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MODELLING WAGES SUBJECT TO BOTH CONTRACTED INCREMENTS AND DRIFT BY MEANS OF A SIMULTANEOUS-EQUATIONS MODEL WITH NON-STANDARD ERROR STRUCTURE

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Modelling wages subject to both contracted increments and drift by means of a simultaneous-equations model with non-standard error structure

by

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ABSTRACT

An asymptotically efficient estimation method of structural parameters is suggested for a non-standard simultaneous-equations model with two error components, one of which has been multiplied by an observable, exogeneous factor. An LM test is derived for the hypothesis of just one error component and a Chow- type test for structural change is discussed.

In an application, a quarterly three-equations model is constructed for wages in Finnish manufacturing. We model the fact that wages are determined in two stages. Contract wages are settled through collective bargaining, but market forces often tend to drive them upwards. This wage drift has in Finland for some years exceeded contracted wage increments. Contracts work on an annual level, implying both an annual and a quarterly error term in the contract wage equation. The annual part of the model - including the annual error term - is multiplied by an observable exogenous allotment factor, which makes our model non-standard. The model also includes separate equations for the drift and for the inflation rate.

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1. INTRODUCTION

The purpose of this paper is to present a feasible estimation technique for simultaneous-equations models with two error components, when one of the error components is multiplied by an observable factor. As far as we know, this very special type of models has never been discussed before in the literature. Estimation problems in simultaneous-equations models with error components have been previously studied by Baltagi (1981), Magnus (1982), Hsiao (1986 , Ch. 5), Balestra and Varadharajan-Krishnakumar (1987) and others. None of these studies can, however, be directly applied to cases where one of the error components has been multiplied by an observable factor. On the other hand, Hsiao (1975), Kelejian and Stephan (1983) and others have studied random coefficients models for panel data, but their models do not allow for simultaneous interdependencies between the dependent variables.

We shall not discuss the identification problems induced by our model in any detail. We just note that Kelejian (1974) found out that in the random parameter case, the indentifiability of equations can be guaranteed by roughly similar conditions as in the constant parameter case. Instead, we shall present an LM- test for testing, whether the two error components are really necessary or not. Because the estimation techniques for error component models are far more cumbersome than the conventional estimation techniques for simultaneous-equations models, it would be nice to know in advance, whether the data actually motivate the use of the more complicated model structure. In addition to this, we also suggest a simple Chow- type test for detecting changes in the structure of the model.

Our interest towards this problem arouse when we wanted to study the wage determination mechanism in Finnish manufacturing industries in the 1980's. In the Scandinavian system (see for instance Holden, 1989), the wages consist of two clearly distinct components, the contract wages and the wage drift. Because the impact of the wage drift has amounted to nearly one half of the total wage increase in some years in Finland, we felt it necessary to model the contract wages separately from the drift. In the model for the contract wages, we had to somehow take into account the transitions between different contract periods. Because the length of the contract period has invariably been one year, we basically formulated our quarterly model for the logarithmic changes of the contract wages in annual terms, with an annual error component attached. During the bargaining, however, the employers' organizations and the trade unions also agree upon the timings of the wage raises. This led us to construct so-called allotment factors for each year that distribute the agreed wage raises across quarters. We calculated the non-negative values of the allotment factors in accordance with the texts of the wage agreements so as to sum up to unity each year. By multiplying the annual part of the model by these allotment factors, we tried to depict the quarterly changes of the contract wage index. On the other hand, we also

included a quarterly error component in the model, because part of the variation in the contract wage index is due to miscellaneous reasons, such as changes in the structure of the work force, ambiguities in the technical recording of the contract wages, etc.
As far as the wage drift was concerned, we made the assumption that for each individual worker, the difference between his "market wage" and his contract wage builds up a certain pressure for drift, and that this pressure will lead to a real wage raise only after exceeding a certain positive threshold.
Finally, it is quite clear that wages have a strong simultaneous interdependence on consumer prices, and that the wage formation mechanism cannot be adequately studied without taking this interdependence into account. This is how we ended up with a three-equations error component model with separate equations for the contract wages, for the wage drift and for the consumer prices. We shall use this wage-price model for the Finnish manufacturing industries as an empirical example to demonstrate the use of the new estimation technique. For further details and for a more thorough economic motivation of the model, see Rahiala and Kovalainen (1992).

The plan of the paper is as follows: In Section 2.1. we shall formulate the basic model and derive the corresponding likelihood function. In Section 2.2. we present a new three-stage estimator for the parameters and prove its consistency and asymptotic efficiency. In Section 2.3. we derive an LM- test for testing the adequacy of a conventional simultaneous-equations model against the more general alternative of two error components. The temporal stability of the model will be discussed in Section 2.4. In Section 3 we illustrate the use of these methods by presenting some results from the Finnish wage determination study. Section 4 concludes.

2. SIMULTANEOUS-EQUATIONS MODELS WITH ERROR COMPONENTS WITH OBSERVABLE MULTIPLIERS

2.1. The model and the likelihood function

For the reasons outlined above, we shall study the following simultaneous-equations model for panel data:

$$Y_{ij} = \mathrm{B}\ Y_{ij} + \Gamma X_{ij} + \psi_{ij} \tag{2.1}$$

where $E(\psi_{ij}) = 0$, $\psi_{ij} = \xi_i S_{ij} + \varepsilon_{ij}$ and

$$\begin{cases} \{\xi_i\} \perp\!\!\!\perp \{\varepsilon_{ij}\}, \\ \xi_i \sim NID(0, \tau^2) \quad , \\ \varepsilon_{ij} = \left(\varepsilon_{ij}^{(1)} \quad . \quad . \quad . \quad \varepsilon_{ij}^{(K)}\right)' \sim NID_K(\mathbf{0}, \Sigma) \\ \qquad i = 1, ..., n \ ; \ j = 1, ..., k \ , \end{cases}$$

where k refers to the number of individuals in the panel, n to the number of time points, or vice versa. The symbol K refers to the number of equations. The K-dimensional column vector Y_{ij} consists of the values of the endogenous variables, X_{ij} denotes the values of the M exogenous variables and S_{ij} consists of K known constants or values of exogenous variables that are completely independent of the error terms. The subscript j refers to the individual and i to the time point, or vice versa. Because ξ_i is assumed one-dimensional, this setup essentially allows for an error component structure in just one of the equations. However, the ideas outlined in this section can be easily extended to the more general case.

Model (2.1) differs from the one studied by Baltagi (1981) and by Balestra and Varadharajan-Krishnakumar (1987), because the error component ξ_i is multiplied by an exogenous, observable factor S_{ij}. Note, that we only allow for two error components, because the model (2.1) is general enough for our application in Section 3. Anyway, the main ideas of this article can be generalized further to the case of three error components as well.

The individual parameters will be denoted by lower case letters whereas parameter vectors and matrices will be denoted by bold face capitals

$$\mathbf{B} = (\beta_{ij}) \quad \text{and} \quad \mathbf{\Gamma} = (\gamma_{ij}) \quad .$$

Further, denote

$$Y_i = \begin{pmatrix} Y'_{i1} \\ \vdots \\ Y'_{ik} \end{pmatrix} \quad , \quad X_i = \begin{pmatrix} X'_{i1} \\ \vdots \\ X'_{ik} \end{pmatrix} \quad ,$$

$$Y_i^* = \text{vec}(Y_i)$$

and

$$Z_{i\nu} = (Y_i \quad X_i)\,\mathbf{C}_\nu$$

where $\mathbf{C}_\nu$ is a $(K+M) \times m_\nu$- matrix of zeros and ones extracting the variables that actually are included in the ν:th structural equation.
Correspondingly, denote by

$$\beta'_{(\nu)} = (\beta_{\nu 1} \quad \dots \quad \beta_{\nu K} \quad \gamma_{\nu 1} \quad \dots \quad \gamma_{\nu M})\,\mathbf{C}_\nu$$

the $m_\nu \times 1$- vector of structural parameters that actually appear in the ν:th equation.

By denoting $p = m_1 + \dots + m_K$,

$$S_i = \begin{pmatrix} S'_{i1} \\ \vdots \\ S'_{ik} \end{pmatrix} \quad , \quad S_i^* = \text{vec}(S_i) \quad ,$$

$$Z_i = \begin{pmatrix} Z_{i1} & 0 & \dots & 0 \\ 0 & Z_{i2} & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & & & \cdot \\ 0 & \dots & 0 & Z_{iK} \end{pmatrix} \quad , \quad \beta = \begin{pmatrix} \beta_{(1)} \\ \cdot \\ \cdot \\ \beta_{(K)} \end{pmatrix} \in R^p \quad \text{and} \quad \varepsilon_i^* = \mathrm{vec}(\begin{pmatrix} \varepsilon_{i1}' \\ \cdot \\ \cdot \\ \varepsilon_{ik}' \end{pmatrix})$$

we can reformulate model (2.1) in a more compact form

$$Y_i^* = Z_i\beta + \xi_i S_i^* + \varepsilon_i^* \tag{2.2}$$

where $\varepsilon_i^* \sim NID_{kK}(0, \Sigma \otimes \mathbf{I})$ and $\xi_i \sim NID(0, \tau^2)$, $\{\xi_i\} \perp\!\!\!\perp \{\varepsilon_i^*\}$.
The error term of model (2.2) is of the special form

$$\psi_i^* = \xi_i S_i^* + \varepsilon_i^* \qquad i = 1, \dots, n$$

and consequently

$$E(\psi_i^*) = 0 \qquad \text{and}$$
$$\mathrm{cov}(\psi_i^*) = \Sigma \otimes \mathbf{I} + \tau^2 S_i^* S_i^{*\prime} \quad .$$

By the matrix inversion lemma $(A+BC')^{-1} = A^{-1} - A^{-1}B(\mathbf{I}+C'A^{-1}B)^{-1}C'A^{-1}$, the inverse of the covariance matrix can be written in the form

$$[\mathrm{cov}(\psi_i^*)]^{-1} = (\Sigma \otimes \mathbf{I})^{-1} - (\Sigma \otimes \mathbf{I})^{-1}\tau S_i^* \left[\mathbf{I} + \tau^2 S_i^{*\prime}(\Sigma \otimes \mathbf{I})^{-1} S_i^*\right]^{-1} \tau S_i^{*\prime}(\Sigma \otimes \mathbf{I})^{-1} \quad . \tag{2.3}$$

The Jacobian determinant of the transformation $\varepsilon_{ij} \longrightarrow Y_{ij}$ in (2.1) is equal to $[\det(\mathbf{I} - \mathbf{B})]^{-1}$ and, consequently, the log- likelihood will be of the form

$$\begin{aligned} l(\beta, \tau^2, \Sigma) &= \sum_{i=1}^{n} \log L_{Y_i|X_i,S_i}(\mathbf{B}, \Gamma, \tau^2, \Sigma) \\ &= nk \log |\det(\mathbf{I} - \mathbf{B})| - \frac{nk}{2} \log(\det\Sigma) - \frac{1}{2}\sum_{i=1}^{n} \log(1 + \tau^2 S_i^{*\prime}(\Sigma \otimes \mathbf{I})^{-1} S_i^*) \\ &\quad - \frac{1}{2}\sum_{i=1}^{n}\{E_i'(\Sigma \otimes \mathbf{I})^{-1}E_i - \frac{\tau^2}{1 + \tau^2 S_i^{*\prime}(\Sigma \otimes \mathbf{I})^{-1} S_i^*}[S_i^{*\prime}(\Sigma \otimes \mathbf{I})^{-1}E_i]^2\} , \end{aligned} \tag{2.4}$$

where $E_i = Y_i^* - Z_i\beta$. Because of the regularity of the likelihood (2.4), it could in principle be straightforwardly maximized with respect to the parameters by any efficient general optimization algorithm, such as the DFP-, GM- or BHHH-algorithms. In practice, however, extremely good starting values are needed to ensure convergence, because the number of parameters easily becomes very large. According to our experience, the exact ML- estimation technique is actually quite impracticable in this case. Furthermore, the maximum of (2.4) can sometimes be attained with $\tau^2 < 0$, i.e. outside the parameter space. We do not discuss this possibility any further, because $\hat{\tau}^2 < 0$ can be taken as rather strong evidence against the fitted model. The problem does not differ from the corresponding problem of negative variance estimates in ordinary error components models.

2.2. The estimation method

Because the ML- method is - according to our experience - computationally infeasible, we are going to suggest a three-stage estimation method that starts by fixing τ, then estimates Σ consistently and then maximizes (2.4) with respect to β. This procedure can then be combined with a grid search in order to maximize the approximate concentrated likelihood with respect to τ^2.

Assuming $\tau^2 \geq 0$ to be fixed, in technical terms, (2.4) very much resembles the likelihood function of an ordinary simultaneous equations model. Because $E\psi_{ij} = 0$ and because we assumed ψ_{ij} and X_{ij} to be mutually independent, we can estimate $\mathbf{B}$ and $\mathbf{\Gamma}$ $\sqrt{n}$- consistently by the usual 2SLS- estimators $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{\Gamma}}$. Denote the corresponding residuals by

$$\tilde{\psi}_{ij} = Y_{ij} - \tilde{\mathbf{B}}Y_{ij} - \tilde{\mathbf{\Gamma}}X_{ij}$$

and estimate Σ by

$$\tilde{\Sigma} = \frac{1}{nk}\{\sum_{i=1}^{n}\sum_{j=1}^{k}\tilde{\psi}_{ij}\tilde{\psi}'_{ij} - \tau^2\sum_{i=1}^{n}\sum_{j=1}^{k}S_{ij}S'_{ij}\} \quad . \tag{2.5}$$

This estimator makes use of the fact that we assumed $\{\xi_i\}$ and $\{\varepsilon_{ij}\}$ to be mutually independent. The $\sqrt{n}$- consistency of $\tilde{\Sigma}$ follows directly from the $\sqrt{n}$- consistency of $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{\Gamma}}$.

In the next step, compute the upper triangular Cholesky- decomposition of $\tilde{\Sigma}$,

$$\tilde{\Sigma} = \mathbf{U}\mathbf{U}'$$

and note that

$$(\tilde{\Sigma}\otimes\mathbf{I})^{-1} = (\tilde{\Sigma}^{-1}\otimes\mathbf{I}) = (\mathbf{U}^{-1}\otimes\mathbf{I})'\ (\mathbf{U}^{-1}\otimes\mathbf{I})$$

where $\mathbf{U}^{-1}$ is an upper triangular matrix as well.

Ignoring for a moment the term $nk\log\ |\ \det(\mathbf{I}-\mathrm{B})\ |$ in (2.4), the maximization of the likelihood $\sum_{i=1}^{n}\log L_{Y_{\cdot}|X_{\cdot},S_{\cdot}}(\mathbf{B},\mathbf{\Gamma},\tau^2,\tilde{\Sigma})$ with respect to $\mathbf{B}$ and $\mathbf{\Gamma}$ would now be equivalent to minimizing the quadratic form

$$Q(\beta) = \sum_{i=1}^{n}\{E_i'(\tilde{\Sigma}\otimes\mathbf{I})^{-1}E_i - \frac{\tau^2}{1+\tau^2S_i^{*\prime}(\tilde{\Sigma}\otimes\mathbf{I})^{-1}S_i^*}\left[S_i^{*\prime}(\tilde{\Sigma}\otimes\mathbf{I})^{-1}E_i\right]^2\}\ , \tag{2.6}$$

where $E_i = Y_i^* - Z_i\beta$.
By defining the following $(kK+1) \times 1$- vectors

$$S_i^{**} = \begin{pmatrix} 1 \\ \tau(\mathbf{U}^{-1} \otimes \mathbf{I})S_i^* \end{pmatrix} \quad , \quad Y_i^{**} = \begin{pmatrix} 0 \\ (\mathbf{U}^{-1} \otimes \mathbf{I})Y_i^* \end{pmatrix} \quad , \quad E_i^{**} = \begin{pmatrix} 0 \\ (\mathbf{U}^{-1} \otimes \mathbf{I})E_i \end{pmatrix}$$

and the $(kK+1) \times p$ matrix

$$Z_i^{**} = \begin{pmatrix} 0 \\ (\mathbf{U}^{-1} \otimes \mathbf{I})Z_i \end{pmatrix} \quad ,$$

the quadratic form (2.6) can be written as

$$Q(\beta) = \sum_{i=1}^{n} E_i^{**\prime}(\mathbf{I} - \mathcal{P}_{S_i^{**}})E_i^{**} \tag{2.6$'$}$$

where

$$\mathcal{P}_{S_i^{**}} = S_i^{**}(S_i^{**\prime}S_i^{**})^{-1}S_i^{**\prime} \quad .$$

Because the dominating part of (2.4) can be formulated as a sum of squared errors (2.6′), it is obvious that (2.4) can be efficiently maximized with respect to β by the Newton-Raphson algorithm without any real danger of indefinite Hessians. Denote

$$Y^{**} = \begin{pmatrix} Y_1^{**} \\ \cdot \\ \cdot \\ Y_n^{**} \end{pmatrix} \quad , \quad S^{**} = \begin{pmatrix} S_1^{**} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & S_2^{**} & \cdot & \cdot & \cdot & 0 \\ \cdot & & \cdot & & & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 & S_n^{**} \end{pmatrix} \quad \text{and} \quad Z^{**} = \begin{pmatrix} Z_1^{**} \\ \cdot \\ \cdot \\ Z_n^{**} \end{pmatrix} .$$

Note that Y^{**} is an $n(kK+1) \times 1$- vector, S^{**} is an $n(kK+1) \times n$- matrix and Z^{**} is an $n(kK+1) \times p$- matrix, where $p = m_1 + \ldots + m_K$.

Denote further

$$X^{**} = (\,Z^{**} \quad S^{**}\,) \qquad \text{and} \qquad \beta^{**} = \begin{pmatrix} \beta \\ \beta^* \end{pmatrix} \quad ,$$

where β^* is an $n \times 1$ vector of regression coefficients for the rows of S^{**}. With this notation, it is obvious that

$$\min_{\beta} Q(\beta) = \min_{\beta^{**}} (Y^{**} - X^{**}\beta^{**})'(Y^{**} - X^{**}\beta^{**}) \quad .$$

In addition to $Q(\beta)$, $nk \log \mid \det(\mathbf{I} - \mathbf{B}) \mid$ is the only term in (2.4) that still depends on the β- parameters. Because $g(\beta^{**}) = nk \log \mid \det(\mathbf{I} - \mathbf{B}) \mid$ does not depend on β^* or $\mathbf{\Gamma}$, the majority of the entries in the gradient vector $Dg(\beta^{**})'$ and in the Hessian matrix $D^2g(\beta^{**})$ will actually be zero.

Formula (2.6′) implies that the maximization of (2.4) with respect to $\mathbf{B}$ and Γ will be equivalent to minimizing

$$(Y^{**} - X^{**}\beta^{**})'(Y^{**} - X^{**}\beta^{**}) - 2g(\beta^{**}) \tag{2.7}$$

with respect to β^{**}. The Newton-Raphson algorithm minimizing (2.7) will consist of the following steps:

$$\beta^{**}_{(\nu)} = \{X^{**\prime}X^{**} - D^2g(\beta^{**}_{(\nu-1)})\}^{-1}\{X^{**\prime}Y^{**} - D^2g(\beta^{**}_{(\nu-1)})\beta^{**}_{(\nu-1)} + Dg(\beta^{**}_{(\nu-1)})'\} \tag{2.8}$$

$$\nu = 1, 2, \ldots .$$

Starting from the obvious initial value

$$\beta^{**}_{(o)} = (X^{**\prime}X^{**})^{-1}X^{**\prime}Y^{**}$$

this algorithm will converge *very* quickly. For a fixed $\tau^2 \geq 0$ we thus have

$$\hat{\beta}_\tau = (\mathbf{I} \quad 0) \lim_{\nu \to \infty} \beta^{**}_{(\nu)} \quad , \tag{2.9}$$

as our estimator for β. The corresponding value of (2.4)

$$l_{\max}(\tau^2) = l(\hat{\beta}_\tau, \tau^2, \tilde{\Sigma}) = \sum_{i=1}^{n} \log \; L_{Y_i|X_i,S_i}(\hat{\mathbf{B}}_\tau, \hat{\Gamma}_\tau, \tau^2, \tilde{\Sigma})$$

can then be calculated.

At the final stage we find the maximum of $l_{\max}(\tau^2)$ by a grid search, and denote the maximizing value by $\hat{\tau}_{3S}$. Our final three-stage $(3S)$ estimator for β will then be

$$\hat{\beta}_{3S} = \hat{\beta}_{\hat{\tau}_{3S}} \quad . \tag{2.10}$$

Proposition 1: *If no structural restrictions are imposed on* Σ *in the model* (2.1) *, then* $\hat{\beta}_\tau$ *in* (2.9) *will be an asymptotically efficient estimator for* β *for each fixed* $\tau \geq 0$.

Proof: Assume τ to be fixed. Because the 2SLS- estimators $\tilde{\mathbf{B}}$ and $\tilde{\Gamma}$ are $\sqrt{n}$- consistent for $\mathbf{B}$ and Γ, the estimator $\tilde{\Sigma}$ for Σ will share the same property whenever no restrictions are imposed on the elements of Σ. From (2.4) it is easy to see that for a fixed τ^2, the information matrix

$$-ED^2l_\tau(\beta, \Sigma) = -ED^2 \sum_{i=1}^{n} \log \; L_{Y_i|X_i,S_i}(\mathbf{B}, \Gamma, \tau^2, \Sigma)$$

will be block diagonal. This is because

$$\frac{d}{d\beta}l_\tau(\beta, \Sigma) = \frac{d}{d\beta}nk \log \; |\det(\mathbf{I} - \mathbf{B})| \tag{2.11}$$

$$- \sum_{i=1}^{n} Z_i'\{(\Sigma \otimes \mathbf{I})^{-1}E_i - \frac{\tau^2}{1 + \tau^2 S_i^{*\prime}(\Sigma \otimes \mathbf{I})^{-1}S_i^*}(\Sigma \otimes \mathbf{I})^{-1}S_i^*S_i^{*\prime}(\Sigma \otimes \mathbf{I})^{-1}E_i\} ,$$

and consequently

$$
\begin{aligned}
&-E\frac{d^2}{d\beta d\Sigma} l_\tau(\beta,\boldsymbol{\Sigma}) \\
&= \sum_{i=1}^{n} \left[\frac{d}{d\Sigma} Z_i'\{(\Sigma\otimes \mathbf{I})^{-1} - \frac{\tau^2}{1+\tau^2 S_i^{*\prime}(\Sigma\otimes \mathbf{I})^{-1}S_i^*}(\Sigma\otimes \mathbf{I})^{-1}S_i^* S_i^{*\prime}(\Sigma\otimes \mathbf{I})^{-1}\} \right] \; E\; E_i \\
&= 0 \qquad\qquad (2.12)
\end{aligned}
$$

Thus, for *any* $\sqrt{n}$- consistent estimator $\tilde{\Sigma}$, the maximizing value of $l_\tau(\beta,\tilde{\Sigma})$ will be asymptotically efficient for β.

Proposition 2: *With the assumptions of Proposition 1,* $\hat{\tau}_{3S}$ *in* (2.10) *will be* $\sqrt{n}$*- consistent for* τ.

Proof: Because $\tilde{\Sigma}$ is $\sqrt{n}$- consistent for Σ, and because $\hat{\beta}_\tau$ is $\sqrt{n}$- consistent for β for each τ according to Proposition 1, it follows that

$$
\frac{1}{\sqrt{n}}\left[l_\tau(\hat{\beta}_\tau,\tilde{\Sigma}) - l(\beta,\tau^2,\Sigma) \right]
$$

tends to a well defined limiting distribution when n tends to infinity. In turn, this implies the $\sqrt{n}$- consistency of $\hat{\tau}_{3S}$.

Proposition 3: *With the previous assumptions,* $\hat{\beta}_{3S}$ *will be an asymptotically efficient estimator for* β.

Proof: Just as in the proof of Proposition 1, we only have to show that the information matrix is block diagonal with respect to β and τ^2 . It follows from (2.11) that

$$
\begin{aligned}
&-E\frac{d^2}{d\beta d\tau^2} l(\beta,\tau^2,\Sigma) \\
&= \sum_{i=1}^{n} \left[\frac{d}{d\tau^2} Z_i'\{(\Sigma\otimes \mathbf{I})^{-1} - \frac{\tau^2}{1+\tau^2 S_i^{*\prime}(\Sigma\otimes \mathbf{I})^{-1}S_i^*}(\Sigma\otimes \mathbf{I})^{-1}S_i^* S_i^{*\prime}(\Sigma\otimes \mathbf{I})^{-1}\} \right] \; E\; E_i \\
&= 0 \qquad\qquad (2.13)
\end{aligned}
$$

Because $\hat{\tau}_{3S}$ and $\tilde{\Sigma}$ are both $\sqrt{n}$- consistent, the proposition has thus been proved.

2.3. Testing the hypothesis $\tau = 0$

If $\tau = 0$, model (2.1) reduces to an ordinary simultaneous-equations model. Because there are lots of ready-made computer programmes available for ordinary FIML, but none for the model (2.1), it is certainly worth while to test the hypothesis $\tau = 0$ before proceeding. From this point of view, the most attractive testing principle would of course be the LM- principle, because the estimation of model (2.1) under the alternative would not be necessary.

To set up the LM- test, we first have to calculate the gradient vector of the log-likelihood (2.4). We note that the likelihood function becomes much simpler, if the elements of $\mathbf{\Omega} = (\omega_{ij}) = \mathbf{\Sigma}^{-1}$ are used as parameters instead of the error covariances $\mathbf{\Sigma} = (\sigma_{ij})$. Further, just one component of the gradient vector will actually be needed, because

$$\left[\frac{d}{d\beta} l(\beta, 0, \mathbf{\Omega})\right]_{\beta=\hat{\beta}_{ML}, \mathbf{\Omega}=\hat{\mathbf{\Omega}}_{ML}} = 0$$

and

$$\left[\frac{d}{d\mathbf{\Omega}} l(\beta, 0, \mathbf{\Omega})\right]_{\beta=\hat{\beta}_{ML}, \mathbf{\Omega}=\hat{\mathbf{\Omega}}_{ML}} = 0 \quad ,$$

where $\hat{\beta}_{ML}$ and $\hat{\mathbf{\Omega}}_{ML} = \hat{\mathbf{\Sigma}}_{ML}^{-1}$ denote the FIML- estimators of β and $\mathbf{\Omega}$ when $\tau = 0$. We thus only have to calculate the $(p+1)$:th component of the gradient vector,

$$\frac{d}{d\tau^2} l(\beta, \tau^2, \mathbf{\Omega}) = -\frac{1}{2}\sum_{i=1}^{n} \frac{S_i^{*\prime}(\mathbf{\Omega} \otimes \mathbf{I})S_i^*}{1 + \tau^2 S_i^{*\prime}(\mathbf{\Omega} \otimes \mathbf{I})S_i^*} + \frac{1}{2}\sum_{i=1}^{n} \frac{[S_i^{*\prime}(\mathbf{\Omega} \otimes \mathbf{I})E_i]^2}{[1 + \tau^2 S_i^{*\prime}(\mathbf{\Omega} \otimes \mathbf{I})S_i^*]^2}$$

and its value at the FIML- estimators $\hat{\beta}_{ML}$, $\hat{\mathbf{\Omega}}_{ML}$

$$\left[\frac{d}{d\tau^2} l(\beta, \tau^2, \mathbf{\Omega})\right]_{\beta=\hat{\beta}_{ML}, \tau^2=0, \mathbf{\Omega}=\hat{\mathbf{\Omega}}_{ML}} = \tag{2.14}$$

$$\frac{1}{2}\sum_{i=1}^{n}\{\left[S_i^{*\prime}(\hat{\mathbf{\Omega}}_{ML} \otimes \mathbf{I})\hat{E}_i\right]^2 - \left[S_i^{*\prime}(\hat{\mathbf{\Omega}}_{ML} \otimes \mathbf{I})S_i\right]^2\} \quad ,$$

where $\hat{E}_i = Y_i^* - Z_i\hat{\beta}_{ML}$. The asymptotic variance of (2.14) can be calculated as the corresponding diagonal element of the inverse of the information matrix,

$$\hat{i}^{(-1)}_{(p+1),(p+1)} = \left[I(\hat{\beta}_{ML}, 0, \hat{\mathbf{\Omega}}_{ML})^{-1}\right]_{(p+1),(p+1)} \quad . \tag{2.15}$$

By combining the results (2.12) and (2.13), we can see that the information matrix will be of the block diagonal form

$$I(\beta, \tau^2, \Omega) = \begin{pmatrix} I_{\beta,\beta} & 0 & 0 \\ 0 & i_{\tau^2,\tau^2} & I'_{\tau^2,\Omega} \\ 0 & I_{\tau^2,\Omega} & I_{\Omega,\Omega} \end{pmatrix} \quad , \tag{2.16}$$

where $I_{\beta,\beta}$ denotes the $p \times p$- block $I_{\beta,\beta} = -E\frac{d^2}{d\beta d\beta'} l(\beta, \tau^2, \Omega)$ of the information matrix and the other blocks have been defined correspondingly. The detailed derivations of explicit expressions for the blocks i_{τ^2,τ^2} , $I_{\tau^2,\Omega}$ and $I_{\Omega,\Omega}$ will be left to the Appendix. Here, we just note that

$$i^{(-1)}_{(p+1),(p+1)} = \frac{1}{i_{\tau^2,\tau^2} - I'_{\tau^2,\Omega} I^{-1}_{\Omega,\Omega} I_{\tau^2,\Omega}} \quad . \tag{2.17}$$

Finally, by combining (2.14) and (2.15) we get

$$z = \frac{1}{2}\sqrt{\hat{i}^{(-1)}_{(p+1),(p+1)}} \sum_{i=1}^{n} \{ \left[S_i^{*\prime}(\hat{\Omega}_{ML} \otimes \mathbf{I})\hat{E}_i \right]^2 - \left[S_i^{*\prime}(\hat{\Omega}_{ML} \otimes \mathbf{I})S_i \right]^2 \} \tag{2.18}$$

$$\overset{\text{asympt.}}{\sim} N(0,1)$$

as our test statistic. Note, that (2.18) is just the square root of the ordinary LM- test statistic for the hypothesis $\tau = 0$.

As a final remark we point out, that it is not always necessary to calculate all of the elements in the information matrix (2.16) in order to carry out the test. In cases where the data is clearly contradictory to the null hypothesis $H_o: \tau = 0$, we can calculate a lower bound for the absolute value of the test statistic (2.18) by substituting $i^{-1}_{\tau^2,\tau^2}$ for $i^{(-1)}_{(p+1),(p+1)}$. This is because $I^{-1}_{\Omega,\Omega}$ is non-negative definite, and according to (2.17) we thus have

$$i^{(-1)}_{(p+1),(p+1)} \geq i^{-1}_{\tau^2,\tau^2}$$

and

$$| z_{\min} | \leq | z | \quad ,$$

where

$$z_{\min} = \frac{1}{2\sqrt{\hat{i}_{\tau^2,\tau^2}}} \sum_{i=1}^{n} \{ \left[S_i^{*\prime}(\hat{\Omega}_{ML} \otimes \mathbf{I})\hat{E}_i \right]^2 - \left[S_i^{*\prime}(\hat{\Omega}_{ML} \otimes \mathbf{I})S_i \right]^2 \} \quad . \tag{2.19}$$

2.4. Testing for structural change

Assume now that additional observations

$$Y_* = \begin{pmatrix} Y^*_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ Y^*_{n+h} \end{pmatrix} \quad , \quad Z_* = \begin{pmatrix} Z_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ Z_{n+h} \end{pmatrix} \quad , \quad S_* = \begin{pmatrix} S^*_{n+1} & 0 & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & 0 & S^*_{n+h} \end{pmatrix} \quad , \tag{2.20}$$

not included in the estimation set, would be available. The symbols used in (2.20) conform to the notation defined in Section 2.1. One way of testing for structural change at the time point $i = n+1$ would be to calculate the forecast errors

$$E_* = Y_* - \hat{Y}_* = Y_* - Z_*\hat{\beta}_{3S}$$

corresponding to the new observations Y_*. According to model (2.1), the asymptotic distribution of E_* with increasing n should be multivariate normal

$$E_* \sim N_{Kkh}(0, \Omega) \quad , \tag{2.21}$$

where

$$\begin{aligned} \Omega = \operatorname{cov}(Y_* - \hat{Y}_*) &= \operatorname{cov}\left((Y_* - Z_*\beta) - Z_*(\hat{\beta}_{3S} - \beta)\right) \\ &= \operatorname{cov}(Y_* - Z_*\beta) + Z_*\operatorname{cov}(\hat{\beta}_{3S} - \beta)Z_*' \\ &= \mathbf{I} \otimes \Sigma \otimes \mathbf{I} + \tau^2 S_* S_*' + Z_*\operatorname{cov}(\hat{\beta}_{3S} - \beta)Z_*' \; . \end{aligned} \tag{2.22}$$

The asymptotic normality simply follows from the fact that $\hat{\beta}_{3S}$ is $\sqrt{n}$- consistent for β and from the normality assumptions in (2.1). Furthermore, because $\tilde{\Sigma}$ is $\sqrt{n}$- consistent for Σ and $\hat{\tau}_{3S}$ is $\sqrt{n}$- consistent for τ, we can deduce from (2.21) and (2.22) that the asymptotic distribution of

$$q_h = E_*' \left[\mathbf{I} \otimes \tilde{\Sigma} \otimes \mathbf{I} + \hat{\tau}^2_{3S} S_* S_*' + Z_*\hat{\operatorname{cov}}(\hat{\beta}_{3S} - \beta)Z_*'\right]^{-1} E_* \tag{2.23}$$

should be χ^2_{Kkh} under the null hypothesis of no change in the model (2.1). The quadratic form q_h can thus be used as a test statistic for testing whether a structural change has occurred in the DGP at $i = n+1$. The estimated covariance matrix $\hat{\operatorname{cov}}(\hat{\beta}_{3S} - \beta)$ can be obtained as a by-product from the estimation method suggested in Section 2.2.

3. AN EMPIRICAL EXAMPLE: MODELLING THE WAGE DETERMINATION IN FINNISH MANUFACTURING INDUSTRIES

3.1. The economic background

3.1.1. The determination of contract wages

In Finland, the contracts on the terms of work are formally made between the trade unions and the corresponding employers' organizations within each branch of the industry. The agreements have mostly been made for one year at a time. The starting date of a new period has usually been March 1. In most years, however, the essential contents of the contract has already beforehand been agreed upon in a collective bargaining between the Central Organization of the Trade Unions and the Employers' Confederation. In later years, there has been a strong tendency towards abandoning centralized agreements. Even in years when the central organizations have not been able to find a mutual agreement, the final contracts for different branches have been astonishingly alike in terms of wage increases. This is why we shall treat the whole industry as one branch and assume that the wage raises during the contract period are actually agreed upon by the central organizations.

Because each agreement on the terms of work has been unique and both the amounts and the timings of the wage raises have varied, we have made use of the following strategy: In the logical derivation of the model, our starting point is the *total* amount of increase of the contract wages within the whole contract period. To distribute the total increase among the four quarters, we have constructed a kind of "allotment factor" variable to indicate the timings of the wage raises within each contract period. The values of the allotment factor are assigned on the basis of the formal texts of the contracts. We shall treat this variable as exogenous, because there seems to be no way of predicting its values in advance. This research strategy can be further motivated by noting that the expectations held by the key economic research institutes in Finland on inflation, productivity, taxes, etc. that actually make up the information set accessible to the negotiators, are also formed on an *annual* basis. The model thus has to first explain, how the total *annual* increases of the contract wages are determined, and only then describe, how the timings of the raises are settled and how the total annual wage changes are allotted among the four quarters.

By thinking of all manufacturing industries as a kind of "large firm" and by concentrating on the total *annual* wage raises, we can actually use the so-called "right to manage"- model to describe the negotiations between the central organizations. For different versions of this well established model, see for instance Nickell and Andrews (1983) or Hoel and Nymoen (1988). Following the reasoning of Nickell and Andrews (1983), we shall assume that the employers' utility function depends on real profits, which in turn depend on expected producer prices $\tilde{P}_t^e$, expected consumer prices P_t^e, expected productivity A_t^e, number of persons employed L_t, real wages w_t, average employment tax rate s_t^e and on fixed costs. The employers are first assumed to maximize their profits with respect to L_t.

The utility function of the trade unions on the other hand is assumed to depend above all on the real wages w_t and on the expected average income tax rate t_t^e. The nominal contract wages W_t^c are then thought to be determined as an unsymmetric Nash solution of the bargaining problem, i.e. through maximization of a logarithmic convex combination of the two utility functions. Nickell and Andrews (1983) deduced that the expected value of the logarithm of the contract wages $E \log W_t^c$ should then depend approximately linearly on the variables $\log P_t^e$, $\log \tilde{P}_t^e$, $\log(1+s_t^e)$, $\log(1-t_t^e)$, $\log A_t^e$ and on the unemployment rate UE_t. Furthermore, $E \log W_t^c$ will depend on which convex combination of the utilities has actually been maximized, i.e. on the relative bargaining powers of the negotiating partners. Because the present *level* of real wages in relation to productivity and the unemployment rate may affect the relative bargaining powers, we also included the most recent unemployment figures and an error correction term in our model that we otherwise formulated in terms of quarterly differences:

$$\bar{w}_{ij}^c = (\gamma_{1o} + \gamma_{11} p_i^e + \gamma_{12} a_i^e + \gamma_{13} ue_i + \gamma_{14} wed_i + \gamma_{15} ec_i^* + \xi_i) \cdot s_{ij} + \varepsilon_{ij}^{(1)} \qquad (3.1)$$

where

$$i = \mathrm{int}(\frac{t-1}{4}) + 1 \qquad , \qquad j = t - 4 \cdot (i-1)$$

and t refers to time. The meanings of the symbols are as follows:

$\bar{w}_{ij}^c =$ the logarithmic change of the contract wage index $\bar{W}_t^c$ during quarter t, i.e. $\log \bar{W}_t^c - \log \bar{W}_{t-1}^c$

$p_i^e =$ the expected rate of inflation for the year i based on the information available at the end of the previous year

$a_i^e =$ the expected, logarithmic change of productivity for the year i

$ue_i =$ the annual change in the unemployment rate at the beginning of the year i

$wed_i =$ the expected annual logarithmic change of the tax wedge $\log \frac{1+s_i^e}{1-t_i^e} - \log \frac{1+s_{i-1}}{1-t_{i-1}}$

$ec_t =$ $\log \bar{W}_t - \hat{\kappa}_1 \log P_t - \hat{\kappa}_2 \log A_t$, where

- $\bar{W}_t =$ the average industrial wage index at quarter t
- $P_t =$ consumer price index at quarter t
- $A_t =$ productivity at quarter t
- $\hat{\kappa}_1, \hat{\kappa}_2$ are the OLS estimates of the parameters defining the cointegration vector between $\log \bar{W}_t$, $\log P_t$ and $\log A_t$ (cf. Engle and Granger, 1987)

$ec_i^* =$ ec_t at the end of the previous year, i.e. $ec_i^* = ec_{4(i-1)}$

$s_{ij} =$ the allotment factor that measures, how large a share of the total annual increase of the contract wages is allotted to quarter j in the year i ($s_{i1} + s_{i2} + s_{i3} + s_{i4} = 1$)

$\xi_i =$ the annual error term
$\varepsilon_{ij}^{(1)} =$ the error term depicting the quarterly miscellaneous variation of $\bar{w}_{ij}^e$

We shall make the following distributional assumptions concerning the error terms :

$$\varepsilon_{ij}^{(1)} \sim NID(0, \sigma_1^2) \quad , \quad \xi_i \sim NID(0, \tau^2) \quad , \quad \{\xi_i\} \perp\!\!\!\perp \{\varepsilon_{ij}^{(1)}\} \ . \tag{3.2}$$

It is to be noted that the expected average payroll tax variable $\log(1 + s_i^e)$ and the average income tax variable $\log(1 - t_i^e)$ have been merged into one entity, the logarithmic tax wedge $\log(1 + s_i^e) - \log(1 - t_i^e)$. This was partly motivated by the empirical finding, that the estimated coefficients of these two variables were almost equal, but of opposite signs.

As proxies for the expectation variables p_i^e and a_i^e , we used the figures published in the most recent Economic Outlook of the Ministry of Finance. According to the interviews with several participants in the wage negotiations, both sides actually base their standpoints on the information supplied by the Finance Ministry. The productivity expectations were first scaled in order to remove the systematic bias. The scaling factor was calculated by the least squares method.

We dropped the expected producer prices $\log \tilde{P}_t^e$ from the model, because we had difficulties in finding reliable data. At first, we also included the expected logarithmic change in the terms of trade into the model (3.1), because large fluctuations in export prices can certainly affect the relative bargaining powers of the negotiating partners. Anyway, the inclusion of this variable did not improve the fit significantly, so we decided to drop it from our final model.

For further details of the logic behind model (3.1) + (3.2), see Rahiala and Kovalainen (1992).

3.1.2. Generating the wage drift

In Rahiala and Kovalainen (1992) we developed a theory for the emergence of the wage drift by assuming that individual workers can put forth new wage claims to their employer even after the collective agreement has been reached. We assumed that an implicit "market wage" $W_{j,t}^*$ could be defined for each worker j as a result of these hypothetical wage negotiations, whether the worker actually makes any new claims or not. The mathematical model we used was largely similar to the model depicting the centralized negotiations, and the nominal "market wages" were taken as the corresponding Nash solutions for each worker. The actual concurrent values of the consumer prices P_t and productivity A_t were assumed known to the negotiating partners. We also assumed that the "market wages" for workers within each firm would be settled one by one in some systematic order - e.g. in the order of seniority. We further assumed that the negotiations between each employee and his employer break down as soon as either side withdraws from the bargaining. According to the vocabulary of Binmore et al. (1986), we used an "outside option point" as the threat point by assuming that the worker would have to find another job whenever the negotiations break down. In such a case, the worker could even face the possibility of getting unemployed. Our approach differs from that of Holden (1989) and Holmlund and Skedinger (1990), who used the contract wage as the fall-back position for employees in the firm-level bargaining. In their concept, the Nash solution would always exceed the contract wage. Our assumption, however, allows the solution to go below the contract wage, because the risk of getting unemployed is always present.

In Rahiala and Kovalainen (1992), we called the relative difference between a worker's "market wage" $W^*_{j,t}$ and his contract wage $W^c_{j,t}$

$$l^*_{j,t} = \frac{W^*_{j,t} - W^c_{j,t}}{W^c_{j,t}} \tag{3.3}$$

the *pressure for wage drift* for worker j. We assumed that the pressure $l^*_{j,t}$ will lead to a real wage raise only after exceeding a positive threshold $c_t > 0$. In other words, we assumed that the actual observed nominal wage $W_{j,t}$ for individual j would be of the form

$$W_{j,t} = \begin{cases} (1 + l^*_{j,t})W^c_{j,t} & \text{if} \quad l^*_{j,t} > c_t \\ W^c_{j,t} & \text{if} \quad l^*_{j,t} \leq c_t \end{cases} \quad . \tag{3.4}$$

We further assumed that the pressure for drift l^*_t for a randomly chosen individual would be log-normally distributed,

$$\log{(1 + l^*_t)} \sim N(\mu_t, \sigma^2_t) \tag{3.5}$$

The parameters $\mu_t = E(\log\ W^*_t - \log\ W^c_t)$ and $\sigma^2_t = \text{var}(\log\ W^*_t - \log\ W^c_t)$ were allowed to vary over time, but the threshold c_t was simultaneously assumed to change so that the probability $\Pi_t = 1 - \Phi(\frac{c^*_t - \mu_t}{\sigma_t})$ of an individual surpassing the threshold would stay fairly constant. Here $c^*_t = \log{(1 + c_t)}$, Φ denotes the cumulative distribution function of the standard normal distribution and ϕ is the corresponding density function. Assumptions (3.4) and (3.5) together imply that the expectation of the *logarithmic wage drift*

$$\begin{aligned} E(\log\ W_t - \log\ W^c_t) &= \ 0 \cdot P(l^*_t \leq c_t) + E\left[\log{(1 + l^*_t)} \mid l^*_t > c_t\right] \\ &= \mu_t + \sigma_t \cdot \frac{\phi(\frac{c^*_t - \mu_t}{\sigma_t})}{1 - \Phi(\frac{c^*_t - \mu_t}{\sigma_t})} \end{aligned} \tag{3.6}$$

would depend approximately linearly on *both* parameters μ_t and σ_t.

We can summarize our conclusions by saying that according to Nickell and Andrews (1983), $E(\log\ W^c_t)$ should depend linearly on $\log\ P^e_t$, $\log\ A^e_t$, and on some other variables. Analogously, $E(\log\ W_t^*)$ should depend on $\log\ P_t$, $\log\ A_t$ and on the bargaining power of individual workers. The workers' bargaining power, the employers' adjustment costs and the workers' fall-back utilities will all certainly depend on the state of the labour market. This is why we decided to include the number of vacancies vac_t in the model as a sensitive indicator of the state of the labour market. Consequently, the expectation of the logarithmic pressure for drift $\mu_t = E(\log\ W_t^*) - E(\log\ W_t^c)$ should be linearly dependent on $\log\ P_t^e$, $\log\ P_t$, $\log\ A_t^e$, $\log\ A_t$ and vac_t.

As a measure of the aggregate wage drift emerging at quarter t we used

$$d_t = \nabla D_t = D_t - D_{t-1} \quad \text{, where} \quad D_t = \log\ \bar{W}_t - \log\ \bar{W}_t^c \quad . \tag{3.7}$$

Here ∇ denotes the quarterly difference operator, $\bar{W}_t$ the average industrial wage index and $\bar{W}_t^c$ the average contract wage index. By definition (3.7), $Ed_t = ED_t - ED_{t-1}$ should be linearly dependent on $\nabla \log\ P_t^e, \nabla \log\ P_t, \nabla \log\ A_t^e$, $\nabla \log\ A_t, \nabla vac_t, \nabla \sigma_t$ and ec_{t-1}. The inclusion of the error correction term ec_{t-1} is motivated by the fact that the employers have to take the *level* of the productivity into account, and the deviation from the equilibrium thus affects the relative bargaining power of the negotiating partners. All the other variables in the list are directly observable, but σ_t constitutes a problem. We simply used the standard deviation of the average hourly wages $stdw_t$, taken over branches, as a proxy for σ_t. We divided the manufacturing industries into 8 branches, and the standard deviations of these 8 averages were assigned as values for $stdw_t$.

One further problem that confounds the data is that the values of the average wage index $\bar{W}_t$ actually drop at the third quarter every year. This is due to the structural changes in the work force during the holiday season, when great numbers of low-paid substitutes temporarily join the work force. This makes it virtually impossible to give an estimate for the wage drift in the third quarter of each year.

To overcome this problem, we defined a new operator ∇^* as follows:

$$\nabla^* D_t = \begin{cases} D_t - D_{t-1} & \text{if} \quad t \neq 4 \cdot \text{int}(\frac{t-1}{4}) + 4 \\ D_t - D_{t-2} & \text{if} \quad t = 4 \cdot \text{int}(\frac{t-1}{4}) + 4 \end{cases} .$$

This means that the value of D_t in the fourth quarter of each year will not be compared to the corresponding value in the third quarter, but in the second quarter.

To allow for an exceptional level of $D_t - D_{t-1}$ in the third quarters, we formulated our final model simply as

$$d_t^* = \nabla^* D_t = \beta_{21} \nabla^* \log\ P_t + \gamma_{20} + \gamma_{21} \delta_{3,t} + \gamma_{22} \nabla^* \log\ P_t^e \tag{3.8}$$

$$+ \gamma_{23} \nabla^* \log\ A_t + \gamma_{24} \nabla^* \log\ A_t^e + \gamma_{25} \nabla^* vac_t + \gamma_{26} \nabla^* stdw_t + \gamma_{27} ec_{t-1} + \varepsilon_t^{(2)}$$

where ec_{t-1} is the error correction term. The symbol $\delta_{3,t}$ denotes a dummy for the third quarter, i.e. $\delta_{3,t} = 1$ if $t = 4 \cdot \text{int}(\frac{t-1}{4}) + 3$, and zero otherwise.

The error term $\varepsilon_t^{(2)}$ was assumed to be normally distributed noise

$$\varepsilon_t^{(2)} \sim NID(0, \sigma_2^2) \quad , \tag{3.9}$$

but it was allowed to correlate simultaneously with $\varepsilon_t^{(1)}$ in (3.1).

3.1.3. Modelling the inflation rate

As far as the price equation of our model is concerned, we merely followed the lines of Branson and Myhrman (1976), Sargan (1980) and Saikkonen and Teräsvirta (1985) without penetrating any deeper into the theory of inflation. We adopted the "markup pricing" view on the pricing mechanism of Finnish products and ended up with the following model

$$p_t = \gamma_{30} + \gamma_{31} i_t + \gamma_{32} p_{t-2} + \beta_{30} \bar{w}_t + \gamma_{33} \bar{w}_{t-1} + \gamma_{34} \bar{w}_{t-2} + \gamma_{35} tax_t + \gamma_{36} a_{t-1} + \varepsilon_t^{(3)} \tag{3.10}$$

where

$p_t =$ $\nabla \log P_t =$ the (logarithmic) inflation rate
$\bar{w}_t =$ $\nabla \log \bar{W}_t =$ the (logarithmic) wage increase at quarter t
$a_t =$ $\nabla \log A_t =$ the annual (logarithmic) change in productivity
$i_t =$ the (logarithmic) rate of increase of the import prices
$tax_t =$ the (logarithmic) change in the rate of indirect taxes

The error term $\varepsilon_t^{(3)}$ was assumed to be normally distributed noise

$$\varepsilon_t^{(3)} \sim NID(0, \sigma_3^2) \quad , \tag{3.11}$$

but it was allowed to correlate simultaneously with $\varepsilon_t^{(1)}$ and with $\varepsilon_t^{(2)}$.

Note that

$$\bar{w}_t = \bar{w}_t^c + d_t^* - \delta_{3,t-1} d_{t-1}^*$$

where $\bar{w}_t^c = \nabla \log \bar{W}_t^c = \bar{w}_{ij}^c$ whenever $i = \text{int}(\frac{t-1}{4}) + 1$ and $j = t - 4 \cdot (i-1)$. This makes the system of three simultaneous equations (3.1), (3.8) and (3.10) complete.

While searching for a suitable specification of the price equation, we started with a fairly complete model including up to four lags of p_t and $\bar{w}_t$, but by dropping out non-significant variables we ended up with the form (3.10).

3.2. Empirical results:

3.2.1. Specification of the model

To start with, we estimated the combined model $(3.1)+(3.8)+(3.10)$ with data covering the time span 1980 - 1988 by the three-stage method developed in Section 2.2. It soon turned out, however, that in the wage drift equation (3.8), the estimated coefficients of $\nabla^* \log P_t^e$, $\nabla^* \log A_t$ and $\nabla^* \log A_t^e$ were not significant and of the wrong signs, whereas the *observed* inflation rate $\nabla^* \log P_t$ seemed to have a clear, positive impact on the drift. Neither had the surprise variables $\nabla^* \log P_t - \nabla^* \log P_t^e$ or $\nabla^* \log A_t - \nabla^* \log A_t^e$ any considerable explanatory power. This is why we dropped the variables $\nabla^* \log P_t^e$, $\nabla^* \log A_t$ and $\nabla^* \log A_t^e$ from the wage drift equation. The simplified joint model thus became

$$\left\{\begin{array}{l} \tilde{w}_{ij}^c = (\gamma_{1o} + \gamma_{11} p_i^e + \gamma_{12} a_i^e + \gamma_{13} ue_i + \gamma_{14} wed_i + \gamma_{15} ec_i^* + \xi_i)\cdot s_{ij} + \varepsilon_{ij}^{(1)} \\ d_t^* = \nabla^* D_t = \beta_{21} \nabla^* \log\ P_t + \gamma_{20} + \gamma_{21}\delta_{3,t} + \gamma_{25}\nabla^* vac_t \\ \qquad\qquad\qquad\qquad\qquad + \gamma_{26}\nabla^* stdw_t + \gamma_{27} ec_{t-1} + \varepsilon_t^{(2)} \\ p_t = \gamma_{30} + \gamma_{31} i_t + \gamma_{32} p_{t-2} + \beta_{30}\tilde{w}_t + \gamma_{33}\tilde{w}_{t-1} + \gamma_{34}\tilde{w}_{t-2} + \gamma_{36} tax_t + \gamma_{37} a_{t-1} + \varepsilon_t^{(3)} \\ \xi_i \sim NID(0,\tau^2) \quad , \quad \{\xi_i\} \perp\!\!\!\perp \{(\varepsilon_{ij}^{(1)} \quad \varepsilon_t^{(2)} \quad \varepsilon_t^{(3)})'\} \\ (\varepsilon_{ij}^{(1)} \quad \varepsilon_t^{(2)} \quad \varepsilon_t^{(3)})' \sim NID_3(0,\Sigma) \qquad \text{where} \\ \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix} \quad \text{and} \quad i = \operatorname{int}(\tfrac{t-1}{4}) + 1 \quad , \quad j = t - 4\cdot(i-1) \ . \end{array}\right. \tag{3.12}$$

It is very easy to see that this model is of the type (2.1) and that the estimation method of Section 2.2. and the LM- test of Section 2.3. can be applied. The parameter estimates have been displayed in Table 1. Because the preliminary inspection of the residuals did not reveal any deviations from the assumptions (3.12), we next wanted to test, whether the inclusion of the annual error term ξ_i was really necessary or not. To this end, we first calculated the $z_{\min}$- statistic in (2.19). The value $z_{\min} = 3.83$ shows that the data are clearly contradictory to the hypothesis $H_o:\ \tau = 0$.

Table 1 here

The most apparent thing in the first equation is that the estimates of the coefficients of p_i^e and wed_i are almost equal to 1 and -1, respectively. This conforms well with our prior expectations, because the tax rates have invariably been set well before the decisive stages of the wage negotiations. The tax wedge variable

wed_i is thus a part of the information set that is accessible to the negotiators. On the other hand, the estimate of the coefficient for the productivity expectations a_i^e is somewhat greater than 1, but the deviation is not large.
We did not drop the error correction term ec_i^* from the model, although it did not turn out to be significant.

In the wage drift equation, the explanatory power of the wage heterogeneity $\nabla^* stdw_t$ and of $\nabla^* vac_t$ seemed rather good. This fact lends some support to our theory on the emergence of the wage drift developed in Section 3.1.2. The error correction term ec_{t-1} did not prove to be significant here either.

In the third equation, it turned out that the changes in the average rate of indirect taxes tax_t and the lagged changes in productivity a_{t-1} lacked explanatory power. Because these variables should logically be incorporated in the price equation and because the estimates were of the right signs, we still accepted these variables to our maintained model.

To test, whether the negotiators seem to anticipate the future wage drift during the negotiations, we tentatively included the total annual drift $\nabla_4 D_i = D_{4i} - D_{4(i-1)}$ in the contract wage equation. The result of the test was negative in the sense that $\nabla_4 D_i$ seemed to be superfluous. Still, the negative constant term of the first equation might of course be interpreted as a kind of reservation for future wage drift.

We also wanted to test, whether large raises of the contract wages have any restraining effects on the wage drift. To this aim, we included the variables $\nabla^* \log \bar{W}_t^c$ and $\nabla^* \log \bar{W}_{t-1}^c$ to the second equation of the model. The concurrent variable $\nabla^* \log \bar{W}_t^c$ had of course to be handled as endogenous. No instantaneous interdependence between the variables $\nabla^* \log \bar{W}_t^c$ and $\nabla^* D_t$ could be found. On the other hand, the lagged variable $\nabla^* \log \bar{W}_{t-1}^c$ might have some weak negative effects on the drift $\nabla^* D_t$. This finding seems quite plausible, because the wages are not likely to drift much right after a steep raise in the contract wages.

One more thing we wanted to test was whether the proportion of workers, who actually benefit from the drift, stays invariant or not. As explained in Section 3.1.2. , we assumed this proportion $\Pi_t = 1 - \Phi(\frac{c_t^* - \mu_t}{\sigma_t})$ to stay constant over time. The constancy of Π_t is obviously equivalent to the constancy of the coefficient of σ_t in the wage drift model. Because we had to substitute a proxy for σ_t in the second equation, we cannot unfortunately derive the value of Π_t from the value of γ_{26}. What we can do instead, is to test the constancy assumption against some specific alternative, such as a linear trend in the coefficient of $\nabla^* stdw_t$. To this end, we calculated the t- value of the variable $(t-28) \cdot \nabla^* stdw_t$, after it had been tentatively included into the model. According to the results presented in Table I, there seems to be no obvious trend in γ_{26}. We can thus say, that the observed data are not contradictory to the assumption of Π_t staying constant.

It is also interesting to see, how much and how fast the wage drift affects the domestic inflation rate p_t. Assume that an exogenous shock would have been added to the wage drift d_t^* at time $t = t_o$. The corresponding estimates for the impulse responses $\omega_o, \omega_1, \omega_2, \ldots$ in the inflation rate p_t would then be

$$\hat{\omega}_o = 0.433 \ , \quad \hat{\omega}_1 = 0.551 \ , \quad \hat{\omega}_2 = 0.197 \ , \quad \hat{\omega}_3 = -0.362 \ , \ldots\ldots$$

for the first year. In order to draw any conclusions about the longer term effects of this exogenous shock, we would of course have to make some further assumptions concerning the expectation formation mechanism that generates the inflation expectations p_t^e. Anyway, we can formally estimate the total response by summing up the impulse response estimates

$$\sum_{i=0}^{\infty} \hat{\omega}_i = (\hat{\beta}_{30} + \hat{\gamma}_{33} + \hat{\gamma}_{34}) \cdot (1 - \hat{\beta}_{30}\hat{\beta}_{21} - \hat{\gamma}_{32} - \hat{\gamma}_{34}\hat{\beta}_{21} - \hat{\gamma}_{33}\hat{\beta}_{21})^{-1} = 0.877 \ .$$

These calculations reveal that the effects of exogenous shocks in the wage drift tend to carry over to domestic prices very quickly indeed. The firms seem to seek full compensation for the additional costs caused by the drifting wages through raising the domestic prices correspondingly within six months time.

3.2.2. Model diagnostics

The goodness of fit of model (3.12) has been visualized in Figures 1-3, where the actual and the fitted values of $\tilde{w}_{ij}^c$, d_t^* and p_t can be compared. The corresponding R^2- measures are 0.929 for the first equation, 0.865 for the second and 0.942 for the third.

Figures 1 - 3 here

In the first equation, the fitted values were calculated simply as

$$\hat{\tilde{w}}_{ij}^c = (\hat{\gamma}_{1o} + \hat{\gamma}_{11}p_i^e + \hat{\gamma}_{12}a_i^e + \hat{\gamma}_{13}ue_i + \hat{\gamma}_{14}wed_i + \hat{\gamma}_{15}ec_i^*) \cdot s_{ij} \ .$$

This means that the residual sum of squares actually comprises both the annual and the quarterly error variations.

Perhaps the most important assumptions in model (3.12) concern the behaviour of the disturbances, namely

$$\begin{cases} \varepsilon_t = (\varepsilon_{ij}^{(1)} \quad \varepsilon_t^{(2)} \quad \varepsilon_t^{(3)})' \sim NID_3(0, \Sigma) & \text{where} \\ i = \text{int}(\frac{t-1}{4}) + 1 \quad , & j = t - 4 \cdot (i-1) \end{cases}$$

It is fairly straightforward to test the serial independence of ε_t by means of the residuals by estimating their cross-correlations or by stepwise VAR- fits as suggested in Tiao and Box (1981). To obtain estimates for the quarterly disturbances $\varepsilon_{ij}^{(1)}$ in the contract wage equation, we first calculated

$$\tilde{e}_{ij}^{(1)} = \tilde{w}_{ij}^{c} - \hat{w}_{ij}^{c}$$

and estimated the parameters α_i $(i = 1, ..., n)$ by least squares from the equation

$$\tilde{e}_{ij}^{(1)} = \alpha_i \; s_{ij} + \tilde{\varepsilon}_{ij}^{(i)} \tag{3.13}$$

where the s_{ij}'s denote the values of the allotment factor variable. The corresponding OLS- residuals $e_{ij}^{(1)} = \tilde{e}_{ij}^{(1)} - \hat{\alpha}_i \; s_{ij}$ were then used as estimates for $\varepsilon_{ij}^{(1)}$ $(i = 1, ..., n \; ; \; j = 1, ..., k)$.

The cross correlation matrices of the residual series $e_t = \left(e_t^{(1)} \; e_t^{(2)} \; e_t^{(3)}\right)'$ for lags $1-5$ have been displayed in Table 2. Significant values (values outside the range $\pm\; 2 \cdot \frac{1}{\sqrt{kn}}$) have been denoted by asterisks. There is only one cross correlation that slightly exceeds the significance limit, namely $\hat{\text{corr}}(e_t^{(1)}, e_{t+1}^{(2)}) = -0.36$. Quite obviously, this correlation only confirms the previous finding that there might be a weak negative association between $\nabla^* \log \bar{W}_{t-1}^{c}$ and d_t^*. All the higher order cross correlations from lag 6 on were very small indeed.

Table 2 here

Table 2 also reveals some weak autocorrelation in the residual series $e_t^{(2)}$ at lags 2 and 4, probably induced by the use of the ∇^*- operator. Anyway, these autocorrelations are so small that it would hardly be worth while to complicate the model any further in order to take this serial dependence into account. Such a weak correlation should not affect the estimates of the structural parameters too much.

We also calculated the Box-Ljung test statistics for the five first autocorrelations for the individual series $e_t^{(1)}$, $e_t^{(2)}$ and $e_t^{(3)}$. The test statistics attained the values B-L $= 8.9$ $(p = 0.113)$ for the first equation, B-L $= 14.8$ $(p = 0.011)$ for the second equation and B-L $= 7.8$ $(p = 0.168)$ for the third equation.

Furthermore, we tentatively fitted VAR- models of increasing orders to the residual vectors e_t in order to test, whether any significant serial dependencies could be found in the e_t- series. The LR- test statistics (see Tiao and Box, 1981) and the corresponding p- values have been listed in Table 3. There is no need to augment the model (3.12) with a VAR error structure.

Table 3 here

The normality assumption of ε_t was tested by applying the Shapiro-Wilk test separately to $e_t^{(1)}$, $e_t^{(2)}$ and to $e_t^{(3)}$. The resulting test statistics were S-W $= 0.951$ $(p = 0.182)$ for $e_t^{(1)}$, S-W $= 0.976$ $(p = 0.733)$ for $e_t^{(2)}$ and

S-W $= 0.945$ $(p = 0.129)$ for $e_t^{(3)}$. The corresponding cumulative probit plots seemed very linear thus confirming the results of the normality tests.

The homoskedasticity of the error terms were tested against the alternatives that the error variances would be somehow associated either to time or to the level of the corresponding fitted values. We used the test suggested by Breusch and Pagan (1979). The test statistics attained the values B-P $= 0.217$ $(p = 0.897)$ for the first equation, B-P $= 0.074$ $(p = 0.964)$ for the second equation and B-P $= 0.673$ $(p = 0.714)$ for the third equation.

With the exception of the weak autocorrelation in the series $e_t^{(2)}$, all assumptions concerning the behaviour of the disturbances in the model (3.12) seemed thus quite realistic.

3.2.3. Testing for structural change at the end of the 1980's

In 1989, the Finnish economy started to show symptoms of overheating. This was partly caused by the recent deregulation of the Finnish money market and by an excess supply of credit. In combination to a rigid exchange rate policy, the overheating symptoms severely deteriorated the competitiveness of Finnish products on the Western export market. On the other hand, the Eastern European export market almost vanished during the years 1990 and 1991, which in conjunction to the increased production costs and to some other factors led to an unprecedentedly sharp decline in the economic activity in 1991 and 1992. For this reason, it was very plausible that the wage determination mechanism and the inflation mechanism would have changed at the end of the decade.

As a formal test for the structural change, we calculated the q_1- statistic defined in (2.23) by adding the information from the year 1989. The test statistic attained the value $q_1 = 20.03$ corresponding to a p- value $p = 0.067$. For the two-year period 1989-1990 we got $q_2 = 46.05$ corresponding to $p = 0.004$, which shows that a structural change had indeed occurred in 1989. To see, whether the values of the parameters had changed, or whether there had been a more fundamental qualitative change in the wage-price mechanism, we updated the estimates of Table 1. The estimates had not changed much, so the obvious explanation was that there had been a more profound change in the wage-price interrelationship. The predictive performance of the model for the contract wages was still fairly good, but the forecasts for the drift and for the inflation rate were clearly less accurate than before.

Table 4 here

4. CONCLUSIONS

According to our experience, the ML estimation method is not computationally feasible for simultaneous-equations models with several error components, when some of the error components appear as products with exogenous, observable factors. It is possible, however, to mimic the ML- method by a simpler three-stage estimation procedure that shares the same asymptotic properties with the ML- estimator. For instance, the asymptotic covariance matrix of the simpler estimator can be evaluated by inverting the Hessian of the log-likelihood function. Although we formulated the new estimator for the case of two error components only, it is possible to generalize the technique for models involving three error components as well. The grid search applied in the estimation of the variance parameters would just become two-dimensional. Models with three error components should be general enough to cover all cases one can think of in connection to panel data.

In our empirical application we did not analyze panel data, but the quarterly wage-price interrelationship in Finland. A major share of the wage increases are determined by collective bargaining between the trade unions and the employers' organizations, whereas the rest can be attributed to wage drift. The fact that the central bargaining basically determines the wages at an annual level motivated the inclusion of two error components in our model. In fact, the necessity of the annual error component was overwhelmingly confirmed by the LM- test.

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Table 1:

Estimation results for the model (3.12)

EQUATION	Variable	Parameter	Estimate	(t- value)
	constant	γ_{10}	-0.0792	(-2.52)
Eq. I:	p_i^c	γ_{11}	0.958	(5.14)
Contract	a_i^c	γ_{12}	1.661	(3.49)
wages	ue_i	γ_{13}	-0.0297	(-2.84)
$\tilde{w}_t^c$	wed_i	γ_{14}	-1.176	(-3.62)
	[$\nabla_4 D_i$]	[γ_{16}]	[-0.428]	[(-0.60)]
	ec_i^*	γ_{15}	-0.115	(-0.43)
	Annual disturbance	τ	0.0039	
	Quarterly disturbance	σ_1	0.0030	
	constant	γ_{20}	0.0099	(12.82)
Eq. II:	$\delta_{3,t}$	γ_{21}	-0.0173	(-8.07)
Wage	$\nabla^* \log P_t$	β_{21}	0.0854	(2.40)
drift	[$\nabla^* \log \bar{W}_t^c$]	[β_{22}]	[0.0617]	[(0.668)]
d_t^*	[$\nabla^* \log \bar{W}_{t-1}^c$]	[γ_{23}]	[-0.087]	[(-1.48)]
	$\nabla^* vac_t$	γ_{25}	0.00000270	(2.68)
	$\nabla^* stdw_t$	γ_{26}	0.00146	(3.14)
	ec_{t-1}	γ_{27}	0.147	(0.17)
	[$(t-28)\cdot\nabla^* stdw_t$]	[γ_{27}]	[-0.000093]	[(-1.53)]
	Disturbance	σ_2	0.0028	
	constant	γ_{30}	0.00077	(0.14)
	i_t	γ_{31}	0.191	(3.26)
Eq. III:	p_{t-2}	γ_{32}	-0.688	(-8.32)
Inflation	w_t	β_{30}	0.418	(5.12)
rate	w_{t-1}	γ_{33}	0.512	(4.67)
p_t	w_{t-2}	γ_{34}	0.447	(4.75)
	tax_t	γ_{35}	0.361	(0.90)
	a_{t-1}	γ_{36}	-0.0594	(-1.37)
	Disturbance	σ_3	0.0062	
	Log- likelihood		**-43.59**	

Table 2:

Cross correlation matrices of the residuals

		$e_t^{(1)}$	$e_t^{(2)}$	$e_t^{(3)}$
Lag 1	$e_{t+1}^{(1)}$	0.34	-0.08	0.00
	$e_{t+1}^{(2)}$	-0.36*	-0.03	-0.05
	$e_{t+1}^{(3)}$	-0.23	-0.19	0.23
Lag 2	$e_{t+2}^{(1)}$	0.32	-0.14	0.07
	$e_{t+2}^{(2)}$	-0.02	-0.46*	0.10
	$e_{t+2}^{(3)}$	-0.12	0.19	0.25
Lag 3	$e_{t+3}^{(1)}$	0.08	-0.22	-0.13
	$e_{t+3}^{(2)}$	-0.03	-0.08	0.10
	$e_{t+3}^{(3)}$	-0.01	0.23	-0.25
Lag 4	$e_{t+4}^{(1)}$	-0.10	0.02	-0.10
	$e_{t+4}^{(2)}$	-0.03	0.39*	0.22
	$e_{t+4}^{(3)}$	0.01	0.25	0.12
Lag 5	$e_{t+5}^{(1)}$	-0.12	0.09	-0.11
	$e_{t+5}^{(2)}$	-0.08	0.13	-0.12
	$e_{t+5}^{(3)}$	0.12	-0.02	-0.12

Table 3:

VAR- fits of increasing orders to the residual series $\left(e_t^{(1)} \; e_t^{(2)} \; e_t^{(3)}\right)'$

Lag	LR-test statistic	AIC	p- value
1	12.11	-32.93	0.21
2	15.67	-33.17	0.074
3	13.91	-33.46	0.13
4	7.13	-33.42	0.62
5	10.15	-33.83	0.34

Table 4:

Forecasts for the period 1989/1 - 1990/4

	Forecasts		
	Quarter	Observed value	Forecast
Contract wages	1989/1	0.005	0.007
	1989/2	0.016	0.020
	1989/3	0.000	0.001
	1989/4	0.016	0.019
	1990/1	0.009	0.007
	1990/2	0.028	0.023
	1990/3	-0.001	0.000
	1990/4	0.022	0.017
Wage drift	1989/1	0.017	0.007
	1989/2	0.021	0.020
	1989/3	-0.000	-0.006
	1989/4	0.005	0.009
	1990/1	0.016	0.011
	1990/2	0.019	0.008
	1990/3	-0.003	-0.010
	1990/4	0.012	-0.001
Inflation rate	1989/1	0.008	-0.013
	1989/2	0.024	0.044
	1989/3	0.010	0.019
	1989/4	0.014	0.007
	1990/1	0.020	0.011
	1990/2	0.013	0.027
	1990/3	0.009	0.237
	1990/4	0.008	0.026

APPENDIX

Calculation of the elements of the information matrix
that are needed in the LM- test statistic
for the hypothesis $\tau = 0$:

In this appendix, we shall use the elements of $\mathbf{\Omega} = (\omega_{ij}) = \mathbf{\Sigma}^{-1}$ as parameters instead of the error covariances $\mathbf{\Sigma} = (\sigma_{ij})$. Further, we define the new notations

$$\tilde{S}_{il} = (\, \delta_{1l}\mathbf{I} \quad \delta_{2l}\mathbf{I} \quad \ldots \quad \delta_{Kl}\mathbf{I}\,) S_i^* \quad \text{and} \quad \tilde{E}_{il} = (\, \delta_{1l}\mathbf{I} \quad \delta_{2l}\mathbf{I} \quad \ldots \quad \delta_{Kl}\mathbf{I}\,) E_i$$
$$l = 1, \ldots, K \quad ,$$

where δ_{il} symbolizes Kronecker's delta. The submatrix of the element ω_{jl} will be denoted by $\mathbf{\Omega}_{jl}^{(S)}$, and $\mathbf{\Omega}_{ju,lv}^{(SS)}$ will denote a $(K-2) \times (K-2)$ submatrix of $\mathbf{\Omega}$ where rows j and u and columns l and v have been deleted.
With this notation the first derivatives can be formulated as

$$\frac{d}{d\tau^2} l(\beta, \tau^2, \mathbf{\Omega}) =$$

$$-\frac{1}{2}\sum_{i=1}^{n} \frac{S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*}{1+\tau^2 S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*} \;+\frac{1}{2}\sum_{i=1}^{n} \frac{\left[S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})E_i\right]^2}{\left[1+\tau^2 S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*\right]^2}$$

and

$$\frac{d}{d\omega_{jl}} l(\beta, \tau^2, \mathbf{\Omega}) =$$

$$\frac{nk}{2}\cdot\frac{2^{1-\delta_{jl}}}{\det\mathbf{\Omega}}\cdot(-1)^{j+l}\cdot\det\mathbf{\Omega}_{jl}^{(S)} \;-\; \frac{1}{2}\sum_{i=1}^{n} \frac{\tau^2 \tilde{S}_{ij}'\tilde{S}_{il}\cdot 2^{1-\delta_{jl}}}{1+\tau^2 S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*}$$

$$-\frac{1}{2}\sum_{i=1}^{n}\Big\{\, 2^{1-\delta_{jl}}\cdot\tilde{E}_{ij}'\tilde{E}_{il} \;+\; 2^{1-\delta_{jl}}\cdot\frac{\tau^4\tilde{S}_{ij}'\tilde{S}_{il}\cdot\left[S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})E_i\right]^2}{\left[1+\tau^2 S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*\right]^2}$$

$$-\frac{\tau^2}{1+\tau^2 S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*}\cdot 2\cdot S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})E_i\cdot\left[\tilde{S}_{ij}'\tilde{E}_{il} + (1-\delta_{jl})\tilde{E}_{ij}'\tilde{S}_{il}\right]\Big\} \quad ,$$

$$j, l = 1, \ldots, K \quad , \quad j \leq l \; .$$

The second order derivatives will be

$$\frac{d^2}{d\tau^4} l(\beta, \tau^2, \mathbf{\Omega}) = \frac{1}{2}\sum_{i=1}^{n} \frac{\left[S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*\right]^2}{\left[1+\tau^2 S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*\right]^2} - \sum_{i=1}^{n} \frac{S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*\cdot\left[S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})E_i\right]^2}{\left[1+\tau^2 S_i^{*\prime}(\mathbf{\Omega}\otimes\mathbf{I})S_i^*\right]^3} \quad ,$$

$$\frac{d^2}{d\tau^2 d\omega_{jl}} l(\beta, \tau^2, \Omega) =$$

$$- \frac{1}{2} \sum_{i=1}^{n} \frac{2^{1-\delta_{jl}} \cdot \tilde{S}_{ij}' \tilde{S}_{il}}{\left[1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*\right]^2}$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \left[1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*\right]^{-4} \cdot$$

$$\cdot \{ \left[1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*\right]^2 \cdot 2 \cdot S_i^{*\prime} (\Omega \otimes \mathbf{I}) E_i \cdot \left[\tilde{S}_{ij}' \tilde{E}_{il} + (1 - \delta_{jl}) \tilde{E}_{ij}' \tilde{S}_{il}\right]$$

$$- 2 \cdot 2^{1-\delta_{jl}} \cdot \tau^2 \cdot \tilde{S}_{ij}' \tilde{S}_{il} \cdot \left[1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*\right] \cdot \left[S_i^{*\prime} (\Omega \otimes \mathbf{I}) E_i\right]^2 \}$$

and

$$\frac{d^2}{d\omega_{jl} d\omega_{uv}} l(\beta, \tau^2, \Omega) =$$

$$- \frac{nk}{2} \; 2^{1-\delta_{jl}} \; 2^{1-\delta_{uv}} \; \{ \; \frac{1}{(\det\Omega)^2} (-1)^{u+v} (-1)^{j+l} \det\Omega_{uv}^{(S)} \det\Omega_{jl}^{(S)}$$

$$- \delta_{ju} \cdot \delta_{lv} \cdot (-1)^{j+l+u+v} \frac{1}{\det\Omega} \cdot \det\Omega_{ju,lv}^{(SS)} \; \}$$

$$+ \frac{1}{2} \sum_{i=1}^{n} 2^{1-\delta_{jl}} \; 2^{1-\delta_{uv}} \frac{\tau^4 \tilde{S}_{ij}' \tilde{S}_{il} \tilde{S}_{uv}' \tilde{S}_{uv}}{\left[1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*\right]^2}$$

$$+ \frac{1}{2} \sum_{i=1}^{n} 2^{1-\delta_{jl}} \; \{ \frac{-2\tau^4 \tilde{S}_{ij}' \tilde{S}_{il} \tilde{S}_{uv}' \tilde{S}_{uv}}{\left[1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*\right]^3} \cdot 2^{1-\delta_{uv}} \cdot \left[S_i^{*\prime} (\Omega \otimes \mathbf{I}) E_i\right]^2$$

$$+ \frac{2\tau^4 \cdot \tilde{S}_{ij}' \tilde{S}_{il} \cdot \left[S_i^{*\prime} (\Omega \otimes \mathbf{I}) E_i\right]}{\left[1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*\right]^2} \cdot \left[\tilde{S}_{iu}' \tilde{E}_{iv} + (1 - \delta_{uv}) \tilde{E}_{iu}' \tilde{S}_{iv}\right] \; \}$$

$$+ \frac{1}{2} \sum_{i=1}^{n} 2 \cdot \left[\tilde{S}_{ij}' \tilde{E}_{il} + (1 - \delta_{jl}) \tilde{E}_{ij}' \tilde{S}_{il}\right] \cdot \{ \frac{-2^{1-\delta_{uv}} \; \tau^4 \tilde{S}_{iu}' \tilde{S}_{iv} \cdot S_i^{*\prime} (\Omega \otimes \mathbf{I}) E_i}{\left[1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*\right]^2}$$

$$+ \frac{\tau^2}{1 + \tau^2 S_i^{*\prime} (\Omega \otimes \mathbf{I}) S_i^*} \cdot \left[\tilde{S}_{iu}' \tilde{E}_{iv} + (1 - \delta_{uv}) \tilde{E}_{iu}' \tilde{S}_{iv}\right] \; \} \quad ,$$

$$j, l, u, v = 1, \ldots, K \quad , \quad j \leq l \quad , \quad u \leq v \quad .$$

Because

$$E\left[S_i^{*\prime}(\Omega\otimes\mathbf{I})E_i\right]^2 = S_i^{*\prime}(\Omega\otimes\mathbf{I})S_i^* \quad ,$$

$$(\mathbf{\Omega}\otimes\mathbf{I})\cdot E(\boldsymbol{E}_i\tilde{\boldsymbol{E}}_{ij}') = \begin{pmatrix} \delta_{1j}\mathbf{I} \\ \cdot \\ \cdot \\ \cdot \\ \delta_{Kj}\mathbf{I} \end{pmatrix}$$

and

$$S_i^{*\prime}\begin{pmatrix} \delta_{1j}\mathbf{I} \\ \cdot \\ \cdot \\ \cdot \\ \delta_{Kj}\mathbf{I} \end{pmatrix} = \tilde{S}_{ij}' \quad .$$

we get

$$\left[-E\frac{d^2}{d\tau^4}l(\beta,\tau^2,\Omega)\right]_{\tau=0} = \frac{1}{2}\sum_{i=1}^{n}\left[S_i^{*\prime}(\Omega\otimes\mathbf{I})S_i^*\right]^2 \quad , \tag{A.1}$$

$$\left[-E\frac{d^2}{d\tau^2 d\omega_{jl}}l(\beta,\tau^2,\Omega)\right]_{\tau=0} = -\frac{1}{2}\sum_{i=1}^{n}2^{1-\delta_{jl}}\tilde{S}_{ij}'\tilde{S}_{il} \tag{A.2}$$

and

$$\left[-E\frac{d^2}{d\omega_{jl}d\omega_{uv}}l(\beta,\tau^2,\Omega)\right]_{\tau=0} =$$

$$\frac{nk}{2}\,2^{1-\delta_{jl}}\,2^{1-\delta_{uv}}\,\{\,\frac{1}{(\det\mathbf{\Omega})^2}(-1)^{u+v}(-1)^{j+l}\det\mathbf{\Omega}_{uv}^{(S)}\det\mathbf{\Omega}_{jl}^{(S)} - \delta_{ju}\cdot\delta_{lv}\cdot(-1)^{j+l+u+v}\frac{1}{\det\mathbf{\Omega}}\cdot\det\mathbf{\Omega}_{ju,lv}^{(SS)}\,\} \tag{A.3}$$

$$j,l,u,v = 1,\ldots,K \quad , \quad j\leq l \quad , \quad u\leq v \quad .$$

The elements of the blocks $i_{\tau^2,\tau^2}, I_{\tau^2,\mathbf{\Omega}}$ and $I_{\mathbf{\Omega},\mathbf{\Omega}}$ of the information matrix (2.16) are now given in equations $(A.1)-(A.3)$.

SVENSK SAMMANFATTNING

Här beskrivs en genomförbar estimeringsteknik för modeller med simultana ekvationer som innehåller två felkomponenter, av vilka den ena multiplicerats med en observerbar faktor. Vårt intresse för denna problemställning uppstod då vi försökte modellera mekanismen för lönebestämningen i den finländska industrin på 1980-talet. I det skandinaviska systemet består lönen av två klart skilda komponenter: kontraktslön och löneglidning. Den senare har vissa år stått för närapå hälften av lönestegringen i Finland. Då alltså bägge komponenterna är viktiga, men bestäms på helt olika sätt ansåg vi det vara nödvändigt att modellera dem skilt för sig.

I modellen för kontraktslönerna måste vi försöka beskriva övergången mellan två kontraktsperioder. Eftersom periodens längd hela tiden varit ett år så formulerade vi en kvartalsmodell för de logaritmiska differenserna på årsbas och med en felkomponent som även den definierades på årsbas. Men under löneförhandlingarna besluter parterna även om tidpunkterna för löneförhöjningarna. I modellen fördelar andelstal årets löneförhöjning över kvartalen. Värdena fick vi ur avtalstexterna. Genom att multiplicera den årsbaserade delen med dessa andelar försökte vi beskriva de kvartalsvisa förändringarna i kontraktslöneindex. Dessutom infördes en kvartalsbaserad felkomponent, för en del av variationen i kontraktslöneindex beror på ett mångfald av orsaker, såsom förändringar i arbetskraftens sammansättning, oklarheter i bokföringen av kontraktslöner mm. Den ovanliga felstrukturen gör att vår modell inte är av matematiskt standardsnitt.

Löneglidning antogs uppstå som ett resultat av att skillnaden mellan en enskild arbetares "marknadslön" och kontraktslönen bygger upp ett tryck, som utlöses i en reallönehöjning då trycket passerat ett givet positivt tröskelvärde.

Det stod också klart att det föreligger starkt simultant ömsesidigt beroende mellan konsumentpriser och löner. Detta förklarar varför vi specificerat tre simultana ekvationer, en för kontraktslöner, en för löneglidning och en för konsumentpriser. Mer detaljerade uppgifter om modellen liksom även en grundlig ekonomisk motivering finns i Rahiala och Kovalainen (1992).

Det finns ingen färdig estimeringsteknik för denna modell. Vi försökte oss först på maximum likelihood, men den metoden visade sig inte kunna tillämpas i praktiken. Vi utvecklar istället en enklare, men asymptotiskt efficient tre stegs estimator för modellens strukturparametrar. Dessutom härleds ett *LM*-test som testar nödvändigheten av att ha en dubbel felstruktur. Slutligen tar vi fram ett enkelt Chow-test som testar strukturella förändringar efter sampelperioden.

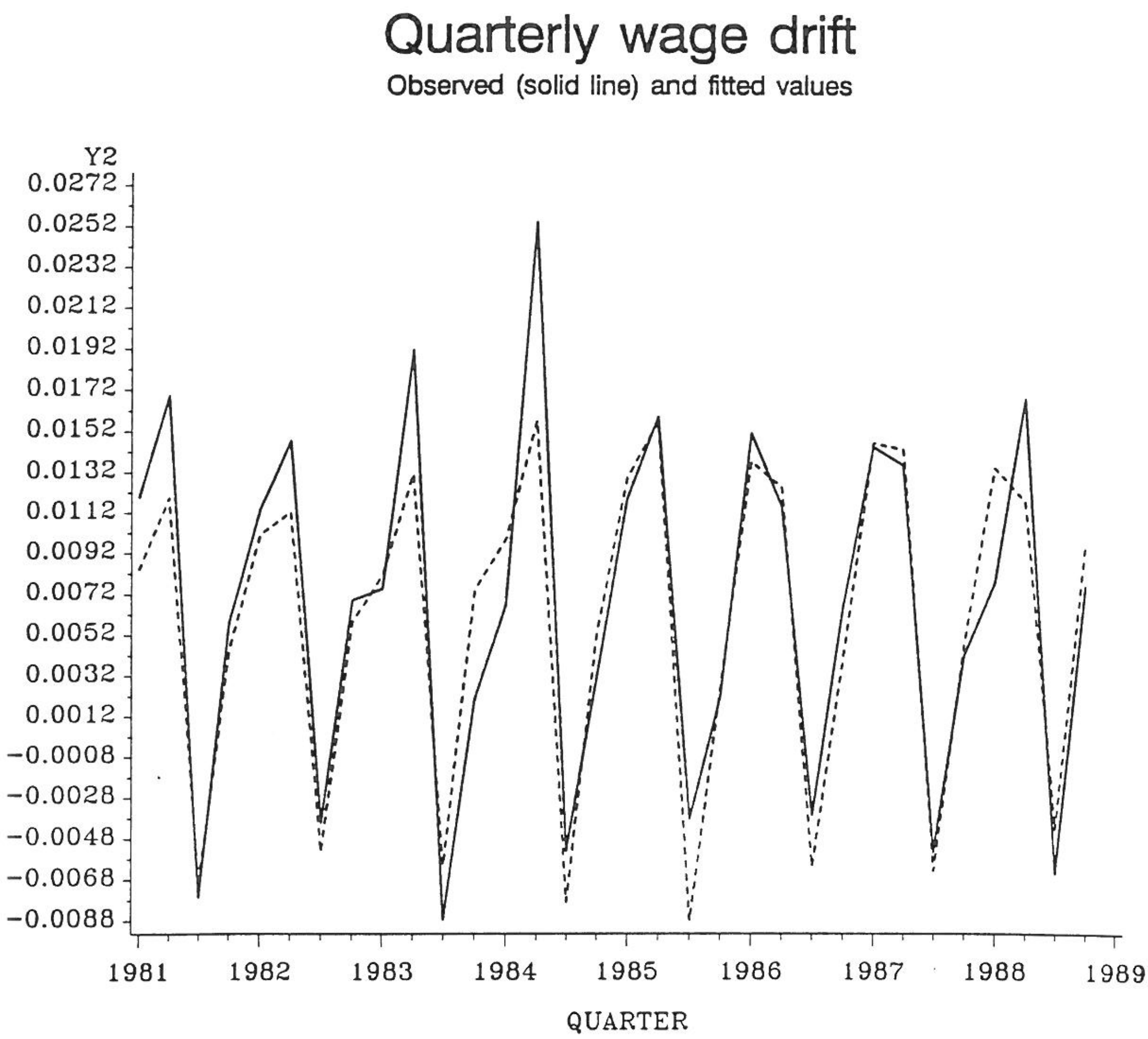
I en empirisk tillämpning användes kvartalsobservationer som gäller Finlands industri samt konsumentpriserna för åren 1980-1988. Åren 1989-1990 reserverades för tester utanför samplet. Resultaten lämnar föga tvivel om att både en kvartals- och en årsbaserad felterm behövs i ekvationen över kontraktslönerna. Modellens tillpassning visade sig vara mycket god även i övriga hänseenden. De strukturella parameterestimatens såväl tecken som storlek befanns väl överensstämma med vad man förväntat sig enligt den ekonomiska teorin.

Modellen användes bl a. för att testa ifall det finns något samband mellan kontraktslöner och löneglidning, men inget samband kunde påvisas. En stor del av löneglidningen ser ut att spilla över i inflation redan inom ett halvår.

Testningen utanför samplet visade på en klar förändring i lönebestämningsmekanismen år 1990. Enligt modellen borde alla objektiva förutsättningar för löneglidning då ha försvunnit, men trots det fortsatte lönerna att glida uppåt.


Quarterly changes of contract wages
Observed (solid line) and fitted values
Y1
0.040
0.038
0.036
0.034
0.032
0.030
0.028
0.026
0.024
0.022
0.020
0.018
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
1981
1982
1983
1984
1985
1986
1987
1988
198
QUARTER

Figure 1



Quarterly wage drift
Observed (solid line) and fitted values
Y2
0.0272
0.0252
0.0232
0.0212
0.0192
0.0172
0.0152
0.0132
0.0112
0.0092
0.0072
0.0052
0.0032
0.0012
-0.0008
-0.0028
-0.0048
-0.0068
-0.0088
1981
1982
1983
1984
1985
1986
1987
1988
1989
QUARTER

Figure 2

Quarterly inflation rate

Observed (solid line) and fitted values

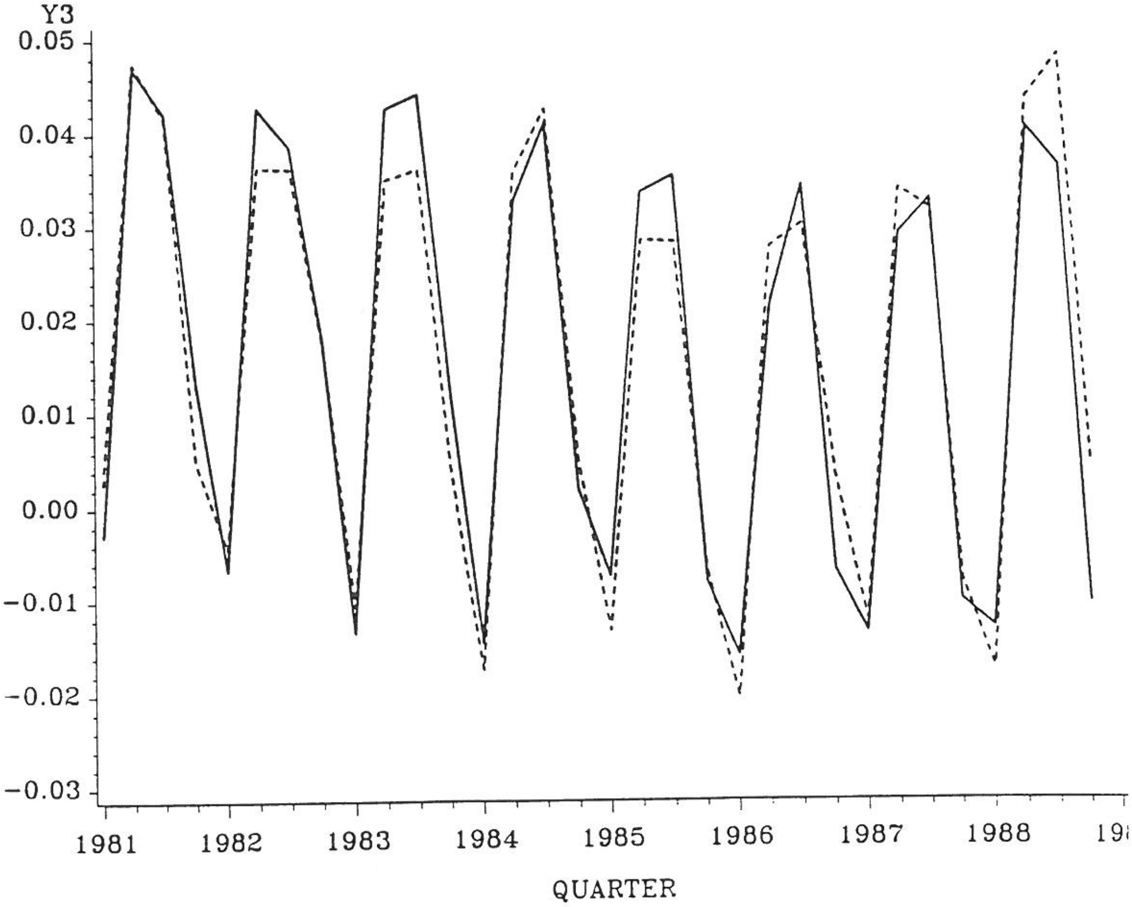


Figure 3

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