CURRENT ACCOUNT AND BUSINESS CYCLES: STYLIZED FACTS FOR SWEDEN

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ABSTRACT

The primary purpose of the paper is to establish some "stylized facts" about current account and business cycles in Sweden. Tests for the existence of stochastic trends in the relevant time series are performed, and the descriptive statistics are confined to the stationary ("transitory") components of the series.

The tests suggest that investments and output contain stochastic trends, whereas the current account does not. This suggests that savings and investments are cointegrated, but the evidence on this issue is mixed. It is also suggested that the decrease in the volatility of Swedish output noted by Sheffrin may be due to lower volatility in the changes of both the permanent and transitory components. However, the level of the transitory component of output, which is a natural measure of "the business cycle", seems as volatile as ever. The current account is positively correlated with (lagged) values of this component and thus appears to be "pro-cyclical".

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1. Introduction.

Recent discussions in business cycle theory have been associated with a renewed interest in descriptions of business cycle phenomena. In particular, the "stochastic trend" feature of many macroeconomic time series pointed out by Beveridge and Nelson (1981), Nelson and Plosser (1982) and Perron (1988) suggests that fluctuations in aggregate economic activity may reflect variations which are of a permanent, rather than transitory, nature. Accounting for the variations in the trend(s) is thus important, e.g., when "stylized facts" about business cycle phenomena are compared with the properties of theoretical models (see, e.g., King, Plosser and Rebelo (1988a,b)).

The primary purpose of this paper is to establish some stylized facts about current account and business cycles in Sweden. This focus of our attention can be justified on several grounds. First, the current account is frequently – rightly or wrongly – used as an information variable or intermediate target for macroeconomic policy, especially in small open economies. To identify and interpret the nature of current account movements accordingly is an important task for empirical research.

Second, the Swedish business cycle deserves special attention, given the finding by Sheffrin (1988) that Sweden, as opposed to other countries, has experienced a substantial reduction in the severity of the business cycle following World War II. It is an open question, however, whether the dampening of aggregate fluctuations is due to lower variation in the long run trend or in the short run cycle (or both).

Third, in order to interpret and evaluate empirical evidence on current account and business cycles, the stochastic properties of the relevant data must be carefully examined. For example, intertemporal optimization theory teaches that private savings and investments (and, hence, the current account) will respond differently to transitory and permanent changes in economic conditions. Economic theory thus provides a motivation for particular attention to stochastic trends in empirical studies of the current account.

Furthermore, standard techniques for statistical analysis presumes that data are stationary. If data contain (stochastic or deterministic) trends, the series should either be appropriately transformed into stationary series, or non–standard techniques will have to be used. Stock and Watson (1988b) discuss this matter at a general level, and some examples from empirical literature related to our study may be used to illustrate its relevance in this context:

– Studies of a number of countries (using various detrending techniques) suggest that investments are highly volatile (in relation to output and consumption) and pro–cyclical, and that the current account (or the trade balance) is counter–cyclical; see, e.g., Englund and Vredin (1989), Söderlind (1989), Danthine and Girardin (1989), and Backus and Kehoe (1988). It has been noted that the latter finding is surprising, since intertemporal optimization models predict that transitory increases in income should have a rather strong *positive* effect on savings. The current account may still be counter–cyclical, of course, if investments are sufficiently pro–cyclical. Discussions of such issues obviously have to be based on explicit assumptions (and, ideally, tests) as to the stochastic properties of trends and cycles.

– Conditioning on particular operationalizations of the growth and business cycle phenomena, Sachs (1981) has argued that international differences in current account experiences after the oil crises in the 1970's can be explained by the behavior of investments, rather than savings. Feldstein and Horioka (1980), on the other hand, have argued that savings and investments are strongly correlated, which they interpret as reflecting a causal link from savings to investments, and as evidence of low international capital mobility. If savings and investments are driven by stochastic trends, these interpretations are open to question; since the standard inference techniques are invalid, it remains to be shown that the estimated correlations are not spurious.

– Rather than being entirely spurious, the correlation between savings and investments may reflect "cointegration" in the sense of Engle and Granger (1987), i.e. a common stochastic trend. This argument has received some empirical support in a study on

U.S. data by Miller (1988). If savings and investments have a common trend, these aggregates will be perfectly correlated in a *long run* sense, although the transitory fluctuations around the trend may be rather weakly correlated and quite consistent with high capital mobility and current account "imbalances" in the *short run*.

– A failure to distinguish between long and short run co–movements may be the reason behind the difficulties to find significant relations between the external balance and various "fundamentals" (see Englund and Vredin (1987), Backus (1986) and Rose and Yellen (1987)). In particular, if the *difference* between savings and investments does not have a trend (which requires these aggregates to have a common trend), a regression of the current account against any single non–stationary variable will asymptotically yield a regression coefficient of zero. In a multivariate analysis the regression coefficients cannot be interpreted in a meaningful way unless the explanatory variables are cointegrated. Tests for stochastic trends and cointegration may thus provide useful information for the specification of econometric models of the current account.[1]

The paper is organized as follows. In section 2 we discuss time series models with stochastic trends. In section 3 we examine the empirical relevance of stochastic trends against a background of univariate tests on more than hundred years of Swedish data. We also discuss the implications of stochastic trends for estimated correlations between savings, investments and the current account. In section 4 we describe empirical estimates of the transitory components of savings, investments, and some other business cycle indicators. Tests for common trends are presented in section 5. Section 6 contains some concluding comments which relate our findings to those of earlier studies. The paper also contains two appendices, giving a detailed presentation of the methods used in sections 3 – 5.

[1] Englund and Vredin (1987) test (on Swedish data) whether the current account, terms of trade, government consumption and the real wage have stochastic trends, but not whether these variables are cointegrated. Rose and Yellen (1987), studying US data, report that net exports, the real exchange rate, and domestic and foreign income are not cointegrated, but no detailed account of their tests is offered.

2. Stochastic trends and business cycles: representation and testing.

In this section we will provide the general analytical framework for our time series analyses of business cycles. The particular estimation procedures and tests used in subsequent sections are described in Appendix A and B.

Suppose that a single time series variable x_t may be decomposed into a non-stationary ("growth") part μ_t, and a stationary process φ_t comprising "irregular" and, possibly, "cyclical" parts, i.e., $x_t = \mu_t + \varphi_t$. If μ_t is given by a linear deterministic trend, $\mu_t = \beta t$, we have a "trend stationary" (TS) model. (The terminology is adapted from Nelson and Plosser (1982)). If, on the other hand, we assume that the trend component is stochastic, e.g.,

(1) $$\mu_t = \mu_{t-1} + \eta_t,$$

where the innovation to growth, η, is assumed to be a stationary stochastic process, we arrive at the "difference stationary" (DS) model

(2) $$x_t - x_{t-1} = \nu_t,$$

where $\nu_t = \varphi_t - \varphi_{t-1} + \eta_t$.

To determine whether a specific time series follows a deterministic or stochastic trend is important for both statistical and economic reasons. According to the TS model x has finite variance, whereas the DS model implies that there is no bound on the uncertainty about future values of x. In analogy, the innovations to x in the TS model have only temporary effects, whereas the effects of ν are permanent according to the DS model. These properties of the DS model are most easily seen if it is rewritten on the form

(3) $$x_t = \Sigma_1^t \nu_t = \Sigma_1^t \eta_t + \varphi_t$$

where it is assumed that $x_t = \varphi_t = \nu_t = 0$ for $t \leq 0$. The TS model suggests that an unexpected change in x is due to cyclical and/or irregular disturbances, whereas the DS model implies that it also reflects disturbances to the stochastic trend.

Univariate tests for stochastic trends have been developed by Dickey and Fuller (1979). A test may e.g. be based on a "t–test" on the first–order autoregressive term in a model which nests the TS and DS models, such as

$$(4) \qquad x_t = \beta t + \rho_1 x_{t-1} + \epsilon_t,$$

but the distribution for the test statistic is non–standard under the null hypothesis $\rho_1 = 1$. Applying the test procedures proposed by Dickey and Fuller, Nelson and Plosser (1982) have shown that for many macroeconomic time series the DS model cannot be rejected.

In general, standard distributions are inapplicable if a hypothesis involving non-stationary variables is to be tested; cf. Granger and Newbold (1974) and Phillips (1986). In order to avoid spurious correlations many empirical studies are based on detrended data. For example, univariate time series analyses in the tradition of Box and Jenkins usually proceed on the assumption that data are difference stationary. In a multivariate setting, however, certain linear combinations of non–stationary series may be stationary. Evidence of such relations may then be used in the empirical "identification" of time series models.

If a variable is stationary in first differences (and fulfils certain other conditions), Engle and Granger (1987) define it as integrated of order 1, denoted I(1), and if some linear combinations of I(1) variables are I(0), i.e. stationary in levels, the variables are said to be cointegrated of order 1,1, denoted CI(1,1). Let the nx1 vector X denote a set of variables which are all I(1), and let X be given by the vector moving average (VMA), or vector DS, model

$$(5) \qquad \Delta X_t = \delta + C(L)\epsilon_t,$$

where L is the lag operator and Δ ($\equiv 1 - L$) is the difference operator.[2] δ is a vector of constants, C(L) an nxn matrix lag polynomial, and ϵ an nx1 vector of white noise disturbances with zero mean and contemporaneous covariance matrix Σ. Let $Z = \alpha'X$ denote an rx1 vector of stationary linear combinations of X. The columns of the nxr matrix α are called the cointegrating vectors. It can be shown that the stationarity of Z implies, i.a., that $\alpha'C(1) = 0$, i.e., that C(1) is not of full rank.

Stock and Watson (1988a) show that cointegration also implies that the VMA model may be rewritten as a common trends (CT) model, given by

(6) $$X_t = X_0 + A\tau_t + C^*(L)\epsilon_t,$$

where $C^*(L)$ is defined according to

(7) $$C(L) = C(1) + (1 - L)C^*(L),$$

and the $k = n - r$ common trends are given by the kx1 vector τ_t,

(8) $$\tau_t = \kappa + \tau_{t-1} + v_t,$$

where v is a vector of white noise processes. By comparing (5) and (8) with (2) and (1), respectively, the VMA and CT models are easily seen to be generalizations of the univariate DS model. (The exact relations between (A, κ, τ, v) and (C(1), δ, ϵ) are specified by Stock and Watson (1988a) and King, Plosser, Stock and Watson (1987). See also Warne (1989).)

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[2] Since ΔX is assumed to be stationary with mean δ, we know from a multivariate version of Wold's theorem (cf. Hannan (1970)) that a VMA representation of the form specified in (5) exists. For our purposes, this result cannot be applied in its most general form, as we need more restrictions on C(L), thereby reducing the set of admissible stochastic processes (see also appendix A).

The CT representation (6) seems to be a natural starting point for empirical business cycle analysis. The "business cycle" may thus be identified with $C^*(L)\epsilon_t$, while $A\tau_t$ captures the "growth cycle". The CT model is a multivariate generalization of the Beveridge–Nelson (1981) decomposition of a single time series into its permanent and transitory components. It should be pointed out that the transitory and permanent components are not, in general, uncorrelated, since ϵ and v are not independent. Other (univariate) procedures for making the business cycle concept operational are discussed in Stock and Watson (1988b).

There are various ways to arrive at estimates of ϵ and C(L). King, Plosser, Stock, and Watson (1987) exploit the vector error–correction representation (VEC) of (5) derived in Engle and Granger (1987):

$$\Delta X_t = \theta + B(L)\Delta X_{t-1} - \gamma Z_{t-1} + \epsilon_t. \tag{9}$$

This specification requires some prior knowledge of the dimension and elements of the cointegrating vectors in α, since $Z_t = \alpha' X_t$. Tests for cointegration (common trends) have been suggested by, e.g., Engle and Granger (1987) and Stock and Watson (1988a). These tests may be described as generalizations of the Dickey–Fuller test to bivariate and multivariate cases, respectively. Engle and Granger suggest a two–step procedure where the cointegrating vector is first estimated by a regression of one variable against the other, whereafter a Dickey–Fuller test may be used to examine whether the residual has a stochastic trend. The test for common trends developed by Stock and Watson is a test of the hypothesis that the nx1 vector X contains k vs n–m common trends, and is based on an examination of the (n–m+1)st largest root of the first–order autocorrelation matrix of a transformation of X (derived under the null hypothesis of k common trends). A detailed account of the Stock–Watson test is given in Appendix B.

In the subsequent sections of this paper we will first, in section 3, investigate whether our time series are all I(1) (difference stationary, stochastic trends), i.e., whether

(5) is a valid representation of our data. In section 4 we then describe our estimates of the transitory component(s) $C^*(L)\epsilon_t$. These are derived by first estimating ϵ from a vector autoregressive moving-average representation,

$$A(L)X_t = \theta + \epsilon_t, \tag{10}$$

which may be derived from (5) (see Engle and Granger (1987) or Warne (1989)). The second step will be to estimate the permanent and transitory component of each series from a multivariate generalization of (3). (The details are given in Appendix A.) In these estimations, no cross-equation restrictions will be imposed. However, the idea of some, possibly a few, common trends seems natural in this context. Hence, tests for cointegration are presented in section 5.

3. Stochastic trends: univariate tests and implications.

In table 1 we report the results of univariate tests for stochastic trends on data from 1876 – 1986. (Initial values are provided by observations from 1871 – 1875.) The time series are national accounts data on real savings (S), investments (I), the current account (CA), and gross domestic product (Y). The original nominal series have all been deflated by the GDP price index (P). Savings are defined from the identity $S \equiv CA + I$.[3] Data on Y, I and P from 1970 – 1986 have been taken from the official national accounts publications, while the figures for the earlier period are from Krantz and Nilsson (1975) (who in turn draw on Johansson (1967)). It should be noted that Y and I, due to lack of historical data on stocks, both are defined exclusive of changes in inventories. Current account data until 1966 are from Ohlsson (1969), and for the later period from national accounts statistics. The export

[3] Note that savings S thus include net transfers from abroad whereas the national income measure Y does not.

Table 1

Tests for stochastic trends

Series	$q_f^{\tau 2}$	q_f^{τ}	q_f^{μ}	τ^{τ}	τ^{μ}
ln Y	–17.56	–12.02	.12	–2.74	.33
Δln Y	–	–97.36***	–96.77***	–4.95***	–4.93***
ln I	–14.83	–15.40	–.06	–3.02	–.06
Δln I	–	–98.45***	–98.57***	–5.81***	–5.84***
ln S	–26.93*	–25.00**	–.68	–3.35*	–.62
Δln S	–	–116.23***	–116.23***	–5.47***	–5.50***
ln p	–25.42*	–12.22	–8.94	–1.97	–2.06
Δln p	–	–103.73***	–102.73***	–5.77***	–5.70***
ln P	–12.38	–3.62	2.28	–.98	1.91
Δln P	–	–45.59***	–38.53***	–4.60***	–3.86***
I/Y	–10.46	–11.09	–1.87	–1.95	–.87
Δ(I/Y)	–	–102.66***	–102.43***	–5.44***	–5.46***
S/Y	–24.79*	–20.94*	–3.93	–2.66	–1.39
Δ(S/Y)	–	–101.33***	–101.27***	–6.20***	–6.21***
CA	–62.83***	–52.09***	–49.82***	–3.23*	–3.03**
ΔCA	–	–135.46***	–135.69***	–5.83***	–5.88***
CA/Y	–46.21***	–35.78***	–31.86***	–3.40**	–3.27**
Δ(CA/Y)	–	–107.40***	–107.45***	–6.43***	–6.45***

Notes: Significant at the *** 1% ** 5% * 10% level. All statistics are based on regressions with 5 lags. The tests are explained in the text.

and import price indices necessary to compute the terms of trade (p) are also from the latter two sources.[4]

The last two columns of table 1 report Dickey–Fuller tests of the hypothesis that ρ_1, in different versions of (4), is unity. The test statistics are equal to the ratio between $(\hat{\rho}_1-1)$ and the estimated standard error of $\hat{\rho}_1$, i.e. conventional "t–statistics". The distributions for these statistics, under the null hypothesis of a stochastic trend, are however non–standard, but relevant tables are provided by Fuller (1976). In the regressions of (4), five lags of x have been included to allow for serial correlation. An intercept has also been included. The τ^{τ}–statistic refers to a regression which includes a deterministic trend, whereas the τ^{μ}–statistic is calculated from a regression where $\beta = 0$. Similarly, the univariate Stock–Watson tests in the second and third columns (i.e. q_f^{τ} and q_f^{μ}) are based on regressions where a linear trend is included or excluded, respectively. The $q_f^{\tau^2}$–statistic in column 1 even allows for a quadratic deterministic trend in x. The test statistics have been calculated using the "Stockwat" procedure supplied with RATS, version 3.01. (The Stock–Watson test is described in Appendix B.)

The hypothesis of a stochastic trend cannot be rejected for the logarithms of output, investments, and the price level (lnY, lnI, and lnP, respectively). In the case of terms of trade (lnp), the Stock–Watson test rejects the null at the ten per cent significance level if a quadratic deterministic trend is allowed for. There is less evidence of a stochastic trend in savings (lnS): if no restriction on the appearance of a linear (or quadratic) deterministic trend is imposed, the null hypothesis is rejected, albeit only at the ten per cent significance level according to the Dickey–Fuller (τ^{τ}) test. In terms of GDP shares, the hypothesis of a stochastic trend cannot be safely rejected either for investments (I/Y) or savings (S/Y), although the Stock–Watson tests suggest rejection in the latter case, at the ten per cent significance level, if a linear or quadratic deterministic trend is included in the regression.

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[4] The data base has been prepared by Judit Weibull, Institute for International Economic Studies.

Finally, the stochastic trend hypothesis is consistently rejected in the case of the current account (CA and CA/Y).

There is generally stronger evidence against stochastic trends in differenced data than in levels. Insofar as the level was found to have a stochastic trend in the first place, the time series thus appear to be I(1). These results are by and large in accordance with international evidence. Tests for stochastic trends in U.S. data on, i.a., the price level and output have been reported by, e.g., Nelson and Plosser (1982) and Perron (1988); King, Plosser, Stock and Watson (1987) also report tests on investments data; and Miller (1988) has reported tests on savings and investments. We are aware of no other test on terms of trade than that on Swedish data reported by Englund and Vredin (1987).

To see how the evidence on stochastic trends may be used to interpret and evaluate econometric evidence on correlations between savings and investments over the business cycle, consider the following regression equations:

$$(11) \qquad (I/Y)_t = a_0 + a_1(S/Y)_t + e_{1,t}$$

$$(12) \qquad (CA/Y)_t = b_0 + b_1 t + b_2(I/Y)_t + b_3 \ln\hat{Y}_t + e_{2,t}$$

where $\ln\hat{Y}$ is the deviation of $\ln Y$ from a linear trend. The first regression is in the spirit of Feldstein and Horioka (1980), although their study is based on cross-country data rather than time series. The second regression has been estimated on time series data from several countries (although not for Sweden) by Sachs (1981).

When (11) and (12) are fitted to the same Swedish data which were subject to tests in table 1, we obtain the following results:

$$(13) \qquad (I/Y)_t = .047 + .724(S/Y)_t$$
$$R^2 = .777 \qquad DW = .56$$

$$(14) \qquad (CA/Y)_t = .388 - .004t - .260(I/Y)_t - .041\ln\hat{Y}_t$$
$$R^2 = .151 \qquad DW = .63$$

The results are similar to those obtained for other countries and time periods. $\hat{a}_1$ is close to unity, which Feldstein and Horioka (1980) – in their cross–country framework – interpret as evidence of low capital mobility; and $\hat{b}_2$ is negative, a finding which Sachs (1981) uses to support his argument that current account movements are due to changes in investments.[5] Furthermore, $\hat{b}_3$ is negative, which may be interpreted as evidence of a counter–cyclical pattern in current account movements. These interpretations are, of course, valid only insofar as the (implicit) structural assumptions are valid, and it should be emphasized that the different interpretations are mutually exclusive. Feldstein and Horioka (1980) view savings as exogenous with respect to investments, whereas the opposite assumption is made by Sachs (1981). We will not discuss the estimated parameters in terms of structural models, however, but merely view them as convenient summary statistics on various partial correlations.

Nevertheless, the tests reported in table 1 call for some caution. The tests suggest that S/Y and I/Y may be driven by stochastic trends. The strong correlation between savings and investments in terms of output shares may thus be spurious. On the other hand, if I/Y and S/Y are cointegrated, the (normalized) cointegrating vector should be close to $(1, -\hat{a}_1)$, since any other linear combination of I/Y and S/Y than the true cointegrating relation would imply an infinite variance of the residual and, hence, would not be selected in a least squares estimation procedure (cf. Stock (1987)). In any case, the distribution for a "t–test" of $\hat{a}_1$ will be non–standard, which makes it hard to draw inferences about the true correlation between savings and investments (and the current account), e.g., in comparisons of stylized facts across different countries or time periods. Similar caveats apply to the second regression, (14). First, detrending of data through the inclusion of a linear deterministic trend receives no support from the univariate tests in table 1. Second, unless I/Y and $\ln\tilde{Y}$ are cointegrated, no parameters from (14) can be evaluated in terms of standard significance tests (see Stock and Watson (1988b) for a more detailed discussion of

[5] The regression coefficients in (13) – (14) are deliberately reported without any estimates of standard errors, by reasons provided in the next paragraph.

such issues). Third, the failure to reject the stochastic trend hypothesis in the case of $\ln Y$, irrespective of whether a deterministic trend is included or not, suggests that $\ln \bar{Y}$ is not a good measure of the business cycle.

It might be argued that empirical analyses of first differences, which appear to be stationary, would make the regression results more meaningful. This approach is followed by Obstfeld (1986), who reports that the values on a_1 differ substantially between countries, and that the correlation between savings and investments appears to decrease over time. However, there are reasons not to work simply with differenced version of (11) and (12). If the levels (or GDP shares) data contain stochastic trends, changes are not the most natural measure of the business cycle, since (unexpected) changes are not necessarily due to movements in the transitory component rather than the "growth cycle". Furthermore, if there are common trends, i.e., cointegration, a regression equation in first differences will be misspecified (this is evident from the error–correction representation (9), as cointegration implies that the n x r matrix γ is non–zero). There are thus both economic and statistical reasons for taking a closer look at how changes are related to fluctuations in the permanent and transitory components of the time series.

4. Stylized facts on current account and business cycles.

In section 2 we argued that the common trends representation (6) is a natural starting point for empirical analysis of business cycle phenomena, since it suggests how a vector time series may be decomposed into one stationary "business cycle" component and one non–stationary "growth cycle" component. In this section we will present estimates of the transitory and permanent components of output, and of the correlation between the current account and various business cycle indicators. The estimation procedure is described in appendix A. The first step involves fitting a VAR(10) model to the data, so the empirical analysis cover the period 1881 – 1986.

Figure 1

Output and its permanent component

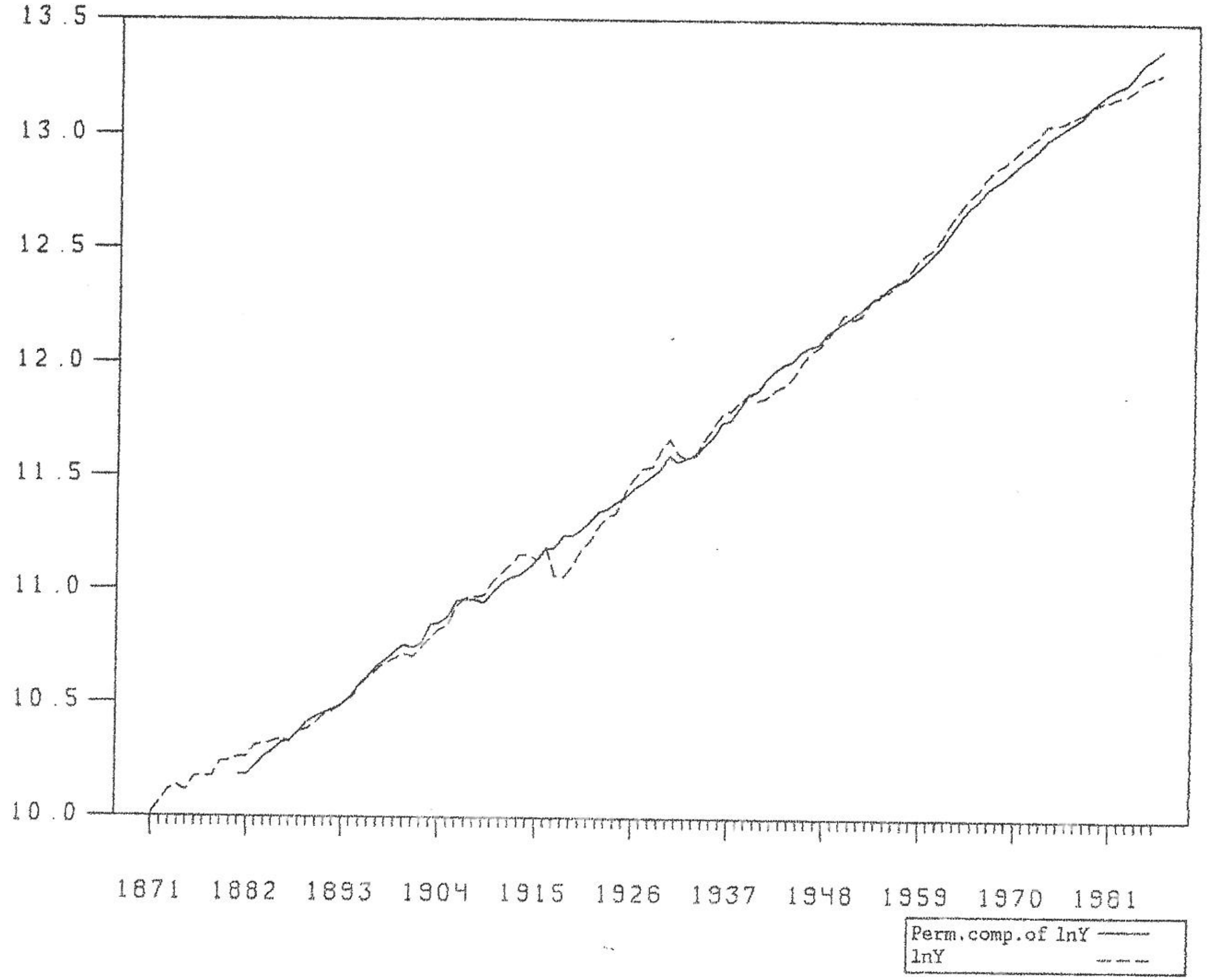


Figure 2

The transitory component of output

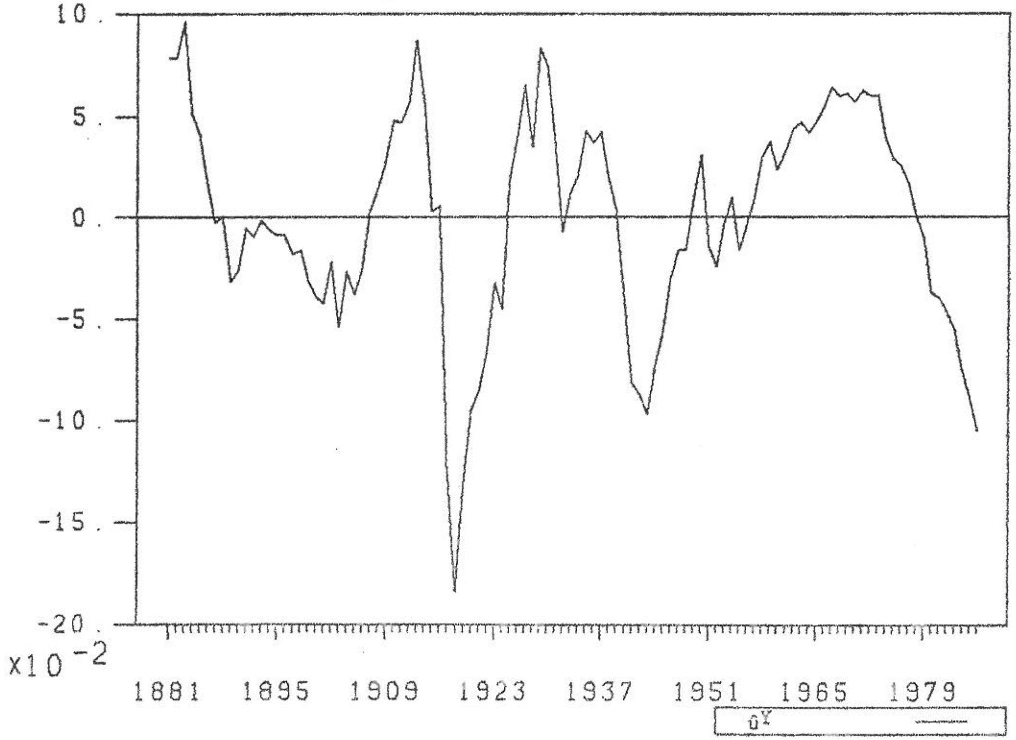


Figure 3

The rate of change of output

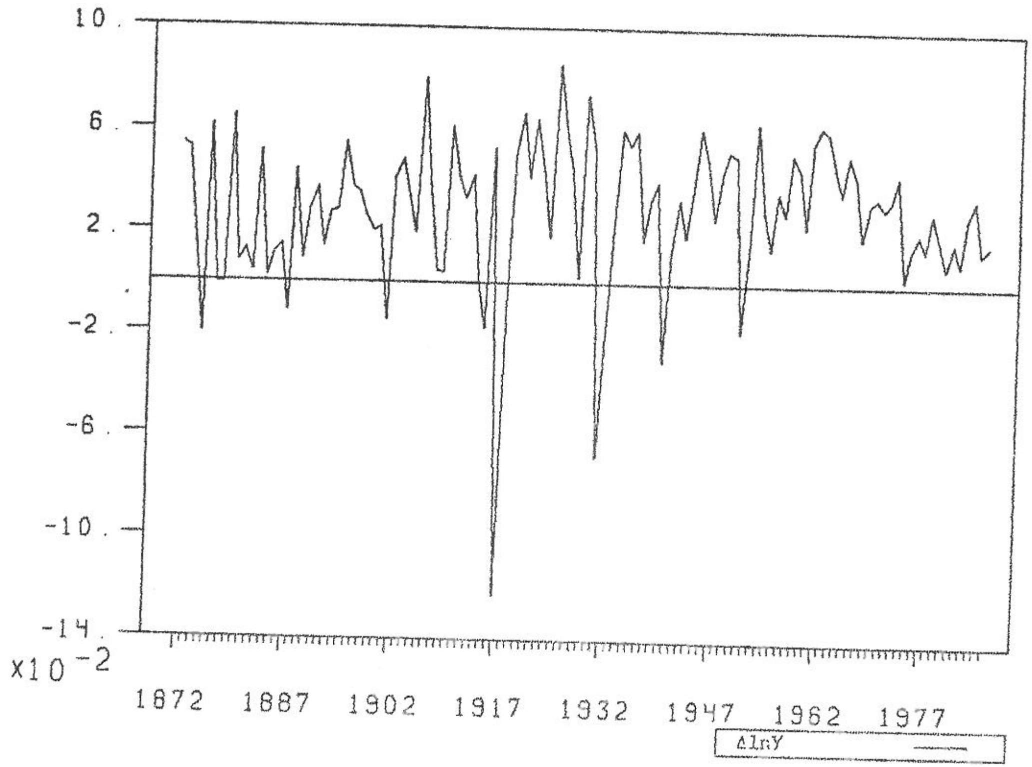


Figure 4

The change in the permanent component of output

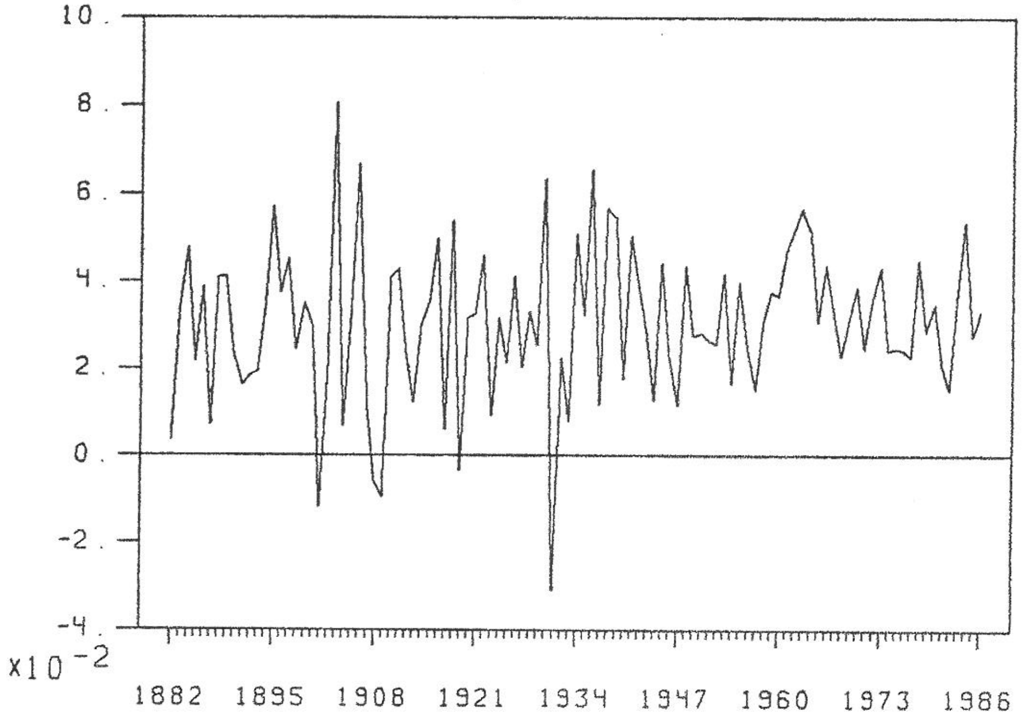
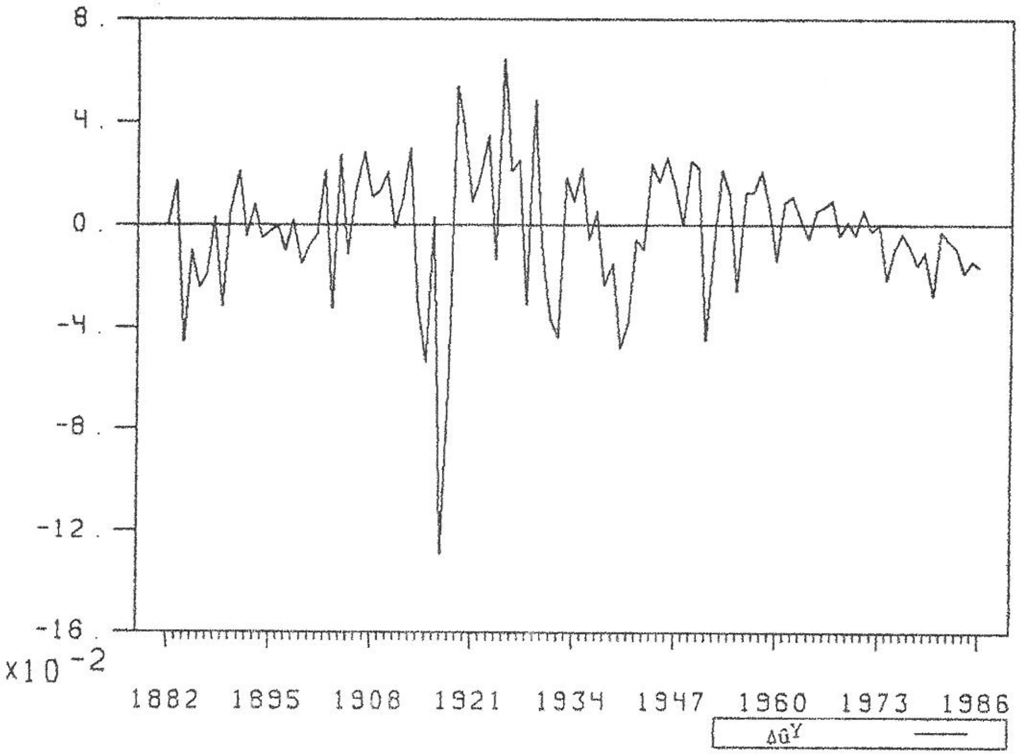


Figure 5

The change in the transitory component of output



In figure 1 the logarithm of output is depicted together with its estimated permanent component. Negative changes in the permanent component are recorded at four occasions: in 1901 (–1.2 per cent), 1908 – 1909 (–.6 and –1.0), 1919 (–.4), and 1931 (–3.1). An inspection of the time series suggests that the decline at the second of these occasions had a long term effect on the trend level of output. The larger loss in output at the last occasion, on the other hand, was regained rather quickly. There also appears to have been an upward shift in the permanent component in the early 1960's. In figure 2 our estimate of the business cycle, i.e. the transitory component of output (the difference between actual output and its permanent component) is depicted. There are four extended periods during which actual output is below the (stochastic) trend: 1889 – 1906, 1917 – 1924, 1940 – 1948, and 1979 – 1986. Although the major slumps in output around 1920 and in the early 1930's, which are evident in figure 1, both were associated with a stagnating or decreasing permanent component, the nature of the two depressions were quite different. The drastic fall in output in 1917 (–12.2 per cent) started from a level quite close to the long term trend, whereas the depression in the 1930's, due to an outstandingly rapid growth in the 1920's, started from a level far above the trend.

Figure 3 shows what recently has been stressed by Sheffrin (1988), namely that the volatility in the rate of change of output is lower in the post–war period than earlier. Since our analysis allows the total change of output to be decomposed into the changes in the permanent and transitory components, it is natural to ask whether the dampening of fluctuations stems from the business cycle or the growth cycle, or both. In figures 4 – 5 the changes of the permanent and transitory components are depicted. There appears to have been a decrease in the volatility of both series, although the change is more pronounced for the transitory component.

However, in terms of the volatility of the estimated transitory component in figure 2 there is no sign of any stabilization. An important question is thus whether the *rate of change* of output or the *level of the transitory component* of output is the most relevant measure of the business cycle. If the only reason for looking at rates of change is that the

underlying data are non–stationary, an approach which attempts at separating the non–stationary and stationary components should be preferred. On the other hand, the development of the level of the transitory component of output does not seem to give much support to the conventional wisdom about recurrent short run business cycles. In our estimate of the business cycle, the "long waves" are more striking than the frequent but short–lived minor booms and troughs.[6] One implication of the long waves is to make any comparison of subperiods very sensitive to the necessarily arbitrary choice of cut–off points. The variance of (the level of) the transitory component for the period 1951 – 1984, which is discussed by Sheffrin (1988), is lower than for the rest of the sample period; but if we look at the period 1943 – 1986, the variance appears to be as large as ever.

Turning to the question about the relation between the current account and the business cycle, we may first note that the first and last of the periods of temporarily low output (1889 – 1906 and 1979 – 1986) have been associated with persistent current account deficits (1898 – 1910 and 1974 – 1983); cf. figure 6. Similarly, the high levels of output recorded in the interwar period occurred in conjunction with a sequence of current account surpluses. Thus, the current account seems to be *pro–cyclical*. (The most remarkable period of deficits, 1873 – 1892, seems to have been associated with a transitory upsurge in output, but we do not have any estimates of the transitory and permanent components of output prior to 1881.)

In table 2 we present some descriptive statistics for the transitory components of (the logarithms of) output, savings, investments, terms of trade, and the price level. The transitory component of output is denoted $\hat{u}^Y$, etc. It can be seen that the standard deviation of savings is higher than the standard deviation of investments, and that the current account (CA/Y) is more strongly correlated with $\hat{u}^S$ than with $\hat{u}^I$. In some sense, therefore, movements in the current account are more associated with fluctuations in savings than in investments.

[6] The low frequency movements in actual data is not only at odds with the conventional wisdom about the properties of business cycles, but also with theoretical (real) business cycle models; cf. King, Plosser and Rebelo (1988a).

Figure 6

Current account as share of output

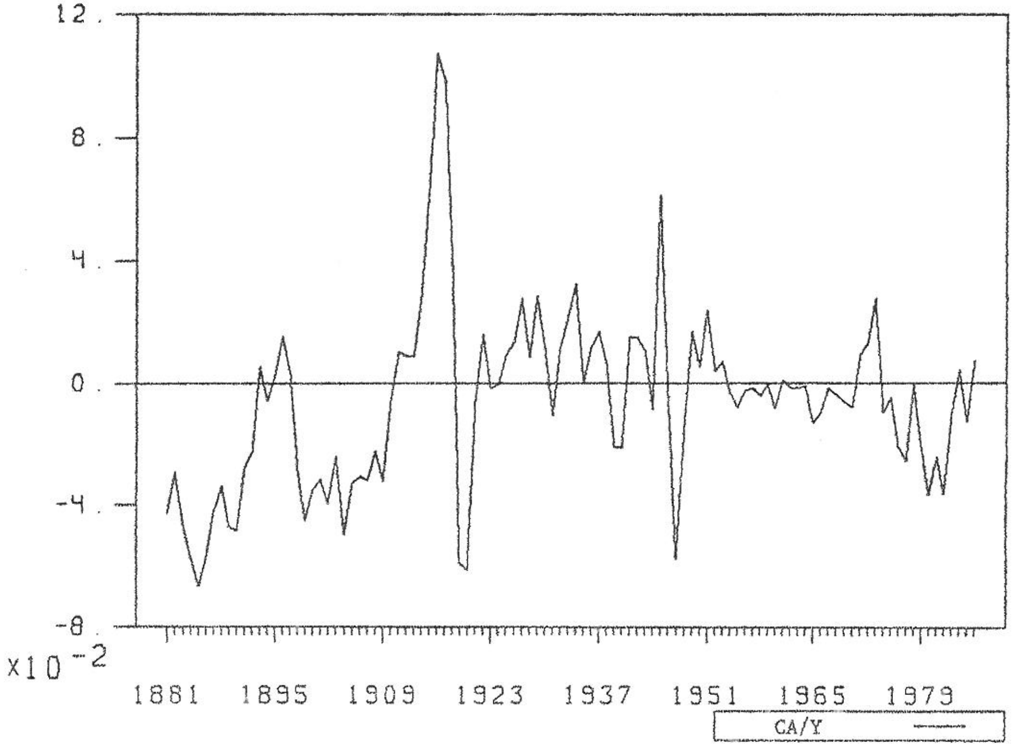


Table 2

Descriptive statistics for transitory components, 1881 – 1986

	Stand. dev.	Corr. coefficients					Corr. with $\hat{u}^{Y}_{t-i}$ i–1	i–2	i–3	i–4	i–5
$\hat{u}^{Y}$	.052	1.0					.875	.701	.514	.307	.093
$\hat{u}^{I}$	.144	.588 [.493,.708]	1.0				.507	.394	.309	.215	.082
$\hat{u}^{S}$	.247	.316 [.116,.536]	.401 [.221,.566]	1.0			.506	.510	.391	.239	.097
$\hat{u}^{p}$	.093	.358 [.198,.553]	.410 [.245,.555]	.444 [.294,.584]	1.0		.372	.198	.013	–.157	–.275
$\hat{u}^{P}$	.247	–.439 [–.664.–.274]	–.585 [–.695,–.455]	–.569 [–.734,–.409]	–.091 [–.296,.119]	1.0	–.522	–.463	–.346	–.180	–.081
CA/Y	.029	–.003 [–.278,.227]	–.187 [–.357,–.032]	.653 [538,.808]	.165 [–.405,.025]	–.175	.255	.331	.290	.210	.128

Note: The confidence intervals have been estimated by Torsten Dahlquist and Erik Mellander, IUI, by means of a bootstrap procedure, constructed as follows. Let z_t, $t=1,...,106$ be the set of 2x1 vectors of any pair of variables, e.g. $(\hat{u}^{Y}_{t}, \hat{u}^{I}_{t})$. One draw $\{z^*_1,...,z^*_{106}\}$ was executed by randomly selecting 106 integers $i_1,...,i_{106}$ with replacement from $\{1,...,106\}$ and the correlation coefficient corresponding to this draw, r^*, was computed. This procedure was repeated 500 times, yielding a distribution for $r-r^*$, where r denotes the point estimate of the correlation coefficient given in the table. The 95% confidence interval was then constructed by choosing constants k_1 and k_2 such that for this distribution $P(k_1, r-r^*) = P(r-r^*, k_2) = 0.975$.

It must be recalled, that the series on investments and output (and, hence, savings) are defined exclusive of changes in inventories. However, Englund and Vredin (1989) show that when Sweden is compared to six other OECD countries, the standard deviation of the rate of change of savings is just as high as that of investments (8.7 per cent) in Sweden, while investments are more volatile than savings in the other countries (except for West Germany). Thus, the volatility of savings in relation to investments does seem to be fairly high in Sweden in an international comparison.[7]

Table 2 also shows that, in terms of contemporaneous correlations with the transitory component of output, the transitory components of savings, investments and terms of trade are pro–cyclical, whereas the price level, perhaps surprisingly, appears to be counter–cyclical. These patterns show up also at different lags, with the exception of the correlation between $\hat{u}_t^p$ and $\hat{u}_{t-i}^Y$, which turns negative at $i = 4$. Furthermore, transitory movements in savings are (contemporaneously) positively correlated with terms of trade. In terms of the contemporaneous correlation between CA/Y and $\hat{u}^Y$, the current account is counter–cyclical, although the correlation is very low. This comes as no surprise given the regression coefficient in (14), but it is contradictory to what our casual inspection of the time series suggested; cf. figures 2 and 6. The reason for this contradiction is, of course, that the simple contemporaneous correlation does not give the whole picture of the comovements between the current account and the business cycle. In particular, the correlation between $(CA/Y)_t$ and $\hat{u}_{t-i}^Y$ is positive for $i = 1 - 6$, with a peak of .331 at lag 2. In a sense, therefore, the current account is really pro–cyclical.

It might be tempting to conclude, e.g., that the positive correlation between savings and terms of trade is a reflection of a positive "Harberger–Laursen–Metzler effect", and

[7] In the present data set, where changes in inventories are excluded, the standard deviations of $\Delta \ln S$ and $\Delta \ln I$ are 18.5 and 10.3 per cent, respectively, for the whole sample while the standard deviations drop to 6.7 and 4.0 per cent 1950 – 1986. In terms of output shares, i.e., when the numerator as well as the denominator is defined inclusive or exclusive of changes in inventories, our historical data set exhibit similar characteristics to that in Englund and Vredin (1989). For example, the correlation between the current account–GNP share and the savings–GNP (investments–GNP) share is .54 (.13) in post–war national accounts data, and .59 (.13) 1872 – 1986, and the savings share is more volatile than the investments share in both data sets.

that the negative correlation between $\hat{u}_t^P$ and $\hat{u}_t^S$ reflects an effect of inflation on savings via the real interest rate. It might also be tempting to conclude that the Swedish deficits on current account since the first oil price shock are quite natural, given the historical pro–cyclicality of the current account. The deficits may be a reflection of an optimal intertemporal allocation of savings and investments in the face of a transitory drop in output. Such interpretations are, of course, questionable for many reasons. First, in order to give an economic interpretation to the correlation between, e.g., savings and the terms of trade, we would at least like to control for the influence of other variables such as output and the price level. In other words, *partial* correlation coefficients would be more telling than the simple correlations i table 2. Second, although partial correlations and regression coefficients may be calculated from the information in table 2,[8] the stylized facts can be unambiguously interpreted only in terms of a formal econometric model (at best). Identifying assumptions are thus necessary in order to be able to speak of, e.g., the Harberger–Laursen–Metzler *effect*. Third, when estimating e.g. the effects on savings and the current account from various (exogenous) changes, it is important to take both permanent and transitory changes into account (see, e.g., Ahmed (1987)).

Although the formulation of a structural econometric model is beyond the scope of this paper, we will look at one further aspect of our data, namely the evidence on cointegration and common trends. Like the empirical evidence on stochastic trends and permanent and transitory components, any evidence of cointegration is useful information in the process of formulation and estimation of an econometric model.

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[8]Consider, e.g., the models $\hat{u}_t^I = \alpha_0 + \alpha_1\hat{u}_t^S$ and $(CA/Y)_t = \beta_0 + \beta_1\hat{u}_t^I + \beta_2\hat{u}_t^Y$, which are versions of (11) and (12) but which includes stationary variables only. OLS coefficients may be calculated from the information in table 2, which gives $\hat{\alpha}_1 = .234$, $\hat{\beta}_1 = -.056$ and $\hat{\beta}_2 = .089$. It can be seen that the correlations between the transitory components of investment, on the one hand, and the current account and the transitory component of savings on the other, are much lower than the correlations reported in (13) and (14). Furthermore, the current account appears to be pro–cyclical.

5. Tests for common trends.

Some tests of the hypothesis that savings and investments are cointegrated have already been reported. First, under the maintained hypothesis that the cointegrating vector is equal to (1, −1), the tests for stochastic trends in CA and CA/Y in table 1 are tests of the hypothesis that S and I, and S/Y and I/Y, respectively, are cointegrated. Since the stochastic trend hypothesis can be rejected in these cases, the results may be interpreted as evidence of cointegration. Second, the Feldstein–Horioka regression equation (13) offers a test of cointegration between S/Y and I/Y, which involves no restriction on the cointegrating vector (except for the normalization). Specifically, the DW–statistic of .56 implies that the hypothesis of no cointegration can be rejected at the 1 per cent significance level, according to the critical values given by Engle and Granger (1987). On the basis of tests on quarterly post–war US data, Miller (1988) also concludes that S/Y and I/Y are cointegrated, with a cointegrating vector reasonably close to (1, −1) (note that standard hypothesis tests about these parameters are invalid), at least for the fixed–exchange rate period.

It can be argued, however, that the hypothesis of a trend in S/Y and I/Y can be ruled out on a priori grounds, at least under the maintained hypothesis that the parameter(s) characterizing the growth process is constant. It also seems unduly restrictive to limit the attention to tests where Y only appears (if at all) in output shares. On the other hand, the hypothesis of a common trend in the logarithms of of savings, investment and output seems highly plausible (cf. King, Plosser, Stock and Watson (1987)). We will thus apply the multivariate Stock–Watson test, which is described in Appendix B, to examine whether there are common trends in X' = (lnS, lnI, lnY, lnP, lnp).

The Stock–Watson test of the hypothesis that the nx1 vector X contains k rather than n–m trends is based on an examination of the (n–m+1)st largest root of the first–order autocorrelation matrix of some transformation of X (derived under the null hypothesis of k trends; cf. Appendix B). In our case, n = 5, and the null hypothesis is that there are as many trends as there are variables, i.e., k = n. The roots λ_f as well as the test stati

Table 3

Tests for common trends

Re	Im	n–m	$q_f^{\tau}(5,n-m)$	critical value (10%)
.7159		4	–29.83	–49.2
.8438		3	–16.40	–32.6
.9260	.0313	2	–7.77	–21.9
.9260	–.0313	1	–7.77	–14.0
.9573		0	–4.48	–8.7

Note: The critical values are from Stock and Watson (1988a), table 3. Tests are based on a VAR(9) regression, i.e., $\tau = 105$.

stics $q_f^T(k,n-m)$ for various alternative hypotheses ($n-m = 0, 1, 2, 3, 4$) are given in table 3. Based on the critical values calculated by Stock and Watson (1988a) the null hypothesis cannot be rejected in favor of any alternative. The common trends hypothesis is thus not supported. This result is somewhat surprising, given that S and I (and S/Y and I/Y) appeared to be cointegrated from the univariate test on CA (and CA/Y) in table 1 and the bivariate test based on (13).

A more powerful test could perhaps be designed if a certain null hypothesis where $k \neq n$ could be motivated on a priori or theoretical grounds. For example, it may be argued that output and the price level are governed by two independent stochastic trends driven by shocks to, e.g., technology and money supply, respectively; that any trend in terms of trade is independent of domestic factors (due to Sweden being a small open economy); and that savings and investments have a common trend with output. This would then lead us to test the hypothesis of $k = 3$ trends against $n-m = 2$ (or $k = 4$ against $n-m = 3$).[9] Ideally, a test should be based on a theoretical model of savings and investments in an open economy, like the closed-economy study by King, Plosser, Stock and Watson (1987). This task is left for future research, however.

6. Concluding comments.

The purpose of this paper has been to establish some stylized facts about current account and business cycles in Sweden. Given that the aim of the study is descriptive, it is quite natural not to draw any far-reaching "conclusions" from the figures presented. Nevertheless, some remarks about how our findings are related to earlier literature may be in order.

[9] OLS regressions of (linearly detrended) savings and/or investments against different sets of explanatory variables do give indications of cointegration when the residuals are subjected to the Dickey — Fuller, Stock — Watson and Durbin — Watson tests. The results are available from the authors upon request.

First, our tests suggest that investments and output, as well as the ratio between these variables, contain stochastic trends. Inference in regression equations which include investments and output as explanatory variables, as in Sachs (1981), is thus a treacherous exercise. Second, the current account does not appear to follow a stochastic trend. Given that investment does, this requires that savings and investments are cointegrated, and that savings also have a stochastic trend. The evidence on this point is however somewhat mixed, since the univariate tests on savings and the various tests for common trends give no clear picture. If savings and investments really are cointegrated these aggregates will be perfectly correlated in the long run. This is consistent with the empirical findings by Feldstein and Horioka (1980), although not with their conclusion that capital mobility is low, since transitory fluctuations may not be (and, in fact, are not) perfectly correlated. Third, if output, investments, etc. contain stochastic trends, the (rates of) change in these variables reflect the stochastic shocks not only to the transitory components but also to the permanent components. Our findings suggest that the decrease in the volatility of Swedish output noted by Sheffrin (1988) may be due to lower volatility in the changes of both components. However, the *level* of the transitory component seems as volatile as ever; in this sense, the fluctuations in the "business cycle" have *not* been dampened. Fourth, the current account is positively correlated with the (lagged) transitory component of output. This "pro-cyclical" pattern of the external balance should be consistent with intertemporal optimization models. Other studies which have reported a counter-cyclical pattern should thus perhaps not be taken as a starting point for developments of new theories of the current account; rather, some further discussion of how the cyclicality concept should be defined might be in order. Fifth, the transitory fluctuations in savings appear to be more volatile than those in investments. In a sense, therefore, current account fluctuations are mainly associated with movements in savings rather than investments. This "result" is contrary to that for other countries reached by Sachs (1981). Judging from the figures presented by Englund and Vredin (1989) this difference seems more due to specific Swedish conditions than to differences in econometric methods. Finally, the finding that the current

account does not appear to have a stochastic trend whereas some "fundamental" explanatory variables do, *may* explain the lack of significant relations in the empirical models of the external balance of Englund and Vredin (1987), Backus (1986) and Rose and Yellen (1987).

APPENDIX A: Estimation of permanent and transitory components of I(1) time series.

The purpose of this appendix is to relate some model representations, which are equivalent under certain assumptions, to a fundamental specification and thereby provide the instruments that we need in order to estimate permanent and transitory components of I(1) time series. To achieve this objective, let X_t denote an $n \times 1$ real valued vector time series, which is assumed to be I(1) such that it has a Wold vector moving average (VMA) representation of the form

$$\text{(A.1)} \qquad \Delta X_t = \delta + C(L)\epsilon_t, \quad t = 1,2,...$$

where ϵ_t is an $n \times 1$ vector of random variables with mean zero and contemporaneous covariance matrix Σ, assumed to be positive definite, L is the lag operator, $\Delta \equiv 1-L$, the $n \times n$ matrix lag polynomial $C(L)$ is defined by $C(L) \equiv \Sigma_{k=0}^{\infty} C_k L^k$, where $\Sigma_{k=1}^{\infty} k|C_k|$ converges, $C(0) = I_n$, and the function $\det[C(\lambda)] = 0$, has all solutions on or outside the unit circle (cf. Hannan (1970)).

The convergence and the I(1) assumptions imply that $C(L)$ may be written in the form

$$\text{(A.2)} \qquad C(L) = C(1) + \Delta \cdot C^*(L),$$

where $C_j^* = -\Sigma_{k=j+1}^{\infty} C_k$ for $j \in \{0,1,...\}$. Furthermore, it can be shown that $C^*(L)$ is well defined in the sense that $\Sigma_{j=0}^{\infty}|C_j^*|$ converges, that is, $C^*(L)$ is absolutely summable (see e.g. Stock (1987)). Finally, we assume that $C^*(1)$ is a nonzero matrix. Substituting (A.2) into (A.1) for $C(L)$, we get after recursive substitutions that

$$X_t = X_0 + \delta t + C(1)\Sigma_{k=1}^{t}\epsilon_k + C^*(L)\cdot(\epsilon_t - \epsilon_0), \quad t = 1,2,...$$

Suppose that we initialize this equation by letting $\epsilon_0 = E[\epsilon_t] = 0$. Moreover, let us assume that X_0 is a constant. Letting $\xi_t \equiv \Sigma_{k=1}^{t}\epsilon_k$ and $u_t \equiv C^*(L)\epsilon_t$ for all $t \geq 1$, we find that the vector time series X_t may be decomposed as

(A.3) $$X_t = X_0 + \delta t + C(1)\xi_t + u_t, \quad t = 1,2,...$$

where $(X_0 + \delta t)$ is a deterministic vector which contains a constant vector, X_0, and a linear trend vector, δt. The stochastic components are $C(1)\xi_t$, which is I(1) and represents the stochastic trends of X_t, and u_t, which is I(0). If we can obtain a consistent estimator of $\{\epsilon_t\}_{t=1}^{T}$ and, hence, a consistent estimator of $\{\xi_t\}_{t=1}^{T}$, then (A.3) may be used to estimate $\{u_t\}_{t=1}^{T}$. Since X_t and ξ_t are I(1), OLS estimates of the possibly serially correlated errors u_t will be consistent.

For the purpose of estimating the vector time series $\{\epsilon_t\}_{t=1}^{T}$ and, thus, $\{\xi_t\}_{t=1}^{T}$, let us assume that X_t has a vector autoregressive represetation of order p, denoted VAR(p), of the form

(A.4) $$A(L)X_t = \theta + \epsilon_t, \quad t = 1, 2,...$$

where $A(0) = I_n$. If the vector time series X_t is *not* cointegrated of order (1,1) then

(A.5) $$C(L)^{-1}\Delta X_t = \theta + \epsilon_t,$$

In this case $\text{rank}[C(1)] = n$, while $\text{rank}\,[A(1)] = 0$. Still, estimation of (A.4) will provide us with consistent estimates of $\{\epsilon_t\}_{t=1}^{T}$. When X_t is not cointegrated, estimates of $\{\epsilon_t\}_{t=1}^{T}$ using (A.5) will be asymptotically efficient to estimates of this time series using (A.4). By asymptotical efficiency in this context, it is understood that the difference between the asymptotic covariance matrix of the parameter estimates in (A.5) and the asymptotic

covariance matrix of the parameter estimates in (A.4) equals a negative semidefinite matrix. Similarly, when X_t is cointegrated of order (1,1), (A.4) provides us with a model from which we can acquire consistent estimates of $\{\epsilon_t\}_{t=1}^{T}$ and $\{\xi_t\}_{t=1}^{T}$. In that case, (A.5) is a model misspecification since cointegration implies that C(1) is singular. Moreover, cointegration implies certain linear constraints on the A(L) matrix lag polynomial (see Engle and Granger (1987)), which means that estimates of $\{\epsilon_t\}_{t=1}^{T}$ using such information will be efficient in comparison to the estimates based on (A.4).

In summary, in order to estimate the time series $\{u_t\}_{t=1}^{T}$ when X_t is I(1) we propose the following procedure: (i) estimate the VAR(p) equation (A.4), which yields a time series $\{\hat{\epsilon}_t\}_{t=1}^{T}$, and (ii) letting $\hat{\xi}_t = \Sigma_{k=1}^{t} \hat{\epsilon}_k$ for $t = 1,...,T$ we estimate X_t in equation (A.3) by OLS, using the regressors $(1,t,\hat{\xi}_t)$, thereby providing us with the time series $\{\hat{u}_t\}_{t=1}^{T}$. As a byproduct, we get consistent estimates of the deterministic component and the stochastic trends. Note also that our strategy to estimating transitory and permanent components of X_t allows the matrix lag polynomial $C^*(L)$ to be of an arbitrary order q.

APPENDIX B: The univariate and multivariate Stock–Watson filter tests for the order of integration and cointegration.

This appendix describes how the multivariate Stock–Watson filter tests for the order of cointegration and the univariate counterparts for the order of integration may be implemented in practise. Primarily, we shall be concerned with the most general case for I(1) real valued vector time series. Before we come to that, however, we shall discuss the univariate filter tests. A reason for considering such a special case is the simplicity by which the testing strategy works. It may be noted that version 3.01 of RATS is supplied with a procedure, Stockwat.src, which performs both the univariate filter tests and the augmented Dickey–Fuller tests (for an analysis of this test, cf. Fuller (1976), Dickey and Fuller (1979,1981) or Said and Dickey (1984)). We have used an updated version of this procedure, provided by Ossian Ekdahl at the Stockholm School of Economics.

Let x_t denote a univariate time series which is supposed to be I(1) under the null and I(0) under the alternative. By assumption, x_t has an invertible ARMA(p,q) representation in first differences under the null. To perform the Stock–Watson filter tests for such a time series, we shall also assume that the MA scalar lag polynomial is of order 0, while the AR scalar lag polynomial is of order p by assumption. Hence, we may represent x_t by the following stochastic process when the mean for Δx_t is constant for all t

(B.1) $$\pi(L)\Delta x_t = \beta + \epsilon_t, \quad t = 1,2,...$$

where $\pi(L)$ has all roots outside the unit circle (which follows by assumption), and β is a constant related to the mean for Δx_t. Specifically, if $E[\Delta x_t] = \mu$, then $\pi(1)\mu = \beta$. Finally, the disturbance, ϵ_t, is an i.i.d. random variable with mean zero and variance σ_ϵ^2. Letting $x_t^* \equiv x_t - x_0 - \mu t$, where x_0 is assumed to be constant, $\nu_t \equiv \sigma_\epsilon^{-1}\epsilon_t$ and $\pi^*(L) \equiv \sigma_\epsilon^{-1}\pi(L)$, we may rewrite (B.1) as

(B.2) $$\pi^*(L)\Delta x_t^* = \nu_t, \quad t = 1,2,...$$

The scalar lag polynomial $\pi^*(L)$ is an autoregressive filter of order p such that Δx_t^* is transformed into a white noise process with zero mean and unit variance.

To see how a test of the null that x_t is I(1) against the alternative that x_t is I(0) can be constructed, assume first that the parameters $(x_0,\mu,\sigma_\epsilon^2,\pi(L))$ are known. Letting $\zeta_t \equiv \pi^*(L)x_t^*$, we may rewrite (B.2) as

(B.3) $$\zeta_t = \varphi\zeta_{t-1} + \nu_t, \quad t = 1,2,...$$

Under the null, $\varphi = 1$, implying that ζ_t is a random walk whose innovation has zero mean and unit variance, while under the alternative $\varphi < 1$. To test the null hypothesis, one may simply regress ζ_t on ζ_{t-1} by OLS. Letting $\bar{\varphi}_f$ denote this estimate, the Stock–Watson filter test statistic is

(B.4) $$q_f(1,0) = T[\bar{\varphi}_f - 1],$$

where T is the sample size. The asymptotic distribution for q_f depends on the exact relationship between x_t^* and x_t. If μ is unrestricted, then the distribution for q_f^τ (see Stock and Watson (1988a), table 3) should be employed and if $\mu = 0$, then the distribution for q_f^μ (table 2, same source) is valid under the null.

In practise, it may well be that the deterministic trend in x_t is quadratic rather than linear. To allow for this possibility we simply include this term in (B.1)

(B.5) $$\pi(L)\cdot(\Delta x_t - \tau t) = \beta + \epsilon_t, \quad t = 1,2,...$$

In this case, $x_t^* \equiv x_t - x_0 - \mu t - \tau t^2$. The asymptotic distribution for q_f when x_t contains a

quadratic trend differs from the distributions when $\tau = 0$. The appendix in Stock and Watson (1989) contains the empirical distribution for $q_f^{\tau^2}$.

In practise, $(x_0,\mu,\tau,\sigma_\epsilon^2,\pi(L))$ are unknown and therefore have to be estimated. Consistent estimates of x_t^* can be obtained by regressing x_t on $(1,t,t^2)$. Let $\hat{x}_t$ denote the OLS estimate of x_t^*. By AR(p) regression of $\Delta\hat{x}_t$, we get estimates of $\pi(L)$ and σ_ϵ^2, which are consistent under the null. Letting $\hat{\pi}(L)$ denote the estimate of $\pi^*(L)$, we find that $\hat{\zeta}_t = \hat{\pi}(L)\hat{x}_t$. We then regress $\hat{\zeta}_t$ on $\hat{\zeta}_{t-1}$ by OLS and thereby provide ourselves with an estimate of φ, denoted $\hat{\varphi}_f$. Replacing $\tilde{\varphi}_f$ by $\hat{\varphi}_f$ in (B.4), we may calculate the statistic for the filter test. It may be noted that $\hat{\pi}(L)$ converges to $\pi^*(L)$ under the null, while it converges to a scalar lag polynomial $\pi_a(L)$ under the alternative. For an analysis of the consistency of q_f, see Stock and Watson (1988a), p. 1100.

Let us now focus our attention on the multivariate tests. Letting X_t denote an n dimensional real valued time series which is I(1) and applying the assumptions stated in appendix A, we know that

$$X_t = X_0 + \delta t + C(1)\xi_t + u_t, \quad t = 1,2,... \tag{B.6}$$

where C(1) is singular when X_t is CI(1,1), such that there may exist a reduced number of cointegrated variables $Z_t \equiv \alpha' X_t$. Let us consider a null hypothesis of r such variables, $r \in \{0,1,...,n-1\}$, against the alternative of m, where $r < m$. From Granger's representation theorem (see Engle and Granger (1987)) we know that $\text{rank}[C(1)] = n-r$ under the null, and n–m under the alternative.

A problem for many strategies to testing such hypotheses is that we must be able to identify variables representing the common trends feature of X_t. A solution, suggested by Stock and Watson (1988a) (SW), is that instead of examining X_t directly, we can examine functions of regression statistics of a linear transformation of X_t, denoted Y_t, which is chosen so that, under the null, the first n–r elements of Y_t are I(1), while the remaining r

elements are I(0). A natural candidate to the final r elements is Z_t as it satisfies the I(0) property. To be more specific, let $Y_t \equiv DX_t$, where $D \equiv [\alpha^* \;\; \alpha]'$, and α^* is an $n\times(n-r)$ matrix of constants.

How do we know that $\alpha^{*\prime}X_t$, denoted by W_t, is I(1) under the null? To see this, first consider $Z_t = \alpha' X_t$, which is an r dimensional vector of I(0) random variables under the null and the alternative. Premultiplying both sides in (B.6) by α', we get

(B.7) $$Z_t = \alpha' X_0 + \alpha'\delta t + \alpha' C(1)\xi_t + \alpha' u_t, \quad t = 1,2,...$$

Since Z_t is I(0), we find that $\alpha' C(1) = 0$, thereby eliminating the stochastic trends ξ_t. Under the null and the alternative C(1) has rank equal to n−r and n−m, respectively. Thus, we know that there exists at least r linearly independent vectors in the n dimensional Euclidean space which are orthogonal to the columns of C(1). These r vectors are contained in the columns of α. Furthermore, $\alpha'\delta = 0$ since the mean of Z_t is a constant vector. This, of course, implies that δ lies in the column space of C(1), i.e. $\delta = C(1)\tilde{\delta}$ for some vector $\tilde{\delta}$. We thus find that

(B.8) $$Z_t = \alpha' X_0 + \alpha' u_t, \quad t = 1,2,...$$

which is I(0). Because the columns of α^* are orthogonal to the columns of α under the null, i.e. $\alpha^{*\prime}\alpha = 0$, we find that $\alpha_j^{*\prime} C(1) \neq 0$ for all $j \in \{1,...,n-r\}$, where α_j^* denotes the j:th column of α^*. Accordingly, under the null, every element of W_t will contain stochastic trends, implying that W_t is I(1). The requirement that $\alpha^{*\prime}\alpha^* = I_{n-r}$ simply means that we normalize the coefficients of α^*. Premultiplying both sides in (B.6) by $\alpha^{*\prime}$, we get

(B.9) $$\begin{aligned} W_t &= \alpha^{*\prime} X_0 + \alpha^{*\prime}\delta t + \alpha^{*\prime} C(1)\xi_t + \alpha^{*\prime} u_t = \\ &= W_0 + Mt + \alpha^{*\prime} C(1)\xi_t + \alpha^{*\prime} u_t, \quad t = 1,2,... \end{aligned}$$

In contrast, under the alternative, $\alpha_j^{*\prime}C(1) \neq 0$ for $j \in \{1,...,n-m\}$, while $\alpha_i^{*\prime}C(1) = 0$ for $i \in \{n-m+1,...,n-r\}$, implying that only n−m elements of W_t are I(1) while m−r elements are I(0). Following SW we shall assume that ΔW_t, which is given by

$$(B.10) \qquad \Delta W_t = M + \alpha^{*\prime}C(1)\Delta\xi_t + \alpha^{*\prime}C^*(L)\Delta\epsilon_t = M + \alpha^{*\prime}C(L)\epsilon_t,$$

has a VAR(p) representation. Accordingly, there exists an $(n-r)\times(n-r)$ matrix lag polynomial $\Pi(L)$ of order p with all roots outside the unit circle such that

$$(B.11) \qquad \Pi(L)\Delta W_t = B + \eta_t, \quad t = 1,2,...$$

where $\Pi(1)M = B$ and $\Pi(L)\alpha^{*\prime}C(L)\epsilon_t = \eta_t$. Note that the dimension of η_t is n−r, while that of ϵ_t is n. It may be worth pointing out that when the null is given by $r = 0$, i.e. X_t is *not* CI(1,1), we are allowed to choose $\alpha^* = I_n$. In that case, $W_t = X_t$ thereby making the decomposition of X_t trivial. In general, this is the first null to consider when checking whether a vector time series X_t is CI(1,1) or not.

Following the notation employed for the univariate test, let $W_t^* \equiv W_t - W_0 - Mt$, $\nu_t \equiv K\eta_t$, where the $(n-r)\times(n-r)$ nonsingular matrix K is chosen so that $E[\nu_t\nu_t'] = I_{n-r}$ while $\Pi^*(L) \equiv K\Pi(L)$. We may then rewrite (B.11) as

$$(B.12) \qquad \Pi^*(L)\Delta W_t^* = \nu_t, \quad t = 1,2,...$$

The matrix lag polynomial $\Pi^*(L)$ is an autoregressive filter of order p such that ΔW_t^* is transformed into n−r white noise processes, each with zero mean and unit variance.

To see how a multivariate test may be constructed, assume that $(D,W_0,M,\Sigma,K,\Pi(L))$ are known. Letting $\zeta_t \equiv \Pi^*(L)W_t^*$, we may write (B.12) as

(B.13) $$\zeta_t = \Phi_f \zeta_{t-1} + \nu_t, \quad t = 1,2,...$$

Under the null, $\Phi_f = I_{n-r}$, implying that the (n−m+1) largest eigenvalue is equal to 1. A reason for considering eigenvalues is, of course, that since Φ_f is diagonal its eigenvalues correspond to its diagonal elements. In contrast, under the alternative, the (n−m+1) largest eigenvalue is associated with an I(0) stochastic process and, hence, must be less than 1. To test the null of n−r common trends (or, equivalently, r cointegrating vectors) against an alternative of n−m such trends, one may simply regress ζ_t on ζ_{t-1} by OLS. Letting $\tilde{\Phi}_f$ denote this estimate, the SW filter test statistic is

(B.14) $$q_f(n-r,n-m) = T[\mathrm{Re}(\tilde{\lambda}_{f,n-m+1}) - 1],$$

where T is the sample size and $\mathrm{Re}(\tilde{\lambda}_{f,n-m+1})$ denotes the n−m+1 largest real part of the eigenvalues of $\tilde{\Phi}_f$.

It may not be obvious why only the real part of the eigenvalues of $\tilde{\Phi}_f$ need to be considered. However, it can be shown that the imaginary parts converge to zero in probability when multiplied by T under the null *and* the alternative, while $q_f(n-r,n-m)$ converges to zero in probability under the null and approaches minus infinity under the alternative (for analysis of the consistency of $q_f(n-r,n-m)$, see SW p. 1100). The intuitive explanation for considering the real eigenvalues is that Φ_f is diagonal and, hence, its eigenvalues are real.

In practise, $(D,W_0,M,\Sigma,K,\Pi(L))$ have to be estimated. SW propose estimating the matrix D by principal components methods, where the estimated r cointegrating vectors are given by those linear combinations corresponding to the r smallest principal components of X_t, while α^* is estimated by those linear combinations corresponding to the n−r largest principal components of X_t. Thus, by construction the basis for the largest principal components is orthogonal to the estimate of α. Such estimates are consistent under the

null *and* the alternative. One may perhaps think that OLS estimates of α (which, as Stock (1987) has proven, are consistent under both hypothesis) may be used with recursive estimates of α^* (using the constraints on α^*) to construct an estimator of D. However, such an estimator may not be consistent under the alternative.

Regression of the estimates of W_t on (1,t) provides us with consistent estimates of W_t^*, which we denote by $\hat{W}_t$. By VAR(p) regression of $\Delta\hat{W}_t$, we obtain estimates of $\Pi(L)$ and Σ, which are consistent under the null. Letting $\hat{\Sigma}$ denote the estimator of Σ, Choleski decomposition of the estimated contemporaneous covariance matrix provides us with a consistent estimator of K. Thus, a consistent estimator (under the null) of $\Pi^*(L)$, denoted $\hat{\Pi}(L)$, is obtained by premultiplying our estimator of $\Pi(L)$ by the estimator of K. Letting $\hat{\zeta}_t \equiv \hat{\Pi}(L)\hat{W}_t$, we can estimate Φ_f by regressing $\hat{\zeta}_t$ on $\hat{\zeta}_{t-1}$ using OLS. This estimated matrix, denoted $\hat{\Phi}_f$, and the real parts of its eigenvalues may be used in (B.14), where $Re(\hat{\lambda}_{f,n-m+1})$ denotes the n−m+1 largest real part of the eigenvalues of $\hat{\Phi}_f$.

Theorems 3.1 and 5.2 in SW establish the asymptotic distributions for the real parts of the eigenvalues of $\hat{\Phi}_f$ under various assumptions on the specification of the deterministic component of W_t. The empirical distributions for $q_f(n-r,n-m)$ are tabulated in tables 1 to 3 of SW and table 7 in the appendix of Stock and Watson (1989) for the case of quadratic deterministic trends.

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