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TESTING THE CONSTANCY OF REGRESSION PARAMETERS AGAINST CONTINUOUS CHANGE[†]

by

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Abstract. A standard explicit or implicit assumption underlying many parameter constancy tests in linear models is that there is a single structural break in the sample. In this paper that assumption is replaced by a more general one stating that the parameters of the model may change continuously over time. The pattern of change is parameterized giving rise to a set of parameter constancy tests against a parameterized alternative. The power properties of the LM type tests in small samples are compared to those of other tests like the CUSUM and Fluctuation Test by simulation and found very satisfactory. An application is considered.

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1. INTRODUCTION

Parameter constancy is a key assumption in econometric models. If it is violated, inference about the parameters may be misleading as well as any policy implications drawn from the model. The accuracy of post-sample forecasting is affected as well. Not surprisingly, testing parameter constancy in linear models has attracted considerable attention in the literature. A recent bibliography of this area is Hackl and Westlund (1989); for surveys, see Krämer and Sonnberger (1986, chapter 4), and Tsurumi (1988).

A standard assumption underlying some parameter constancy tests is that if the parameters change, they change once during the observation period, so that the linear model contains a structural break. The classical test of Chow (1960) assumed the possible break-point to be known. This may often be too strong an assumption in practice and considerable efforts have been made in the literature to develop tests without it. Brown et al. (1975) derived the CUSUM and CUSUMSQ tests based on recursive residuals assuming that the break-point or break-points were unknown and that all regressors were independent of the disturbances. Krämer et al. (1988) showed that the CUSUM test retains its asymptotic significance level even if the model contains lags of the dependent variable. Ploberger and Krämer (1992) showed how the CUSUM test can be carried out using OLS residuals thus avoiding the recursive estimation of regression coefficients.

Recently, the theory of weak convergence and functional central limit theorems have become important in the statistical theory of testing parameter constancy. Sen (1980) and Ploberger et al. (1989) considered a test called the fluctuation test based on comparisons between parameter estimates from the partial samples and those of the complete sample. The regressors of the model were assumed stationary. Andrews (1989) derived the asymptotic null distribution of the sequential likelihood ratio test (Quandt, 1960) of parameter constancy. He also showed that this test has nontrivial local asymptotic power against all alternatives of nonconstant parameters. Ploberger et al. (1989) contained a corresponding result for the fluctuation test. Chu and White (1991) extended the distribution results of Andrews to nonstationary regressors. They then derived tests for the constancy of the trend and the parameters in the cointegration relationship. All the above tests are based on the assumption that if the parameters are nonconstant they are deterministic.

Another strand of literature is concerned about the case in which the alternative to constancy is that the parameters are stochastic and fluctuate according to some time series model. LaMotte and McWhorter (1978) assumed that if the null is not true the parameters follow a random walk and constructed an exact F test for testing against that alternative. Assuming the same alternative Nyblom and Mäkeläinen (1983) derived locally most powerful tests. Nyblom (1989) continued that work considering tests based on maximum likelihood estimation and also showed that both the random walk type variation and a single break-point can be regarded as special cases of a more general set-up. Hansen (1990) further extended Nyblom's (1989) work to cover other than maximum likelihood estimators.

The starting-point of the present paper is that the alternative to parameter constancy is deterministic change in parameters over time. A major difference between the approach here and that taken in several other papers is that the change in parameters (if any) is assumed to be smooth over time. This may often be a more realistic assumption than that of a single structural break. Discrete change in parameters emerges as a special case within this more general framework. Another difference is that the alternative to the null of parameter constancy is a parametric one. Although a rejection of the null hypothesis can in practice hardly be taken to mean that the alternative is true, it is possible to estimate the alternative and try to find out in which part of the sample the constancy assumption seems to break down. This should often be helpful in respecifying an inadequate model. Our tests resemble the one discussed in Farley et al. (1975) in which the alternative to parameter constancy was that the parameters change as a linear function of time. This was taken to be an approximation to a single structural break with an unknown break-point. The present paper gives another justification to that approximation. The plan of the paper is as follows. In section 2 we introduce the nonlinear model that provides the framework for testing parameter constancy. In section 3 tests for testing this hypothesis are derived. Section 4 discusses the specification of the changing parameter model in the case parameter constancy is rejected. In section 5 we investigate the small-sample behavior of our tests by simulation and compare that with power properties of some competitors. Section 6 contains an example and section 7 concludes.

2. THE SMOOTH TRANSITION REGRESSION MODEL

Consider the following smooth transition regression (STR) model

$$y_t = x_t'\pi_1 + x_t'\pi_2 F(z_t) + u_t, \quad t = 1,\dots, T \tag{1}$$

where $x_t = (1, y_{t-1},\dots, y_{t-p}, x_{1t},\dots, x_{qt})'$ is an $m \times 1$ vector, $m = p + 1 + q$, $\pi_1 = (\pi_{11},\dots, \pi_{1m})'$, $\pi_2 = (\pi_{21},\dots, \pi_{2m})'$ are $m \times 1$ parameter vectors and u_t is an error term with $Eu_t = 0$, $Ex_tu_t = 0$ and $Ez_tu_t = 0$. The function $F(z_t)$ is a transition function allowing the model to change from $E(y_t|x_t) = x_t'\pi_1$ to $E(y_t|x_t) = x_t'(\pi_1 + \pi_2)$ with z_t. Bacon and Watts (1971) considered (1) in modelling a transition from one regression to another. In their application, the only regressor x_t was also the transition variable. Maddala (1977, p. 396) also suggested (1) and more recently, Granger and Teräsvirta (1993) discussed the specification and estimation of STR models.

In the parameter stability or structural change literature the transition variable is a function of time; $z_t = t$. It has been popular to assume F to be a Heaviside function: $F(t) = 0,\ t \le t_0$; $F(t) = 1,\ t > t_0$. However, Tsurumi (1980) applied the idea of smooth transition to modelling parameter change and we already mentioned Farley et al. (1975) who assumed $F(t) = t$ as the alternative to constant parameters. Ohtani et al. (1990) also considered a model which is a variant of (1), and Varoufakis and Sapsford (1991) had a model in which F(t) changes monotonically from zero to unity over time.

The STR model with $z_t = t$ is more suitable for testing parameter constancy in dynamic linear models than representing a data generating process. Nevertheless, if parameter constancy or linearity of the model is rejected, the parameters of (1) may be estimated, and they may provide information about where in the sample the linear model seems to run into trouble and how the parameters seem to change. That is an important reason for having the STR model (1) as the alternative hypothesis to parameter constancy. However, this requires a sufficiently flexible parameterization of F(t) and that issue will be considered in the next section.

3. TESTING PARAMETER CONSTANCY

To parameterize (1) fully we consider the following cases. Either

$$F(t) = F(t, \gamma) = (1 + \exp(-\gamma (t^k + \alpha_1 t^{k-1} + \dots + \alpha_{k-1} t + \alpha_k)))^{-1}, \ k = 1, 3 \quad (2)$$

or

$$F(t) = F(t, \gamma) = 1 - \exp(-\gamma (t - \alpha)^2). \quad (3)$$

The alternative (3) in some sense corresponds to k = 2 in (2) but is more parsimonious. We shall refer to it as "k = 2" for brevity. When k = 1, the parameters γ and α_1 in (2) have a clear interpretation. The reciprocal of the latter is a parameter representing the (average) location of the parameter change. The parameter γ is a slope parameter indicating how rapid the change is. When $\gamma \to \infty$, the change becomes abrupt (a single structural break). When $\gamma \to 0$, (1) becomes linear. The change is monotonic as (2) with k = 1 is a monotonic function of t. This is illustrated in Figure 1. However, for $k = 3$ the change need no longer be monotonic in t, and rather different types of structural change can then be described by (2). If F is represented by (3) the change is nonmonotonic and symmetric about α.

For the purpose of deriving the test we modify F in (2) by replacing it by $\tilde{F}(t, \gamma) = F(t, \gamma) - 1/2$ without any loss of generality. This makes $\tilde{F}(t, 0) = 0$ so that H_0: $\gamma = 0$ in (1) becomes a natural hypothesis for parameter constancy for both the modified (2) and (3). The alternative is H_1: $\gamma > 0$ in both cases. (To identify (1) under H_1, one has to assume either $\gamma > 0$ or $\gamma < 0$.). If the null hypothesis is true, parameters π_2 and $\alpha_1, \ldots, \alpha_k$ (or α) remain unidentified. For other examples of lack of identification of similar kind see e.g. Luukkonen et al. (1988) and Granger and Teräsvirta (1993, chapter 6).

This difficulty may be circumvented by finding a suitable approximation of $\tilde{F}$ and using it as a substitute. If $k = 1, 3$, an obvious candidate is the first order Taylor approximation

$$A_1(t, \gamma) = \tilde{F}(t, 0) + \tilde{F}'(t, 0)\gamma = a_1\gamma(t^k + \alpha_1 t^{k-1} + \ldots + \alpha_{k-1}t + \alpha_k). \tag{4}$$

Substituting (4) for $F(t)$ in (1) yields

$$y_t = x_t'\pi_1 + \gamma a_1 x'\pi_2(t^k + \alpha_1 t^{k-1} + \ldots + \alpha_{k-1}t + \alpha_k) + u_t. \tag{5}$$

If $k = 2$, we similarly obtain

$$A_2(t, \gamma) = a_2\gamma(t - \alpha)^2$$

which gives an equation corresponding to (5) with k = 2. Combining terms in (5) yields a reparameterized model

$$y_t = s_t'\lambda + (s_t \otimes v_t)'\varphi + u_t \qquad (6)$$

where $s_t = (1,\ t,\ t^2,\ t^3)'$, $\lambda = (\lambda_0,\ \lambda_1,\ \lambda_2,\ \lambda_3)'$, $v_t = (y_{t-1},\dots,\ y_{t-p};\ x_{1t},\dots,\ x_{qt})'$, $(s_t \otimes v_t) = (v_t',\ tv_t',\ t^2v_t',\ t^3v_t')'$ and $\varphi = (\varphi_0',\ \varphi_1',\ \varphi_2',\ \varphi_3')'$, with $\varphi_0 = (\varphi_{0t},\dots,\ \varphi_{0p};\ \varphi_{0,p+1},\dots,\ \varphi_{0,p+q})'$. Then the null hypothesis $\gamma = 0$ becomes

$$H_o:\ \lambda_1 = \lambda_2 = \lambda_3 = 0,\quad \varphi_1 = \varphi_2 = \varphi_3 = 0. \qquad (7)$$

Note that by adopting (6) we give up information about the coefficients of (5). In return, we obtain a test statistic whose asymptotic distribution under suitable conditions is known when (7) holds. The following theorem is needed to obtain this result.

Theorem 1. *Consider model (6) when (7) holds and assume that the roots of the characteristic polynomial $\varphi_0(L) = L^p - \varphi_{01}L^{p-1} - \dots - \varphi_{op} = 0$ lie inside the unit circle, the observation vector v_t is I(0), $Ev_t = \mu$ and $cov(v_t) = C_{oo}$. Assume furthermore that $\{u_t\}$ is a martingale difference sequence with respect to an increasing sequence of σ-fields $\{\mathcal{F}_t\} = \{y_t,\ y_{t-1},\dots;\ x_{1,t-j},\dots,\ x_{q,t-j},\ j = -1,\ 0,\ 1,\dots\}$ such that $\sup_t E\{|u_t|^\alpha|\mathcal{F}_{t-1}\} < \infty$ for some $\alpha > 2$ and $\lim_{t\to\infty} E\{u_t^2|\mathcal{F}_{t-1}\} = \sigma^2$ almost surely. Assume that z_t at time t is $\mathcal{F}_{t-1}$ measurable, i.e., that $x_{1t},\dots,\ x_{qt}$ also depend on the previous observations. Then the least squares estimator*

$$\hat{\psi}_T = (\sum_{j=1}^{T} h_t h_t')^{-1} \sum_{j=1}^{T} h_t' y^{(T)}$$

of $\psi = (\lambda', \varphi')'$ *in (6) where* $\hat{\psi}_T = (\hat{\lambda}_T', \hat{\varphi}_T')'$, $h_t = (s_t', (s_t \otimes v_t)')'$, $y^{(T)} = (y_1, \ldots, y_T)'$ *is asymptotically normally distributed in the sense that* $(\sum_{t=1}^{T} h_t h_t')^{1/2} (\hat{\psi}_T - \psi) \xrightarrow{d} N(0, \sigma^2 I)$.

Proof. See the Appendix.

After establishing asymptotic normality of $\hat{\psi}_T$ it can be shown that the customary test for linear restrictions $R\psi = r$ in (6) has an asymptotic χ^2 distribution under the null hypothesis. In our case the null hypothesis is (7) and the number of the degrees of freedom in the null distribution is km. In practice we recommend the use of the F statistic. In small samples its empirical size remains close to the nominal one even in cases where the number of degrees of freedom in the numerator is large. At the same time, the power of the test remains reasonable. If the null hypothesis is large with respect to the sample size the empirical size of the χ^2 test may be far away from the nominal one.

We shall call the test statistics LM_1 LM_2 and LM_3 for k = 1, 2, 3, respectively. They are not Lagrange Multiplier statistics but are related in the sense that testing does not require the estimation of (1) under the alternative. Furthermore, the tests can be carried out by means of a simple auxiliary regression. Hence we also call them "LM type" tests.

4. MODEL SPECIFICATION

As underlined above, there is usually no reason to believe that the data have been generated by (1) if the null hypothesis is rejected. However, for reasons already given estimating the parameters of the alternative may still be worthwhile. This raises the problem of choosing F(t) among the alternatives corresponding to $k = 1, 2, 3$. As discussed in section 2, these choices allow us considerable flexibility. Normally there is no economic theory available for choosing k, and we suggest a statistical selection technique based on a short sequence of nested tests as in Teräsvirta (1993) and Granger and Teräsvirta (1993, chapter 7). The outline of the procedure is as follows. Assume $k = 3$ in (6). If (7) is tested and rejected, take (6) as the maintained model and test

$$H_{03}: \lambda_3 = 0, \varphi_3 = 0 \tag{8}$$

in (6) against its complement. If it is rejected, choose (2) with $k = 3$. If (8) is accepted, test

$$H_{02}: \lambda_2 = 0, \varphi_2 = 0 \mid \lambda_3 = 0, \varphi_3 = 0 \tag{9}$$

Rejecting (9) means choosing (3) whereas accepting it means selecting (2) with $k = 1$. Of course, even if (9) is rejected, we can still test

$$H_{01}: \lambda_1 = 0, \varphi_1 = 0 \mid \lambda_2 = \lambda_3 = 0, \varphi_2 = \varphi_3 = 0$$

but this test is our original constancy test against $k = 1$. It is advisable to carry out all these tests to obtain as clear a picture of the situation as possible through the p-values of the tests. After choosing k, the parameters of (1) with (2) or (3) can be estimated by nonlinear least squares. If $k = 3$, the nature of parameter change implied by the estimated coefficients in $\gamma(t^3 + \alpha_1 t^2 + \alpha_2 t + \alpha_3)$ is often difficult to figure out just by looking at the estimates. A graph of the estimated transition function is therefore very helpful and should accompany any estimation of the parameters of the model. The technique is demonstrated by an application in section 6.

5. MONTE CARLO EXPERIMENTS

The small sample properties of our LM type tests were considered by simulation and compared to those of some other tests. They included the well-known CUSUM and CUSUM square (CUSUMSQ) tests (Brown et al., 1975), the test against the random walk hypothesis suggested in Nyblom (1989), hereafter called the N-test, and a fluctuation test in Chu (1989). All the values for the random variables were generated using the random number generator in Gauss 2.0. The random variables sampled were all assumed normal. In generating time series from linear or nonlinear models, the first 100 observations were always discarded to avoid initialization effects. The total of 1000 replications were performed for each design.

In size simulations we found that the parameterization of the null model affected the empirical size of the test. If y_{t-1} was included and had a large positive coefficient then LM_2 and LM_3 in particular rejected too often for $T < 300$. This tendency was reversed if this coefficient was large in absolute value but negative. The empirical sizes of the other tests were also somewhat sensitive to the structure of the null model.

Our tests of course performed very well when the true alternative was a STR model, so that those results are not reported here. We focus instead upon two special cases where the change is either (a) a single or (b) a double structural break. As an example, consider the following model

$$y_t = 0.7 + 0.7\ y_{t-1} + 0.4\ x_t + u_t,\ \text{(a)}\ t < \eta T,\ \text{(b)}\ t < \eta_1 T,\ t > \eta_2 T \qquad \textbf{(10a)}$$

$$y_t = 0.5 + 0.5\ y_{t-1} + 0.2\ x_t + u_t,\ \text{(a)}\ t \geq \eta T,\ \text{(b)}\ \eta_1 T \leq t \leq \eta_2 T \qquad \textbf{(10b)}$$

$$x_t = 0.8\ x_{t-1} + \xi_t \quad t = 1, \ldots, T$$

where $(u_t\ ,\ \xi_t)' \sim NID\ (0,\ 0.1^2 I)$.

The empirical powers of the tests for T = 100 appear in Table 1. If there is a single structural break the LM type tests are the most powerful ones but the N-test also has very satisfactory power. LM_1 might be expected to be the most powerful of our tests but enlarging the alternative as implied by LM_2 and LM_3 seems to improve power in small samples. The CUSUM and the fluctuation tests are clearly less powerful than LM type tests. Typically, the CUSUM test loses power if the change occurs late and the fluctuation test if it appears early in the sample. The CUSUMSQ test has generally lower power than the other tests for reasons explained in Ploberger and Krämer (1990).

Bleaney (1990) recently investigated the power of LM_1 (he called it the "linear test") in a linear model with an intercept and another regressor. The other tests were some variants of the Chow (1960) test, and the design contained a single structural break. Not surprisingly, LM_1 was the most powerful test if the break-point assumed in applying the Chow test was not sufficiently close to the true break-point.

In the double break-point case the parameters change at $\eta_1 T$ and the second time back to their original values after $\eta_2 T$. Since the parameter change is no longer monotonic, LM_1 should be inferior to LM_2 and LM_3. The results for T = 100 are in Table 1. An interesting feature is that the power of LM_1 is very low if $\eta_1 = 1 - \eta_2$ or close to it. This is due to the properties of the coefficients ψ_{21} in (6) as functions of the coefficients in the original model (1) when k = 1; see Teräsvirta (1990) for discussion. If $\eta_2 - \eta_1$ is fairly small and both η_1 and η_2 are close to either zero or one, LM_1 recognizes only one structural break and the power improves. As expected, LM_2 and LM_3 are mostly more powerful than LM_1 in this experiment. Interestingly enough, the N-test behaves very much like LM_1. This type of a double structural break generally seems to be difficult to detect by a CUSUM or a fluctuation test, although the latter works well for $\eta_1 = 0.7$, $\eta_2 = 0.9$, i.e., when the test basically recognizes a single break late in the sample. The CUSUMSQ test again has low power throughout.

Other experiments showed that the STR alternative was flexible enough to lead to powerful tests even when the true alternative to parameter constancy was a random walk. If the "parameters" were assumed to follow a stationary AR(1) process the power of the LM type and other tests diminished with the (positive) AR coefficient while the power of the CUSUMSQ tests increased.

6. APPLICATION

In this section we shall demonstrate the use of the tests, the specification techniques and and the STR model (1). We consider the logarithmed volume index of Dutch industrial production, 1961(i) to 1986(iv), (denoted y_t) from *OECD Economic Indicators* which is graphed in Figure 2. The series is modelled using four-quarter or seasonal differences ($\Delta_4 y_t$) as in Teräsvirta and Anderson (1992) and first differences (Δy_t) with seasonal dummy variables. From Figure 2 it appears that there is at least one break in the trend which would correspond to a change in the intercept in the autoregressive model for four quarter differences. The change in seasonality should manifest itself as a change in coefficients of the seasonal dummy variables.

We begin by selecting an autoregressive model for $\Delta_4 y_t$. When this is done using AIC, an AR(1) model is chosen. Because the fourth-order lag is almost significant, we choose the AR(4) model as a basis for our tests. This gives a considerably larger alternative hypothesis than an AR(1) model would allow. The estimated AR(4) model is

$$\underset{}{\Delta_4 y_t} = \underset{(0.0038)}{0.0097} + \underset{(0.10)}{0.90}\Delta_4 y_{t-1} - \underset{(0.14)}{0.11}\Delta_4 y_{t-2} + \underset{(0.14)}{0.17}\Delta_4 y_{t-3} - \underset{(0.10)}{0.19}\Delta_4 y_{t-4} + \hat{u}_t \qquad (11)$$

$s_L = 0.0262$, LB(8-4) = 5.9 (0.21), ML(2) = 8.6 (0.013), sk = -0.027, ek = -0.26,
JB = 0.29 (0.87)

where s_L is the residual standard deviation, LB is the Ljung-Box test of no error autocorrelation, ML is the McLeod-Li test of no autoregressive conditional heteroskedasticity (ARCH) in the errors, sk is skewness, ek excess kurtosis and JB the Lomnicki-Jarque-Bera test of normality of the errors. The figures in parentheses after the test statistics are p-values.

Note that the null hypothesis of no ARCH is rejected at the 5% level of significance, a first indication that there is something wrong with (11). Instead of re-estimating the model by assuming ARCH in the errors we carry out our parameter constancy tests. The p-values of LM_1, LM_2, LM_3 and the specification tests for $\Delta_4 y_t$ are in Table 2. Parameter constancy is strongly rejected. The sequence of specification tests suggests a third-order polynomial as already H_{03} is rejected at the 5 % level of significance. A specification of (1) with $x_t = (1, \Delta_4 y_{t-1}, \Delta_4 y_{t-2}, \Delta_4 y_{t-4})'$ and (2) with k = 3, estimated by nonlinear least squares is

$$\underset{}{\Delta_4 y_t} = \underset{(0.0045)}{0.019} + \underset{(0.0091)}{0.65}\Delta_4 y_{t-1} + \underset{(0.099)}{0.27}\Delta_4 y_{t-2} - \underset{(0.055)}{0.22}\Delta_4 y_{t-4}$$

$$+ (\underset{(0.0045)}{-0.019} + \underset{(0.13)}{0.19}\Delta_4 y_{t-1} - \underset{(0.13)}{0.54}\Delta_4 y_{t-2})\hat{F}(z_t) + \hat{u}_t \qquad (12)$$

s = 0.0241, s^2/s_L^2 = 0.85, LB(4) = 2.9, ML(2) = 0.67 (0.71), sk = -0.067, ek = -0.0086, JB = 0.074 (0.96)

with

$$\hat{F}(t) = (1 + \exp\{-\underset{(690)}{240}(t^{*3} - \underset{(3.3)}{8.5}t^{*2} + \underset{(4.6)}{10.28}t^{*} - \underset{(1.5)}{3.2})\})^{-1}$$

where $t^* = t/100$, s is the residual standard deviation.

The null of no ARCH in the errors cannot be rejected. The large standard deviation of the estimate of γ in F does not indicate insignificance. It is due to the fact that there is a wide range of values of γ yielding almost the same $\hat{F}$; see e.g. Bates and Watts (1988, p. 87) or Teräsvirta (1993) for discussion. Restriction $\pi_{10} = -\pi_{20}$ is supported by the data. The transition function is graphed in Figure 3. It shows two sharp structural breaks, the first one in 1975 where the function rapidly grows from zero to unity and the second one, signifying the return to the first regime, in 1984. The first regime (F = 0) is characterized with a positive intercept (a positive trend in levels) and a complex pair of roots in the characteristic polynomial with modulus 0.79 and period 18 quarters. The second regime (F = 1) has a zero intercept (no trend) and the corresponding complex pair of roots has modulus 0.86 and period 10 quarters. That means much shorter cycles than those in the first regime. All of this can be compared with the visual information in Figure 2.

Note that (12) does not give any indication about changing seasonality. This is due to the fact that by modelling the series using seasonal differences one can at least in some situations adequately describe changing seasonality with constant parameters. If the series is modelled using first differences, AIC selects an AR(4) model with seasonal dummies. The test results are found in Table 2 and show an overwhelming rejection of the null hypothesis of parameter constancy. The specification search again yields k = 3. Note, however, that if the choice is strictly between k = 1 and k = 2 then the former alternative is not rejected at the 5% level of significance. This suggests that F may be monotonic.

The estimated STR model is

$$\Delta y_t = \underset{(0.0059)}{0.14} - \underset{(0.0088)}{0.20}\, d_{1t} - \underset{(0.0095)}{0.088}\, d_{2t} - \underset{(0.0069)}{0.19}\, d_{3t} + (\underset{0.028)}{0.074} + \underset{(0.10)}{0.22}\, \Delta y_{t-2}$$

$$+ \underset{(0.10)}{0.22}\, \Delta y_{t-4} - \underset{(0.043)}{0.14}\, d_{1t} - \underset{(0.052)}{0.25}\, d_{2t} + \underset{(0.025)}{0.027}\, d_{3t})\hat{F}(t) + \hat{u}_t \qquad (13)$$

where d_{1t}, d_{2t}, d_{3t} are the seasonal dummy variables, $s = 0.0216$, $s^2/s_L^2 = 0.69$, LB(4) = 2.4, ML(2) = 4.4 (0.11), sk = 0.067, ek = 0.19, JB = 0.23 (0.89)

and

$$\hat{F}(t) = (1 + \exp\{-\underset{(54)}{147}(t^{*3} - \underset{(0.053)}{1.4}t^{*2} + \underset{(0.053)}{0.65}t^{*} - \underset{(0.015)}{0.11})\})^{-1}.$$

The coefficient estimates in (13) indicate that the seasonality becomes more pronounced as F grows from zero to unity. In Figure 2 that is seen to happen gradually over time. The graph of $\hat{F}$ in Figure 4 agrees with this impression. The change is monotonic, slows temporarily down in early 1970s and is over by 1980. The first differences and seasonal dummies representation appears far too rigid a model for Dutch industrial production. Additional examples of applying the tests and estimating the alternative after rejecting parameter constancy can be found in Rahiala and Teräsvirta (1993).

7. CONCLUSIONS

Although the example amply demonstrates the potential of the present approach it has to be stressed that the nonlinear least squares estimation of the alternative may often cause problems. Local minima are likely and good starting-values are therefore an important prerequisite to successful parameter estimation. Sometimes an adequate model may not be found at all but at any rate the tests will have told the investigator that the original linear model is misspecified. In our opinion, the tests against smooth structural change based on the STR model are a useful addition to the econometrician's toolkit. They represent a reinterpretation of the test Farley et al. (1975) suggested. Such a reinterpretation is useful in the sense that successful estimation of the alternative when the null hypothesis is rejected does provide information about where in the sample and how the assumption of constancy breaks down. This information may often be valuable in respecifying constant parameter models that did not pass these tests.

APPENDIX. Proof of Theorem 1

Proof. We prove Theorem 1 for $k = 3$ and apply Theorem 3 in Lai and Wei (1981). Therefore, we have to show that there exists a nonrandom positive definite symmetric matrix B_T such that $B_T^{-1}(\sum_{t=1}^{T} h_t h_t')^{1/2} \xrightarrow{p} I$ and $\max_{1\le t<T} \| B_T^{-1} h_t \| \xrightarrow{p} 0$ ($\xrightarrow{p}$ means convergence in probability). As $v_t \sim I(0)$,

$$T^{-(i+j+1/2)} \sum_{t=1}^{T} t^{i+j}(v_t - \mu) \Rightarrow \int_0^1 r^{i+j} \, dB(r) \tag{A.1}$$

where $B(r)$ is a $(p + q)$ - dimensional Brownian motion and $\Rightarrow$ denotes weak convergence of the probability measures; see e.g. Sims et al.(1990).

From (A.1) it follows

$$T^{-(i+j+1)} \sum_{t=1}^{T} t^{i+j}(v_t - \mu) \xrightarrow{p} 0 \tag{A.2}$$

$$T^{-(i+j+1)} \sum_{t=1}^{T} t^{i+j}(v_t - \mu)(v_t - \mu)' \xrightarrow{p} C_{ij} \tag{A.3}$$

Furthermore,

$$T^{-(i+j+1)} \sum_{t=1}^{T} t^{i+j} \xrightarrow{p} (i + j + 1)^{-1} \tag{A.4}$$

(Sims et al. , 1990). From (A.2) and (A.4) it follows that

$$T^{-(i+j+1)} \sum_{t=1}^{T} t^{i+j} v_t \xrightarrow{p} (i + j + 1)^{-1}\mu. \tag{A.5}$$

Combining (A.3), (A.4), and (A.5) yields

$$T^{-(i+j+1)}\sum_{t=1}^{T} t^{i+j} v_t v_t' \xrightarrow{p} C_{ij} - (i + j + 1)^{-1}\,\mu\mu' = M_{ij}.$$

Let

$$M = \begin{bmatrix} M_{00} & M_{01} & M_{02} & M_{03} \\ & M_{11} & M_{12} & M_{13} \\ & & M_{22} & M_{23} \\ & & & M_{33} \end{bmatrix}, \qquad U = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 \\ & 1/3 & 1/4 & 1/5 \\ & & 1/5 & 1/6 \\ & & & 1/7 \end{bmatrix},$$

$$D = \begin{bmatrix} U & (U \otimes \mu') \\ (U \otimes \mu) & M \end{bmatrix},$$

$\tilde{T}_4 = \text{diag}(T^{1/2}, T^{3/2}, T^{5/2}, T^{7/2})$ and $\tilde{T} = \text{diag}(\tilde{T}_4, (\tilde{T}_4 \otimes I_{m-1}))$. Then $\tilde{T}^{-1}(\sum_{t=1}^{T} h_t h_t')\tilde{T}^{-1} \xrightarrow{p} D$, where D is positive definite because $\tilde{T}^{-1}(\sum_{t=1}^{T} h_t h_t')\tilde{T}^{-1}$ is positive definite for all T. Let $B_T = (\tilde{T}D\tilde{T})^{1/2}$ (positive definite). Then $B_T^{-1}(\sum_{t=1}^{T} h_t h_t')^{1/2} \xrightarrow{p} I_d$. Furthermore, $\|B_T^{-1}h_t\|^2 = h_t'\tilde{T}^{-1}D^{-1}\tilde{T}^{-1}h_t = T^{-1}O_p(1) = O_p(T^{-1})$, because $\tilde{T}^{-1}h_t = T^{-1/2}(\tilde{s}_t', (\tilde{s}_t \otimes z_t)')'$ where $\tilde{s}_t = (1, t/T, (t/T)^2, (t/T)^3)'$. Thus $\max_{1\le t\le T}\|B_T^{-1}h_t\| \xrightarrow{p} 0.$ □

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Table 1. The empirical power of several parameter constancy tests against a single and a double structural break based on 1000 replications generated from model (10) when the nominal significance level is 0.05 and the sample size T = 100.

Test	η 0.1	0.3	0.5	0.7	0.9	η_1 / η_2	0.1 / 0.3	0.1 / 0.5	0.1 / 0.7	0.1 / 0.9	0.3 / 0.7	0.3 / 0.9	0.5 / 0.9	0.7 / 0.9
LM_1	0.470	0.791	0.823	0.763	0.400		0.443	0.464	0.236	0.149	0.053	0.167	0.262	0.326
LM_2	0.499	0.822	0.864	0.796	0.459		0.359	0.524	0.723	0.706	0.640	0.715	0.520	0.292
LM_3	0.531	0.777	0.828	0.792	0.516		0.419	0.770	0.837	0.636	0.613	0.757	0.761	0.421
FLUCTU-ATION	0.008	0.033	0.468	0.830	0.523		0.029	0.099	0.308	0.122	0.049	0.031	0.287	0.524
CUSUM	0.269	0.453	0.399	0.249	0.076		0.124	0.156	0.181	0.221	0.323	0.398	0.333	0.151
CUSUMSQ	0.100	0.062	0.116	0.266	0.246		0.074	0.052	0.093	0.087	0.061	0.063	0.129	0.229
N-test	0.208	0.763	0.793	0.699	0.292		0.334	0.507	0.451	0.102	0.079	0.222	0.350	0.288

Table 2. p-values of LM_1, LM_2, LM_3 and model specification tests based on the AR(4) models for the four-quarter differences and the AR(4) model with seasonal dummies for the first differences, respectively, of the logarithmed index of Dutch industrial production.

Department Variable	Test			Specification Hypothesis	
	LM_1	LM_2	LM_3	H_{03}	H_{02}
$\Delta_4 y_t$	0.031	0.0076	0.0010	0.018	0.039
Δy_t	5×10^{-4}	2×10^{-4}	8×10^{-6}	0.0016	0.058

Figure 1. The graph of two transition functions for k = 1 in the smooth transition regression model (1), transition speed γ = 0.1 (---), γ = 1.0 (—); location parameter α_1 = -50, sample size T = 100.

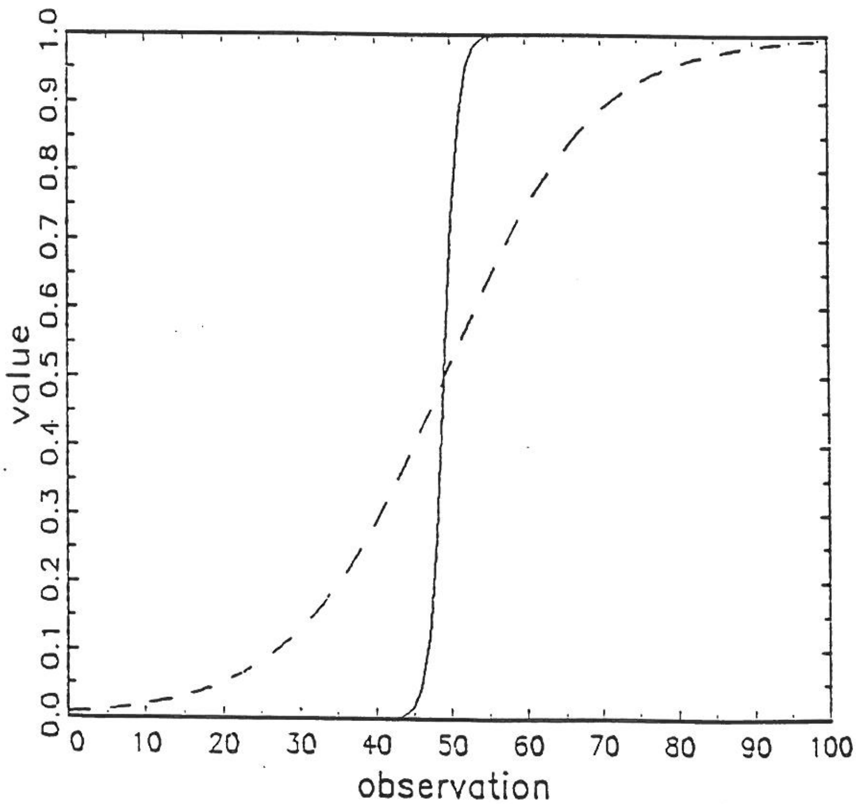


Figure 2. The logarithmic values of the quarterly seasonally unadjusted index of Dutch industrial production, 1960(i) to 1986(iv).

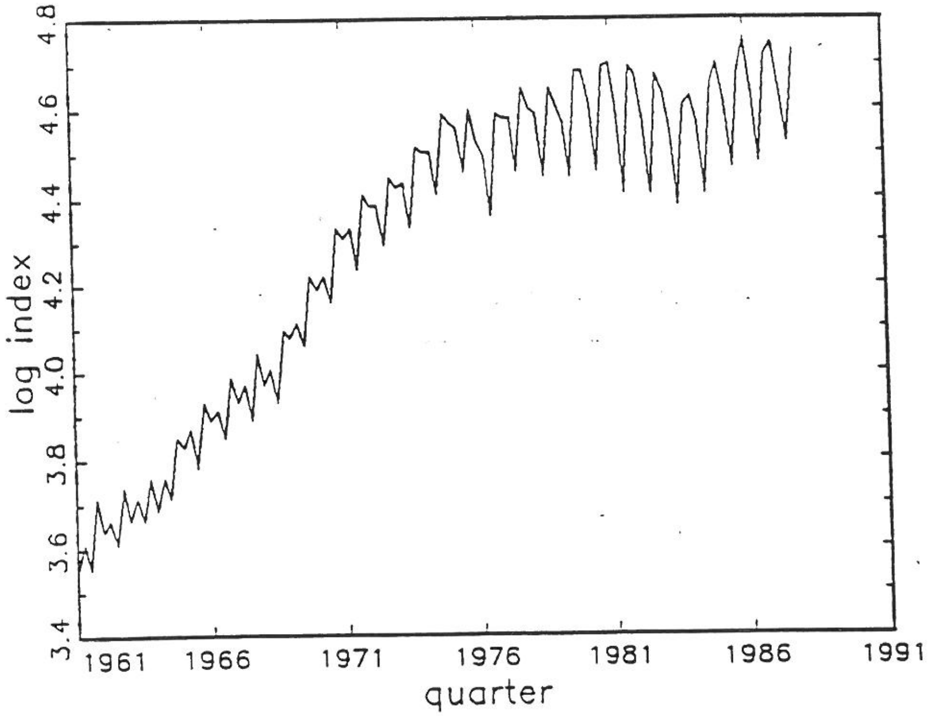


Figure 3. The graph of the transition function in the estimated STR model (12) for the quarterly four-quarter differences of the logarithmed volume index of Dutch industrial production, 1962(i) to 1986(iv)

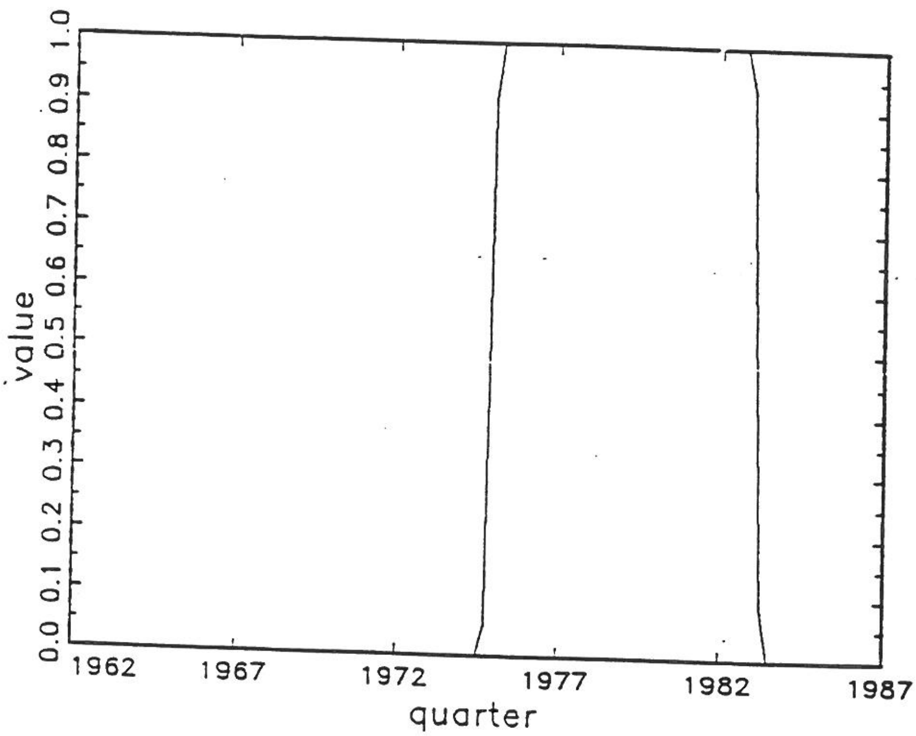
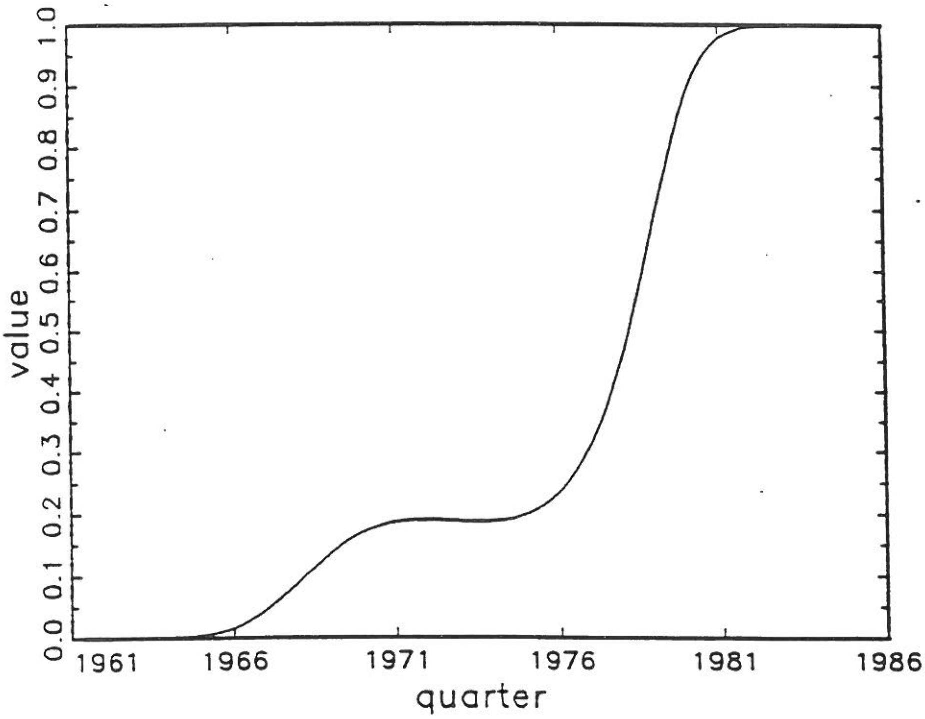


Figure 4. The graph of the transition function in the estimated STR model (13) for the first differences of the logarithmed volume index of Dutch industrial production, 1961(ii) to 1986(iv)



SAMMANFATTNING

Antagandet om att parametrarna i en lineär modell är konstanta är väsentligt i all statistisk inferens och tillämpas också när man gör prognoser med hjälp av sådana modeller. Därför är det viktigt att antagandet också kan testas. Många olika tester för detta ändamål existerar i litteraturen, och ett stort antal av dem är tester mot den situationen, att parametrarna i modellen förändras precis en gång. Detta kallas ett strukturbrott. Det finns också tester i vilka mothypotesen till konstanta parametrar är, att parametrarna är stokastiska. Testerna i denna uppsats skiljer sig från de flesta i att mothypotesten tillåter en parametervektor, som kan förändras kontinuerligt och deterministiskt i tiden. Den alternativa modellen är fullt parametriserad, vilket inte är helt vanligt heller. Fördelen med ett parametriserat alternativ till konstanta parametrar är, att om man förkastar nollhypotesen, kan man fortsätta med att skatta alternativet. Den vägen utvinner man information om var i urvalet antagandet om konstanta parametrar bryter samman och om huruvida detta sker mycket snabbt (strukturbrott) eller mera långsamt i tiden. Sådan information kan hjälpa modellbyggaren i att förbättra specifikationen av sin modell. I uppsatsen härleder man teststorheterna för tre olika alternativa parametriseringar och bevisar, att de under några ganska sedvanliga antaganden har en välkänd asymptotisk nollfördelning, nämligen en χ^2-fördelning. I praktiken bör man dock undvika χ^2-testet och helst använda det motsvarande F-testet, eftersom det senare testet i små urval brukar ha bättre statistiska egenskaper än det förra. Simuleringarna i uppsatsen visar, att F-testet faktiskt fungerar bra och oftast har bättre styrka än vissa andra tester, som förekommit i litteraturen och blivit tillämpade i praktiken. Uppsatsen innehåller också ett praktiskt exempel, där testerna tillämpas på två autoregressiva modeller för en kvartalsserie med ca. 100 observationer av den holländska industriproduktionen. Nollhypotesen om konstanta parametrar blir förkastad, den alternativa modellen skattas och resultaten diskuteras.

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