Second Draft
88-11-18

An Approach to

THE SUPPLY SIDE OF THE ECONOMETRIC MODEL OF THE NIER

Alfred Kanis

Alek Markowski

National Institute of Economic Research

Stockholm, September 1988

CONTENTS

1. INTRODUCTION[1]

The econometric model of the NIER, KOSMOS, includes to date nine domestic markets: the labor market and eight markets for goods and services (see below). A number of markets for financial assets will be added later. For each of the goods traded in the goods markets, a foreign market is also postulated, thus increasing the number of markets to fourteen.

In each period each market achieves short-run equilibrium in the sense that there is no rationing. At the same time, an adjustment process towards a long-run equilibrium is postulated for each market. Since the adjustment most probably depends on the degree of disequilibrium, we have to define a disequilibrium measure for each market.

A complete market description should then comprise the following variables:

domestic demand
foreign demand (exports)
part of domestic demand met by foreign producers (imports)
price
domestic supply
productive capacity
demand for factors of production (determining future capacity)
a disequilibrium measure.

For the time being, only the first four variables are endogenized in KOSMOS.

As usual, non-public domestic demand is divided into private consumption, fixed investment and inventory investment. If prices of these three demand components are to be differentiated, the number of markets in the model increases rapidly.

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[1] This paper draws partly on a first draft written by A.Markowski.

The supply in goods markets can be modelled either directly or through productive capacity. If supply is modelled directly, the amount of involuntary inventory investment can be computed as supply minus demand (planned inventories being included in one of the two latter terms) and treated as a measure of disequilibrium. If the supply side is confined to productive capacity, the capacity utilization rate can be used as a disequilibrium measure.

The first approach corresponds to the situation when producers each period decide how much to supply. In the second case, the level of supply is determined by demand. The supply decision is then limited to adjusting productive capacity, given the target rate of capacity utilization. Obviously, the first approach is more general (but much more difficult), since demand can be exactly known only ex post and some short-run supply decision is needed even if the level of demand is seen as the main constraint.

The purpose of the present paper is to outline a possible approach to the supply side in KOSMOS. Notwithstanding the relatively low level of sophistication, it remains to see whether the suggested approach is a viable way of modelling the economy. Since very few regressions have yet been performed, the suggested specifications - and their alternatives - should be viewed as hypotheses to be tested. Future development should include a more explicit and more sophisticated expectation formation and, possibly, a vintage production function approach.

Some definitional conventions are warranted before proceeding. "Demand" or "quantity demanded at given price" will be synonymous with "purchases" (in the absence of rationing), assuming purchasers are price-takers. Similarly, "supply" or "quantity supplied at given prices" will mean "sellers' sales target plus planned inventory change", since inventories basically are assumed to be held by producers. Producers' sales target may differ from actual sales leading to unplanned inventory changes. The supply decision is understood here as the output decision, though in principle producers' inventory stocks should also be included in the general notion of supply.

2. GENERAL OUTLINE OF THE APPROACH

The starting point of our approach is the view, that supply decisions are taken separately from expenditure decisions. This is different from the traditional econometric models of the Keynesian type, where output is determined by demand. According to our view of the economy, the supply decisions, taken by the producers, need not match exactly the expenditure decisions taken by the consumers. Furthermore, in the short run prices do not adjust enough to ensure the equality of quantity supplied and demanded.

When a mismatch between supply and demand occurs, several adjustment processes are initiated. Prices, output, inventories, imports and demand for factors of production are simultaneously adjusted. Furthermore, since the identity

(1) imports + domestic output = exports + domestic demand

holds by definition, either imports or inventories play the role of the ultimate buffer. (Some degree of rationing can possibly be introduced, through increasing stocks of unfilled orders.)

Two possible approaches in this spirit are exemplified by the London Business School (LBS) model and the OECD Interlink model (cf Appendix 1 and 2). In the LBS model supply is determined through the price decision and imports are computed as a residual. Domestic output equals demand at the prevailing price and the producers implicitly determine the share of imports in domestic demand.

In the OECD model the producers determine both the volume of output and the price. If the demand for domestic output at the prevailing price is not equal to current production, an inventory change takes place. Thus, inventory investment is computed as a residual. The adjustment of prices (and output) is assumed to take time in both models.

Our approach has several features in common with the OECD

model. The starting point is a production function

(2) $$Q = f(K,L,t)$$

where Q - real output, K - capital stock, L - labor input and t - time trend proxying technical progress.

Producers are assumed to perceive factor prices and the price of output as given. Price adjustment is here seen as a process that takes place at an aggregate level. Each individual producer "follows the others" in respect of the price in the (output and factor) market.

The long-run demand for production factors is determined by the profit maximising conditions, subject to the production function and real factor prices. In the short run factor demand is assumed to adjust slowly towards the long-run solution.

The supply decision determines Helliwell's intensity of factor utilization[2] (IFU), i.e. the ratio of actual output to "normal" output. Normal output is determined upon substitution of actual labor input and capital stock into the production function. It defines the output that would be obtained with the actual factor inputs used at "normal" intensity.The IFU is employed in the model as a measure of market disequilibrium (analogously to the capacity utilization rate). In the long-run the IFU is equal to one:

(3) $$Q/QN = 1 \ ,$$

where QN - normal output.

In the short run output adjusts to the ratio of actual inventory stock to desired inventory stock, the ratio of normal sales to normal output and changes in profitability. Although it might seem to be a difficult task to assess the desired inventory stock, the OECD approach, that estimates it as the long-run solution to the IFU equation, is in that respect no better.

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[2] Cf J.Helliwell, P.Sturm, P.Jarrett and G.Salou, "The supply side in the OECD macroeconomic model", OECD Economic Studies No. 6, Spring 1986.

The intuition behind the notion of IFU is that both labor and capital should be treated as quasi-fixed factors. This means that in the very short run factor inputs adjust very little to variation in output. Rather, the existing factor inputs are employed at different levels of intensity, resulting in deviations of actual output from normal output. The assumption that labor is a quasi-fixed factor is, however, somewhat problematic when labor input is expressed in hours that include overtime.

The formulation of the output equation implies, that inventory stocks are assumed to be held by the producers of the goods in question. Inventory investment is obtained as the difference between output and the sum of domestic demand and net exports (cf equation (1) above).

The short-run domestic market price is determined by costs, excess demand (IFU[3]) and inflationary expectations. In the long run the price level is determined by the ratio of the money stock to the real demand for goods and services. The price of domestic output can then be obtained from the GDP identity.

Sector output prices can be explained in relation to the aggregate domestic output price, under the condition that the long-run growth rates of all the sector prices should be equal to that of the aggregate output price.

The proposed price determination mechanism requires the introduction of the money stock into the model. For the moment, the money stock can be either exogenous to the model or can be endogenised on an ad hoc basis.

An interim solution would be to make the price level equal to costs in the long run. It is, however, possible that the downward adjustment of prices would in that case be extremely slow. This adjustment would take place through the short-run effect of decreasing output on prices and - possibly - through the Phillips curve. In the price model proposed in

[3] According to Jarrett and Torres (cf P.Jarrett, R.Torres, "A revised supply block for the major seven countries in Interlink", OECD Working Papers No. 41, April 1987) IFU is insensitive to short run shocks and should be replaced by a capacity utilization rate.

the first place, adjustment takes place through the current balance effect on the money stock.

The EFO approach, i.e. setting domestic prices equal in the long run to the foreign price level, eliminates from the model important short-run adjustment mechanisms and gives it a long-run character.

3. THE PRODUCTION FUNCTION

The analysis in the present and the following three sections (i.e. sections 3-6) pertains to one sector of the economy. Thus, for each sector there are postulated a production function, two factor demand functions and an inventory investment identity .

The production function is postulated to be of the CES type:

$$Q = g[\delta K^{-r} + (1-\delta)L^{-r}]^{-n/r} \tag{4}$$

where n is the degree of homogeneity (n=1 means constant returns to scale); g is a scale parameter, that can be used to describe neutral technical change; δ describes to what extent the production process is capital intensive and r is the substitution parameter, such that the elasticity of substitution between capital and labor, σ, equals

$$\sigma = 1/(1 + r) \ . \tag{5}$$

The scale parameter, g, is assumed to incorporate neutral technical progress:

$$g = g_0 e^{g_1 t} \ , \tag{6}$$

where t - time trend.

An alternative approach would be to define technical change as purely labor-augmenting (Harrod-neutral) and to express

labor input in efficiency units[4], as:

$$(7) \qquad L = g_0 e^{g_1 t} E \ ,$$

where E - employment (hours).

The production function can be estimated directly, upon multiplication of the factor inputs by some capacity utilization measures, or from the marginal conditions[5]. The former approach involves either non-linear estimation or linearisation of the production function. It is not always easy to obtain reasonable results, so future difficulties at this stage of the work should not be underrated. A method of estimation from the marginal productivity conditions is suggested in the next section.

The production function is employed to compute normal output (QN), upon substitution of the actual labor input and the capital stock. While the production relation holds at different levels of intensity of factor utilisation, normal output describes the level of production that would be obtained at the "normal" intensity (cf section A3.3 in Appendix 3 for a discussion of normal output and capacity output).

4. FACTOR DEMAND

The producers are assumed to perceive factor prices and the price of the output as given. Prices are determined in the respective market and producers cannot affect them through their decisions.

Assuming profit maximisation (under perfect competition), we

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[4] Cf P.Jarrett, R.Torres, op.cit.

[5] Cf J.Helliwell, P.Sturm, P.Jarrett and G.Salou, op.cit.

obtain the marginal productivity conditions[6]:

(8) $$\delta Q/\delta K = n\delta g^{-r/n}\, Q^{1+r/n}\, K^{-(1+r)} = c/p$$

$$\delta Q/\delta L = n(1-\delta)g^{-r/n}\, Q^{1+r/n}\, L^{-(1+r)} = w/p\ ,$$

where p - output price, c - price of capital, w - price of labor and $\delta A/\delta B$ is the partial derivative of A with respect to B.

Solving for K and L, respectively, we get:

(9) $$K = (c/p)^{-\sigma}\, Q^{\sigma(1+r/n)}\, [n\delta g^{-r/n}]^{\sigma}$$

$$L = (w/p)^{-\sigma}\, Q^{\sigma(1+r/n)}\, [n(1-\delta)g^{-r/n}]^{\sigma}\ .$$

Allowing for the technical change in accordance with equation (6) above, we have:

(10) $$K = (c/p)^{-\sigma}\, Q^{\sigma(1+r/n)}\, e^{(\sigma-1)ng_1 t}\, [n\delta g_0^{-r/n}]^{\sigma}$$

$$L = (w/p)^{-\sigma}\, Q^{\sigma(1+r/n)}\, e^{(\sigma-1)ng_1 t}\, [n(1-\delta)g_0^{-r/n}]^{\sigma}\ .$$

Equations (10) define the profit-maximising real demand for the factors of production. They are thus postulated to constitute the long-run solutions to our factor demand equations. In the short run factor inputs can differ from the optimal ones, due to adjustment costs.

The volume of output, Q, is in this context an endogenous variable, that is controlled by the producers. In equations (10) Q is subject to the technological constraint, i.e. it is equal to the output from the production function with inputs K and L, as defined by (10).

In the short run demand for capital is affected by the variation in output, real cost of capital (c/p) and relative profitability (RPROF). The latter variable describes the profitability of production in relation to the profitability in alternative uses of capital, e.g. financial investment. The relation between the profit rate (PROF) and the interest

[6] Cf K.Wallis, Topics in applied econometrics (London:Gray-Mills Publishing Ltd., 1973).

rate (R) should be expressed as the (logarithmic) deviation from the normal value[7] of PROF/R, so that the average value of ln(RPROF) be equal to zero.

Demand for labor is in the short run affected by the variation in output and in real labor cost (w/p).

The equations to be estimated have the error-correction form[8]:

$$(11)\qquad \begin{aligned} DlnK = -a_4[&lnK_{-1} - a_0 - a_1 ln(c/p)_{-1} - a_2 lnQ_{-1} - a_3 t] \\ &+ \Sigma a_{5,i} Dln(c/p)_{-i} + \Sigma a_{6,i} DlnQ_{-i} \\ &+ \Sigma a_{7,i} DlnK_{-i} + \Sigma a_{8,i} RPROF_{-i} \end{aligned}$$

$$\begin{aligned} DlnL = -b_1[&lnL_{-1} - b_0 - a_1 ln(w/p)_{-1} - a_2 lnQ_{-1} - a_3 t] \\ &+ \Sigma b_{2,i} Dln(w/p)_{-i} + \Sigma b_{3,i} DlnQ_{-i} \\ &+ \Sigma b_{4,i} DlnL_{-i} \ , \end{aligned}$$

where a_i, $a_{j,i}$, b_i , $b_{j,i}$ - coefficients, $DX = X - X_{-1}$ and

$$(12)\qquad DlnK = ln(1 + \text{real gross fixed investment}/K_{-1}).$$

It should be noted that the coefficients a_1 , a_2 and a_3 are defined in (10) and are common to both equations. Equations (10) define also the relation between a_0 and b_0. A comparison of (11) and (10) will show that a_1 equals the elasticity of substitution with the opposite sign. We can also note, that non-increasing (but positive) returns to scale imply[9]:

$$(13)\qquad a_2 \geq 1 > \sigma \quad \text{or} \quad a_2 \leq 1 < \sigma \ .$$

The coefficient a_2 equals 1 in the case of constant returns to scale.

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[7] The "normal" value is here understood as a sort of equilibrium value and not necessarily the sample mean.

[8] The long-run solution to the equations is given explicitly. The estimated form can be obtained upon multiplying $-a_4$ and $-b_1$ by the terms in the respective square bracket.

[9] Since from the definition of a_2
$n=(1-\sigma)/(a_2-\sigma)$
we obtain that
$n>0$ if $1>\sigma<a_2$ or $1<\sigma>a_2$.
Furthermore,
$n\leq 1$ if $1\leq a_2>\sigma$ or $1\geq a_2<\sigma$ and
$n>1$ if $1>a_2>\sigma$ or $1<a_2<\sigma$.

If the production function (4) is estimated separately, the coefficients a_0 , a_1 , a_2 , a_3 and b_0 should be imposed on (11) in accordance with their definitions in (10). Alternatively, equations (11) can be estimated simultaneously and the parameters of the production function can be identified from the estimates of a_0 , a_1 , a_2 , a_3 and b_0. While the computation of σ , n and g_1 is straightforward, we can note that:

(14) $$\delta = [\ \exp(a_0/\sigma)/[\exp(a_0/\sigma) + \exp(b_0/\sigma)]\ ]/n\ .$$

$$g_0 = [\ [\exp(a_0/\sigma)/n]/\delta\ ]^{-n/r}\ .$$

Besides, it can be easily seen from equation (4) above (after substitution of (6)), that computation of δ and g_0 is really not necessary, since $\exp(a_0/\sigma)/n$ and $\exp(b_0/\sigma)/n$ can directly be used as weights for K and L, respectively, in the production function.

Should simultaneous estimation of equations (11) fail or give unreasonable results, it is possible to estimate the parameters of the production function from the labor demand equation and then impose them on the investment equation[10].

Finally, let us note that it is important to give reasonable empirical content to c and w. The variable c should be computed as appropriately defined user cost of capital. The variable w represents labor cost, including employers' contribution to social security and wage-cost taxes paid by the employers.

5. OUTPUT DECISION

The profit maximising level of output is given in the model by normal output (cf section 3). The latter is obtained at the "normal" level of intensity of factor utilisation. Following Helliwell[11], we describe here the output decision in terms of the intensity of factor utilisation (IFU), i.e. the ratio of actual output to normal output.

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[10] Labor demand equations of the above form have been estimated by A.Markowski. Cf A.Markowski, "Demand for labor" in Kosmos - An econometric model for Sweden, Konjunkturinstitutet, 1988-05-31, mimeo.

[11] Cf J.Helliwell,P.Sturm,P.Jarrett,G.Salou, op.cit.

In the short run, this ratio is assumed to be affected by changes in the profitability of production, the deviation of inventory stocks from their desired value and the deviation of sales from their long-run value. Sales are here defined as final domestic demand for the aggregate in question (consumption plus fixed investment) minus imports plus exports. The approach employed in the 1985 version of the OECD model[12], where only a part of imports was subtracted, is quite appealing , but it is difficult to define a long-run value for such an aggregate. The long-run value for our sales variable is normal output minus the appropriately defined "normal" inventory investment.

The normal intensity of factor utilization is by definition the profit-maximising one. The long-run solution to our output equation gives Q = QN, i.e. the normal intensity of factor utilisation.

The output equation to be estimated has the following form (note the zero intercept):

$$(15) \qquad \begin{aligned} DlnQ = & -a_1 ln(Q/QN)_{-1} + \Sigma a_{2,k} DlnPROF_{-k} \\ & + a_3 ln[\, SALES/(QN - NII)\,] \\ & + a_4 ln(ISTOCK/NISTOCK)_{-1} + \Sigma a_{5,i} DlnQ_{-i} \end{aligned}$$

where Q - real output, QN - normal (real) output, PROF - profitability of production index (cost over price), SALES - real final demand for domestic output, i.e. real output minus actual (real) inventory investment, NII - "normal" (real) inventory investment (compatible with the definition of NISTOCK below), ISTOCK - real inventory stock (end-of-period value), NISTOCK - desired real inventory stock (end-of-period value).

The desired inventory stock equals normal output times the desired stock/output ratio, defined as NISTOCK/QN. The latter ratio can be postulated to be constant , trendwise declining or a function of the long-run rate of inflation and the output[13]. It may be extremely difficult to assess the desired stock/output ratio and in that case we should follow the OECD

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[12] Ibid.

[13] Cf P.Jarrett,R.Torres, op.cit.

approach[14] and include its determinants (multiplied by QN) in the equation. In each case the definition of NII should be made consistent with the definition of NISTOCK.

The ratio Q/QN can be used in the demand and price equations as a measure of excess demand. As already mentioned, Jarrett and Torres[15] point out, that this measure is insensitive to short-run shocks, due to the fact that both its numerator and denominator include actual (rather than potential) employment. Jarrett and Torres suggest, that a capacity utilisation measure could be used instead of Q/QN .

6. RECONCILIATION OF SUPPLY AND DEMAND

Output is to be determined in the model through the output function defined in the preceding section. The demand for goods and services is determined by the investment equation, described in section 4 above, and a consumption function. The import function is to determine the share of imports in total domestic demand. The latter function (as well as the export function) should react to the variation in capacity utilisation or IFU.

The reconciliation of supply and demand is then obtained through residual computation of inventory investment (cf equation (1) above):

(16) Inventory investment = output + imports - exports - fixed investment - private consumption.

Inventory investment includes here both desired and undesired inventories.

This approach may lead to large forecasting errors in inventory investment, since e.g. a small relative error in private consumption implies a corresponding error in the inventory investment that is very large by the standards of the latter variable. Inventory investment is difficult to forecast in any case. The advantage of our approach is that inventory investment is consistent with the buffer stock

[14] Ibid.

[15] Ibid.

assumption. Given the generally asserted poor quality of the real inventory data, the discrepancy between the simulated inventory investment and the actual data need not be a problem.

7. PRICE DETERMINATION

The suggested approach to the determination of the overall price level follows that of the Riksbank's Minimac model[16]. We may note that Minimac experience indicates that the model is prone to inflation in simulations.

According to our approach, we determine first the overall (non-public) price level in the domestic market. The sectoral prices are then defined in relation to the overall price level.

Price inflation is here considered to be in the long run a monetary phenomenon, resulting from the asymmetry in the monetary and real expansion. Monetary expansion, that is not accompanied by a proper increase in real sales, results inevitably in a higher rate of inflation.

The domestic market price level is thus postulated to be determined in the long run by a real money demand function, which is linearly homogeneous in demand:

$$(17) \qquad \ln(M/P) = c_0 + \ln DD + c_1 R$$

where M - money stock (M_3), DD - real domestic demand (non-public GDP minus exports plus imports), P - implicit deflator for DD, R - money market interest rate.

The money stock is here defined as M_3, including time deposits and saving accounts. A more narrow definition would probably be more appropriate, but is difficult to obtain, owing to the peculiarities of the Swedish banking system[17].

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16 Cf A.Markowski, Minimac - A small econometric model of the Swedish economy from a central bank's perspective, Sveriges Riksbank, forthcoming 1988.

17 There is no clear difference between demand deposits and time and saving deposits.

The demand variable in equation (17) includes inventory investment. The latter comprises - as already mentioned - both desired and undesired inventories (cf section 5 above). Although inventories are here assumed to be held by the producers of the goods in question, we are inclined to consider desired inventory investment as part of the demand. Undesired inventories obviously do not represent any demand, but we conjecture that their share in total inventory investment is rather small.

The implication of this conjecture is, that the coefficient a_4 in equation (15) is rather large (greater than 0.6 ?). Should that coefficient prove to be small, it will be more appropriate to exclude inventory investment from DD.

In the short run, price inflation is affected by the variation in unit cost (UCOST), import prices (PM), indirect taxes (VAT), excess demand (IFU) and inflationary expectations ($DlnP_{-i}$).

The equation to be estimated has the form:

(18)
$$DlnP = -c_2[lnM_{-1} - ln(P \cdot DD)_{-1} - c_0 - c_1 R_{-1}] + \Sigma c_{3,i} DlnUCOST_{-i} + \Sigma c_{4,i} DlnPM_{-i} + \Sigma c_{5,i} D(VAT)_{-i} + \Sigma c_{6,i} D(IFU^2)_{-i} + \Sigma c_{7,i} D(R)_{-i} + \Sigma c_{8,i} DlnP_{-i}$$

where UCOST - unit cost of domestic output, PM - implicit deflator for imports, VAT - value added tax rate (fraction) and other variables are defined above.

The long-run effect of the import price variable should be consistent with the share of imports in domestic demand and the price elasticity of imports. The long-run effect of the VAT variable should be consistent with the share of goods subject to VAT in domestic demand.

The IFU is squared in equation (18) in order to obtain large responses at high levels of capacity utilisation. The experience with Minimac indicates, that capacity effects are limited.

The market price of the domestic output supplied to the domestic market can be obtained from P by weighting out imports. The producer price can then be computed upon subtracting changes in VAT.

Sectoral price changes are in the short run determined in relation to the overall price inflation, on the basis of relative costs. In the long run, sectoral price growth is assumed to be equal to the overall price growth. It seems unduly restrictive to assume that the sectoral price levels are in the long run equal to the overall price level.

A typical equation to be estimated could have the form:

$$(19) \qquad \begin{aligned} DlnP^i - DlnP = {} & e_1(DlnUCOST^i - DlnUCOST) \\ & + e_2(DlnPM^i - DlnPM) \\ & + e_3[D(IFU^{i2}) - D(IFU^2)] \\ & + \Sigma a_{4,i}(DlnP^i - DlnP)_{-i} \ , \end{aligned}$$

where the superscript i denotes the i-th sector.

If the long-run value of any of the relative terms on the right-hand side of equation (19) is not 1, the equation should be complemented with an intercept.

For K sectors, K-1 sectoral prices can be determined using equation (19). One sectoral price must be computed as a residual, in order to ensure consistency of the sectoral prices with the overall market price.

8. MARKET DEFINITIONS

The markets for goods and services in KOSMOS comprise for the moment:

1) Agriculture, hunting, forestry and fishing
2) Mining and "half-manufactures" excluding coal and crude petroleum[18]
3) Coal, crude petroleum, petroleum refining and manufacture of products of petroleum and coal

[18] Mining and quarrying excluding coal mining and crude petroleum production; manufacture of food beverages and tobacco; saw mills, planing mills, wood preserving plants; wood pulp industries; non-ferrous metal basic industries.

4) Manufacturing[19]
5) Electricity, gas and water
6) Construction
7) Services excluding dwellings[20]
8) Letting of dwellings and use of owner-occupied dwellings.

Thus, market 1 comprises agriculture, which is mainly regulated, and production of timber , that is not regulated. By the same token, market 2 includes both protected (i.e. regulated) and import-competing food manufacturing. Market 4 includes iron and steel basic industries (non-ferrous metals are in market 2!), that produce the same products as their competitors abroad, and machinery and equipment, that usually is assumed to be "distinguished by the place of production".

While it is difficult to define homogeneous products at this level of aggregation, it would be desirable to define markets that are homogeneous with respect to the institutional environment. In this case, this would mean markets that comprise either regulated or non-regulated production and products either facing one world price or distinguished by the place of production. This kind of homogeneity would make modelling supply and demand (including foreign demand) much easier.

One possible approach to market definitions, that better complies with the above requirements, would be to define markets for:

a)agriculture and protected food manufacturing
b)mining and basic industries facing a world price (excluding energy sources)
c)energy sources
d)manufacturing of products more or less distinguished by the place of production

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[19] Textile,wearing apparel and leather industries; manufacture of wood products (excluding saw mills,planing mills,wood preserving plants and wood pulp industries),paper and paper products; printing and publishing; manufacture of chemicals and chemical,rubber and plastic products (excluding coal and petroleum products); manufacture of non-metallic mineral products (except products of petroleum and coal); iron and steel basic industries; manufacture of fabricated metal products,machinery and equipment; other manufacturing industries including public semi-industrial activities (samhällsföretag).

[20] Wholesale and retail trade,restaurants and hotels; transport storage and communication; financing, insurance, real estate and business services (excluding letting of dwellings and use of owner occupied dwellings); personal services.

e)services (including electricity, gas and water and excluding dwellings)
f)dwellings

Construction can either be included in the service market or constitute a separate market.

The sector division proposed above is still fairly disaggregated. Consequently, it requires extensive data computation and data updating. Furthermore, it is quite obvious that the supply system suggested above can hardly be expected to be estimable for each of the sectors. Rather, a much higher level of aggregation is appropriate on the supply side. Given the persistent data problems, there is no reason why the demand side shouldn't be aggregated accordingly. In our opinion, a division into

a)manufactures
b)non-manufactures (i.e. all the rest)

would be sufficient and adequate. It is understood that the production of manufactures is much more capital-intensive than the production of non-manufactures. The degree of capital-intensity is, in our view, the most important distinction for a fairly general description of the structure of the economy.

APPENDIX 1. THE SUPPLY SIDE OF THE LBS MODEL

The description of the London Business School model is based on A.Budd,E.Dinenis,S.Holly, "Supply-side developments in the LBS model", paper presented to the Second International Conference on Economic Modelling, London, March 1988. In order to give a better account of the model, the main relations on the demand side are also described. We start by the supply side.

A1.1 Output

Output is determined in the LBS model for a)manufacturing, b)non-manufacturing. The supply of energy & water and the output of the public sector are treated as exogenous to the model. Below, we derive only the estimated model for manufacturing. The model for non-manufacturing is analogous, the main exception being that raw materials & energy are replaced by energy only.

Firms are assumed to operate in imperfectly competitive markets.

The sector uses three factors of production - labor L, capital K and raw materials M - to produce gross output O. Part of the output, namely X, is exported, the remainder, Y, is supplied to the domestic market.

Exports and goods supplied to the domestic market are not perfect substitutes, so the production frontier is described by the convex transformation function

$$(A1.1) \qquad \Phi(L,K,M,Y,X) = 0$$

or an equivalent restricted cost function

$$(A1.2) \qquad C(w,p,K,Y,X) = \min_{L,M}(wL + p_r M \mid \Phi \geq 0)$$

where w is the nominal wage and p_r is the price of raw materials including energy. The capital stock is assumed to be constant in the short run and is therefore not included in the minimisation process.

The domestic demand for domestic production is given by

(A1.3) $Y = Y(p, p^f, Q)$

where p is the price of domestic production (of manufactures) supplied to the home market, p^f is the price of imports and Q is domestic absorption (final domestic expenditure). We can note, that the above function is the complement of a traditional import function.

The demand for exports is given by

(A1.4) $X = X(p^X, x^{WX}, Z)$

where p^X is the domestic currency price of exports, p^{WX} is the domestic currency price of world manufactures and Z is an index of world demand.

Firms are assumed to maximise profits given by

(A1.5) $\pi = pY + p^X X - C(w, p_r, Y, X)$.

Optimisation is assumed to be for a single period. The profit maximising supply of both export and goods supplied to the home market are given by the first order conditions:

(A1.6) $p^X[1 + 1/E(X, p^X)] = \delta C/\delta X$

$p\,[1 + 1/E(Y, p\,)] = \delta C/\delta Y$

where $\delta A/\delta B$ is the partial derivative of A with respect to B and $E(\alpha, \beta)$ is the elasticity of α with respect to β . The latter price elasticities are assumed to be constant.

The estimated model includes the two demand equations ((A1.3) and (A1.4)) and the two price equations ((A1.6)). The transformation function (A1.1) is assumed to be separable. This implies that there is a joint cost structure (the level of X depends on the level of Y) and that the production process consists of two stages. First, firms produce total output and then total output is sold to different markets in such a way as to maximise profits.

The cost function (A1.2) is assumed to have the Cobb-douglas form:

(A1.7) $\quad C = a_0 w^{\alpha} p_r^{\beta} K^{g} Y^{\theta} X^{\phi}$.

While the level of Y depends on the level of X (joint cost structure), one implication of a separable cost structure is that the output mix depends only on the relative price[21]:

(A1.8) $\quad X/Y = (\theta/\phi)(p/p^X)$.

Cost separability is assumed only in the long run. In the short run, unit labor cost and raw material price are allowed to affect the output mix.

Equations (A1.6) represent the long-run solutions to the estimated equations. Given the Cobb-Douglas form of the cost function,equations (A1.6) have the form:

$$p^X = [E(X,p^X)/(1 + E(X,p^X))]\phi a_0 C(w,p_r,Y,X)/X ,$$

$$p = [E(Y,p)/(1 + E(Y,p))]\theta a_0 C(w,p_r,Y,X)/Y ,$$

or

(A1.9) $\quad \ln p = a' + \alpha \ln w + \beta \ln p_r + g \ln K + (\theta - 1) \ln Y + \phi \ln X$,

$$\ln p^X = a'' + \alpha \ln w + \beta \ln p_r + g \ln K + \theta \ln Y + (\phi - 1) \ln X .$$

The four equations were estimated simultaneously (using FIML) with the following restrictions imposed (and tested):

- the price equations must be linearly homogeneous with respect to input prices, i.e. $\alpha + \beta = 1$,

- the coefficients on input prices in each price equation must be the same, $E(p,w) = E(p^X,w)$,

- the sum of the coefficients on the output terms in the price equations must be equal, $E(p,Y)+E(p,X)=E(p^X,Y)+E(p^X,X)$,

[21] Cf equations (A1.6) and (A1.7).

- constant returns to scale, $1 + g = -(\theta+\phi)$,
  (the equations include thus the ratio terms: X/K and Y/K).

In estimation Q was defined as domestic final expenditure (C+I+II+G), Z - as OECD exports of manufactures, w - as productivity adjusted wage cost, i.e. the unit labor cost.

A1.2 Private consumption

The long-run solution to the private consumption equation is of the form:

$$(A1.10) \quad C/Y = a_0(Dpc/pc_{-1})^{\alpha}(W/pc)^{\beta}[RSLA(1 - \tau)]^{g}$$

where C - real private consumption, Y - real personal disposable income, pc - consumer price index, W/pc - real financial wealth plus the real value of the housing stock, RSLA - short-term interest rate, τ - the basic tax rate and $Dx = x - x_{-1}$.

A1.3 Fixed investment

The long-run solution to the fixed investment equation is of the form:

$$(A1.11) \quad I = a_0 INVOX^{\alpha} CM^{\beta} p_r^{g}$$

where I - real fixed investment, INVOX - share of profits in total output, CM - user cost of capital, p_r - price of raw materials.

A1.4 Inventory investment (finished goods)

The long-run inventory-sales ratio is defined as follows:

$$(A1.12) \quad V/S = a_0(CV)^{\alpha}(K/S)^{\beta}$$

where V - inventory stock, S - sales, CV - financial cost of holding inventories.

The dynamic equation is of the form:

(A1.13) $D\ln V = f[D\ln S,\ \ln(S/V)_{-1},\ CV,\ D_3\ln K,\ \ln(K/S)_{-3},\ D\ln(ULC/PIMO)_{-1}]$.

A1.5 Reconciliation of supply and demand

The LBS model includes both a determination of the expenditure components and of the (gross) domestic product (Y + X).

An increase in demand gives rise to a price adjustment (through the output terms) and to an adjustment of inventories, since an increase in sales reduces inventories on impact. This adjustment, however, does not ensure that the following identity holds:

(A1.14) output + imports = final expenditure + exports.

For this reason, imports are computed as the difference between final expenditure and domestic supply (transformed to value added terms).

A1.6 Labor market

Wages and employment are determined in the public, manufacturing and non-manufacturing sectors.

It is assumed that firms and unions bargain over real wages and then firms decide on the level of employment.

Wages are affected by factors determining the bargaining strength of firms and unions. Trade union power, unemployment benefits, taxes, higher wages in other sectors and mismatching between workers and jobs tend to increase wages. Incomes policies and unemployment (especially short-term unemployment, long-term unemployed tend to be deskilled and discouraged) reduce wages.

Unemployment depends on output, real wage costs and real input prices.

As far as the labor supply is concerned, male participation is independent of wages, while the female participation rate is affected by relative level of male and female earnings, average level of real earnings, unemployment rate and a time trend.

A1.7 Exchange rate and asset markets

The prices of equities, gilts and foreign exchange are determined within a market clearing (and highly disaggregated) model of asset demand and supplies. Forward-looking expectations of changes in asset prices affect the expected rate of return from holding financial assets.

APPENDIX 2. THE SUPPLY SIDE OF THE OECD MODEL

The description of the supply side of the OECD model is based on J.Helliwell, P.Sturm, P.Jarrett, G.Salou, "Aggregate supply in Interlink: model description and empirical results", OECD Working Papers No 26, Nov. 1985. The description of the revisions introduced in the 1987 version of the model is taken from P.Jarrett, R.Torres, "A revised supply block for the major seven countries in Interlink", OECD Working Papers No 41, April 1987.

A2.1 Production structure (choice of technology)

The supply block is defined at the highest level of aggregation with exogenous central government and housing investment.

The production structure is of a nested type. Capital and energy are bundled together in an inner CES function. That bundle is then combined with efficiency units of labor in a CES outer function. Energy was included as a third production factor for the major seven OECD countries, but not for the other countries.

Corresponding to the three inputs chosen, the output measure is business value added plus the value of business energy inputs.

The three-factor production function cannot be estimated directly, since the utilization rate for the employed factors cannot be measured independently. However, if the sample is long enough then the average utilization rate can be taken to be normal. The strategy for choosing the parameter values was to derive them as far as possible from the requirement that the production function should hold on average over the sample period and that the cost-minimizing factor ratios should on average equal the actual factor ratios (given the conventional assumption of constant returns to scale).

A2.2 Output decision

The output decision is defined in terms of the utilization rate for employed factors, i.e. the ratio of actual to normal output. Normal output is computed from the estimated production function, using the current values for the capital-energy bundle and employment measured in efficiency units (i.e. multiplied by an index of labor efficiency, representing the technical progress).

The output equation in the 1985 version had the form:

$$(A2.1) \quad \ln(QBV/QBSV) = a_0 + a_1\ln(CQB) + a_2\ln\{[SALES - a_4(MGSV - MESV)]/QBSV\} + a_3\ln(STOCKV_{-1}/QBSV)$$

where QBV - actual output, QBSV - normal output, CQB - ratio of actual unit cost to output price (normalised to equal 1 on average), SALES - final sales (final domestic demand plus exports), MGSV-MESV - real non-energy imports of goods and services, STOCKV - end-of-period inventory stock.

The theory behind the equation explains the choice that the producer face in case of an unexpected change in demand or cost conditions. In such a case, the two possibilities are a change in output (i.e. adjustment of the factor utilization rate, QBV/QBSV) or change in inventories. The choice is affected by sales (representing a shift in the demand function), profitability (CQB) and the ratio of actual to desired inventories.

The inventory term was in most cases insignificant or had the wrong sign. This was believed to depend on the implicit assumption that the equilibrium stock/output ratio is constant and equals the sample mean value.

In the 1987 version of the model, the equilibrium stock/output ratio is assumed to be a function of 1)the expected long-term interest rate (IRLRE), as a proxy for the opportunity cost of stockholding, 2)the expected long-run rate of inflation, via expected capital gains on inventories,

3)output itself, reflecting possible scale economies in stockholding and 4)time trend (T).

The sales variable is in the new version of the model redefined to include normal inventory investment (NIG), instead of the actual one (ISKV). The new demand variable is defined as:

(A2.2) $NQBV = QBV - ISKV + NIG$

and equals in equilibrium (when ISKV = NIG) the actual output.

Normal inventory investment is defined as:

(A2.3) $NIG = g \cdot STOCKV$,

where g is the output growth rate on a balanced-growth path, here proxied by the annual growth rate of the labor force in efficiency units over the previous five-year period. Thus, the stock-output ratio is assumed to be constant along a balanced-growth path.

The output equation has the following form in the new version of the model:

(A2.4)
$$\begin{aligned} \ln(QBV/QBSV) = {} & a_0 + a_1\ln(CQB) + a_2\ln(CQB_{-1}) \\ & + a_3\ln(NQBV/QBSV) + a_4\ln(NQBV/QBSV)_{-1} \\ & + a_5\ln(STOCKV_{-1}/QBSV) \\ & + a_6\ln(STOCKV_{-2}/QBSV_{-1}) + a_7\ln(QBSV) \\ & + a_8\ln(QBSV)_{-1} + a_9\ln(IRLRE) + a_{10}T \\ & + a_{11}(1/T) + a_{12}T^2 \ . \end{aligned}$$

The inflation term was excluded due to insignificance. The coefficients a_5+a_6 can be said to measure the cost of changing activity levels. The larger the sum of the coefficient values the lower the cost of adjusting output.

A2.3 Factor demand

Factor demand is determined using the desired future output level, QSTAR:

$$(A2.5) \qquad QSTAR = QBV^{W} \cdot QBSV^{(1-W)} \cdot LFG^{4} \cdot (PQB/CKEL)^{0.3}$$

where LFG - growth rate of the labor force in efficiency units over the past five years, PQB - price of QBV and CKEL - cost dual coming out of the production structure.

The last term in (A2.5) means that aggregate supply is affected by profitability: the higher the sales price relative to normal cost, the larger desired output will be.

Since QSTAR can be written as

$$(A2.6) \qquad QSTAR = QBV \cdot (QBV/QBSV)^{(W-1)} \cdot LFG^{4} \cdot (PQB/CKEL)^{0.3} \ ,$$

the coefficient W can assume values greater than 1. Such values mean that factor utilization rates in excess of unity raise output expectations. The coefficient values for the last two terms (4 and 0.3) are imposed.

The investment function in the 1985 version had the form:

$$\begin{aligned} (A2.7) \qquad \ln(IBV/IBV_{-1}) = {} & a_0 + a_1 \ln(KBSTAR/KBSTAR_{-1}) \\ & + a_2 \ln(KBSTAR_{-1}/IBV_{-1}) \\ & + a_3 \ln(KBSTAR_{-1}/KBV_{-1}) + a_4 PROFR \\ & + a_5 \ln(QBV/QBSV) \ , \end{aligned}$$

where IBV - real business investment, KBV - business gross fixed capital stock, PROFR - transformation of the profitability measure CQB, such that PROFR varies round zero with the gross operating surplus per unit of gross fixed capital and KBSTAR is defined as

$$(A2.8) \qquad KBSTAR = QSTAR \cdot KQBSTAR$$

with KQBSTAR being the expected cost-minimising capital-output ratio derived from the underlying production structure and relative expected input prices.

The investment equation (A2.7) implies that, when profitability is at its normal value (PROFR = 0), the capital stock will converge to the values sufficient to produce QSTAR at normal values of intensity of factor utilization. When profitability diverges from its normal value, the desired factor input is modified.

In practice, the integral adjustment term (KBSTAR/KBV) was insignificant and thus omitted. After this change, the equation has an error correction specification, where investment adjusts toward desired capital stock. Since this stock is related to expected output, the model still implies a constant capital-expected output ratio.

In the 1987 version of the model, the investment equation is specified as a capital stock equation. This enables the introduction of the KBSTAR/KBV term, ensuring that actual and desired capital stock are equal in long-run equilibrium. The equation has the form:

$$\text{(A2.9)} \qquad \ln(KBV) = a_0 + a_1\ln(KBV_{-1}) + (1-a_1-a_2-a_3)\ln(KBV_{-2}) + a_2\ln(KBSTAR) + a_3\ln(KBSTAR_{-1}) + a_4 PROFR_{-i} + a_5\ln(QBV/QBSV)_{-j}$$

$$IBV = 2[KBV - (1 - RSCRB/200)KBV_{-1}] \ ,$$

where RSCRB - scrapping rate.

The restriction on the coefficient for KBV_{-2} ensures the homogeneity of KBV with respect to KBSTAR.

PROFR can be seen as a proxy for (the inverse of) Tobin's q, on the assumption that current profits are a proxy for the present value of the expected future profits.

The labor demand function in the 1985 version was specified

as an error correction equation:

$$(A2.10) \quad \ln(ETB/ETB_{-1}) = a_0 + a_1\ln(EBSTAR/EBSTAR_{-1}) + a_2\ln(EBSTAR_{-1}/ETB_{-1}) + a_3\ln(CQB) + a_4\ln(QBV/QBSV) ,$$

where EBSTAR - desired employment, defined as the number of workers required to produce the expected future profitable output QSTAR with the desired capital stock KBSTAR and the corresponding energy input; it is calculated by inverting the aggregate production function.

In the 1987 version of the model the equation has the following form:

$$(A2.11) \quad \ln(ETB) = a_0 + a_1\ln(ETB_{-1}) + a_2\ln(ETB_{-2}) + (1-a_1-a_2-a_3-a_4)\ln(EBSTAR) + a_3\ln(EBSTAR_{-1}) + a_4\ln(EBSTAR_{-2}) + a_5\ln(CQB_{-i}) + a_7\ln(QBV/QBSV)_{-j} .$$

A2.4 Reconciliation of supply and demand

Inventory investment is computed as the difference between output plus imports and total demand. The authors of the 1985 description prefer to state it as follows: "In an accounting sense inventory change is the residual element" (p.8). (...) "In fact, the responses are mutually dependent..." (p.8).

A2.5 Price determination

Output price is determined by factor utilization rate, cost of inputs and prices of foreign competitors, the latter influencing the cost mark-up that producers are able to charge. A term representing cyclical effects of inventory disequilibrium proved insignificant.

Prices of foreign competitors are approximated by import prices reweighted to correspond to the overall production structure (the structure of imports may be different from the structure of total output).

The price equation has the form:

$$(A2.12)\quad \ln(PQB) = a_0 + \Sigma a_{1,i}\ln(PQB_{-i}) + \Sigma a_{2,j}\ln(CKEL_{-j}) + \Sigma a_{3,k}\ln(COST_{-k}) + \Sigma a_{4,n}\ln(PMQ_{-n}) + \Sigma a_{5,m}\ln(QBV/QBSV)_{-m}$$

where PQB - deflator for gross business output (gross value added plus energy input), CKEL - cost index computed from the dual of the aggregate production function, COST - domestic cost measure that does not assume that prices are set based on full adjustment of factor inputs to changes in relative factor prices (total actual cost/QBSV), PMQ - reweighted import price.

Long-run homogeneity was imposed with respect to cost and import price:

$$(A2.13)\quad \Sigma a_{1,i} + \Sigma a_{2,j} + \Sigma a_{3,k} + \Sigma a_{4,n} = 0 \ .$$

Dummy variables and time trends were included in the estimation, but not in the simulation equations.

The ratio QBV/QBSV - proxying excess demand - proved to be insensitive to simulated shocks. This is due to the excessive cyclical sensitivity of QBSV, depending on the fact that actual employment is used in computing QBSV. In the 1987 version of the model an attempt is made to compute potential output, using cost-minimising labor demand and actual labor-energy bundle. It is not clear whether potential output has been substituted for QBSV in the simulation equations.

A2.6 Labor supply

Equations for male and female labor force participation rates were estimated using OLS, since it was believed that probit/logit procedures would give only trivial differences in parameters. Double logarithmic specification was employed.

A number of social/demographic and economic variables was tested. The former group included age structure, rate of family formation and child bearing, continuing education and employment opportunities for women.

The economic factors comprised aggregate unemployment rate, factor utilization rate (QBV/QBSV), real wage (wages per employee net of direct taxes and social security contributions, deflated by the consumer price index), real non-wage income per capita (excluding interest on consumer debt and government transfers) and real government transfers per capita.

In the simulation equations, the social/demographic variables are replaced by a high-order polynomial function on time and its inverse.

APPENDIX 3. THE SUPPLY SIDE OF THE BANK OF ENGLAND MODEL

This description is based on K. Patterson, I. Harnett, G. Robinson and J. Robinson,"Bank of England quarterly model of the UK economy", Economic Modelling No. 4, October 1987.

A3.1 Overview

The BOE model was originally based on the LBS model, which was overtaken in whole in 1973 and then became subject to its own evolution. It is quite disaggregated and has a large monetary side. The source description of the functioning of the system mentions theoretical underpinnings only briefly, partly by reference to other works. The behavioral equations appear to be an eclectic work of art and cumulated experience in modelling rather than the implementation of a dominating and perhaps constraining theoretical vision. The modellers seem quite willing to allow empirical performance to influence model design, given an intuitive substantiation and/or heuristic support in the literature, without resorting to complex on-the-spot optimization mathematics. Wide usage of the term "proxy" for critical variables such as "output" or "supply" makes the supply side somewhat obscure.

The economy is overlappingly sectorized to integrate prices, production, trade, etc according to different statistical, institutional and functional divisions. Roughly, the production economy can be divided into manufacturing, "other business" (trade, services), North Sea energy (actually part of other business), and the non-trading public sector (authorities) . The exogenous North Sea component will not be mentioned further below, nor will the monetary side be treated. Food, drink and tobacco (important imports) do not appear to belong to manufacturing, whereas public enterprises overlap the first two sectors.

The BOE model was apparently originally a Keynesian demand-determined system, containing standard influences from capacity limitations (eg. the labour market), into which supply side elements have later been introduced, or simply interpreted into the system. GDP is essentially determined

from the expenditure side, and sector level outputs proxy-constructed from that measure.

The major explicit, but limited, supply-side ingredient is a 2-stage CES production function of the OECD-Helliwell type for the determination of "normal output" (QN). The ratio of actual to normal output is a "capacity utilization" variable (CAP=Q/QN), which affects pricing and factor adjustments. The production function otherwise plays no role in determining current output or supply (prices and quantities offered). From this standpoint, the model contains no explicit supply side.

On the other hand, from the standpoint that pricing is the essence of the supply decision, it would appear that the BOE contains a very rich and complex, but only implicit, supply-side. It is ad hoc, having no basis in optimization based on a particular production function. The rest of this appendix will briefly describe some of the explicit and implicit supply features.

A3.2 The Production Function and Capacity Utilization

There is no need for detail here, since BOE and OECD seem to have the same approach. A CES-function with constant returns and labour-augmenting technical progress is estimated for manufacturing and other business, respectively. Labour input is measured in manhours and the other input is the capital-energy bundle "produced" by an inner CES function. The Central Statistical Bureau's official capital stocks were adjusted for presumed premature scrappage in earlier years and OECD retrofitting concept provides a putty/semi-putty feature. The approach to estimation seems to have been aimed more at sound practicality than econometric elegance. Given reasonable assumptions as to substitution elasticities, the scale parameters were estimated to achieve a fit to average values of variables over the period 1967-85. The rate of technical progress was set to a constant after examining variable rates implied from the function over the period.

Substitution of employed capital, energy and manhours into the function results in estimated "normal output" over

history and for projections. The ratio of actual to normal output measures capacity utilization interpreted as the rate of utilization of employed factors, CAP= Q/QN, which appears in pricing and factor adjustment equations.

A3.3 Normal Output and "Full Employment"

It is important to focus on some implications of CAP. The authors in at least one instance slip into referring to QN also as "potential output" (call it "QP"). This term has the wrong connotation, since QN does not measure what could (at any point in time) be produced at full utilization of the economy's resources -- including those which may be idle outside of enterprises -- but only of those already employed. Thus, in contrast to the BOF's "CUT" (=Q/QP, Appendix 4), CAP can be equal to unity at high rates of labour unemployment. However, presumably the same function could be used to measure potential output (in the Finnish manner) by substituting extraneously estimated full employment levels of the factor inputs into the CES function. This of course presumes that the natural rate of unemployment can be determined outside of the model (which BOF presumes).

The distinction between normal and potential output (or between CAP and CUT) raises both practical and conceptual problems. Suppose that the (UK) economy has been operating at less than most observers would regard full employment of resources, but near "normal" levels, ie., that Q has been fluctuating around the path of QN, which has been less than QP. Then normal output is less than potential output and the achievement of full employment (Q=QP) under current circumstances would imply "abnormally" high output. Moreover, since CAP=1 signifies equilibrium in the BOE model and disequilibrium CAP>1 induces higher price increases and a factor adjustment upswing, inflation would be abnormally high at rather high unemployment (higher than what might be regarded as "natural unemployment").

Thus, the achievement of long run equilibrium in the BOE sense (CAP=Q/QN=1) as well as the BOF sense (CUT=Q/QP=1) requires the convergence of normal output to potential output as well as the convergence of actual output to normal output.

If this presumes low unemployment, some equilibrating process is required which provides for lower inflationary thresholds at low unemployment rates. Otherwise, a movement toward full employment (potential output) could induce an accelerating inflation which moved activity in the opposite direction. Alternatively, it requires that extraneous notions of potential output be redefined as equivalent to normal output by regarding high levels of idleness (unemployment) as "normal" (or "natural").

This is of course a central problem confounding policy makers and modellers. The UK and continental approach of tolerating high levels of unemployment to avoid inflation seems to entail extremely slow equilibration (reduction) in unemployment, while the Swedish or Finnish low unemployment policies seem to be undermining the equilibration process by generating unsustainable inflation rates.

The advantage of the BOE concept of normal output would appear to be that it takes the existing economic structure as given. It does not attempt to define a long run equilibrium (steady state) in terms of a prespecified unemployment level -- since that level might be mathematically inconsistent with the existing economic structure or, if consistent, might necessitate a period of equilibration which, under an unchanged structure, extends over decades.

A3.4 Factor Adjustment

It should be noted that although CAP enters the equations for adjustment of manhours and investment, neither of these equations is derived on the basis of the postulated and estimated CES production functions underlying CAP; nor are the labour and investment functions mutually consistent on some other explicit theoretical basis.

The investment equations are apparently ad hoc. Gross investment in manufacturing is essentially a distributed lag function of output changes, CAP, a long and a short real interest rate, and the relative price of capital (rental) to labor. Only (a proxy for) output changes drives investment in other business.

Manhours are similarly, but not so drastically, ad hoc. Manhours are a distributed lag in output, CAP, relative factor price, the real wage (in consumer prices!) and a trend. Here there are elements of both cost-minimization and profit-maximization models. Employment in persons is in turn modelled as a distributed lag adjustment to manhours, normal working hours and a trend.

A3.5 Wages

Wage formation is obviously a critical element in the equilibration problem mentioned above. The BOE system-approach seems applicable to Swedish circumstances because it, in principle, deals with several potentially destabilizing elements which bear on convergence to a long term non-inflationary, low unemployment equilibrium. In a highly unionized economy, wage setting by collective bargaining is a labour-supply decision, or at least alters the nature of the labour supply which firms face and bears on the corresponding decisions of firms to supply output. An important question which is not addressed in the source text is whether the wage formation sector of the model could be regarded as inherently inflationary at low unemployment and high marginal tax rates or under low wage differentials, ie., whether it makes the combination of low unemployment and inflation unsustainable.

There are essentially three sectors: manufacturing, the non-trading public sector,and other business. Wage increases in the manufacturing and public sector, respectively, are dependent on not only expected consumer price increases and other variables mentioned below, but also on distributed lag wage increases in the other of these two sectors. Thus, there is no leader-follower relationship here, but the simultaneous and recursive one characteristic of the wage-wage spiral. However, the other business sector is a follower of the manufacturing sector.

The mentioned "other items" are labor productivity, the unemployment rate of the short term unemployed, the after-tax wedge, and an incomes policy item representing the equivalent of Swedish government wage increase guidelines (4%, etc). The

role of short term unemployment seems relevant in view of contentions that other groups of unemployed (the long term unemployed, outsiders) have lesser importance for wage formation. Absorption of outsiders into employment by demand expansion may cause overemployment of insiders to trigger inflationary impulses which preclude low total unemployment without excessive inflation, ie. prevent convergence to a noninflationary low unemployment equilibrium, at least in a reasonably short period.

A3.6 Prices

Pricing is characterized as "the" supply decision. Firms face imperfect competition and are regarded as deciding simultaneously prices and sales targets which are consistent with profit maximization. The manufacturing wholesale price is the "key" price in the system. The domestic manufacturing price is a function of unit labour costs, prices of imported inputs, a relative price of domestic to imported manufactures, and CAP, which is a disequilibrium measure that represents adjustments in profit margins to pressures on capacity. In steady state equilibrium, relative prices and capacity utilization are unity, and domestic price is a markup on unit variable costs.

The export price of manufactures has a similar structure to that for domestic price, but with a larger effect for relative price. Both the export and import prices of manufactures depend on the levels and the changes in world market prices and domestic costs. Whereas the levels of these items in steady state appear to have the correct coefficient signs, the coefficients for the corresponding changes have the opposite signs, which is perplexing.

A3.7 Foreign Trade

Exports of manufactures are modelled in terms of UK-weighted world trade in manufactures and relative unit labour costs (RULC). The use of RULC rather than relative prices is empirically motivated and is rationalized in terms of firms' desired profit margins; firms may in some cases absorb cost

increases in lower margins and in others prefer higher margins at given costs.

The highly complex import function for finished manufactures is an illustration of the implicit and ad hoc nature of the BOE supply side. In the BOE summary rendition, the total demand for manufactures is derived from final expenditure using I/O and relative prices of manufactures to total goods. Subtraction of exports from total demand yields total domestic demand, which is met by domestic and foreign sources. The domestic share of total domestic demand (one minus the foreign share) is a function of domestic demand relative to its moving mean, of domestic production relative to its moving mean, of the moving mean in relative manufacturing prices and RULC and of a trend.

In each case in which an explanatory variable is measured relative to its moving mean, the mean is interpreted as representing capacity, as from a production function. When demand or production exceed the corresponding moving mean, there is a tendency for a portion of demand to be supplied from domestic sources and for a spillover to foreign sources. There is thus an element of hysteresis in the system, modelled in a crude way. The mentioned spillovers could occur at high unemployment rates and low rates of capacity utilization (measured not as CAP, but in traditional ways). Whether the spillovers occur depends on recent events rather than on the degree of slack in the economy at the current time.

The moving means in relative prices and RULC merely represent the delayed effects of these variables on trade, where the negative trend effect represents the unexplained trend loss of market shares for manufactures in the UK. The trend component, which has also appeared in research, is of course a practical modelling problem, but it is also of theoretical significance. If the possible obstacles to a noninflationary low unemployment equilibrium were absent, what would happen to the trend?

APPENDIX 4. THE SUPPLY SIDE OF THE BOF3 MODEL

This description is based on J.Tarkka, A.Willman (eds), The BOF3 Quarterly Model of the Finnish Economy, D:59, Helsinki: Bank of Finland, 1985. BOF3 is the 3rd BOF version. In this English language version the authors apparently did not notice several typographical errors in basic formulas which they regarded as correct and used as inputs into further derivations. In some instances this obscures the train of argument, which, however, appears to be mainly based on correct formulations in the original text.

A4.1 Overview

BOF3 is characterized by its authors as short run Keynesian and long run neoclassical, ie., as relatively fixed-price and fixed-output, respectively, from short to long run. The behavioral equations are essentially log-linear OLS-estimates. The monetary (LM) side of the (IS-LM) framework is characterized by the regulation of credit, which entails administered interest rates and (apparently) policy action to achieve long term monetary and balance of payments equilibrium. The monetary aspects of the supply side are not treated below.

Production and prices are structured along EFO-lines with an open sector consisting of manufacturing and of forest industries, and a sheltered (S-) sector consisting of (priv. and pub.) services and of agriculture. The open sector is wage-leader and the S-sector is a slavish follower.

The industries in both sectors maximize profits under imperfect competition in the short run. This holds also for the S-sector in the long run. In adherence to the small-open-economy idea, the open sector is modelled as moving to perfect competition with the international tradeable economy in the long run: its price elasticity of demand moves to infinity and its prices move to parity with (exogenous) foreign prices in the long run. By "parity" is meant that relative prices of tradeables are constant, though not necessarily unity; a constant differing from unity presumably

stands for long term quality and/or assortment differences relative to the world or to deficiencies in measurement.

Whereas the short run involves the usual sorts of qualitative disequilbria and sluggish price and factor adjustments, the long run steady state (stationary or growing) entails monetary and external balance, price parity with the external economy, and an economy-wide capacity utilization rate of unity (CUT=1).

CUT is the ratio of actual to potential output (GDP), where the latter is defined as that output value which results from substitution of actual capital stock and full employment manhours into a Cobb-Douglas (CD-) production function with constant returns to scale and technical progress as a trend (T). Thus, CUT is not merely the utilization rate of producers, but encompasses also the labour force, and is achieved at full employment (natural unemployment). A better name for CUT might be the "economy resource utilization rate". The CD-function produces value-added (Q), which together with domestic and imported materials inputs (M) produces gross output (X) in an I/O-framework.

A4.2 Output and Technology

The I/O model (1970) links gross output and intermediate inputs (domestically produced and imports) to final demand and value added. The Leontieff framework implies constant returns. Given recourse to imports, there should be no intermediate input shortages that could constrain gross output. Constant returns are also posited for value added in the CD function for the total economy:

(A4.1) $\ln Q = a_0 + gT + a\ln L + (1-a)\ln K$

The parameters (a_0, g, a) are estimated using OLS where K is replaced by <CE·K>. CE is the capital utilization rate proxied by the energy utilization rate. (Presumably the equation above holds only when CE=1, but CE is never mentioned in other contexts when this formula appears).

L is, in principle, manhours and in practice remunerated manhours (LW). LW relates to employed persons (N) by LW=h·N where h is working hours per person. Employed persons are related to the (endogenous) work force (NF) by N=(1-UR)NF, where UR is the unemployment rate. When UR = the natural rate (URbar, extraneous), we have the number of fully employed persons, and given h, we obtain the full employment value for LW or L . Substitution of full employment L into the above equation yields potential output lnQP and

(A4.2) CUT= Q/QP ,

the capacity utilization rate of the economy. CUT=1 is characteristic of long term equilibrium, whereas it otherwise operates as a disequilibrium variable.

A4.3 Short Run Adjustments - Output, Supply and Inventories

It is not quite clear in what sense or to what degree supply decisions are explicitly modelled in BOF. The presentation in BOF3 containes a pedagogical overview in which a simplified supply function appears. It deals also with inventories, in which the output decision seems to be implicit, and in a particular case (goods exports) a supply decision is explicit. It is worth holding in mind that an output decision need not necessarily be regarded as a supply decision in the very short term; the output decision can be aimed at an inventory target as well as at sales, whereas a supply decision aims at a quantity of sales at a particular price. Alternatively, a supply decision is a price decision at a particular level of sales.

To make the concepts of supply and output decision compatible, it might be useful to regard producer's planned inventory increases as "internal sales", i.e. a supply component to satisfy internal demand. In steady state growth, the excess of output over sales (= purchases) would be this internal component, part of supply being external and part internal.

Pedagogical overview

The pedagogical presentation aims at providing a simplified overview of the model. Here, the whole economy is treated as an open sector, intermediate inputs are abstracted away (X=Q) and perfect competition is assumed (price=marginal revenue). The supply function is obtained by taking the first order profit-maximizing condition with respect to labor input,

(A4.3) $w/p = aQ/L$ (= Marginal product of labour)

This is solved for L which is substituted into the production function (A4.1) to solve for supply (QS as Q). Supply is a function of relative price (p/w), capital stock and the level of technical progress:

(A4.4) $\ln QS = \text{const} + \ln K + [a/(1-a)]\ln(p/w) + [g/(1-a)]T$

Materials input prices should also appear, but are theoretically proxied by wages, and the capital stock is exogenous as in the standard short run paradigm. Actually, this appears to be essentially what happens when price determination is actually modelled for the open sector (see sect. A4.4 below).

The price equation for supply of tradeables is based on profit-maximization over an infinite horizon in which the capital stock is always exogenous and in which the price of materials inputs is absent. As concerns materials, it would appear that the price of materials inputs did not function well empirically for which reason materials inputs were abstracted out of the optimization problem. Moreover, since capital stock is always exogenous, even when current pricing depends on expectations for the distant future, there is only a short run supply decision (A4.4). Thus, short run supply "gropes" its way toward long run equilibrium in a myopic manner via the capital stock adjustment mechanism (investment function, below), which shifts K over time.

Goods Exports -- explicit supply decision

The model for exports of goods contains an explicit rendition of the supply decision. Exports (x) are the difference between domestic supply (S) and domestic demand for domestically produced goods(D). Supply is defined here as in (A4.4) above. The rate of change in exports is a function of the rate of change in export market size and the level of relative prices (foreign/domestic). If relative price were at parity, exports would depend only on export market growth. Given foreign prices, a change in relative price is in effect a change in domestic export prices. Relating the change in exports to the level of relative prices implies that the elasticity of exports with respect to relative prices tends to infinity over time, but is finite in the short run. As specified, the export function has the chracteristic that a price change induces an acceleratin in exports (i.e. a change in export change), which means a growing export reaction. Hence, relative price changes can result in eventual collapse or explosion of goods exports.

The rate of change in relative price is a constant elasticity function of relative excess demand in the open sector, ie. of the ratio of the sum of exports and domestic demand to supply, (x+D)/S; zero excess demand (ratio = 1) implies zero relative price change. Operationally, the excess demand variable is (approx.) the error in regression equation (A4.4). That is, the predicted values from that equation as a regression are lnS, the actuals are lnQ; and the residuals ln(Q/S) measure "excess demand". Total demand, x+D, is thus equivalent to Q. The interpretation is that when (demand =) sales (Q) exceed quantity supplied (S), relative export prices tend to rise, which in turn causes a gradual decline in exports under the influence of an ever increasing demand elasticity. The erosion of exports and price increase in turn reduces excess demand by decreasing Q and increasing S, which should tend to return the economy to equilibrium.

The disconcerting aspect of the rendition is the uncertainty as to whether the dependent variable in the mentioned regression (Q) is an output measure or sales defined as exports plus domestic demand (x+D). Since the former includes

inventory investment, the latter seems correct. Otherwise an unintended inventory increase corresponding to a relatively high output level would perversely signify excess demand. If the dependent variable of the regression is x+D, however, the production function is in effect estimated on sales instead of output. Inventories were not mentioned in the context of goods exports.

Inventories -- implicit output decision

The output decision is implicit in the chapter on inventories. As the authors point out, given the identity relating sales, output and inventory investment, an explicit model for two of these items yields an implicit model for the third. Since demand and inventory are modelled explicitly in BOF3, the output decision is implicit. Inventory investment is modelled as the outcome of a cost-minimizing trade-off between achieving target inventory and output levels. Optimal inventories depend (inter alia) on normal sales which are equal to normal output which is a moving average of actual sales.

The resulting inventory investment equation contains sales, stocks, etc in standard fashion. Output -- ie. the output decision -- is residually determined from the mentioned identity. No breakdown into planned and unplanned inventory investment is achieved theoretically or empirically. The quantity of supply does not appear in the inventory model except in the concept of normal sales = normal output, which is a moving average of past sales and not economically determined as a function of prices and costs.

A4.4 Pricing Behavior and Supply

Since imperfect competition reigns in the short run, there is really no such thing as a supply function, but only the quantity supplied at a profit-maximizing price. The supply decision, as conceived by BOF, appears to be a profit-maximizing simultaneous price-setting and sales-offering decision based on firms' demand and cost functions. As explained above, whether firms (industries) succeed in selling the (optimal) quantities corresponding to the

(optimal) prices they set depends on demand fluctuations and unintended inventory changes.

Quantity demanded in the S-sector industries is a constant elasticity function of own-price relative to the general price level and to income. Profit maximization results in price as a markup on unit variable cost defined as the weighted sum of unit labor and unit material costs. Demand elasticity determines the size of the markup. The elasticity is finite in the long run for the S-sector.

In the open sector the CHANGE in quantity demanded depends on level of relative price (domestic/foreign) and on the CHANGE in income. Abstracting from the income effect, as long as domestic and foreign prices deviate from parity, the quantity demanded will change, whereas achievement of parity will stop this change.

As mentioned earlier, on the production/cost side, the distinction between gross and net output is dropped and with it materials inputs and their prices; capital stock is exogenous. Costs then depend positively on wages (proxy also for materials prices) and output and negatively on capital stock and technical progress. A lengthy maximization of the present value of present and future profits results in an error-correction equation for the change in domestic prices of tradeables as a function of changes in foreign prices, domestic wages, income, the level of relative prices and the change in capital stock (the latter with a negative sign). The significant feature of the price function is that its steady state form is consistent with parity between domestic and foreign prices of tradeables. As with exports of tradeables, deviations from parity result in the changes in sales to the domestic market, eg. the loss of market share. Like the effects on export markets, deviations from parity can affect the level of activity, income, demand and unemployment thereby inducing the equilibrating wage and price adjustments that eventually move prices back toward parity.

A4.5 Wages

The open sector is wage leader in this EFO-framework and its wages affect open sector prices as just described above. The rate of wage increase depends on labour demand and supply elements bearing on wage drift and on the negotiated component. That is, wage increase depends positively on the negotiated increase and on wage disequilibrium, defined as the ratio of the Cobb-Douglas marginal revenue product to initial wages (p·MPL/W from equ. A4.3), and negatively on the "unnatural" part of unemployment. The negotiated increase in turn depends on increases in consumer prices and on the change in the unemployment rate. Outside the steady state, the disequilibrium items play an obvious role in a two sector wage-price spiral which affects competitiveness on domestic and foreign markets and should induce equilibration. In long run equilibrium, the disequilibrium arguments vanish; wage increases depend on a negotiated component which depends on consumer price increases. The latter must of course be consistent with price parity in the open sector and with monetary and external balance.

A4.6 Factor Adjustments

Adjustments of capital stock and manhours are motivated by profit maximization. They are induced by disequilibria between respective Cobb-Douglas marginal revenue products and factor prices. The dynamic adjustment mechanisms are essentially stock-adjustment models. The product price expectations underlying the marginal revenue products are static. The marginal product component of marginal revenue product, being based on current output, is also static -- as if the current output were to be produced in the future. No connection is apparent between the amount of current output which would be forthcoming by substitution of current factor levels into the production function (A4.1) and the output level implied from the inventory model (section A4.3).

On the other hand, potential output (QP) can be computed each period (as described above) and together with actual output, this yields the economy-wide capacity utilization rate: CUT=Q/QP (cf. equ. A4.3). This variable in turn regulates the

rate of change in employed persons (N) relative to the level of manhours (L or LW). In steady state equilibrium CUT=1 and Q=QP is produced by factors for which their marginal revenue products equal factor prices.

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