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TESTING FOR SHORT MEMORY IN A VARMA PROCESS

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Abstract

We generalize the short term memory test of an ARMA model. presented in Öller (1985), to the multivariate VARMA cases. In a study on Swedish exports and OECD demand we demonstrate how the multivariate setting extends the short memory.

Key words: Prediction horizon. memory, VARMA models.

1. INTRODUCTION

How far into the future can changes in general business activity be forecasted and how much information is there in the forecast? An F-test of Parzen's prediction variance horizon (Parzen, 1982) of an ARMA model yields the number of steps ahead that a forecast contains information. In analysing the Finnish GDP, Öller (1985) found that a three-years ahead forecast is statistically informative: for a ten-years ahead forecast there are strong doubts about both model memory and model validity. The classification of time series as possessing *no memory* (or white noise), *short memory* and *long memory* is widely explored in the literature, see Parzen (1981b). As far as forecasting is concerned it can be said that a time series has a short memory if it is only partially predictable into the future: long memory if it can be predicted far or indefinitely into the future; and no memory if the future cannot be predicted at all from the past.

As it is of both practical and theoretical interest to know the l-step ahead forecastability of a series, it has been suggested by Öller (1985), that the short memory of a series is the last step for which a forecast information measure is significantly greater than zero. This is a statistical rule that makes it possible to determine the horizon $l = L$ after which the forecasted series contains no more information. In the present study we generalize this decision rule to vector autoregressive moving average, VARMA processes; and we construct an equivalent of this information measure for multivariate time series.

Our reasoning is illustrated using a simple vector autoregressive moving average model for quarterly OECD demand and quarterly Swedish exports. We found that the forecast horizon of Swedish exports is extended by relying on its relationship to OECD demand.

2. THE DECISION RULE

2.1. The Univariate case

Let Z_t be a scalar valued, stationary time series, $t = 1, 2, \ldots, T$. Then there exists an ARMA(p, q) model

$$\phi(B)Z_t = \theta(B)a_t, \tag{2.1}$$

where ϕ and θ are polynomials in the back shift operator B with all their roots outside the unit circle, and a_t is i.i.d. noise. The Wold transformation of (2.1) is

$$Z_t = \psi(B)a_t, \ \psi(B) = \phi^{-1}(B)\theta(B). \tag{2.2}$$

The l-step ahead forecast error variance is:

$$V(l) = \sigma_a^2(1 + \psi_1^2 + \ldots + \psi_{l-1}^2), \tag{2.3}$$

whereas the total variance of Z_t is:

$$\sigma_Z^2 = \sigma_a^2 \sum_{l=0}^{\infty} \psi_l^2. \tag{2.4}$$

With analogy to the R^2 of ordinary regression, see Nelson (1976), Parzen (1982), Harvey (1984) and Öller (1985), we define the fraction of the variation in Z_t that *can* be forecasted up to the horizon l as:

$$I(l) = 1 - \frac{V(l)}{\sigma_Z^2}. \tag{2.5}$$

It is well known that, in ordinary regression, the hypothesis that the true value of R^2 is zero can be tested by comparing the F statistic:

$$F = \frac{N-k-1}{k}\frac{R^2}{1-R^2} \tag{2.6}$$

to the critical values of Fisher's F-distribution, with k and $N-k-1$ degrees of freedom respectively. Substituting an estimated $I(l)$, $\hat{I}(l)$, for R^2 and $p+q$ for k we get:

$$F = \frac{N-p-q-1}{p+q}\frac{\hat{I}(l)}{1-\hat{I}(l)}, \tag{2.7}$$

which may be considered as approximately F-distributed, see Nelson (1976). Using results from Öller (1985) the statistical decision rule may be formulated as follows: the short memory of model (2.1) is that $l = L$ for which:

$$\hat{I}(L) \geq \omega_r, \ \hat{I}(L+1) < \omega_r \tag{2.8}$$

with r as the significance level and

$$\omega_r = \frac{F_r(p+q, N-p-q-1)}{\left[F_r(p+q, N-p-q-1) + \frac{N-p-q-1}{p+q}\right]}. \tag{2.9}$$

2.2. The multivariate case

Now let the stationary vector time series $\mathbf{Z}_t$ of order (p,q), be represented by the VARMA model, see Tiao and Box (1981):

$$\mathbf{Z}_t = \mathbf{\Phi}_1\mathbf{Z}_{t-1} + ... + \mathbf{\Phi}_p\mathbf{Z}_{t-p} + \boldsymbol{\eta}_t + \mathbf{\Theta}_1\boldsymbol{\eta}_{t-1} + ... + \mathbf{\Theta}_q\boldsymbol{\eta}_{t-q}, \ t = 0, 1, 2.... \tag{2.10}$$

where $\mathbf{Z}_t = (\mathbf{Z}_{1,t}, ..., \mathbf{Z}_{K,t})'$ is a $(K \times 1)$ random vector, $\mathbf{\Phi}_i$ and $\mathbf{\Theta}_j$ are fixed $(K \times K)$ coefficient matrices. The $\boldsymbol{\eta}_t = (\eta_{1,t}, \eta_{2,t}, ..., \eta_{K,t})'$ is a K-dimensional *white noise* or *innovation process*, subject to: $E(\eta_{i,t}) = 0$ for all i. $E(\eta_t\eta_s') = \Sigma_\eta$ for $t = s$, $E(\eta_t\eta_s') = 0$ for $t \neq s$ and Σ_η is a non-singular matrix that may be diagonalized using a Cholesky decomposition. Suppose that each individual Z_{it} is a non-deterministic stationary series: the generalization of the Wold's decomposition theorem then yields:

$$\mathbf{Z}_t = \mathbf{c}(B)\eta_t, \tag{2.11}$$

where $\mathbf{c}(B)$ is a $(K \times K)$ matrix polynomial in the back-shift operator B, with typical element:

$$c_{ij}(B) = \sum_{m=0}^{\infty} c_{ij,m}B^m. \tag{2.12}$$

The typical series in (2.11) is actually a weighted sum of current and past values of each of the K uncorrelated white noise series η_t, so that:

$$Z_{i,t} = \sum_m c_{i1,m}\eta_{1,t-m} + \sum_m c_{i2,m}\eta_{2,t-m} + \ldots + \sum_m c_{iK,m}\eta_{1,t-m}. \tag{2.13}$$

For $\mathbf{c}(B)$ of (2.11) there exist matrix polinomials $\Phi(B)$ and $\Theta(B)$ of appropriate dimensions such that:

$$\Phi(B)\mathbf{c}(B) = \Theta(B), \tag{2.14}$$

where $\Phi(B) = \sum_{j=0}^{p} \phi_j B^j$, $\Theta(B) = \sum_{j=0}^{q} \theta_j B^j$ and $\mathbf{c}(B) = \sum_{j=0}^{\infty} \mathbf{c}_j B^j$ as above so that $\Phi(B)$ is taken to possess an inverse. Now consider $\mathbf{Z}_t(l)$ as the forecast of $\mathbf{Z}_{t+l}$, made at the time origin t, l periods ahead. Then $\mathbf{Z}_t(l)$ is based on the information set $\mathbf{I}_t = \{\mathbf{Z}_{t-j};\, j \geq 0\}$. Thus:

$$\begin{aligned} \mathbf{Z}_t(l) &= \sum_{j=0}^{\infty} \lambda_{j,l} \mathbf{Z}_{t-j} \\ &= \lambda_l(\mathbf{B})\mathbf{Z}_t \\ &= \sum_{j=0}^{\infty} \mathbf{c}_j B^j \eta_t, \end{aligned} \tag{2.15}$$

where $\lambda_{j,l}$ is a $K \times K$ matrix, and (2.15) represents infinite sums in η_t. The error series of the l-step ahead forecast is known as:

$$\mathbf{e}_{t,l} = \mathbf{Z}_{t+l} - \mathbf{Z}_t(l), \tag{2.16}$$

and the covariance matrix of this forecast error is:

$$\mathbf{V}(l) = \sum_{j=0}^{l-1} \mathbf{c}_j \mathbf{\Sigma}_\eta \mathbf{c}_j', \tag{2.17}$$

which is a multivariate equivalent of (2.3). As in the univariate case it is of interest to know $V(l)$ for a single series $Z_{i,t}$. Since $Z_{i,t}$ comes from the system $\mathbf{Z}_t$, its forecast variance will potentially be affected by other series in the system. Using the weighted sum representation in (2.13) the respective error variances are given by:

$$\begin{aligned} V^{(1)}(l) &= \sigma_{\eta 1}^2 \sum_{j=0}^{l-1} c_{11,j}^2 + \sigma_{\eta 2}^2 \sum_{j=0}^{l-1} c_{12,j}^2 + \ldots \sigma_{\eta K}^2 \sum_{j=0}^{l-1} c_{1K,j}^2 \\ V^{(2)}(l) &= \sigma_{\eta 1}^2 \sum_{j=0}^{l-1} c_{21,j}^2 + \sigma_{\eta 2}^2 \sum_{j=0}^{l-1} c_{22,j}^2 + \ldots \sigma_{\eta K}^2 \sum_{j=0}^{l-1} c_{2K,j}^2 \end{aligned}$$

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$$V^{(K)}(l) \;=\; \sigma_{\eta 1}^2 \sum_{j=0}^{l-1} c_{K1,j}^2 + \sigma_{\eta 2}^2 \sum_{j=0}^{l-1} c_{K2,j}^2 + \ldots \sigma_{\eta K}^2 \sum_{j=0}^{l-1} c_{KK,j}^2.$$

Denoting by $V^i(l)$ the l-step ahead error variance of the i-th component of $\mathbf{Z}_t$, we may write compactly:

$$V^{(i)}(l) = \sum_{j=0}^{l-1}\sum_{k=1}^{K} c_{ik,j}^2 \sigma_{\eta k}^2, \; i = 1, \ldots, K. \tag{2.18}$$

For the short term forecast of horizon l, a measure of the amount of information for the i-th component of $\mathbf{Z}_t$ is

$$\xi_i(l) = 1 - \frac{\sum_{j=0}^{l-1}\sum_{k=1}^{K} c_{ik,j}^2 \sigma_{\eta k}^2}{\sum_{j=0}^{\infty}\sum_{k=1}^{K} c_{ik,j}^2 \sigma_{\eta k}^2}, \tag{2.19}$$

which is the multivariate analogue of (2.5). Using the same reasoning as in the univariate case, we substitute an estimated $\xi_i(l)$, $\widehat{\xi}_i(l)$, for R^2 in (2.5) and get, for a single series $Z_{i,t}$, with ν_i as the number of estimated parameters:

$$F_i = \frac{N - \nu_i - 1}{\nu_i} \frac{\widehat{\xi}_i(l)}{1 - \widehat{\xi}_i(l)}, \tag{2.20}$$

and the same procedures follow as in the univariate case. In analogy with the approximation by Nelson (1976) in the univariate case, (2.20) may also be seen as approximately F-distributed. A reason is that the second term of the right hand side of (2.20) is just a quotient of two sums of squares which may be seen as approximately independent χ^2 variates with ν_i and $N - \nu_i - 1$ degrees of freedom, respectively.

3. OECD demand and Swedish exports

For an illustration of the measurement of the amount of information in a model. we use data on total OECD demand and quarterly Swedish exports of goods. see Main Economic Indicators, OECD. As a proxy for OECD demand we have chosen quarterly total industrial production. There are 100 observations ranging from the first quarter of 1970 to the fourth quarter of 1994. The idea is simply to check if Swedish exports can be better predicted if we take into account influences from other OECD countries. We start by analysing both variables separately in a univariate scheme, where an arsenal of model selection criteria is used, see for example Granger and Newbold (1986) or Vandaele (1983). Secondly, a multivariate analysis is conducted; we also test for cointegration using the Johansen (1995) cointegration test. Plots of the indexes of both series, with 1990 as a reference year, are provided in Figures B1 and B2 in the appendix.

There are two dominant components in both series: growth and a seasonal. We used Hylleberg, Engle, Granger and Yoo (1990) to test for unit roots on trend and seasonal frequencies. This analysis suggests that one ordinary difference is enough to stationarize both series, and the seasonal component may be modelled by four dummies. We also tested for cointegration using the Johansen Likelihood Ratio test. We found the eigenvalues $(\lambda_1; \lambda_2) = (.0452; .0009)$ with Likelihood Ratio = $(4.626; .089)$ and 5% critical values $(15.41; 3.76)$ respectively. This means that cointegration is rejected at the 5% level.

We first specify an univariate model for Swedish exports. After one differencing an AR(1) model with four seasonal dummies, D_i, adequately describes the data.

The estimated model is (with standard errors in parentheses):

$$SWEXPORTS_t = \underset{(1.310)}{10.2} - \underset{(.098)}{.495} * SWEXPORTS_{t-1} - \underset{(2.63)}{7.07} * D_1 - \underset{(1.421)}{9.67} * D_2 - \underset{(1.763)}{18.46} * D_3 \qquad (3.1)$$

$$R^2 = .819, R^2 adj = .811$$

The results of the Ljung-Box Q-test, in Table 1 below, show little evidence against the null hypothesis of no serial autocorrelation of the residuals from the estimated model (3.1) above.

Table 1: Ljung-Box Q Statistic

Lag	$LB-Q$	Lag	$LB-Q$
1	.25	13	19.04
2	1.21	14	19.04
3	1.40	15	22.44
4	9.78	16	24.10
5	9.83	17	24.54
6	11.24	18	26.35
7	11.27	19	26.90
8	12.23	20	27.12
9	12.26	21	27.13
10	16.18	22	27.25
11	18.16	23	27.34
12	18.81	24	27.98

Next the series is modelled as a bivariate VAR(1) where, again, first differences of the individual series are taken to induce stationarity; the seasonal patterns that characterize these series is again expressed in the dummies. The estimated system of equations is:

$$\begin{aligned}
OECD_t &= \underset{(.099)}{.348} *OECD_{t-1} + \underset{(.023)}{.006} * SWEXPORTS_{t-1} \\
&\quad + \underset{(.415)}{6.291} - \underset{(.954)}{8.845} *D_1 - \underset{(.402)}{5.231} *D_2 - \underset{(.540)}{9.853} *D_3 \\
R^2 &= .90, R^2adj = .894 \\
SWEXPORTS_t &= \underset{(.411)}{1.201} *OECD_{t-1} - \underset{(.096)}{.547} *SWEXPORTS_{t-1} \\
&\quad + \underset{(1.720)}{13.640} - \underset{(3.953)}{15.846} *D_1 - \underset{(1.665)}{12.454} *D_2 - \underset{(2.238)}{22.734} *D_3 \\
R^2 &= .834, R^2adj = .825 \qquad (3.2)
\end{aligned}$$

As could be expected (3.2) shows no feedback from Swedish exports. Now to use results of models (3.1) and (3.2) for testing the short memory of Swedish exports we need the following results: for an univariate AR(1): $\Delta Z_t = \phi_1 \Delta Z_{t-1} + \eta_t$, it is known that $c_i = \phi^{i-1}, c_0 = 1$, hence

$$\xi(l) = 1 - \left[\sum_{i=0}^{l-1} (\phi_1^{2i}) / \sum_{i=0}^{\infty} (\phi_1^{2i}) \right]. \qquad (3.3)$$

For the determination of the c_i in a given ARMA(p,q), see Lyhagen (1997). Exact expressions for $c_{ik,j}^2$ in (2.19) can be found for a VARMA by using a generalization of the Yule-Walker equations. Derivations for a bivariate VAR(1) is provided in the appendix, and the same procedure may be used for higher dimension models.

Thus, for Swedish exports we obtain:

$$\xi_2(L) = 1 - \frac{\sigma^2_{\eta_{1t}} \sum_{j=0}^{l-1} U_j^2 + \sigma^2_{\eta_{2t}} \sum_{j=0}^{l-1} V_j^2}{\sigma^2_{\eta_{1t}} \sum_{j=0}^{\infty} U_j^2 + \sigma^2_{\eta_{2t}} \sum_{j=0}^{\infty} V_j^2}. \tag{3.4}$$

where $U_j = U_{j-2}(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + U_{j-1}(\phi_{11} + \phi_{22})$, $V_j = V_{j-2}(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + V_{j-1}(\phi_{11} + \phi_{22})$ and the ϕ_{ij} are simply autoregressive coefficients in the bivariate VAR(1). The values of the estimated information measure of Swedish exports are given in Table 2 below for both the univariate and the bivariate model.

Table 2: Estimates of the information measure of Swedish exports in the univariate and the bivariate cases.

l	*Univariate* Model (3.1)	*Bivariate* Model (3.2)
1	.245	.699
2	.060	.664
3	-	.075
4	-	.044
5	-	-
6	-	-
	-	-
	-	-
	-	-

In the univariate case the short term memory is statistically informative up to two periods ahead. In the multivariate case the forecastability of Swedish exports may be extended to four periods ahead: in either cases the critical value is $\omega_{.05} = .040$.

4. Conclusion

The univariate test for short memory in Öller (1985) is generalized to the case of multivariate time series. The concept is an analogy to the R^2 in ordinary regression. We have established a statistical rule to determine the horizon $l = L$ after which the forecasted series contains no more information.

We compare the forecast horizons in the univariate and in the multivariate cases. A theoretical expression of the information measure is derived: real life data is then used to illustrate the test for short memory. For this, Swedish exports is modelled, first univariately and then by relating it to foreign demand. In the univariate case the forecast horizon covers just two periods ahead. In the bivariate setting this forecast horizon is extended to four periods ahead. Next one would like to derive the short term memory measure for cointegrated variables and apply it to real life data with more than two variables.

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APPENDIX I

Consider the stationary vector time series of order (p, q) as defined in (2.10)

$$\mathbf{Z}_t = \Phi_1 \mathbf{Z}_{t-1} + ... + \Phi_p \mathbf{Z}_{t-p} + \eta_t + \Theta_1 \eta_{t-1} + ... + \Theta_q \eta_{t-q} t = 0, 1, 2, ... \quad (4.1)$$

We know that the covariance matrix of the forecast errors is:

$$\mathbf{V}(l) = \sum_{j=0}^{l-1} \mathbf{c}_j \Sigma \mathbf{c}_j'. \quad (4.2)$$

The l-step ahead error variance of the i-th component of $\mathbf{Z}_t$ is:

$$V^{(i)}(l) = \sum_{j=0}^{l-1} \sum_{k=1}^{K} c_{ik,j}^2 \sigma_{\eta k}^2, \; i = 1, ..., K. \quad (4.3)$$

As mentioned in the text, the explicit expression of the component $c_{ik,j}^2$ becomes quite complicated with the order of the VARMA process. However using a generalization of the Yule-Walker equations these expressions can be found recursively. With analogy to the argument for the univariate case given in Box and Jenkins (1970) the $\mathbf{c}_j$ matrices may be obtained by writing:

$$\begin{aligned} \mathbf{Z}_t &= (\phi(B))^{-1}(\theta(B))\eta_t & (4.4) \\ &= \sum_{j=0}^{\infty} \mathbf{c}_j B^j \eta_t. & (4.5) \end{aligned}$$

Hence in a VARMA(p, q), the following recurrence relationships for the $\mathbf{c}_j$ matrices are deduced by equating powers in (4.4) and (4.5) so that:

$$\begin{aligned} \mathbf{c}_1 &= \phi_1 - \theta_1 \\ \mathbf{c}_2 &= \phi_1 \mathbf{c}_1 + \phi_2 - \theta_2 \\ \mathbf{c}_j &= \phi_1 \mathbf{c}_{j-1} + ... + \phi_p \mathbf{c}_{j-p} - \theta_j, & (4.6) \end{aligned}$$

In (4.6) we note that the moving average coefficients vanish for $j > q$. Consider now a stationary bivariate VAR(1):

$$Z_{1,t} = \phi_{11} Z_{1,t-1} + \phi_{12} Z_{2,t-1} + \eta_{1t} \quad (4.7)$$

$$Z_{2,t} = \phi_{21} Z_{1,t-1} + \phi_{22} Z_{2,t-1} + \eta_{2t} \quad (4.8)$$

which can be written in a matrix form as:

$$\begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} = \begin{bmatrix} 1-\phi_{11}B & -\phi_{12}B \\ -\phi_{21}B & 1-\phi_{22}B \end{bmatrix}^{-1} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}. \quad (4.9)$$

Suppose now that we are interested in the single series $Z_{2,t}$, which in the examples above may stand for the Swedish export. Thus from (4.9) we get:

$$\begin{aligned} Z_{2,t} &= (\phi_{21}B)\left[(1-\phi_{11}B)(1-\phi_{22}B) - \phi_{12}\phi_{21}B^2\right]^{-1}\eta_{1t} \\ &\quad +(1-\phi_{11}B)\left[(1-\phi_{11}B)(1-\phi_{22}B) - \phi_{12}\phi_{21}B^2\right]^{-1}\eta_{2t} \\ &= \mathrm{U}(B)\eta_{1t} + \mathrm{V}(B)\eta_{2t}, \end{aligned} \quad (4.10)$$

where $\mathrm{U}(B) = (\phi_{21}B)\left[(1-\phi_{11}B)(1-\phi_{22}B) - \phi_{12}\phi_{21}B^2\right]^{-1}$ and $\mathrm{V}(B) = (1-\phi_{11}B)\left[(1-\phi_{11}B)(1-\phi_{22}B) - \phi_{12}\phi_{21}B^2\right]^{-1}$ respectively. Both $\mathrm{U}(B)$ and $\mathrm{V}(B)$ can also be represented as infinite sums in the back-shift operator B. Thus by equating powers of B we obtain:

$$\begin{aligned} U_1 &= \phi_{21} \text{ and } V_1 = \phi_{11} + \phi_{22} \\ U_2 &= \phi_{21}(\phi_{11} + \phi_{22}) \text{ and } V_2 = \phi_{12} + \phi_{21} - \phi_{11}\phi_{22} + V_1(\phi_{11} + \phi_{22}) \\ U_3 &= \phi_{21}(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + \phi_{21}(\phi_{11} + \phi_{22})^2 \\ &= U_1(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + U_2(\phi_{11} + \phi_{22}) \end{aligned}$$

$$
\begin{aligned}
\text{and } V_3 &= V_1(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + V_2(\phi_{11} + \phi_{22}) \\
U_4 &= U_2(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + U_3(\phi_{11} + \phi_{22}) \\
\text{and } V_4 &= V_2(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + V_3(\phi_{11} + \phi_{22}) \\
&\;\vdots \\
U_j &= U_{j-2}(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + U_{j-1}(\phi_{11} + \phi_{22}) \\
\text{and } V_j &= V_{j-2}(\phi_{12}\phi_{21} - \phi_{11}\phi_{22}) + V_{j-1}(\phi_{11} + \phi_{22}),
\end{aligned}
$$

so that in this case, for a four period ahead forecast we obtain:

$$
\xi_2(4) = 1 - \frac{\sigma^2_{\eta_{1t}}(1 + U_1^2 + U_2^2 + U_3^2) + \sigma^2_{\eta_{2t}}(1 + V_1^2 + V_2^2 + V_3^2)}{\sigma^2_{\eta_{1t}} \sum_{j=0}^{\infty} U_j^2 + \sigma^2_{\eta_{2t}} \sum_{j=0}^{\infty} V_j^2} \tag{4.11}
$$

with $U_0 = 1$ and $V_0 = 1$.

Fig. A1: Demand of Goods from all OECD Area

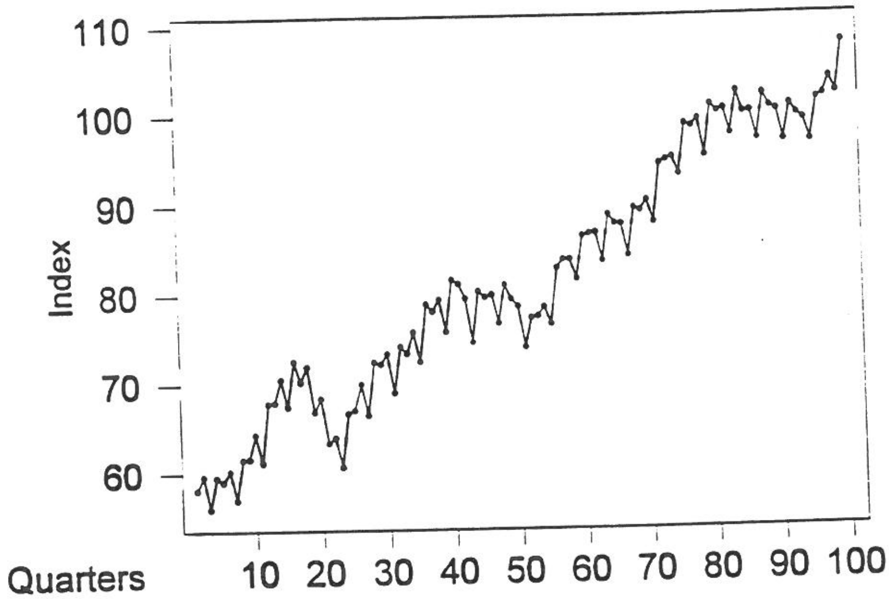
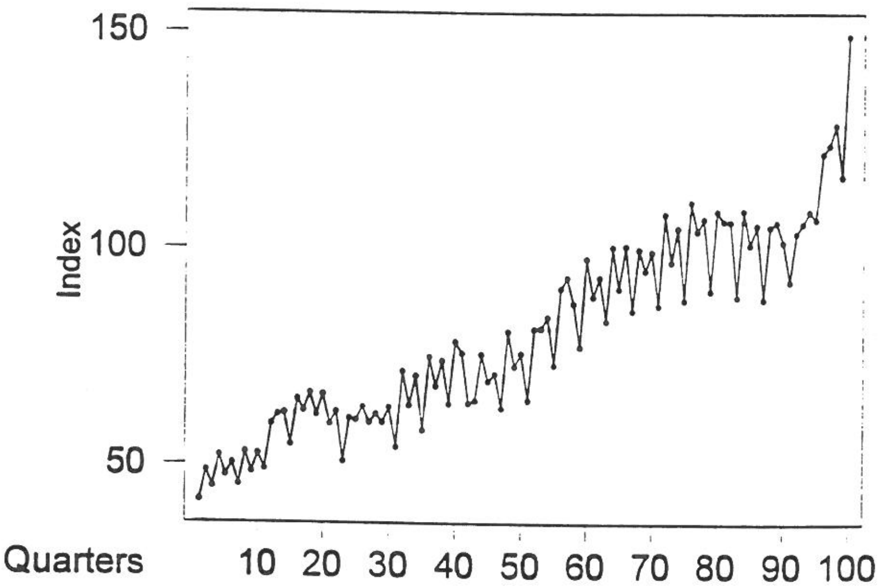


Fig. A2: Total Swedish Export of Goods



Svenskt sammandrag

Hur man testar längden på det korta minnet i en *VARMA*-process

Hur långt in i framtiden kan man prognosticera konjunkturförloppet och hur mycket information finns det i prognosen? I Öller (1985) förslås att man använder sig av Parzens horisont för prediktionsvariansen i en *ARIMA*-modell och ett *F*-test. I ibid. befinnes prognoser på Finlands *BNP* tre år framåt i tiden vara statistiskt informativa; trots en rätt klar tioårig devalveringscykel ställer man sig tveksam till längre prognoser, både av statistiska skäl och pga. osäkerhet gällande modellens invarians i tiden. En modell kan sägas ha *inget minne, kort minne* eller/och *långt minne*. Det sista brukar förknippas med enhetsrötter och eventuellt med *ARFIMA*-modeller och behandlas inte här.

Det kan sägas vara både av praktiskt och av teoretiskt intresse att känna en series prognostiserbarhet. I Öller (1985) föreslås att minnet sträcker sig till det sista steget som statistiskt signifikant minskar prognososäkerheten. För detta ändamål användes ett informationsmått som påminner om förklaringsgraden i en regressionsekvation och som även testas med samma test. I föreliggande uppsats *generaliseras* måttet och testen till även gälla vektoriala tidsserier, dvs. *VARMA*-processer.

Metoden illustreras med data över *OECD*:s efterfrågan på svensk export. Porgnosminnet kan förlängas med hjälp av information om efterfrågan, jämfört med att man enbart använder information om exportens egen historia.

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