WORKING PAPER No. 55
May, 1997

THREE SEMINAR PAPERS ON OUTPUT GAP

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Svensk resumé på sid 44.

CONTENTS

The Output Gap: Measurement, Related Concepts and Policy Implications[1]

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[1] The author would like to thank Bharat Barot, Mats Kinwall and Per Jansson for pointers to relevant literature. He has gained from discussions with them, Håkan Frisen, Alfred Kanis, Lasse Koskinen, Alek Markowski and Lars-Erik Öller. Detailed editorial comments were provided by Lars-Erik Öller, who holds no responsibility for errors that have escaped his argus eyes.

I. Introduction

It has sometimes been said that the Non-accelerating Inflation Rate of Unemployment, the NAIRU, is not carved in stone[1]. The same holds for the concept of potential output which has undergone considerable reformulation over the years; this has been true even independently of the way the NAIRU is conceived, influencing in turn the measurement of the output gap which is the difference between potential and actual output.

The concept of an output gap that is linked to inflationary tendencies is present in Keynes' early manuscript, "How to pay for the war", but gained currency really only in the inflationary seventies. Until then, the widely used concept of potential output had a kind of engineering intonation: it is simply the maximum capacity output, occurring when all factors of production are fully utilized. The peak to peak trend-fitting method embodied this concept: capacity output was assumed to be reached at the peak of each cycle, leaving open the question whether all factors were indeed employed fully, or to the same extent, at all peaks.

Still, the trend-fitting method was thought to provide estimates of potential output at least as good as the somewhat more involved method based on Okun's law, a basic macroeconomic relationship popularized by Arthur Okun (1962) in his work for the U.S. President's council of economic advisors. This relationship between the changes in unemployment - of labour, just one of the factors of production - and output growth may be considered to be more of a theorem, rather than a rigid law, since the so-called Okun's Coefficient that quantifies this link has been subject to quite a bit of variation over the years. This variation has been mostly attributed to the non-inclusion of the other production factors, which meant for instance that the effects on capital formation of the oil shocks of the seventies were not captured. To correct for this "misspecification", Okun's law has sometimes been embedded in a more general formulation that encompasses the input of other factors of production as well.

The complete production function approach for the measurement of potential output has, of course, been used independently, without any reference to the Okun's law which has had a foothold basically only in the United States. It has been used - at least by some of the research staff - at the International Monetary Fund

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[1] cf. Gordon (1997).

(IMF), and is now in vogue at the OECD's Economic Secretariat. The primary advantage of this approach lies in the fact that all the various sources of output growth are represented, though sometimes measurement is arbitrary, as is usually the case with total factor productivity.

However, the concept of potential output for which the production function approach is being applied is no longer that of maximum capacity output. That was discarded hastily after the supply shocks of the seventies brought the necessity of controlling inflation to a level of urgency. Since then, potential output has been increasingly defined as the level of output consistent with non-accelerating inflation. Accordingly, the NAIRU determines the level of equilibrium unemployment that is used in the OKUN's law and production function approaches to the estimation of potential output. It may be added that the popular concept of the NAIRU need not always correspond to that of the NARU, the natural rate of unemployment that Friedman (1969) wrote about, which could have very long-run built-in equilibrium conditions.

In fact, in some of the so-called "fully structural" approaches, used for instance by the IMF, the NAIRU and potential output are determined simultaneously. At the other end of the spectrum is the fully stochastic approach that relies on univariate filters. Occupying a promising fertile middle ground are the "structural-stochastic" methods, espoused particularly by the Bank of Canada, which take in considerable structural information for use in their multi-variate filters which are described in a later section.

All these diverse methods for determining potential output and the output gap have their own advantages as well as disadvantages, but a discussion of these is deferred to the end of the paper, which is structured as follows. The next short section disposes of the trend-fitting method, which does not attract much attention now. Section III discusses the OKUN's law approach, and the following section extensions of it. Section V describes the production function-based method in some detail, followed by an examination of the common ways of estimating the NAIRU. Section VII takes up the "fully structural" or "system" approach, followed by sections describing the univariate and extended multivariate filter, or the "stochastic-structural" method of the Bank of Canada, respectively. Before the final section which provides some recommendations, there is a section (X) which discusses the methods adopted by the various national and international organizations, and the respective merits and drawbacks of these methods.

II. Estimating potential output by the use of linear trends

This method, which is based on a maximum capacity notion of potential output, was commonly used until the 1970s, because of its simplicity, and because anyway the other concomitant approaches did depend to varying degrees on trend-fitting. Linear trends are fitted through consecutive peaks, the assumption being that capacity output is reached at all peaks - and that output grows loglinearlybetween successive peaks. No consideration is given to the possibility that the employment and utilization of production factors can differ between peaks. As something of a compromise, trends are sometimes fitted through centre points in the cycles instead.

In none of its specifications does the method take any account of inflationary pressures, nor is any macroeconomic relation an input into the procedure. During periods of supply shocks, this pure time trend method invariably produces inflated estimates of potential output, the use of time dummies for the supply shock periods being not of much help when a continuous string of supply shocks occur. However, in many of the other estimation methods such as the production function approach, time trend-fitting is resorted to partially, due to its simplicity, providing a weak link in the method.

III. Okun's Law and the output gap

Okun's law is a well-established macroeconomic relationship dating back to the formulation by Okun (1962), who advanced the proposition that a one percent rise in the unemployment rate in the United States will translate into a three percent fall in the growth rate of GNP. Thus, in equation (1), Okun's Coefficient $a = 3$.

$$y_t - y_t^* = -a\,(u_t - u_t^*), \qquad (1)$$

where 'y' is the log change in output, 'u' the unemployment rate, and the subscript t denotes time. The superscript * denotes potential or natural rate values, that are assumed to be purely exogenous in this approach, fixed at some benchmark level, or obtained using time trends.

Kinwall (1996) has applied this approach to estimate the GNP gap for Sweden, and compared the results with those derived from an Hodrick-Prescott (H-P) filter, cf. Section VIII . That study uses the form

$$y_t^* = y_t + a \, \delta u_t \, , \tag{2}$$

with 1986 as the benchmark year when equality is assumed between actual and potential output on the one hand, and the actual unemployment rate and the natural rate NAIRU on the other. This assumption follows the usual practice when using this approach, that of making arbitrary assumptions about 'normal years' during which actual and potential output (and unemployment) have coincided. He attributes this to the fact that with the H-P method, the restrained growth of actual output in the recession of the early 1990s would have even affected potential output due to the end-point problem with this method; however, when he adopted a higher NAIRU - quite realistically - from 1993 onwards, the estimated output gap with the Okun method came closer to that obtained with the H-P filter. Clearly, the exogenous inputs of the potential values are crucial when using the Okun relationship.

There are other difficulties associated with the Okun method. Wage and price dynamics play no role whatsoever, and this is of course the reason why the equilibrium level of unemployment has to be fed in exogenously, unless the Okun relation is presented as part of a larger system. Also, in any 'abnormal' period, such as that of the supply shocks of the seventies, this approach falters since labour is the only production factor that is explicitly considered; so, e.g., the increased scrapping rate of the capital stock due to higher energy prices, lowering the growth of potential output, will not be captured. All the same, Okun's law continues to be regarded as an important macroeconomic relationship between unemployment and growth.

IV. Okun's Law: extensions and related approaches

As noted already, Okun's law has been subject to criticism because of its focus on only one of the factors of production. This exclusion of other (factor) influences in fact means that the estimated Okun's Coefficient actually represents a *composite* effect, rather than the sole effect of unemployment changes, as Prachowny (1993) points out. He embeds Okun's relation in a composite production function relationship:

$$y_t = \alpha\ (k_t + c_t) + \beta\ (n_t + h_t) + f_t\ , \tag{3}$$

where y=output, k=capital stock, c=utilization rate of capital, n=no. of workers, h=no. of hours, and f =total factor productivity, all in logs. t denotes time as before. A similar expression holds for potential output, with stars representing potential values of the variables in equation (3):

$$y_t^* = \alpha\ (k_t^* + c_t^*\ ) + \beta\ (n_t^* + h_t^*) + f_t^*. \tag{4}$$

Then, assuming that capital stock is at its potential, $k = k^*$ from (3) and (4),

$$y_t - y_t^* = \alpha(c_t - c_t^*) + \beta(n_t - n_t^*) + \beta(h_t - h_t^*) + (f_t - f_t^*)\ , \tag{5}$$

or, writing in terms of labour supply e and the unemployment rate u,

$$y_t - y_t^* = \alpha\ (c_t - c_t^*\ ) + \beta\ (e_t - e_t^*) - \beta\ (u_t - u_t^*) + (f_t - f_t^*). \tag{6}$$

From (6), it can be seen that even when unemployment is at its natural level, output can deviate from its potential level. It follows that in Okun's Coefficient a in equations (1) and (2), the effects of changes in factor utilization and total factor productivity growth are also incorporated although they do not appear explicitly in the Okun representation. In his empirical work, Prachowny (1993) found that the composite specification (6) provided better estimates than those using (1). The potential values in (6) needed for the estimates were obtained using trend methods.

Perry (1977) follows Okun's tradition in his emphasis on the labour market when estimating potential output, but works at a more disaggregated level. He considers altogether ten age groups, differentiating by sex as well, and determines (weighted) total potential man hours, based on the estimates of potential participation rates and potential average hours for each group. Estimates of potential output are then made from total potential man hours and (linear) trend labour productivity. A study on the natural rate of unemployment for Canada (Fortin (1989)) has a similar approach, with a stable NAIRU being estimated only for males over 25, with the aggregate NAIRU being more volatile due to labour market composition effects. A detailed description of the methods adopted for the determination of equilibrium unemployment is deferred to Section VI.

Even in the extended form discussed above, Okun's approach is lacking in potential wage and price dynamics that may influence equilibrium unemployment levels. An early attempt at the inclusion of these influences was made by Friedman and Wachter (1974), who included expectational variables in an unemployment equation, which in bare elements was an Okun's law formulation. In the system approach to be described in Section VII, Okun's relationship forms part of a system of equations that includes wage and price equations, potential output and the natural rate of unemployment being determined simultaneously. In Vector Autoregression (VAR) approaches, e.g. Blanchard and Quah (1989), Okun's relation is part of a whole system, just one of the VAR relations between innovations. The VAR methodology enables a distinction to be made between supply and demand shocks, the former having permanent effects, which are generated by common stochastic trends.

V. The Production Function approach

In this approach, estimates of potential output are obtained from the production relation between factor inputs and output when factor inputs are at their potential level, as they are when technical progress is of the disembodied form. The definitions of factor inputs used in the estimate are not self-evident; e.g., there is considerable ambiguity about the definition of equilibrium labour inputs, and often it is the actual capital stock, rather than an estimated potential, that is used. This method is exemplified by the OECD (Giorno et al. (1995)) approach, which is now being used and is preferred to the H-P filter approach that was opted for earlier.

Modelling the business sector production in Cobb-Douglas form gives (lower case signifying logs)

$$y_t = a + \alpha n_t + (1 - \alpha) k_t + f_t , \tag{7}$$

where y_t is output, n_t labour input, k_t capital stock, α is the average labour share, a is a constant and f_t is total factor productivity. From (7) the factor productivity series is estimated and then smoothed with the H-P filter to get the potential series, which is substituted back into the function, together with *actual* capital stock and potential labour input to get potential output. Thus

$$y_t^* = a_t + \alpha\, n_t^* + (1 - \alpha)\, k_t + f_t^*, \tag{8}$$

stars representing potential values of variables. For obtaining the potential labour input used in (8), the natural rate of unemployment is pinned down first. The next section is devoted to a discussion of the concept and estimation of the natural rate of unemployment. Here we may just note that while the OECD has chosen to work with the concept of NAWRU, the non-accelerating wage rate of unemployment, the NAIRU is often the preferred concept: primarily because price inflation is a policy target variable, while the control of wages is not a goal in itself.

To get at the NAWRU, it is first assumed that the change in wage inflation is proportional to the unemployment gap:

$$D^2 \, Log \, W_t = -b\, (U_t - NAWRU_t) + a_t, \tag{9}$$

where D is the difference operator, W_t the wage rate, U_t the actual unemployment rate and a_t a residual. For a period of stable NAWRU, the value of b is obtained from (9), and then substituted back to get a series for the NAWRU, which is then smoothed.

The NAWRU estimates are then used in the following way to get the labour input used in (8) in the determination of potential output:

$$N_t^* = L_t\, (1 - NAWRU_t) - EG_t, \tag{10}$$

where L_t = smoothed labour force = working population times trend participation rate, and EG is government employment. Potential output for the economy as a whole is obtained by adding *actual* value added in the government sector to the business sector estimates obtained by using (10) in (8).

The OECD also uses a more detailed production function approach that can incorporate the effects of energy price changes. Torres and Martin (1990), in their work for the OECD, applies the following nested CES specification for the business sector:

$$Q_t = (a\ (L_t\ e_f\ )^{\rho} + b\ (KE)_t^{\rho})^{1/\rho}, \tag{11}$$

where Q_t is production, KE_t is the actual capital-energy bundle, L_t is actual employment, e_t is the labour efficiency index, and *a* and *b* are scale parameters. In the production function, the scrapping rate has been endogenized, thus supplying the vintage element; an estimated proportion of the capital stock is adjusted in each period according to shifts in real energy prices. Technical progress is labour-embodied, represented by the labour efficiency index, and is assumed to follow the productivity development in the U.S. for other OECD countries, a kind of "catch-up" hypothesis.

Potential output is obtained from equation (11) using the actual capital-energy bundle, potential employment and the labour efficiency index. This will be the level of output consistent with non-accelerating wage inflation, since that is the definition (NAWRU) of equilibrium unemployment adopted. The labour force variable used there is normalized using a geometric moving average. It may be mentioned that the concept of the NAIRU has also been used by the OECD elsewhere (Coe, 1985), along the lines of the discussion in the next section.

The incorporation of energy as a production factor - or, rather, along with capital as a composite factor, to be precise - and the endogenization of the scrapping rate seem to be necessary steps to capture the effects of the energy price shocks of the seventies, and the estimates of potential output for the OECD countries are lower than those obtained from time trend methods. But it may be mentioned that similar effects on capacity growth for Sweden in the middle of the 1970s have been obtained by Markowski (1988), based solely on real wage costs, using a model similar to Knoester and Sinderen (1980).

The merits and drawbacks of the general production function approach are discussed in Section X. An innovation of the OECD approach is the use of the "catch-up" hypothesis for the estimation of potential labour productivity, which bypasses the common problem associated with the time trend estimation of total factor productivity.

In KOSMOS, the macroeconometric model of the National Institute of Economic Research, Stockholm , the concept of potential output is akin to that of *capacity* output, with *full* employment (actual employment + unemployment) labour input being used in the production function. In the case of industry, actual output is derived using an endogenous capacity utilization index,- which is the ratio of actual to potential output given as (cf. Ernsäter and Markowski, 1994)

$$CU = f\left(\frac{S}{y_p}, \frac{RIS}{DIS}, PR\right),$$

where S is total sales, y_p is potential output, RIS is real inventory stock, DIS is desired inventory stock and PR is profitability. As in Helliwell (1984), actual output will be decided by the level of capacity utilization. But note that in Helliwell, the denominator in the capacity utilization index expression is *normal* output, derived using actual employment; utilization deviates from the normal at the actual employment level due to demand shocks. A case can be made for measuring potential output using equilibrium labour inputs consistent with an estimated natural rate of unemployment, rather than full employment; then the capacity utilization index will represent deviations from an equilibrium level of employment, rather than from a level which is never ever reached.

VI. Determining the natural rate of unemployment - consistent with non-accelerating inflation

The usual way of obtaining the NAIRU is from an augmented Philips Curve equation (Gordon, 1997 and Fortin, 1989). Such an estimate need not be synonymous with Friedman's (1968) natural rate of unemployment, which is the level of equilibrium unemployment ground out by a fully-specified general equilibrium equation system, unless additional long-run equilibrium conditions are imposed as will be discussed later in this section. The estimation of NAIRU as is commonly done from wage and price equations is as follows in the simplest specification:

$$w_t = a_0 + a_1 p_{te} - a_2 u_t, \tag{12}$$

$$p_t = b_0 + b_1 (w_t - q_t) + b_2 z_t \quad . \tag{13}$$

Lagged terms have been omitted for simplicity. Also, there will be an additional equation representing price expectations formation based e.g. on lagged prices. Equation (12) is a Philips Curve relation for wages -

in log form ; w is the wage rate, u actual unemployment, and subscript e in prices p represents an expectation. Equation (13) is a price mark-up relation, where q is the constant trend productivity growth and z a vector of other relevant variables. The left-hand side variables are regarded as expected values. In (12) and (13) it is assumed that cointegration prevails between the variables that are not stationary.

In the "long-run" equilibrium with stable inflation, realized expectations, and full nominal wage adjustment to expectations as well as full mark-up for wage costs, we have

$$p_{te} = p_t = p_{t-1} \; ; \quad a_1 = 1 \; , \; b_1 = 1 \; . \tag{14}$$

The NAIRU, the level of unemployment consistent with this non-accelerating inflation equilibrium is solved out so as to be independent of the price level - representing a vertical long-run Philips Curve. Consequently the NAIRU depends only on the coefficients in the equations. However, it is influenced by trend labour productivity. Also, when employers' payroll taxes are taken into account, (w-q) in the price mark-up equation will be replaced by (w+s-q), where s is the payroll tax rate, and the NAIRU will be seen to vary with changes in s as well. Now, substituting (13) and (14) into (12), we can solve for the NAIRU as

$$u_t^* = \frac{a_0 + a_1 b_0 - (1 - a_1 b_1)\, w_t - a_1 b_1 q_t}{a_2}.$$

With a_1 and b_1 equal to unity, the natural rate of unemployment is independent of wage inflation.

Equation (12) may also contain a vector of other relevant variables; this could include variables such as the degree of unionization, profits etc., which may be justified by bargaining models of wage determination. This vector may also contain structural variables relating to labour market composition.

The price mark-up equation (14) could be formulated in error correction terms , as in Torres and Martin (1990):

$$p_t = b_0 + b_1\, c_{te} - b_2\, (p_{t-1} - c_{t-1}) + b_3\, (R_t - 1) + a_t \; , \tag{15}$$

where c represents total unit cost, including capital cost (in (13), capital costs would be submerged in the constant term). R is the ratio of actual to potential output, end a_t is white noise around zero. Since the error correction term is in level terms, price will equal cost in long-run equilibrium. Also, then, actual output will equal potential, so that there is consistency between factor and product markets.

The open-economy aspects of NAIRU determination are given prominence in Joyce & Wren-Lewis (1991). Basically, they use a Layard & Nickell (1986) type model, with imperfect competitionin goods and labour markets - with a bargaining model of wage determination - complemented by endogenous real exchange rate determination via current account imbalances. But, we can even extend the framework represented by equations (12) - (14) to arrive at the insights that an explicit open economy extension has to offer.

Rewrite (13) as

$$p_t = b_0 + b_1 (w_t - q_t) + b_2 \, p_{mt} \, , \tag{16}$$

where p_m is the import price that is expected to affect the mark-up of firms. Solving as before by substituting (16) and (14) into (12), the NAIRU is obtained as

$$u_t^* = \frac{a_0 + a_1 b_0 - (1 - a_1 b_1) w_t - a_1 b_1 q_t + a_1 b_2 \, p_{mt}}{a_2} \, . \tag{17}$$

With a_1 and b_1 equal to unity, the NAIRU is again independent of the wage rate, but now it varies with the import price. Hence, in a period of supply shocks - with permanent effects, which is the sense in which they are usually distinguished from demand shocks even in the stochastic filter approaches - the NAIRU will not be constant. For a constant NAIRU, i.e., invariant with respect to import price changes, the following assumptions have to be made:

$$w_t = p_{mt} \; ; \quad a_1 = 1 \; ; \quad 1 - b_1 = b_2 \, . \tag{18}$$

Note that the domestic inflation rate has to equal the foreign rate.

Joyce and Wren-Lewis (1991) derive the following relationship from their wage and price equations:

$$w_t^* + p_t^* = a_1 c_{ut} - a_2 u_t + a_3 s_t + a_4 (p_{mt} - p_t), \tag{19}$$

where starred w and p are target real wage and target profit margin, respectively (all in log form) , c_u is capacity utilization, s is the payroll tax rate (they have other taxes as well), p_m is import price, and p domestic price. In steady state, when inflation is constant, the LHS of (19) will be zero, capacity utilization will be unchanging, and then the level of unemployment consistent with this scenario is the NAIRU, which is seen to depend on the real exchange rate (and the payroll tax rate).

The implications of this dependence can easily be illustrated. Suppose there is a current account surplus as there is in Sweden now (1997); with an endogenous real exchange rate that moves to eliminate trade imbalances (as is in Joyce-Wren-Lewis's larger model), there will be a *real* exchange appreciation (a rise in domestic prices relative to import prices), which from equation (20) above, *ceteris paribus* is seen to *reduce* the NAIRU. In other words, a current account surplus is an indication of a scenario where reductions in actual unemployment *and* the NAIRU are possible (of course, if the current account is *always* constrained to be balanced, as in Forslund (1995), then this possibility vanishes; but note that the definition of NAIRU does not incorporate current account balance).

Hence the NAIRU can be time-varying. There may be other structural factors which cause this variation in addition to the supply shocks discussed above, for instance, changes in labour market composition. A study by the Bank of Canada (Cote and Hostland (1996)) found trend unemployment to be cointegrated with the degree of unionization in the labour market, as well as with payroll tax rates.

Gordon (1997) estimates a time-varying NAIRU in a simple fashion, complementing a Philips Curve equation with the following hysteresis - type (random walk) specification:

$$u_t^* = u_{t-1}^* + \epsilon_t , \tag{20}$$

where, if the standard deviation of the error term is not zero, a varying NAIRU series is obtained, even if the Philips Curve had implied a constant one. The choice of the standard deviation is here a key step somewhat similar to the choice of the smoothing parameter in the H-P filter method that will be discussed later.

For estimating the natural rate or structural rate of unemployment, Koskinen & Öller (1997) and Assarson and Jansson (1996) use the method of additive decomposition of the unemployment rate into two unobservable components: a *natural rate* or a trend component, and a cyclical component. The natural rate here is not synonymous with the NAIRU, since there is no reference to the rate of inflation. The decomposition is produced by the unobserved components model: along with the additive decomposition of the unemployment rate into the natural and cyclical components, they have a random walk specification for the cyclical rate, and a formulation where the natural rate depends on its own lagged value and the lagged cyclical rate - so that hysteresis is present. The natural rate is determined from the path of observable variables, and for identification the authors use two relationships (in fact, one is sufficient): that between capacity utilization and cyclical unemployment, and that between world demand and domestic cyclical unemployment. The authors choose these relationships as versions of the Okun's Law, with cyclical output-related measures co-varying with cyclical unemployment. The Kalman Filter method is used to estimate the unobserved components.

VII. The system approach[1]

The IMF's (Adams and Coe, 1990) methodology represents the comprehensive "system approach" for determination of potential output and the NAIRU. These are determined *jointly,* based on a system of simultaneous equations. Essentially, what is done is the following: a number of single-equation estimates are made from wage, price, unemployment and output equations, and then these equations are combined and estimated as a system, with appropriate cross-equation restrictions. Some inputs which are exogenous for the single-equation estimates are endogenized under this approach (otherwise, one is often assumed when estimating the other; for instance, when using Okun's relationship to estimate potential output, equilibrium unemployment is often an exogenous input). The estimation is carried out for the U.S only.

The equation system used is:

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[1] For an application of the econometric simultaneous model KOSMOS to estimate the output gap for Sweden, cf. the second essay in this working paper.

$$U_t = \alpha_0 + \alpha_1(y_t - y_t^*) + \alpha_2(Z_{ut}) + \epsilon_{ut} . \tag{21}$$

$$dw_t = dp_{et} + \beta_1(U_t - U_t^*) + \beta_2\, dq_t^* + \beta_3\, Z_{wt} + \epsilon_{wt} . \tag{22}$$

$$y_t = \delta_0 + \delta_1 h_t + \delta_2 k_t + \epsilon_{yt} . \tag{23}$$

$$dp_t = (dw_t - dq_t^*) + \delta_1(y_t - y_t^*) + \epsilon_{pt} . \tag{24}$$

Lower case letters represent logarithms, and *dx* stands for a change in variable *x*. *w*, *q*, *y*, *U* and *p* represent the wage rate, productivity, output, the unemployment rate and the price level respectively. The superscript * signifies a natural or trend value, while the subscript *e* represents expectations. *Z* contains other - often structural - variables. (21) is Okun's relation, (22) is a Philips Curve equation. The other two relationships represent a production function (*h* and *k* are labour and capital inputs respectively) and a price equation where the output gap and unit labour costs determine the growth of prices.

The way the single-equation estimates are combined within a system can be seen by focusing on , say, the unemployment equation. From equation (21), the natural rate of unemployment is estimated. This is done by including a number of structural variables related to the labour market, but without incorporating any information on wage and price developments. Some assumption about trend output will also be required. The equation is estimated in a dynamic form, the steady-state equilibrium of which is

$$U_t^* = a_0 + a_1\, NWLC_t + a_2\, UIRR_t + a_3\, RMW_t + a_4\, UNN_t , \tag{25}$$

UNN being the degree of unionization, *UIRR* the average ratio of weekly unemployment insurance payment to income when employed, *NWLC* employer's social security contributions as a proportion of total wage costs, and *RMW* the minimum wage - multiplied by the labour force share of the age group 16-24, who are the ones most likely to be affected.

Then, when the system is estimated simultaneously, the natural rate term in the Philips Curve equation (22) for wages is replaced by expression (25). Similarly, in (21) and (24), the obtained trend output term is replaced by the expression for potential output derived from the production function - which is from an individual equation assuming equilibrium factor inputs. These substitutions, which amount to cross-equation restrictions, ensure that the estimates of potential output and the equilibrium unemployment rate obtained are consistent with each other.

But though the estimated potential values are consistent, the weaknesses of the individual equations are carried over into the system estimation. Thus, the assumed level of equilibrium factor inputs - note here there is no energy input specified - in the production function still do influence the derived potential values. Though a proxy variable, the input of research and development, is used to capture total factor productivity growth, a time trend is used in addition, so that this approach is really not "fully structural".

VIII. Stochastic approaches: univariate filters

The system approach described in the previous section attempts to come up with structural determinants of the shifts in potential output and the natural rate of unemployment, avoiding the use of shift dummies and *ad hoc* trend fitting (as far as feasible, at least). However, there is presumably always insufficient knowledge about the important structural determinants (a common criticism of any macromodelling activity), especially since modelling the political decision making process - even that which leads to supply shocks such as energy price increases - is next to impossible. This has led to the increased adoption of less detailed models that still use structural information, but are well-suited for the stochastic environment. But before coming to the description of such multi-variate filters complemented with structural inputs, used e.g. by the Bank of Canada, the univariate approach for the determination of potential output will be discussed.

In the use of *univariate* filters for such as the Hodrick-Prescott (HP) filter, the underlying assumption is that the potential output is driven by a stochastic process. The filter helps to decompose an observed shock into a supply (permanent) component and a demand (temporary) component - the identifying difference being that supply shocks have lasting, permanent effects, while demand shocks have only transitory effects. Of course, in practice, when the demand shocks are of a longer duration, it will be difficult to separate the two. Nevertheless, such a mode of classification has been used popularly since the work of Blanchard and Quah (1989).

The HP filter is derived by minimizing the sum of the squared deviations of output from its trend, subject to a smoothness constraint that penalizes deviations in the trend. So HP trend values are those that minimize

$$\phi_t = \Sigma(y_t - y_{p,t})^2 + \lambda\Sigma(\Delta^2 y_t)^2 \quad , \tag{26}$$

where Δ^2 is a double difference. In (26), y is output, with the subscript p representing the trend or potential value. The first term is the so-called global distance, while the second term represents the fluctuating part, the fluctuations assumed to be dominantly transitory, due to demand disturbances. The choice of the smoothing parameter, the multiplicator for the second term that penalizes deviations, plays a key role. With a very large value chosen for the smoothing parameter, the restriction becomes dominant, and potential output is just modelled as a linear trend - the assumption being that demand disturbances have relatively large variances which are thus minimized, leaving only the supply or the permanent component. A very small value of the smoothing parameter will mean almost no restriction at all, and the potential series will follow the actual one very closely.

The Central Bank of Sweden (Apel, Hansen and Lindberg, 1996) has used the HP filter method for measuring the output gap, and compared the results with those obtained from an unobserved components method. As mentioned in section II, Kinwall (1996) has made a comparison of the results for the output gaps obtained using the HP filter method and the Okun's Law. The relative merits of these different methods are taken up in section X, which also discusses the approaches of various national and international organizations to this issue.

The *unobserved components* method may be considered to be a combination of a univariate filter approach with appropriate - often ad hoc - structural information that helps in locating the path of unobservable potential variable values. The work of Assarson and Jansson (1996) differentiates between permanent and cyclical components of unemployment. Apel, Jansen and Lindberg (1996) of the Central Bank of Sweden (Riksbanken) have a similar approach, drawing inferences about unobserved potential output from observable changes in actual output and inflation:

$$y_{p,t} = \mu + y_{p,t-1} + \epsilon_t \tag{27}$$

$$dy_t = \beta_0 + \beta_1 (y_{p,t-1} - y_{t-1}) + u_t \; . \tag{28}$$

$$d\,\pi_t = \gamma_1 (y_{p,t-1} - y_{t-1}) + e_t \; . \tag{29}$$

In (27) - (29), subscript p represents a trend value, while the operator d stands for a one-period change. The last terms in all equations represent random influences. Equation (28) implies that output tends to return to a trend, and resembles the minimization of the deviation of actual from trend values represented by the first term in the HP filter. Equation (29) relates the change in inflation to the output gap - an effect which may not be all that strong for Sweden, according to Kinwall (1996) and Koskinen and Öller (1997) - who are discussing the unemployment gap instead. The observable variables, actual output and its change, and the change in inflation, are used to determine the level of potential output. The estimation of potential output is carried out using a Kalman Filter, which is basically an iterative process: it proceeds by guessing the initial value of potential output in period t-1, and then makes estimates of changes in output and inflation. The prediction errors with respect to these variables are used to update the initial estimate of potential output, and the process is repeated until the latest observation is reached, thus generating a time series for potential output.

IX. The stochastic-structural approach: Multivariate filters with structural inputs

Essentially, if a univariate filter method is complemented by the addition of structural information, it is termed a multivariate filter. But in more elaborate versions the filter is applied to more than just one variable in the process of estimating the potential value for the final target variable.

Laxton and Tetlow (1992) present a *multivariate filter* in which the univariate HP filter with its fluctuation restriction is complemented by information from a Philips curve inflation rate equation and an unemployment rate equation. The general minimization problem has the usual HP filter's penalty function and the global distance term for the output gap, but also included are gap terms for inflation and unemployment, all the gap terms attached with time-varying weights. Thus, information is incorporated about variables which are useful in determining potential output levels. The intuitive idea is illustrated by the following reasoning: if inflation is rising , and it is not due to other factors, then output must be above its potential level.

The HP filter identifies supply and demand shocks in a rather mechanical fashion, which may falter when the shock occurs towards the end of the sample. Laxton's and Tetlow's extension also does not include much information about the determination of output. However, Butler (1996), in a technical report for the Bank of Canada, uses a decomposition of output combined with considerable structural information to distinguish and demand shocks, in an extensive combination of the stochastic and structural approaches. The method of Kutner (1992) is similar to that of Laxton and Tetlow (1992), adding Philips curve information on inflation to an unobserved components model. Other work in this area follow the lead set by Blanchard and Quah (1989), and use VAR impulse responses to estimate the supply and demand components of each innovation, only the supply components considered to feed into potential output.

The extended multivariate filter of Butler (1996), in contrast to Laxton's and Tetlow's filter, which is directly applied to potential output, decomposes output into its components and applies the filter at disaggregated levels. The decomposition has some practical measurement advantages.

Starting out from a Cobb-Douglas production function

$$Y_t = (TFP_t)\, N_t^{a}\, K_t^{1-a} \,, \tag{30}$$

where Y is output, TFP is total factor productivity, N is labour input, K is capital and a is the labour share or labour-output elasticity, the marginal product of labour is obtained from (30) as

$$\frac{\delta Y_t}{\delta N_t} = a\, \frac{Y_t}{N_t} \tag{31}$$

Expressed in *log terms,*

$$y_t = n_t + \mu_t - \alpha_t \,, \tag{32}$$

where the three right hand side variables stand for labour input, marginal product of labour (MPL) and labour-output elasticity respectively. *Equilibrium* values of these variables are needed for the estimation of the potential level of output.

The equilibrium labour input is given as the multiple of the total population, equilibrium participation rate and the equilibrium employment (1-equilibrium unemployment) rate. The *equilibrium* participation rate is obtained by fitting it to the *observed* participation rate and a *trend estimate* of the participation rate. Separate weights are given to these two gaps, and the smoothness parameter - similar to the HP filter smoothness parameter - is also chosen appropriately.

The general method used is to specify an objective function that constrains the weighted average of the squared errors (deviations) from each of the conditioning terms. The value of the unobservable variable that best explains the conditioning term, or minimizes the squared error terms - subject to restrictions, here an end-of sample growth restriction and a smoothness constraint similar to that in the HP filter - is the filter's estimate of the unobserved variable. Thus, the filter's estimate of the equilibrium participation rate is that rate which minimizes the sum of the weighted, squared deviations of the gaps of the equilibrium rate from a trend estimate and an observed participating rate, subject to the restrictions imposed.

The NAIRU estimate - needed for calculating equilibrium labour input, n, - uses a structural estimate of the trend unemployment rate developed in a cointegration analysis by Cote and Hostland of the Bank of Canada (1996), the previous period's NAIRU, and a Philips Curve relation. The objective function that is minimized is given as

$$F_t = (u_t - u_{nt})^2 W_1 + (u_{n,t} - u_{n,t-1})^2 W_2 + (c_t - u_{nt})^2 W_3 + e_t^2 W_4 , \qquad (33)$$

where u_t is the actual unemployment rate, c is the trend unemployment rate, and subscript n denotes the NAIRU, e_t is the residual from a Philips curve equation and the Ws are the - a priori - chosen weights There is also an end-of-the sample growth restriction, and a smoothness restriction similar to that used in the HP filter.

The growth restriction (the second term) in the objective function (33) reduces the importance of the last observation. The residual e_t is from an asymmetric Philips Curve with the property that a negative gap between actual unemployment and the NAIRU is more inflationary - in absolute terms - than the disinflationary effect of an identically sized positive gap.

The estimation of potential output in (32) also requires an estimate of the equilibrium values of MPL and the labour-output elasticity. Under the CD production function assumption and perfect competition, labour's share of income is equal to the labour-output elasticity. To remove cyclical variations in this, the HP filter is applied with the smoothness parameter set to a high value of 10000 (the usual choice is 1600) to the measured income share to get the equilibrium value.

The *equilibrium marginal product of labour* is estimated more elaborately, the following conditioning information being used, with appropriate weights given to the squared gaps: previous period's equilibrium MPL, the real producer wage W, the residual from an inflation-MPL relation, and the residual from a modified Okun's relation relating changes in the output gap to lagged changes in the MPL gap. Okun's modified relation is included since it can be a pointer, as when the marginal product rises above the equilibrium level, firms hire more, reducing the unemployment gap. The inflation-MPL relation provides another similar pointer. The usual growth and smoothness restrictions are also applied.

Butler (1996) notes that in comparison with HP filter estimates of potential output for Canada, the extended multivariate filter estimates fare well: current estimates for each period in recursive runs are seen to be closer to full-sample estimates for the same periods, so that less updating is necessary with the multivariate filter. But it may be possible to improve the quality of the structural information provided (some of the conditioning information in Butler (1996) is of a somewhat indirect nature), taking some inputs from the simultaneous system approach that uses basic macroeconomic relationships with clear causality. Also, open economy aspects may be incorporated more, particularly in NAIRU determination.

X. The various methods: their use in organizations, merits and drawbacks

The so-called pure smoothing or trend-fitting methods - which find scarce use now anywhere - are characterized by lack of information concerning the determinants of potential output or the natural rate of unemployment. These totally atheoretical methods derive estimates of potential values by fitting trends to output data, usually through the peaks of cycles, embodying the notion of a maximum capacity output as the level of potential output. So the cyclical position of the last observation becomes very important, even when the fit is through a limited number of successive peaks only. The basic advantage of this method is its simplicity, requiring no information at all about economic relationships, the level of factor employment etc. But this also constitutes its basic disadvantage in times of economic disturbances, and is bound to give inflated estimates in periods of, say, supply side shocks, regardless of the degree of smoothing and sophistication. In stable periods, the trend estimates thus obtained may be used as rough bench marks to note whether labour and product markets are getting heated.

Okun's law, considered a basic macroeconomic relationship between changes in unemployment and the growth rate of output, has been used extensively in the U.S. - e.g. by the President's Council of Economic Advisors and economists attached to The Brookings Institution - to derive estimates of potential output. Okun's coefficient is quite stable over fairly long periods marked by the absence of supply shocks, which together with the underlying relatively constant output growth and unemployment rates even prompted the belief that simple trend-fitting could do the job as well. The demerits of the Okun's method are clear: no consideration is given to factor markets other than the labour market; and even for the labour market, the wage and price dynamics which provides the key influences, are ignored. So any period of large relative price changes will push this method's estimates wide off the mark. But authors using this broad approach , e.g. Perry (1977), have often worked at disaggregated levels of the labour market, isolating important structural determinants of unemployment.

The *production function approach*, in contrast, can claim the merit of decomposing potential output into all its components, and is, in this way, a fully disaggregated approach, especially if aggregate labour input is also disaggregated into its various determinants. In fact, extended versions of Okun's law use a production function - in which Okun's relation is embedded - to obtain the *isolated* effects of changes in unemployment. Yet, use of a production function for estimating potential output values does not amount to a *fully* structural approach, since equilibrium or potential values of the various inputs are exogenous. Often, the natural rate of unemployment that is needed to get the equilibrium labour input is assumed to be the fairly constant rate that has or had prevailed over a period, and potential total factor productivity is determined - at least partially, even when proxies such as the input of research and development are used - by the trend method. Also, the assumption of a particular form of production function itself can be unduly restrictive. The problem of determining the potential input of capital is usually got around by the use of the actual stock of capital. The production function approach is now being used by the OECD, in preference to the HP filter method that was in use earlier, and has also been used by the staff at IMF. Furthermore, the Central Bank of Sweden (Apel, Jansen and Lindberg, 1996) has applied the production function method and compared results with those obtained using HP filter and unobserved components methods.

The *system approach* is more comprehensive than the production function method, in that much of the exogenous inputs in the latter are endogenized here - and is thus put forward as being a fully structural, simultaneous approach. As seen in section VIII, the IMF's system method determines the potential level of output and the natural rate of unemployment simultaneously, including inputs from the wage-price sector, and considering structural factors in the labour market. Yet, while consistency can be thus maintained between the estimates of potential output and unemployment, some of the weaknesses of the individual equations remain. The production function depends at least partially on an estimate of trend productivity from outside the system; in this sense the approach is not fully structural, and an important part of the estimation does not possess the advantage claimed for it.

Thus, not only do methods devoted to comprehensive modelling of the supply side have the irredeemable drawback of being unable to include *all* relevant structural information; they are all dependent to some extent or other on time trend-fitting. But it had been noted earlier that pure time trend methods that tried to handle periods of supply disturbances by using dummy variables didn't work well either. A continuous string of supply shocks permanently lowered potential output in a way that time trends couldn't capture, even with spline and kink modifications. This led to increased adoption of methods that allow for unit roots, or at least long memory, in the time series considered, and where permanent changes in output can be caused by supply shocks.

Univariate stochastic methods such as the HP filter method, while simple to apply, have the disadvantage of using only very limited information - even when additional infomation may be there just for the taking. Only the information from realized output is used to estimate the level of potential output. Given this lack of information, which also implies an inability to distinguish between demand and supply shocks, arbitrary smoothing, based on assumptions about the relative variance of supply disturbances, has to be resorted to for the estimation. There is also the difficulty of separating out persisting demand-side effects from supply disturbances, particularly at the end of the sample.

Multivariate filter methods, in contrast, tend to take in considerable additional information, not restricted to the sole output variable. Often, each of aggregate output's individual components are smoothed by filter techniques, where components are also disaggregated into structural determinants. Reliance on any pure time trend is avoided, and basic macroeconomic relations as well as ad hoc structural information are used so that these are "stochastic-structuralmethods", using *structural* modelling rather than arbitrary assumptions about demand and supply shocks. Sometimes informed judgements are also incorporated as inputs, which may be important particularly at end-of-sample points. The Bank of Canada works extensively with these methods for the estimation of potential output and the natural rate of unemployment. This procedure seems to be able to side-step the disadvantages associated with pure structural, time trend, or stochastic methods.

XI. Conclusions

A totally structural approach - even of the simultaneous system type - as well as the univariatefilter methods seem to have drawbacks that are avoidable. So do methods based on Okun's law. In fact, this relationship may be better utilized to predict changes in unemployment using estimates of potential output derived elsewhere. The output gap may be a better predictor of the unemployment gap than of inflationary surges: the output gap - inflation correlation seems weak at least for Sweden.

An extended multivariate filter method along the lines of Butler (1996) of the Bank of Canada, with additive structural information replacing the a priori restrictions of univariate methods, seems a good candidate for producing better estimates of potential output. It also has the advantage that the filter is applied to each of the components - and often their determinants - of potential output, at a very disaggregated level. The method also avoids having to deal with estimations of trend total factor productivity and capital stock (but puts some restrictions on the production function). But it should be possible to further improve the quality of structural information fed in, using well-established relationships such as those used in the simultaneous system approach, to replace some of the more indirect or *ad hoc* relations.

There may be a case for replacing the concept of capacity output in KOSMOS, the macroeconometric model of the National Institute of Economic Research, with that of potential output based on an estimated natural rate of unemployment. Currently, the level of actual output in industry is derived using an index of deviations in factor utilization from the level of output that corresponds to full employment - that is never ever reached. It may be more intuitive to work with an index of deviations in utilization from an equilibrium level of employment - that is attainable.

As regards the estimation of the equilibrium level of unemployment, the NAIRU, open-economy influences should be fully incorporated - which is usually not the case even in multivariate filter approaches. Clearly, the NAIRU, as is commonly defined, depends on the real exchange rate. A longer-run equilibrium concept, that imposes strict continuous current account balance may be smacking more of Friedman (1968), and may not be resonant with the stochastic approaches where supply shocks are deemed to have permanent effects. This does have some identifiable policy implications: when there is a current account surplus that leads to, or allows a fall in the (endogenous) *relative* price of imports, the NAIRU is lowered as described in Section

V. Hence there seems to be an unemployment - current account trade-off which is now in a favourable constellation for beneficial policy action in Sweden.

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A note on finding a *NAIRU* for Sweden using univariate filtering

Lasse Koskinen and Lars-Erik Öller

1. Introduction

For a long time there has been an urge to find "hidden components" in the more or less irregular statistical time series on the economy, the idea being that these time series would be easier to understand if they are (log-) additively decomposed into a trend (or a constant level), a harmonic oscillation and a symmetric fully random disturbance. In the case of unemployment the trend has been assumed to be a constant that has been assigned an economic meaning: the rate of utilization of labour that keeps the inflation from accelerating *(NAIRU)* (cf. Layard et al 1991).

A glance at the Figures 1 and 3 will make even a convinced adherer to these simple explanations to unemployment sceptical. Except for possibly the North American series, there seems to be no constant level, and no harmonics. Recent research has questioned the (constant) NAIRU concept even in USA.[1] Evidence of hysteresis, appering as level shifts, in unemployment rates have been found e g. in Assarson and Jansson (1995), and Jacobson et al. (1997) for the Scandinavian countries, and Jaeger and Parkinson (1994) for some other European countries and Canada.

Assuming the simple theoretical model, attempts have been made to estimate *NAIRU*. This has been accomplished either within a multivariate setting, or else using some univariate filtering. Here we will be studying univariate approaches. We shall start by describing how *NAIRU* has been defined in the literature in Section 2. In the following section we compare two well known filters: the *Moving Average* and the *Hodrick-Prescott* filters to the *Median Filter*. In Section 4 we apply the three filters, choosing the one that provides the highest degree of determination in the Philips equation, relating excess unemployment to accelerating inflation. We find that only for the US and possibly for Canada and Germany can a constant *NAIRU* be regarded as having some reasonable content of the "stylized fact" character. Some weak connection also seems to apply in some other countries, in case *NAIRU* is allowed to change over time. If there is any such connection in Sweden, we show that the median filter would be the one to be applied. However, in the concluding section we suggest that given the huge rise in unemployment in Sweden in the beginning of the 1990's, as compared to the rest of the variance in this series, not much is gained from resorting to a decomposition.

[1]For more readings on this debate , cf. Galbraith (1997), Gordon (1997), and Staigner (1997

2. Defining *NAIRU*

The standard definition of *NAIRU* is the degree of unemployment u^*. for which the price level p_t at no point t in time will accelerate (or decelerate), $p_t''=0$:

$$p_t'' = -b(u_t - u^*) + a_t, \qquad (1)$$

where b is a constant and a_t is a random term. Friedman (1996) emphasized that *NAIRU* can change over time, but to have any meaning it must be less volatile than observed unemployment. This gave rise to various low pass filtering techniques for constructing a time dependent *NAIRU*, u_t^*, so that instead of (1) we study:

$$p_t'' = -b(u_t - u_t^*) + a_t'. \qquad (1')$$

Next we take a look at filters discussed in the literature, and suggest a new one.

3. Moving Average, Hodrick-Prescott and the Median Filter

3.2. General comments on filters

A filter is a mathematical operation which results in a decomposition of a time series into two or more components, having some desired properties. A slowly changing trend may be one such component. The filters operate on many successive observations at a time, trying to find a characteristic value for these observations, moving from one end of the data to the other. The weight function that operates on this string of observations is called a *window*, and the length of the string is the same as the *length* of the window. To give an example, a three terms moving average is a window of length three and with the weight function (1,1,1). The filtered value represents the middle observation.

Here only symmetric filters, containing an odd number of terms, are used because otherwise one shifts the time scale of the filtered series as compared to the original one. Because of symmetry one looses data points in both ends of the series. This dilemma can be handled in the same way as when using moving averages in seasonal adjustment. Here the end values are substituted for the missing observations.

3.2. The Moving Average (MaF)

The well known moving average is the weighted mean of the observations in a window of length k, where k is smaller than the total length of the time series, n . When this window is stepwise moved from the start to the end of the time series, the method generates a series of averages.

Here we use only equal weights, $1/k$, which means that the window is an arithmetic mean.

3.3. Hodrick-Prescott (HP)

The Hodrick-Prescott (1980) filter is a widely used smoothing method (it can be shown to be a weighted moving average), defined by the formula

$$\min_{u^*_t}(\Sigma[u_t - u^*_t)]^2 + \mu\Sigma[u^*_{t+1} - 2u^*_t + u^*_{t-1}]^2), \qquad (2)$$

where, as before, u_t is observed unemployment and u_t* is the filtered (smoothed) series. The first term to be minimized requires the smoothed series to be as close as possible to the observed values. The second term minimizes the *second derivative*, implying that the smoothed series is not allowed steep changes, the smoothing effect depending on the value given to the parameter μ. A large value of μ generates a very smooth filtered series; a small μ means that one hardly wants any smoothing at all, cf. Figure 1. The idea behind *HP* filters is close to *smoothing splines*, where u just belongs to the group of smooth functions (cf. Luo et al 1996).

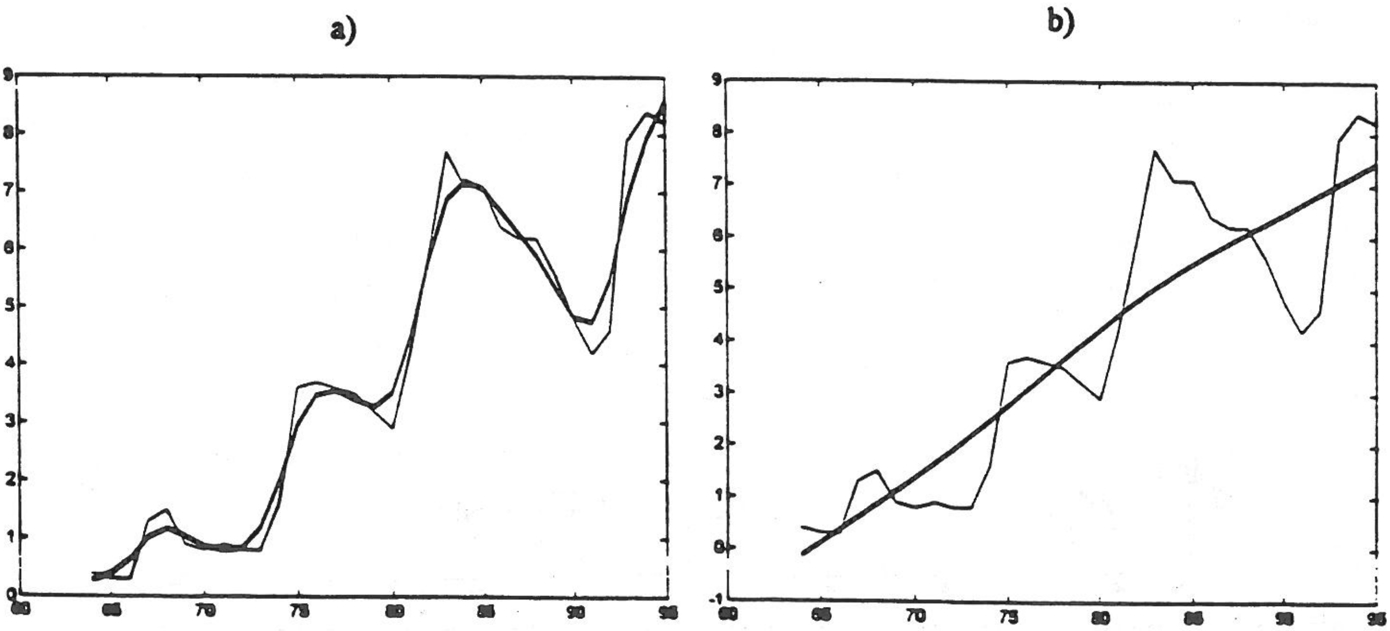

Figure 1. The unemployment in Germany and a) the output of *HP* with $\mu = 1$ (thick curve) and b) the output of *HP* with $\mu = 1000$ (thick curve).

3.3 The Median (MedF)

The sample median is defined as the middle value when the observations are in increasing order. It has been widely used as a location parameter, especially for skewed distributions (cf. income distributions). *MedF* is robust to ouliers[2].

3.4 Comparing the filters

MaF and *HP* are both linear. Without here trying to explain what "linearity" means in this case, suffice it to say that linear filters are easy to analyse, eg. using spectral methods. In contrast, the *MedF* filter is non-linear and its properties have to be analyzed using simulations, etc.

MaF will smooth the edges of a ramp. In the case of unemployment, the smoothed series would *anticipate* the rise in unemployment and reach the higher level much after the observed series. Trying to rationalize this behaviour one would imply that a *NAIRU* based on the smoothed curve would show some "potential" unemployment, growing *before* the rise occurs, and after this, one would regard the observed values as an overreaction, continuing until the smoothed and the unsmoothed curves meet again. No evidence has been produced hinting at such a behaviour.

A characteristic feature of *HP* is that the effect of the filter varies with the series filtered, and this is accomplished through varying values of the parameter μ in (2). For a series with a huge ramp, like the one in Figure 2 this would mean that a compromise μ would be chosen that would let through some of the small oscillations before the ramp, but still "cut" the edges of the ramp, like *MaF* did. While *MaF* uses only part of the series at a time, *HP* chooses the smoothing parameter μ using the entire length of the time series. Because of this we do not think that letting the smoothing parameter vary, as in Razzak and Dennis (1996), is a consistent way of improving the robustness of the filter.

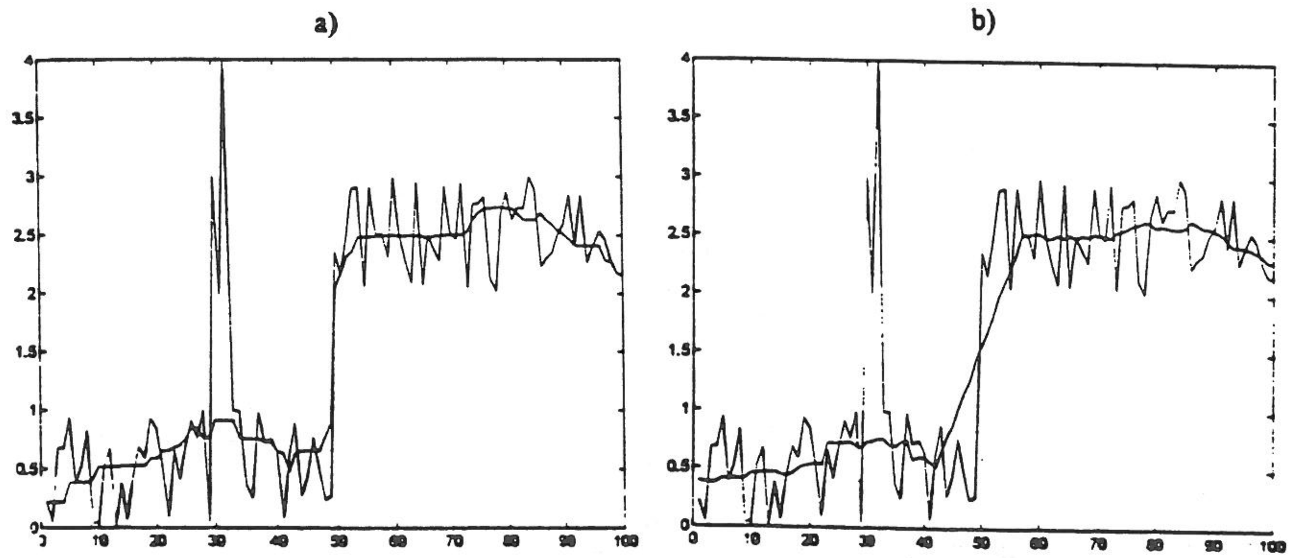

Figure 2. Original sequence and the result of a) *MedF*, b) *MaF* (window length is 15).

[2]For more readings on *MedF*, cf. Huber (1980) and Hampel (1986).

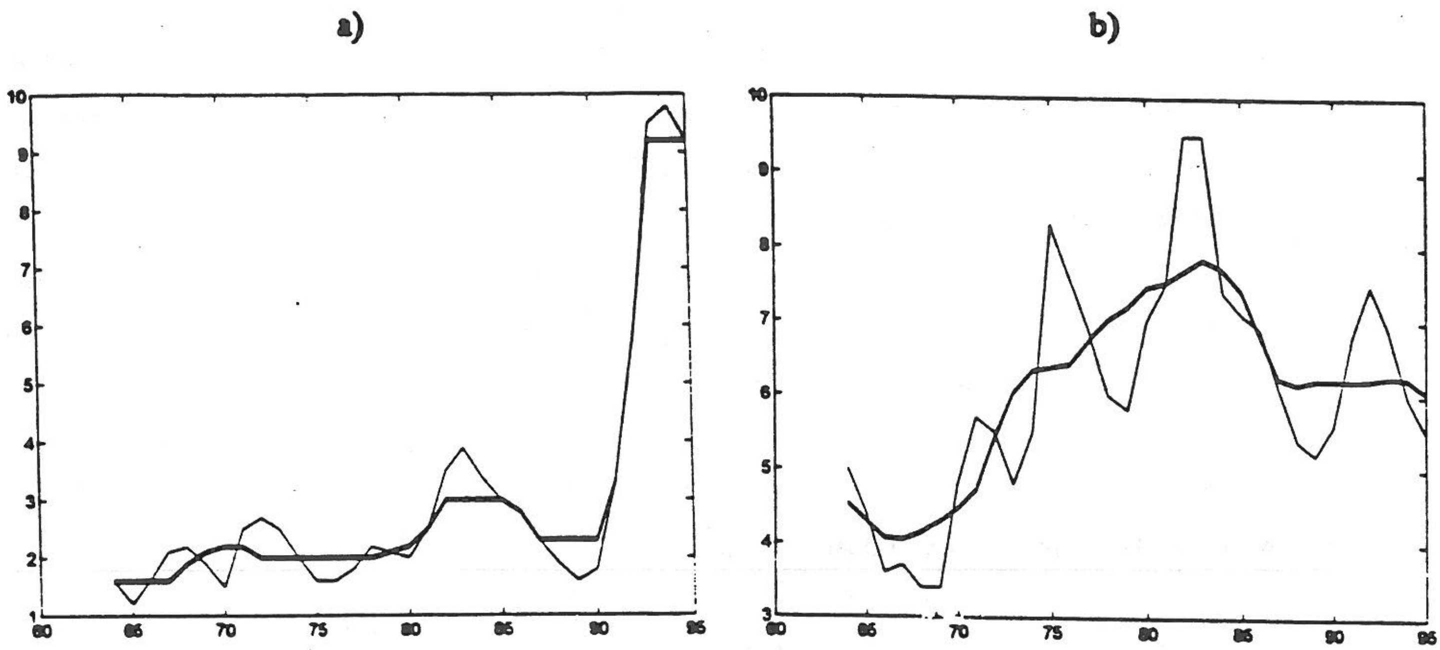

Figure 3. a) The unemployment series of Sweden and the *NAIRU* estimate (thick curve), b) the unemployment series of USA and the *NAIRU* estimate (thick curve).

Table 1. NAIRU statistics for 12 OECD countries (* significant at 1% level).

Country	100xR^2 *for constant NAIRU*	*Filter and its parameter*	100xR^2 *for time varying NAIRU*
USA	35*	*MaF* k= 7	61*
Japan	6	*HP* μ=½	46*
Germany	17*	*MaF* k=11	48*
France	11	*HP* μ=1000	21*
UK	13	*MaF* k=1, HP= ½	48*
Italy	4	*MedF* k=3	14
Canada	22*	*MaF* k=7	40*
Australia	10	*HP* μ=1000, *MaF* k= 5	20*
Finland	3	*MedF* k=10	10
Norway	5	*MaF* k =5	12
Spain	7	*HP* μ=1000	8
Sweden	6	*MedF* k=7	18*

MedF was introduced in Tukey (1971). As an imaginary example, consider a time series with a temporary outlier and a ramp, like that in Figure 2. *MedF* (like *MaF*) ignores the temporary outliers and reproduces the original series where its level shifts (which *MaF* does not) and then remains constant on the new level. This is a typical feature of *MedF*: it reproduces all monotone functions and has a tendency to remain constant for small changes (cf. Figure 2 a)). It may also reproduce small ramps, which may sometimes be an undesirable feature.

If one thinks that a constant *NAIRU* is an unrealistic description of reality, then the next thing that comes to one's mind is a *NAIRU* that switches regimes, staying constant within the regime. *MedF* comes very close to that concept. One could add that no linear, time invariant filter can accomplish this.

4. Estimating *NAIRU* from OECD data by filtering

4.1 Trying to find a reasonable estimated NAIRU

A simple way to test if there is any content in the *NAIRU* concept is to substitute observed values of p_t and u_t into (1) and, using ordinary least squares, to study the coefficient of determination R^2. No a priori assumption need to be done concerning u^* whose value is estimated in the process. If there is any connection between unemployment (deviations) and acceleration of inflation, then R^2 should be statistically significant[3]. Next one can allow for variation in *NAIRU*, using different filters to obtain u_t^* in (1'). The filter that produces the largest R^2, provided it's significant, could be regarded as the most reasonable estimated *NAIRU*. The data used here are annual observations of CPI and unemployment between the years 1964-1995, as they appear in *Economic Outlook*, published by OECD.

4.2 A constant NAIRU

From Table 1, the first column of numbers, we see that the assumption of a constant *NAIRU* can possibly find some support only in the cases of USA, Canada and Germany. Note, however that the values of R^2 are very low for both Canada and Germany.

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[3] Naturally (1) and (1') are unidentified, since both variables are endogenous. Probably important variables and lags are missing, and hence there may be both proxying and residual autocorrelation inflating the value of R^2. The normality of residuals can also be challenged. Hence test values can only be regarded as very approximate. To somewhat cope with these shortcomings we use only the 1 % significance level of the F test.

4.3 Varying NAIRU

The second column in Table 1 shows the filter and parameters that produced the highest R^2 when *NAIRU* is allowed to vary. We see that *MaF* results in the highest degree of determination and this is statistically significant in four cases, *HP* in three and *MedF* in one. Note that in almost one half of the cases *NAIRU* does not work, using any of the filters. *MedF* is the filter chosen for Sweden. This means that if one insists on using a *NAIRU* for Sweden, the median is the best choice. However, R^2 is very low, only 0.18. Figure 3 a) shows the resulting estimates.

5. Concluding remarks

The most important result from this study is that it is possible to find some substance in the *NAIRU* concept. However, the idea of a global constant level has to be abandonded. Only for USA and possibly for Canada and Germany can a constant *NAIRU* be given any empirical content. Assuming a varying *NAIRU* considerably increases the degree of determination even for these countries. This assumption also produced signifance for a few other countries, although only in slightly more than half of the countries analyzed did the *NAIRU* connection prove significant at all. It is of some interest to note that for Sweden a nonlinear median filter produced a significant connection.

Our contribution is that we use the given *NAIRU* definition as an objective function when searching for a suitable filter, where a nonlinear filter is one alternative. Especially for Sweden, the nonlinear median filter seems to produce a smoothed *NAIRU* that could be entertained, keeping in mind that the connection in the case of Sweden is very weak. It is important to emphasize that this study does not show that *NAIRU* would be an important concept for understanding inflation and unemployment. However, our results support the observations of Staigler et al (1997) that NAIRU can have only limited role in predicting inflation.

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THE OUTPUT GAP COMPUTED USING THE RELATIONS IN KOSMOS

by

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February 1997

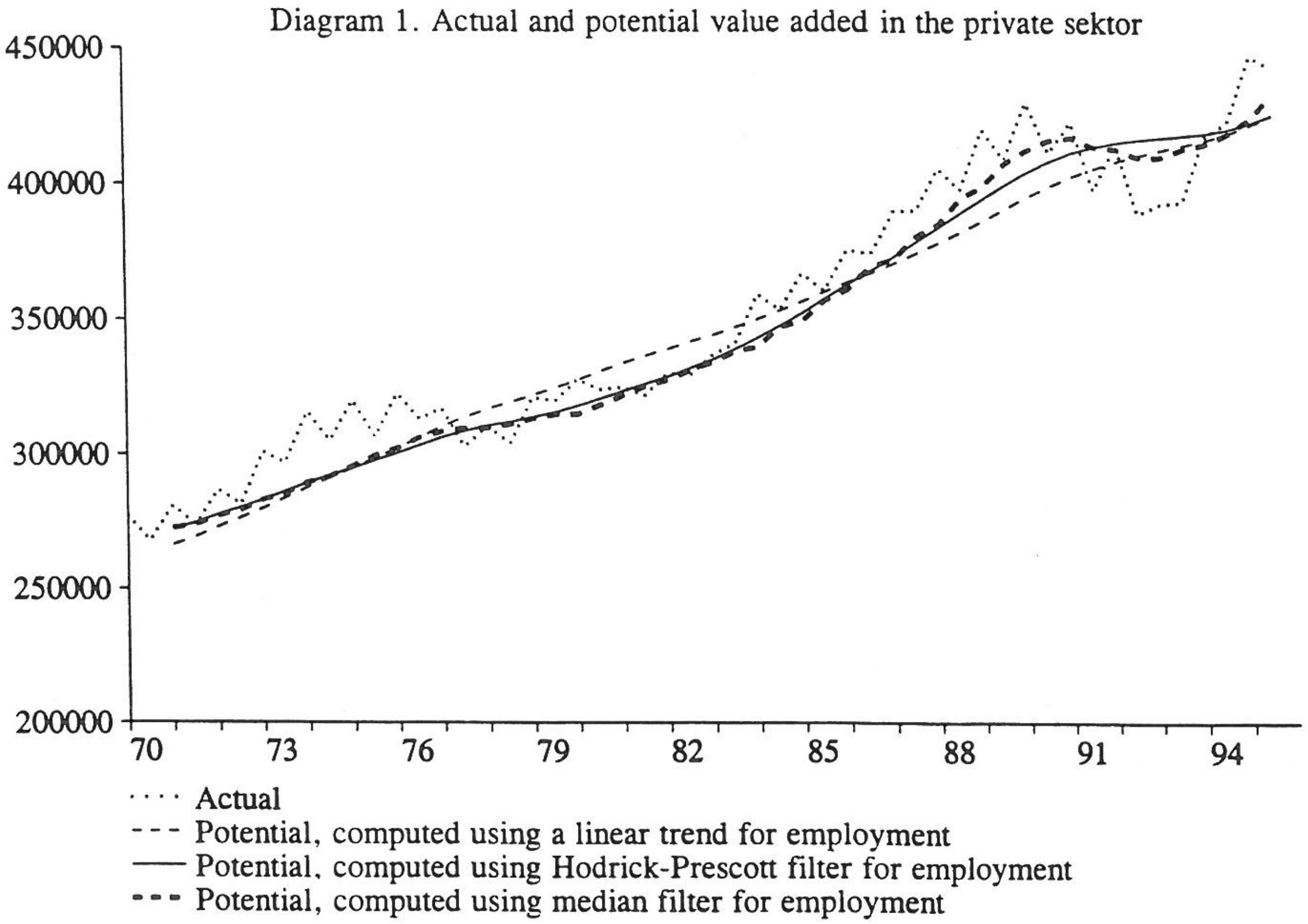
Diagram 1. Actual and potential value added in the private sektor
450000
400000
350000
300000
250000
200000
70
73
76
79
82
85
88
91
94
Actual
Potential, computed using a linear trend for employment
Potential, computed using Hodrick-Prescott filter for employment
Potential, computed using median filter for employment

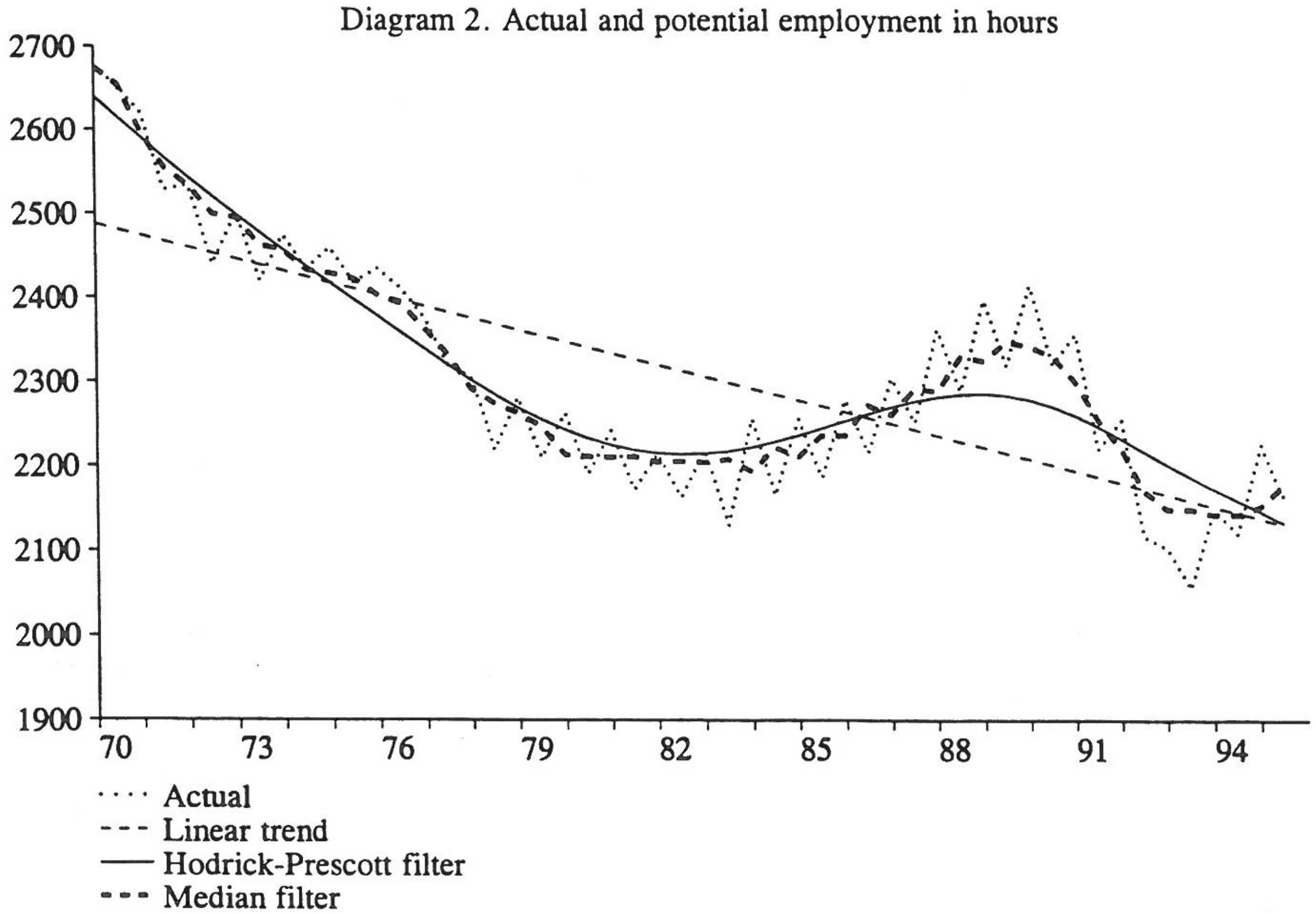
Diagram 2. Actual and potential employment in hours
2700
2600
2500
2400
2300
2200
2100
2000
1900
70
73
76
79
82
85
88
91
94
Actual
Linear trend
Hodrick-Prescott filter
Median filter

The National Institute of Economic Research in Stockholm has recently been investigating different ways of computing potential output and the thereby implied output gap* . The latter is employed to assess the possibility of economic expansion without accelerating inflation.

One way of computing potential output is by introducing "potential values" for the factors of production into a production function. Output gap is subsequently computed as the ratio of actual output to potential output.

Below, the output gap for Sweden is computed using the relations of KOSMOS , the econometric model developed at the Institute. KOSMOS has two private, producing sectors: industry and other business. For each of them, the model includes a CES production function, which determines value added using two production factors: capital and labour. Potential output (value added) was computed for each sector separately, using the actual capital stock[1] and three different definitions of the potential labour input. Total potential output was obtained as the sum of potential outputs for the two sectors.

The output gaps implied by the above computations are illustrated in Diagram 1. The diagram shows actual value added and three definitions of potential output, based on three assumptions about the potential labour input. The assumptions are crude, their main aim being to illustrate the approach. The difference between the actual and potential output is the output gap.

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* The author is indebted to Håkan Frisén and Lars-Erik Öller for valuable comments and help and to Nils Hofvander for computational help.

[1] The production functions are of the nested type. The capital stock in industry is defined as a CES aggregate of machinery and construction, the capital stock in other business is an arithmetic sum of the two categories. Cf. Markowski and Ernsäter [1994].

Potential labour input is usually assessed using assumptions about labour supply, unemployment and hours worked. The experience of the last few years indicates that there can be a relation between increasing unemployment and increasing average work week. In order to account for this relation, the potential labour input was assessed upon transformation of labour demand expressed in hours rather than as the number of employed. Alternative potential labour input series were obtained from labour hours for 1970-95 by applying a) the median filter, b) the Hodrick-Prescott filter and c) the linear trend estimator. Potential labour input series were computed separately for industry and other business. Diagram 2 shows the sums of both sectors according to each method.

According to Diagram 1, the output gap was positive during the second half of the 1980-ies and negative in 1991-93. This means that the existing productive resources in the private sector were not utilised in full in the beginning of the 1990-ies. The degree of utilisation increased, however, significantly in 1994-95, when the output gap is again positive. It appears that this was due to higher demand rather than to lower productive capacity.

The above picture appears to be in agreement with our general idea about the development of *capacity utilisation* in the private sector. It is, however, difficult to say, whether it also gives an adequate description of the *inflationary pressures* in the economy. According to the prevalent interpretation of the output gap, the inflationary pressure in 1994 would have been higher than in 1992. An important factor that, however, is not allowed for in this kind of analysis is the expectations formation.

The picture of the output gap is little affected by the method employed to smooth the employment variable. Generally, the linear trend gives larger output gaps. Furthermore, it results in a negative gap 1977-83.

The potential employment above does not allow for the level of unemployment. This gives the resultant output gap a short-term character, which actually is close to the notion of capacity utilisation. The idea behind this approach is that the employment level cannot change much unless the capital stock - and possibly its structure - is adjusted accordingly. However, this does not necessarily mean that *all* unemployment is considered to be structural, since some trade-off between the number of employed and hours worked per week still is possible.

An alternative approach would involve a closer analysis of those unemployed in order to determine their actual potential for increased employment. Potential employment could then include an appropriate part of the unemployed, subject to the usual restriction on inflationary pressure. The capital stock could possibly also be expanded, in order to avoid large changes in the capital intensity of the production process. This approach would give the output gap a longer-term character but at the same time make it more imprecise, since potential output no longer would refer to the potential of today but rather to some vague future when only capital stock and labour input were defined.

As can be seen in Diagrams 1 and 2, as long as potential employment follows the actual one, its exact definition is not crucial for the result[2]. The output gap would, however, change if a large part of those unemployed were to be included in the potential labour input. In fact, the level of potential employment is crucial for potential output. In a way, in the production function approach the main difficulty is shifted to an earlier stage. The difficult question of potential output development is replaced by the equally difficult question of potential labour input.

The problem becomes much more complicated when potential employment is defined in terms of non-accelerating inflation (cf. the NAIRU). One could imagine scenarios where a given employment (or unemployment) level is accompanied by different inflation

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[2] One might believe that direct filtering of the value added series would give almost the same output gap as above, since value added and employment in hours are highly correlated. This is, however, not the case, as illustrated in Diagram 3.

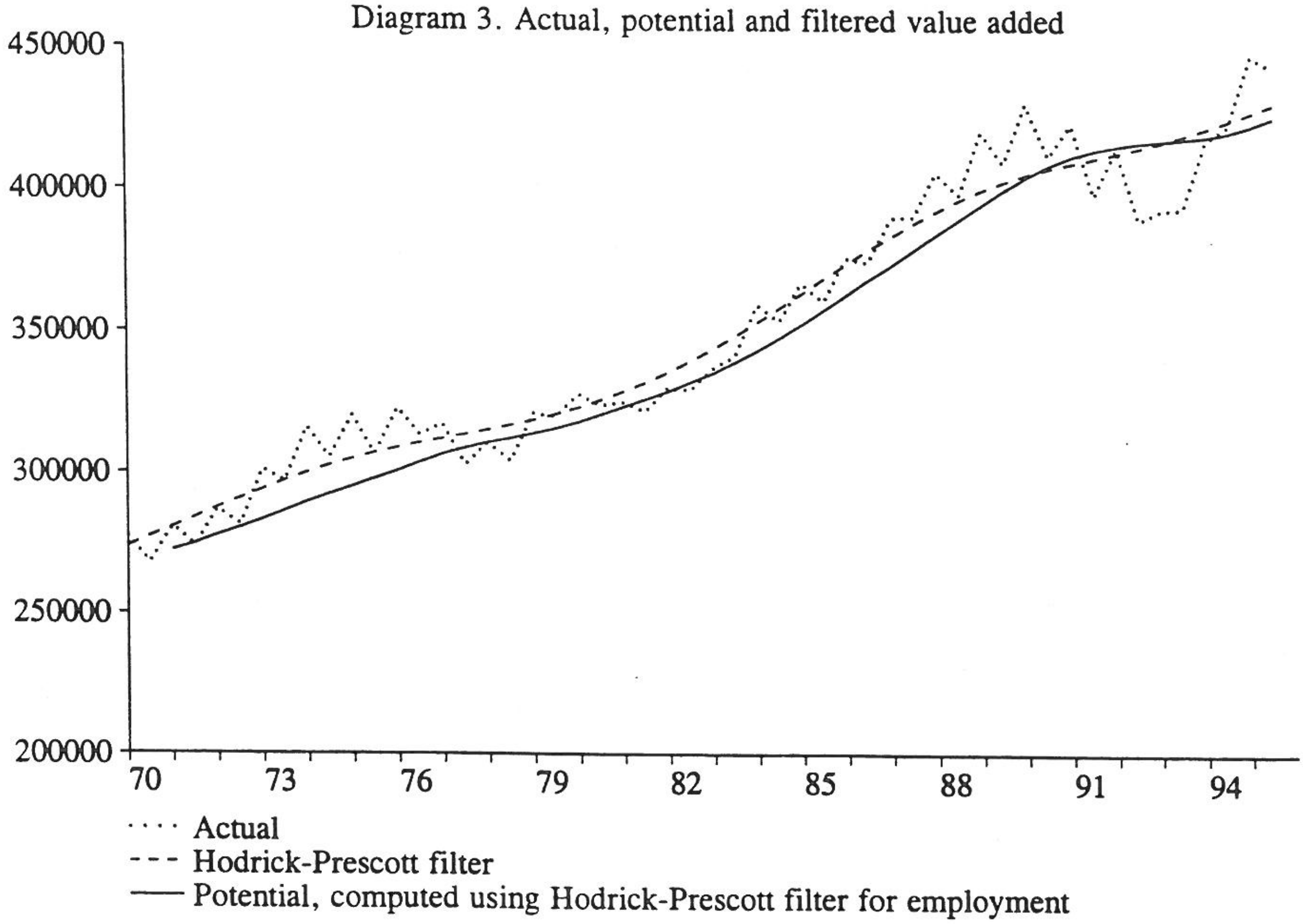
Diagram 3. Actual, potential and filtered value added
450000
400000
350000
300000
250000
200000
70
73
76
79
82
85
88
91
94
Actual
Hodrick-Prescott filter
Potential, computed using Hodrick-Prescott filter for employment

developments, depending on the overall economic situation. Investigations of the output gap appear in this context as attempts to reduce a complicated simultaneous system to a simple one-equation model. In my opinion, much more light will be shed on the questions of interest when a complete simulation with our econometric model KOSMOS (cf. e.g. Ernsäter and Markowski [1997]) is analysed rather than a single output gap variable.

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Svenskt sammandrag

Tre uppsatser om produktionsklyftor

Detta nummer av *Working Paper* serien innehåller tre uppsaster: en kritisk översiktsartikel av litteraturen om begreppet "produktionsklyftan" *(output gap)*, en liten studie av univariata metoder för beräkning av det närliggande begreppet *jämviktsarbetslöshet* samt en kort beskrivning av hur klyftor kan framräknas med hjälp av Konjunkturinstitutets ekonometriska modell *KOSMOS*.

Produktionsklyftan: mätning, relaterade begrepp och implikationer för politiken inleds av en beskrivning av en gammal metod att beräkna hur produktionen avviker från en tänkt produktion som fullt ut utnyttjar befintliga produktionsfaktorer. Metoden går ut på att parvis sammanbinda två konjunkturtopparmed hjälp av räta linjer. Skillnaden mellan linjesegmenten och faktisk produktion har använts som ett mått på klyftan. Utbudschockernapå 1970-talet ledde till missvisande klyftestimat och detta gjorde att många övergav denna enkla metod. En annan lätt förstådd och utförd metod bygger på *Okun's lag* och relaterar produktionens avvikelse från sin trend till arbetslöshetens avvikelse från någon form av jämviktsarbetslöshet. Modellen kalibreras på ett utvalt år som skall vara "normalt". I Okun's lag tas ingen hänsyn till dynamik, utbudschocker och många andra faktorer som kan tänkas ha relevans. Man har därför försökt sig på att bygga ut modellen och har då närmat sig de systemansatser uppsatsen återkommer till.

Vid OECD använder man sig av en metod som bygger på en Cobb-Douglas produktionsfunktion. Arbetsinsatsen bestämmes separat med hjälp av en beräknad jämviktsarbetslösheten, som i sin tur fås genom linjär filtrering. Faktorproduktiviteten estimeras och den resulterande produktionsserien utjämnas linjärt. Eftersom detta är en en-ekvationsansats kan inga återkopplingar beaktas.

För att beräkna produktionsklyftor måste man i modellansatserna alltså först beräkna någon slags jämviktsarbetslöshet.Man brukar då starta med två ekvationer, där den ena är en Philips-kurva och den andra är en ekvation för prisuppskrivning *(mark up)*. På lång sikt blir Philips-kurvanen vertikal linje och man får ett enda värde på det sk. *NAIRU*, dvs. den arbetslöshet som inte får inflationen att accelerera. Genom att öppna modellen för utrikeshandel får man en *NAIRU* som varierar med bytesförhållandet med utlandet *(terms of trade)*. Hypotesen om en konstant jämviktsarbetslöshethar ju inte funnit mycket empiriskt stöd, så genereliseringen är ett försök på en mer realistisk skattning.

När allt fler faktorer skall beaktas blir man snart tvungen att övergå till ett ekvationssystem. *IMF* använder ett sådant för att samtidigt bestämma både produktionsklyftan och jämviktsarbetslösheten. Man får på det sättet fram variabla, men släta jämviktsserier som är sinsemellan konsistenta.

Stokastiska metoder som bygger på filtrering har använts för att ta fram trender i arbetslösheten. Det mest använda filtret under senare tid är det sk. *Hodrick-Prescott-filtret (H-P)*.Till den stokastiska metodgruppen kan man även räkna modeller med oobserverade komponenter. I sådana modeller lägger man släthetsvillkor på de oobserverade jämviktsvariablerna: produktionsklyfta och jämviktsarbetslöshet - och så estimeras de med hjälp av en läges-rymd-modell *(state-space model)*.

Bank of Canada använder sig av multivariata strukturella metoder, där man på faktorinstatserna tillämpar *H-P*-filtret. I dess mer utarbetade form utgår metoden från en Cobb-Douglas produktionsfunktion som deriveras och skrives i log-form. *NAIRU*-estimatet fås som ett vägt medetal av värden som stöder sig dels på en Philips-ekvation, dels på en strukturell skattning med en felkorrigeringsmodell och dels på ett jämnhetsvillkor. Metoderna bygger i alla steg på kvadratminimering och är således linjära.

Nandakumar avslutar sin översikt med några rekommendationer. Att skatta produktionsklyftan med hjälp av trender mellan produktionens lokala maxima får anses vara en alltför godtycklig, om ock enkel metod. Okuns lag utdömes också eftersom den inte beaktar andra insatsfaktorer än arbete, ej heller arbetets pris/löne-dynamik.

En produktionsfunktion har den fördelen att den åtminstone beaktar dekomponeringen: arbete och kapital, ofta även insatstermen forskning och utveckling. Men då ekvationen inte finns i ett system antas insatser, såsom jämviktsarbetslösheten vara exogent givna och där tar man till grova filtreringar. Denna metod tillämpas numera av *OECD* och av *Sveriges Riksbank* som ett alternativ till *H-P*. En fullt ut strukturell bestämning av produktionsklyfta och jämviktsarbetslöshet gav i *IMF*:s ansats konsistens mellan dessa två skattningar, men man kunde ändå inte kringgå behovet av exogen filtrering för att ta fram en jämviktskomponent.

Den multivariata metod som används av *Bank of Canada* har klara fördelar då det gäller att beräkna produktionsklyftan, isynnerhet om man ersätter vissa restriktioner med strukturell information. Filtreringen tillämpas här på disaggregerad nivå. Även i *KOSMOS* kunde man gå in för potentiell och uppnåelig produktion, utgående från jämviktsarbetslöshet, istället för kapacitetsproduktion.

NAIRU måste definieras som varierande i tiden i en liten öppen ekonomi. Vid stort överskott i utrikeshandeln stärkes *ceteris paribus* bytesförhållandet med utlandet och detta leder i sin tur till lägre *NAIRU*.

I den andra uppsatsen av Koskinen och Öller undersökes ifall begreppet *NAIRU* kan ha någon motsvarighet i data, och om så är, vilket univariat filter man isåfall borde använda. Först ser man på det empiriska stödet för en konstant *NAIRU*. En viss korrelation som stöder relevansen i ett sådant begrepp kan hittas i data från Nordamerika och möjligen i tyska data. Tillåts *NAIRU* variera över tid uppnås signifikant korrelation mellan arbetslöshetens avvikelse från sin trend och förändring i inflationen bland majoriteten av länder. Olika filter prövas och det som ger den högsta korrelationen väljes. För Sveriges del kan man nå signifikans om man använder ett olinjärt *medianfilter*. Detta filter följer med i trappvisa förändringar i arbetslösheten av det slag som är vanliga i variabler med drag av *hysteresis*. Men i de fall att de största förändringarna är just av detta slag, frågar man sig vad man vinner på att överhuvudtaget filtrera fram något utjämnat värde.

I den sista uppsatsen använder sig *Markowski* av två samband i *Konjunkturinstitutets* ekonometriska modell *KOSMOS* för att räkna fram produktionsklyftan. Modellen har två privata sektorer: industrin och övrigt näringsliv. Den potentiella produktionen ges av *CES*-funktioner, en för var sektor. Kapitalstocken definieras som den verkliga stocken, medan man använder exogen filtrering av arbetsinsatsen. Tre filter prövas: medianen, *H-P* och en linjär trend. Slutsatsen blir att man genom att lägga in en filtrerad arbetsinsatsserie i en produktionsfunktion bara skjuter problemet framför sig. Vill man ha ett svar på frågan om hur mycket man kan expandera ekonomin utan att inflationen tar fart skall man använda hela *KOSMOS*-modellen för simuleringar som ger bättre underbyggda och fullödigare svar på frågan.

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