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FORECASTING THE SWEDISH UNEMPLOYMENT RATE:
V A R vs. Transfer Function Modelling

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Forecasting the Swedish Unemployment Rate: VAR vs. Transfer Function Modelling

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Abstract

The Swedish unemployment rate is forecast using three time series methods: the ARIMA, transfer function and VAR models. Within this context the choice of modelling strategy is discussed. It is found that the forecasting performance of VAR models is improved by explicitly taking account of cointegration between the variables in the model, despite the fact that unemployment *is not* co-integrated. However, the more parsimonious ARIMA and transfer function models have lower RMSE for all forecasting horizons. It is also found that the additional variables in the VAR models are important for predicting the turning points in the unemployment rate.

1. Introduction

The aim of this paper is to study how the choice of model and modelling strategy affects the forecasting performance of time series methods. The variable we study is the Swedish unemployment rate. We have chosen this variable since it is an important policy variable. Traditionally, the goals of economic policy are formulated as low unemployment rate, low inflation and a high and stable growth rate. The unemployment rate displays two sharp turning points during the evaluation period 1979 to 1990 and can thus be expected to be relatively hard to forecast and provide a challenge for the forecasting methods.

We consider three overlapping model classes: ARIMA-models, transfer function models and VAR-models. A priori we expect the VAR-models to do better than ARIMA and transfer function models since VAR's allow for the interdependence we expect to find among economic variables. The class of VAR-models encompass ARIMA and transfer function models, but the noise is modelled in a much less sophisticated and less parsimonious manner in the VAR-framework. Because of this and the differences in modelling strategy, we would not expect the selected ARIMA and transfer function models to be special cases of the selected VAR-models.

ARIMA and transfer function model building is by now fairly standard and we follow the procedures outlined by Box and Jenkins [1976].

VAR-models are frequently used in the economic literature and have been found to forecast well, especially in the Bayesian formulation (Litterman [1986a]). The issue of model building strategy is far less settled for this type of model than for ARIMA and transfer function models. In particular, we try to shed some light on how the lag length should be determined and how cointegrated and integrated variables should be treated.

We restrict ourselves to the classical framework and do not consider Bayesian formulations of the three model classes. Kadiyala and Karlsson [1990] study how different ways of parameterising the prior beliefs affect the forecasting performance of VAR-models. Nor do we consider the vector ARIMA (VARIMA) model of Tiao and Box [1981]. Öller [1985], using data on the Finnish economy, finds that VARIMA-models provide better forecasts than univariate ARIMA-models.

2. The Data

Our data set consists of quarterly observations for the period 64:1-90:4 for unemployment, the dependent variable, and five explanatory variables shown in Figure 1:

UNEMP is the officially reported unemployment rate in Sweden according to the AKU (work force survey) conducted by Statistics Sweden. The series is not seasonally adjusted. The definition of unemployment was changed in the first quarter of 1987 (87:1). It has been estimated that this change of definition decreased the unemployment rate by one half percentage point. To make the series consistent over time we decided to add 0.5 percentage points to the observations beginning with 87:1. The series shows some cyclical pattern as well as a seasonal pattern.

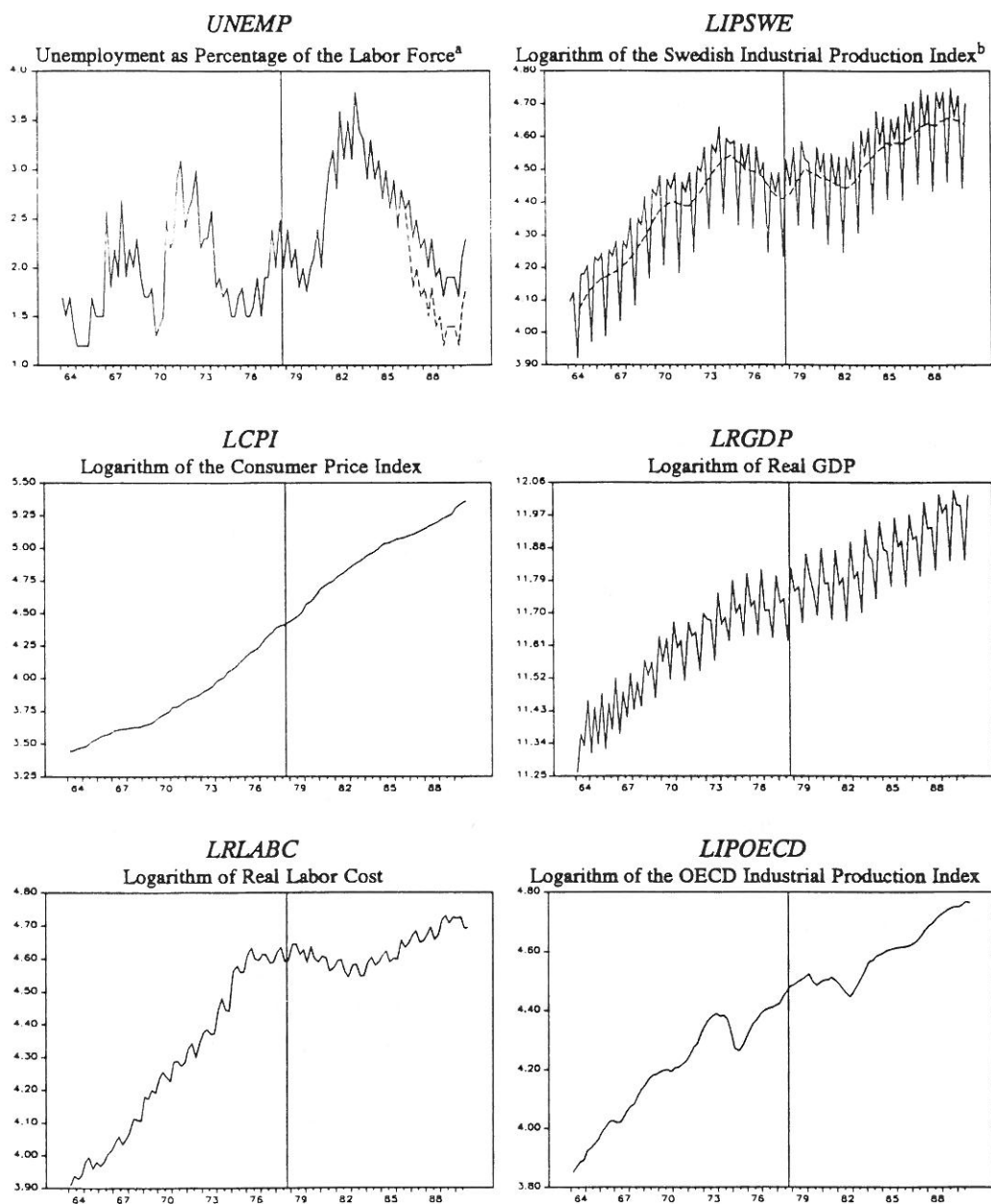
LIPSWE is the logarithm of the index of the Swedish industrial production. The series is shown together with a four quarter moving average. The moving average will be used as the explanatory variable in the transfer function model. We note both a cyclical pattern and a very strong seasonal pattern with a marked dip in the third quarter corresponding to the industrial vacation period. It should also be noted that the seasonal pattern differs from that of the unemployment series. If a large portion of unemployment is in the industrial sector, decreased industrial production would lead to increased unemployment.

LCPI is the logarithm of the consumer price index. The inclusion is based on a Phillips curve argument.

LRGDP is the logarithm of real GDP. *LRGDP* has a seasonal pattern that resembles that of *LIPSWE* but the third quarter vacation 'dip' is not so deep as for industrial production. Even though *LRGDP* and *LIPSWE* series follow the same general trend and seasonal pattern, *LRGDP* may be a better explanatory variable than *LIPSWE* if a large part of unemployment comes from the non-industrial sector. Since the trade and service sector has increased their share of the employed over time this may be an interesting assumption to test.

LRLABC is the logarithm of the real labor cost. The positive trend in the series is broken in 1976. We may suspect that increased labor cost would lead to increased unemployment.

Figure 1. The Data



The vertical bar marks the end of the identification period.

^a) The dashed line is the unadjusted series. ^b) The dashed line is a four quarter moving average.

LIPOECD is the logarithm of the OECD index of industrial production. The series is seasonally adjusted. This series is used as an international leading indicator of economic activity.

All series except real GDP are available on a monthly basis. For these series we use a three month average as our quarterly data. The data on real GDP prior to 70:1 and the labor cost prior to 68:1 was kindly provided by Jan Eklöf (see Eklöf [1990] for data sources). The remainder of the data was collected from the OECD Main Economic Indicator database.

2.1. Unit Roots

As a first step in the specification search, unit-root tests were performed for all variables. The bounded variation of the unemployment rate implies that it can not contain a unit root¹. We do, however, report the results of unit root tests for *UNEMP*. Even though the true data generating process (DGP) does not contain a unit root, a first difference might be a good approximation in the short run and useful for forecasting purposes.

The traditional Dickey-Fuller tests, τ_r and $T(\hat{\rho} - 1)$ (Fuller [1976], Dickey and Fuller [1979]), are reported in Table 1. Since the variables display trends a time-trend was included in the Dickey-Fuller regressions:

$$y_t = \mu + \beta t + \rho y_{t-1} + \sum_{i=1}^p \theta_i (y_{t-i} - y_{t-i-1}) + \varepsilon_t$$

Since most of the variables display seasonality, the tests were also performed using Dickey-Fuller regressions that included seasonal dummies. These tests are also reported in Table 1.

Critical values, conditional on the standardized trend, from De Jong et.al. [1988] were used. The standardized trend is estimated under the null of a unit root as $\hat{\delta} = \hat{\beta}/\hat{\sigma}_\varepsilon$. The estimated standardized drift is small for all the variables and the conclusions are the same if we use the critical values from Fuller [1976], which were obtained under the assumption that the DGP contains no deterministic trend. As pointed out by Sims and Uhlig [1991] the τ_r statistic conveys the same information about the shape of the likelihood as the conventional t statistic even when the DGP is non-stationary. In a Bayesian setting

¹Given the maintained hypothesis that the unemployment rate can be represented as a stochastic linear process, the presence of a unit root in the autoregressive polynomial implies that the variance is infinite or tends to infinity with t , depending on how the process is started up. This is clearly contradicted by the unemployment rate being bounded below by 0 and above by 100.

Table 1. Dickey-Fuller Test for Unit Roots

Variable	Constant Term, Trend					Seasonal Dummies, Trend				
	Lags	$\hat{\rho}$	τ_τ	$T(\hat{\rho}-1)$	$\hat{\delta}$	Lags	$\hat{\rho}$	τ_τ	$T(\hat{\rho}-1)$	$\hat{\delta}$
<i>UNEMP</i>	4	0.690	-3.280 ^a	-50.243 ^d	-0.004	2	0.829	-1.948	-9.599	0.000
<i>LCPI</i>	0	0.993	-0.345	-0.441	0.033	0	0.997	-0.151	-0.186	0.034
<i>LRGDP</i>	3	1.061	0.497	0.859	-0.029	3	1.056	0.498	0.925	0.026
<i>LRLABC</i>	5	0.552	-3.276 ^a	58.238	-0.005	0	0.822	-1.972	-10.527	0.005
<i>LIPSWE</i>	5	0.868	-1.787	-46.357 ^d	-0.009	0	0.959	-0.930	-2.432	0.015
<i>LIPOECD</i>	1	0.899	-2.939	-16.572 ^a	-0.003	1	0.899	-2.865	-16.698 ^a	0.008
$\nabla UNEMP$	3	0.047	-2.757 ^a	-21.840 ^d	0.034	3	0.191	-2.768 ^a	-20.000 ^d	0.060
$\nabla LCPI$	0	0.262	-5.771 ^d	-42.777 ^d	0.011	0	0.330	-5.195 ^d	-38.878 ^d	0.008
$\nabla LRGDP$	2	2.817	-28.339 ^d	117.658	-0.002	2	-2.112	-11.948 ^d	185.217	0.008
$\nabla LRLABC$	3	0.235	-3.126 ^b	-39.529 ^d	-0.046	0	-0.234	-9.276 ^d	-71.581 ^d	0.009
$\nabla LIPSWE$	6	0.032	-3.001 ^b	172.121	0.030	0	-0.099	-7.951 ^d	-63.754 ^d	0.033
$\nabla LIPOECD$	0	0.605	-3.644 ^d	-22.311 ^d	0.003	0	0.618	-3.526 ^c	-22.164 ^d	0.005

Models: $y_t = \mu + \beta t + \rho y_{t-1} + \sum_{i=1}^p \theta_i (y_{t-i} - y_{t-i-1}) + \varepsilon_t$ and $y_t = \mu + \beta t + \rho y_{t-1} + \sum_{i=1}^p \theta_i (y_{t-i} - y_{t-i-1}) + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \varepsilon_t$, p is chosen to produce white noise residuals. The time trend is excluded for tests with differenced data.

a) Denotes significance at the 10% level, b) at the 5% level, c) at the 2.5% level and d) at the 1% level.

with a diffuse prior or in general for inference based on the likelihood principle one could use the ordinary critical values from the t -distribution as guide lines.

When seasonal dummies are included, the unit root can only be rejected for *LIPOECD* and the $T(\hat{\rho}-1)$ test at the 10% level. Without seasonal dummies the null of a unit root is rejected for *UNEMP*, *LRLABC*, *LIPSWE* and *LIPOECD*.

The unit root is rejected in all cases when the tests are performed on first differences.

In general the investigation of non-stationary roots of the AR-representation should not be restricted to the root +1 of the AR-polynomial since any root with a modulus of one will have the same effects on the time series properties, i.e. long memory, infinite variances and non-normal asymptotic distributions for parameter estimates.

The seasonal difference-operator $(1 - B^4)$ is frequently used with quarterly data to render them stationary and this operator can be factored into

$$\begin{aligned}
 (1 - B^4) &= (1 - B)(1 + B)(1 + B^2) \\
 &= (1 - B)(1 + B)(1 - iB)(1 + iB).
 \end{aligned}$$

It is clear that the difference operator corresponds to four roots with modulus one, 1, -1 , i and $-i$, corresponding to the zero frequency, the biannual frequency and the yearly frequency. Since the roots i and $-i$ are complex conjugates, and thus indistinguishable, they are both interpreted as the yearly frequency.

The testing procedure developed by Hylleberg et.al. [1990] (HEGY) is used here. They show that any polynomial $\varphi(B)$ which is finite valued at the points $\theta_i = \{1, -1, i, -i\}$ can be written as

$$\begin{aligned}\varphi(B) = & \lambda_1 B(1+B)(1+B^2) - \lambda_2 B(1-B)(1+B^2) \\ & + \lambda_3 (-iB)(1-B)(1+B)(1-iB) + \lambda_4 (iB)(1-B)(1+B)(1+iB) \\ & + \varphi^*(B)(1-B^4)\end{aligned}\quad (1)$$

for $\varphi^*(B)$ a possibly infinite real valued polynomial. Then $\lambda_i = 0$ if and only if θ_i is a root of $\varphi(B)$. A test for $\lambda_i = 0$ is hence a test for θ_i a root of $\varphi(B)$. Since λ_3 and λ_4 must be complex for $\varphi(B)$ to be real valued, we substitute $\lambda_1 = -\pi_1$, $\lambda_2 = -\pi_2$, $2\lambda_3 = -\pi_3 + i\pi_4$ and $2\lambda_4 = -\pi_3 - i\pi_4$ into (1) to obtain

$$\begin{aligned}\varphi(B) = & -\pi_1 B(1+B+B^2+B^3) + \pi_2 B(1-B+B^2-B^3) \\ & + (\pi_4 + \pi_3 B)B(1-B^2) + \varphi^*(B)(1-B^4)\end{aligned}$$

where all the factors are real valued.

If data is assumed to be generated by

$$\varphi(B)x_t = \mu_t + \varepsilon_t,$$

where μ_t contains a deterministic trend and seasonal components, the tests for θ_i a root of $\varphi(B)$ can be carried out as tests on the π -coefficients in

$$\varphi^*(B)y_{4t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \mu_t + \varepsilon_t \quad (2)$$

where

$$\begin{aligned}y_{1t} &= (1+B+B^2+B^3)x_t, \quad y_{2t} = -(1-B+B^2-B^3)x_t, \\ y_{3t} &= -(1-B^2)x_t, \quad y_{4t} = (1-B^4)x_t.\end{aligned}$$

In this setting a test for a unit root (the zero frequency) is a test of $\pi_1 = 0$ against the stationary alternative $\pi_1 < 0$, a test for a root -1 (the biannual frequency) is a test of $\pi_2 = 0$ against the stationary alternative $\pi_2 < 0$ and a test for a root of $\pm i$ (the yearly frequency) is a test of $\pi_3 = \pi_4 = 0$ against the alternative that not both are zero. The first two tests can be conducted as t -tests and the distributions of the t -statistics for π_1 and π_2

Table 2. HEGY Tests for Seasonal Unit Roots

Variable	Lags	't'-test				'F'-test
		π_1	π_2	π_3	π_4	$\pi_3 \cap \pi_4$
<i>UNEMP</i>	1	-3.280 ^a	-0.491	-3.120 ^d	-0.013	4.868 ^d
<i>LCPI</i>	0	-1.155	-5.511 ^d	-5.266 ^d	-3.210 ^c	26.540 ^d
<i>LRGDP</i>	0	0.497	-0.322	-0.611	-0.503	0.324
<i>LRLABC</i>	2	-3.276 ^a	-3.529 ^d	-0.999	-0.472	0.612
<i>LIPSWE</i>	2	-1.787	0.594	-0.390	-0.238	0.105
<i>LIPOECD</i>	0	-2.868	-4.623 ^d	-1.695	-6.372 ^c	24.867 ^d
$\nabla_4 UNEMP$	0	-3.953 ^d	-4.324 ^d	-8.057 ^d	-2.728 ^c	53.846 ^d
$\nabla_4 LCPI$	0	-2.518	-6.366 ^d	-6.906 ^d	-2.590 ^c	37.508 ^d
$\nabla_4 LRGDP$	0	-2.786 ^a	-4.227 ^d	-5.366 ^d	-1.754 ^a	17.764 ^d
$\nabla_4 LRLABC$	0	-2.558	-5.218 ^d	-7.377 ^d	-1.646	31.929 ^d
$\nabla_4 LIPSWE$	1	-2.557	-3.977 ^d	-6.552 ^d	-4.279 ^d	35.216 ^d
$\nabla_4 LIPOECD$	0	-4.052 ^d	-5.438 ^d	-2.813 ^d	-6.902 ^d	38.291 ^d

Model: $y_{4t} = \mu + \beta t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \sum_{i=1}^p \theta_i y_{4,t-i} + e_t$, p is chosen to produce white noise residuals. The time trend is excluded for tests with differenced data.

a) Denotes significance at the 10% level, b) at the 5% level, c) at the 2.5% level and d) at the 1% level. One-sided tests except π_4 and $\pi_3 \cap \pi_4$.

are tabulated in HEGY. The last test can either be conducted as an F -test of the joint null or as a sequential test where $\pi_4 = 0$ is first tested against the two-sided alternative and $\pi_3 = 0$ is tested against $\pi_3 < 0$ conditionally on $\pi_4 = 0$. The distributions of the F -test and t -tests for π_3 and π_4 are also tabulated in HEGY.

The HEGY-tests were conducted with a time trend and seasonal dummies included in (2). The results with only a time trend included are reported in Table 2 and the results with both time trend and seasonal dummies are reported in Table 3.

When seasonal dummies are included the seasonal roots can be rejected for all variables except *LRGDP* and the zero frequency root can not be rejected for any of the variables.

The tests were also run on seasonal differences of all variables. The presence of seasonal roots was rejected in all cases, but the zero frequency root could not be rejected for $\nabla_4 LCPI$, $\nabla_4 LRLABC$ and $\nabla_4 LIPSWE$.

To summarize, allowing for a deterministic season both the Dickey-Fuller and HEGY tests indicate that *LCPI*, *LRGDP*, *LRLABC*, *LIPSWE* and *LIPOECD* have a zero-frequency root. The HEGY tests indicate that *LRGDP*, in addition to the zero frequency

**Table 3. HEGY Tests for Seasonal Unit Roots,
Seasonal Dummies Included**

Variable	Lags	't'-test				'F'-test $\pi_3 \cap \pi_4$
		π_1	π_2	π_3	π_4	
<i>UNEMP</i>	0	-2.221	-3.419 ^c	-5.946 ^d	-2.828 ^d	31.841 ^d
<i>LCPI</i>	0	-1.474	-6.222 ^d	-5.736 ^d	-2.869 ^c	28.436 ^d
<i>LRGDP</i>	0	0.498	-2.368	-2.394	0.229	2.888
<i>LRLABC</i>	0	-1.390	-4.130 ^d	-5.098 ^d	-2.414	20.294 ^d
<i>LIPSWE</i>	0	-0.833	-3.439 ^c	-4.412 ^c	-4.034 ^c	27.251 ^d
<i>LIPOECD</i>	0	-2.779	-4.402 ^d	-1.667	-6.218 ^c	24.867 ^d
$\nabla_4 UNEMP$	0	-3.853 ^d	-4.239 ^d	-7.868 ^d	-2.620 ^b	51.063 ^d
$\nabla_4 LCPI$	0	-2.447	-6.143 ^d	-6.700 ^d	-2.840 ^c	35.247 ^d
$\nabla_4 LRGDP$	0	-2.753 ^a	-4.148 ^d	-5.505 ^d	-1.691	18.416 ^d
$\nabla_4 LRLABC$	0	-1.613	-5.018 ^d	-7.394 ^d	-2.042	29.914 ^d
$\nabla_4 LIPSWE$	1	-2.432	-3.913 ^d	-6.322 ^d	-4.189 ^d	33.177 ^d
$\nabla_4 LIPOECD$	0	-3.914 ^d	-5.257 ^d	-2.692	-6.705 ^d	35.879 ^d

Model: $y_{4t} = \mu + \beta t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + \sum_{i=1}^p \theta_i y_{4,t-i} + \alpha_1 D_{1t} + \alpha_2 D_{2t} + \alpha_3 D_{3t} + \varepsilon_t$, p is chosen to produce white noise residuals. The time trend is excluded for tests with differenced data.

a) Denotes significance at the 10% level, b) at the 5% level, c) at the 2.5% level and d) at the 1% level. One-sided tests except π_4 and $\pi_3 \cap \pi_4$.

root, has roots on the unit circle at both the seasonal frequencies. It appears as if *UNEMP* is well approximated by a difference stationary variable.

2.2. Cointegration

The inference that *LCPI*, *LRGDP*, *LRLABC*, *LIPSWE* and *LIPOECD* are integrated at the zero frequency open up the possibility that they are cointegrated at this frequency. That is, there exists one or more linear combinations of the variables that do not have a root at the zero frequency.

The maximum likelihood based procedure of Johansen [1988b] and Johansen and Juselius [1990] is used to test for cointegration. This involves estimation of the error correction representation (ECM)

$$\nabla y_t = \mu_t + \sum_{i=1}^{p-1} \Pi_i \nabla y_{t-i} + \Pi y_{t-p} + \varepsilon_t$$

for the vector y_t of m difference stationary variables.

Table 4. Johansen Tests for Cointegration

No. of Cointegrating relations						
r^*	1	2	3	4	5	6
λ_{max}	52.054 ^d	29.861 ^b	15.658	13.692	5.670	1.293
λ_{Trace}	118.229 ^d	66.174 ^d	36.313	20.655	6.963	1.293
Estimated Cointegrated Relations (α)						
<i>UNEMP</i>	0.009	-0.168	-1.881	-0.320	-0.022	0.024
<i>SUMGDP</i>	1.000	1.000	1.000	1.000	1.000	1.000
<i>LIPOECD</i>	-0.901	-1.984	-12.414	2.516	-0.176	-2.888
<i>LIPSWE</i>	-0.803	0.527	56.247	-3.067	-0.576	5.284
<i>LRLABC</i>	-1.831	-1.6634	-65.839	-0.707	-1.652	-15.723
<i>LCPI</i>	0.528	0.827	36.567	-1.191	0.108	12.425

Error correction representation estimated with one lag of first differences and seasonal dummies.

a) Denotes significance at the 10% level, b) at the 5% level, c) at the 2.5% level and d) at the 1% level. Critical values are from Table A1, $T=50$, of Jacobson and Larsson [1991].

From the Granger Representation theorem (Engle and Granger [1987]) we know that the impact matrix Π is the zero matrix if and only if the variables are not cointegrated. If the variables are cointegrated then Π will have reduced rank $r < m$ and can be decomposed into two $m \times r$ matrices α and γ , $\Pi = \gamma\alpha'$, where α contains the r linearly independent cointegrating relations and γ is the matrix of adjustment coefficients. Then the transformation $z_t = \alpha'y_t$ of the integrated variables is stationary and the ECM can be restated as

$$\nabla y_t = \mu_t + \sum_{i=1}^{p-1} \Pi_i \nabla y_{t-i} + \gamma z_{t-p} + e_t$$

or, since the lag of z_t is arbitrary, as

$$\Pi(B)\nabla y_t = \mu_t + \gamma z_{t-1} + e_t.$$

The cointegration tests were performed in a six-variable system containing all the variables and a five variable system where *UNEMP* was excluded. The results were similar and only the tests in the six variable system are reported. Since the presence of seasonal roots in the DGP for *LRGDP* could not be rejected the transformation $SUMGDP = (1 + B + B^2 + B^3)LRGDP$ is used to remove the seasonal roots.

The LR-tests for the rank of Π and the corresponding estimates of the cointegrating relations are reported in Table 4. The test-statistics λ_{max} and λ_{Trace} both test the null $r < r^*$

Table 5. Tests of Hypotheses on the Cointegrating Relations

Hypothesis	χ^2	df
<i>UNEMP</i> does not enter into the cointegrating relations	3.101	2
<i>LCPI</i> does not enter into the cointegrating relations	5.322 ^a	2
<i>LIPOECD</i> does not depend on the cointegrating relations	6.116 ^b	2

The test statistics are asymptotically distributed as a χ^2 conditional on the cointegrating rank $r = 2$.

a) Denotes significance at the 10% level, b) at the 5% level, c) at the 2.5% level and d) at the 1% level.

and differ only in the alternative. For λ_{max} the alternative is $r = r^*$ and for λ_{Trace} , $r \geq r^*$. Both tests indicate that there are two linearly independent cointegrating relations.

To further investigate the nature of the cointegration, a series of LR-tests of α and γ were conducted. The test statistics reported in Table 5 have the usual asymptotic χ^2 -distribution, conditional on the cointegrating rank $r = 2$.

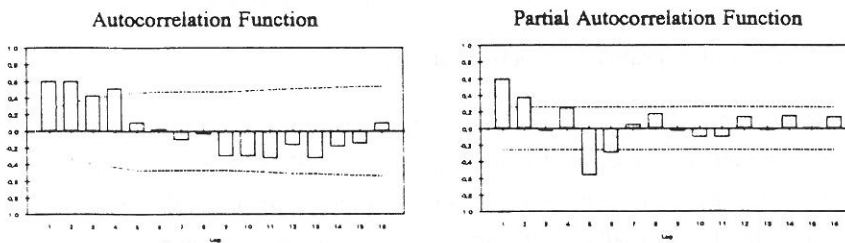
The inference that *UNEMP* is integrated is, as discussed above, questionable on theoretical grounds. We would, consequently, not expect *UNEMP* to enter into the cointegrating relations. The zero restrictions on the row of α corresponding to *UNEMP* can not be rejected at any reasonable significance level.

LCPI is the only nominal variable in the system and it seems unlikely that this variable is cointegrated with the other, real, variables. This hypothesis is rejected at the 10% level and is thus only weakly supported by the data.

Cointegration implies that at least one of the variables is caused (in the Granger sense) by the other variables in the cointegrating relations. It does however seem unlikely that *LIPOECD* is caused by the other, domestic, variables. The absence of causality in this direction implies that the row in γ corresponding to *LIPOECD* is zero, a hypothesis which is rejected by the data.

We conclude that *LRGDP*, *LRLABC*, *LIPOECD* and *LIPSWE* are cointegrated at the zero frequency with cointegrating rank two. We can not rule out the possibility that *LIPOECD* is Granger-caused by the other variables in the cointegrating relations.

Figure 2. Identification of ARIMA Model



Estimates are for the period 64:1 to 78:4.

3. The Forecasting Models

Below we identify forecasting models using the ARIMA, transfer function and VAR model approaches. In all cases we use only the first 60 observations. This means that we only use information up to 78:4 to specify our models.

3.1. Identification of an ARIMA Model for UNEMP

When we tested for unit roots in the *UNEMP* series for the identification period 64:1-78:4 we could reject the unit root at the 10% level if no seasonal dummies were included in the model. This result is in accordance with the automatic identification procedure in AUTOBOX software which suggests that the original series is used. The ACF and PACF are given in Figure 2.

The ACF and PACF suggest an AR(2) model with a seasonal term. The seasonal pattern in the PACF suggests a seasonal MA(1) term but, we have also tried a seasonal AR(1) component.

We obtained the following estimated models for the 64:1-78:4 period (*t*-ratios in parenthesis),

$$(1 - 0.382B - 0.332B^2)(UNEMP_t - 1.965) = (1 + 0.506B^4)a_t$$

$$(2.98) \quad (2.53) \quad (8.88) \quad (3.89)$$

$$\hat{\sigma}_a = 0.3019, \quad AIC = -2.262 \quad n = 60, \quad df = 56$$

$$(1 - 0.464B - 0.318B^2)(1 - 0.610B^4)(UNEMP_t - 2.066) = a_t$$

$$(3.63) \quad (2.38) \quad (5.19) \quad (4.82)$$

$$\hat{\sigma}_a = 0.2859, \quad AIC = -2.371 \quad n = 60, \quad df = 56$$

There are no significant residual autocorrelations in either of the two models. Since the model with a seasonal AR-component is slightly better than the MA-model we decided to use that model as a yardstick for the transfer function and VAR models.

3.2. Identification of a Transfer Function Model

There are several approaches to transfer function identification. The best known method, the pre-whitening method proposed by Box and Jenkins [1976], uses an estimated ARIMA model for the input variable as a filter for both the input and the output variable. After filtering both variables, ordinary cross correlations are computed for different lags. By filtering the input variable to white noise the cross correlations should not reflect the autocorrelation structure of the input variable.

After some initial analysis we decided to use *LIPSWE* as input variable. Using traditional identification techniques we found the following model for *LIPSWE*:

$$(1 - B)(1 - B^4)LIPSWE_t = (1 - 0.599B^4) a_t \quad \hat{\sigma}_a = 0.02196 \quad (5.76)$$

with no significant residual autocorrelations at the 5% level.

We then used this model to filter *UNEMP* and then computed the pre-whitened cross correlation function, shown in the left panel of Figure 3. Note the seasonal pattern in the CCF. The reason for this is that the pre-whitening method works well when the series either are non-seasonal or have a similar seasonal pattern. In the right panel of Figure 3 we show the cross correlation function when both series have been seasonally differenced. The $(1 - B^4)$ differencing can be factored as $(1 - B) \times (1 + B + B^2 + B^3)$ which is a regular differencing of a moving four-quarter sum which removes the seasonality.

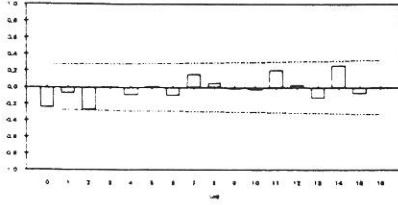
From Figure 3 we identified a transfer function with a lag of 0-3 quarters. We have also used OLS to estimate a multiple regression model between lagged values of *LIPSWE* and *UNEMP*. The model included seasonal dummies to account for different seasonal patterns. The results from the regression are very similar to the results in Figure 3.

The first transfer function model was estimated as

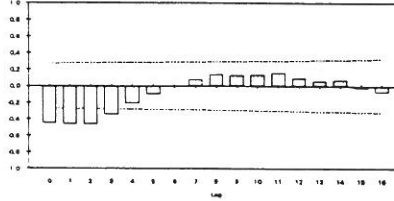
$$UNEMP_t - 2.062 = \frac{-2.271}{(1 - 0.893B)} (1 - B^4) LIPSWE_t + \frac{1}{(1 - 0.347B - 0.461B^2)(1 - 0.731B^4)} a_t \quad \hat{\sigma}_a = 0.2522$$

Figure 3. Identification of Transfer Function Model

Cross Correlation Function between Pre-Whitened *LIPSWE* and *UNEMP*



Cross Correlation Function between $(1-B^4)LIPSWE$ and $(1-B^4)UNEMP$



Estimates are for the period 64:1 to 78:4.

All estimated coefficients are significant at the 5% level. δ_1 (0.893) is not significantly different from 1 ($t = 1.43$) and strongly correlated with the estimated mean of *UNEMP*. As $(1-\delta_1 B)$ is close to $(1-B)$ the model can be simplified by 'dividing' $(1-B^4)$ by $(1-B)$ which gives $(1+B+B^2+B^3)$ *LIPSWE*_{*t*} as the input-variable, resulting in the following model:

$$UNEMP_t - 2.977 = -2.084 [(1+B+B^2+B^3) LIPSWE_t - 17.457] + \frac{1}{(1-0.355B-0.458B^2)(1-0.711B^4)} a_t \quad \hat{\sigma}_a = 0.2552$$

All coefficients are significant. The lag polynomial $(1+B+B^2+B^3)$ has an average lag of 1.5 quarters which seems reasonable with respect to the crosscorrelations in Figure 3 and lead times in the economy. The reduction in $\hat{\sigma}_a$ compared to the ARIMA model is very modest (from 0.286 to 0.255).

Since *LIPSWE* has no a priori upper bounds, the estimated model implies that the *UNEMP* variable also is unbounded, which could not be true in the long run. However, when testing for a cointegrating relationship between the variables in a bivariate model, this was not rejected at the 5% level. We conclude that the model may be reasonable only in the short run.

We have also estimated a model with *LIPSWE* as the input variable (not the moving sum). The best model was obtained with a two quarter lag which is in line with the models above. The model is:

$$UNEMP_t - 2.280 = \frac{-1.616}{(1+0.327B+0.317B^2)} (LIPSWE_{t-2} - 4.354) + \frac{1}{(1-0.553B-0.366B^2)} a_t \quad \hat{\sigma}_a = 0.2414$$

All coefficients are significant. The roots of $(1 - \delta_1 B - \delta_2 B^2)$ are complex with a cycle of 4.92 quarters (not too far from the seasonal periodicity) which, in combination with the missing $(1 - \Phi B^4)$ polynomial, implies that the *LIPSWE* tries to explain the seasonal variation in *UNEMP* even though they have different seasonal patterns. We therefore decided to use the previous model as our forecasting model.

3.3. VAR-Based Models

Several alternative strategies for building VAR-models have been proposed in the literature. We will consider three of these strategies in order to assess the effects of the model building procedure on the forecasting performance.

Strategy 1. The most prevalent procedure in the economic literature is to include the a priori determined variables with the same number of lags in all equations. The lag order is frequently determined by LR-tests (using the Sims [1980] type of correction) or by information criteria such as AIC (Akaike [1974]), SBC (Schwarz [1978]) or LIL (Hannan and Quinn [1979]). Jacobson [1991], in a Monte Carlo study, found that LIL (in conjunction with multivariate Ljung-Box tests (Hosking [1980]) for white noise residuals) performs best of the information criteria and we will primarily rely on this for lag-length selection.

Strategy 2. This strategy allows for different lag length and is a modification of the procedure proposed by Hsiao (Hsiao [1979], Hsiao [1982]). The equation for each variable is specified independently of the other equations using a univariate information criterion (LIL).

In the first step the number of own lags is determined with only a priori included variables (constant term, time trend, seasonal dummies, and cointegrating relations) in the equation. In the second step all variables not already included in the equation are tested for inclusion with different lag lengths. The combination of variable and lag length which gives the largest reduction in the information criteria is added to the model until no further reductions are achieved. To safeguard against overparameterization, the number of lags tested is subject to the restriction that the degrees of freedom for the equation is at least 20. Finally, variables with insignificant parameter estimates are removed from the model if this does not produce a significant Ljung-Box statistic. Note that the *t*-tests used as a criteria for removing individual lags are invalid when the model is estimated using the levels of the variables when the variables contain unit roots.

Strategy 3. In this approach variables are included in the forecasting model based on their predictive power for *UNEMP* as measured by conventional *F*-tests of zero restric-

Table 6. Cointegrating Regressions

Dependent variable	Constant	<i>LIPOECD</i>	<i>LRLABC</i>	R^2	<i>D-W</i>	Obs.
<i>SUMGDP</i>	35.759	1.132	1.347	0.98	0.37	57
<i>LIPSWE</i>	1.536	0.403	0.262	0.54	2.43	60

Estimates are for the pre-forecasting period 64:1 to 78:4

tions on the lags of the variable. In a second step the variables not included in the first step are tested for predictive power for the variables included in the model. In order to reduce the number of possible combinations, the lag length is restricted to be equal for all variables and selected by LIL.

For all three procedures the maximum lag length allowed is eight. The selected models are verified by testing for white-noise residuals with multivariate and univariate Ljung-Box tests and the lag length is increased if necessary.

An additional issue of concern is the treatment of the integrated and cointegrated variables. Engle and Yoo [1987] demonstrate that if two or more of the variables are cointegrated the forecasting performance of an error correction model is superior to an unrestricted VAR for longer lead times.

In our case, the variable of interest - *UNEMP* - is not cointegrated and the gains from using an error correction representation are less obvious. Nevertheless we would expect to achieve a modest gain from improved forecasts of the variables used to predict *UNEMP* at higher lead times.

In order to assess the effect of explicitly imposing the restriction of two cointegrating relations the first and the second approach to model selection above will be used with both the error correction representation and the unrestricted VAR representation. The forecasting models based on the ECM are labeled VAR1A and VAR2A. For these models the seasonal difference of *LRGDP* and the first difference of *UNEMP*, *LIPSWE*, *LCPI*, and *LIPOECD* are used as dependent variables in the estimation. The unrestricted forecasting models estimated on the levels are labeled VAR1B and VAR2B.

Following Engle and Granger [1987] and Engle and Yoo [1987] a two-step approach is used when estimating the ECM, subject to the reduced rank restriction. In the first step, the two cointegrating regressions in Table 6 are estimated by OLS and the residuals from these regressions are our estimates of the stationary transformations, z_{1t} and z_{2t} .

In the second step the ECM is estimated by OLS using the estimated stationary transformations. More efficient estimates could be obtained by the FIML method of Johansen or by using Zellner's SURE in the second step when the explanatory variables differ between equations. The two-step OLS procedure is chosen because of its computational simplicity.

For the VAR1A model (the ECM with equal lag lengths) a lag length of one was selected and seasonal dummies were included. Each equation thus contains a constant term, three seasonal dummies, the stationary linear combinations z_1 and z_2 and the first lag of the six dependent variables in the model. Estimates of the equations in the model are in Table A1 of the appendix. The VAR1B model has a lag length of 2 and seasonal dummies as well as a time trend are included in each equation. Estimates of this model is in Table A2 of the appendix.

For the VAR2 models the set of explanatory variables is different for each equation and for the sake of brevity only the equation for the unemployment rate is reported in the text. Estimates of the full models are in Tables A3 and A4 of the appendix. In the VAR2A model the *LIPSWE* variable only enters the model through the cointegrating relations and the equation for the unemployment rate is

$$\begin{aligned} \nabla UNEMP_t = & \alpha_0 + \alpha_1 D_{2t} + \alpha_2 D_{3t} + \alpha_3 D_{4t} + \alpha_4 z_{1t} + \alpha_5 z_{2t} \\ & + \alpha_6 \nabla UNEMP_{t-1} + \alpha_7 \nabla LRLABC_{t-4} + \varepsilon_{1t}. \end{aligned}$$

For the VAR2B model all six variables remain in the model and we have the following equation for the unemployment rate

$$\begin{aligned} UNEMP_t = & \alpha_0 + \alpha_1 D_{2t} + \alpha_2 D_{3t} + \alpha_3 D_{4t} + \alpha_4 t + \alpha_5 UNEMP_{t-2} \\ & + \alpha_6 LCPI_{t-1} + \alpha_7 LCPI_{t-2} + \alpha_8 LCPI_{t-3} + \alpha_9 LRGDP_{t-2} + \alpha_{10} LRGDP_{t-3} \\ & + \alpha_{11} LRLABC_{t-1} + \alpha_{12} LRLABC_{t-2} + \alpha_{13} LIPSWE_{t-2} + \alpha_{14} LIPSWE_{t-7} \\ & + \alpha_{15} LIPOECD_{t-1} + \alpha_{16} LIPOECD_{t-2} + \alpha_{17} LIPOECD_{t-3} + \varepsilon_{1t}. \end{aligned}$$

The third approach tends to lead to models with a small number of variables. The inclusion of the cointegrating relations requires that separate forecasting models are built for the cointegrated variables, thus negating some of the gains from having a small forecasting model. In addition, the variables are required to be stationary for the *F*-tests to be valid. Stationary differences of the variables (except for *UNEMP* which is in levels) will consequently be used with the third approach and the possible gains from imposing the cointegration restrictions are ignored. This model is referred to as VAR3.

The tests used to determine which variables to include in the model are reported in Tables A5 and A6 of the appendix. Based on these tests it was decided to only include

the first difference of *LIPSWE* in the model. The explanatory variables in the two equations are two lags of *UNEMP* and $\nabla LIPSWE$, a constant term and seasonal dummies. Estimates of the model are shown in Table A7 of the appendix.

4. Forecasting Results

To evaluate the forecasting performance of the specified models we have used a rolling estimation scheme. The models were first estimated with data up to 78:4 and then forecasts were made for each of the following eight quarters. Future values for the explanatory variables were obtained using an ARIMA model to forecast *LIPSWE* in the transfer function model and using the relevant equations for the VAR models. We have thus made the forecasts the way they would be made in a real situation.

After the first set of forecasts had been obtained, the models were re-estimated using data up to 79:1 and then forecasts were made for the following eight quarters. This procedure was repeated for all quarters up to 88:4. The same specified models were used for the whole 10 year period. In practice the models would be re-specified at some pre-determined interval or as the forecast errors grow. Our approach implies model stability over time. Probably, this assumption does not hold for all models. Still, we have judged the positive effects of modifying the models to be of less importance in our case. If we want to re-specify the models we still need some reasonable way of determining when the models are to be re-specified.

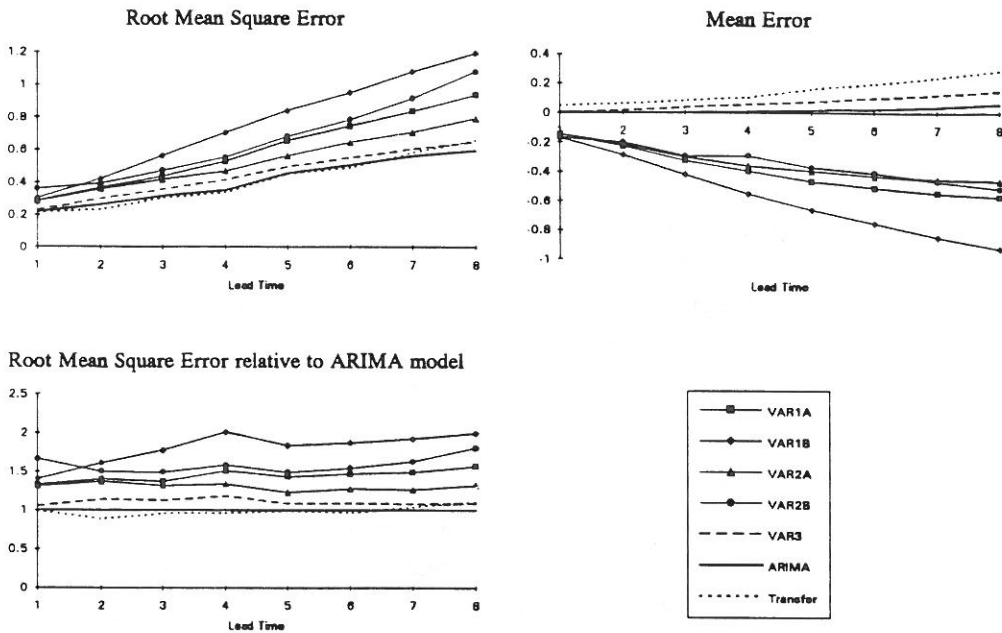
The results will be shown in three different ways. First we present diagrams showing the RMSE and the mean error of the forecasts at different lead times in Figure 4. We also computed the mean absolute error, but since the results are very similar to the RMSE results we do not show these diagrams. Tables A8 to A12 in the appendix show the actual values of the RMSE, mean error as well as the mean absolute error.

We then present graphs showing forecasts with different lead times at selected origins. Finally, we do a pairwise comparison of the forecasting information for the models at the eight lead times, using a test proposed by Fair and Shiller [1990].

4.1. RMSE and Mean Error

Figure 4 shows that the ARIMA and transfer function models perform best over all lead times. Except for the seven and eight-quarter-ahead forecasts the transfer function model is marginally better than the ARIMA model. The gain from including *LIPSWE* in the transfer function model is relatively small, even though the parameters of the transfer function are significant. One reason for this is that we have to make forecasts of future

Figure 4. Forecast Statistics



LIPSWE values in order to forecast *UNEMP*. If the true *LIPSWE* values are used to forecast *UNEMP* (which would not be possible in a real situation) the RMSE for the transfer function model is reduced significantly. At lead time eight the reduction is approximately 20%. This is in accordance with the result of Ashley [1983], who shows that 'For a variety of types of models inclusion of an exogenous variable x_t , ... worsen the y_t forecasts whenever x_t must itself be forecast by \hat{x}_t and $MSE(\hat{x}_t) > Var(x_t)$.'

The VAR3 model RMSE is not too far from that produced by the ARIMA and transfer function models. The more complex VAR1A-VAR2B models have RMSE values up to twice the ARIMA RMSE. The 1A and 2A models generate better forecasts than the 1B and 2B models.

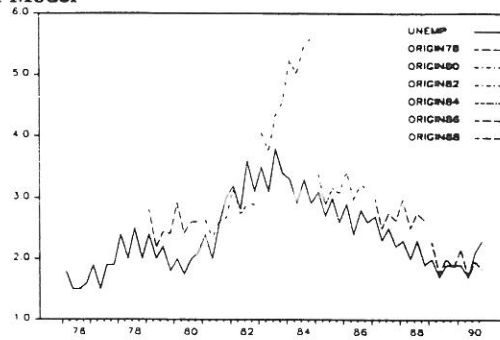
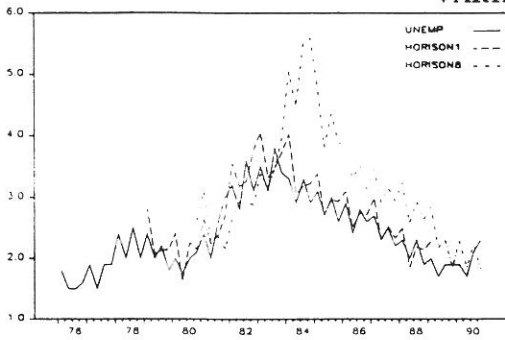
The higher RMSE values for the VAR1 and VAR2 models are partially due to the relatively large negative bias.

4.2. Forecasts at Different Lead Times

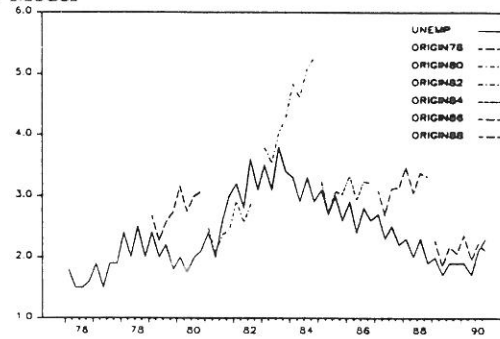
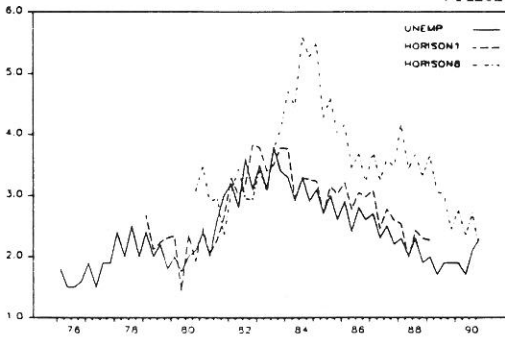
The one step-ahead forecasts in Figure 5 are very close to the true values, except possibly for the VAR1B and VAR2B models. When we look at the eight step-ahead forecasts there are at least three different patterns. The VAR1A, VAR1B and VAR2A models do

Figure 5. Forecasts

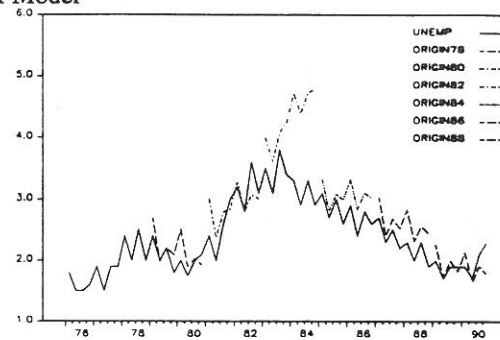
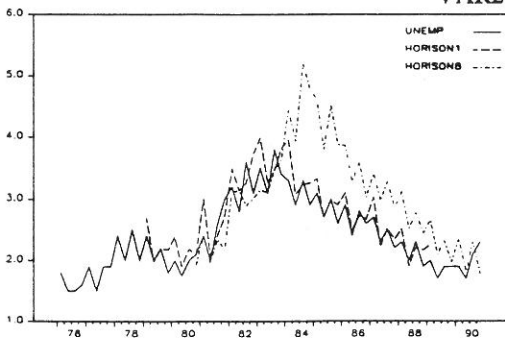
VAR1A Model



VAR1B Model

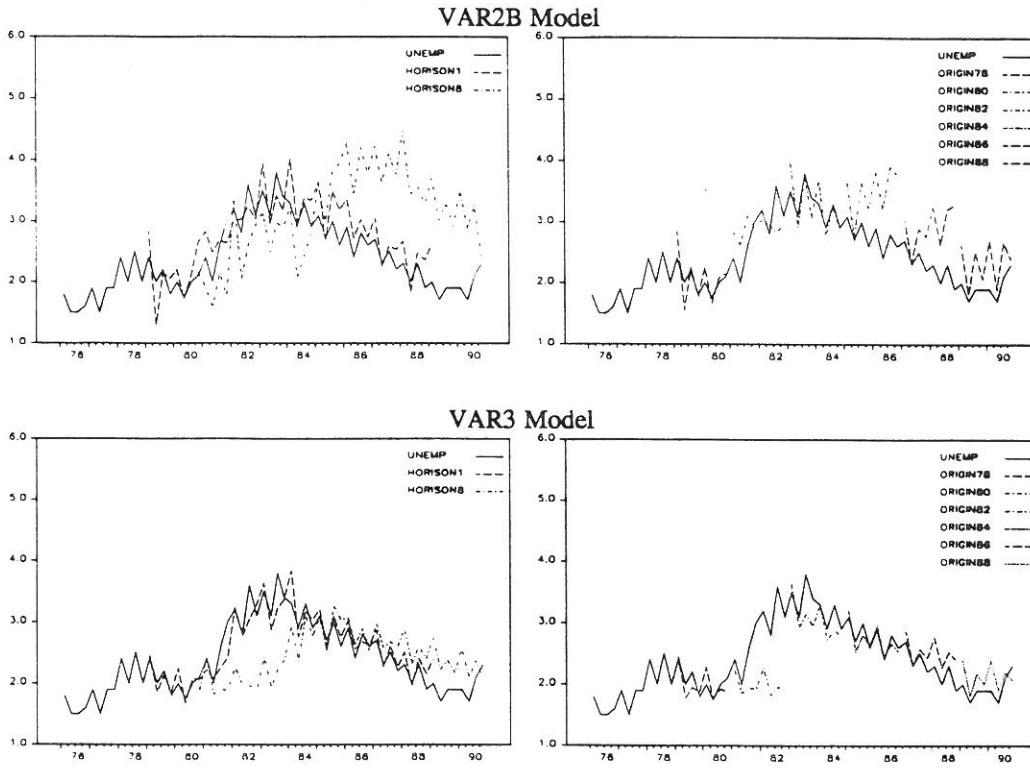


VAR2A Model



Left panels are one step and eight step-ahead forecasts. Right panels are forecasts with origins 78:4, 80:4, 82:4, 84:4, 86:4 and 88:4.

Figure 5. Continued



Left panels are one step and eight step-ahead forecasts. Right panels are forecasts with origins 78:4, 80:4, 82:4, 84:4, 86:4 and 88:4.

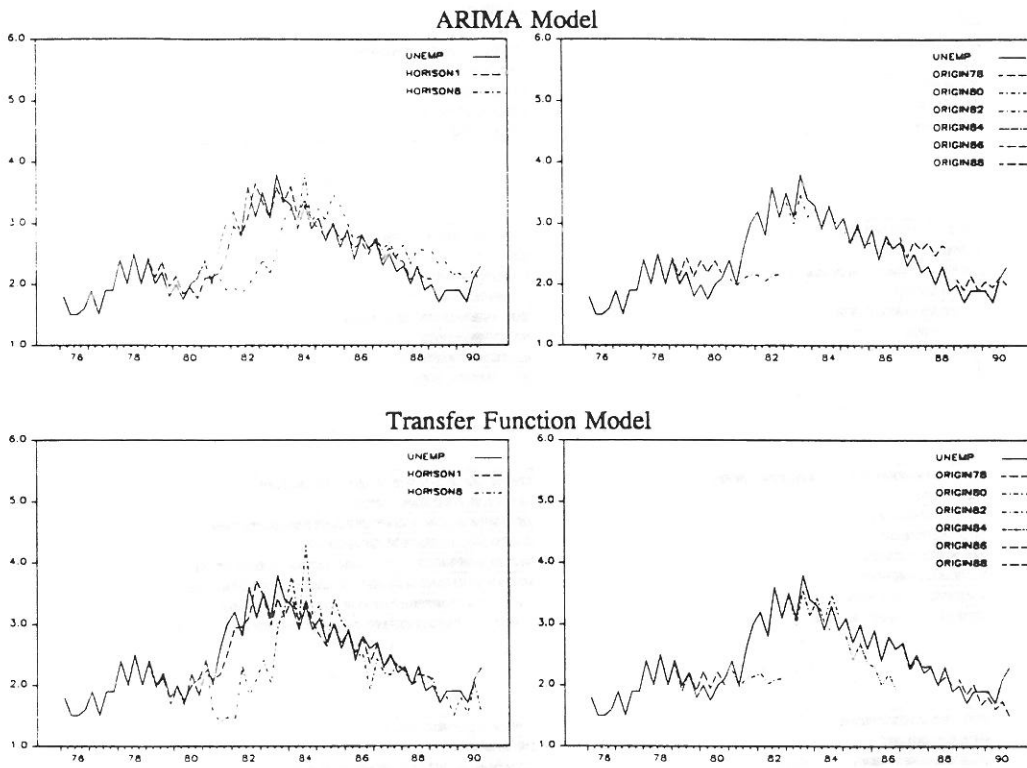
not catch the downswing in unemployment in 1984. The maximum forecasting error is about two percentage units! However, they do catch the rise in 1981. The VAR2B model tracks the rise in 1981 and fall in 1984 with a varying delay. The VAR3, ARIMA and transfer function models also miss the turning point in 1981, but they do not overshoot after 1984, as the VAR2B model does. These models seem to smooth the unemployment series.

4.3. Fair-Shiller Tests

Fair and Shiller [1990] proposed a procedure for testing the forecasting information in forecasts from two or more models. For two forecasting models they estimate the model

$$Y_t - Y_{t-s} = \alpha + \beta_s(\hat{Y}_{1t-s} - Y_{t-s}) + \gamma_s(\hat{Y}_{2t-s} - Y_{t-s}) + u_t \quad (3)$$

Figure 5. Continued

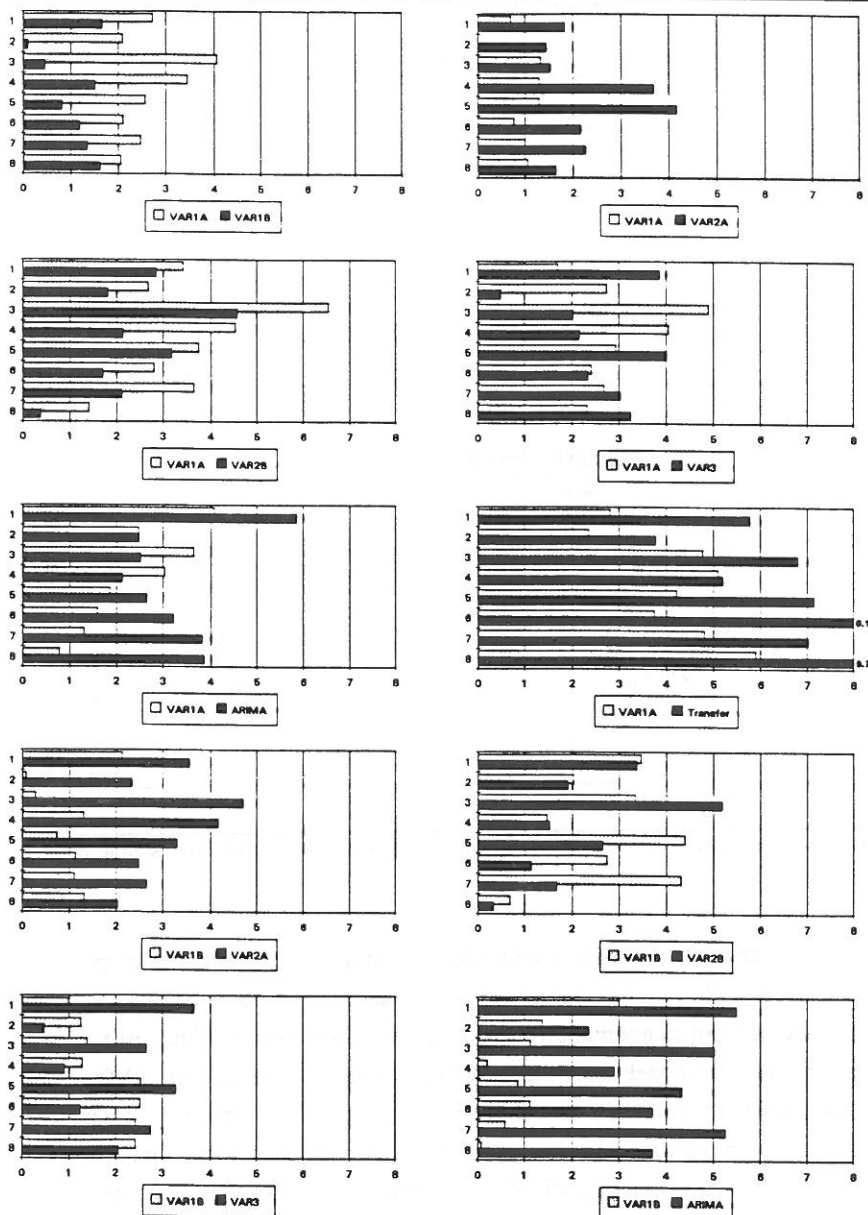


Left panels are one step and eight step-ahead forecasts. Right panels are forecasts with origins 78:4, 80:4, 82:4, 84:4, 86:4 and 88:4.

where Y_t and Y_{t-s} are the true values of dependent variable at time t and $t-s$ respectively and ${}_s\hat{Y}_{t-s}$ is a forecast of Y_t at time $t-s$, s periods ahead, using model i ($i = 1, 2$). The forecasts are based on information known at no later than period $t-s$. If the estimates of β and γ are zero, neither model 1 nor model 2 contains any information useful for s period-ahead forecasting of Y . If only the estimate of β is significant, model 1 contains all relevant information in both models plus some information not available in model 2. If the estimates of both β and γ are significant both models contain unique forecasting

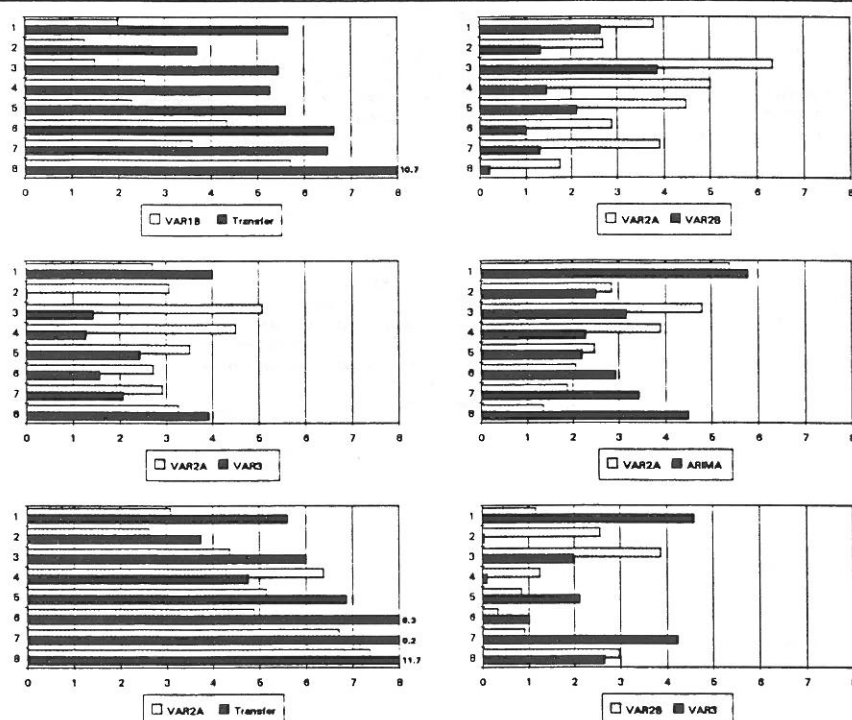
information. (Note that if both models contain exactly the same information we will not be able to estimate the coefficients.) α is an estimate of the forecasting bias. Fair and Shiller propose testing the following hypotheses; $H_1: \beta = 0$ and $H_2: \gamma = 0$ where " H_1 " is the hypothesis that model 1's forecasts contain no information relevant to forecasting s

Figure 6. Fair-Shiller Tests



Absolute value of t -statistics for Fair-Shiller regressions (3) at lead times 1 to 8.

Figure 6. Continued



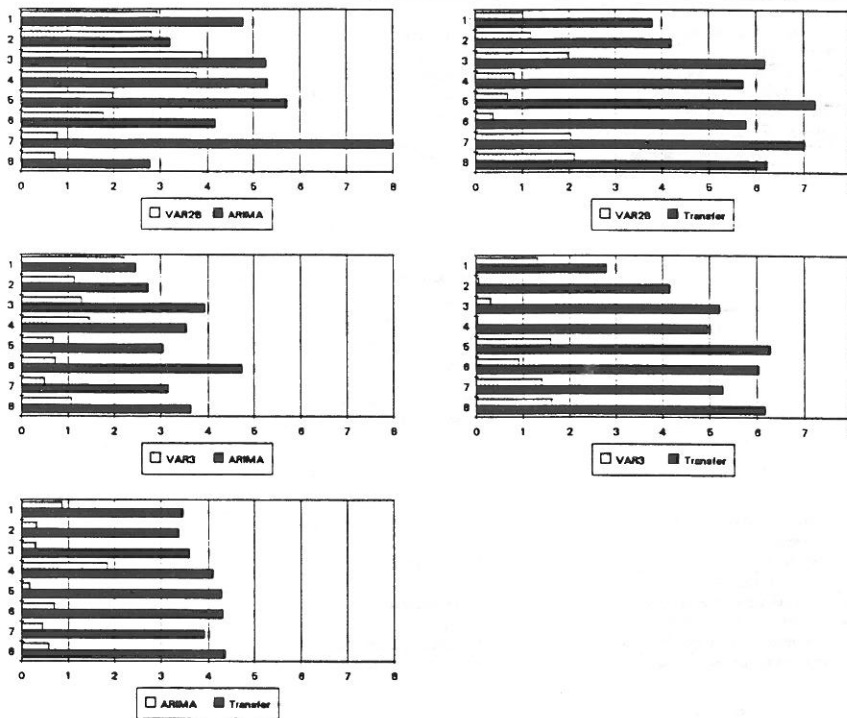
Absolute value of t -statistics for Fair-Shiller regressions (3) at lead times 1 to 8.

period ahead not in the constant term and in model 2, and H_2 is the hypothesis that model 2's forecasts contain no information not in the constant term and in model 1".

Equation (3) was estimated using the Newey and West [1987] consistent variance-covariance estimator to correct for autocorrelated residuals and heteroskedasticity. Figure 6 shows the (absolute) t -values for β and γ at each lead time and for all combinations of forecasting models.

The transfer function model has significant coefficients at all lead times in all comparisons. The same is true for the ARIMA model except when its forecasts are compared to the transfer function forecasts. This is to be expected since the ARIMA model is a subset of the transfer function model. (At least if the forecasts of the *LIPSWE* variable contain some unique forecasting information.) It is also interesting to note that the coefficients for the VAR3 model (with RMSE similar to the ARIMA and transfer function models) are not significant when compared to the ARIMA (with one exception) and transfer function models.

Figure 6. Continued



Absolute value of t -statistics for Fair-Shiller regressions (3) at lead times 1 to 8.

The VAR1A and 2A models seem to contain more unique forecasting information than the VAR1B and VAR2B models. When the A models are compared with the corresponding B model, the A models have significant coefficients in all cases but one and the B models have only one significant coefficient. The A models also do better in the comparisons with the ARIMA and transfer function models than the B models.

Comparing the VAR1 and VAR2 models we find that the VAR2 models contain more forecasting information than the corresponding VAR1 model. The VAR2A model clearly dominates the VAR1A model with four significant coefficients against zero.

5. Conclusions

The VAR3, ARIMA and transfer functions models have the lowest RMSE and mean error values at all leads. According to the Fair-Shiller tests the ARIMA and transfer function models seem to contain more forecasting information than the VAR models. They do not, however, catch the turning points as well as the VAR1 and VAR2 models.

The ARIMA and transfer function models seem to be forecasting the average unemployment rate rather than the actual unemployment rate.

This shows that it is very important to 'look' at the forecasts and not solely rely on the conventional measures of forecasting precision when evaluating different forecasting models.

5.1. Turning Points

It is not surprising that the ARIMA model is not very good at forecasting turning points. The AR-polynomial of the ARIMA model has two real roots and hence the business cycle is not captured by the model. In addition, the cyclical pattern changed during the eighties, first we had a 'mini boom' followed by a short recession and the longest recovery period on record. Clearly an ARIMA-type model will not catch such movements in the data if they are caused by factors external to the model.

The transfer function and the VAR3 models, on the other hand, include a measure of the economic activity (the industrial production). We would therefore expect these models to predict the increase in the unemployment rate better than the ARIMA model. Since the forecasts from the transfer function and the VAR3 models are very similar to the ARIMA forecasts we conclude that the variation in the unemployment rate is, at least partially, due to other factors.

These factors are to some extent included in the VAR1 and VAR2 models. These models do, however, overshoot and miss the second turning point by approximately one year. The economic recovery can largely be explained by an upswing in the international economy and the combined effect of the devaluations of the Swedish krona by 10% in September 1981 and by 16% in November 1982. Since the exchange rates are not included in the models this might be an explanation why the VAR models miss the second turning point.

Due to the inherent complexity in the economy we would expect the VAR models that contain more variables (more information) and allow for interaction to forecast turning points better than the simpler ARIMA and transfer function models. It is, however, clear that some important information is missing in our models.

5.2. Identification Issues

In most applications of transfer function modelling the input and output variables are either non-seasonal or have the same seasonal pattern. Then the pre-whitening cross-correlation method performs well. When the variables have different seasonal patterns, as in

our case, pre-whitening can obscure the relationship between the variables. Ideally, we should be able to remove from the output variable the part of the seasonal variation that is not explained by the input variable, before pre-whitening. Otherwise it is possible that the cross-correlation function will be dominated by the confounded seasonal patterns.

In VAR models it is common practice to include the a priori determined variables with the same number of lags for all variables and in all equations. This is largely due to the simplification of the identification and estimation achieved by imposing this type of constraint. In many cases, as in the VAR1 models, this leads to low lag lengths being selected. Alternatively a very rich parameterization with few remaining degrees of freedom is chosen. Both strategies are potentially damaging to the forecasting performance. In our case the VAR1 models do worse than the corresponding VAR2 models and the VAR3 model. It thus appears that a better strategy is to allow for different lags and to choose a parsimonious model as in VAR2, or to test each variable for inclusion as in VAR3. Differentiating between the VAR2 and VAR3 strategies is harder but we tend to favour the VAR2 strategy since it tends to lead to 'richer' models.

5.3. Cointegration

Our results confirm the finding in Engle and Yoo [1987] that explicitly imposing the cointegrating restrictions rather than estimating the unrestricted VAR in levels improves the forecasting performance, especially for longer leads. This is noteworthy since the variable we forecast, the unemployment rate, is *not* included in the cointegrating relations.

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Appendix: Tables

Table A1. The VAR1A Forecasting Model

	Dependent Variable					
	$\nabla UNEMP$	$\nabla LCPI$	$\nabla_4 LR GDP$	$\nabla LRLABC$	$\nabla LIPSWE$	$\nabla LIPOECD$
<i>Constant</i>	0.3864 (0.501)	0.0728 (0.019)	0.0023 (0.039)	0.0140 (0.044)	-0.0196 (0.044)	-0.0225 (0.023)
D_2	-0.4403 (0.576)	-0.0633 (0.022)	0.0348 (0.045)	0.0167 (0.050)	0.0503 (0.050)	0.0455 (0.027)
D_3	-0.1746 (0.430)	-0.0522 (0.016)	-0.0105 (0.034)	-0.0393 (0.037)	-0.2463 (0.037)	0.0426 (0.020)
D_4	-0.6300 (0.922)	-0.1023 (0.035)	0.0880 (0.073)	0.0153 (0.080)	0.2285 (0.080)	0.0451 (0.043)
z_1	2.6033 (0.917)	-0.0360 (0.035)	-0.1077 (0.072)	0.0514 (0.080)	0.0083 (0.080)	0.1366 (0.043)
z_2	-4.0172 (1.332)	0.0634 (0.051)	0.1472 (0.105)	0.0188 (0.116)	-0.0944 (0.116)	-0.1815 (0.062)
$\nabla UNEMP_{t-1}$	-0.5364 (0.144)	-0.0046 (0.005)	-0.0235 (0.011)	-0.0068 (0.013)	0.0002 (0.013)	0.0020 (0.007)
$\nabla LCPI_{t-1}$	-1.8971 (3.953)	0.1381 (0.150)	-0.6809 (0.312)	0.2403 (0.344)	0.2263 (0.344)	-0.3045 (0.185)
$\nabla_4 LR GDP_{t-1}$	-0.5949 (1.580)	0.0013 (0.060)	0.2311 (0.125)	0.0851 (0.138)	0.3570 (0.137)	-0.1312 (0.074)
$\nabla LRLABC_{t-1}$	1.3931 (1.717)	-0.1273 (0.065)	0.0684 (0.135)	-0.3310 (0.150)	0.1681 (0.149)	0.1459 (0.081)
$\nabla LIPSWE_{t-1}$	1.4053 (1.996)	-0.2089 (0.076)	0.0985 (0.157)	0.1041 (0.174)	-0.0953 (0.174)	0.1651 (0.094)
$\nabla LIPOECD_{t-1}$	-0.6165 (2.389)	-0.0917 (0.091)	0.0169 (0.188)	-0.3572 (0.208)	0.2552 (0.208)	0.5585 (0.112)
R^2	0.740	0.392	0.522	0.643	0.991	0.593
$D-W$	1.845	2.134	2.137	2.000	1.904	1.843
$LB-Q(21)$	26.413	21.766	26.505	16.345	19.059	15.607
Multivariate $LB-Q(648) = 664.564$, $p = 0.323$						

Estimates are for the period 65:2 to 78:4. Standard errors in parenthesis.

Table A2. The VAR1B Forecasting Model

	Dependent Variable					
	<i>UNEMP</i>	<i>LCPI</i>	<i>LRGDP</i>	<i>LRLABC</i>	<i>LIPSWE</i>	<i>LIPOECD</i>
<i>Constant</i>	80.3028 (32.303)	0.1679 (0.895)	23.1794 (2.435)	3.3127 (2.435)	0.1782 (2.404)	1.8880 (1.508)
<i>D</i> ₂	-1.1800 (0.687)	-0.0177 (0.019)	0.1918 (0.052)	0.0359 (0.052)	-0.0028 (0.051)	0.0322 (0.032)
<i>D</i> ₃	-0.4718 (0.472)	-0.0266 (0.013)	0.0053 (0.036)	-0.0327 (0.036)	-0.2756 (0.035)	0.0263 (0.022)
<i>D</i> ₄	-1.2269 (1.025)	-0.0566 (0.028)	0.2984 (0.077)	0.0405 (0.077)	0.0841 (0.076)	0.0356 (0.048)
<i>t</i>	0.2186 (0.072)	-0.0023 (0.002)	0.0273 (0.005)	0.0094 (0.005)	-0.0029 (0.005)	0.0062 (0.003)
<i>UNEMP</i> _{<i>t</i>-1}	0.4045 (0.165)	-0.0048 (0.005)	-0.0100 (0.012)	-0.0111 (0.012)	0.0104 (0.012)	0.0028 (0.008)
<i>UNEMP</i> _{<i>t</i>-2}	0.1419 (0.163)	-0.0010 (0.005)	-0.0058 (0.012)	0.0159 (0.012)	0.0106 (0.012)	0.0023 (0.008)
<i>LCPI</i> _{<i>t</i>-1}	-1.3927 (4.653)	0.7868 (0.129)	-0.1906 (0.351)	0.5875 (0.351)	0.4264 (0.346)	-0.2004 (0.217)
<i>LCPI</i> _{<i>t</i>-2}	-3.5086 (4.989)	0.1915 (0.138)	-0.5513 (0.376)	-0.7221 (0.376)	-0.6475 (0.371)	0.0308 (0.233)
<i>LRGDP</i> _{<i>t</i>-1}	-2.5669 (1.579)	-0.0069 (0.044)	-0.1118 (0.119)	0.0651 (0.119)	-0.0516 (0.118)	0.0228 (0.074)
<i>LRGDP</i> _{<i>t</i>-2}	0.5320 (1.481)	-0.1010 (0.041)	-0.6910 (0.112)	-0.1409 (0.112)	-0.0020 (0.110)	-0.0148 (0.069)
<i>LRLABC</i> _{<i>t</i>-1}	-1.2627 (2.268)	-0.0099 (0.063)	-0.0315 (0.171)	0.4152 (0.171)	0.3955 (0.169)	0.0134 (0.106)
<i>LRLABC</i> _{<i>t</i>-2}	-4.9317 (2.288)	0.2147 (0.063)	-0.0883 (0.173)	0.1931 (0.172)	0.0352 (0.170)	-0.0616 (0.107)
<i>LIPSWE</i> _{<i>t</i>-1}	-1.4832 (2.076)	-0.1095 (0.058)	0.4422 (0.157)	0.1678 (0.156)	0.3940 (0.155)	0.0081 (0.097)
<i>LIPSWE</i> _{<i>t</i>-2}	0.6533 (1.967)	0.0950 (0.055)	-0.1447 (0.148)	-0.0450 (0.148)	0.1828 (0.146)	-0.1027 (0.092)
<i>LIPOECD</i> _{<i>t</i>-1}	1.3662 (2.725)	-0.0517 (0.076)	0.0221 (0.205)	-0.4844 (0.205)	0.0092 (0.203)	1.3812 (0.127)
<i>LIPOECD</i> _{<i>t</i>-2}	-4.4896 (3.125)	0.1671 (0.087)	-0.3052 (0.236)	0.2268 (0.236)	0.3214 (0.233)	-0.6037 (0.146)
<i>R</i> ²	0.805	1.000	0.986	0.995	0.992	0.996
<i>D-W</i>	1.938	2.171	2.308	1.985	1.582	2.208
<i>LB-Q</i> (21)	24.379	18.063	13.851	19.156	13.905	17.979
Multivariate <i>LB-Q</i> (684) = 662.453, <i>p</i> = 0.720						

Estimates are for the period 64:3 to 78:4. Standard errors in parenthesis.

Table A3. The VAR2A Forecasting Model

	Dependent Variable					
	$\nabla UNEMP$	$\nabla LCPI$	$\nabla_4 LR GDP$	$\nabla LRLABC$	$\nabla LIPSWE$	$\nabla LIPOECD$
<i>Constant</i>	0.7732 (0.108)	0.0113 (0.004)	0.0256 (0.010)	0.0641 (0.010)	0.0044 (0.008)	0.0063 (0.005)
D_2	-0.9187 (0.115)	-0.0089 (0.003)	-0.0010 (0.008)	-0.0175 (0.010)	0.0823 (0.007)	-0.0010 (0.006)
D_3	-0.6315 (0.143)	-0.0105 (0.004)	-0.0223 (0.008)	-0.0586 (0.008)	-0.2460 (0.009)	0.0084 (0.005)
D_4	-1.2467 (0.260)	-0.0016 (0.009)	0.0553 (0.021)	-0.0410 (0.021)	0.2698 (0.017)	-0.0263 (0.013)
z_1	2.3056 (0.754)	-0.0025 (0.026)	-0.1828 (0.060)	0.0339 (0.059)	-0.0117 (0.047)	0.0991 (0.039)
z_2	-2.9400 (1.004)	0.0427 (0.034)	0.2505 (0.079)	0.0095 (0.081)	-0.0351 (0.061)	-0.1319 (0.051)
$\nabla UNEMP_{t-1}$	-0.4671 (0.143)					
$\nabla LCPI_{t-1}$			-0.7923 (0.300)			
$\nabla LCPI_{t-2}$					-1.1044 (0.218)	
$\nabla LCPI_{t-3}$		0.6246 (0.121)		-0.7568 (0.275)		
$\nabla LR GDP_{t-1}$			0.2299 (0.132)			
$\nabla LRLABC_{t-1}$				-0.3579 (0.129)		0.1420 (0.083)
$\nabla LRLABC_{t-4}$	-3.5513 (1.526)					
$\nabla LRLABC_{t-8}$					-0.4474 (0.100)	
$\nabla LIPOECD_{t-1}$				-0.4777 (0.176)		0.6665 (0.112)
R^2	0.765	0.462	0.478	0.722	0.995	0.541
$D-W$	2.171	1.708	2.165	2.059	1.888	2.127
$LB-Q(21)$	17.071	13.879	21.720	15.047	14.484	18.758
Multivariate $LB-Q(612) = 617.522$, $p = 0.437$						

Estimates are for the period 66:2 to 78:4. Standard errors in parenthesis.

Table A4. The VAR2B Forecasting Model

	Dependent Variable					
	<i>UNEMP</i>	<i>LCPI</i>	<i>LRGDP</i>	<i>LRLABC</i>	<i>LIPSWE</i>	<i>LIPOECD</i>
<i>Constant</i>	-80.3572 (11.612)	0.2873 (0.150)	12.9117 (2.372)	-2.4031 (1.108)	4.9543 (0.893)	0.6201 (0.150)
D_2	2.3078 (0.252)		0.1000 (0.021)	-0.0320 (0.014)	0.0494 (0.008)	
D_3		-0.0098 (0.003)		-0.0187 (0.008)	-0.2509 (0.007)	
D_4	1.2748 (0.146)		0.0934 (0.018)	-0.0654 (0.013)	0.1065 (0.024)	
t			0.0164 (0.003)	0.0046 (0.002)	0.0150 (0.003)	0.0014 (0.000)
$UNEMP_{t-1}$		-0.0110 (0.002)	-0.0167 (0.007)			0.0093 (0.004)
$UNEMP_{t-2}$	0.6682 (0.093)					
$UNEMP_{t-4}$	-0.2954 (0.092)			0.0184 (0.007)		
$UNEMP_{t-8}$				0.0342 (0.006)		
$LCPI_{t-1}$	1.6615 (3.104)	0.8731 (0.023)	-0.5817 (0.099)		0.7867 (0.228)	
$LCPI_{t-2}$	-5.7565 (3.980)				-0.6514 (0.239)	
$LCPI_{t-3}$	4.6792 (3.030)					
$LCPI_{t-7}$					-0.5149 (0.139)	
$LRGDP_{t-2}$	1.6314 (0.965)	-0.0552 (0.016)	-0.4771 (0.105)	0.1294 (0.080)		
$LRGDP_{t-3}$	7.0070 (0.933)					
$LRGDP_{t-4}$			0.5108 (0.097)			
$LRGDP_{t-8}$				0.3555 (0.068)		

Table A4. Continued

	Dependent Variable					
	<i>UNEMP</i>	<i>LCPI</i>	<i>LRGDP</i>	<i>LRLABC</i>	<i>LIPSWE</i>	<i>LIPOECD</i>
<i>LRLABC</i> _{<i>t</i>-1}	-3.2011 (1.157)			0.5072 (0.096)	0.2161 (0.090)	
<i>LRLABC</i> _{<i>t</i>-2}	-4.0565 (1.435)	0.1370 (0.042)				
<i>LRLABC</i> _{<i>t</i>-3}		-0.1113 (0.046)				
<i>LRLABC</i> _{<i>t</i>-5}				0.1491 (0.098)		
<i>LRLABC</i> _{<i>t</i>-7}					-0.2825 (0.090)	
<i>LRLABC</i> _{<i>t</i>-8}		0.1831 (0.036)			-0.5876 (0.107)	
<i>LIPSWE</i> _{<i>t</i>-1}					0.3274 (0.088)	
<i>LIPSWE</i> _{<i>t</i>-2}	-4.7907 (0.545)					
<i>LIPSWE</i> _{<i>t</i>-7}	4.8248 (0.745)					
<i>LIPOECD</i> _{<i>t</i>-1}	7.6735 (2.077)			-0.8770 (0.182)	0.3952 (0.075)	1.4998 (0.102)
<i>LIPOECD</i> _{<i>t</i>-2}	-13.4901 (3.426)			0.9665 (0.305)		-0.6611 (0.097)
<i>LIPOECD</i> _{<i>t</i>-3}	8.1059 (2.047)			-0.5451 (0.183)		
<i>R</i> ²	0.928	1.000	0.984	0.997	0.996	0.994
<i>D-W</i>	2.263	2.143	1.936	2.054	2.003	2.063
<i>LB-Q</i> (21)	20.788	16.936	13.285	22.408	15.624	19.382
Multivariate <i>LB-Q</i> (612) = 660.931, <i>p</i> = 0.081						

Estimates are for the period 66:1 to 78:4. Standard errors in parenthesis.

Table A5. Bivariate Systems

Variables (stationary differences)	Lags ^a	F-tests for zero coefficients on lags of (p-values)						R^2
		UNEMP	LIPSWE	LRLABC	LRGDP	LCPI	LIPOECD	
UNEMP	2	0.000	0.045					0.765
LIPSWE		0.192	0.272					0.989
UNEMP	4	0.000		0.162				0.821
LRLABC		0.106		0.271				0.640
UNEMP,	2	0.000			0.398			0.768
LRGDP		0.009			0.003			0.356
UNEMP,	3	0.000				0.548		0.753
LCPI		0.307				0.000		0.455
UNEMP	3	0.000					0.559	0.753
LIPOECD		0.554					0.000	0.426

a) LIL selected a lag length of 1 in all cases but this gave unsatisfactory Ljung-Box Q-statistics and the lag length was increased.

Table A6. Trivariate Systems

Variables (stationary differences)	Lags ^a	F-tests for zero coefficients on lags of (p-values)						R ²
		UNEMP	LIPSWE	LRLABC	LRGDP	LCPI	LIPOECD	
UNEMP	2	0.000	0.074	0.658				0.769
LIPSWE		0.206	0.282	0.798				0.990
LRLABC		0.460	0.755	0.149				0.581
UNEMP	1	0.000	0.925		0.290			0.754
LIPSWE		0.014	0.184		0.011			0.991
LRGDP		0.540	0.014		0.014			0.291
UNEMP	2	0.000	0.011			0.209		0.780
LIPSWE		0.190	0.562			0.005		0.992
LCPI		0.558	0.072			0.213		0.329
UNEMP	2	0.000	0.053				0.378	0.774
LIPSWE		0.280	0.312				0.180	0.990
LIPOECD		0.751	0.969				0.000	0.394
UNEMP	1	0.000		0.380	0.250			0.758
LRLABC		0.276		0.074	0.312			0.571
LRGDP		0.830		0.049	0.003			0.258
UNEMP	2	0.000		0.405		0.861		0.743
LRLABC		0.385		0.359		0.154		0.608
LCPI		0.463		0.092		0.011		0.322
UNEMP	2	0.000		0.212			0.171	0.760
LRLABC		0.201		0.044			0.013	0.647
LIPOECD		0.651		0.614			0.000	0.406
UNEMP	2	0.000			0.182	0.418		0.777
LRGDP		0.018			0.200	0.049		0.439
LCPI		0.230			0.150	0.145		0.333
UNEMP	2	0.000			0.493		0.637	0.772
LRGDP		0.011			0.004		0.962	0.357
LIPOECD		0.826			0.625		0.000	0.470

a) LIL selected a lag length of 1 in all cases, when this gave unsatisfactory Ljung-Box Q-statistics the lag length was increased.

Table A7. The VAR3 Forecasting Model

	Dependent Variable	
	<i>UNEMP</i>	<i>VLIPSWE</i>
<i>Constant</i>	0.1739 (0.558)	0.0103 (0.047)
D_2	0.6062 (0.890)	-0.0314 (0.074)
D_3	0.0193 (0.462)	-0.2863 (0.039)
D_4	-0.2776 (0.910)	0.2033 (0.076)
$UNEMP_{t-1}$	0.5918 (0.135)	0.0067 (0.011)
$UNEMP_{t-2}$	0.2977 (0.140)	0.0055 (0.012)
$VLIPSWE_{t-1}$	-2.0809 (1.604)	-0.0570 (0.134)
$VLIPSWE_{t-2}$	-3.7203 (1.567)	0.1977 (0.131)
R^2	0.765	0.990
$D-W$	1.942	2.046
$LB-Q(21)$	19.744	18.049
Multivariate $LB-Q(76) = 78.572$, $p = 0.397$		

Estimates are for the period 64:4 to 78:4.
Standard errors in parenthesis.

Table A8. Root Mean Square Forecast Error

Lead time	Method						
	VAR1A	VAR1B	VAR2A	VAR2B	VAR3	ARIMA	Transfer function
1	0.2888	0.3056	0.2848	0.3614	0.2288	0.2172	0.2145
2	0.3675	0.4224	0.3586	0.3926	0.2992	0.2620	0.2321
3	0.4370	0.5655	0.4191	0.4745	0.3585	0.3181	0.3042
4	0.5302	0.7055	0.4711	0.5566	0.4160	0.3514	0.3369
5	0.6565	0.8415	0.5645	0.6843	0.4977	0.4583	0.4549
6	0.7464	0.9518	0.6483	0.7855	0.5530	0.5083	0.4914
7	0.8376	1.0792	0.7089	0.9161	0.6054	0.5616	0.5825
8	0.9399	1.1940	0.7935	1.0818	0.6513	0.5983	0.6619

Statistics based on 41 sets of forecasts covering the period 79:1 to 90:4.

Table A9. Mean Forecast Error

Lead time	Method						
	VAR1A	VAR1B	VAR2A	VAR2B	VAR3	ARIMA	Transfer function
1	-0.1479	-0.1701	-0.1479	-0.1690	0.0011	-0.0001	0.0474
2	-0.2283	-0.2867	-0.2150	-0.2019	0.0158	-0.0009	0.0626
3	-0.3262	-0.4216	-0.3025	-0.2958	0.0382	-0.0008	0.0828
4	-0.3983	-0.5535	-0.3635	-0.2934	0.0556	0.0075	0.1037
5	-0.4682	-0.6636	-0.3983	-0.3727	0.0739	0.0152	0.1608
6	-0.5103	-0.7552	-0.4322	-0.4099	0.1004	0.0232	0.1956
7	-0.5487	-0.8504	-0.4544	-0.4681	0.1228	0.0376	0.2400
8	-0.5728	-0.9312	-0.4655	-0.5187	0.1481	0.0583	0.2860

Statistics based on 41 sets of forecasts covering the period 79:1 to 90:4.

Table A10. Mean Absolute Forecast Errors

Lead time	Method						
	VAR1A	VAR1B	VAR2A	VAR2B	VAR3	ARIMA	Transfer function
1	0.2378	0.2635	0.2283	0.3052	0.1696	0.1701	0.1565
2	0.2802	0.3521	0.2692	0.3481	0.2322	0.2073	0.1737
3	0.3614	0.5059	0.3405	0.3994	0.2762	0.2566	0.2279
4	0.4422	0.6442	0.4093	0.4765	0.3104	0.2609	0.2467
5	0.5141	0.7555	0.4566	0.5829	0.3808	0.3539	0.3111
6	0.5673	0.8352	0.4799	0.6676	0.4354	0.3888	0.3522
7	0.6583	0.9430	0.5573	0.7916	0.4667	0.4228	0.4157
8	0.7442	1.0278	0.6296	0.9400	0.5063	0.4532	0.4531

Statistics based on 41 sets of forecasts covering the period 79:1 to 90:4.

Table A11. Root Mean Square Error Relative to ARIMA Model

Lead time	Method					
	VAR1A	VAR1B	VAR2A	VAR2B	VAR3	Transfer function
1	1.3296	1.4073	1.3115	1.6641	1.0533	0.9876
2	1.4028	1.6122	1.3688	1.4986	1.1422	0.8859
3	1.3737	1.7773	1.3173	1.4913	1.1270	0.9561
4	1.5087	2.0077	1.3404	1.5838	1.1838	0.9586
5	1.4324	1.8361	1.2316	1.4932	1.0858	0.9926
6	1.4686	1.8726	1.2754	1.5454	1.0880	0.9668
7	1.4915	1.9218	1.2624	1.6313	1.0780	1.0373
8	1.5709	1.9956	1.3263	1.8082	1.0886	1.1063

Statistics based on 41 sets of forecasts covering the period 79:1 to 90:4.

Table A12. Mean Absolute Error Relative to ARIMA Model

Lead time	Method					
	VAR1A	VAR1B	VAR2A	VAR2B	VAR3	Transfer function
1	1.3981	1.5488	1.3422	1.7943	0.9968	0.9198
2	1.3512	1.6982	1.2984	1.6790	1.1201	0.8375
3	1.4086	1.9720	1.3271	1.5568	1.0767	0.8885
4	1.6951	2.4694	1.5688	1.8265	1.1898	0.9455
5	1.4526	2.1347	1.2901	1.6470	1.0759	0.8789
6	1.4591	2.1482	1.2345	1.7173	1.1200	0.9060
7	1.5572	2.2305	1.3181	1.8724	1.1039	0.9832
8	1.6420	2.2679	1.3892	2.0740	1.1170	0.9997

Statistics based on 41 sets of forecasts covering the period 79:1 to 90:4.

Sammanfattning

Syftet med denna studie är att undersöka hur valet av modellansats påverkar prognosegenskaperna för olika tidsseriemodeller. Som beroende variabel har vi använt andelen arbetslösa enligt AKU. Tidsserien för arbetslösheten, som är en viktig policy-variabel, har under prognosperioden 1979 till 1990 haft två skarpa vändpunkter och borde därför vara relativt svår att förutsäga.

Vi har använt tre överlappande modellklasser: ARIMA-, transferfunktions- och vektor-autoregressiva (VAR) modeller. Arbetshypotesen har varit att de mer komplexa VAR-modellerna, som utnyttjar mer information om sambanden mellan variablerna, bör generera mer precisa prognoser. Även om VAR-modellerna innesluter ARIMA- och transferfunktionsmodellerna har de senare mer sofistikerade residualmodeller varför ARIMA- och transferfunktionsmodellerna inte behöver vara specialfall av VAR-modellerna.

ARIMA- och transferfunktionsmodellerna har identifierats och skattats med gängse metoder. Metoderna för att bygga VAR-modeller är inte lika väldefinierade och vi har därför försökt belysa hur lag-längden bör bestämmas och hur ko-integrerade och integrerade variabler bör behandlas.

Som förklarande variabler har vi använt industriproduktionsindex, konsumentprisindex, real BNP, real arbetskraftskostnad och industriproduktionsindex för OECD. För samtliga variabler har vi använt kvartalsdata för perioden 1964 till och med 1990. Samtliga modeller har specificerats (identifierats) med hjälp av data från perioden 64 t.o.m. 78.

Utvärderingen av de specificerade modellerna har gjorts på följande sätt: Modellerna skattades först med data t.o.m. 78:4. Sedan gjordes prognoser för de följande åtta kvartalerna. De förklarande variablerna prognosticerades för transferfunktionsmodellen med en ARIMA-modell och för VAR-modellerna med de relevanta ekvationerna. Prognoserna för arbetslösheten har således gjorts på samma sätt som i en verklig situation. Efter den första uppsättningen prognoser skattades modellen om med data fram t.o.m. 79:1 och prognoserna beräknades på nytt för de följande åtta kvartalerna. Denna procedur upprepades sedan för alla kvartal t.o.m 88:4.

Vi fann att ARIMA- och transferfunktionsmodellerna tillsammans med en enklare VAR-modell (en två-variabelmodell) gav de lägsta prognosfelen (mätt med roten ur medelkvadratfelet) över alla prognoshorisonter. (Transferfunktionsmodellen och den enkla VAR-modellen har industriproduktionsindex som förklarande variabel.) Värde av att

inkludera en förklarande variabel i transferfunktionsmodellen är relativt litet. Om vi ersätter de prognosticerade värdena för industriproduktionen med de faktiska förbättras dock medelkvadratfelet med upp till 20%. Ett sk Fair-Shiller-test visar att de tre bästa prognosmetoderna innehåller mer prognosinformation än de mer komplexa VAR-modellerna.

Även om ARIMA-, transferfunktionsmodellerna och den enkla VAR-modellen har mindre prognosfel än de mer komplexa VAR-modellerna så finner VAR-modellerna vändpunkterna med större precision. De enkla modellerna verkar mer ge prediktioner över den genomsnittliga arbetslösheten. Detta kan eventuellt förklaras med att de mer komplexa modellerna innehåller mer information om det ekonomiska systemet och därför bättre kan fånga in vändpunkterna.

Den vanliga kors-korrelationsmetoden för att identifiera transferfunktionsmodeller fungerade mindre bra på grund av att den förklarande variabeln, industriproduktionsindex, har ett annat säsongmönster än den beroende variabeln, arbetslösheten. För att lindra detta problem skattades korskorrelationsfunktionen från säsongdifferentierade data. Säsongvariationen reducerades och det blev lättare att identifiera sambandet mellan variablerna.

När man specificerar VAR-modeller brukar man, för att förenkla modellbyggandet, ofta ha samma antal laggar för alla variabler i systemet. Detta leder ofta till att modellerna innehåller få laggar. En alternativ metod är att tillåta olika antal laggar för variablerna vilket ger modeller som hushållar med frihetsgraderna. Den senare ansatsen gav säkrare prognoser för arbetslösheten.

Genom att inkludera ko-integrerande restriktioner förbättras prognosförmågan, speciellt på längre sikt, jämfört med att skatta VAR-modeller utan restriktioner på nivådata.

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