WORKING PAPER No.41
October, 1994

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Svensk resumé på sid 34.

A CONVERGING TRIANGULARIZATION ALGORITHM AND THE INTERTEMPORAL SIMILARITY OF PRODUCTION STRUCTURES

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October 1994

Abstract

Triangularization of input-output tables is commonly used as a measure for international and intertemporal comparisons of hierarchical production structures. The triangularization problem is here considered as a maximization problem, and necessary and sufficient conditions are formulated. We presented a branch and bound algorithm that takes advantage of these conditions for bounding the number of permutations that must be examined to find the optimal solution. We also define conditions for suboptimal solutions with the optimal solution as the limit when the number of sectors in the search tree converges towards the total number of sectors. The convergence property of the algorithm is illustrated by computing the suboptimal solutions for the input-output tables of 1970, 1980 and 1985 for four Nordic countries. We find that the hierarchical similarity of production structures holds over time among these Nordic countries.

1 Introduction

Triangularization of input-output tables is commonly used as a measure for international and intertemporal comparisons of hierarchical production structures. The triangularization problem is traditionally solved as a maximization problem; find the greatest sum of elements below the principal diagonal of an input-output table by permuting the industry order of the table. The number of permutations that must be examined to find this optimal solution increases with the dimension of the table. The larger the dimension, the more efficient must an algorithm be in bounding the number of permutations examined.

Heuristic algorithms based on ringshift permutations are mostly used for solving the triangularization problem. The number of permutations that is examined by such algorithms is bounded by the ringshift permutation theorem, e.g. Korte and Oberhofer (1970) and Fukui (1986). This theorem only states a necessary condition for the optimal solution. We cannot know whether the suboptimal solutions produced by such algorithms also are optimal solutions. A sufficiency condition for the branch and bound method was introduced by Haltia (1992). He also showed that an optimal solution, given by the sufficiency condition, cannot be improved by iterative use of the ringshift theorem, whereas the opposite is not true.

Haltia (1992) addressed the triangularization of input-output tables as a minimization problem. Already, Östblom (1986) had presented a branch and bound algorithm that finds a solution of the triangularization problem by minimization, although his algorithm did not take advantage of the sufficiency condition. The present paper shows that the minimization problem is dual to the traditional formulation of the triangularization problem. We state necessary and sufficient conditions for the maximization problem. The algorithm presented takes advantage of these conditions for bounding the number of permutations examined when searching for the optimal solution. When matrices are large, the algorithm also takes advantage of another condition for bounding the number of permutations. We therefor define the conditions for a suboptimal solution. In the suboptimal solution, the number of sectors in the search tree is less than the total number of sectors. As with the heuristic algorithms based on ringshift permutations we now search for approximate solutions to the triangularization problem. With our algorithm the suboptimal solutions converge to the optimal solution as we let the number of sectors in the search tree increase.

The convergence properties of the algorithm are illustrated by computing suboptimal solutions for the input-output tables of 1970, 1980 and 1985 for the four Nordic countries Denmark, Finland, Norway and Sweden. We also examine the intertemporal similarity of hierarchical structures of production for the Nordic countries. The present paper gives some more support to the findings of similar production structures among different countries made in international comparisons by others, c.f. Chenery and Watanabe (1958), Simpson and Tsukui (1965), Korte and Oberhofer (1971) and Fukui (1986). The hierarchical similarity of production structures among countries also holds over time for Denmark, Finland Norway and Sweden.

The paper first introduces an alternative approach to the traditional triangularization problem, where the sum of elements below the diagonal in an input-output matrix is maximized. We then maximize the sum of negative differences between the elements below and the symmetrical elements above the diagonal. Finally, we present a triangularization algorithm and illustrate its convergence properties for large matrices. The paper also reports the result of using the algorithm to study the intertemporal similarity of production structures among Nordic countries. Appendix I includes detailed tables showing the convergence properties and the similarity of production structures. Appendix II gives the computer program and appendix III presents triangularized input-output tables for Denmark, Finland, Norway and Sweden.[1]

2 Background

Analysis of the hierarchy of sectors leading from primary goods to final products forms a basis for a comparison of production structures. The standard method of arranging sectors into hierarchical orders is triangularization of flow matrices or coefficient matrices. The

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[1] The computer program is available from the author.

circular interdependence in production, however, does not permit us to observe a perfect triangular system of production. The circular flows of goods and services are affected by changes in the organisation of production as well as changes in the pattern of final demand. Influences from final demand affect the flow matrix but not the coefficient matrix. The economic significance of the hierarchical interdependence is that effects of changes in final demand spread through the economy from higher to lower sectors, and that reactions in the opposite direction, resulting in a continuing series of repercussions, are quite limited.

A measure of hierarchical interdependence is the degree of linearity. Hierarchical interdependence or circular structure will, however, differ between the flow matrix and the coefficient matrix. When triangularizing the flow matrix, the magnitude of interindustry transactions and thus also the circular structure is affected by the pattern of final demand. After triangularization of an input-output matrix we find that production processes can be grouped into blocks such as: Non-metal final, Non-metal basic, Metal, Energy and Services. Production processes in the block Non-metal final, which is linked directly to final demand, are recorded as processes of higher order. Production processes in the blocks Non-metal basic, Metal and Energy are of lower order, and processes in the Services block, which is linked mostly to intermediate demand, are of lowest order.

Figure A shows a matrix for which rows and columns can be rearranged to give a perfect triangular shape as illustrated by figures B and C. The ordering of sectors π in A is $\pi(1) = 1$, $\pi(2) = 2$, $\pi(3) = 3$, $\pi(4) = 4$, $\pi(5) = 5$. The positions of P, primary input, and of D, final demand, are left unchanged by the triangularization process. When *maximizing* the sum of elements *above* the diagonal, or equally a minimizing the sum of elements below the diagonal, the ordering in figure A is permuted to $\pi(1) = 3$, $\pi(2) = 2$, $\pi(3) = 4$, $\pi(4) = 5$, $\pi(5) = 1$. This is illustrated by figure B. If we instead *maximize* the sum of elements *below* the diagonal, the resulting permutation gives an ordering in figure C reversed to that in figure B. The sum of elements above the diagonal in B is equal to the sum of elements below the diagonal i C. Triangularization of a matrix will in the following refer to the situation depicted in figure C.

A

	1	2	3	4	5	D
1	+	0	0	0	0	+
2	+	+	0	+	+	+
3	+	+	+	+	+	+
4	+	0	0	+	0	+
5	+	0	0	+	+	+
P	+	+	+	+	+	+

B

	3	2	4	5	1	D
3	+	+	+	+	+	+
2	0	+	+	+	+	+
4	0	0	+	+	+	+
5	0	0	0	+	+	+
1	0	0	0	0	+	+
P	+	+	+	+	+	+

C

	1	5	4	2	3	D
1	+	0	0	0	0	+
5	+	+	0	0	0	+
4	+	+	+	0	0	+
2	+	+	+	+	0	+
3	+	+	+	+	+	+
P	+	+	+	+	+	+

The triangularization problem cannot be solved by analytical methods but belongs to a class of NP-complete problems[2]. Insted, we must use some kind of search algorithm to solve the triangularization problem. For small matrices, all possible permutations might be searched and that with the largest sum is chosen. This method, however, becomes less attractive and even most impractical when the number of sectors increases. The number of sectors has a factorial relationsship to the number of possible permutations. With 5 sectors, the number of permutations is $5! = 25$, but with 20 sectors we have $20! = 2{,}4329 \times 10^{18}$ possible permutations.[3] The algorithms proposed for handling the triangularization have used different criteria for bounding the set of permutations. These criteria can produce different solutions, something that has been convincingly demonstrated by Howe (1991).

3 Formulation of the problem

Let π be a permutation of $N_n = \{1,2,\ldots,n\}$. The permutation $\pi = (\pi(1),\pi(2),\ldots,\pi(n))$ generates a rearrangement of the sector order in the $n*n$ non-negative interindustry transaction table $X = \{x_{ij}\}$ into the order $X(\pi) = \{x_{ij}(\pi)\}$. We define the sums $R(X(\pi))$, $R'(X(\pi))$ and $R''(X(\pi))$ for the matrix $X(\pi)$. $R(X(\pi))$ is the sum of negative differences between the elements *below* the diagonal and those symmetrically positioned *above* the diagonal. $R'(X(\pi))$ is the sum of positive differences between the elements *above* and the

[2] The time required to solve NP-complete problems is a non-polynominal function of the size of the problem (e.g. the order of the input-output matrix).

[3] Even by checking 7.7147×10^{10} permutations per second, with 20 sectors it will still take a year to search through all permutations.

symmetrical elements *below* the diagonal. R"(X(π)) is the sum of elements below the diagonal.

The triangularization problem is usually stated as a problem of finding a permutation π* such that $\pi^* = \arg\max_{\pi} R''(\mathbf{X}(\pi))$. Here we will instead search for the permutation π* for which $\pi^* = \arg\max_{\pi} R(\mathbf{X}(\pi))$ and show that its dual formulation is the minimization problem $\pi^* = \arg\min_{\pi} R'(\mathbf{X}(\pi))$ stated by Östblom (1986) and by Haltia (1992). The sum $R(\mathbf{X}(\pi)) = \sum_{i>j} x_{ij}(\pi) - x_{ji}(\pi)$ for $x_{ij}(\pi) < x_{ji}(\pi)$ $(i > j)$ can be rewritten as:

$$R(\mathbf{X}(\pi)) = \tfrac{1}{2}\sum_{i>j} x_{ij}(\pi) - x_{ji}(\pi) - |x_{ij}(\pi) - x_{ji}(\pi)| = \begin{cases} \sum_{i>j} x_{ij}(\pi) - x_{ji}(\pi) & \text{for } x_{ij}(\pi) < x_{ji}(\pi);\ (i > j) \\ 0 & \text{for } x_{ij}(\pi) \geq x_{ji}(\pi);\ (i > j) \end{cases}$$

and the sum $R'(\mathbf{X}(\pi)) = \sum_{j>i} x_{ij}(\pi) - x_{ji}(\pi)$ for $x_{ij}(\pi) > x_{ji}(\pi)$ $(j > i)$ can be rewritten as:

$$R'(\mathbf{X}(\pi)) = \tfrac{1}{2}\sum_{j>i} x_{ij}(\pi) - x_{ji}(\pi) + |x_{ij}(\pi) - x_{ji}(\pi)| = \begin{cases} \sum_{j>i} x_{ij}(\pi) - x_{ji}(\pi) & \text{for } x_{ij}(\pi) > x_{ji}(\pi);\ (j > i) \\ 0 & \text{for } x_{ij}(\pi) \leq x_{ji}(\pi);\ (j > i) \end{cases}$$

The limits for R(X(π)) and R'(X(π)) are: $\sum_{i>j} x_{ij}(\pi) - x_{ji}(\pi) \leq R(\mathbf{X}(\pi)) \leq 0$ and $0 \leq R'(\mathbf{X}(\pi)) \leq \sum_{j>i} x_{ij}(\pi) - x_{ji}(\pi)$. We also note that R(X(π)) = -R'(X(π)) and thus we have the following dual problem: argmax. R(X(π)) = argmax -R'(X(π)) = argmin R'(X(π)).

Elements $x_{ij}(\pi)$ and $x_{ji}(\pi)$ can take different positions in X(π) for different permutations π, but they remain in symmetrical positions. When elements in a symmetrical position pass over the diagonal, the difference $x_{ij}(\pi)$ - $x_{ji}(\pi)$ changes sign but its absolute value will not change . Therefor, the sum of absolute differences, between the elements below and the symmetrical elements above the diagonal $\sum_{i>j} |x_{ij}(\pi) - x_{ji}(\pi)|$, is the same for all permutations π. The sum of symmetrical elements is $\sum_{i>j} x_{ij}(\pi)+x_{ji}(\pi)$ and thus $\sum_{i>j} x_{ji}(\pi) = \sum_{i>j} x_{ij}(\pi)+x_{ji}(\pi) - \sum_{i>j} x_{ij}(\pi)$.

The sum R(X(π)) can now be rewritten as:

$$R(\mathbf{X}(\pi)) = \tfrac{1}{2}\sum_{i>j} x_{ij}(\pi)-x_{ji}(\pi)-|x_{ij}(\pi)-x_{ji}(\pi)| = \sum_{i>j} x_{ij}(\pi) - \tfrac{1}{2}\sum_{i>j} x_{ij}(\pi)+x_{ji}(\pi) - \tfrac{1}{2}\sum_{i>j}|x_{ij}(\pi)-x_{ji}(\pi)|$$

The two last terms on the right hand side, $\tfrac{1}{2}\sum_{i>j} x_{ij}(\pi)+x_{ji}(\pi)-\tfrac{1}{2}\sum_{i>j}|x_{ij}(\pi)-x_{ji}(\pi)|$, vary only with the sum of symmetrical elements and the sum of absolute differences. These sums take the same values for every permutation π and obviously, R(X(π)) will be maximized only when $\sum_{i>j} x_{ij}(\pi) = R''[x_{ji}(\pi)]$ is maximized. Also, R'(X(π)) will be minimized only when $\sum_{i>j} x_{ij}(\pi) = R''[x_{ji}(\pi)]$ is maximized as argmax R(X(π)) = argmin R'(X(π)). Thus R(X(π)) = R(X(π*)) implies both R"(X(π)) = R"(X(π*)) and R'(X(π)) = R'(X(π*)).

4 The solution algorithm

Sector π(k) contributes to R(X(π)) by the subsum $r(z_k|z_{k-1},z_{k-2},\dots,z_1)$, which is dependent of how we choose the sectors π(1),π(2),...,π(k-1) in the permutation π. Suppose we already know the k-1 positions in the permutation π then we can choose among n-k+1 possible sectors for π(k). We define the subsum $r(z_k|z_{k-1},z_{k-2},\dots,z_1) = \sum_{i>k} x_{ik}(\pi)-x_{ki}(\pi)$ for $x_{ik}(\pi) < x_{ki}(\pi)$ $(i>k)$. The operator *r*(.) thus sums only negative differences $z_k = x_{ik}(\pi)-x_{ki}(\pi)$ for *i*>*k* given the differences $z_{k-1}, z_{k-2},\dots,z_1$ and there are n-k+1 different values to chose among for *r*(.) at step *k* as there are n-k+1 possible sectors for π(*k*). The triangularization problem is now stated as to find the maximum $R(\mathbf{X}(\pi)) = R(z_1,\dots,z_{n-1}) = r(z_1)+r(z_2|z_1)+ \dots + r(z_{n-2}|z_{n-3},z_{n-4},\dots,z_1)+r(z_{n-1}|z_{n-2},z_{n-3},\dots,z_1)$.

Before defining the conditions for the maximum we introduce the following sums to be used in the theorem and corollaries. Define the following sums, where an asterisk denotes the optimal solution and $\circ \neq *$:

$$R(z_k^\bullet,\dots,z_{n-1}^\bullet) = r(z_k^\bullet|z_{k-1}^\bullet,\dots,z_1^\bullet)+r(z_{k+1}^\bullet|z_k^\bullet,\dots,z_1^\bullet)+\dots+r(z_{n-2}^\bullet|z_{n-3}^\bullet,\dots,z_k^\bullet,\dots,z_1^\bullet)+r(z_{n-1}^\bullet|z_{n-2}^\bullet,\dots,z_k^\bullet,\dots,z_1^\bullet)$$

$$R(z_k^\circ,\dots,z_m) = r(z_k^\circ|z_{k-1}^\bullet,\dots,z_1^\bullet)+r(z_{k+1}|z_k^\circ,z_{k-1}^\bullet,\dots,z_1^\bullet)+\dots+r(z_{m-1}|z_{m-2},\dots,z_k^\circ,z_{k-1}^\bullet,\dots,z_1^\bullet)+r(z_m|z_{m-1},\dots,z_k^\circ,z_{k-1}^\bullet,\dots,z_1^\bullet)$$

Also, define $R(z_1,\dots,z_{k-1}) = r(z_1) + r(z_2|z_1) + \dots + r(z_{k-2}|z_{k-3}, z_{k-4},\dots,z_1) + r(z_{k-1}|z_{k-2}, z_{k-3},\dots,z_1)$.

For $\pi = \pi^*$ we have that $R(z_1,\dots,z_{k-1}) = R(z_1^\bullet,\dots,z_{k-1}^\bullet)$.

THEOREM 1[4]: The permutation $\pi^* = \arg\max_{\pi} R(\mathbf{X}(\pi))$ is absolutely optimal, if for every k, $1 \le k \le n-2$, there is m, $k \le m \le n-1$, such that $R(\mathbf{X}(\pi^\bullet)) = R(z_k^\bullet,\dots,z_{n-1}^\bullet) > R(z_k^\circ,\dots,z_m)$ where $\circ \ne *$.

PROOF: Theorem 1 states that any of the following inequalities must hold for k:

$$\left\{\begin{array}{ll} m=k & R(z_k^\bullet,\dots,z_{n-1}^\bullet) > R(z_k^\circ) \\ m=k+1 & R(z_k^\bullet,\dots,z_{n-1}^\bullet) > R(z_k^\circ,\dots,z_{k+1}) \\ \cdot & \cdot \qquad\qquad \cdot \\ \cdot & \cdot \qquad\qquad \cdot \\ \cdot & \cdot \qquad\qquad \cdot \\ m=n-2 & R(z_k^\bullet,\dots,z_{n-1}^\bullet) > R(z_k^\circ,\dots,z_{n-2}) \\ m=n-1 & R(z_k^\bullet,\dots,z_{n-1}^\bullet) > R(z_k^\circ,\dots,z_{n-1}) \end{array}\right\}$$

Assume there is z° such that the inequalities above do not hold, i.e.

$R(z_k^\bullet,\dots,z_{n-1}^\bullet) < R(z_k^\circ,\dots,z_m) \Rightarrow R(z_k^\bullet,\dots,z_{n-1}^\bullet) < R(z_k^\circ,\dots,z_{n-1})$ for $m = n-1$. Hence $R(\mathbf{X}(\pi^\bullet)) = R(z_1^\bullet,\dots,z_{k-1}^\bullet) + R(z_k^\bullet,\dots,z_{n-1}^\bullet) < R(z_1^\bullet,\dots,z_{k-1}^\bullet) + R(z_k^\circ,\dots,z_m)$, which is a contradiction to the assumption of π^* and thus Theorem 1 states a necessary condition for π^*.

Theorem 1 also states that m and k should exist for π^* such that the inequality $R(\mathbf{X}(\pi^\bullet))$ $= R(z_1^\bullet,\dots,z_{k-1}^\bullet) + R(z_k^\bullet,\dots,z_{n-1}^\bullet) > R(z_1^\bullet,\dots,z_{k-1}^\bullet) + R(z_k^\circ,\dots,z_m)$ always holds. Then $R(\mathbf{X}(\pi^\bullet))$ $= R(z_1^\bullet,\dots,z_{k-1}^\bullet) + R(z_k^\bullet,\dots,z_{n-1}^\bullet) > R(z_1^\bullet,\dots,z_{k-1}^\bullet) + R(z_k^\circ,\dots,z_m) + r(z_{m+1}|z_m,\dots,z_k^\circ,z_{k-1}^\bullet,\dots,z_1^\bullet)$ is always true as all subsums $r(\cdot) \le 0$. Hence, there is no $R(\mathbf{X}(\pi))$ greater than $R(\mathbf{X}(\pi^\bullet))$ and Theorem 1 also states a sufficient condition for π^*.

COROLLARY 1: A sufficient but not necessary condition for π^* is $R(z_k^\bullet,\dots,z_{n-1}^\bullet) > R(z_k^\circ)$ for $1 \le k \le n-2$, and $\circ \ne *$.

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[4]The corresponding theorem for the dual minimization problem was derived by Haltia (1992).

PROOF: The proof follows from Theorem 1 by putting $m = k$. Corollary 1 is not a necessary condition because $R(z_k^*,\ldots,z_{n-1}^*) < R(z_k^*)$ can still produce $R(z_k^*,\ldots,z_{n-1}^*) > R(z_k^*,\ldots,z_m)$ as all subsums $r(\cdot) \le 0$.

COROLLARY 2. If the sufficiency condition in Corollary 1 holds for all k, the maximum sum $R(\mathbf{X}(\pi*))$ is the sum of submaxima $R^s(\mathbf{X}(\pi)) = R^s(z_1,\ldots,z_k,\ldots,z_{n-1})$ defined as:
$R^s(z_1,\ldots,z_k,\ldots,z_{n-1}) = r^s(z_1)+\ldots+r^s(z_k|z_{k-1},\ldots,z_1)+\ldots+r^s(z_{n-1}|z_{n-2},\ldots z_k,\ldots,z_1)$ where for all k
$r^s(z_k|z_{k-1},\ldots,z_1) = \max_i r_i(z_k|z_{k-1},\ldots,z_1) \quad i \le n-k+1$.

PROOF. For Corollary 1 to hold it must be that $R(z_k^*,\ldots,z_{n-1}^*) > R(z_k^*,\ldots,z_m)$ as all subsums $r(\cdot) \le 0$. This can be true iff $r(z_k^*|z_{k-1}^*,\ldots,z_1^*) = r^s(z_k|z_{k-1}^*,\ldots,z_1^*)$ and when repeated for all k, $R(\mathbf{X}(\pi*)) = R^s(\mathbf{X}(\pi))$.

COROLLARY 3. The maximum $R(\mathbf{X}(\pi*))$ will not necessarily be the sum of submaxima $R^s(\mathbf{X}(\pi))$ and we can have that $R(\mathbf{X}(\pi*)) > R^s(\mathbf{X}(\pi))$.

PROOF. Suppose there is k for which the sufficiency condition of Corollary 1 not holds; $R(z_k^*,\ldots,z_{n-1}^*) \le R(z_k^*)$. Clearly, for the first term of $R(z_k^*,\ldots,z_{n-1}^*)$ we could have either $r(z_k^*|z_{k-1}^*,\ldots,z_1^*) = r^s(z_k|z_{k-1},\ldots,z_1)$ or $r(z_k^*|z_{k-1}^*,\ldots,z_1^*) \ne r^s(z_k|z_{k-1},\ldots,z_1)$. The necessary condition of Theorem 1 must hold, and with $r(z_k^*|z_{k-1}^*,\ldots,z_1^*) \ne r^s(z_k|z_{k-1},\ldots,z_1) = r(z_k^*|z_{k-1}^*,\ldots,z_1^*)$ we have that
$r(z_k^*|z_{k-1}^*,\ldots,z_1^*) + r(z_{k+1}|z_k^*,z_{k-1}^*,\ldots,z_1^*)+\ldots+r(z_{m-1}|z_{m-2},\ldots,z_k^*,z_{k-1}^*,\ldots,z_1^*) + r(z_m|z_{m-1},\ldots,z_k^*,z_{k-1}^*,\ldots,z_1^*) < R(z_k^*,\ldots,z_{n-1}^*)$
At least one subsum of $R(\mathbf{X}(\pi*))$ is thus not a submaximum and it follows that $R(\mathbf{X}(\pi*))$ $> R^s(\mathbf{X}(\pi))$.

The algorithm starts by computing the sum of submaxima $R^s(\mathbf{X}(\pi)) = r^s(z_1) + r^s(z_2|z_1) + \ldots + r^s(z_{n-2}|z_{n-3},z_{n-4},\ldots,z_1) + r^s(z_{n-1}|z_{n-2},z_{n-3},\ldots,z_1)$. In view of Corollary 3 we must verify that the sum of submaxima is a true optimum and satisfies the sufficiency condition. We then examine the sufficiency condition by backtracking to a position $1 \le k \le n-1$ where Corollary

1 does not hold; $r^s(z_k|z_{k-1},...,z_1)+r^s(z_{k+1}|z_k,...,z_1)+...+r^s(z_{n-1}|z_{n-2},...,z_k,...,z_1) \le r^s(z_k^\circ|z_{k-1},...,z_1)$ for $r^s(z_k|z_{k-1},...,z_1) \ne r^s(z_k^\circ|z_{k-1},...,z_1)$. We now have the first k positions in the permutation π; $\pi(1),\pi(2),...,\pi(k-1),\pi(k)$. As Corollary 1 not holds at position k we try the alternative permutation π°; $\pi(1),\pi(2),...,\pi(k-1),\pi^\circ(k),...,\pi(m)$. The permutations that will not satisfy the necessary condition in Theorem 1 are rejected and the alternative permutation that gives a sum $R(z_k^\circ,...,z_m) > R(z_k,...,z_{n-1})$ is expanded further until $m=n-1$. If there is no sum $R(z_k^\circ,...,z_m) \le R(z_k,...,z_{n-1})$ we again backtrack to a position where Corollary 1 does not hold, and try new alternative permutation. This is continued for all k and if still $R(z_k^\circ,...,z_m) \le R(z_k,...,z_{n-1})$ when $k=1$, the process stops with $R(\mathbf{X}(\pi^*)) = R(\mathbf{X}(\pi))$ (initially we have $R(\mathbf{X}(\pi)) = R^s(\mathbf{X}(\pi))$). If we reach the position $m=n-1$, the process starts again but now with the alternative permutation π° in place of π. We are no longer on the path of submaxima and again the process will terminate when position $k=1$ has been examined.

5 Large matrix properties

The process above sets an upper bound for the number of permutations searched by the algorithm. This bound may not be efficient enough (in terms of computer time) when matrices are large. When triangularizing such large matrices, we set an upper bound that gives a suboptimal solution of the triangularization problem. This solution converges towards the optimal solution, as the upper bound increases. The lower the value of the upper bound, the more approximate is the solution of the triangularization problem and the higher the value of the upper bound, the closer we get to the optimal solution.

Let nt be the upper bound for the number of subsums in $R(z_k,...,z_\ell)$. We examine at least two subsums and at most n-1 subsums and thus $2 \le nt \le n-1$ giving $1 \le \ell - k \le nt-1$. For the optimal solution we have especially that $\ell = nt = n-1$. This implies that $1 \le k \le n-2$, $k \le m \le n-1$ and Theorem 1 applies.

DEFINITION: The ordering $\pi^{\triangleright}$ is a relatively optimal ordering if for any ℓ, $nt \le \ell \le n-1$ and any k, $\ell - nt + 1 \le k \le \ell - 1$ there is m, $k \le m \le \ell$ such that $R(z_k^{\triangleright},\ldots,z_\ell^{\triangleright}) > R(z_k^{*},\ldots,z_m)$ where $\triangleright \ne *$. Thus the following inequalities should always be true for ℓ, k and m:

$$\left\{\begin{array}{ll} m = k & R(z_k^{\triangleright},\ldots,z_\ell^{\triangleright}) > R(z_k^{*}) \\ m = k+1 & R(z_k^{\triangleright},\ldots,z_\ell^{\triangleright}) > R(z_k^{*},\ldots,z_{k+1}) \\ \cdot & \cdot \qquad\qquad \cdot \\ \cdot & \cdot \qquad\qquad \cdot \\ \cdot & \cdot \qquad\qquad \cdot \\ m = \ell - 1 & R(z_k^{\triangleright},\ldots,z_\ell^{\triangleright}) > R(z_k^{*},\ldots,z_{\ell-1}) \\ m = \ell & R(z_k^{\triangleright},\ldots,z_\ell^{\triangleright}) > R(z_k^{*},\ldots,z_\ell) \end{array}\right\}$$

THEOREM 2: For any given nt, each ordering π other than $\pi^{\triangleright}$ generates a sum $R(\mathbf{X}(\pi))$ $\le R(\mathbf{X}(\pi^{\triangleright}))$.

PROOF: The proof is given by Theorem 1 with $nt = n-1$.

THEOREM 3: For $\pi = \pi^{*}$ we have $R(\mathbf{X}(\pi^{\triangleright})) \le R(\mathbf{X}(\pi^{*}))$ for any nt, $2 \le nt \le n-1$.

PROOF: We have that $R(\mathbf{X}(\pi^{\triangleright})) = R(\mathbf{X}(\pi^{*}))$ for $nt = n-1$ according to Theorem 1 but $R(\mathbf{X}(\pi^{\triangleright}))^{nt<n-1} > R(\mathbf{X}(\pi^{\triangleright}))^{nt=n-1}$,which contradicts the assumption of $R(\mathbf{X}(\pi^{*}))$.

When applying the algorithm for large matrices we start from $nt < n-1$, say nt^0, instead of $nt = n-1$ and increase nt until $R(\mathbf{X}(\pi^{\triangleright}))_{nt} = R(\mathbf{X}(\pi^{\triangleright}))_{nt^T} = \mathrm{Max}\{R(\mathbf{X}(\pi^{\triangleright}))_{nt^T-1},\ldots$ $,R(\mathbf{X}(\pi^{\triangleright}))_{nt^0}\}$.

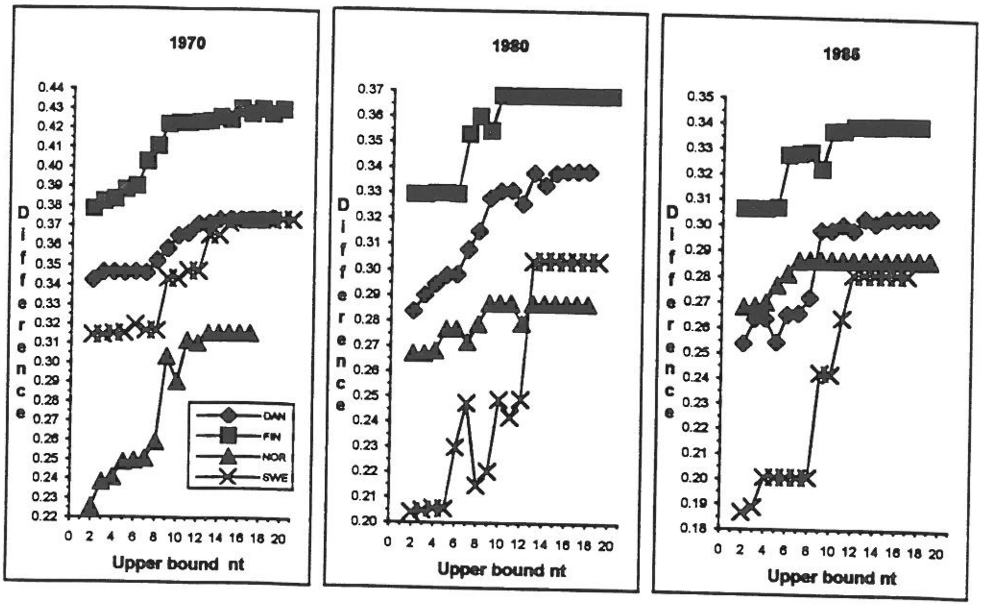
When $nt < n-1$ and we still want to try if $R(\mathbf{X}(\pi^{\triangleright}))_{nt}$ can be increased, we start the process again with $nt^0 = nt^T + 1$. At each step, for nt, put k=ℓ-nt+1. Start from ℓ=n-1 and decrease ℓ until k=1. For decreasing ℓ, take as the rule $R(z_{\ell-m}^{\triangleright},\ldots,z_\ell^{\triangleright}) = 0$ to put ℓ equal to ℓ-m in the next step. For every ℓ continue as with the algorithm described for finding the optimal solution, $nt = n-1$, in preceding sections. The suboptimal solution coincides with the optimal solution at the final step for nt if the permutations $\pi(k),\pi(k+1),\ldots,\pi(\ell-1),\pi(\ell)$ and $\pi^*(k),\pi^*(k+1),\ldots,\pi^*(\ell-1),\pi^*(\ell)$ are rearrangements of the same set for every k>1.

Only the dimension of the matrix and the number of circular relations in the matrix can set a computational time limit before we reach $nt = n-1$. The suboptimal solutions converge to the optimal solution at $nt = n-1$. This convergence can be measured by the degree of linearity, $R''[x_{ji}(\pi)]/\sum_{i \neq j} x_{ij}(\pi)$, for different suboptimal solutions.

6 An example of convergence and intertemporal similarity

The convergence of the algorithm is illustrated in figure 1 for n=24 with the input-output tables of Denmark, Finland, Norway and Sweden for the years 1970, 1980 and 1985. Convergence is reached at different values of *nt*. The Danish table of 1970 (DAN70)

Figure 1 Difference in linearity compared to the initial solution when the upper bound nt increases.



converges for *nt*=14, DAN80 for *nt*=16 and DAN85 for *nt*=17. The Finnish table of 1970 (FIN70) converges for *nt*=16, FIN80 converges for *nt*=13 and FIN85 converges for *nt*=15. The Norwegian table of 1970 (NOR70) converges for *nt*=13, NOR80 converges for *nt*=9 and

NOR85 converges for *nt*=15.The Swedish table of 1970 (SWE70) converges for *nt*=20, SWE80 converges for *nt*=14 and SWE85 converges for *nt*=12. The triangularized tables are given in appendix III.

Table I in appendix I suggests that sectors can be arranged in a similar order among the Nordic countries over time. Foods, beverages and tobacco products and Textiles and lether, which link directly to final demand are recorded as sectors at the top of the production hierarchy. The energy sectors, Petroleum refineries and Utilities, together with Communications are recorded as sectors at the bottom of the production hierarchy. We note that several solutions with the same value for the degree of linearity can exist. This is due to zero differences between above-diagonal and below-diagonal symmetrical elements.[5] We have four solutions for FIN70, NOR70, NOR80, SWE80 and SWE85, three for SWE68, two for NOR85. For DAN70, DAN80, DAN85, FIN80 and for FIN85 unique orderings are identified.

Table 2 Spearman rank correlation coefficients among pairs of countries

	1970	1980	1985
FIN-SWE	.6191	.8783	.8730
DEN-FIN	.5661	.8348	.8730
DEN-SWE	.8461	.7452	.8252
NOR-SWE	.7817	:8261	.8783
NOR-FIN	.5104	.8270	.9687
NOR-DEN	.8600	.7713	.8591

Production structures for pairs of countries are compared by Spearman rank correlation coefficients in table 2. Correlation coefficients for pairs of countries are high and increasing over compared years. This gives further support to the findings of hierarchical similarity between different countries reported in international comparisons made by others (c.f. Chenery and Watanabe (1958), Simpson and Tsukui (1965), Korte and Oberhofer (1971) and Fukui (1986)). Inspection of table 2 also confirm the impression in table I of the Appendix,

[5] The existence of zero differences is examined outside the algorithm by a recursive procedure (cf. Östblom (1986), p.102).

that the hierarchical similarity between countries holds also over time for Denmark, Finland Norway and Sweden.

7 Conclusions

The triangularization problem was here considered as a maximization problem, and necessary and sufficient conditions were formulated in theorem 1. We presented a branch and bound algorithm that takes advantage of these conditions for bounding the number of permutations that must be examined to find the optimal solution. We also defined conditions for a suboptimal solution (theorem 2) with the number of sectors in the search tree being less than the total number of sectors. It was proved (theorem 3) that the optimal solution is the limit of the suboptimal solutions when the number of sectors in the search tree converges towards the total number of sectors.

The convergence property of the algorithm was illustrated by computing the suboptimal solutions for the input-output tables of 1970, 1980 and 1985 for the four Nordic countries Denmark, Finland, Norway and Sweden. We also examined the intertemporal similarity of hierarchical structures of production for the Nordic countries. We found that the hierarchical similarity of production structures among countries holds over time for Denmark, Finland Norway and Sweden.

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APPENDIX I

Table I Ordering of sectors in triangular arrangement.

Label of sector	DAN70	FIN70	NOR70	SWE68	DAN80	FIN80	NOR80	SWE80	DAN88	FIN85	NOR85	SWE85
1 Agriculture	6	7	5	6	6	6	5	6	6	7	7	6
2 Forestry	5	24	6	5	20	5	6	17	5	6	6	5
3 Fishing	1	6	1	1	7	1	20	5	1	5	5	1
4 Mining, quarring&drilling*	20	5	20	20	2	3	1	1	3	1	1	20
5 Foods	3	3	17	9	17	20	3	3	20	3	3	17
6 Beverages&tobacco prod.	13	1	3	3	8	7	15	20	17	20	20	3
7 Textiles & leather	4	20	15	8	5	17	16	9	8	17	17	9
8 Wood and wood products	15	13	16	2	13	9	17	8	7	9	9	8
9 Paper & paper products	8	21	14	15	1	8	14	2	13	8	15	2
10 Printing & publishing	2	10	8	13	3	2	8	15	2	13	8	15
11 Chemicals	7	17	7	16	15	13	7	16	15	2	2	16
12 Rubber&plastics	17	9	11	7	16	15	11	13	16	15	16	14
13 Non-metal mineral products	16	8	13	17	14	16	13	12	12	16	14	13
14 Basic metals	14	2	4	14	11	14	4	14	14	12	13	4
15 Machinery&transport equipmen	11	11	21	4	21	12	21	4	11	14	21	7
16 Electrical prod.	21	22	12	21	12	11	12	7	21	11	12	11
17 Other manufactures	12	15	10	22	22	4	10	11	22	4	11	21
18 Petroleum refineries	22	16	9	24	24	21	9	21	24	21	18	22
19 Utilities	24	14	22	12	10	22	22	22	10	22	4	24
20 Construction	10	4	2	11	9	24	2	24	9	24	22	12
21 Trade	9	12	18	10	23	23	18	10	19	19	24	10
22 Transport	19	23	24	23	19	19	24	19	23	18	10	19
23 Communicatons	23	19	19	19	18	10	19	23	18	10	19	23
24 Services	18	18	23	18	4	18	23	18	4	23	23	18

Table II Degree of linearity when the number of sectors in the search tree (nt) increases.

NT	DAN70	FIN70	NOR70	SWE68	DAN80	FIN80	NOR80	SWE80	DAN85	FIN85	NOR85	SWE85
INITIAL	.4122	.3744	.4494	.4168	.4263	.4420	.4768	.4648	.4601	.4567	.4753	.4732
2	.7548	.7526	.6748	.7312	.7102	.7712	.7439	.6690	.7140	.7631	.7436	.6601
3	.7587	.7565	.6876	.7312	.7166	.7713	.7439	.6697	.7236	.7632	.7440	.6621
4	.7587	.7578	.6900	.7321	.7210	.7719	.7452	.6706	.7238	.7632	.7454	.6740
5	.7584	.7626	.6979	.7323	.7244	.7719	.7538	.6704	.7147	.7638	.7522	.6740
6	.7590	.7644	.6988	.7366	.7249	.7715	.7538	.6948	.7256	.7842	.7566	.6740
7	.7584	.7770	.7000	.7336	.7343	.7952	.7487	.7125	.7262	.7849	.7617	.6740
8	.7645	.7851	.7085	.7336	.7415	.8021	.7559	.6796	.7322	.7855	.7619	.6740
9	.7708	.7962	.7525	.7603	.7543	.7965	.7641	.6853	.7583	.7790	.7619	.7156
10	.7771	.7970	.7396	.7603	.7573	.8104	.7641	.7139	.7583	.7936	.7619	.7151
11	.7785	.7970	.7612	.7644	.7574	.8102	.7641	.7069	.7605	.7941	.7619	.7374
12	.7833	.7976	.7598	.7644	.7524	.8104	.7562	.7140	.7583	.7958	.7619	.7543
13	.7839	.7981	.7652	.7822	.7644	.8105	.7641	.7684	.7631	.7958	.7619	.7543
14	.7857	.8003	.7652	.7822	.7597	.8105	.7641	.7686	.7612	.7958	.7619	.7543
15	.7857	.7989	.7652	.7884	.7644	.8105	.7641	.7686	.7632	.7961	.7619	.7543
16	.7857	.8047	.7652	.7900	.7652	.8105	.7641	.7685	.7632	.7961	.7619	.7543
17	.7857	.8023	.7652	.7900	.7652	.8105	.7641	.7686	.7637	.7961	.7622	.7543
18	.7857	.8047		.7902	.7652	.8105	.7641	.7686	.7637	.7961	.7621	
19	.7867	.8023		.7904		.8105		.7686	.7637		.7622	
20		.8047		.7909		.8107						
21				.7909								

APPENDIX II

The computer program

GAUSS-386i VM Version 3.01

```
#linesoff;
load path = c:\triang\data;
print "Name of io-matrix";
file1=cons;
print "Number of sectors";
n=con(1,1);
print "Number of sectors in the search tree";
nt=con(1,1);
print "Name of permutation vector";
file3=cons;
bsl=chrs(92);
stig = "c:"$+bsl$+"triang"$+bsl$+"data"$+bsl;
file3 = stig $+ file3;
loadm x[n,n] = ^file1;
output file = ^file3 reset;
output file = ^file3 off;
start = -9999999999999999;
ppmax = 0;
rnkmax = 0;
pmax = zeros(1,n);
test = start;
nt0 = nt;
rtime0 = 0;
y1=ones(1,n);
y2 = y1;
nynt:
time0=date;
z=zeta(x);
m = n - 1;
ittn = n;
if nt > n;
nt = n;
endif;
ittm = ittn - nt;
if ittm < 1;
ittm = 1;
endif;
ittp3 = m;
m4=m+4;
m3=m+3;
m1=m+1;
m2=m+2;
p2=zeros(1,n);
p3 = p2;
ceros=zeros(n,1);
cc0=zeros(n,m4);
p00 = talserie(n);
nystart:
cc=zeros(n,m4);
c1=zeros(1,n);
p1=zeros(1,n);
v1=zeros(1,n);
p0 = p00;
itt=1;
y0=0;
do until itt > ittp3;
     v1=urval(y0,p0);
     z0=submat(z,v1,v1);
     c=sumc(z0);
     cind=rev(sortind(c));
     c1[itt]=c[cind[y1[itt]]];
     p1[itt]=v1[cind[y1[itt]]];
/****skapar testmatris****/
     it=1;
     do until it > itt;
     cc[it,itt]=c1[itt];
```

```
      it=it+1;
      endo;
      kant=ittp3-itt+1;
      if yl[itt] >= kant;
      ii = 1;
      else;
      ii=yl[itt]+1;
      endif;
      cc[itt,m2]=c[cind[ii]];
      cc[itt,m4] = vl[cind[ii]];
      cc[itt,m3]=ii;
      cc[.,ml]=sumc(cc[.,1:m]');
 /****skapar testmatris****/
      if test >= cc[1,ml];
      testl = 0;
      nyrad:
      cl[itt]=0;
      pl[itt]=0;
      yl[itt] = 1;
      cc[1:m,itt]=ceros[1:m,1];
      cc[itt,m2]=0;
      cc[itt,m3]=0;
      cc[itt,m4]=0;
      itt=itt-1;
         if itt < ittm;
         break;
         endif;
      pitt=pl[itt];
      let vnames= vl pitt;
      vlp=mergevar(vnames);
      vl=sortc(vlp',1);
      vl=vl';
      p0=vl;
      y0=0;
      if itt > ittn;
      goto nyrad;
      endif;
 /*****sufficiency test*****/
      if itt > 1;
      testl = testl + cl[itt];
      test2 = sumc(cl[1:itt-1]') + cc[itt,m2];
      if testl > cc[itt,m2] AND test >= test2;
 /*    if testl > cc[itt,m2];*/
 /*    if test >= test2;*/
      goto nyrad;
      elseif cc[1,itt] == 0;
      goto nyrad;
      endif;
      endif;
 /*****sufficiency test*****/
      if cc[itt,m3] > kant;
      goto nyrad;
      else;
      yl[itt] = cc[itt,m3];
      endif;
      else;
          y0=vl[cind[yl[itt]]];
          p0=vl;
          itt=itt+1;
     endif;
 endo;
 if itt >= ittp3;
 test = cc[1,ml];
 cc0 = cc;
 c2 = cl;
 p2 = pl;
 p2[ittn] = setdif(p00',p2',1);
 p4 = p2 + p3;
 y2 = yl;
 format /rd 4,0;
 print;
 print "ittm" ittm;
 print "ittn" ittn;
 zz=submat(x,p4,p4);
 zz0=lowmatl(zz);
 pp=sumc(zz0);
 pp=sumc(pp)-n;
 prank=pp/(sumc(sumc(lowmatl(zz)+upmatl(zz)))-2*n);
 print;
 print p4;
```

```
format /rd 8,1;
print "Sum of elements below diagonal=     " pp;
goto nystart;
endif;
ittk0 = ittn;
do while cc0[ittk0,m1] == 0;
ittk0 = ittk0 - 1;
if ittk0 < 1;
break;
endif;
endo;
ittn0 = ittk0 + 0;
if ittn0 > ittn;
ittn0 = ittn;
elseif ittn0 < nt;
ittn0 = ittn;
endif;
p3[ittn0:ittn] = p2[ittn0:ittn];
p00p3 = setdif(p00',p3',1);
p00 = p00p3';
ittn = ittn0 - 1;
ittm = ittn - nt;
if ittm < 1;
ittm = 1;
endif;
ittp3 = ittn - 1;
if ittn > 1;
    y5 = ones(1,n);
    y2 = y5;
    p5 = zeros(1,n);
    c5 = p5;
    y00 = 0;
    p000=p00;
    ittk = 1;
    do until ittk > ittn;
    v5=urval(y00,p000);
    z0=submat(z,v5,v5);
    cc5=sumc(z0);
    cind5=rev(sortind(cc5));
    kantc5 = ittn - ittk + 1;
    do while y5[ittk] <= kantc5;
    p5[ittk]=v5[cind5[y5[ittk]]];
    if p5[ittk] == p4[ittk];
    break;
    endif;
    y5[ittk] = y5[ittk] + 1;
    endo;
    c5[ittk]=cc5[cind5[y5[ittk]]];
    y00=v5[cind5[y5[ittk]]];
    p000=v5;
    ittk=ittk+1;
    endo;
    test = sumc(c5[1:ittn]');
    itto = ittm - 1;
    if itto < 1;
    y1 = ones(1,n);
    else;
    y1[1:itto] = y5[1:itto];
    endif;
    y1 = y5;
/*    test=start;*/
goto nystart;
endif;
yes = "yes";
no = "no";
time1=date;
rtime=ethsec(time0,time1);
rtime=rtime/100;
rtime0 = rtime0 + rtime;
if pp >= ppmax;
output file = ^file3 on;
print "**************************************************************";
print "Runtime for this search in secs =          " rtime;
print "Total runtime in secs           =          " rtime0;
print;
format /rd 8,1;
print "Greatest sum in preceeding searches:       " ppmax;
print "Sector ordering with greatest sum:         ";
format /rd 6,4;
print "Degree of linearity:                       " rnkmax;
```

```
format /rd 4,0;
print  pmax;
print;
format /rd 8,1;
print "Greatest sum in this search:                " pp;
print "Sector ordering in this search:             ";
format /rd 6,4;
print "Degree of linearity:                        " prank;
format /rd 4,0;
print  p4;
print;
print "Initial number of sectors in search tree:" nt0;
print "Current number of sectors in search tree:" nt;
print;
print "*************************************************************";
output file = ^file3 off;
print "Continue?";
print "Answer Yes or No";
answer = cons;
ppmax = pp;
rnkmax=prank;
pmax = p4;
else;
if nt < m;
answer = yes;
else;
answer = no;
endif;
endif;
if answer $== yes;
    y5 = ones(1,n);
    y2 = y5;
    p5 = zeros(1,n);
    c5 = p5;
    y00 = 0;
    p000=talserie(n);
    ittk = 1;
    do until ittk > n;
    v5=urval(y00,p000);
    z0=submat(z,v5,v5);
    cc5=sumc(z0);
    cind5=rev(sortind(cc5));
    kantc5 = n - ittk + 1;
    do while y5[ittk] <= kantc5;
    p5[ittk]=v5[cind5[y5[ittk]]];
    if p5[ittk] == pmax[ittk];
    break;
    endif;
    y5[ittk] = y5[ittk] + 1;
    endo;
    c5[ittk]=cc5[cind5[y5[ittk]]];
    y00=v5[cind5[y5[ittk]]];
    p000=v5;
    ittk=ittk+1;
    endo;
   if pp == ppmax;
   y1 = ones(1,n);
    else;
    y1 = y5;
    endif;
    nt = nt + 1;
    test = start;
time0=date;
goto nynt;
elseif answer $== no;
goto stopp;
endif;
stopp:
output file = ^file3 on;
print "*************************************************************";
print "Runtime for this search in secs:            " rtime;
print "Total runtime in secs:                      " rtime0;
print;
format /rd 8,1;
print "Greatest sum for all searches:              " ppmax;
print "Sector ordering with greatest sum:          ";
format /rd 6,4;
print "Degree of linearity:                        " rnkmax;
format /rd 4,0;
print  pmax;
```

```
print;
print "Initial number of sectors in search tree:" nt0;
print "Current number of sectors in search tree:" nt;
print "************************************************************";
output file = ^file3 off;
end;


#linesoff;
PROC (1) = talserie(n);
/* Procedure till algtri.prg */;
    LOCAL i,serie;
    i=0;
    serie=zeros(1,n);
    do while i < n;
        i=i+1;
        serie[1,i]=i;
    endo;
    RETP(serie);
ENDP;


#linesoff;
Proc (1) = urval(ynoll,pnoll);
/* Procedure till algtri.prg */;
/* v„ljer sektorer till matris z0 */;
    LOCAL vnoll,vett,vtva,vtre;
    vnoll=pnoll./=ynoll;
    vett=vnoll.*pnoll;
    vtva=miss(vett,0);
    vtre=packr(vtva');
    vett=vtre';
    RETP(vett);
ENDP;


#linesoff;
PROC (1) = zeta(x);
/* Procedure till algtri.prg */
    LOCAL l,u,d,dabs,zij,zji,z;
    l=lowmatl(x);
    u=lowmatl(x');
    d=l-u;
    dabs=abs(d);
    zij=d-dabs;
    zji=d+dabs;
    z=zij-zji';
    RETP(z);
ENDP;
```

APPENDIX III

Triangularized input-output tables

Triangularized Input-Output Table 1970 for Denmark

Degree of linearity: 0.7867

	6	5	1	20	3	13	4	15	8	2	7	17	16	14	11	21	12	22	24	10	9	19	23	18
6 Beverages&tobacco prod.	45	15	13	2				2			1				15	10		2	185	1				
5 Foods	104	3319	956	1	4			1	2		16				143	1	1		599		2			
1 Agriculture	61	9254	1319	187		4	1	17	2	1	10	1	9	1	11	34	1	25	183	13	12	4	6	1
20 Construction	16	55	220	33	2	26	1	41	11	18	11	2	11	3	22	472	5	253	1654	19	6	219	259	1
3 Fishing		397	25		14														12					
13 Non-metal mineral products	47	25	8	1934		273		21	8		1	1	3	9	17	33	2		1		1			5
4 Mining,quarring&drilling*		8	3	41		74	3	1						1	2							1		72
15 Machinery&transport equipmen	84	290	455	1185	67	55	7	1201	56	3	24	8	141	47	80	110	8	238	145	19	11	31	15	21
8 Wood and wood products	15	15	7	687	10	5		68	289		22	6	3	1	10	21	3	4	28		11			
2 Forestry				19				9	156												3			
7 Textiles & leather	3	3	13	33	10	1		27	48		690	3	2		3	38	6	3	21	7	3	1	1	
17 Other manufactures		4	1	22		1		7	3		11	10	1		4	12	1	2	7	6	2	1	1	
16 Electrical prod.	3	15	14	603	3	14	3	147	6		5	1	171	35	8	36	3	18	71	4	4	12	6	5
14 Basic metals	3	11	3	180	1	20	1	371	4		1	9	191	203	3	10	1	3	4	5	4	3	1	1
11 Chemicals	9	71	351	189	2	12	1	108	41		19	10	28	5	139	66	80	20	77	38	15	6	3	3
21 Trade	65	375	822	1079	29	128	4	712	147	1	126	37	155	283	71	284	21	245	782	85	68	41	15	43
12 Rubber&plastics	4	64	9	61	2	6	1	97	24		25	12	25	1	88	104	57	18	26	4	4	3	2	1
22 Transport	74	349	59	593	50	144	39	166	82	8	61	8	32	30	99	1365	20	1303	163	130	3	11	82	8
24 Services	85	187	415	1424	11	42	13	249	75	6	101	17	75	5	120	775	31	695	4646	182	32	40	72	6
10 Printing & publishing	32	47	26	54	2	10	2	66	17	1	26	4	20	1	38	304	9	73	802	806	13	12	15	2
9 Paper & paper products	16	234	7	49		38	3	63	21		30	14	30	2	37	144	18	6	32	166	107	1	1	5
19 Utilities	18	101	133	43		56	5	84	27		30	5	21	19	40	86	18	15	107	18	17	17	7	24
23 Communicatons	26	51	38	35	3	11	4	77	20	2	31	5	24	2	41	172	10	81	380	45	10	13	10	2
18 Petroleum refineries	14	50	75	389	16	35	3	18	4		6	1	2	4	27	64	3	112	53	13	7	161	6	105

Triangularized Input-Output Table 1980 for Denmark

Degree of linearity: 0.7652

	6	20	7	2	17	8	5	13	1	3	15	16	14	11	21	12	22	24	10	9	23	19	18	4
6 Beverages&tobacco prod.	240	7	8			1	42	2	29		9	2		27	29	1	8	656	5	2	2	2	1	
20 Construction	45	68	31	11	5	4	147	53	583	8	171	36	20	67	1210	13	651	6982	53	11	877	1127	4	3
7 Textiles & leather	1	45	687		9		7	3	8	13	29	5	1	9	30	5	5	27	4	1	1	2	5	
2 Forestry		35						5			3													
17 Other manufactures	2	59	19		65	1	11	3	2	1	27	6	1	6	23	2	6	22	15	2	1	2	1	1
8 Wood and wood products	22	1683	19		16		30	20	6	1	195	19	5	18	6	2	4	54	2	2		1	1	1
5 Foods	164	12	855		3	390	13505	7	2826	10	22	2	1	308	11	3	5	1414	4	27	1	2	2	1
13 Non-metal mineral products	98	4431	68		1	5	102	682	6	1	59	9	25	53	27	5	3	19	1	1		1	34	10
1 Agriculture	324	382	163	1	10	7	25700	21	4733	1	63	34	7	38	165	5	74	479	40	547	20	22	3	2
3 Fishing			734			533	1269		5	20								27						
15 Machinery&transport equipmen	205	3185	189	10	45	13	860	190	1338	213	4027	359	115	254	373	38	1010	422	69	30	12	71	83	30
16 Electrical prod.	19	1116	24	1	15	4	74	50	24	15	561	523	36	44	72	13	104	213	16	8	4	26	26	11
14 Basic metals		193	4		6		15	36	1		852	148	109	7	8	3	1	8	3	6			1	
11 Chemicals	23	444	72		55	7	205	60	962	9	384	63	19	637	134	272	42	182	119	44	3	8	12	4
21 Trade	135	2409	514	3	123	32	1168	375	2569	72	2196	344	421	278	704	82	914	2398	328	277	47	202	101	14
12 Rubber&plastics	25	206	71		36	18	256	26	43	35	225	64	4	145	294	126	65	83	17	19	5	9	10	10
22 Transport	269	1139	322	20	43	36	1176	369	152	132	796	145	62	379	4062	98	5234	694	570	115	171	64	19	236
24 Services	239	4271	301	23	57	69	1230	312	1240	26	1438	318	64	571	3330	120	2731	17128	672	128	337	352	58	60
10 Printing & publishing	41	188	65	5	12	14	237	59	59	7	283	59	12	147	778	44	242	2706	2658	65	53	71	15	9
9 Paper & paper products	77	61	145		30	23	777	60	27	1	136	54	4	136	477	48	16	106	364	278	3	5	3	9
23 Communicatons	41	90	49	3	10	11	188	53	55	4	243	55	11	100	404	20	222	949	102	22	67	62	10	8
19 Utilities	72	139	144	1	21	34	475	167	424		304	73	131	195	677	70	110	750	83	42	74	121	68	11
18 Petroleum refineries	34	1089	55	1	7	40	293	145	454	128	134	33	47	95	337	30	1123	233	20	18	40	905	731	25
4 Mining,quarrying&drilling*		134	5			1	31	162	30		1		6	14				5					477	33

Triangularized Input-Output Table 1985 for Denmark

Degree of linearity: 0.7637

	6	5	1	3	20	17	8	7	13	2	15	16	12	14	11	21	22	24	10	9	19	23	18	4
6 Beverages&tobacco prod.	312	121	42		12	1	5	3	2		17	4	3	1	50	53	15	1040	10	1	6	6	1	2
5 Foods	278	13064	3404	6	13	7	23	56	9		18	2	6	1	470	14	4	2393	8	46	2	1	1	1
1 Agriculture	286	36595	6474	2	364	20	21	35	35	4	133	65	10	12	84	382	144	1061	72	120	56	33	3	10
3 Fishing		1990	8	2											1			60						
20 Construction	70	217	943	30	133	22	47	45	55	16	328	75	22	20	103	1967	1209	11403	90	30	1849	1559	5	79
17 Other manufactures	5	27	6	2	81	132	12	36	6		71	12	7	5	14	45	12	52	16	7	6	4	2	3
8 Wood and wood products	28	50	6	1	2422	47	1411	27	25		259	25	3	12	15	8	3	117	6	49	2		1	1
7 Textiles & leather	2	8	12	24	48	15	200	2006	4		54	6	8	1	14	39	40	50	15	2	4	3	8	1
13 Non-metal mineral products	133	136	8		6413	3	208	6	931		144	9	20	24	71	20	3	13	1	15	2		41	15
2 Forestry			1		12		616	1	2		5									10				
15 Machinery&transport equipmen	320	1188	2079	414	4547	112	378	77	253	13	8432	726	61	184	424	542	1068	836	130	81	156	29	64	57
16 Electrical prod.	40	131	94	21	1140	41	26	29	83	1	1244	1124	25	40	84	155	124	384	41	30	83	16	22	30
12 Rubber&plastics	24	293	47	31	403	57	67	69	33	1	487	150	212	4	183	324	104	184	31	29	12	11	12	18
14 Basic metals	3	35	3	1	160	11	15	5	57		1272	144	5	204	14	17	4	13	8	11	2		1	1
11 Chemicals	26	282	1470	9	774	92	212	145	81	1	683	97	448	30	1211	231	83	321	193	79	20	11	22	10
21 Trade	255	1889	3807	128	3950	295	1228	1040	604	4	4456	673	137	765	503	1257	1288	4597	706	658	242	80	98	40
22 Transport	432	1577	149	262	2903	82	355	414	438	22	1347	259	161	77	609	6995	11202	1432	982	183	83	367	32	1278
24 Services	401	1838	2075	55	9291	128	709	526	467	39	2841	643	244	109	1129	6016	4568	33154	1200	193	743	975	109	274
10 Printing & publishing	57	270	97	9	328	23	101	75	67	5	426	98	60	13	206	1039	320	5200	4637	63	132	133	19	34
9 Paper & paper products	168	1233	45	1	75	103	135	73	91		268	83	97	6	234	653	24	180	695	583	10	9	7	21
19 Utilities	113	733	591		219	37	189	122	279	1	529	114	122	204	331	1042	225	1229	173	110	420	121	115	16
23 Communicatons	73	292	153	6	446	23	114	88	80	4	503	116	42	18	206	792	379	2486	189	33	142	109	19	27
18 Petroleum refineries	70	494	422	290	1520	11	56	66	146	3	235	65	55	35	167	592	2406	369	36	41	900	70	926	30
4 Mining,quarring&drilling*		50	60		394			2	246		10			8	16			11		3	964		4016	80

Triangularized Input-Output Table 1970 for Finland

Degree of linearity: 0.8047

	7	24	6	5	1	3	20	13	21	10	17	9	8	2	11	22	15	16	14	4	12	23	19	18
7 Textiles & leather	628	16	3	3	2		27	1	5	2	4	10	24		4	2	12	2	1		10		2	
24 Services	38	628	17	62	111		242	15	647	32	3	41	29	16	33	302	69	14	13	4	10	27	15	14
6 Beverages&tobacco prod.		135	31	7			1					1			3	2								
5 Foods	11	416	24	1868	506	1	1		2			1	1		4	12	1						1	
1 Agriculture	9	46	42	2867	1232									45		2								
3 Fishing		8		19		1																		
20 Construction	4	271	2	8	138		324	3	35	2		17	6	10	5	56	20	3	7	2	2	67	11	3
13 Non-metal mineral products	1	11	26	6	8		609	36	5		1	10	7		9	2	12	9	3	6	1		6	1
21 Trade	18	467	7	55	166	2	365	8	86	5	2	17	15	48	13	165	50	13	9	4	6	10	47	2
10 Printing & publishing	15	294	11	20	12		4	5	103	224	1	12	8	2	22	10	25	5	4	1	3	16	4	2
17 Other manufactures	12	11		1			4		3	2	2	1	1			1	2							
9 Paper & paper products	19	34	8	98	4		44	13	26	159	3	1636	24	3	71	5	24	16	2	2	9	1	68	8
8 Wood and wood products	14	42	2	18			549	9	3	1	6	148	271		3	1	39	9	2	1	2	1	27	
2 Forestry		263	1	6	84		36				1	1026	928	26	1		2					1	1	
11 Chemicals	37	35	4	12	264		93	7	8	12	1	125	37	15	234	4	31	5	17	6	10		5	1
22 Transport	13	42	9	129	3		461	72	408	16	2	168	72	2	38	186	55	6	50	12	4	17	20	7
15 Machinery&transport equipmen	12	70	11	41	31	2	627	28	14	9	3	93	61	5	33	169	571	41	167	18	7	6	16	11
16 Electrical prod.	8	18	5	11	3		234	9	8	6	2	18	12		8	2	111	70	6	6	2	1	20	2
14 Basic metals	2	2	1	1	3		171	33	1	1	9	10	5		9		432	148	1133	3	2		6	
4 Mining,quarrying&drilling*	1	1		1	13		44	50	1			14	1		21	1	39		230	8	1		1	1
12 Rubber&plastics	31	13	8	38	5		57	5	10	1	3	9	11	1	21	11	25	6	2	4	6		2	
23 Communicatons	10	117	2	12	25		20	4	116	31	1	7	7	4	5	23	24	3	2		3	4	5	1
19 Utilities	22	216	6	42	31		20	24	92	6	1	355	56	1	48	13	62	10	35	18	11	7	556	9
18 Petroleum refineries	6	193	2	15	42	2	113	12	26	1		21	7	19	13	85	13	2	10	3	1	3	80	21

Triangularized Input-Output Table 1980 for Finland

Degree of linearity: 0.8107

	6	5	1	3	20	7	17	9	8	2	13	15	16	14	12	11	4	21	22	24	23	19	10	18
6 Beverages&tobacco prod.	203	50	7		5	1		5	2		1	6	1	1		19	1	1	3	477		3	2	
5 Foods	117	7107	1986	16	18	72	1	28	12	1	3	19	4	4	4	34	2	5	43	1098		17	4	2
1 Agriculture	129	9848	932			9				68							1		7	191				
3 Fishing		83	66	17																26				
20 Construction	9	46	146	2	72	16	3	93	33	33	24	135	14	48	6	31	7	141	373	1180	324	198	21	17
7 Textiles & leather	4	25	21	1	70	1730	10	40	102		6	46	11	9	27	21	3	19	17	35	1	6	9	1
17 Other manufactures	1	6	1		20	47	24	4	5	1	1	17	5	10	2	4	2	17	5	23	1	2	16	1
9 Paper & paper products	49	490	14		212	82	12	7337	154	6	62	122	58	18	29	300	10	139	36	148	5	429	942	20
8 Wood and wood products	2	25	4		2586	49	30	694	1400	1	46	171	51	21	13	29	4	16	7	77	4	225	4	2
2 Forestry	11	21	113		141			3246	4150	38		2		2					20	19	25	13		1
13 Non-metal mineral products	62	40	113		2751	22	3	94	132	2	300	114	35	74	24	119	7	35	10	49	2	47	13	7
15 Machinery&transport equipmen	49	211	246	22	2551	40	14	625	359	31	131	3134	114	325	29	171	35	73	482	197	30	81	137	48
16 Electrical prod.	10	55	15		1157	12	4	112	37	3	36	707	400	95	18	32	17	49	20	110	52	65	29	14
14 Basic metals	13	11	11		482	8	55	74	69	1	137	2834	317	4344	13	55	7	5	2	11	1	52	15	5
12 Rubber&plastics	10	226	32		242	81	7	63	43	1	36	118	28	17	26	128	7	34	101	69	1	12	9	8
11 Chemicals	10	122	864	3	896	112	13	772	317	29	85	246	97	61	285	1604	4	84	55	229	3	104	87	31
4 Mining,quarrying&drilling*	1	10	25		70	2		92	7		148	12	2	630	4	98	30	4	1	4		81	1	6
21 Trade	62	405	782	24	1819	107	25	164	114	103	79	408	126	116	46	224	31	643	924	1361	49	339	81	21
22 Transport	61	665	68		1920	46	7	848	551	11	357	229	28	273	15	305	39	2487	736	437	66	175	365	27
24 Services	103	464	310	9	1228	207	17	321	319	51	124	968	259	169	46	246	84	2976	2154	4653	154	282	573	71
23 Communicatons	9	52	110		94	40	6	48	34	13	17	93	30	15	8	28	7	677	141	931	23	32	74	5
19 Utilities	48	242	166		122	121	8	2298	366	2	134	328	82	404	55	547	108	564	95	1528	39	7807	54	105
10 Printing & publishing	61	113	57		25	57	11	107	48	5	24	151	43	12	15	89	5	593	87	1861	85	66	1434	22
18 Petroleum refineries	49	250	345	21	478	93	10	307	130	72	138	342	54	186	23	738	54	308	1543	998	15	978	59	221

Triangularized Input-Output Table 1985 for Finland

Degree of linearity: 0.7961

	7	6	5	1	3	20	17	9	8	13	2	15	16	12	14	11	4	21	22	24	19	18	10	23
7 Textiles & leather	2417	3	23	33	9	88	6	69	137	7		78	25	25	15	38	4	22	32	52	16	9	18	6
6 Beverages&tobacco prod.	2	336	53	22		14		9	2	3		8	2	1	2	31	2	5	17	539	9	2	2	4
5 Foods	160	186	12215	3266	23	25	1	60	14	4	1	22	7	5	6	122	2	8	84	1473	23	7	6	2
1 Agriculture	30	240	15601	1276							81			3		1		1	23	325				
3 Fishing			134	41	25															53				
20 Construction	24	8	61	257	18	125	4	114	31	34	56	216	28	10	70	60	9	217	672	2521	155	10	38	509
17 Other manufactures	55	1	7	1	2	37	20	5	5	2	1	16	8	2	9	6	3	24	14	67	2	1	28	1
9 Paper & paper products	100	56	688	29		294	24	10531	145	75	10	157	83	58	54	462	14	157	105	317	57	36	1693	9
8 Wood and wood products	11	3	24	8		3538	39	1247	2057	66	1	257	65	24	28	45	5	16	7	159	366	8	10	5
13 Non-metal mineral products	13	86	52	16		3985	12	84	162	631	1	182	16	15	114	68	15	54	33	133	99	8	18	6
2 Forestry		6	30	151	1	193	1	4508	4531	1	55	3			4	1			24	36	85			27
15 Machinery&transport equipmen	73	59	229	434	17	3416	54	842	453	233	77	6029	215	46	527	299	49	133	1053	373	216	58	251	82
16 Electrical prod.	25	7	76	31		1407	5	153	52	51	5	1194	900	20	54	61	27	35	166	237	161	10	53	42
12 Rubber&plastics	74	15	223	51		373	14	65	37	27	4	203	71	57	16	179	6	23	206	124	23	12	11	6
14 Basic metals	35	22	56	28		641	46	137	98	222	3	3881	408	29	6508	82	16	3	7	13	45	6	50	2
11 Chemicals	174	14	280	1292	1	1085	27	1232	418	107	60	400	133	384	98	2176	6	44	138	274	52	46	184	7
4 Mining,quarring&drilling*	2	2	25	114		105	1	114	8	208	1	16	2	5	758	235	53	9	3	5	457	7	3	
21 Trade	109	53	557	1192	13	2849	29	206	121	96	105	543	189	66	158	276	49	935	831	2751	452	63	107	16
22 Transport	58	73	1065	42	10	3319	13	1310	681	571	6	365	51	38	360	564	115	3837	801	743	576	56	616	153
24 Services	336	172	1144	481	18	2739	52	878	523	320	119	2109	515	171	418	487	226	5558	2445	13213	967	319	1207	344
19 Utilities	140	63	415	300		189	14	4499	482	184	3	546	134	90	611	784	141	793	235	3659	11627	157	98	81
18 Petroleum refineries	61	26	298	494	24	1113	8	224	91	165	144	299	31	20	244	766	74	439	1767	2232	946	281	34	59
10 Printing & publishing	76	58	176	88		27	18	163	61	44	5	203	79	32	49	146	27	571	278	2849	140	96	2594	64
23 Communicatons	50	10	68	190	9	123	6	74	47	29	17	143	78	17	24	41	12	1011	272	2190	55	10	473	102

Triangularized Input-Output Table 1970 for Norway

Degree of linearity: 0.7652

	5	6	1	20	3	17	15	16	14	8	7	11	13	4	21	12	10	9	2	22	18	24	19	23
5 Foods	2330	16	1132	17	12	4	40	5	9	12	37	45			8	2		2				346		
6 Beverages&tobacco prod.	6	21		1								4										66		
1 Agriculture	3059	13	1610	141	1		2		3		27	1		1	26			2	73	9		66	1	2
20 Construction	34	8	152	10	3	1	45	12	31	9	10	15	8	19		5	9	16	38	141	7	896	137	142
3 Fishing	1277		18	5	4		1		2		7	6			6			2		17		17	1	1
17 Other manufactures				9		5	4	3			7				6							7		
15 Machinery&transport equipmen	107	10	28	1074	83	6	1571	65	91	91	43	46	19	31	157	19	4	71	2	740	3	62	15	26
16 Electrical prod.	4		6	379	4	1	126	207	45	8	4	8		4	19		1	10		6	1	12	7	7
14 Basic metals				188		17	432	86	324	5		7	7	9	7	1	1	7		1	1	5		3
8 Wood and wood products	50	8	6	1493	2	8	80	22	9	642	7	17	4	1	38	4		87		3	1	4	1	2
7 Textiles & leather	19	1	5	57	12	3	46	6	12	47	331	7	1		32	15	4	14		10	1	21	1	1
11 Chemicals	21	3	159	133	5	3	87	12	155	28	42	231	7	17	14	65	7	39		57	6	23	1	2
13 Non-metal mineral products	12	1	8	763			39	13	20	23	1	9	100		4	3		8		1	2	7		
4 Mining, quarrying&drilling*	1		3	147			1		129			31	77	5	3			17			28	4		1
21 Trade		8	187	773	29	12	387	76	318	208	83	121	81	18	257	36	62	274	4	171	36	247	15	22
12 Rubber&plastics	20	1	3	84	1	6	49	8	6	20	15	10	3	1	178	56	2	15		10	2	18	1	3
10 Printing & publishing	43	14	3	7	1	2	69	19	47	26	19	35	17	2	197	12	420	33	1	57	15	264	5	7
9 Paper & paper products	175	28	9	120	2	7	27	12	19	64	22	102	18	3	316	14	225	1040		10	13	14	2	6
2 Forestry	4			96		1				245		1						426				1		
22 Transport	106	41	82	203	2	8	205	58	145	74	57	100	46	41	1219	34	83	66		600	47	224		57
18 Petroleum refineries	32	3	35	64	36		19	3	107	8	6	42	38	11	74	4	3	38	1	133	26	40	4	5
24 Services	118	43	75	300	18	7	200	56	159	75	57	106	52	39	1516	34	152	78	1	657	46	3127	42	55
19 Utilities	64	5	53	60	1	2	56	9	364	25	15	89	29	23	14	9	9	106		29	13	249	1320	7
23 Communicatons	35	12	10	166	5	2	61	18	46	22	18	33	14	14	166	10	25	24	1	112	13	360	22	77

Triangularized Input-Output Table 1980 for Norway

Degree of linearity: 0.7641

	5	6	1	20	3	17	15	16	14	8	7	11	13	4	21	12	10	9	2	22	18	24	19	23
5 Foods	7321	57	2658	50	57	3	322	28	17	57	50	131	9	46	36	5				7	2	1154	1	4
6 Beverages&tobacco prod.	18	71	1	4			2		1			5		2	6							247		1
1 Agriculture	8922	26	3527	492	5		16	2	8	5	57	9	2	7	127	1	7	4	84	29	1	181	4	4
20 Construction	107	16	401	131	14	2	177	23	96	32	17	54	38	263	4	9	46	36	84	346	18	2804	610	661
3 Fishing	3110		23	9	70		1		4	1	22	19	1	2	15	1	1	3		31	2	62	2	5
17 Other manufactures	2		1	42		6	36	8		8	13		1	2	25	3	7	1			1	16	2	1
15 Machinery&transport equipmen	335	29	89	3006	388	35	6247	278	339	494	118	210	137	544	566	48	33	147	16	1670	54	290	97	196
16 Electrical prod.	31	2	12	1066	7	8	592	409	53	58	12	36	13	41	82	6	10	19	2	11	8	45	17	32
14 Basic metals	14		10	483	4	57	1003	155	918	31	9	47	26	52	68	5	5	23	1	9	13	30	14	28
8 Wood and wood products	104	9	39	4866		15	622	58	43	1973	22	39	32	52	117	7	2	293		4	4	9	2	6
7 Textiles & leather	16	1	4	142	8	4	154	17	12	100	407	9	7	18	46	22	9	11		27	2	62	4	6
11 Chemicals	56	17	409	448	14	25	264	55	645	100	109	1130	36	68	54	374	30	151		53	32	90	10	15
13 Non-metal mineral products	38	5	50	2678		7	158	26	34	152	6	27	532	17	23	8		15		2	11	31	5	4
4 Mining,quarring&drilling*	5		12	666	2		28	2	309	8	2	85	196	81	35	2	2	31		3	1416	8	7	15
21 Trade		29	504	2515	106	18	1097	159	495	618	134	357	207	94	871	111	182	459	18	499	304	1182	134	98
12 Rubber&plastics	80	9	29	359	2	13	129	21	13	78	27	37	19	10	458	90	8	23		21	13	49	4	7
10 Printing & publishing	201	34	8	108	1	17	346	73	81	126	65	118	54	87	720	34	1500	81	2	201	17	888	33	35
9 Paper & paper products	408	54	19	276	5	9	87	37	50	118	20	231	58	28	359	37	555	1936	1	11	24	46	15	16
2 Forestry	15			234		2	10			1082					1			616				4		
22 Transport	487	89	207	1328	6	36	943	194	225	305	134	308	147	463	3361	89	452	134		2550	49	749	1	196
18 Petroleum refineries	188	19	181	380	268	4	129	22	558	51	27	473	231	117	435	22	23	222	17	988	790	319	51	36
24 Services	770	118	260	2153	34	49	2620	316	369	427	175	417	243	674	4446	138	859	192	11	2078	71	13936	243	261
19 Utilities	200	23	195	505	19	8	222	41	1107	101	32	305	74	93	64	37	41	274	1	94	40	956	6420	67
23 Communicatons	162	28	19	690	16	12	305	64	93	95	44	115	53	169	401	30	143	56	8	369	19	1576	131	326

Triangularized Input-Output Table 1985 for Norway

Degree of linearity: 0.7622

	6	7	5	1	3	20	17	9	15	8	2	16	14	13	21	12	11	18	4	22	24	10	19	23
6 Beverages&tobacco prod.	35	2	40	1		2			53	3		4	10	2	8	2	23		40		309			
7 Textiles & leather	2	379	9	2	10	117	2	11	132	96		19	9	7	45	29	13	1	70	29	88	5	3	5
5 Foods	138	75	11636	2866	225	39	2	10	410	23		37	34	21	110	10	192	3	322	7	2102	9	6	9
1 Agriculture	22	59	12757	5020	19	435		4	18	4	102	3	8	2	88	1	11	1	5	34	325	12	7	9
3 Fishing		3	4115	23	521	9		5	3	1			5	1	13	1	30	2	5	40	105	2	5	11
20 Construction	25	16	191	706	34	126	4	50	200	57	145	51	331	29	1	11	69	35	448	519	6168	69	1047	973
17 Other manufactures		18	4	2		76	8	2	44	8		10	2	2	31	3	3		16	4	29	11	3	4
9 Paper & paper products	74	26	530	25	7	312	17	2934	116	110	1	30	53	60	494	27	276	18	91	24	95	991	19	23
15 Machinery&transport equipmen	75	100	433	113	584	3103	47	200	9320	341	16	332	410	200	598	83	347	61	2827	2690	555	53	178	369
8 Wood and wood products	18	11	133	24	1	6885	13	447	411	2510		31	45	21	195	11	53	4	125	6	44	7	5	10
2 Forestry	1		11			287		1113	17	1349					1						8			
16 Electrical prod.	7	11	47	23	14	1299	29	36	937	38	4	731	94	22	122	10	88	11	246	29	122	19	42	89
14 Basic metals	5	12	23	11	2	707	53	20	1998	42		187	1385	59	56	7	88	6	202	9	61	6	10	22
13 Non-metal mineral products	6	4	47	80		4794	23	17	288	207		56	73	777	38	18	56	22	73	9	84	10	7	8
21 Trade	78	142	612	825	232	3594	21	686	1736	842	27	241	684	272	1329	150	481	443	220	1075	2121	292	199	254
12 Rubber&plastics	10	28	92	31	3	508	34	24	206	87		42	10	16	591	96	57	10	29	30	78	21	5	9
11 Chemicals	41	102	101	720	17	547	16	291	499	157		95	966	75	107	482	1958	25	209	85	171	50	12	15
18 Petroleum refineries	25	24	232	267	377	1191	3	131	157	53	18	20	527	202	567	18	572	770	185	1536	625	33	79	73
4 Mining,quarring&drilling*	45	70	195	63	8	1175	14	92	2296	137	4	205	560	367	91	37	354	10885	2400	13	65	7	39	80
22 Transport	134	148	830	338	7	1116	82	208	1794	442		381	456	239	4153	141	582	259	1739	3632	1704	927	1	395
24 Services	185	210	1242	409	57	4901	113	331	4987	641	8	493	623	400	10533	221	736	155	3290	3831	27626	1729	712	616
10 Printing & publishing	57	74	383	16	4	595	39	101	691	188	3	150	177	96	1606	55	238	46	185	409	2531	2104	74	78
19 Utilities	48	56	479	400	140	1445	16	497	527	207	2	95	1510	161	177	90	439	84	276	238	1601	109	16570	169
23 Communicatons	43	48	272	30	32	1314	27	83	548	140	6	119	164	81	1850	46	194	40	196	643	3141	281	297	892

Triangularized Input-Output Table 1968 for Sweden

Degree of linearity: 0.7909

	6	5	1	3	20	9	8	2	15	13	16	7	17	14	4	21	22	24	12	11	10	23	19	18
6 Beverages&tobacco prod.	65	1																80						
5 Foods	55	3277	745			2	12		1			7						787	1	49				1
1 Agriculture	70	4989	33		5				1		1	15	1	1		5	1	97		3		1	1	1
3 Fishing		42	9															15		1				
20 Construction	9	40	610			45	30	43	129	30	24	26	4	50	27	112	254	2183	8	28	15	849	352	7
9 Paper & paper products	15	220	11		149	1044	73		87	31	25	26	7	24		307	16	51	16	68	432	7	2	1
8 Wood and wood products	2	3	6	5	2498	335	933	2	114	6	26	13	1	14	1	71	10	35	3	9	4	4		
2 Forestry		1	3		9	1166	1371	45	11	3		4		3	2				8	15	5			
15 Machinery&transport equipmen	45	181	70	20	2136	149	259	76	3793	49	107	22	6	292	74	35	429	329	76	92	11	6	7	27
13 Non-metal mineral products	26	61	10		1895	10	4		127	540	10	1		67	4			55	2	32				
16 Electrical prod.	1	7	9		540	24	7	1	311	20	390	4	1	47	13	7	2	36	5	15	1	1	16	
7 Textiles & leather			3	12	77	51	62		54	2	11	1917	6	4		5	41	23	55	12	2			
17 Other manufactures			2	1	14		1		13		1	16	17			4	3	21			1	1		
14 Basic metals	10	4			749	14	15		1978	69	442		74	1193	27			21	4	20	5		1	1
4 Mining,quarring&drilling*			12		295	16			47	35	1			374	432					21				1
21 Trade	12	311	380	11	841	278	215	40	517	67	37	24	5	558	10	188	164	690	12	31	88	8	43	9
22 Transport	15	304	59		905	309	542		186	88	30	43	6	73	45	1953	152	117	15	36	44	94	5	4
24 Services	31	175	242		1628	106	97	37	548	67	138	96	12	94	25	2223	664	2051	29	65	255	127	37	12
12 Rubber&plastics	7	78	11		82	1	5	2	289	1	58	23		14		94	89	50	18	46				1
11 Chemicals	4	21	267	16	191	219	157	3	262	36	24	70	10	104	29	88	11	119	124	526	57			5
10 Printing & publishing	23	94	5		42	32	40	6	129	27	47	56	41	21	2	720	121	344	23	112	218	46	25	12
23 Communicatons	5	22			96	16	25		109	12	29	25	5	23	2	151	100	440	7	13	112	69	14	
19 Utilities	8	92	86		64	291	83	37	255	69	37	43	4	209	60	162	102	428	23	146	31	18	47	10
18 Petroleum refineries	3	16	59	8	192	33	9	21	46	25	5	6	1	61	10	49	144	35	3	53	3	2	32	76

Triangularized Input-Output Table 1980 for Sweden

Degree of linearity: 0.7686

	6	17	5	1	3	20	9	8	2	15	16	13	12	14	4	7	11	21	22	24	10	19	23	18
6 Beverages&tobacco prod.	[illegible]		5														4	17	37	275				
17 Other manufactures		[illegible]				15	2	3		26	5		1	1		60	3	41	10	131	3		5	
5 Foods	109	7	[illegible]	2903		12	43	4		35		6	5	1		99	178	308	69	1538				1
1 Agriculture	141		14115	[illegible]		22				4			1			28	14	219	7	159				
3 Fishing			139	23						1								11	4	46				
20 Construction	29	11	175	880		[illegible]	188	112	93	557	152	64	29	173	145	45	110	412	915	9069	93	1694	743	18
9 Paper & paper products	74	43	818	24	4	315	[illegible]	146	4	374	102	78	84	33	5	127	405	758	126	771	1919	20	32	6
8 Wood and wood products	6	75	24	33	18	6529	1853	[illegible]	36	902	57	277	14	73	25	23	21	205	249	543	3	17	25	1
2 Forestry		1	19	16		80	4117	5016	[illegible]	12		18	1	1	2	14	40	27		28		42		3
15 Machinery&transport equipmen	238	58	331	670	37	5092	494	609	472	[illegible]	355	145	99	868	216	91	400	637	1326	1165	81	293	50	202
16 Electrical prod.	2	7	11	5	2	1049	112	45		875	[illegible]	21	9	71	6	6	30	236	57	299	3	152	57	8
13 Non-metal mineral products	57	3	64	50		4458	46	277		391	69	[illegible]	5	145	34	13	51	77	9	364		12		9
12 Rubber&plastics	30	26	298	33		153	21	30	18	1241	210	39	[illegible]	8	8	30	170	263	177	277	25	10	5	17
14 Basic metals	8	171	71			1114	53	95		6081	575	162	22	[illegible]	7	15	92	109		156	3	17		2
4 Mining,quarring&drilling				14		439	12			111	7	314	1	1315	320	4	19	7		69		16		9
7 Textiles & leather	7	13	36	19	14	86	114	216	25	175	14	4	66	13	14	[illegible]	7	69	92	181	1	4	36	
11 Chemicals	33	75	222	857	3	1093	848	432	5	1157	319	84	433	179	106	156	[illegible]	91	83	402	118	130	24	113
21 Trade	120	109	917	979	10	2962	1841	1237	64	4397	634	369	144	2023	60	248	775	[illegible]	703	1633	154	235	54	80
22 Transport	106	21	859	115	5	2286	302	765	14	1516	371	430	88	402	182	160	338	9313	[illegible]	1752	224	40	239	9
24 Services	254	56	1222	713	10	2786	849	827	244	3092	1125	415	194	491	215	281	1274	6748	3759	[illegible]	1035	621	1399	137
10 Printing & publishing	12	8	88	24		317	94	68	24	518	150	43	32	85	17	39	108	1011	306	2180	[illegible]	26	429	8
19 Utilities	33	23	397	368		199	1534	394	13	1028	193	209	104	942	283	120	646	896	300	4033	88	[illegible]	91	240
23 Communicatons	27	8	148	75		247	91	97	24	638	201	59	36	79	20	39	185	1066	644	2140	476	112	[illegible]	11
18 Petroleum refineries	21	17	198	280	16	662	408	110	83	417	48	231	24	658	106	51	618	387	1870	893	22	1244	27	[illegible]

Triangularized Input-Output Table 1985 for Sweden

Degree of linearity: 0.7543

	6	5	1	3	20	17	9	8	2	15	16	14	13	4	7	11	21	22	24	12	10	19	23	18
6 Beverages&tobacco prod.	329	10														1	43	46	453					
5 Foods	201	13377	4266		16	10	97	5		86	2	1	9		211	364	570	60	2652	4				1
1 Agriculture	197	20735	219		30					1					7	17	337	1	311					1
3 Fishing		289	28							2							20		99					
20 Construction	36	268	1049		322	5	302	163	189	845	286	254	101	156	59	246	724	1446	15670	49	124	3630	990	33
17 Other manufactures					18	23	3	6		41	14				51	6	57	15	212	1	4		11	
9 Paper & paper products	94	1200	29	5	398	67	6729	175	6	696	198	48	146	7	180	598	1050	246	1236	138	3544	45	90	3
8 Wood and wood products	10	39	46	28	6361	72	2582	4131	65	1275	94	145	339	38	27	34	323	533	547	20	4	38	65	
2 Forestry		40	17		83	1	7492	7695	432	14	1	8	14	2	3	75	55		62	1	1	409		1
15 Machinery&transport equipmen	346	499	999	35	7411	81	719	798	650	15823	561	1299	196	277	118	655	799	2851	1435	157	147	562	139	360
16 Electrical prod.	3	18	9	3	1365	8	211	68		1341	1067	119	34	8	10	56	311	133	397	16	8	342	187	17
14 Basic metals	12	120			1695	207	98	173		9855	930	6226	180	10	19	161	185	2	272	42	3	20		4
13 Non-metal mineral products	59	95	84		6010	3	54	345		584	97	218	963	58	22	109	113	20	317	5	1	21		9
4 Mining,quarrying&drilling			47		559	5	27	1		74	11	1651	415	326	9	77	12		100	1		36		124
7 Textiles & leather	8	23	23	18	72	20	159	322	28	269	22	42	7	27	1160	13	79	163	239	77	1	5	65	
11 Chemicals	44	304	806	4	1744	92	1144	570	16	1863	535	327	120	132	213	3357	154	208	675	625	208	221	71	168
21 Trade	169	1343	1572	17	4115	133	3178	1728	86	6977	1088	4699	464	83	336	1168	1030	828	2867	215	266	264	93	106
22 Transport	175	1405	183	9	3736	31	501	1143	20	2842	708	693	605	298	238	617	13334	11201	2940	138	556	81	315	20
24 Services	370	1860	885	13	4160	64	1167	1138	334	5327	2007	721	545	287	391	2388	12094	7553	65675	308	1941	981	3538	201
12 Rubber&plastics	34	414	43		187	31	34	42	25	2093	375	13	52	11	47	278	383	337	468	230	43	26	12	32
10 Printing & publishing	19	136	36		437	12	156	92	40	953	286	184	62	24	51	194	1649	690	4254	52	7400	77	855	11
19 Utilities	45	694	468		236	20	2484	593	85	1785	342	1507	293	382	169	1077	1203	613	6438	178	161	900	159	370
23 Communicatons	44	230	112		347	9	137	146	38	1147	375	126	78	28	57	358	1805	1391	4532	66	853	207	200	20
18 Petroleum refineries	37	281	548	66	1146	19	530	136	195	535	67	1015	247	166	78	1156	994	3603	1219	37	28	1598	94	732

Svensk resumé

Triangularisering av input-outputtabeller används vanligen som ett medel att göra internationella och intertemporala jämförelser av hierarkiska produktionsstrukturer. Triangulariseringsproblemet löses traditionellt som ett maximeringsproblem; nämligen att finna den största summan av element under huvuddiagonalen i en input-outputtabell genom att permutera ordningen mellan tabellens sektorer. Det antal permutationer som måste undersökas för att hitta denna optimala lösning ökar med antalet sektorer i tabellen. Ju fler sektorer desto effektivare måste en lösningsalgoritm kunna begränsa antalet permutationer som undersöks.

Triangulariseringsproblemet betraktas här som ett maximeringsproblem och nödvändiga samt tillräckliga villkor formulerades i teorem 1. Vi presenterade en branch-and-bound-algoritm som drar fördel av dessa villkor för att begränsa antalet undersökta permutationer för att finna den optimala lösningen. Vi definierade också villkor för en suboptimal lösning, i teorem 2, med ett antal sektorer i sökträdet som är mindre än det totala antalet sektorer. Det bevisades, i teorem 3, att den optimala lösningen är gräns för de suboptimala lösningarna närt antalet sektorer i sökträdet går mot det totala antalet sektorer.

Algoritmens konvergensegenskaper illustreras genom att beräkna suboptimala lösningar till input-outputtabellerna för de fyra nordiska länderna Danmark, Finland, Norge och Sverige. Vi undersöker också den intertemporala likheten i hierarkiska produktionsstrukturer för de nordiska länderna. Uppsatsen stöder resultaten från andra undersökningar om att produktionsstrukturer liknar varandra olika länder. Den hierarkiska likheten i produktionsstrukturer bland olika länder står sig också över tiden för Danmark, Finland, Norge och Sverige. Appendix I innehåller detaljerade tabeller som visar kovergensegenskaperna och likheten i produktionsstrukturer. Appendix II ger datorprogram och appendix III redovisar triangulariserade input-outputtabeller för Danmark, Finland, Norge och Sverige.

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