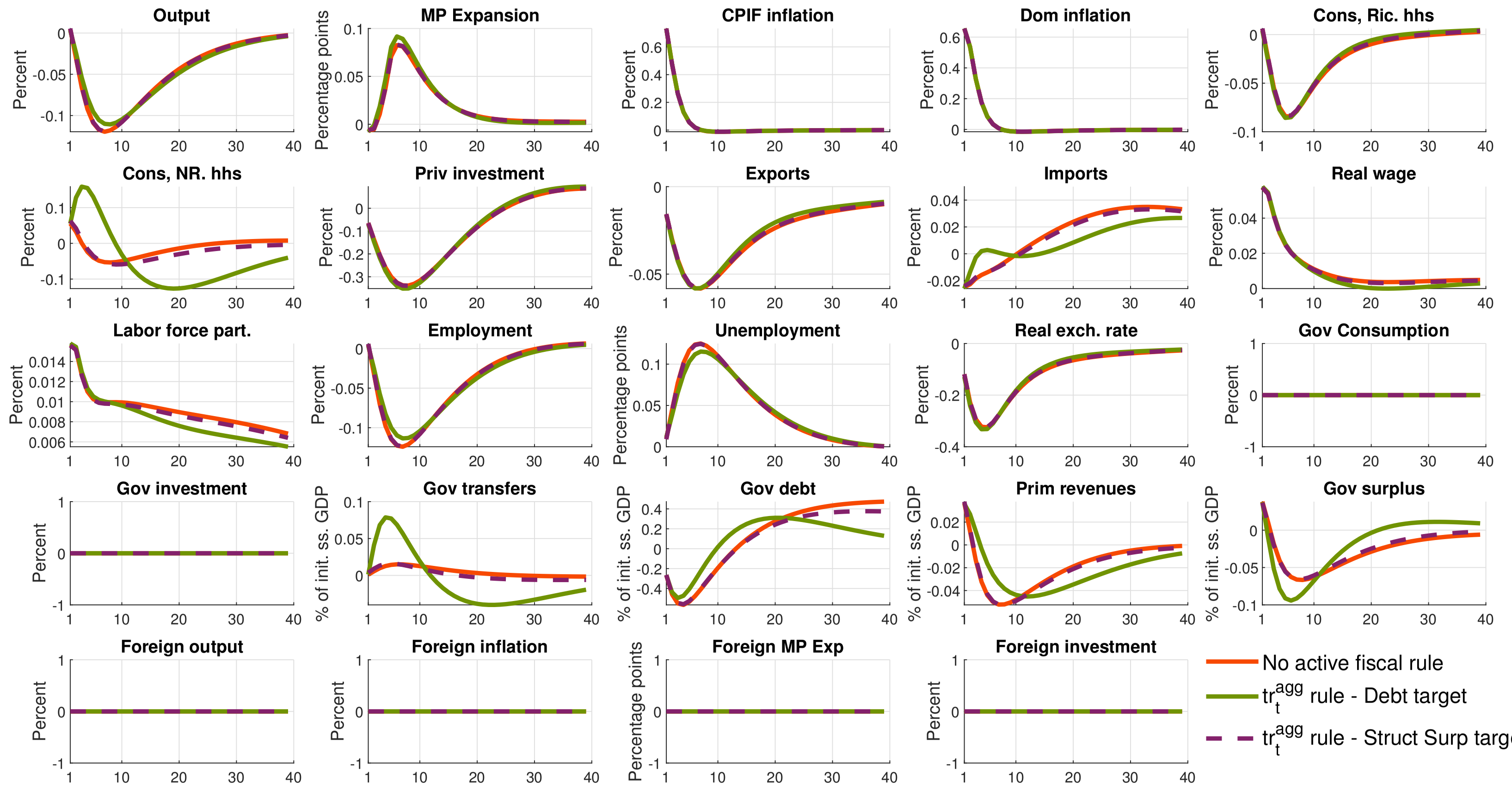
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 29: Temporary investment shock



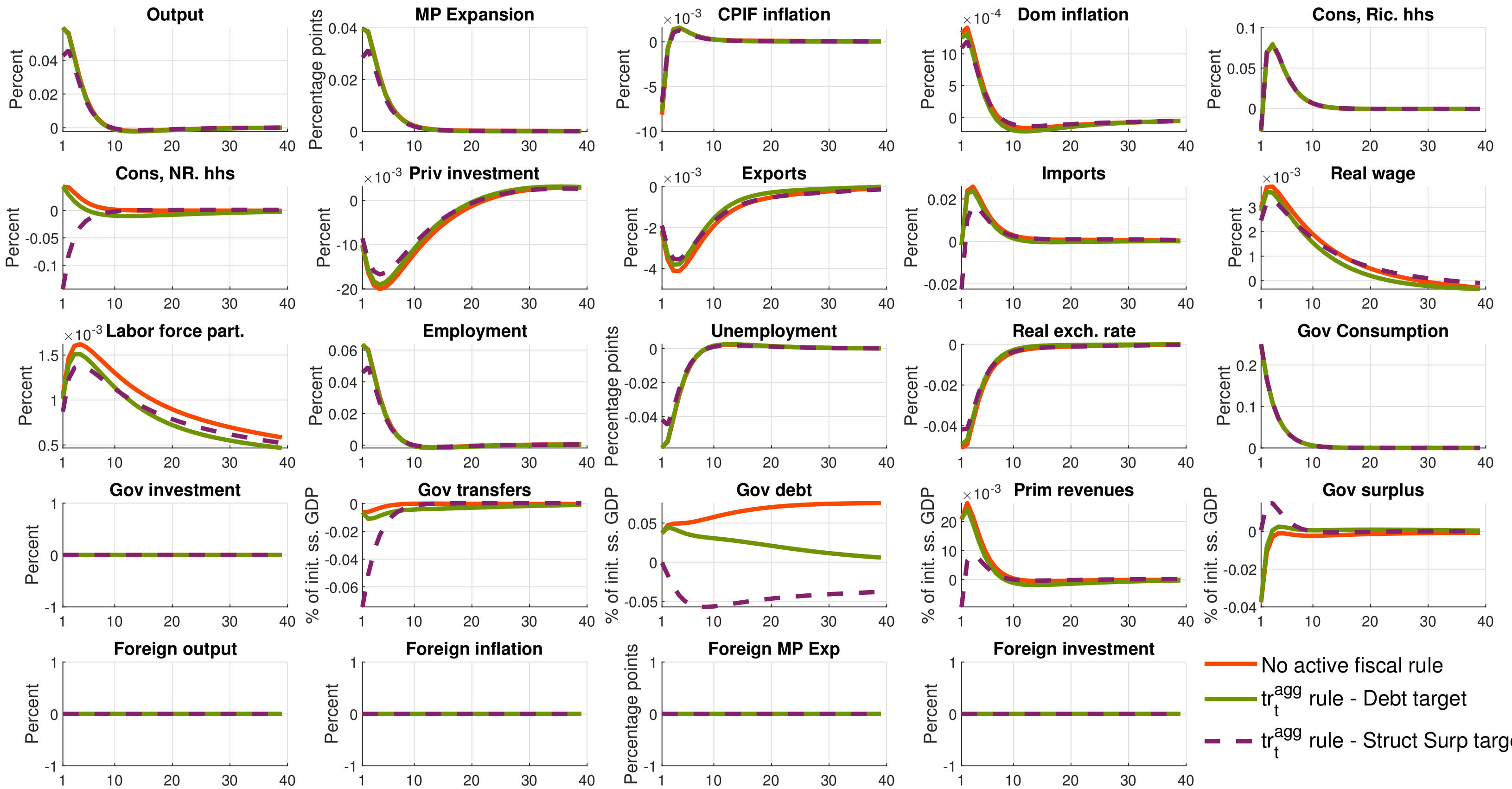
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 30: Domestic Energy Price Shock



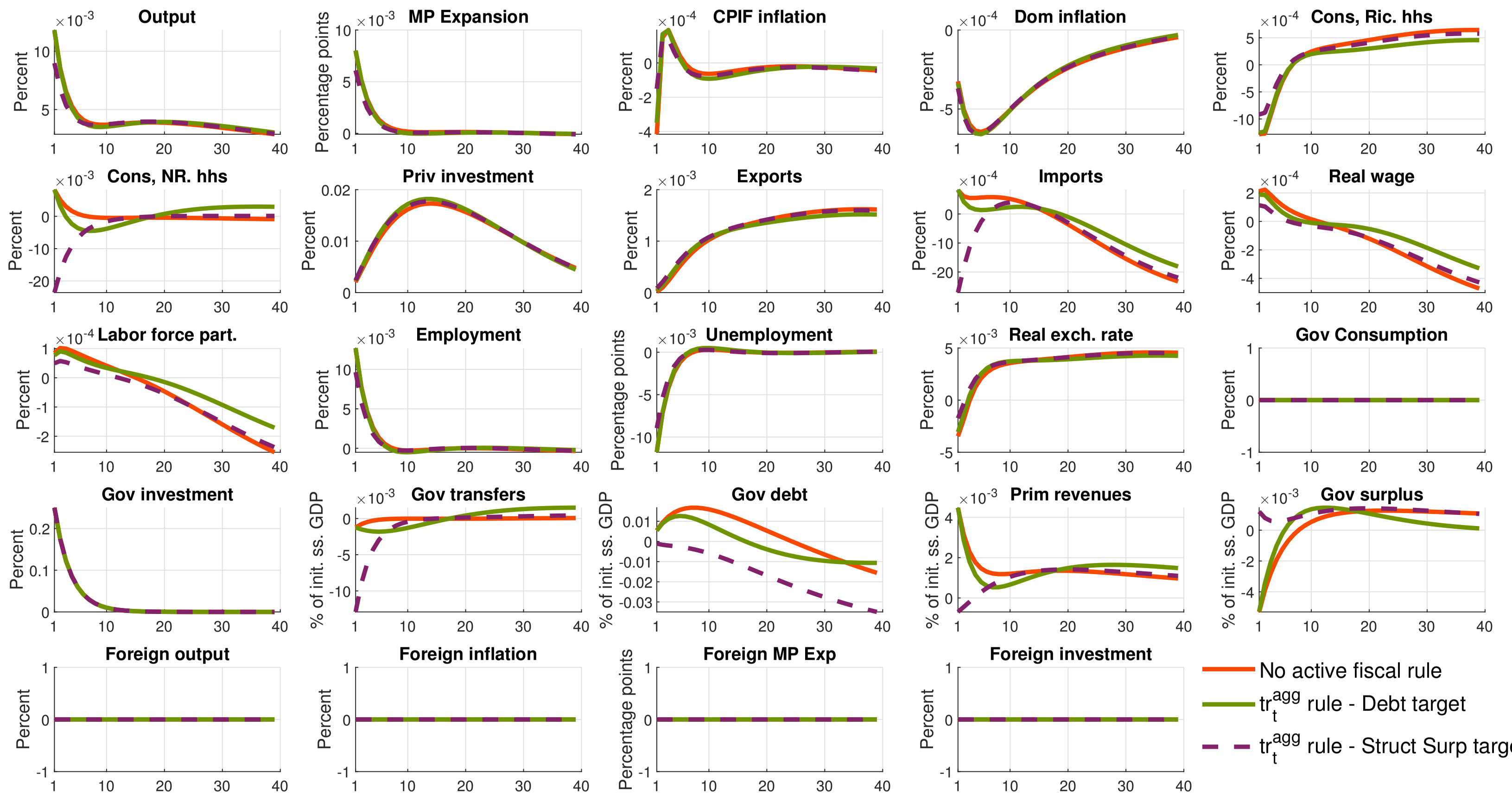
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 31: Inflation Trend Shock



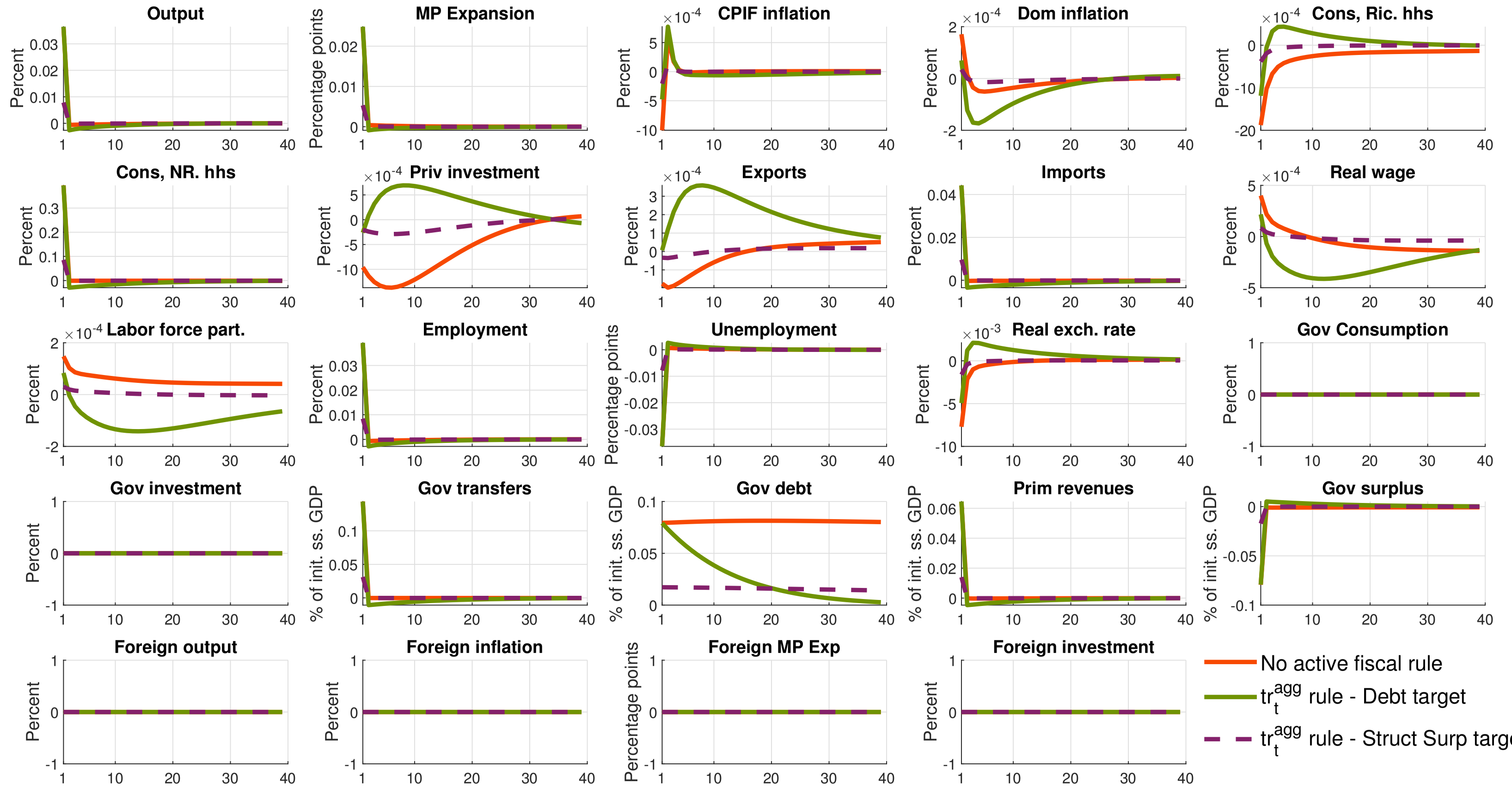
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 32: Government Consumption Shock



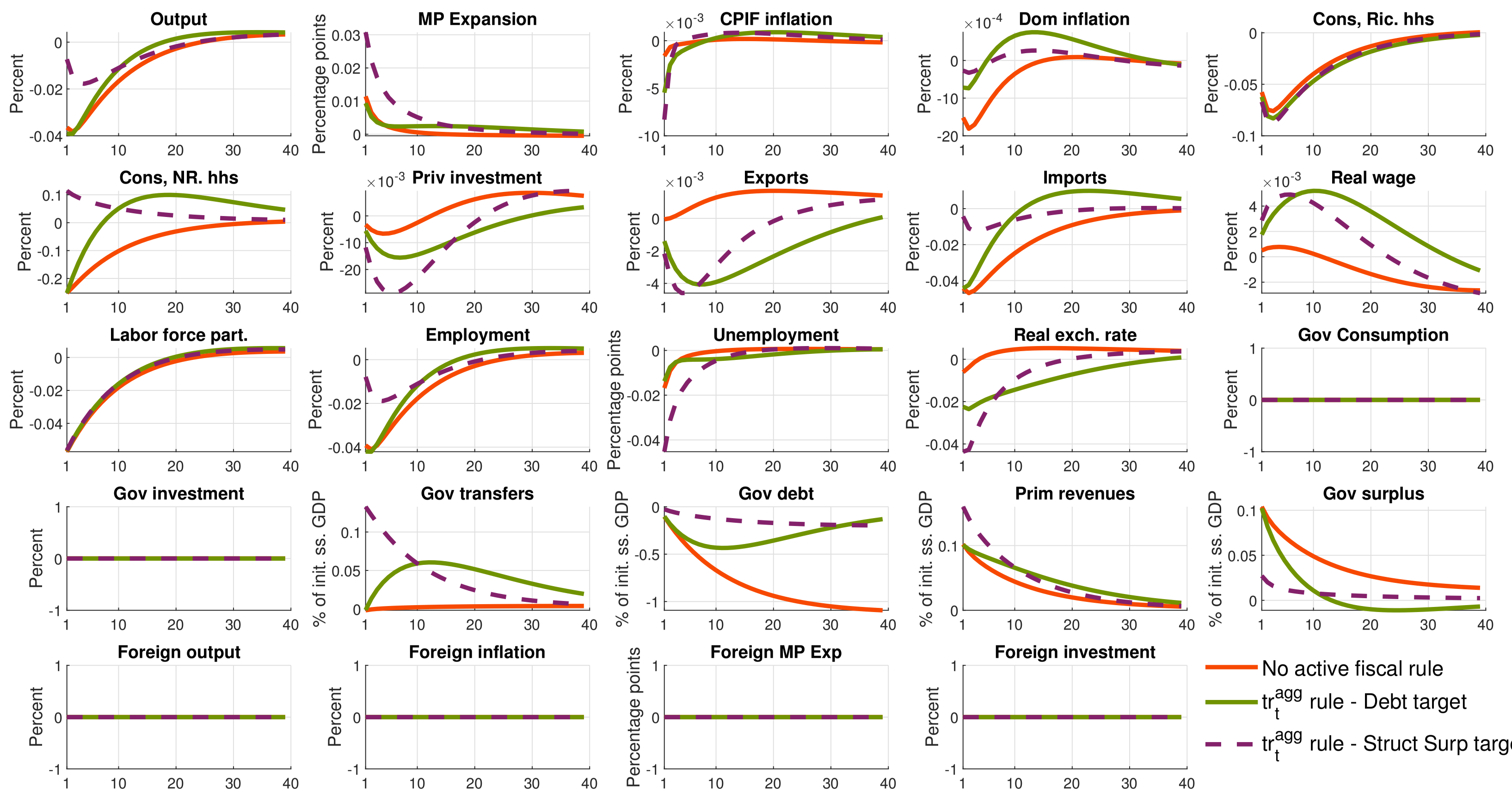
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 33: Government Investment Shock



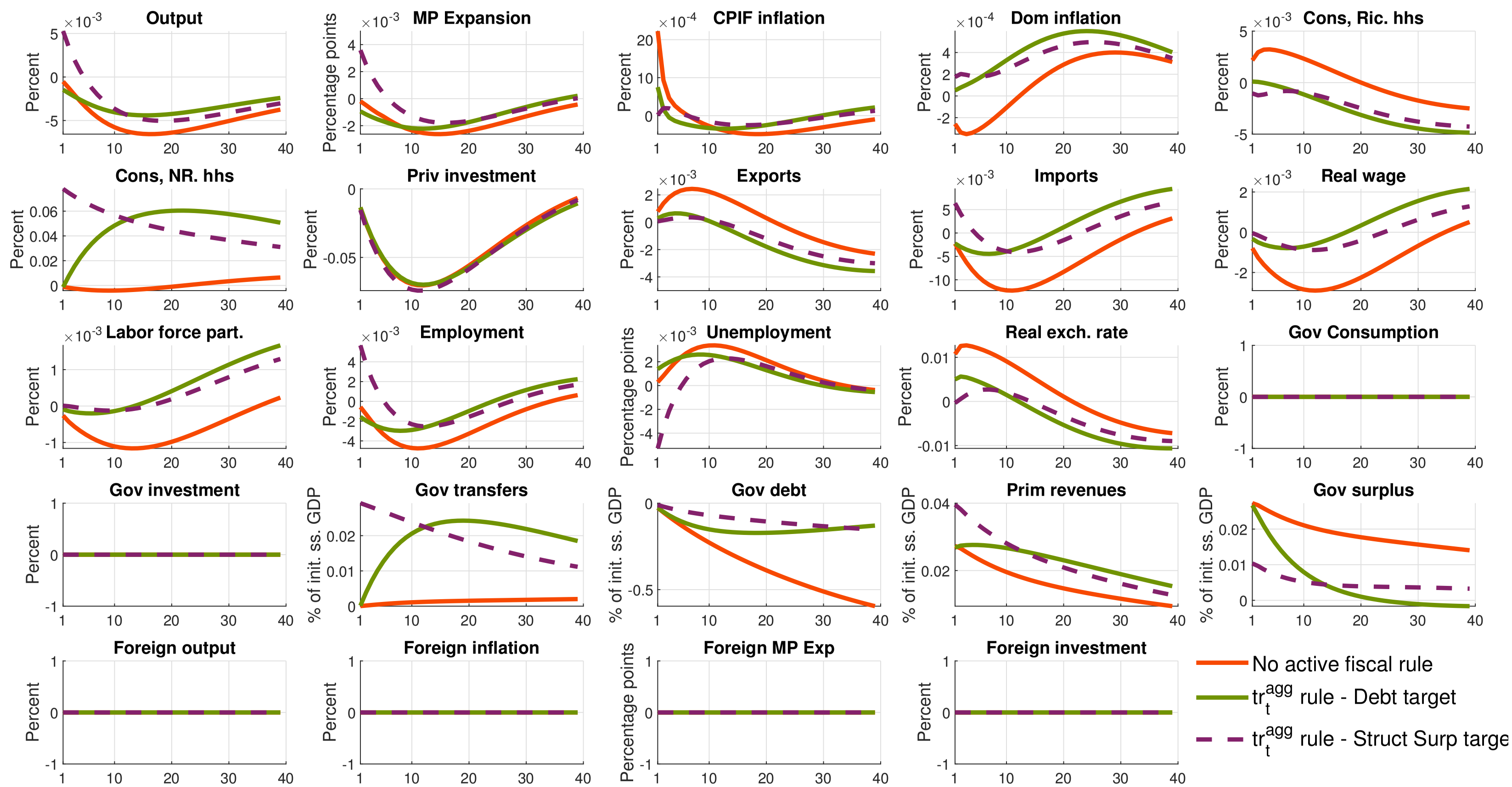
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 34: Aggregate Transfer Shock



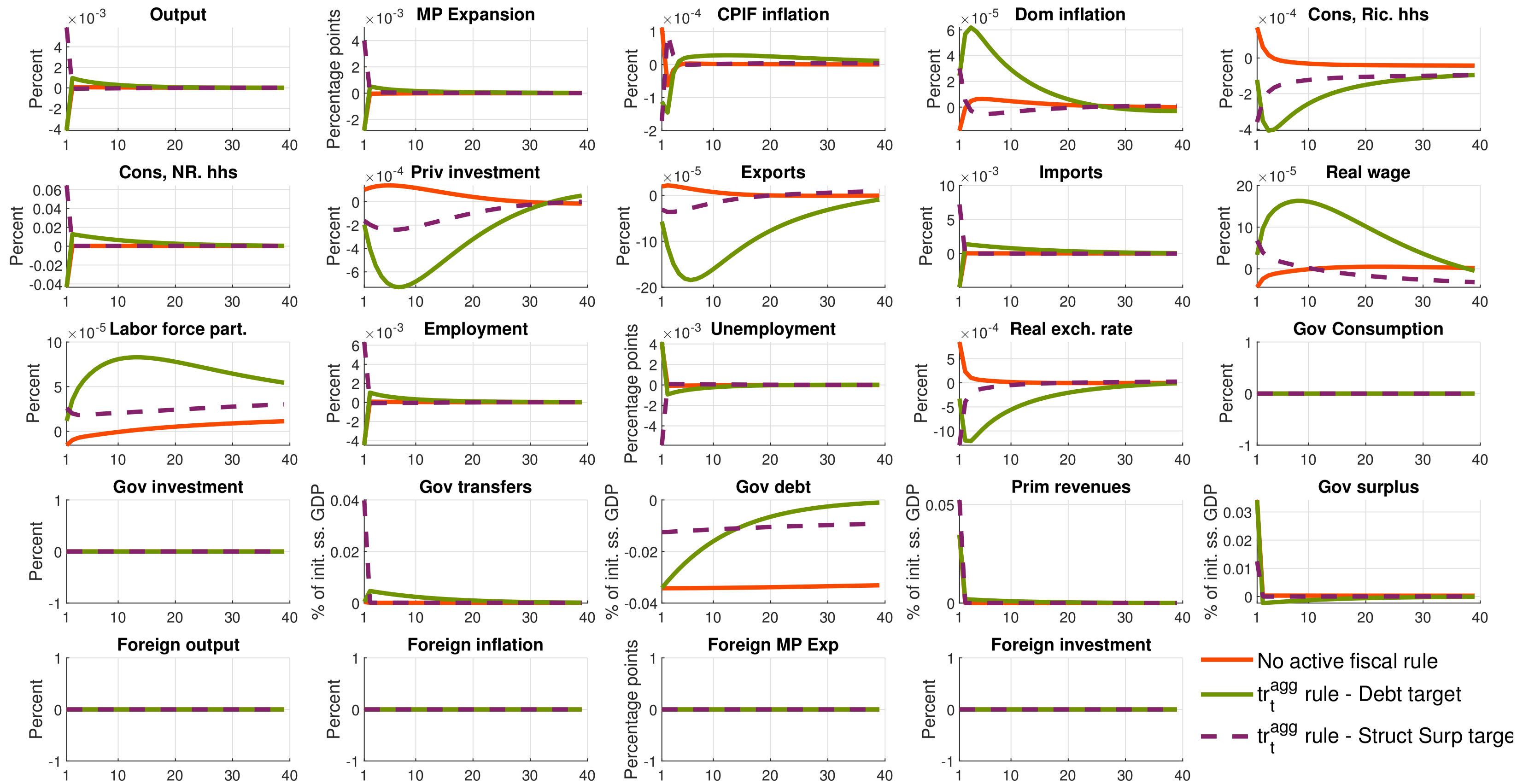
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 35: Consumption Tax Shock



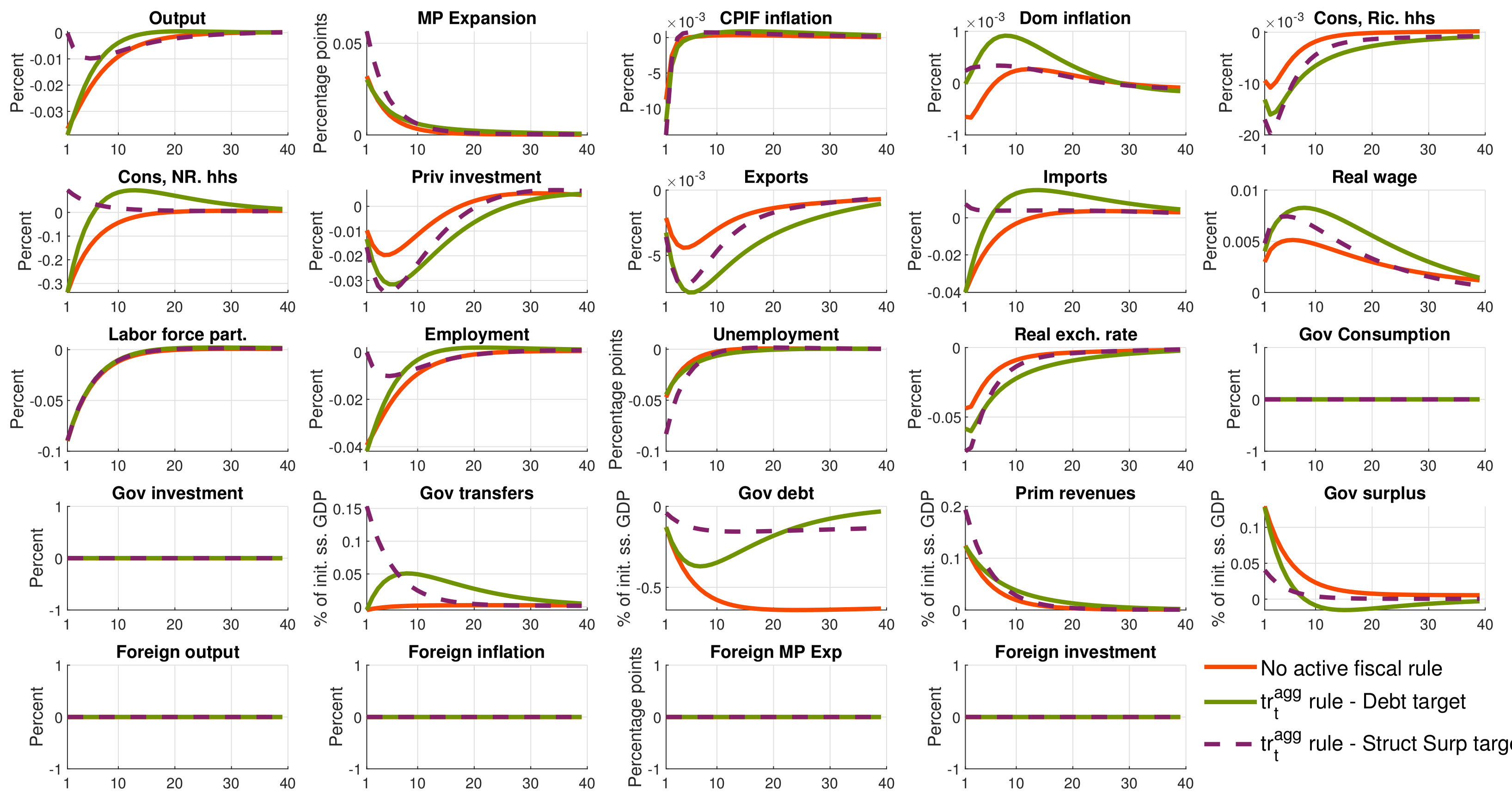
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 36: Capital Income Tax Shock



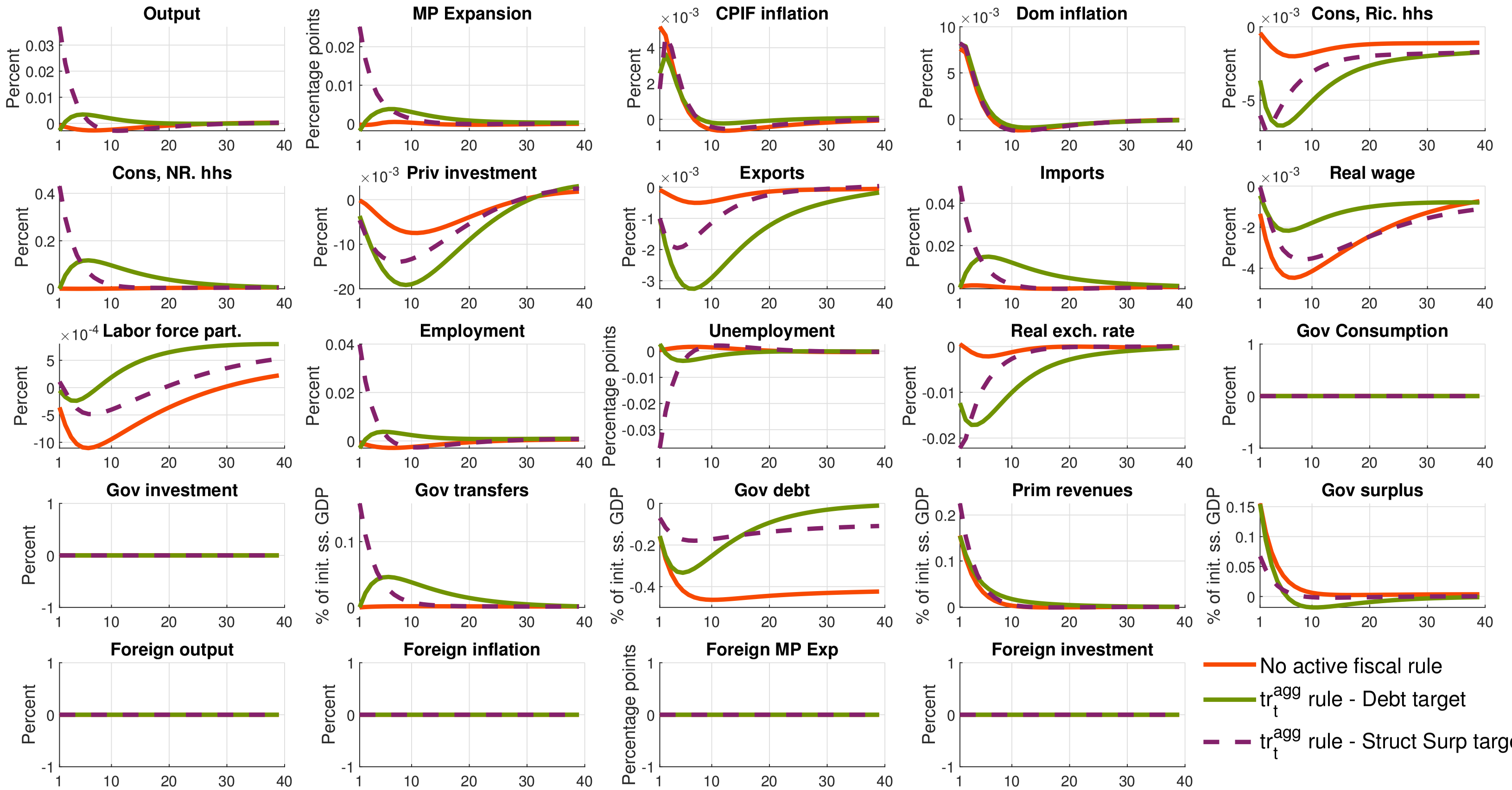
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 37: Transfer Tax Shock



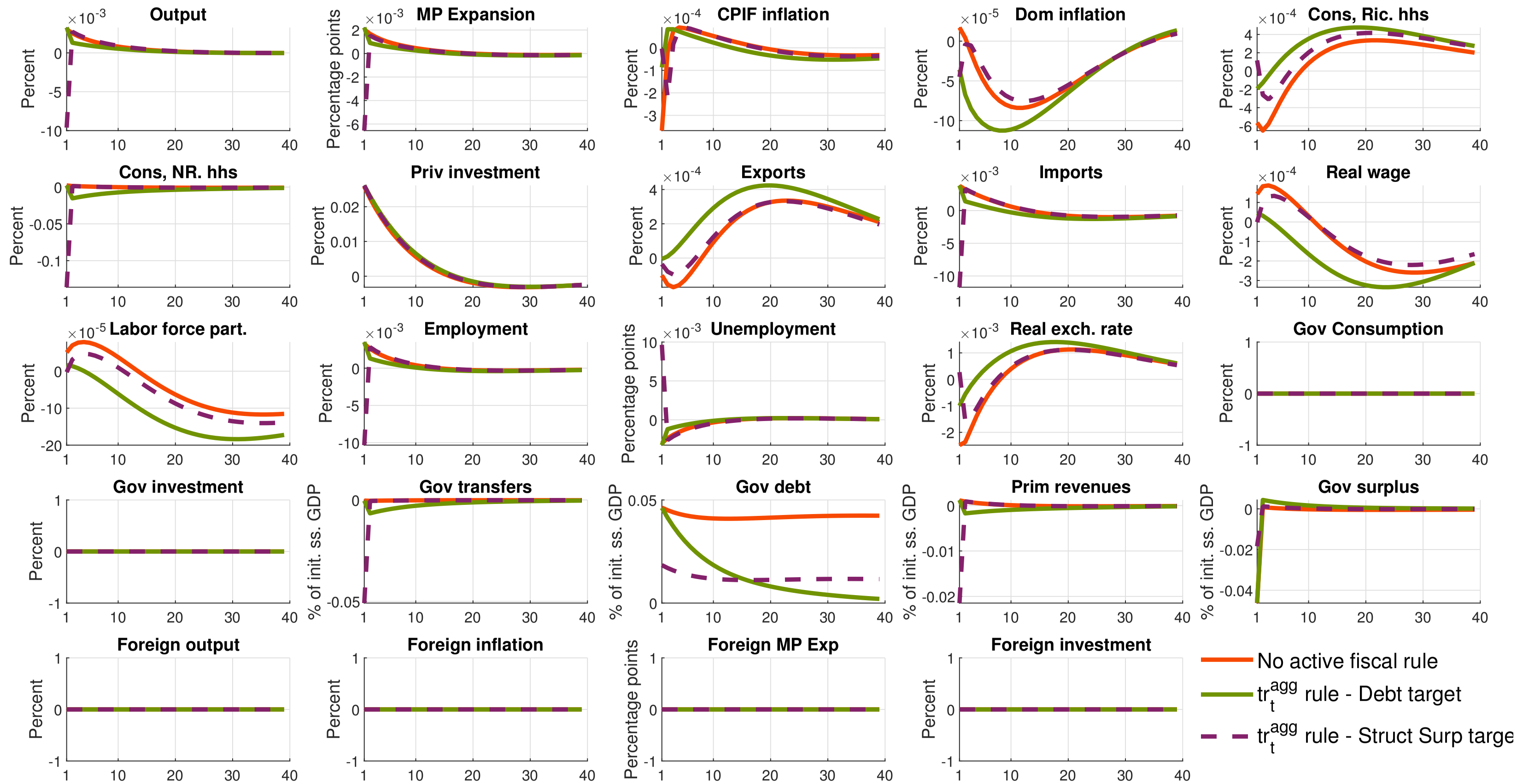
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 38: Labor Income Tax Shock



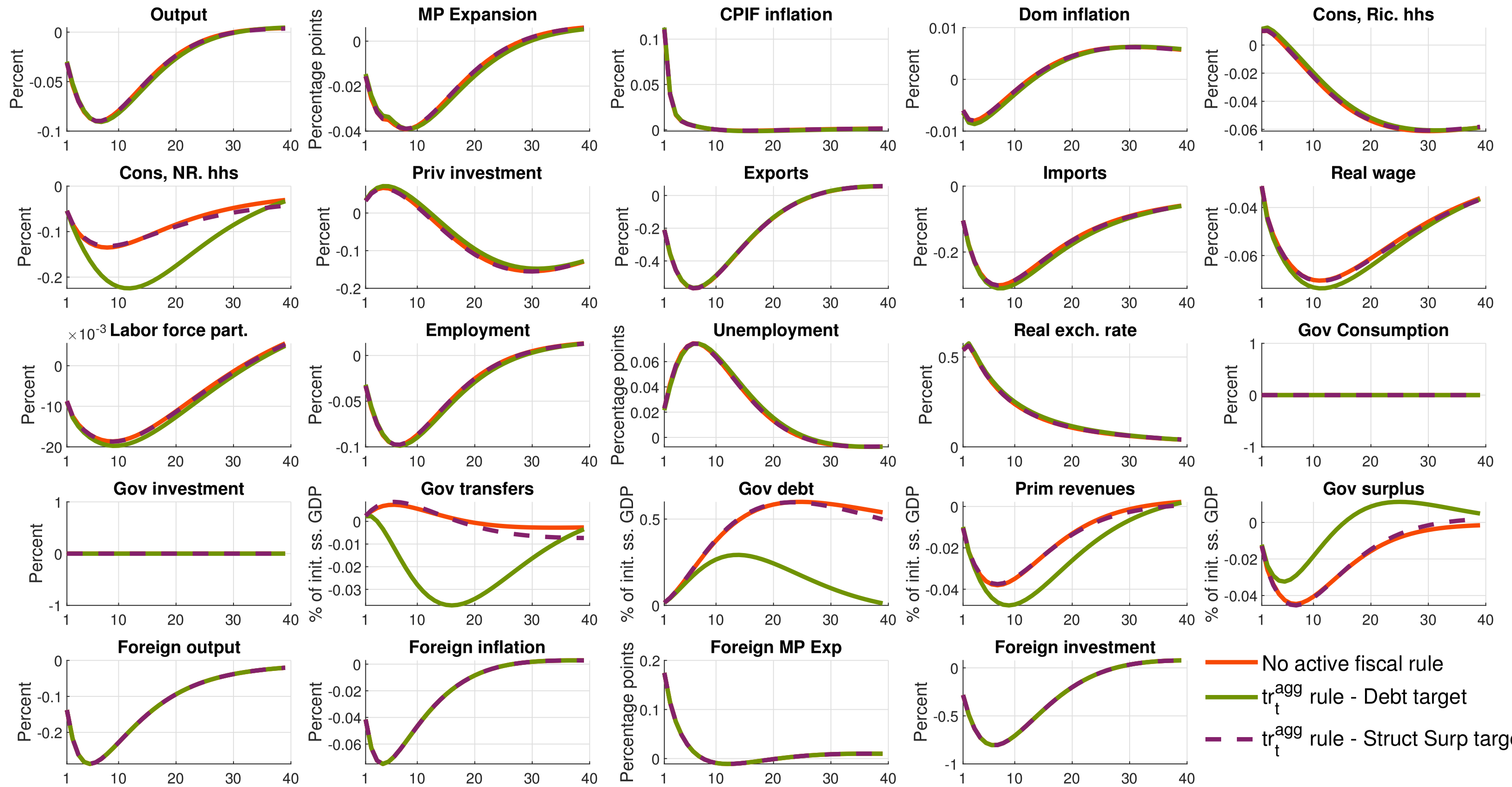
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 39: Social Security Contribution Shock



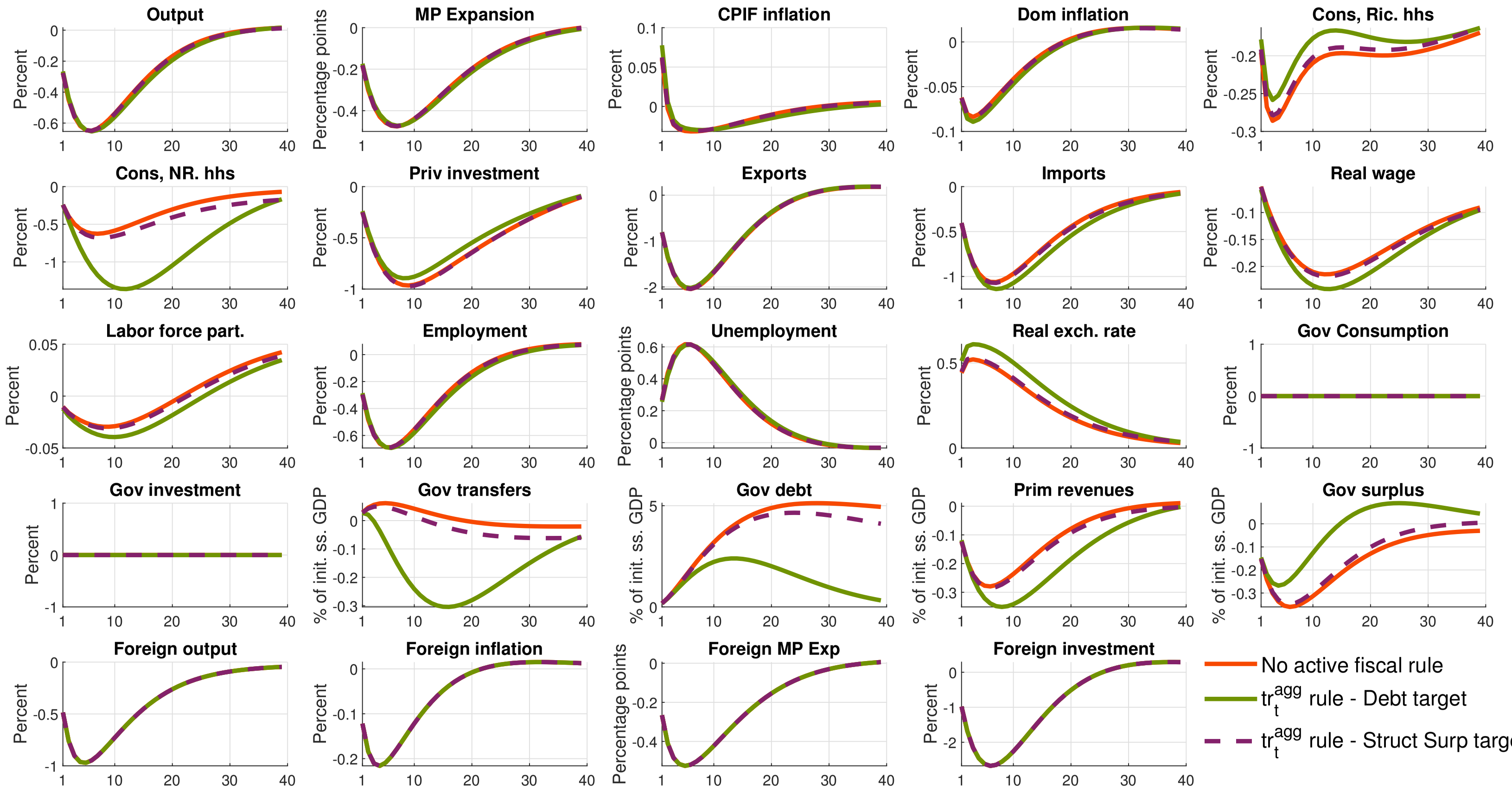
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 40: Investment Subsidy Shock



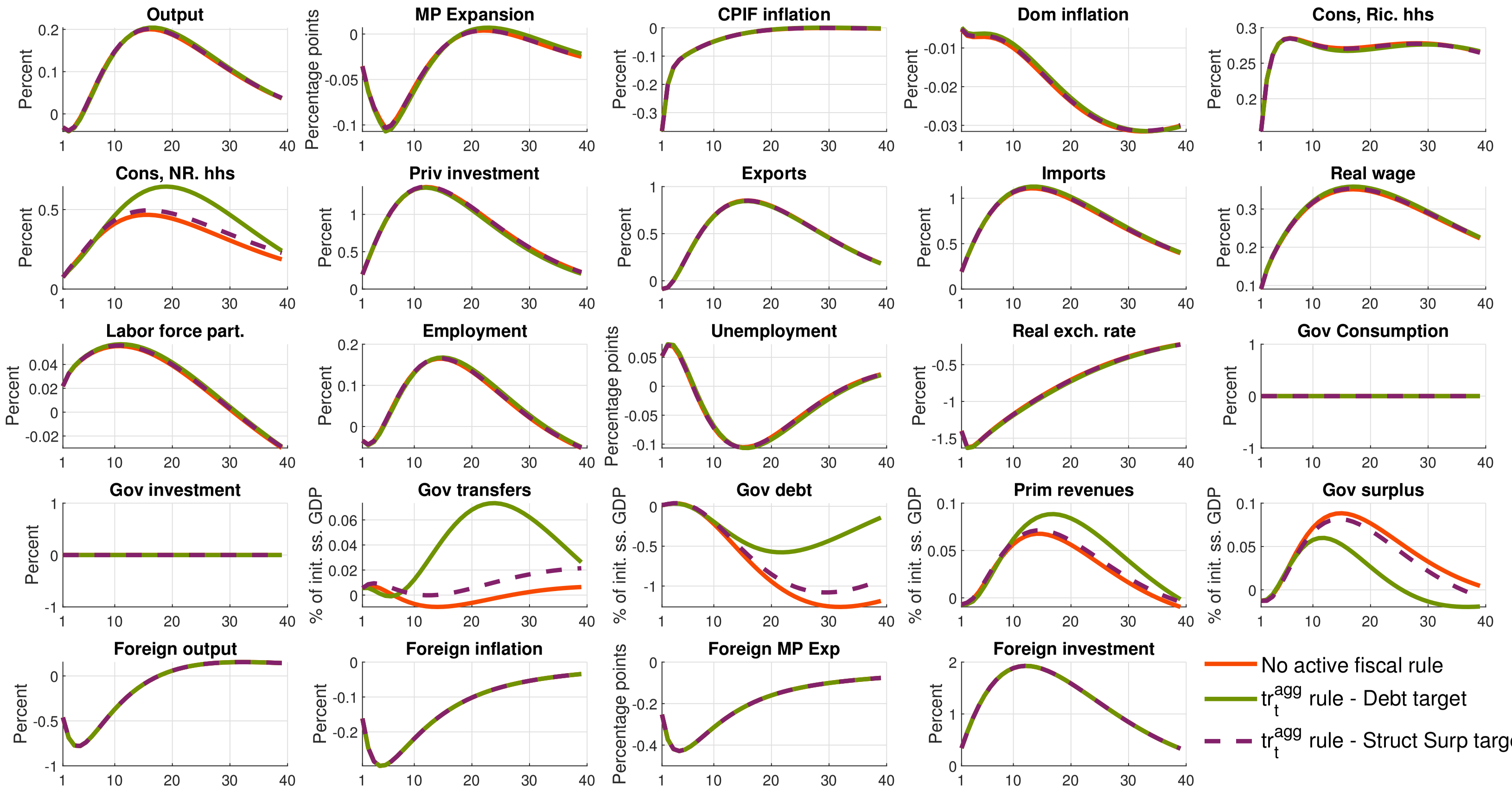
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 41: Foreign Nominal Interest Rate Shock



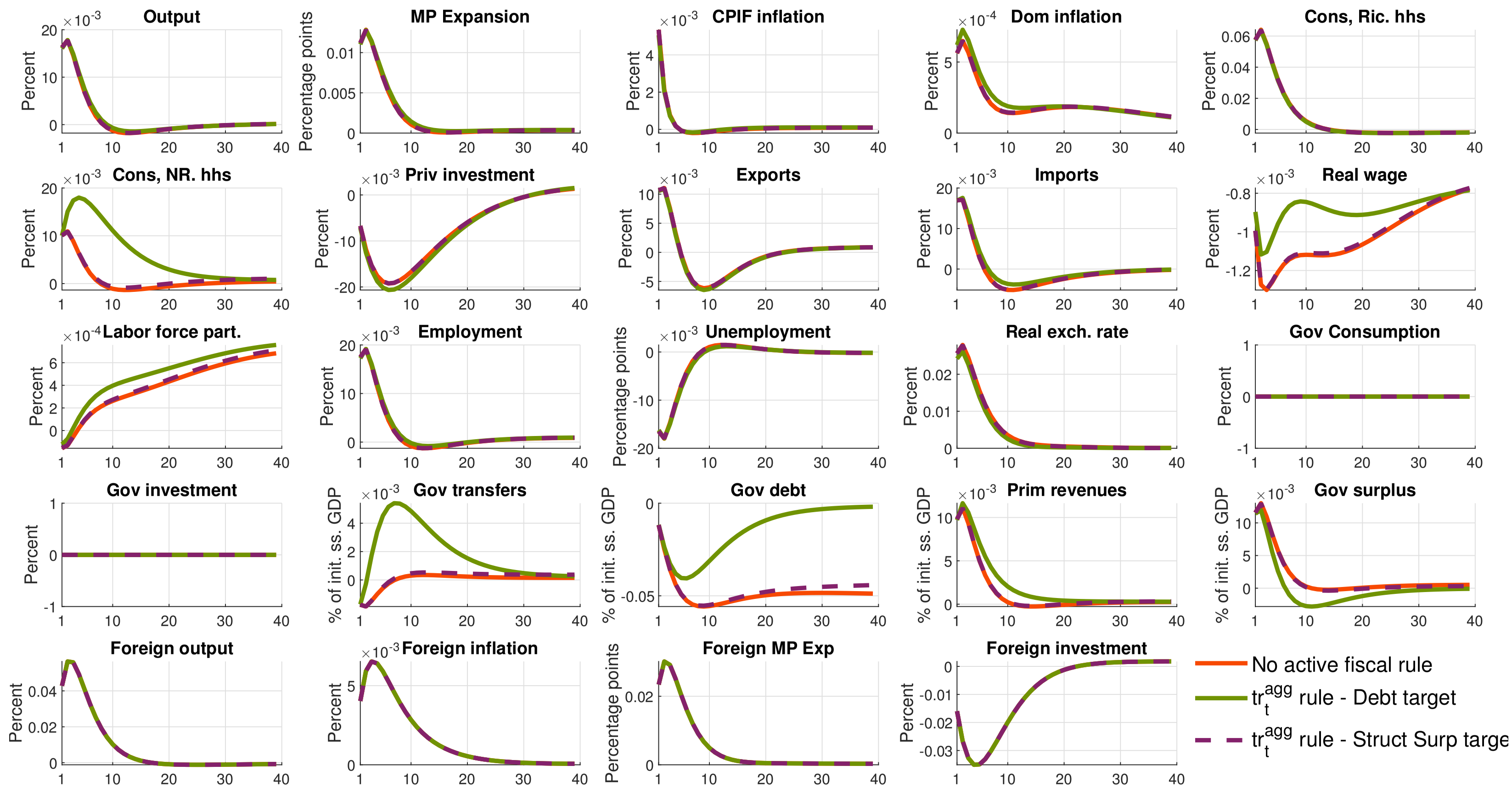
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 42: Foreign Risk Premium Shock



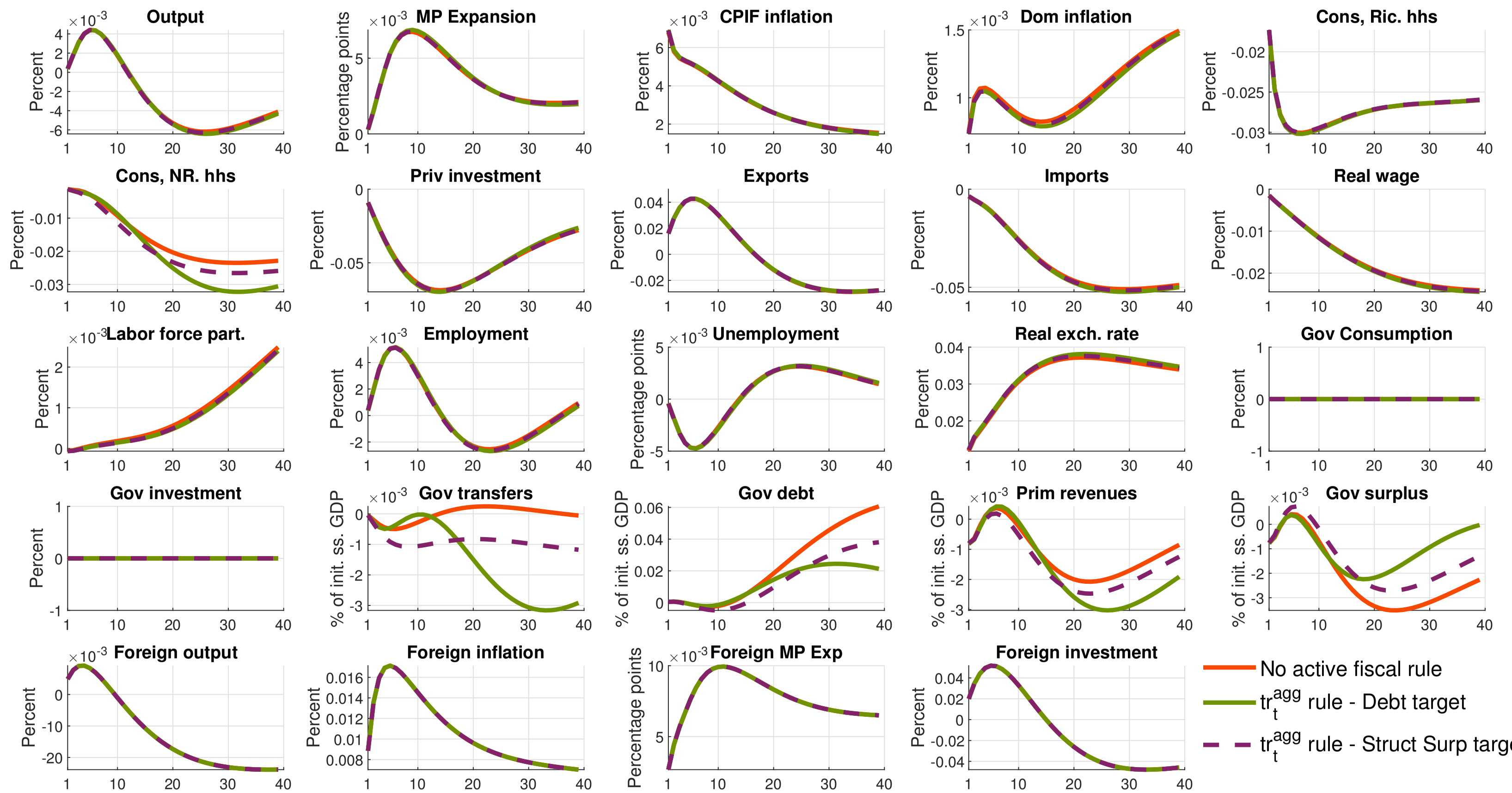
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 43: Foreign Discount Factor Shock



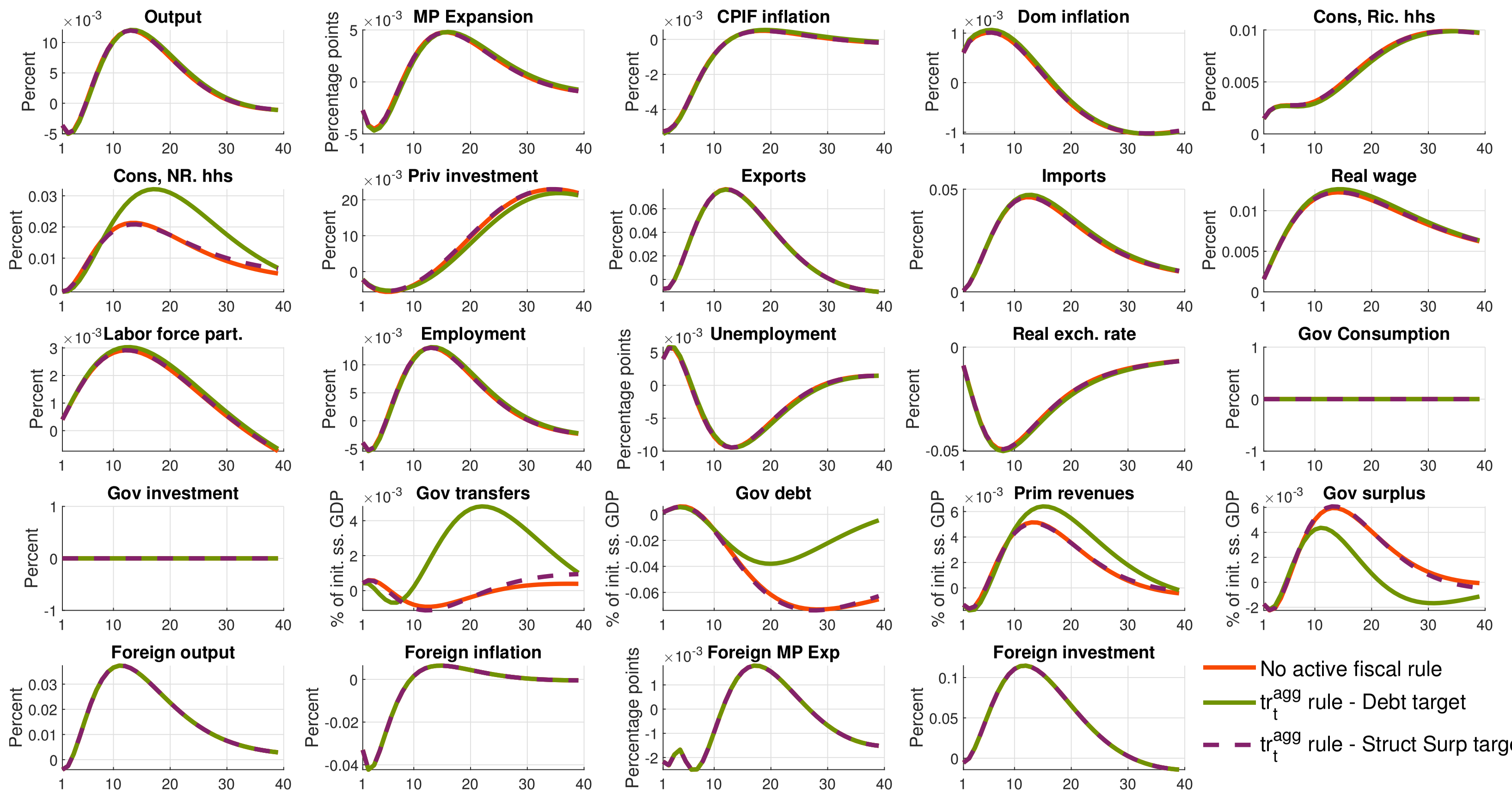
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 44: Foreign Consumption Preference Shock



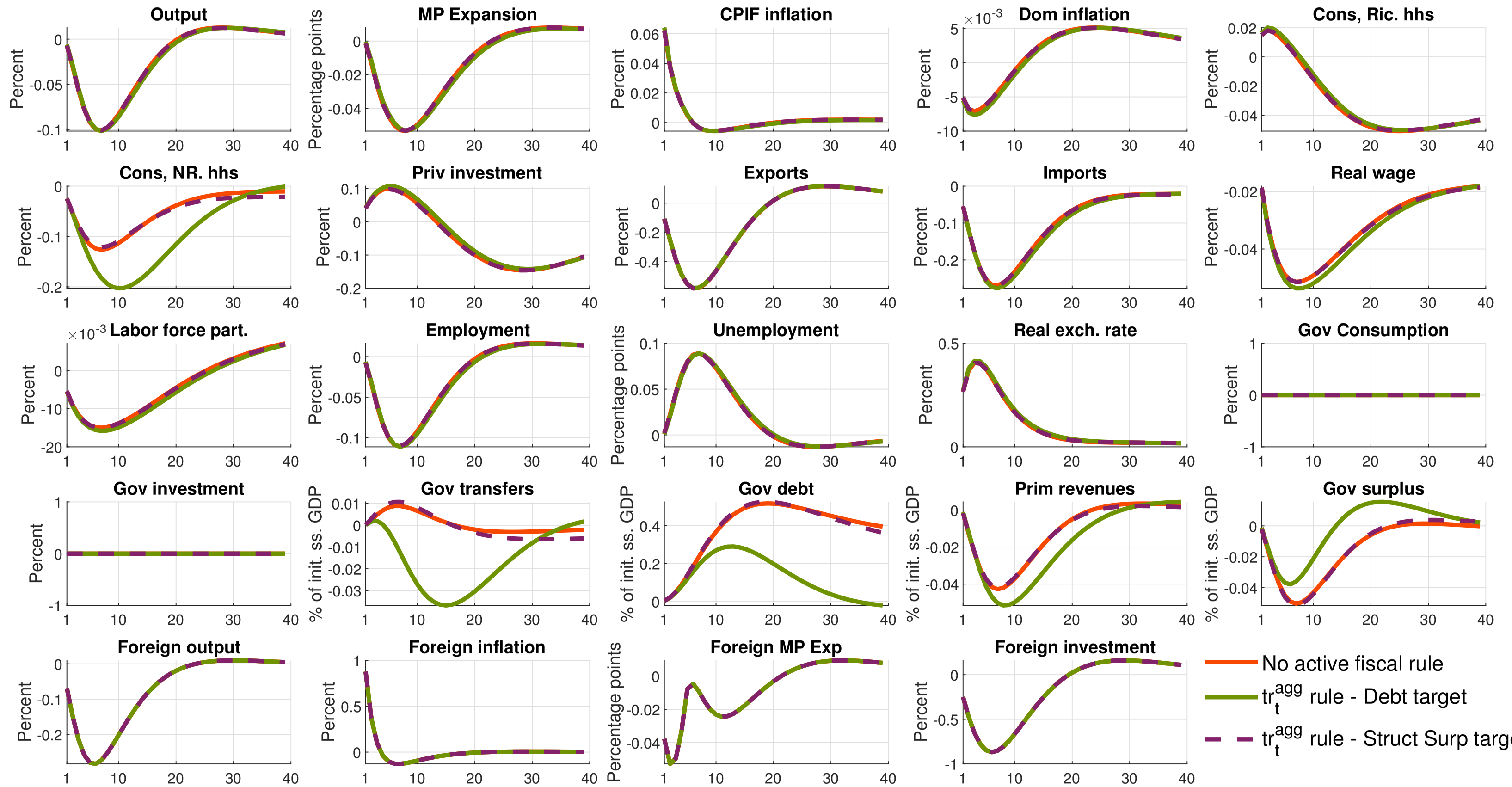
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 45: Foreign Labor Preference Shock



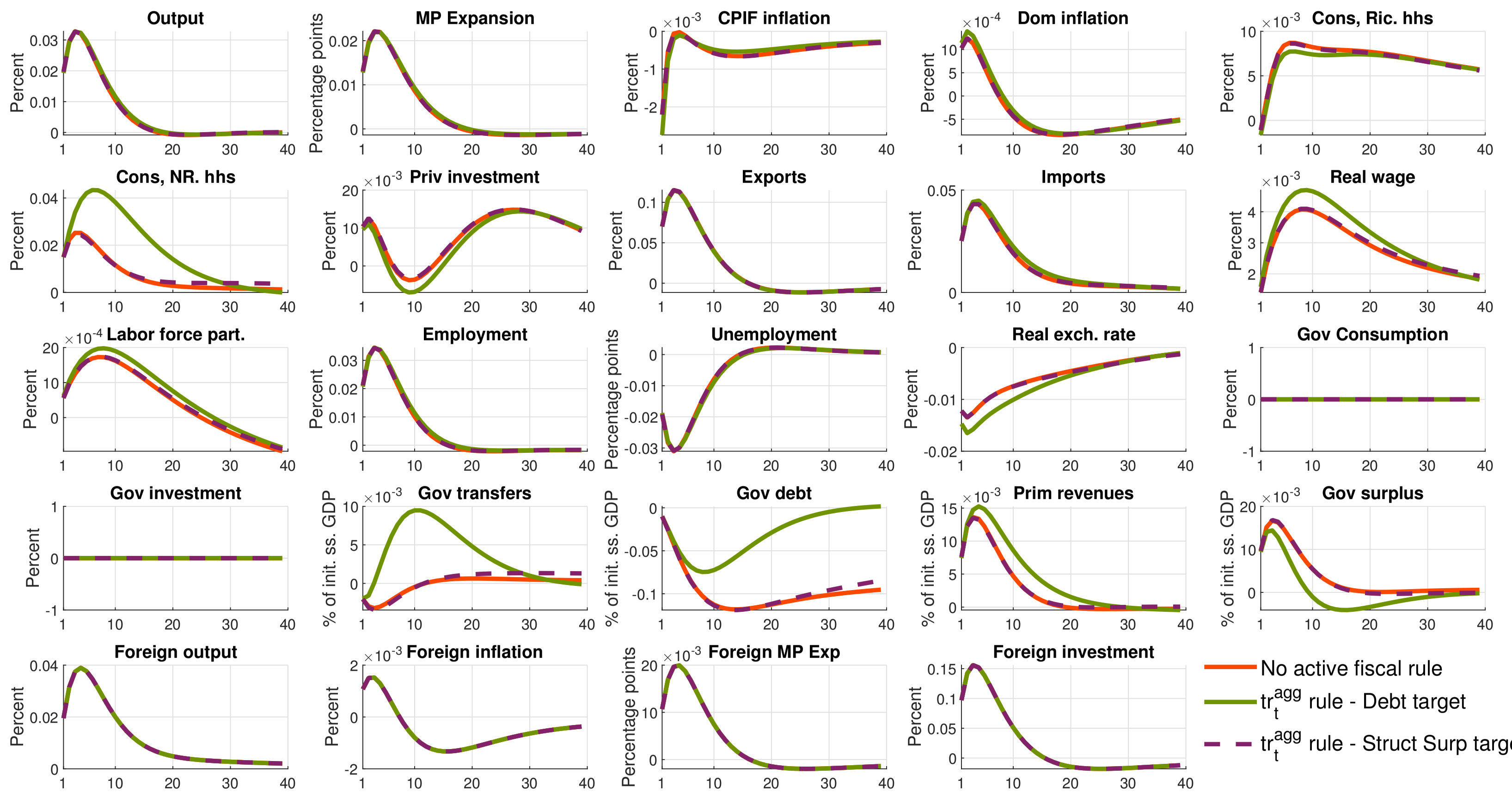
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 46: Foreign Stationary Technology Shock



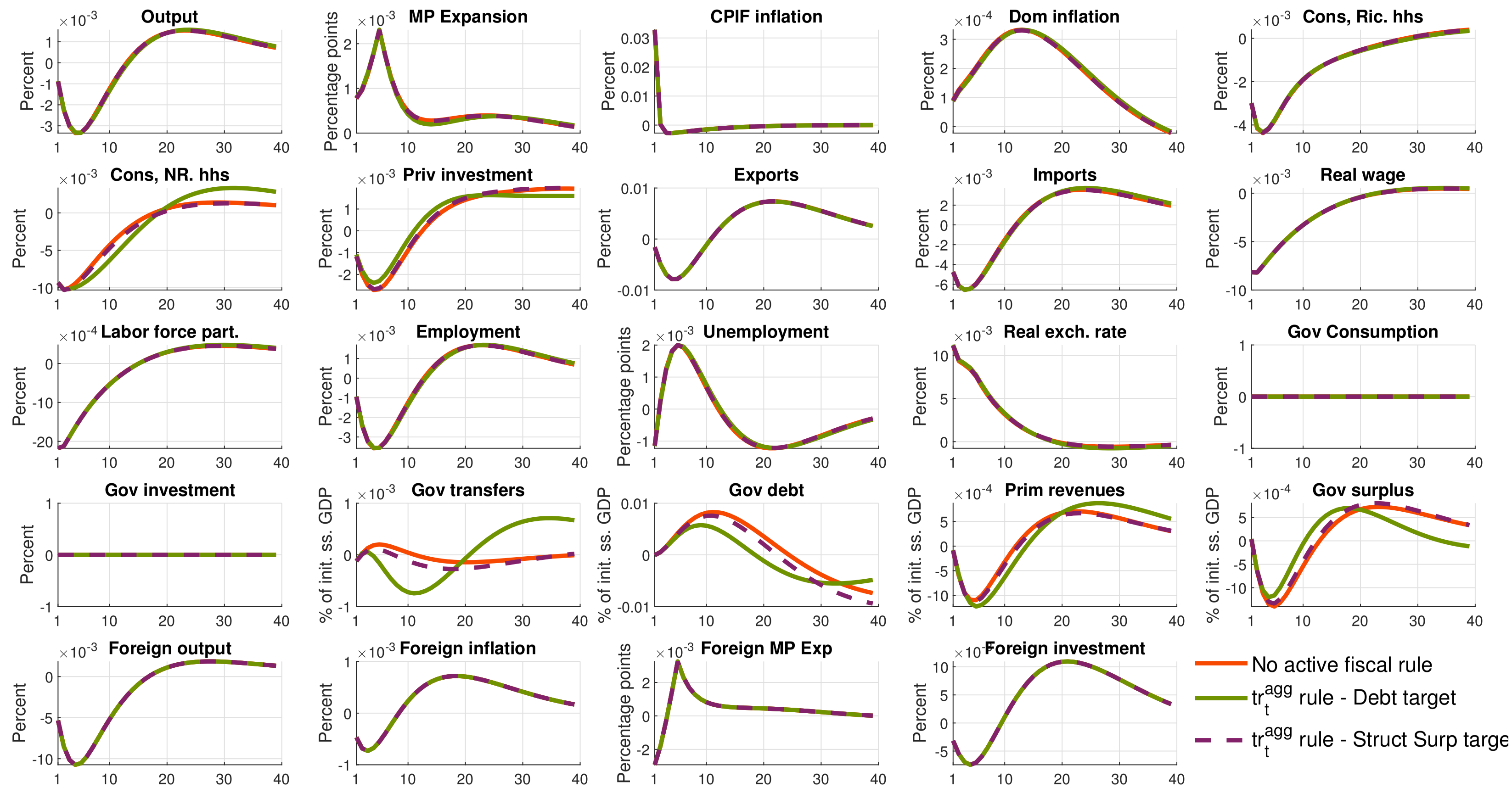
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 47: Foreign Intermediate Goods Price Markup Shock



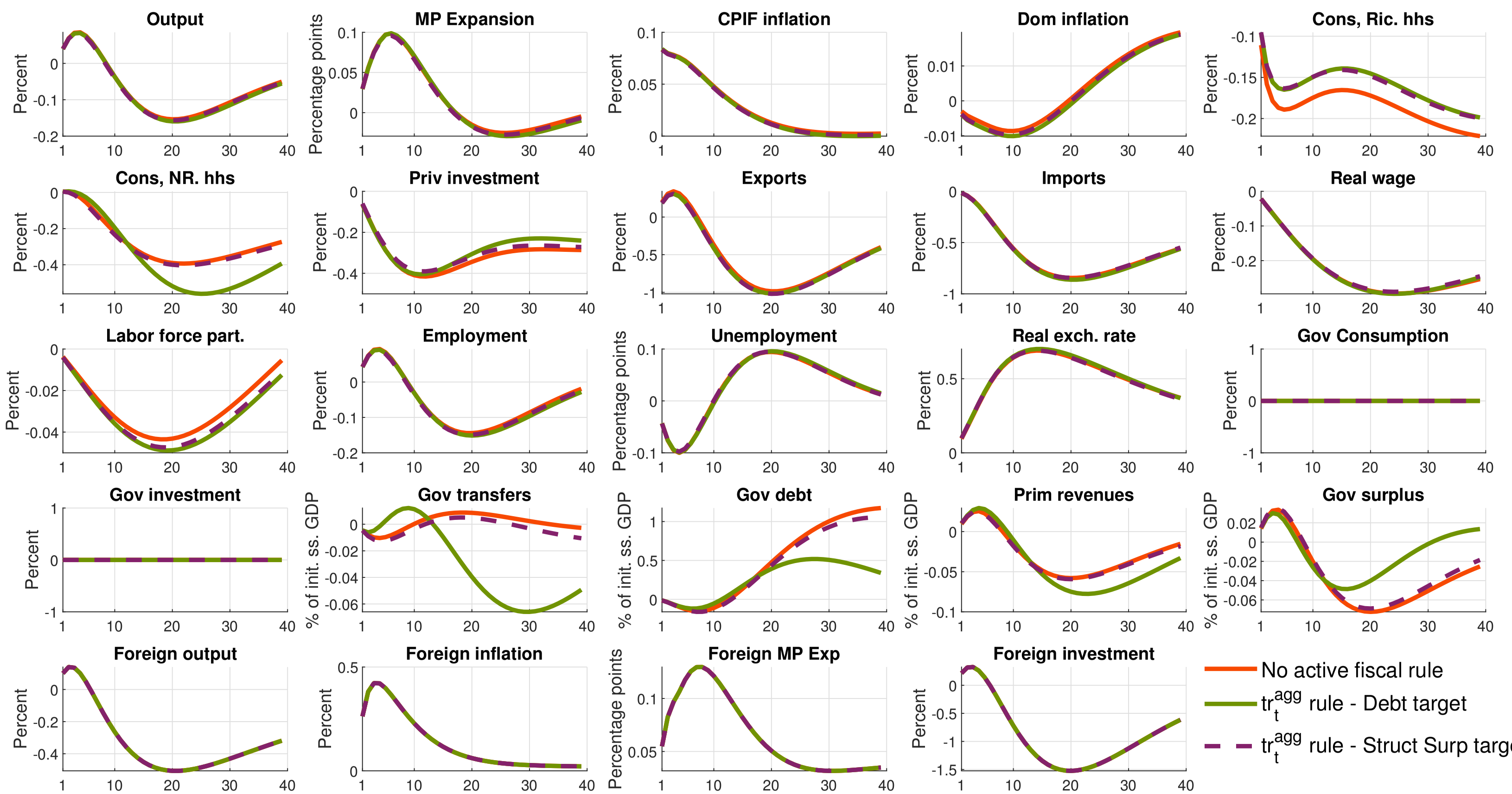
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 48: Foreign Stationary Investment Shock



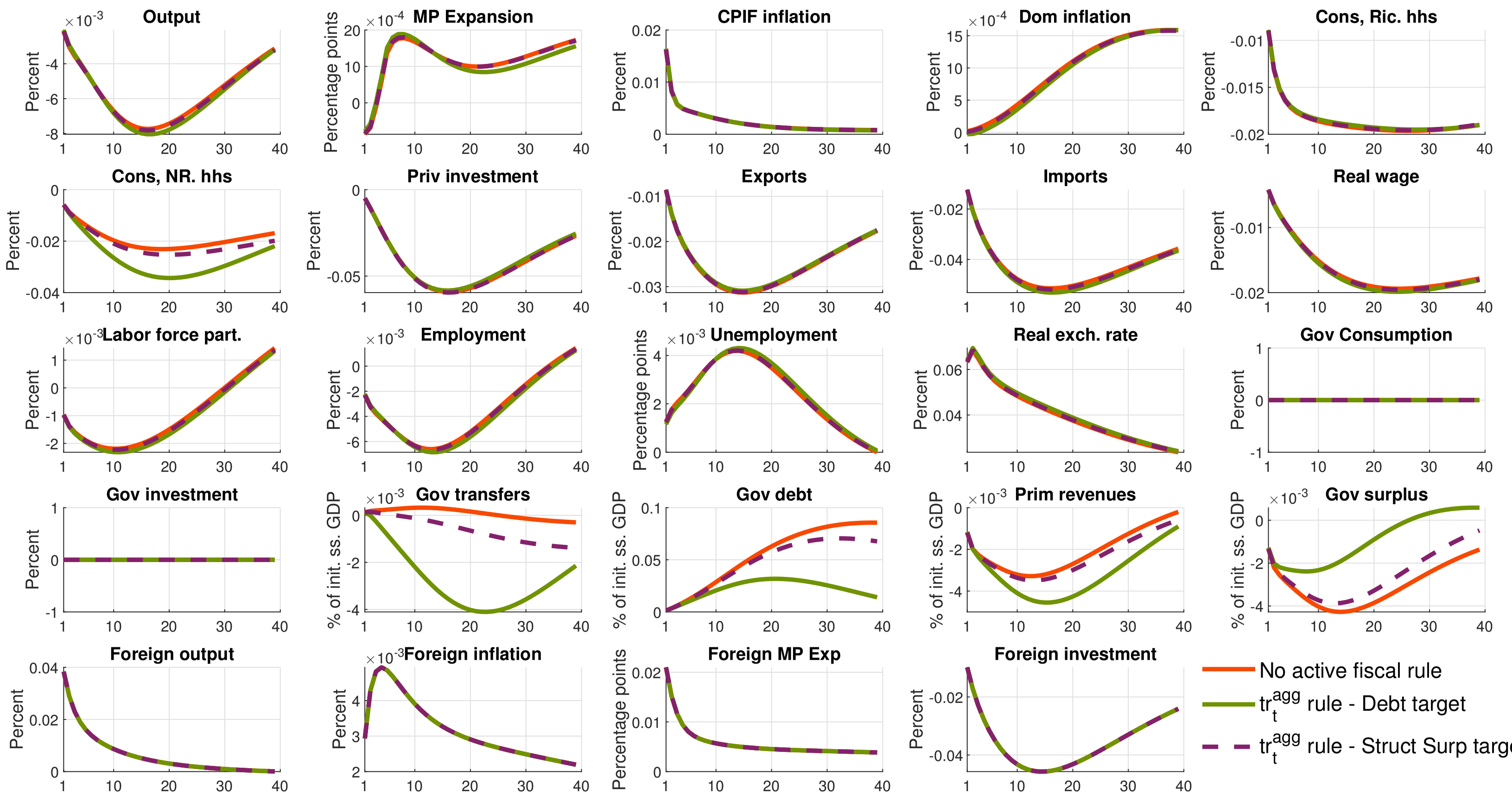
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 49: Foreign Energy Price Shock



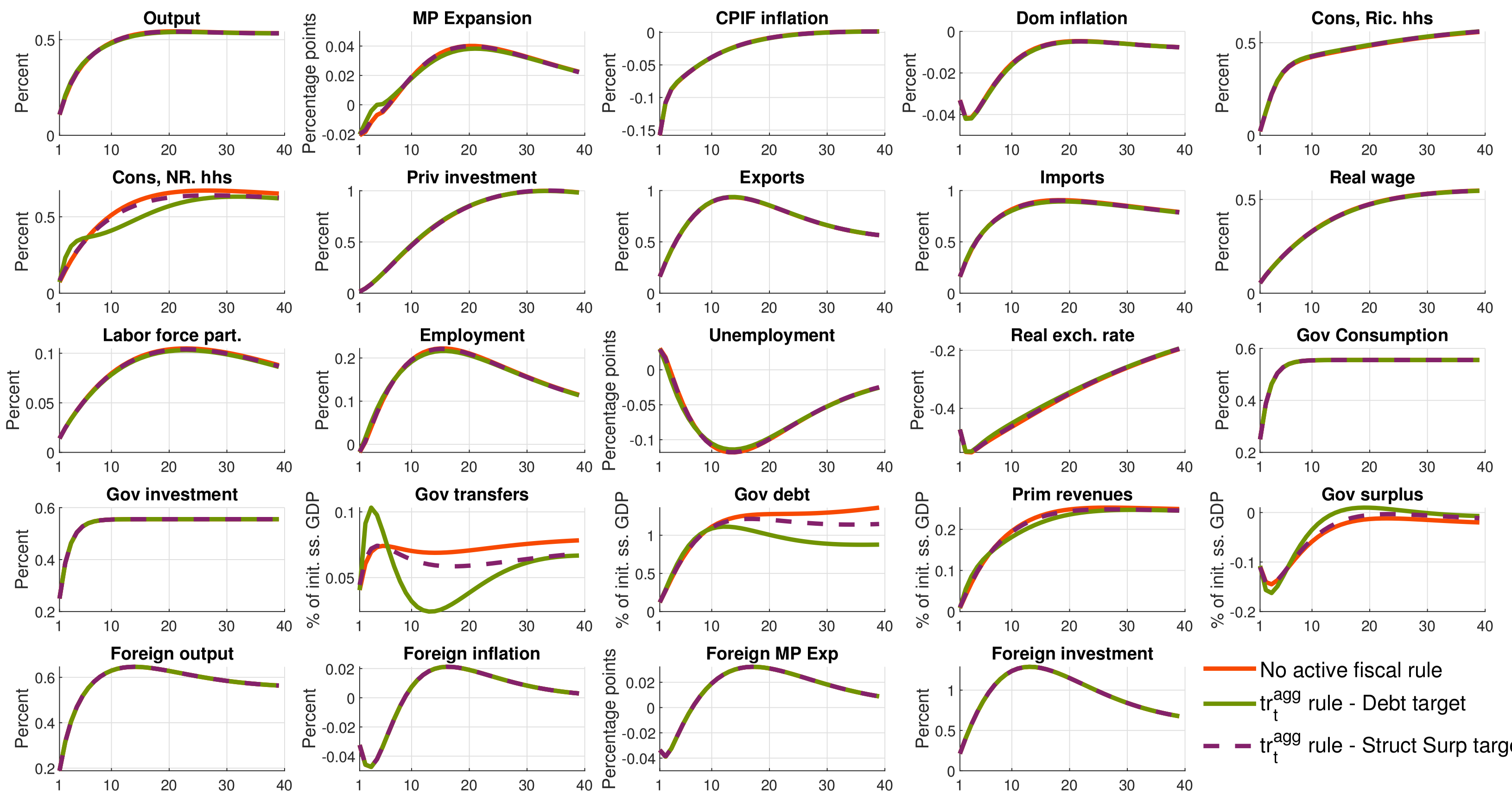
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 50: Foreign Inflation Trend Shock



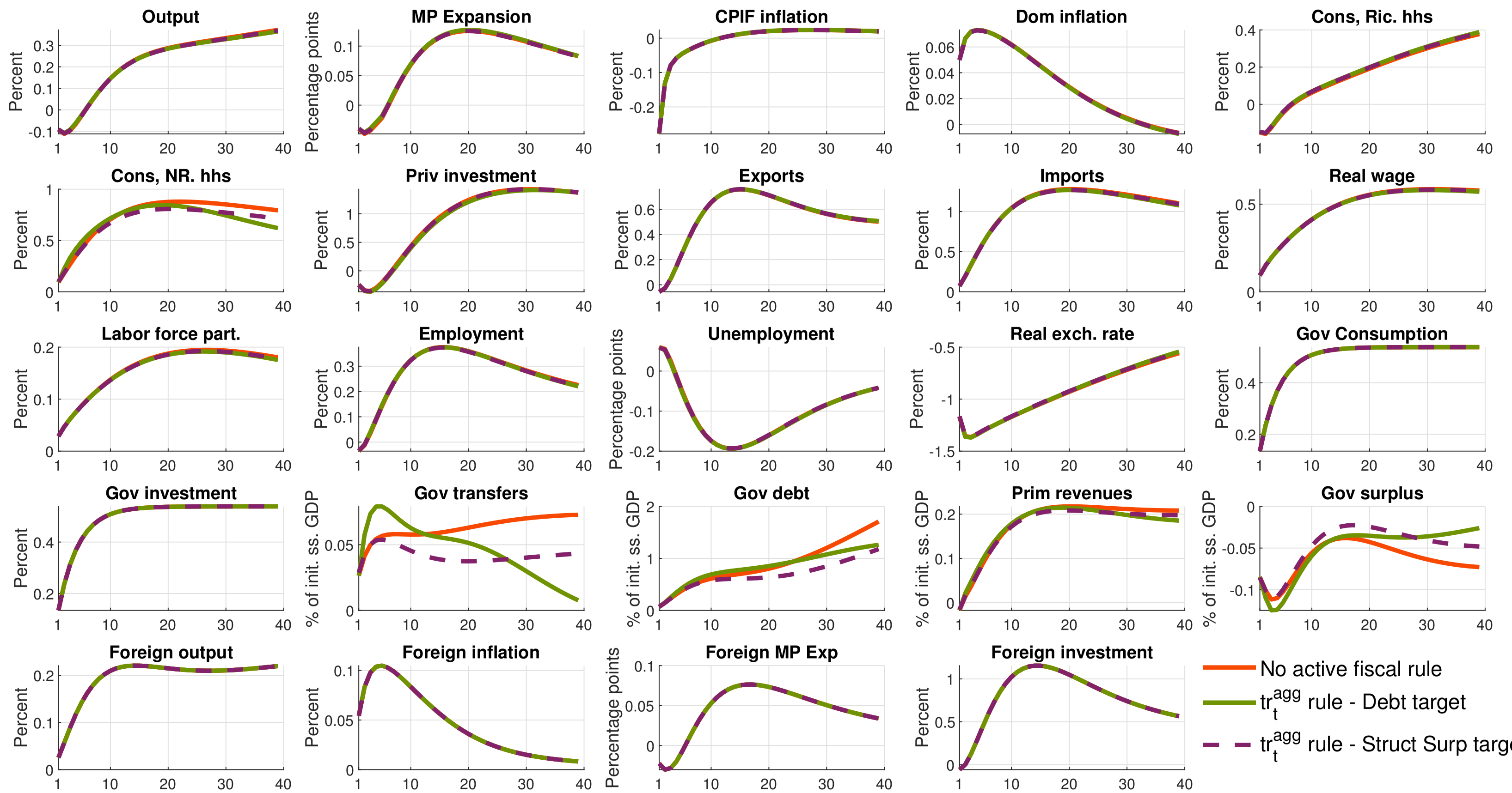
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 51: Foreign Government Consumption Shock



Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 52: Global Labor Technology Shock



Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 53: Global Investment Technology Shock