SELMA
Svensk Ekonomisk Lineariserad Modell för samhällsekonomisk Analys
Technical Documentation*

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Abstract

This document contains a full description of The National Institute of Economic Research's (NIER) model SELMA, a DSGE model intended to support macroeconomic analysis and forecasting at the NIER and at the Ministry of Finance. The model consists of a small open economy; Sweden, and a large economy that represents the rest of the world; Foreign. Sweden consists of a household sector with Ricardian households that have access to financial markets and Non-Ricardian households that do not, a production sector that is subject to price and wage stickiness, and a central bank that conducts monetary policy according to a Taylor rule. In addition, the Swedish economy is characterized by a detailed fiscal sector, with a fiscal authority that conducts fiscal policy and has several fiscal instruments at its disposal. Sweden engages in trade with Foreign. The Foreign economy consists of a household sector, a production sector that is subject to price and wage stickiness, and a central bank that conducts monetary policy. This technical documentation entails a description of the model, a description of the parametrization of the model, and a presentation of impulse response functions for selected shocks. The document also contains a full list of the dynamic and steady state equations, and derivations of these equations.

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1 Introduction

The model described in this documentation is named SELMA. SELMA is a two country version of a New-Keynesian DSGE model which depicts the Swedish economy and a Foreign economy, where the latter represents the rest of the world. The non-fiscal blocks of the model build on well-known contributions by, among others, Christiano, Trabandt, and Walentin (2011), Christiano, Eichenbaum, and Evans (2005a), Smets and Wouters (2003) and Adolfson et al. (2008). The most closely related model for the non-fiscal part of SELMA is the model described in Corbo and Strid (2020), whereas the most closely related model for the fiscal part is the model described in Coenen, Straub, and Trabandt (2013). The Swedish economy in SELMA is characterized by a detailed fiscal sector, with a fiscal authority that conducts fiscal policy and has several fiscal instruments at its disposal as well as the ability to issue government debt. With a detailed fiscal sector, it is possible to analyze the effects of fiscal policy, taking into account general equilibrium effects as well as analyzing the interaction between monetary and fiscal policy. In addition, an energy sector similar to Corbo and Strid (2020) is included.

In the Swedish economy, unemployment is modelled following Galí (2011) and Galí, Smets, and Wouters (2012).

The main difference between SELMA and its earlier counterparts in the models in Adolfson et al. 2008 and Coenen, Straub, and Trabandt (2013), except for the presence of unemployment in SELMA, is the structure of the Foreign economy. Both Adolfson et al. (2008) and Coenen, Straub, and Trabandt (2013) assume a vector auto-regressive (VAR) representation of the Foreign economy, while in SELMA it is modelled as a structural economy with optimizing, forward-looking households and firms, in a similar manner as in Corbo and Strid (2020). The main advantage of modelling the Foreign economy as structural is that shocks that originate in the Foreign economy can be interpreted in terms of the model mechanisms.

The layout of the rest of the document is as follows: the model is presented in Section 2, the parametrization of the model is presented in Section 3, and impulse response functions for selected shocks are presented in Section 4. The stationarized and log-linearized model equations are presented in Appendix A, the steady state equations are in Appendix B, derivations of the model equations are in Appendix C, D and E, while the variable and parameter definitions are in Appendix H and Appendix I respectively. The estimation methodology is described in Appendix J, the details of data transformations in estimation is given in Appendix K, the observation equations for the model estimation process are given in Appendix L. Finally, model-implied statistics of interest and the impulse response functions of the selected model variables to model shocks are reported in the Appendix M.

2 The model

The world economy consists of two economies, Sweden, and the rest of the world, called Foreign.[1] Sweden is a small open economy, which means that Sweden relies heavily on trade with other countries. At the same time, Sweden is a sufficiently small economy relative to the rest of the world that changes in the economic environment or economic decisions in Sweden do not affect Foreign. In contrast, Foreign is a large economy, which means that changes in the economic environment or the economic decisions in Foreign have an impact on Sweden. Households and firms in both economies make decisions based on optimizing behavior and rational, forward-looking expectations. We assume trade in goods and bonds between the two economies, but we abstract from the possibility of labor mobility between countries.

In the Swedish economy, the household sector is composed of two types of households: Ricardian and Non-Ricardian. Both types of households consume and work. The difference between them is that the Ricardian households have access to financial markets, which implies that they can save and borrow. Non-Ricardian households do not have access to financial markets and can neither save nor borrow (which implies that they cannot smooth their consumption over time). Production is carried out by intermediate good firms that rent capital and labor from households. Domestically produced intermediate goods are then combined with imported goods to produce final goods, which are sold either on the domestic market, or on the export market. Separate firms specialize in the business of importing and exporting. Furthermore, Sweden has a detailed fiscal sector with a government that uses several sources of tax revenue to finance government consumption, investment and transfers to households. Figure 1 shows an overview of SELMA; the structure of the Swedish economy and how it connects with the Foreign economy.

Foreign is partly a mirror-image of Sweden. However, as the main focus of the model is the analysis of Sweden, Foreign is modelled with a more sparse structure. In particular, the fiscal sector in Foreign is modelled in much less detail compared to the Swedish economy,[2] and there is only one type of household, the Ricardian household.

Both Sweden and Foreign are affected by two non-stationary technology shocks for each, which determine the long-run path for productivity. They are denoted z_t and γ_t for Sweden, and $z_{F,t}$ and $\gamma_{F,t}$ for Foreign.

[1] For a list of all variables and parameters, see Appendix H and I respectively.

[2] In Foreign, all of the proceeds from taxation are spent on transfers to households and the government runs a balanced budget every period.

For Sweden, z_t and γ_t may be interpreted, respectively, as a labor augmenting technological process and a technological process specific to the production of investment goods. z_t^+, which is a function of z_t and γ_t, summarizes the compound effect of technology on the level of production along the balanced growth path. For the Foreign economy, the similar variable that summarizes the compound effect of technology is denoted by $z_{F,t}^+$. In addition to these non-stationary technology shocks, each of the two economies are affected by a number of country-specific shocks, where some of the shocks are allowed to be correlated.

The remainder of this section describes the problems solved by optimizing agents in the two economies, as well as the policy rules that govern monetary and fiscal policy. A complete list of the equilibrium conditions and the derivations can be found in Appendix A, and Appendices C, D and E respectively.

2.1 The Swedish household sector: Ricardian households

The Swedish household sector consists of a continuum of households with total mass equal to one and indexed by k. They can be divided into two types of representative households, Ricardian households, with mass $(1-s_{nr})$ and Non-Ricardian households with mass s_{nr}. In this section, we describe the Ricardian households. A representative Ricardian household earns income from wages and from the return on its savings, and it decides how much to consume and how to allocate its remaining resources between different kinds of savings. There are four kinds of assets that the household can save in: 1) capital, which is owned by households and rented to firms on a period-by-period basis, 2) private bonds denominated in Swedish currency, 3) private bonds that are denominated in the currency of Foreign, and 4) a portfolio of government bonds denominated in Swedish currency.

A representative household is a large structure with many members who are represented by the unit square $(h, j) \in [0, 1] \times [0, 1]$, where each member is indexed by h according to their type of labor service they are specialized in and indexed by j according to their degree of disutility of work. We drop the household's index k because all households have the the same optimization problem. The objective of representative large household is to maximize the following expected discounted life time utility:

$$E_0 \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \int_0^{N_{h,t}} j^\eta \, dj dh \right], \tag{1}$$

where ρ_h is the consumption habit formation parameter, ζ_t^c is the consumption preference shock and the composite consumption $\tilde{C}_t$ of household is defined as a constant elasticity of substitution (CES) aggregate:

$$\tilde{C}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} C_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1 - \alpha_G)^{\frac{1}{\upsilon_G}} G_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}$$

where C_t denotes the household's aggregate consumption of private consumption goods which is obtained by integrating over all household members' consumption, $C_t = \int_0^1 \int_0^1 C_{h,j,t} dj dh$. We assume full risk sharing of consumption among household members which implies $C_t = C_{h,j,t}$ for all (h, j). G_t measures government consumption. Note that α_G is a share parameter and $\upsilon_G > 0$, where υ_G measures the elasticity of substitution between private consumption and government consumption. $\upsilon_G \to 0$ implies perfect complementarity, $\upsilon_G \to \infty$ gives perfect substitutability, and $\upsilon_G \to 1$ yields the Cobb-Douglas (CD) case. Following Coenen, Straub, and Trabandt (2013), Bouakez and Rebei (2007), Leeper, Walker, and Yang (2009) and others, we allow government consumption to enter household utility in a non-separable way. This feature has several implications. First, changes in government consumption affect optimal private consumption decisions directly, as opposed to the indirect wealth effect in case of separable government consumption. Second, conditional on the degree of complementarity, a co-movement of private and government consumption may be obtained, which is observed in macro data, see for example the discussion in Galí, López-Salido, and Vallés (2007). Intuitively, examples of government consumption goods that represent complements to private consumption goods are public security provision such as defense or police, and education. The term $\rho_h \tilde{C}_{t-1}$ in the utility function captures an external habit formation, which implies that households dislike to deviate from the last period's average consumption.

The term $N_{h,t}$ denotes the employment level for profession h and by integrating disutility of work over j the household utility can be written in the following way:

$$E_0 \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \frac{N_{h,t}^{1+\eta}}{1+\eta} dh \right],$$

The functions $u(\cdot)$ is twice continuously differentiable function and β_t is the subjective discount factor. The term ζ_t^n denotes an economy-wide preference shock to the disutility of labor that evolves stochastically and that causes exogenous shifts in the supply of labor. The term Θ_t^n is an endogenous shifter defined as

$$\Theta_t^n = Z_t^n U_{c,t} \tag{2}$$

where Z_t^n is an approximation for the trend of marginal utility of consumption $U_{c,t}$ and defined by

$$Z_t^n = (Z_{t-1}^n)^{1-\chi_n}(U_{c,t})^{-\chi_n} \tag{3}$$

where $\chi_n \in [0,1]$ and determines the persistency of Z_t^n. The formulation of Θ_t^n implies a "consumption externality" to the labor force participation. When the marginal utility of consumption $U_{c,t}$ is below its trend value Z_t^n, marginal disutility of work goes down for an individual household member through the value of Θ_t^n. This mechanism helps to reduce the short-run "wealth effect" on labor force participation, the magnitude of which is determined by the value of parameter χ_n. The lower the value of χ_n the lower is the "wealth effect" in the short-run.

The household budget constraint is the following:

$$\begin{aligned}
&\underbrace{(1+\tau_t^C)P_t^C C_t}_{\text{Consumption expenditure}} + \underbrace{(1-\tau_t^I)\frac{P_t^I}{\gamma_t}I_t + P_t^K\triangle_t^K}_{\text{Investment expenditure}} + \underbrace{\frac{B_{t+1}^{priv}}{R_t\zeta_t} + B_t^n + \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t\Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)}}_{\text{Bond savings}} + T_t = \underbrace{\int_0^1 (1-\tau_t^W)W_{h,t}N_{h,t}\,dh}_{\text{Labor income}} + \\
&\underbrace{(1-\tau_t^K)\left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t}a(u_t)K_t\right) + \iota^K\tau_t^K\delta P_{t-1}^K K_t}_{\text{Capital income}} + \underbrace{B_t^{priv} + \left(\alpha_B + (R_{t-1}^B - 1)\right)B_t + S_t B_t^{FH}}_{\text{Bond income}} \\
&+ \underbrace{(1-\tau_t^{TR})TR_t}_{\text{Transfer income}} + \Xi_{B,t} + \Xi_{B^{FH},t} + \Psi_t, \qquad (4)
\end{aligned}$$

P_t^C is the price index of private consumption goods, P_t^I is the price index of investment and P_t^K is the price of capital. R_t is the gross nominal interest rate on private bonds denominated in Swedish currency and $R_{F,t}$ is the gross nominal interest rate of bonds denoted in the currency of Foreign. S_t is the nominal exchange rate, expressed as the price in Swedish currency of one unit of Foreign currency. There are different types of taxes levied on the household: τ_t^C denotes the consumption tax rate, τ_t^W the labor income tax rate and τ_t^K the capital income tax rate. Moreover, τ_t^{TR} denotes the tax rate levied on transfers from the government. We also allow for the possibility of investment tax credit/subsidy τ_t^I.[3]

In the budget constraint, Equation (4), the left-hand side items represent expenditure on private consumption $\left(1+\tau_t^C\right)P_t^C C_t$, investment $\left(1-\tau_t^I\right)\frac{P_t^I}{\gamma_t}I_t$, newly installed capital $P_t^K\triangle_t^K$, domestic private bonds B_{t+1}^{priv}, newly issued debt by the government B_t^n, and private bonds denominated in the currency of Foreign B_{t+1}^{FH}. Following Smets and Wouters (2007), we also include a risk-premium shock ζ_t which affects the household's return on bonds, hence also the Euler equation. $\Xi_{B,t} + \Xi_{B^{FH},t}$ denote lump-sum rebates of financial intermediation costs associated with the risk premium shocks on domestic private bonds and foreign private bonds. The function $\Phi(\cdot)$ represents a premium on Foreign bond holdings, which we will refer to as an external risk premium.[4] Its presence in the budget constraint is motivated below in the discussions of financial assets (Section 2.1.2) and net foreign assets (Section 2.7.4). The term $1/(R_t\zeta_t)$ is the effective price of domestic private bonds, while the effective price of private bonds denominated in the currency of Foreign is $S_t/(R_{F,t}\zeta_t\Phi(\cdot))$. The right-hand-side terms represent labor income net of taxes $(\int_0^1(1-\tau_t^W)W_{h,t}N_{h,t}dh)$, rental income from the capital stock $\left(1-\tau_t^K\right)(R_t^K u_t K_t)$ and the gross return on bonds carried from the previous period $B_t^{priv} + (\alpha_B + (R_{t-1}^B - 1))B_t + S_t B_t^{FH}$.

The maintenance cost of the stock of capital is $(\frac{P_t^I}{\gamma_t}a(u_t)K_t)$, where $a(u_t)$ is the cost of capital utilization and K_t is the capital stock. The expression $\tau_t^K\frac{P_t^I}{\gamma_t}a(u_t)K_t$ captures the notion that the maintenance cost of capital can be deducted from the capital tax bill. Moreover, $\tau_t^K\delta P_{t-1}^K K_t$ captures the notion that depreciation of capital can be deducted from the capital tax bill at its historical cost. The allowance of tax deduction of depreciation of capital is contingent on the indicator variable $\iota^K \in \{0,1\}$ being set to 1. TR_t and T_t denote lump-sum transfers and taxes, respectively. The last term on the right-hand side, Ψ_t, denotes the sum of profit transfers from firms. Each individual Ricardian household owns an equal share of the domestic firm sector and any profits or losses are returned on a period-by-period basis to the household sector. Since the access to financial markets and the possibility to save is reserved for the Ricardian households, we now describe the average interest rate on government bonds and the capital accumulation equation.

To capture the empirical fact that government bonds have different maturities, which, among other things, leads to an incomplete pass-though of a change in the monetary policy rate to the interest payments for the government in the following period, we follow the approach of Krause and Moyen (2016) and allow the government bonds to have stochastic maturity. The government issues bonds that mature with probability α_B in a given

[3] Note that the modeling approach of τ_t^C adopted here implies that there is a complete pass-through of changes in the consumption tax rate into the sales price. In other words, the consumption tax rate modeling approach resembles a sales tax as in the U.S.

[4] As will be clear later, the external risk premium does not affect the Foreign household's return on their savings. Hence, $\Phi(\cdot)$ can also be interpreted as a pure exchange-rate shock.

period. Until stochastic maturity, the bond pays a non-state contingent interest rate. The portfolio of government bonds B_{t+1} that the household holds evolves according to

$$B_{t+1} = (1 - \alpha_B) B_t + B_t^n \tag{5}$$

where B_t^n denotes the newly issued debt by the government in period t. Following Krause and Moyen (2016), households are assumed to buy a representative portfolio of government bonds with all possible stochastic maturities. The average interest rate R_t^B on outstanding government debt bought by the household is given by

$$\left(R_t^B - 1\right) B_{t+1} = (1 - \alpha_B) \left(R_{t-1}^B - 1\right) B_t + \left(R_t^{B,n} - 1\right) B_t^n \tag{6}$$

where the interest rate on newly issued government debt is denoted by $R_t^{B,n}$.

2.1.1 Investment and capital services

The stock of capital K_{t+1} owned by the household evolves according to the following accumulation expression:

$$K_{t+1} = (1 - \delta) K_t + \Upsilon_t F (I_t, I_{t-1}) + \triangle_{k,t}^K, \tag{7}$$

where δ is a constant rate of depreciation. The stock of capital in $t + 1$ is given by the previous period's stock of capital that survives the depreciation $(1 - \delta) K_t$, the stationary investment-specific technology shock Υ_t, the new investment net of adjustment costs regulated by the function $F (I_t, I_{t-1})$, and the amount of capital traded between household k and the other households in Sweden $\triangle_t^K$.

In particular, it is assumed that adjustments in the rate of investment are costly. Hence, the price of one unit of installed capital, P_t^K, may differ from the cost of one unit of investment, which is denoted by $\frac{P_t^I}{\gamma_t}$. The presence of a market where households can trade capital $\triangle_t^K$ allows us to conveniently derive the price P_t^K.[5]

Firms in the intermediate goods sector rent capital services K_t^s from Ricardian households. The amount of capital services rented and used in the intermediate goods production depends on the household's chosen degree of utilization u_t and the household's chosen level of capital K_t. In every period, the individual household observes the going rental rate of capital services, R_t^K, and decides how intensively to use its current stock of capital. A higher degree of utilization u_t implies that more capital services are rented to the firm sector. The cost of a higher utilization rate is higher maintenance costs. In the current version of the model, the households' ability to vary the degree of capital utilization is de-activated, see Section 3.1.

2.1.2 Financial assets

We assume that there exists a set of contingent claims that allows an individual household member to diversify the component of idiosyncratic risk that is associated with its wage income and employment status, which allows full risk-sharing within the household. However, we also assume that individual members take into account household utility rather than their personal utility while giving thier decisions. This second assumption coming with the first assumption is crucial because under the full consumption risk-sharing being not working (or being unemployed) gives more utility than being employed for an individual member, and thus not internalizing the benefits to the household of members' employment would lead to no participation in the labor market.

Swedish private bonds purchased in period t yield a gross, nominal return of R_t, set by the Riksbank, times an exogenous risk premium ζ_t in the subsequent period, which creates a wedge between the Riksbank policy rate and the return that the household gets. This rate of return is known with certainty at the time of investment. The gross return on Foreign bonds earned by Swedish households, in terms of Foreign currency, is determined by the nominal interest rate in Foreign, $R_{F,t}$, the risk premium ζ_t and by the external risk premium, $\Phi(\cdot)$. The presence of the external risk premium is motivated by two concerns, the first of which is to ensure the existence of a well-defined steady state (see e.g. Schmitt-Grohe and Uribe (2001)). The second concern has to do with model dynamics around the steady state and the empirical failure of the standard uncovered interest parity (UIP) condition. Outside of the steady state, the external risk premium will cause deviations from the standard UIP condition, helping the model to better fit the data, e.g. the behavior of the real exchange rate after a monetary policy shock. We follow Adolfson et al. (2008) and specify the external risk premium as a function of the (aggregate) net foreign asset position of Sweden, of the expected change in the nominal exchange rate and of an exogenous shock, $\widetilde{\phi}_t$.[6]

[5] In equilibrium, all Ricardian households will want to hold the same quantity of capital. The market for capital will thus clear at a price at which the individual household wants neither to buy nor to sell any units.

[6] The functional form of $\Phi(\cdot)$ will be discussed further below, in Section (2.7.4).

2.1.3 Wage setting

As in Erceg, Henderson, and Levin (2000), each individual member of the Ricardian household is assumed to supply a differentiated labor service to the intermediate firm sector. The labor market is characterized by monopolistic competition and by staggered nominal wage contracts.
A representative employment agency rents differentiated labor services from Ricardian households and aggregates them into a homogeneous labor service which can be written as $N_t = \left[\int_0^1 (N_{h,t})^{\frac{\varepsilon_{w,t}-1}{\varepsilon_{w,t}}} dh\right]^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}}$, which is sold to intermediate firms. The labor type h charges a wage rate $W_{h,t}$ for its differentiated labor service $N_{h,t}$, and the employment agency optimizes the input of different labor services in order to minimize costs. When doing so, it takes the wage rates of differentiated labor services and homogeneous labor service as given. The minimum expenditure required to produce one unit of the homogeneous labor service is given by $W_t = \left[\int_0^1 (W_{h,t})^{(1-\varepsilon_{w,t})} dh\right]^{\frac{1}{1-\varepsilon_{w,t}}}$ and W_t can be interpreted as the aggregate wage index. The agency's demand for labor from the labor type h,

$$N_{h,t} = \left(\frac{W_{h,t}}{W_t}\right)^{-\varepsilon_{w,t}} N_t, \tag{8}$$

is derived from this cost minimization problem. $\varepsilon_{w,t}$ is the wage-elasticity of demand for $N_{h,t}$, $\lambda_t^W = \frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}$ is each labor type's desired wage markup and N_t is the aggregate employment.

The wage setting by the individual household is subject to Calvo-style frictions. At the beginning of each period, labor type h learns if it is allowed to reset its wage in that period or not. The opportunity to reset the wage occurs with constant probability $(1-\xi_w)$. This probability is independent of the number of periods that passed since the last time the household had the possibility to reset its wage.[7] In periods when the wage cannot be reset, it is indexed by a factor $\overline{\Pi}_t^W = \left(\Pi_{t-1}^W\right)^{\chi_w} \left(\Pi_t^{trend}\right)^{1-\chi_w}$, where $\Pi_t^W = \frac{W_t}{W_{t-1}}$ is the aggregate wage inflation in period t and Π_t^{trend} is the inflation trend in the economy. $\chi_w \in [0,1]$ governs the weight on previous period's inflation in the inflation indexation. The higher χ_w is, the higher is the wage inflation inertia. Suppose labor type h has the opportunity to reset its wage in period t. Also recall that it considers households utility rather than its individual utility. It then chooses the optimal wage rate $W_{h,t}^{\text{opt}}$ that maximizes (1), subject to the budget constraint (4), the labor demand schedule (8) and the constraint that the wage rate $W_{h,t+k}$ in any future period $(t+k)$ evolves according to:

$$W_{h,t+k} = \begin{cases} \overline{\Pi}_{t+k}^W W_{h,t+k-1} & \text{with probability } \xi_w, \\ W_{h,t+k}^{\text{opt}} & \text{with probability } (1-\xi_w). \end{cases} \tag{9}$$

We assume that Non-Ricardian households set their wage equal to the average wage of Ricardian households and face identical labor demand. This assumption implies that the group of Ricardian and the group of Non-Ricardian households will have the same average wage rate and supply the same amount of labor.[8]

2.1.4 Labor supply and unemployment

We follow Galí (2011) and Galí, Smets, and Wouters (2012) in modelling labor force participation. Given the assumption that household members take into account the household welfare and their own personal disutility of work, the individual household member (h,j) will find it optimal to participate in the labor market in period t if and only if

$$\Omega_t^c(1-\tau_t^W)\left(\frac{W_{h,t}}{P_t^C}\right) \geq \zeta_t^n \Theta_t^n A_n j^\eta,$$

where Ω_t^c is a modified marginal utility of consumption defined below. Denote the labor supply of the marginal supplier j by $L_{h,t}$. Labor force participation condition is then written as the following:

$$\Omega_t^c(1-\tau_t^W)\left(\frac{W_{h,t}}{P_t^C}\right) = \zeta_t^n \Theta_t^n A_n L_{h,t}^\eta \tag{10}$$

This condition is a unique feature of the Galì approach that enable us to incorporate unemployment into the model in a theoretically coherent way. The condition says that household members are willing to participate to the labor force as long as the consumption utility they receive from their wage income is bigger than or equal to their disutility of work. Aggregate labor supply of the representative household is then given by

[7] The opportunity to reset the wage in any given period is also independently distributed across different labor types.
[8] Note that the alternative assumption that both Ricardian and Non-Ricardian households supply their labor services via unions that act as wage setters subject to the demand for labor services would give the same result that wages and labor supply are identical across both groups, see Coenen, Straub, and Trabandt (2013).

$$L_t = \int_0^1 L_{h,t} dh.$$

Having market power enables each labor type (or labor unions) to set its wage with a positive markup over marginal rate of substitution. This results in wages that are higher than in the competitive equilibrium, implying that markets do not clear and that unemployment exists in the model. Unemployment rate now can be written by its standard definition:

$$un_t = \frac{L_t - N_t}{L_t} \tag{11}$$

2.1.5 First-order conditions

In every period t, the household chooses C_t, I_t, u_t, $\triangle_t^K$, K_{t+1}, B_{t+1}^{priv}, B_{t+1}, B_t^n and B_{t+1}^{FH} in order to maximize Equation (1) subject to (4)-(7). The first-order conditions associated with this problem are presented next. Denote $\Omega_{h,t}^C$ as the marginal utility of consumption including the tax on consumption:

$$\Omega_t^C \equiv \frac{\zeta_t^c u_{C_t}(\tilde{C}_t, \tilde{C}_{t-1})}{1+\tau_t^C} = \frac{U_{c,t}}{1+\tau_t^C}.$$

$\beta_{t+1}^r \equiv \frac{\beta_{t+1}}{\beta_t}$ represents the change in the subjective discount factor between two consecutive periods. θ_t^b, θ_t^S, θ_t^R and θ_t^k are the Lagrange multipliers associated with the budget constraint (4), the equation for the stock of government bonds (5), the equation for the average rate of return on government bonds (6) and the capital accumulation equation (7), respectively.

$$C_t: \quad \theta_t^b P_t^C = \Omega_t^C \tag{12}$$

$$I_t: \quad \theta_t^b \frac{P_t^I}{\gamma_t}\left(1-\tau_t^I\right) = \theta_t^k \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \theta_{t+1}^k \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right], \tag{13}$$

$$u_t: \quad R_t^K K_t = \frac{P_t^I}{\gamma_t} a'(u_t) K_t, \tag{14}$$

$$\triangle_t^K: \quad \theta_t^b P_t^K = \theta_t^k, \tag{15}$$

$$K_{t+1}: \quad \theta_t^k = E_t \beta_{t+1}^r \left[\left(1-\tau_{t+1}^K\right)\theta_{t+1}^b \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1})\right) + \theta_{t+1}^b \iota^K \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^k (1-\delta)\right], \tag{16}$$

$$B_{t+1}^{priv}: \quad \theta_t^b = E_t \beta_{t+1}^r \theta_{t+1}^b R_t \zeta_t, \tag{17}$$

$$B_{t+1}: \quad E_t\beta_{t+1}^r\theta_{t+1}^b\left(\alpha_B + \left(R_t^B - 1\right)\right) = \theta_t^S - E_t\beta_{t+1}^r\theta_{t+1}^S\left(1-\alpha_B\right) + \left(\theta_t^R - \left(1-\alpha_B\right)E_t\beta_{t+1}^r\theta_{t+1}^R\right)\left(R_t^B - 1\right) \tag{18}$$

$$B_t^n: \quad \theta_t^b \beta_t = \theta_t^S \beta_t + \beta_t \theta_t^R \left(R_t^{B,n} - 1\right) \tag{19}$$

$$R_t^B: \quad \theta_t^R E_t B_{t+1} = E_t \beta_{t+1}^r \theta_{t+1}^b B_{t+1} + E_t \beta_{t+1}^r \theta_{t+1}^R \left(1-\alpha_B\right) B_{t+1} \tag{20}$$

$$B_{t+1}^{FH}: \quad \theta_t^b S_t = E_t\left[\beta_{t+1}^r \Phi\left(\bar{a}_t, s_t, \widetilde{\phi}_t\right) R_{F,t}\zeta_t S_{t+1} \theta_{t+1}^b\right]. \tag{21}$$

In periods when there is an opportunity to reset the wage, the household also chooses $W_{h,t}^{opt}$. To simplify notation, let $W_{h,t+k|t} = W_{h,t}^{opt}\overline{\Pi}_t^W \overline{\Pi}_{t+1}^W \ldots \overline{\Pi}_{t+k-1}^W$ denote the wage of labor type h in future period $(t+k)$, given that the household last had the opportunity to reset its wage in period t. The first-order condition of the wage optimization problem may then be written:

$$E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^k \beta_{t+i}^r\right) N_{h,t+k|t}\, \theta_{h,t+k}^b \left[\left(1-\tau_{t+k}^W\right) W_{h,t+k|t} - \lambda_{t+k}^W\, \zeta_{t+k}^n A_n \Theta_{t+k} \frac{N_{h,t}^{(\eta)}}{\theta_{h,t+k}^b}\right] = 0, \tag{22}$$

where $\prod_{i=1}^k \beta_{t+i}^r = \beta_{t+1}^r \beta_{t+2}^r \ldots \beta_{t+k}^r$ and $\prod_{i=1}^0 \beta_{t+i}^r \equiv 1$.[9]

[9] For future reference, we also derive the corresponding condition for the case of flexible prices and wages. When the household is

2.2 The Swedish household sector: Non-Ricardian households

Non-Ricardian and Ricardian households have identical preferences. The difference between the two types of households is that Non-Ricardian households have no access to capital or bonds markets. In addition, it is assumed, for simplicity, that the wage and employment supplied by Non-Ricardian household equals the average wage and employment supplied by Ricardian households.

Since a Non-Ricardian household has no ability to save nor borrow, its nominal consumption expenditure equals its after-tax wage income plus the transfers it gets from the government. We index Non-Ricardian household with m, but for notational convenience we drop the index. Formally,

$$(1+\tau_t^C)\,P_t^C C_t = (1-\tau_t^W)W_t N_t + \left(1-\tau_t^{TR}\right)TR_t. \tag{24}$$

Note the assumption that the lump-sum taxes are only paid by Ricardian households. Furthermore, note that an increase in government consumption has no direct effect on the consumption decision of the Non-Ricardian household.

2.3 Aggregation of individual household variables

The private and government bonds owned by Ricardian households sum to the following aggregates:

$$B_{t+1}^{priv} = \int_0^{1-s_{nr}} B_{k,t+1}^{priv}dk$$

$$B_{t+1} = \int_0^{1-s_{nr}} B_{k,t+1}dk$$

$$B_t^n = \int_0^{1-s_{nr}} B_{k,t}^n dk$$

$$B_{t+1}^{FH} = \int_0^{1-s_{nr}} B_{k,t+1}^{FH}dk$$

Aggregate consumption and transfers can be expressed as follow:

$$C_t^{agg} = \int_0^{1-s_{nr}} C_{k,t}dk + \int_{1-s_{nr}}^1 C_{m,t}dm$$

$$TR_t^{agg} = \int_0^{1-s_{nr}} TR_{k,t}dk + \int_{1-s_{nr}}^1 TR_{m,t}dm$$

Aggregate private investments, aggregate capital traded between households, the aggregate capital stock and the aggregate capital services respectively sum to:

$$I_{t+1} = \int_0^1 I_{k,t+1}dk$$

$$\triangle_{t+1}^K = \int_0^1 \triangle_{k,t+1}^K dk$$

$$K_{t+1} = \int_0^1 K_{k,t+1}dk$$

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free to optimize its wage in every period, the first-order condition becomes:

$$\left(1-\tau_t^W\right)W_{h,t}^{fp} = \lambda_t^W \zeta_t^n A_n \Theta_t^n \frac{(N_{h,t}^{fp})^{(\eta)}}{\theta_{h,t}^{b,fp}}. \tag{23}$$

where $W_{h,t}^{fp}$, $N_{h,t}^{fp}$ and $\theta_{h,t}^{b,fp}$ is the flexible-price equivalent expressions of wages, labor supplies and the Lagrange multiplier for the budget constraint. When wages are flexible, labor type h achieves the desired markup λ_t^W in every period.

$$K_{t+1}^s = \int_0^1 K_{k,t+1}^s dk$$

2.4 The Swedish firm sector

Several different types of firms operate in the Swedish economy. Some of these firms are price setters and others are price takers.

Six types of firms operate in monopolistically competitive markets and they face nominal frictions in their price setting. These are producers of domestic intermediate goods, import firms for non-energy consumer goods, import firms for energy consumer goods, import firms for investment goods, import firms for export goods, and export good producers. The rationale for including three different types of import firms is to be able to better match the macro data on Swedish imports and import prices.

Four types of representative firms operate under perfect competition. These firms take both the prices of their inputs and the prices at which they sell their output as given. Two representative firms produce private final consumption goods and private investment goods, while two other representative firms produce government consumption goods and government investment goods, respectively. The private consumption and investment goods firm use domestically produced intermediate goods as well as imported goods while the public consumption and investment good firms only use domestically produced intermediate inputs.

The optimization problems of the different types of firms are described below. In addition, the firm sector also consists of a number of aggregator firms that aggregate the different varieties of goods that are produced within each of the markets characterized by monopolistic competition. All aggregator firms operate under perfect competition and their problems are not explicitly discussed in the text. Instead, the standard input demand functions and price indices associated with these aggregators are stated as restrictions in the problems of other firms.

2.4.1 Swedish intermediate good producers

A continuum of firms produce domestic intermediate goods, each of which is differentiated from other intermediate goods produced in the sector. The total mass of these firms is unity and they operate in a market characterized by monopolistic competition. Each firm sets its price to minimize the costs of producing the associated output.

A representative aggregator firm buys the different varieties of goods and aggregates it into a homogeneous intermediate good that is sold to the firms producing consumption goods, investment goods and export goods. The demand for the individual variety i, $Y_t(i)$, is a function of the relative price of that variety and of total demand for the homogeneous, intermediate good:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{\frac{\lambda_t}{1-\lambda_t}} Y_t. \tag{25}$$

$P_t(i)$ denotes the price charged by firm i, $P_t = \left(\int_0^1 P_t(i)^{\frac{1}{1-\lambda_t}} di\right)^{1-\lambda_t}$ is the price associated with the homogeneous, intermediate good and Y_t denotes total demand. λ_t is a time-varying markup over marginal cost that evolves according to an exogenous, stochastic process.[10]

The individual intermediate good firm takes the rental rate of capital services R_t^K, the wage rate $\left(1+\tau_t^{SSC}\right) W_t$ including social security contributions τ_t^{SSC}, and the public capital stock $K_{G,t}$ as given when it decides on an optimal input of production factors: $K_t^s(i)$ and $N_t(i)$. In addition to these two variable costs, firms also incur a fixed cost $z_t^+\phi$ in each period. The cost-minimization problem of firm i is given by

$$\min_{K_t^s(i), L_t(i)} \left\{R_t^K K_t^s(i) + \left(1+\tau_t^{SSC}\right) W_t N_t(i)\right\}$$

s.t.

$$Y_t(i) = \varepsilon_t \left[\tilde{K}_t^s(i)\right]^\alpha \left[z_t N_t(i)\right]^{1-\alpha} - z_t^+\phi.$$

where $\tilde{K}_t^s(i)$ denotes a composite capital service input made up by private capital services $K_t^s(i)$ and public capital $K_{G,t}$. We assume the following constant elasticity of substitution (CES) aggregator of private capital services $K_t^s(i)$ and the public capital stock $K_{G,t}$:

[10] Note that λ_t may be interpreted as a function of a time varying elasticity of substitution between the different varieties of intermediate goods. One natural interpretation, therefore, of shocks to λ_t is of an exogenous change in the degree of market power enjoyed by the individual firms in this sector.

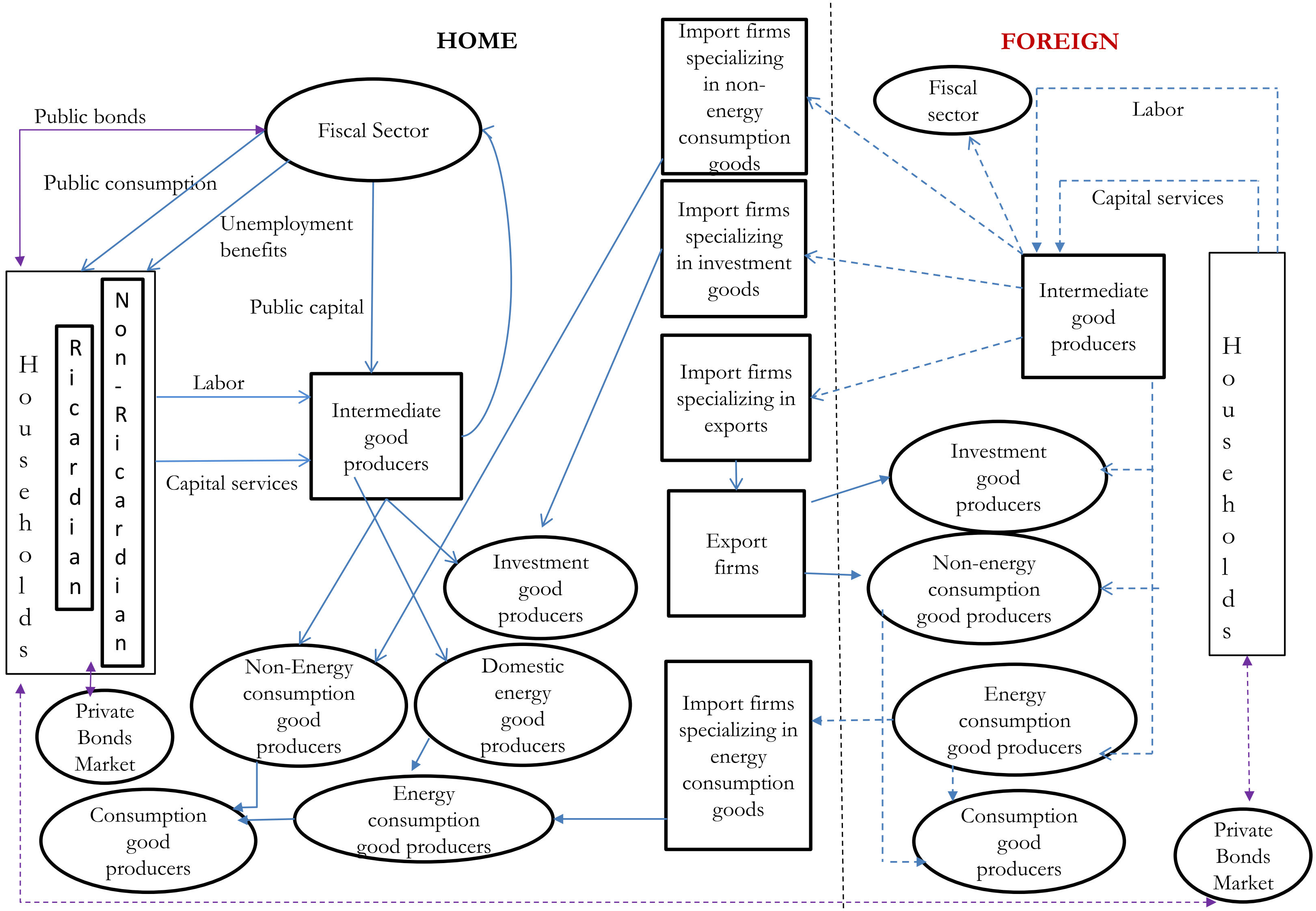


Figure 1: Overview of SELMA

$$\tilde{K}_t^s(i) = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(K_t^s(i)\right)^{\frac{\upsilon_K - 1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}} \left(K_{G,t}\right)^{\frac{\upsilon_K-1}{\upsilon_K}} \right)^{\frac{\upsilon_K}{\upsilon_K-1}}.$$

Hence, we assume that each intermediate good firm i has access to the same public capital stock. We also assume that public capital grows at the same rate as private capital services along the balanced growth path. The parameter υ_K is the elasticity of substitution between private capital services and the public capital stock, and α_K is a share parameter. ε_t is a stationary stochastic process, with an unconditional mean of unity, that is common to all firms in the Swedish intermediate good sector. The shock captures temporary changes in total factor productivity of the firms. The stochastic process that governs the labor augmenting technology, z_t, is growth-stationary. Let $\mu_{z,t} = \frac{z_t}{z_{t-1}}$ denote the growth rate of z_t.

The variable z_t^+,which is multiplied by the fixed cost, ensures that the fixed cost grows in proportion to output. It consists of a combination of the labor augmenting technology variable z_t and an investment-specific productivity variable γ_t and is given by

$$z_t^+ = z_t \gamma_t^{\frac{\alpha}{1-\alpha}}. \tag{26}$$

The solution to the cost minimization problem can be expressed in terms of a marginal cost function:

$$MC_t(i) = \frac{\left(\frac{(1+\tau_t^{SSC})W_t}{z_t}\right)^{1-\alpha} \left(R_t^K\right)^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\varepsilon_t \Gamma_{G,t}(i)}. \tag{27}$$

A relationship between the rental rate for capital services and the optimal capital-to-labor ratio:

$$R_t^K = \alpha \varepsilon_t z_t^{1-\alpha} MC_t(i) \left(\frac{\tilde{K}_t^s(i)}{L_t(i)}\right)^{\alpha-1} \left(\Gamma_{G,t}(i)\right)^{\frac{1}{\alpha}}. \tag{28}$$

where

$$\Gamma_{G,t}(i) = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}},$$

The firms face a Calvo style price friction when they set their prices. In every period t, there is a probability $(1-\xi)$ that the individual firm i gets the opportunity to reset its price. With complementary probability ξ the firm does not have this opportunity. In the latter case, the non-reset price $P_{t-1}(i)$ will instead be indexed by $\overline{\Pi}_t$ such that $P_t(i) = \overline{\Pi}_t P_{t-1}(i)$, where $\overline{\Pi}_t = (\Pi_{t-1})^{\chi}\left(\Pi_t^{trend}\right)^{1-\chi}$ is a weighted average of previous period's gross inflation Π_{t-1} and the inflation trend Π_t^{trend}. The inflation trend does in turn follow a stochastic autoregressive process which is specified later. $\chi \in [0,1]$ represents the weight on previous period's inflation in indexation. Suppose firm i has the opportunity to reset its price in period t and let $P_{t+k|t}(i) \equiv P_t^{opt}\overline{\Pi}_{t+1}\cdots\overline{\Pi}_{t+k}$ denote the price that will apply in period $(t+k)$, conditional on the firm not having any opportunity to reset its price between periods t and $(t+k)$. When choosing P_t^{opt}, the firm seeks to maximize the expected, discounted sum of present and future profits, which may be written as

$$E_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \left\{ P_{t+k|t}(i) Y_{t+k|t}(i) - TC_{t+k|t}\left[Y_{t+k|t}(i)\right] \right\}, \tag{29}$$

where $Y_{t+k|t}(i)$ is the demand in period $(t+k)$ for the output of firm i, conditional on the price $P_{t+k|t}(i)$. $\Lambda_{t,t+k}$ represents the firm's stochastic discount factor and $TC_{t+k|t}[\cdot]$ denotes total cost, as a function of output.[11] The first-order condition associated with this problem may be written as[12]

$$E_t \sum_{k=0}^{\infty} \xi^k \Lambda_{t,t+k} \frac{Y_{t+k|t}}{(\lambda_{t+k}-1)} \left(P_{t+k|t} - \lambda_{t+k} MC_{t+k}\right) = 0. \tag{31}$$

As mentioned above, private consumption goods, investment goods and export goods are assumed to be composites of domestically produced intermediate goods and of imported goods. Before we proceed to describe the firms that are active in the markets for these final goods, we outline the problem of Swedish import firms.

[11] In keeping with the assumption that Ricardian households own the firms, firms discount future profits at the same rate as households discount future income: $\Lambda_{t,t+k} = \frac{\beta_{t+k}\Omega_{t+k}^C P_t^C}{\beta_t \Omega_t^C P_{t+k}^C}$.

[12] All firms that have an opportunity to reset their price in period t will face the same problem. As a consequence, all such firms will choose the same optimal reset price and they will produce the same quantity of output in that period. Therefore, index i is dropped in this equation. In the equilibrium with flexible prices and wages, the corresponding (standard) first-order condition of the firm gives a price equal to the desired markup times the marginal cost:

$$P_t^{\text{fp}} = \lambda_t MC_t^{\text{fp}}. \tag{30}$$

2.4.2 Swedish import firms

Following Corbo and Strid (2020), there are four types of Swedish import firms. One type of firm specializes in the business of importing intermediate goods from Foreign and transforming those imported goods into inputs that are suitable for the production of export goods. A second type of import firm transforms imported goods to inputs suited for the production of private investment goods. The third type of import firm transforms imported goods to inputs suited for the production of non-energy consumption goods, and the fourth specializes in transforming the energy good from Foreign into an input suited for the production of the energy consumption good. We capture the local currency pricing through the import firm's price setting. The import firms face sticky prices, allows for incomplete pass-through from the exchange rate to prices in the importing country. There exists a continuum of individual import firms of each type, and each of these individual firms owns a technology to make one-to-one transformations of the homogeneous Foreign export good into a differentiated import good. The individual heterogeneous import goods are then again transformed into a homogeneous import good by an aggregator firm. Let $n \in \{X, I, \{C, xe\}, \{C, e\}\}$ index the type of import firm, and let $M_t^n(i)$ represent the quantity produced by the individual firm i of type n. The cost to firm i of producing $M_t^{n^{xe}}(i)$ units of the differentiated import good of type $n^{xe} \in \{X, I, \{C, xe\}\}$ is $S_t P_{F,t}\left[M_t^{n^{xe}}(i) + z_t^{+}\phi^{M,n^{xe}}\right]$, where $P_{F,t}$ is the price of the homogeneous Foreign intermediate good and $S_t P_{F,t} z_t^{+}\phi^{M,n}$ denotes the fixed cost of production. The cost to firm i of producing $M_t^{C,e}(i)$ units of the differentiated energy import good is given by $S_t P_{F,t}^{C,e}\left[M_t^{C,e}(i) + z_t^{+}\phi^{M,C,e}\right]$, where $P_{F,t}^{C,e}$ is the price of the Foreign energy good and $P_{F,t}^{C,e} z_t^{+}\phi^{M,C,e}$ denotes the fixed cost of production. The price of the differentiated product of the individual import firm i of type n is denoted by $P_t^{M,n}(i)$ and $P_t^{M,n} = \left(\int_0^1 P_t^{M,n}(i)^{\frac{1}{1-\lambda_t^{M,n}}} di\right)^{1-\lambda_t^{M,n}}$ denotes the price of the homogeneous import good of type n. $\lambda_t^{M,n}$ is a time-varying, exogenous markup that is specific to all import firms of type n. The individual firm faces the following demand for its differentiated product:

$$M_t^n(i) = \left[\frac{P_t(i)^{M,n}}{P_t^{M,n}}\right]^{\frac{\lambda_t^{M,n}}{1-\lambda_t^{M,n}}} M_t^n. \tag{32}$$

M_t^n represents the total demand for the homogeneous import good of type n. Like firms in the intermediate good sector, import firms face pricing frictions. With probability $(1 - \xi_{M,n})$, individual firm i will be able to reset its price in period t. The optimal reset price is denoted $P_{t,opt}^{M,n}$. With complementary probability $\xi_{M,n}$, the price from the previous period will instead be indexed according to $P_t^{M,n}(i) = \overline{\Pi}_t^{M,n} P_{t-1}^{M,n}(i)$. The indexing factor $\overline{\Pi}_t^{M,n}$ is defined as $\overline{\Pi}_t^{M,n} = \left(\Pi_{t-1}^{M,n}\right)^{\chi_{m,n}} \left(\Pi_t^{trend}\right)^{1-\chi_{m,n}}$, where $\Pi_{t-1}^{M,n} = \frac{P_{t-1}^{M,n}}{P_{t-2}^{M,n}}$. $\chi_{m,n} \in [0,1]$ represents the weight on previous period's inflation of import goods. Let $P_{t+k|t}^{M,n} = P_{t,opt}^{M,n}\,\overline{\Pi}_{t+1}^{M,n} \cdots \overline{\Pi}_{t+k}^{M,n}$ denote the price that will apply in period $(t+k)$, conditional on the firm not having any opportunity to reset its price between periods t and $(t+k)$. In periods when the firm does have an opportunity to reset its price, it chooses $P_{t,opt}^{M,n}$ in order to maximize:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \Lambda_{t,t+k} \left\{P_{t+k|t}^{M,n^{xe}}(i)\, M_{t+k|t}^{n^{xe}}(i) - S_t P_{F,t+k} M_{t+k|t}^{n^{xe}}(i) - S_{t+k} P_{F,t+k} z_{t+k}^{+} \phi^{M,n^{xe}}\right\} \tag{33}$$

for the non-energy firms and

$$E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \Lambda_{t,t+k} \left\{P_{t+k|t}^{M,C,e}(i)\, M_{t+k|t}^{C,e}(i) - S_t P_{F,t+k}^{C,e}\, M_{t+k|t}^{C,e}(i) - S_{t+k} P_{F,t+k}^{C,e} z_{t+k}^{+} \phi^{M,C,e}\right\} \tag{34}$$

for the energy firms. If we write the marginal costs of the firms as

$$\begin{aligned} MC_t^{M,n^{xe}}(i) &= S_t P_{F,t} \\ MC_t^{M,C,e}(i) &= S_t P_{F,t}^{C^e}, \end{aligned} \tag{35}$$

then the first-order condition associated with the firm's maximization problem may be written:[13]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \Lambda_{t,t+k} \frac{M_{t+k|t}^n}{\left(\lambda_{t+k}^{M,n} - 1\right)} \left(P_{t+k|t}^{M,n} - \lambda_{t+k}^{M,n}\, MC_{t+k}^n(i)\right) = 0, \quad n \in \{X, I, \{C, xe\}, \{C, e\}\}. \tag{37}$$

[13] The corresponding first-order condition in the flexible price and wage equilibrium is:

$$P_{t,\text{fp}}^{M,n} = \lambda_t^{M,n}\, MC_t^n(i), \quad n \in \{X, I, C^{xe}, C^e\}. \tag{36}$$

2.4.3 Swedish export firms

Firms in the Swedish export sector use domestically produced intermediate goods and imported goods as inputs in their production of export goods. Export firms act as price takers in the markets for their input goods and as price setters in the market for their output goods. There are infinitely many export good producers, each of which produce a differentiated good that is sold in a market characterized by monopolistic competition. The different export firms share a common production technology and minimize the costs of production by choosing an optimal mix of inputs. Let $D_t^X(i)$ and $M_t^X(i)$ denote, respectively, the quantity of the domestically produced intermediate good and of the imported good used as inputs by individual firm i in the export good sector. The cost minimization problem is given by

$$\min_{D_t^X(i),\, M_t^X(i)} \left\{ P_t\, D_t^X(i) + P_t^{M,X}\, M_t^X(i) \right\}$$

s.t.

$$X_t(i) = \left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x - 1}{\nu_x}} + (1-\psi^X)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}} \right]^{\frac{\nu_x}{\nu_x - 1}} - z_t^+ \phi^X.$$

$X_t(i)$ denotes the quantity produced by the individual firm i and ν_x represents the elasticity of substitution between domestically produced and imported inputs in the production of export goods. Furthermore, $\psi^X = \vartheta^X + \frac{1}{1+\omega}\left(1-\vartheta^X\right)$ is the weight of the domestically produced intermediate good in production, where $\vartheta^X \in [0,1]$ may be interpreted as an index of home bias. $z_t^+\phi^X$ is the fixed cost of production. The marginal cost of the export goods is given by[14]

$$MC_t^X = \left[\psi^X \left(P_t\right)^{(1-\nu_x)} + \left(1-\psi^X\right)\left(P_t^{M,X}\right)^{(1-\nu_x)} \right]^{\frac{1}{1-\nu_x}}. \tag{38}$$

A representative aggregator firm buys the different varieties of export goods and aggregates them into a homogeneous export good that is sold to import firms in Foreign. The demand for the individual variety i, $X_t(i)$, is a function of the relative price of that variety and of total demand for Swedish exports: $X_t(i) = \left[\frac{P_t^X(i)}{P_t^X}\right]^{\frac{\lambda_t^X}{1-\lambda_t^X}} X_t$. $P_t^X(i)$ denotes the price charged by firm i, $P_t^X = \left(\int_0^1 P_t^X(i)^{\frac{1}{1-\lambda_t^X}} di\right)^{1-\lambda_t^X}$ is the price of the homogeneous export good and X_t is total demand. λ_t^X denotes the desired markup of Swedish export firms and is governed by an exogenous, stochastic process. The pricing frictions faced by the individual export good producers are of the same type as those faced by firms in the intermediate good sector and the import good sector. The probability that firm i has an opportunity to reset its price in any given period is denoted $(1-\xi_x)$ and the optimal reset price is represented by $P_t^{X,opt}$. The objective function of the export firm may be written

$$E_t \sum_{k=0}^{\infty} (\xi_x)^k \Lambda_{t,t+k} \left\{ P_{t+k|t}^X(i) S_{t+k} X_{t+k|t}(i) - TC_{t+k|t}^X \left[X_{t+k|t}(i)\right] \right\}, \tag{39}$$

where S_t enters the function due to local currency pricing[15], $TC_{t+k|t}^X\left[X_{t+k|t}(i)\right]$ represents total costs and $P_{t+k|t}^X = P_t^{X,opt}\, \overline{\Pi}_{t+1}^X \cdots \overline{\Pi}_{t+k}^X$. $\overline{\Pi}_t^X = \left(\Pi_{t-1}^X\right)^{\chi_x} \left(\Pi_{F,t}^{trend}\right)^{1-\chi_x}$ denotes the indexing factor, $\Pi_{t-1}^X = \frac{P_{t-1}^X}{P_{t-2}^X}$ and $\Pi_{F,t}^{trend}$ is the trend inflation in Foreign. In order for the reset price to be optimal, it must satisfy:[16]

$$E_t \sum_{k=0}^{\infty} (\xi_x)^k \Lambda_{t,t+k} \frac{X_{t+k|t}}{(\lambda_{t+k}^X - 1)} \left(P_{t+k|t}^X S_{t+k} - \lambda_{t+k}^X\, MC_{t+k}^X \right) = 0. \tag{41}$$

[14] For future reference, note that the first-order conditions from this problem may be used to derive input demand equations $D_t^X(i) = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t(i) + z_t^+\phi^X\right]$ and $M_t^X(i) = (1-\psi^X)\left(\frac{MC_t^X}{P_t^{M,X}}\right)^{\nu_x} \left[X_t(i) + z_t^+\phi^X\right]$. $D_t^X(i)$ represents the demand for the domestically produced, intermediate good from the individual export firm i. $M_t^X(i)$ denotes the demand for the imported good from the same firm.

[15] For the export good producer, local currency pricing implies that the export producers price their goods in the currency of Foreign. They are, however, interested in maximizing profits in Swedish currency, which is why the exchange rate enters the equation.

[16] In the equilibrium with flexible prices and wages, the corresponding first-order condition instructs the firm to set its price (times the exchange rate to denote it into Swedish currency) equal to the desired markup times the marginal cost:

$$P_t^{X,\text{fp}} S_t = \lambda_t^X MC_t^{X,\text{fp}}. \tag{40}$$

2.4.4 Swedish investment good producers

After describing the sectors for intermediate goods and for imports and exports, we now turn to the production of private investment goods. The investment good production sector consists of a continuum of investment good firms that operate on a market characterized by perfect competition. This means that they act as price takers, both in the market for their inputs and in the market for their output. The representative investment good producer use domestically produced intermediate goods and imported goods used for investment as inputs in its production of investment goods. Let D_t^I and M_t^I denote, respectively, the quantity of the domestically produced homogeneous intermediate good and of the homogeneous imported good used as inputs by the representative investment good firm. Furthermore, let $V_t^I = \frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]$ be the output of the investment good firm (meaning that the utilization cost is payed for via investment goods), and P_t^I be the price of investment goods. The maximization problem of the firm is then given by

$$\max_{V_t^I,\, D_t^I,\, M_t^I} \left\{P_t^I\, V_t^I - P_t\, D_t^I - P_t^{M,I}\, M_t^I\right\}$$

s.t.

$$V_t^I = \left[\left(\psi^I\right)^{\frac{1}{\nu_I}} \left(D_t^I\right)^{\frac{\nu_I - 1}{\nu_I}} + (1-\psi^I)^{\frac{1}{\nu_I}} \left(M_t^I\right)^{\frac{\nu_I - 1}{\nu_I}}\right]^{\frac{\nu_I}{\nu_I - 1}}.$$

$\psi^I = \vartheta^I + \frac{1}{1+\omega}\left(1-\vartheta^I\right)$ is the weight of the domestically produced intermediate good in the production of the investment good, and $\vartheta^v \in [0,1]$ may be interpreted as an index of home bias. The first-order conditions from this problem yield input demand functions $D_t^I = \psi^I\left(\frac{P_t^I}{P_t}\right)^{\nu_I} V_t^I$ and $M_t^I = \left(1-\psi^I\right)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I} V_t^I$. Note that ν_I may be interpreted as the price-elasticity of demand for the two respective inputs. P_t^I is the minimum expenditure needed to produce one unit of each investment good:

$$P_t^I = \left[\psi^I \left(P_t\right)^{1-\nu_I} + \left(1-\psi^I\right)\left(P_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{42}$$

Given the assumption of perfect competition, the representative investment good firm make zero profits. Therefore we interpret P_t^I as the appropriate price index for the investment good.

2.4.5 Swedish consumption good producers

In the modelling of private consumption, we follow Corbo and Strid (2020) and let the private consumption goods C_t^{agg} be created by a combination of non-energy consumption goods C_t^{xe} and energy consumption goods C_t^e. These goods are in turn created by combining domestic and imported non-energy goods, $D_t^{C,xe}$ and $M_t^{C,xe}$, and domestic and imported energy goods, $D_t^{C,e}$ and $M_t^{C,e}$, respectively. All firms face perfect competition, which means that they are price takers both regarding their inputs and their outputs. The maximization problem for the representative private consumption good firm is given by

$$\max_{C_t^{agg},\, C_t^{xe},\, C_t^e} \left\{P_t^C\, C_t^{agg} - P_t^{C,xe}\, C_t^{xe} - P_t^{C,e}\, C_t^e\right\}$$

s.t.

$$C_t^{agg} = \left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}} \left(C_t^{xe}\right)^{\frac{\nu_C - 1}{\nu_C}} + (1-\vartheta^C)^{\frac{1}{\nu_C}} \left(C_t^e\right)^{\frac{\nu_C - 1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C - 1}}$$

where $P_t^{C,xe}$ is the price of non-energy consumption goods and $P_t^{C,e}$ is the price of energy consumption goods. ϑ^C is the weight of non-energy consumption good in the production function. The first-order conditions from this problem yield input demand functions

$$C_t^{xe} = \vartheta^C\left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C} C_t^{agg} \tag{43}$$

$$C_t^e = \left(1-\vartheta^C\right)\left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C} C_t^{agg}. \tag{44}$$

Note that ν_C may be interpreted as the price-elasticity of demand for the two respective inputs. P_t^C is the minimum expenditure needed to produce one unit of each consumption good:

$$P_t^C = \left[\vartheta^C\left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}}. \tag{45}$$

The non-energy good producers face the following maximization problem:

$$\max_{C_t^{xe},\, D_t^{C,xe},\, M_t^{C,xe}} \left\{ P_t^{C,xe}\, C_t^{xe} - P_t\, D_t^{C,xe} - P_t^{M,C,xe}\, M_t^{C,xe} \right\}$$

s.t.

$$C_t^{xe} = \left[\left(\vartheta^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + (1-\vartheta^{C,xe})^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}$$

where $\vartheta^{C,xe}$ is the weight of the domestically produced intermediate good in the production of goods. The first-order conditions from this problem yield input demand functions

$$D_t^{C,xe} = \vartheta^{C,xe} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_{C,xe}} C_t^{xe} \tag{46}$$

$$M_t^{C,xe} = \left(1-\vartheta^{C,xe}\right) \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}} C_t^{xe} \tag{47}$$

Note that $\nu_{C,xe}$ may be interpreted as the price-elasticity of demand for the two respective inputs. $P_t^{C,xe}$ is the minimum expenditure needed to produce one unit of each non-energy consumption good:

$$P_t^{C,xe} = \left[\psi^{C,xe} \left(P_t\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C,xe}\right) \left(P_t^{M,C,xe}\right)^{1-\nu_{C,xe}} \right]^{\frac{1}{1-\nu_{C,xe}}}. \tag{48}$$

The energy good producers face the following maximization problem:

$$\max_{C_t^{e},\, D_t^{C,e},\, M_t^{C,e}} \left\{ P_t^{C,e}\, C_t^{e} - P_t^{D,Ce}\, D_t^{C,e} - P_t^{M,C,e}\, M_t^{C,e} \right\}$$

s.t.

$$C_t^{e} = \left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}} \left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + (1-\vartheta^{C,e})^{\frac{1}{\nu_{C,e}}} \left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}$$

where $\vartheta^{C,e}$ is the weight of the domestically produced intermediate good in the production of goods. The first-order conditions from this problem yield input demand functions

$$D_t^{C,e} = \vartheta^{C,e} \left(\frac{P_t^{C,e}}{P_t^{D,C,e}} \right)^{\nu_{C,e}} C_t^{e} \tag{49}$$

$$M_t^{C,e} = \left(1-\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{M,Ce}} \right)^{\nu_{C,e}} C_t^{e} \tag{50}$$

Note that $\nu_{C,e}$ may be interpreted as the price-elasticity of demand for the two respective inputs. $P_t^{C,e}$ is the minimum expenditure needed to produce one unit of each non-energy consumption good:

$$P_t^{C,e} = \left[\vartheta^{C,e} \left(P_t\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right) \left(P_t^{M,C,e}\right)^{1-\nu_{C,e}} \right]^{\frac{1}{1-\nu_{C,e}}}. \tag{51}$$

Note that the price of energy follows a stochastic process which is defined in Section 2.8.

2.4.6 Swedish government consumption and government investment good producers

Two representative firms, a government consumption good producer and a government investment good producer, use only domestically produced inputs in their production of final goods. These representative firms act as price takers, both in the markets for their inputs and in the markets for their respective outputs. There are no pricing frictions in the markets for government consumption and investment goods. Let $D_t^{v^P}$ be the quantity of the domestically produced intermediate goods used as inputs by the representative firm in sector $v^P \in \{G, I^G\}$. Furthermore, let V_t^P, $P \in \{G, I^G\}$ denote the output of such a representative firm. The profit maximization problem of such a representative firm is given by

$$\max_{V_t^P,\, D_t^{v^P}} \left\{ P_t^{v^P} V_t^P - P_t D_t^{v^P} \right\}$$

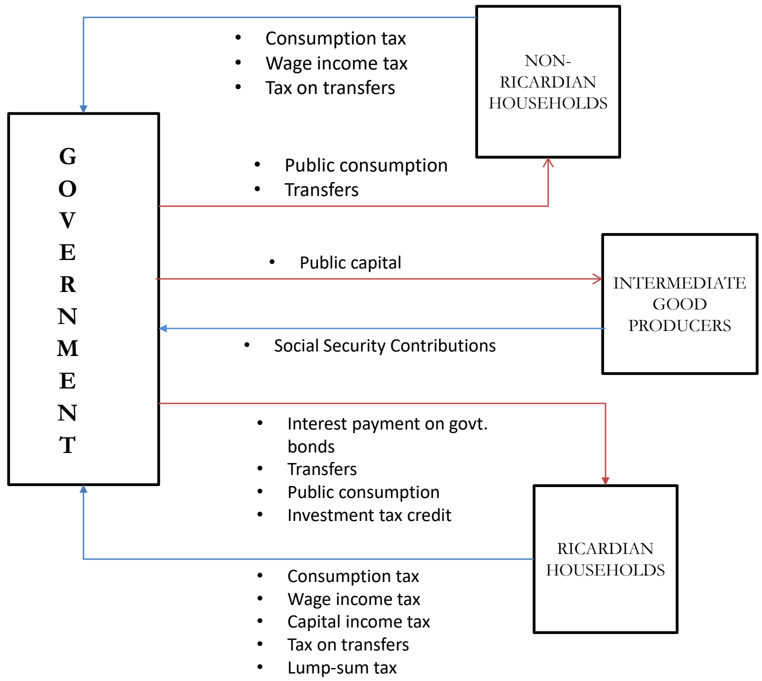
s.t.

$$V_t^P = D_t^{v^P}.$$

which implies that the price of both types of goods is given by

$$P_t^{v^P} = P_t.$$

Figure 2: Fiscal policy block



2.5 Fiscal authority and central bank in Sweden

In Sweden, a fiscal authority controls a large set of fiscal instruments (described in detail below) and the central bank, Riksbank, controls the nominal interest rate on private bonds.The interest rate is set according to a Taylor rule, taking into account the zero or effective lower bound for the nominal interest rate.

2.5.1 The Swedish fiscal authority

The government in Sweden collects taxes levied on household labor income, transfers, private consumption, household capital income, as well as lump-sum taxes. Furthermore, it collects social security contributions from the intermediate good firms. The government uses the tax revenue and issues bonds to finance expenditures. The expenditures consist of government consumption, government investment, lump-sum transfers and an investment tax credit as well as interest payments on government debt. Figure 2 illustrates the fiscal sector and its flows. The government budget constraint is given by

$$\tau_t^C P_t^C C_t^{agg} + (\tau_t^{SSC} + \tau_t^W) W_t N_t + \Upsilon_t^K + B_t^n + T_t = \left(\alpha_B + (R_{t-1}^B - 1)\right) B_t + \tau_t^I \frac{P_t^I}{\gamma_t} I_t + P_t G_t + P_t \frac{I_t^G}{\gamma_t} + (1 - \tau_t^{TR}) TR_t^{agg} \tag{52}$$

where

$$\Upsilon_t^K = \tau_t^K \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t \right) - \iota^K \tau_t^K \delta P_{t-1}^K K_t. \tag{53}$$

$\tau_t^C P_t^C C_t^{agg}$ denotes the aggregate revenue from the tax on private consumption, while $\left(\tau_t^{SSC} + \tau_t^W\right) W_t N_t$ denotes the aggregate revenue from tax on labor income. Υ_t^K denotes the capital income tax revenue, B_t^n denotes the newly issued debt by the government in period t and T_t denotes lump-sum taxes. On the right hand side of the government budget equation, $\left(\alpha_B + \left(R_{t-1}^B - 1\right)\right) B_t$ denotes interest rate payments on previously issued government bonds, where R_t^B is the average interest rate on outstanding government debt. $\tau_t^I \frac{P_t^I}{\gamma_t} I_t$ denotes the expenses due to the investment tax credit. $P_t G_t$ and $\frac{P_t}{\gamma_t} I_t^G$ denote expenses on government consumption and government investment, respectively. Finally $\left(1 - \tau_t^{TR}\right) TR_t^{agg}$ denotes aggregate lump-sum transfers net of taxes.

The government owns and maintains the public capital stock in the economy:

$$K_{G,t+1} = (1 - \delta_G) K_{G,t} + I_t^G$$

where $K_{G,t+1}$ denotes the public capital stock in the next period and I_t^G denotes government investment.[17] [18]

Government surplus: We define primary revenues $PREV_t$ as

$$PREV_t = \tau_t^C P_t^C C_t^{agg} + \left(\tau_t^{SSC} + \tau_t^W\right) W_t N_t + \Upsilon_t^K + \tau_t^{TR} TR_t^{agg} + T_t \tag{54}$$

and primary expenditure $PEXP_t$ as

$$PEXP_t = \tau_t^I \frac{P_t^I}{\gamma_t} I_t + P_t G_t + P_t \frac{I_t^G}{\gamma_t} + TR_t^{agg}. \tag{55}$$

Given the primary revenues and the primary expenditure, we use the government budget constraint and define the government surplus $SURP_t$ as[19]

$$SURP_t \equiv \underbrace{\underbrace{PREV_t - PEXP_t}_{\text{primary surplus}} - \underbrace{\left(R_{t-1}^B - 1\right) B_t}_{\text{interest payments}}}_{\text{total surplus (or gov. balance)}} = \alpha_B B_t - B_t^n$$

Hence, the surplus equals the incoming government debt that matures in period t minus the newly issued government debt.

The fiscal instruments: Fiscal policy can be conducted using the following different instruments:

$$x_t \in \left\{g_t,\ I_t^G,\ \tau_t^I,\ \tau_t^C,\ \tau_t^W,\ \tau_t^K,\ \tau_t^{TR},\ \tau_t^{SSC}\right\},$$

and tr_t^{agg}. g_t and I_t^G are the government transfers, government consumption and government investment per capita, and τ_t^I, τ_t^C, τ_t^W, τ_t^K, τ_t^{TR}, τ_t^{SSC} are the different tax rates in the economy. tr_t^{agg} is the aggregate transfers in units of domestically produced intermediate goods. The equations for each of the instruments can be divided into two different parts: an ARMA(1,1) process and a fiscal feedback rule, so that $x_t = x_t^{ARMA} + x_t^{Rule}$. The ARMA(1,1) part for all instruments except for the government transfers can be described by

$$x_t^{ARMA} = (1 - \rho_x)x + \rho_x x_{t-1} + \varepsilon_t^x + \eta_x \varepsilon_{t-1}^x. \tag{56}$$

For the government transfers the AR component is adjusted with one component of x_t^{Rule}, see below.

The fiscal feedback rule consists of three elements: the deviation of the government debt level as percent of GDP from its target $b_{\bar{y},t} - b_{\bar{y},t}^{Target}$, the deviation of the structural government surplus as percent of steady state GDP from its target $Stsurp_{\bar{y}.t} - Stsurp_{\bar{y},t}^{Target}$, and log deviation of GDP from its steady state level $\hat{y}_t$. On the other hand, the feedback rule for tr_t^{agg}consists of $b_{\bar{y},t} - b_{\bar{y},t}^{Target}$, $Stsurp_{\bar{y}.t} - Stsurp_{\bar{y},t}^{Target}$ and log deviation of unemployment from its steady state level $\hat{un}_t$. In the fiscal rules the surplus target is defined on the structural surplus, that is the cyclically adjusted budget balance.

The structural surplus is defined as the difference between the structural primary revenue, $Stprev_t$ and the structural primary expenditure, $Stpexp_t$, net of the interest payments on the current debt $(R_{t-1}^B - 1)B_t$. The structural primary expenditure is calculated by removing the business-cycle component (i.e. the output gap or unemployment gap reactions of the variables in the fiscal rules) from the actual primary expenditure while the structural primary revenues are calculated by multiplying all tax rates with their respective structural tax bases. Hence the structural surplus is given as[20]:

$$Stsurp_t = Stprev_t - Stpexp_t - (R_{t-1}^B - 1)B_t \tag{57}$$

with

$$Stprev_t = \tau_t^C P^C C^{agg} + \left(\tau_t^{SSC} + \tau_t^W\right) WN + \tau_t^K K \left(R^K - \iota^K \delta \frac{P^K}{\Pi}\right) + \tau_t^{TR} \left(TR_t^{agg} - F_{tr,un} Y \breve{un}_t\right) + T \tag{58}$$

where $P^C C^{agg}$ denotes the steady state consumption tax base while WN is the wage income tax base at the steady state. $K(R^K - \iota^K \delta \frac{P^K}{\Pi})$ and $TR_t^{agg} - F_{tr,y} Y \hat{un}_t$ denote the steady state capital income tax base and structural transfer tax base, respectively. $Stpexp_t$ is defined as:

[17]Note that there is no investment adjustment cost for public capital.

[18]The aggregate investments in the economy can be written as $I_t^{agg} = I_t + I_t^G$.

[19]Government surplus is sometimes referred to as government net lending. Here we do, however, use government surplus to be in line with the literature.

[20]For more detailed information on the structural surplus calculation see Appendix C.7.1.

$$\frac{Stpexp_t}{P_t} = \left(\frac{TR_t^{agg}}{P_t} - F_{tr,un} Y \hat{un}_t\right) + \left(\frac{I_t^G}{\gamma_t} - \mathcal{F}_{IG,y}\frac{I^G}{\gamma Y}(Y_t - Y)\right) + \left(G_t - \mathcal{F}_{g,y}\frac{G}{Y}(Y_t - Y)\right) + \tau_t^I \frac{P^I}{\gamma P_t} I \quad (59)$$

where $\frac{P^I}{\gamma} I$ denotes the steady state level for investment tax base and all other government expenditure terms are adjusted for their respective cyclical component.

A fiscal rule equation has been defined for eight of the instruments.[21] The investment subsidy has not been assigned a rule as there is no such subsidy in Sweden at present. For government consumption and government investment, $x_t^{Rule} \in \{g_t, I_t^G\}$, the rule is given by

$$x_t^{Rule} = \mathcal{F}_{x,b}\left(b_{\bar{y},t} - b_{\bar{y},t}^{Target}\right) + \mathcal{F}_{x,surp}\left(Stsurp_{\bar{y},t} - Stsurp_{\bar{y},t}^{Target}\right) + \mathcal{F}_{x,y}\hat{y}_t. \quad (60)$$

The first two terms on the right-hand-side of the equations are supposed to capture the Swedish fiscal framework, which includes a surplus target and a debt anchor. This kind of feedback for the debt level can be found also in e.g. Coenen, Straub, and Trabandt (2013) and Erceg and Lindé (2013). The third and last part of the equation is supposed to capture automatic stabilizers. For the tax rates, $x_t^{Rule} \in \{\tau_t^C, \tau_t^W, \tau_t^K, \tau_t^{TR}, \tau_t^{SSC}\}$, the rule is given by

$$x_t^{Rule} = \mathcal{F}_{x,b}\left(b_{\bar{y},t} - b_{\bar{y},t}^{Target}\right) + \mathcal{F}_{x,surp}\left(Stsurp_{\bar{y},t} - Stsurp_{\bar{y},t}^{Target}\right). \quad (61)$$

The eighth rule is the transfer rule which is normalized by steady-state GDP, $\bar{y}$:

$$tr_t^{agg,Rule} = \bar{y}F_{tr,b}\left(b_{\bar{y},t} - b_{\bar{y},t}^{Target}\right) + \bar{y}\mathcal{F}_{tr,surp}\left(Stsurp_{\bar{y},t} - Stsurp_{\bar{y},t}^{Target}\right) + \bar{y}\mathcal{F}_{tr,un}\breve{un}_t. \quad (62)$$

The ARMA(1,1) component of government transfers is then given by

$$tr_t^{agg,ARMA} = (1-\rho_x)tr^{agg} + \rho_{tr}(tr_{t-1}^{agg} - \bar{y}\mathcal{F}_{tr,un}\breve{un}_{t-1}) + \varepsilon_t^x + \eta_x\varepsilon_{t-1}^x. \quad (63)$$

where AR component is adjusted with the automatic stabilizer component of the rule to make sure that persistence in transfers are only derived by fiscal stabilization component.

There is a mapping between the debt-target and the surplus target. This mapping needs to hold in the steady state, since a certain level of debt in percent of GDP in the long run implies a unique surplus in percent of GDP. The mapping between the debt target and the surplus target is defined as

$$Stsurp_{\bar{y},t}^{Target} = \left(\frac{1}{\mu_{z+}\Pi} - 1\right) b_{\bar{y},t}^{Target}.$$

Debt and surplus target shocks: The debt and surplus target are also fiscal policy variables. The fiscal authority might want to deviate from the rules temporarily. This is captured by the debt target shock $\varepsilon_t^{b^{Target}}$.[22] After such a shock, the debt target follows an AR(2) process which is described as

$$b_{\bar{y},t}^{Target} - b_{\bar{y}} = \left(\rho_{1,b^T} + \rho_{2,b^T}\right)\left(b_{\bar{y},t-1}^{Target} - b_{\bar{y}}\right) - \rho_{1,b^T}\rho_{2,b^T}\left(b_{\bar{y},t-2}^{Target} - b_{\bar{y}}\right) + \epsilon_t^{b^{Target}}. \quad (64)$$

Aggregate transfer distribution: The share of aggregate transfers that goes to Ricardian and Non-Ricardian households respectively off the steady state is governed by the following equation:

$$\varpi_{dyn}\breve{tr}_t = (1-\varpi_{dyn})\breve{tr}_t^{nr},$$

where $\breve{tr}_t$ and $\breve{tr}_t^{nr}$ are the deviations in transfers to Ricardians and Non-Ricardians in units of domestically produced intermediate goods. The equation implies that the steady-state distribution of transfers between Ricardian and Non-Ricardian households might differ from the distribution off the steady state.

[21] We calibrate the fiscal rule parameters of all instruments except for the government transfers to zero, thus they are kept inactive in our benchmark estimation. Moreover, the debt coefficient in the government transfers rule is also set to zero so that only structural surplus target is used for the fiscal budget stabilization.

[22] Note that the debt target shock can also be used to capture a shock to the surplus target, since the debt target and the surplus target are mirror images in the steady state.

2.5.2 The Swedish central bank

The Riksbank sets the policy interest rate according to a Taylor rule. We follow Corbo and Strid (2020), and let the interest rate be affected by the deviations of inflation and unemployment from their steady-state levels, and by the changes in inflation and unemployment. The rule is written in deviations from steady state, where $\breve{i}_t$ is defined as the deviation of the policy rate from the neutral interest rate, defined below. $\hat{\Pi}_t^C \equiv ln\left(\frac{\Pi_t^C}{\Pi^C}\right)$ is the deviation of inflation from the Riksbank target rate Π^C which is also the steady state inflation rate. The Riksbank reacts to annual inflation rather than quarterly inflation, where the annual inflation is defined as $\hat{\Pi}_t^{a,C} = \frac{1}{4}\left(\hat{\Pi}_t^C + \hat{\Pi}_{t-1}^C + \hat{\Pi}_{t-2}^C + \hat{\Pi}_{t-3}^C\right)$. $\breve{un}_t$ is the deviation of unemployment rate from its steady state level. Furthermore, there is a lower bound on the interest rate $\breve{\underline{i}}$. If the Taylor rule implies an interest rate level below the lower bound, the interest rate is set to the lower bound. The following two equations govern the interest rate:

$$\breve{i}_t^{notional} = \rho \breve{i}_{t-1}^{notional} + (1-\rho)\left(r_\pi \hat{\Pi}_{t-1}^{a,C} + r_{un}\breve{un}_{t-1}\right) + r_{\triangle\pi}\left(\hat{\Pi}_t^C - \hat{\Pi}_{t-1}^C\right) + r_{\triangle un}\left(\breve{un}_t - \breve{un}_{t-1}\right) + \epsilon_t^i, \quad (65)$$

$$\breve{i}_t^{ss} = max(\breve{\underline{i}}, \breve{i}_t^{notional} + \breve{i}_t^{nat}) \quad (66)$$

where $\breve{i}_t^{notional}$ denotes the notional nominal policy rate, i.e. the nominal interest rate absent the effective lower bound constraint, $\breve{i}_t^{ss}$ is the deviation of the actual interest rate from its steady state value, and $\breve{i}_t^{nat}$ is the neutral interest rate, both in deviations from their respective steady state values. ϵ_t^i is an exogenous, stochastic shock. The second equation introduces the zero or effective lower bound constraint into the model.

2.5.3 The neutral interest rate

We follow Corbo and Strid (2020) and introduce a neutral interest rate into the model. The neutral rate is introduced for empirical reasons, given the observation that global interest rates have declined over time, at the same time as it is difficult to argue that actual monetary policy have become more and more expansionary. As such, we interpret $\breve{i}_t$ as being the policy rate deviation from the neutral rate rather than the deviation from its steady state level, such that

$$\breve{i}_t = \breve{i}_t^{ss} - \breve{i}_t^{nat} \quad (67)$$

A consequence of this assumption is that the resulting model simulations (except for the simulated policy rate) are not affected by the introduction of a neutral interest rate, except for when the neutral interest rate lies below the lower bound of the interest rate. Furthermore, we assume that the inflation rate in the neutral rate remains constant, so that changes in the neutral rate happens only via changes in the neutral real interest rate. Hence, we can write the neutral interest rate in a similar manner as in Corbo and Strid (2020), as

$$\breve{i}_t^{nat} = r_\mu \hat{\mu}_{z+,t} - r_\zeta \hat{\zeta}_t + \hat{z}_t^R \quad (68)$$

where $\hat{\mu}_{z+,t}$ is the log-deviation of growth rate of from its steady-state level, $\hat{\zeta}_t$ is the log deviation of the risk-premium shock from its steady-state level, and $\hat{z}_t^R$ is a shock process introduced to capture factors that are not introduced into the model explicitly, but that can be assumed to change the neutral rate, such as demographic factors. Given how the neutral rate is introduced into the model, we interpret an interest rate that is lower than the neutral rate as expansionary monetary policy, while an interest rate that is higher than the neutral rate as contractionary monetary policy.

2.6 The Foreign economy

We model Sweden as a small open economy. Due to its size relative to Sweden, the Foreign economy instead behaves like a closed economy. From the perspective of the Foreign economy, any transactions between the two countries will be arbitrarily small, compared to the total quantities of goods that are produced and consumed within Foreign. Formally, we assume that the size ω of Foreign tends to infinity, $\omega \to \infty$ implying that the relative size of the Swedish economy in relation to world economy; $\frac{1}{1+\omega}$, tends to zero. Given the size of Foreign, we abstract from modelling the Foreign export and import sectors. The reasons are the following: Firstly, since the exports and imports from Sweden are arbitrarily small compared to aggregate Foreign output, they will not have any effect on the equilibrium allocations and prices in Foreign. Secondly, the modelling of Foreign exports and imports adds an additional layer of complexity, but does not give any additional information to the evolution of Swedish imports and exports that can not be captured by the modelling of the Swedish import and export sectors. Note however, that we still need to model the demand for Swedish exports and supply of Swedish imports. Both are discussed in the market clearing section. The derivation of the export demand is however presented in Section E.

The Foregin firms' optimization problems are to a great extent identical to those in Sweden, up to a scaling factor. There are also, however, important differences between the two economies. Compared to the Swedish economy, the fiscal sector in Foreign is modelled in much less detail. In addition, intermediate goods producers use only private capital as physical capital in their production. Moreover, it is assumed that all households in Foreign are Ricardian households, and that they can only save in bonds denominated in the currency of Foreign. The following sections describe the different agents in Foreign and their decision problems. Because of the similarities with Sweden, the explanations given here are relatively sparse and emphasis is given to areas where the two economies differ.

2.6.1 Foreign households

We model the foreign households slightly different from the households in Sweden. We assume a standard representative household setup which is the most commonly used in DSGE literature, where households (or its members) differentiate from each other only by their labor type but not by their disutility of labor. We also use hours worked as the unit of labor in foreign economy (intensive margin) while we use employment for the Swedish economy (extensive margin). The total mass of households in Foreign is ω. Foreign households' preferences over private consumption and hours worked are identical to the Ricardian households in Sweden. Furthermore, we assume that Foreign households are able to hold assets that yield a risk-free return in terms of Foreign currency, just like the Ricardian households in Sweden are able to hold assets with a risk-free return in terms of Swedish currency. Foreign households are however not allowed to hold bonds in Swedish currency. The problem of the individual household f in Foreign is to choose private consumption $C_{f,t}$, physical capital $K_{f,t+1}$, Investment I_t, capital utilization $u_{f,t}$, the change in capital stock by trading in the market $\triangle^K_{f,t}$, domestic nominal bonds that are denominated in the Foreign currency $B^{FF}_{f,t+1}$ and the nominal wage $W_{f,t}$, in order to maximize

$$E_0 \sum_{t=0}^{\infty} \beta_{F,t} \left[\zeta^c_{F,t} u(C_{f,t}, C_{F,t-1}) - \zeta^n_{F,t} \nu(N_{f,t}) \right], \tag{69}$$

subject to the following budget constraint:

$$P^C_{F,t} C_{f,t} + \frac{P^I_{F,t}}{\gamma_t} I_{f,t} + P^K_{F,t} \triangle^K_{f,t} + \frac{B^{FF}_{f,t+1}}{R_{F,t}\zeta_{F,t}}. = (1-\tau^w_F) W_{f,t} N_{f,t} + R^K_{F,t} u_{f,t} K_{f,t} - \frac{P^I_{F,t}}{\gamma_t} a(u_{f,t}) K_{f,t} + B^{FF}_{f,t} + \Xi_{B^{FF},t} + \Psi_{f,t} + TR_{f,t}, \tag{70}$$

and the capital accumulation process:

$$K_{f,t+1} = (1-\delta_F) K_{f,t} + \Upsilon_{F,t} F(I_{f,t}, I_{f,t-1}) + \triangle^K_{f,t}, \tag{71}$$

$C_{F,t}$ denotes aggregate consumption in Foreign. δ_F is the depreciation rate of the foreign capital and $\Upsilon_{F,t}$ is the stationary investment-specific technology shock. In periods when the household has an opportunity to reset its wage, it also chooses $W^{\text{opt}}_{f,t}$ subject to the following condition:

$$W_{f,t+k} = \begin{cases} \overline{\Pi}^W_{F,t+k-1} W_{f,t+k} & \text{with probability } \xi^F_w \\ W^{\text{opt}}_{f,t+k} & \text{with probability } 1-\xi^F_w \end{cases} \tag{72}$$

for all $k \geq 0$, and taking $N_{f,t+k} = \frac{1}{\omega}\left(\frac{W_{f,t+k}}{W_{F,t+k}}\right)^{-\varepsilon^F_w} N_{F,t}$ as given. Let $W_{f,t+k|t} = W^{opt}_{f,t} \overline{\Pi}^W_{F,t+1} \ldots \overline{\Pi}^W_{F,t+k}$ denote the wage of household f in future period $(t+k)$, given that the household last opportunity to set the wage was in period t. The first-order conditions associated with this problem are presented next. $\Omega^C_{f,t}$ denotes the marginal utility of consumption and $\beta^r_{F,t+1} = \frac{\beta_{F,t+1}}{\beta_{F,t}}$ represents changes in the subjective discount factor between consecutive periods. $\theta^{\text{b}}_{f,t}$ and $\theta^k_{f,t}$denote the Lagrange multipliers associated with the budget constraint (70) and the capital accumulation equation (71), respectively.

$$C_{f,t} : \theta^b_{f,t} P^C_{F,t} = \Omega^C_{f,t}, \tag{73}$$

$$B^{FF}_{f,t+1} : \theta^b_{f,t} P^C_{F,t} = E_t \left[\beta^r_{F,t+1} \theta^b_{f,t+1} P^C_{F,t} R_{F,t} \zeta_{F,t} \right] \tag{74}$$

$$K_{f,t+1} : \theta^k_{f,t} = E_t \beta^r_{F,t+1} \left[\theta^b_{f,t+1} \left(R^K_{F,t+1} u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}} a(u_{f,t+1}) \right) + \theta^k_{f,t+1}(1-\delta_F) \right] \tag{75}$$

$$I_{f,t} : \theta^b_{f,t} \frac{P^I_{F,t}}{\gamma_t} = \theta^k_{f,t} \Upsilon_{F,t} F_1(I_{f,t}, I_{f,t-1}) + E_t \left[\beta^r_{F,t+1} \theta^k_{f,t+1} \Upsilon_{F,t+1} F_2(I_{f,t+1}, I_{f,t}) \right] \tag{76}$$

$$u_{f,t} : R^K_{F,t} K_{f,t} = \frac{P^I_{F,t}}{\gamma_t} a'(u_{f,t}) K_{f,t} \tag{77}$$

$$\triangle^K_{f,t} : \theta^b_{f,t} P^K_{F,t} = \theta^k_{f,t}. \tag{78}$$

The first-order condition associated with $W^{opt}_{f,t}$ is given by

$$E_t \sum_{k=0}^{\infty} \left(\xi_w^F\right)^k \left(\prod_{i=1}^{k} \beta^r_{F,t+i}\right) N_{f,t+k|t}\, \theta^b_{f,t+k} \left[(1-\tau_F^w)\, W_{f,t+k|t} - \lambda_F^W\, \zeta^n_{F,t+k} \frac{\nu'\left(N_{f,t+k|t}\right)}{\theta^b_{f,t+k}} \right] = 0, \tag{79}$$

where $\prod_{i=1}^{k} \beta^r_{F,t+i} = \beta^r_{F,t+1}\beta^r_{F,t+2}\ldots\beta^r_{F,t+k}$ and $\prod_{i=1}^{0} \beta^r_{F,t+i} \equiv 1$.[23]

2.6.2 Foreign intermediate good producers

The intermediate good sector in Foreign is consist of a continuum of firms with total mass ω. As in Sweden, a representative aggregator firm buys the different varieties of goods and produces a homogeneous, intermediate good that is sold to firms in other sectors. The demand for the individual variety j, $Y_{F,t}(j)$ is a function of the relative price of that variety and of total demand for the homogeneous, intermediate good: $Y_{F,t}(j) = \frac{1}{\omega}\left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} Y_{F,t}$. $P_{F,t}(j)$ denotes the price charged by firm j, $P_{F,t} = \left(\frac{1}{\omega}\int_0^{\omega} P_{F,t}(j)^{\frac{1}{1-\lambda_{F,t}}} dj\right)^{1-\lambda_{F,t}}$ is the price index associated with the homogeneous, intermediate good and $Y_{F,t}$ denotes total demand. $\lambda_{F,t}$ is a time-varying, stochastic markup. Intermediate good firms in Foreign use labor and capital as inputs in their production. The cost-minimization problem of firm j is:

$$\min_{K_{F,t}(j) L_{F,t}(j)} \left\{ R^K_{F,t} K^s_{F,t}(j) + W_{F,t} N_{F,t}(j) \right\}$$

s.t.

$$Y_{F,t}(j) = \varepsilon_{F,t}\left[K^s_{F,t}(j)\right]^{\alpha_F} \left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z^+_{F,t}\phi_F,$$

where $\varepsilon_{F,t}$ is a stationary stochastic process with an unconditional mean of unity, that is common to all firms

in the sector. As in Swedish economy, $z^+_{F,t}$ is a function of the two stochastic variables $z_{F,t}$ and $\gamma_{F,t}$ and is given by

$$z^+_{F,t} = z_{F,t}\gamma_{F,t}^{\frac{\alpha_F}{1-\alpha_F}}. \tag{80}$$

The cost-minimization problem yields the following expression for nominal marginal cost:

$$MC_{F,t} = \frac{\left(\frac{W_{F,t}}{z_t}\right)^{1-\alpha_F} \left(R^K_{F,t}\right)^{\alpha_F}}{\alpha_F^{\alpha_F}\left(1-\alpha_F\right)^{1-\alpha_F} \varepsilon_{F,t}}. \tag{81}$$

The rental rate of capital services can be written as a function of the marginal cost and the optimal capital-to-labor ratio:

$$R^K_{F,t} = \alpha_F \varepsilon_{F,t} z_t^{1-\alpha_F} MC_{F,t} \left(\frac{K^s_{F,t}}{N_{F,t}}\right)^{\alpha_F - 1}. \tag{82}$$

The price setting problem of intermediate good firms in Foreign is identical to that of the corresponding firms in Sweden. Therefore, we only state the first-order condition associated with that problem and refer the reader to Section 2.4.1 for more details:[24]

$$E_t \sum_{k=0}^{\infty} (\xi^F)^k \Lambda^F_{t,t+k} \frac{Y_{F,t+k|t}}{(\lambda_{F,t+k}-1)} \left(P_{F,t+k|t} - \lambda_{F,t+k}\, MC_{F,t+k}\right) = 0. \tag{83}$$

[23] When the household is free to optimize its wage in every period, as is the case in the equilibrium with flexible prices and wages, the first-order condition becomes:

$$\left(1-\tau_w^F\right) W^{\text{fp}}_{f,t} = \lambda_F^W \zeta^n_{F,t} \frac{\nu'\left(N^{\text{fp}}_{f,t}\right)}{\theta^{\text{b,fp}}_{f,t}}.$$

[24] In the equilibrium with flexible prices and wages, the corresponding first-order condition is:

$$P^{\text{fp}}_{F,t} = \lambda_{F,t}\, MC^{\text{fp}}_{F,t}.$$

2.6.3 Foreign consumption good producers

A representative firm produces consumption goods that are sold to households in Foreign. As in Sweden, the consumption good consists of a combination of non-energy and energy goods. The markets for the inputs and outputs of this representative firm are characterized by perfect competition, flexible prices and zero profits. The production function is similar to that of Sweden, i.e. given by

$$C_{F,t}=\left[\left(\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}}\left(C_{F.t}^{xe}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}+\left(1-\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}}\left(C_{F.t}^{e}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}.$$

The respective demand functions for energy and non-energy goods are given by

$$C_{F.t}^{xe}=\vartheta_F^C\left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}}\right)^{\nu_{F,C}}C_{F,t} \tag{84}$$

$$C_{F.t}^{e}=\left(1-\vartheta_F^C\right)\left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}}\right)^{\nu_{F,C}}C_{F,t} \tag{85}$$

and the price of the Foreign consumption good is given by

$$P_{F,t}^C=\left[\vartheta_F^C\left(P_{F,t}^{C,xe}\right)^{1-\nu_{F,C}}+\left(1-\vartheta_F^C\right)\left(P_{F,t}^{C,e}\right)^{1-\nu_{F,C}}\right]^{\frac{1}{1-\nu_{F,C}}}. \tag{86}$$

Similarly to Sweden, non-energy consumption is produced by combining domestic and imported non-energy goods according to the production function

$$C_{F,t}^{xe}=\left[\left(\vartheta_F^{C,xe}\right)^{\frac{1}{\nu_{F,C,xe}}}\left(D_{F.t}^{C,xe}\right)^{\frac{\nu_{F,C,xe}-1}{\nu_{F,C,xe}}}+\left(1-\vartheta_F^{C,xe}\right)^{\frac{1}{\nu_{F,C,xe}}}\left(M_{F.t}^{C}\right)^{\frac{\nu_{F,C,xe}-1}{\nu_{F,C,xe}}}\right]^{\frac{\nu_{F,C,xe}}{\nu_{F,C,xe}-1}}.$$

However, due to Foreign being so large compared to Sweden, the share of Swedish imports in the production of Foreign non-energy consumption goes to zero, meaning that $\vartheta_F^{C,xe}\rightarrow 1$.This reduces the production function to

$$C_{F,t}^{xe}=D_{F.t}^{C,xe}. \tag{87}$$

Furthermore, due to the assumption of perfect competition, the price of the non-energy good is given by

$$P_{F,t}^{C,xe}=P_{F,t}. \tag{88}$$

Finally, the Foreign energy consumption good is produced by transforming Foreign domestic intermediate goods to energy goods. Their production function is given by

$$C_{F,t}^{e}=D_{F.t}^{C,e}. \tag{89}$$

2.6.4 Fiscal authority and central bank in Foreign

The fiscal authority in Foreign is modelled in a sparse manner. The fiscal authority levies a tax on labor income, and all tax income is returned to households using transfers. The government budget is balanced every period, and there is no government debt. The fiscal authority's budget constraint is given by

$$W_{F,t}N_{F,t}\tau_F^w=TR_{F,t}+G_{F,t}. \tag{90}$$

Concerning monetary policy, it is assumed that the central bank in Foreign sets its policy interest rate according to a Taylor rule, where $\check{i}_{F,t}$ is the policy rate deviation from the neutral interest rate, following Corbo and Strid (2020). The Taylor rule is similar to Sweden, but reacts to output rather than unemployment, since unemployment is not modelled in Foreign. The Foreign central bank also reacts to annual inflation rather than quarterly inflation, where the annual inflation is defined as $\hat{\Pi}_{F,t}^{a,C}=\frac{1}{4}\left(\hat{\Pi}_{F,t}^C+\hat{\Pi}_{F,t-1}^C+\hat{\Pi}_{F,t-2}^C+\hat{\Pi}_{F,t-3}^C\right)$. As in Sweden, the interest rate in Foreign is restricted by its lower bound $\underline{i_F}$. The interest rate follows the following equations:

$$\check{i}_{F,t}^{notional}=\rho_F\check{i}_{F,t-1}^{notional}+(1-\rho_F)\left(r_{F,\pi}\hat{\Pi}_{F,t}^{a,C}+r_{F,y}\hat{y}_{F,t}\right)+r_{F,\triangle\pi}\left(\hat{\Pi}_{F,t}^C-\hat{\Pi}_{F,t-1}^C\right)+r_{F,\triangle y}\left(\hat{y}_{F,t}-\hat{y}_{F,t-1}\right)+\epsilon_t^{i_F}, \tag{91}$$

$$\check{i}_{F,t}^{ss}=max(\underline{i_F},\check{i}_{F,t}^{notional}+\check{i}_{F,t}^{nat}) \tag{92}$$

Just as in Sweden, we define the monetary policy expansion as the difference between the actual rate $\breve{i}_{F,t}^{ss}$ and the the neutral rate $\breve{i}_{F,t}^{nat}$:

$$\breve{i}_{F,t} = \breve{i}_{F,t}^{ss} - \breve{i}_{F,t}^{nat} \tag{93}$$

where the neutral rate is defined as

$$\breve{i}_{F,t}^{nat} = r_{F,\mu}\hat{\mu}_{z^+,t} - r_{F,\zeta}\hat{\zeta}_{F,t} + \hat{z}_t^R \tag{94}$$

2.7 Market clearing

In equilibrium, decisions taken by individual households and firms must be consistent with market clearing in the markets for goods, bonds and capital. For most types of goods and assets, the markets need to clear within each country. In principle, the markets for traded goods between the country also needs to clear. For the export goods, this is done by equalizing the supply of export goods, described in Section 2.4.3, and the demand for Swedish exports, which is defined below. The goods that are imported to Sweden from Foreign does however either consist of Foreign domestic goods or Foreign energy goods, which are created by Foreign domestic goods. Since Sweden is so small compared to Foreign, the demand for Foreign domestic goods and energy goods by Swedish firms do not have any effect on the aggregate output in Foreign. Therefore, we abstract from the purchases by Swedish import firms in the Foreign market clearing conditions. The international payments between Sweden and Foreign must however balance, which is achieved via the Balance of Payments equation. The expressions in this section are derived in Appendix E.

2.7.1 Aggregate resources

As a necessary condition for the Swedish market for domestically produced intermediate goods to clear, the sum of output from individual intermediate good producers must equal Swedish final good producers' (i.e. private and government consumption, private and government investment and export good producers) demand for domestically produced intermediate goods. Let Y_t be the amount of domestically produced homogeneous intermediate goods. Also, let $\overleftrightarrow{P}_t^X = \int_0^1 \left(\frac{P_t^X(i)}{P_t^X}\right)^{\frac{\lambda_t^X}{1-\lambda_t^X}} di$ be a measure of price dispersion among firms in the export good sector. Furthermore, let $N_t^D = \int_0^1 N_t(i)di$ denote total demand for labor services from the intermediate good producers. The aggregate resource constraint for Sweden may then be written as

$$Y_t = \psi^{C,xe}\left(\frac{P_t^{C,xe}}{P_t^C}\right)^{\nu_{C,xe}} C_t^{xe} + D_t^{C,e} + \psi^I\left(\frac{P_t^I}{P_t}\right)^{\nu_I}\left[\frac{I_t}{\gamma_t} + a(u_t)\frac{K_t}{\gamma_t}\right] + \psi^X\left(\frac{MC_t^X}{P_t}\right)^{\nu_x}\left[X_t\overleftrightarrow{P}_t^X + z_t^+\phi^X\right] + G_t + \frac{I_t^G}{\gamma_t} \tag{95}$$

Corresponding definitions are used to write the Foreign aggregate resource constraint. For example, $\overleftrightarrow{P}_{F,t} = \int_0^\omega \frac{1}{\omega}\left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} dj$ is a measure of price dispersion in the Foreign market for intermediate goods, and $N_{F,t} = \int_0^\omega N_{F,t}(j)dj$ denotes the total (or aggregate) demand for labor services in Foreign. Aggregate resources in Foreign are used to satisfy the demand for non-energy consumption goods, energy consumption goods and investment goods. Remember, however, that there is no government consumption and government investment. Therefore, the Foreign aggregate resource constraint contains fewer terms than that of the Swedish economy:

$$\begin{aligned}\varepsilon_{F,t}\left[K_{F,t}^s\right]^{\alpha_F}\left[z_t N_{F,t}\right]^{1-\alpha_F} = {} & \overleftrightarrow{P}_{F,t}\psi_F^{C.xe}\left(\frac{P_{F,t}^{C,xe}}{P_{F,t}}\right)^{-\nu_{F,C}} C_{F,t}^{xe} + \overleftrightarrow{P}_{F,t}C_{F,t}^{e} \\ & + \overleftrightarrow{P}_{F,t}\psi_F^I\left(\frac{P_{F,t}^I}{P_{F,t}}\right)^{-\nu_{F,I}}\left[\frac{I_{F,t}}{\gamma_t} + a(u_{F,t})\frac{K_{F,t}}{\gamma_t}\right] + G_{F,t} + z_{F,t}^+\omega\phi_F.\end{aligned} \tag{96}$$

For future reference, we also define Swedish and Foreign output (GDP), where GDP is the same as the domestically produced homogeneous input goods Y_t and $Y_{F,t}$, where

$$\overleftrightarrow{P}_t Y_t = \int_0^1 \left(\varepsilon_t\left[K_t^s(i)\right]^\alpha\left[z_t L_t(i)\right]^{1-\alpha} - z_t^+\phi\right) di \tag{97}$$

$$\overleftrightarrow{P}_{F,t}Y_{F,t} = \int_0^\omega \left(\varepsilon_{F,t}\left[K_{F,t}^s(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z_{F,t}^+\phi_F\right) dj \tag{98}$$

In the definition of Swedish and Foreign GDP given by Equation (97) and Equation (98), respectively, capital utilization costs are included, since some of the output goes to paying these costs. These equations implicitly include utilization costs as a part of final demand. This definition is consistent with an interpretation of these costs as a form of investment. Christiano, Trabandt, and Walentin (2011), however, suggest that a second and alternative definition of output be used for the purpose of matching model variables to the data. We adopt their approach and use Y_t^m to denote 'measured output' in Sweden and $Y_{F,t}^m$ in Foreign, which are equal to output less of utilization costs. The capital utilization costs are represented by the term $\psi^I \left(\frac{P_t^I}{P_t}\right)^{-\nu_I} a(u_t) \frac{K_t}{\gamma_t}$ in Equation (95) and by the term $\psi_F^I \left(\frac{P_{F,t}^I}{P_{F,t}}\right)^{-\nu_{F,I}} a(u_{F,t}) \frac{K_{F,t}}{\gamma_t}$ in Equation (98). Hence we have

$$Y_t^m = Y_t - \psi^I \left(\frac{P_t^I}{P_t}\right)^{-\nu_I} a(u_t) \frac{K_t}{\gamma_t} \tag{99}$$

$$Y_{F,t}^m = Y_{F,t} - \psi_F^I \left(\frac{P_{F,t}^I}{P_{F,t}}\right)^{-\nu_{F,I}} a(u_{F,t}) \frac{K_{F,t}}{\gamma_t} \tag{100}$$

With this alternative concept of output, capital utilization costs are treated as a proper cost and are not included in gross fixed capital formation.

2.7.2 Market clearing for bonds

There are three different bond markets that have to clear. The first is the market for private bonds denominated in Swedish currency. The market clearing condition is given by

$$B_{t+1}^{priv} = 0. \tag{101}$$

The second is the market for Foreign bonds. First define $B_{t+1}^{FH} = \int_0^{1-s_{nr}} B_{k,t+1}^{FH} dk$, the aggregate value of purchases by all Swedish households of such bonds in period t. Since the bonds are traded across the two countries, the clearing condition is given by

$$B_{t+1}^{FH} + \int_0^{\omega} B_{f,t+1}^{FF} df = 0. \tag{102}$$

The third bond market is the market for government bonds. In that market, the total amount of newly issued debt by the government B_t^n must equal the total household demand for newly issued government debt. The market clearing condition is given by

$$B_t^n = \int_0^{1-s_{nr}} B_{k,t}^n dk. \tag{103}$$

2.7.3 International trade in goods

Let X_t denote aggregate demand for Swedish exports. The consumption and investment good firms in Foreign are using inputs from Sweden in their production. The demand function for Swedish export goods is derived in Appendix E.3.5, and is given by

$$X_t = \left(1 - \psi_F^{C,xe}\right) \left(\frac{P_t^X}{P_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} C_{F,t}^{xe} + \left(1 - \psi_F^I\right) \left(\frac{P_t^X}{P_{F,t}^I}\right)^{-\nu_{F,I}} I_{F,t}. \tag{104}$$

We now move on to the import sector. Since Sweden is arbitrarily small in relation to Foreign, the demand for imports does not affect the supply of goods in Foreign, meaning that Foreign will supply any number of goods that the Swedish households demand for the given price. Define all Swedish non-energy imports M_t^{xe} as

$$M_t^{xe} = \int_0^1 M_t^{C,xe}(i) di + \int_0^1 M_t^I(i) di + \int_0^1 M_t^X(i) di + z_t^+ \phi^{M,C,xe} + z_t^+ \phi^{M,I} + z_t^+ \phi^{M,X}. \tag{105}$$

Then total Swedish imports are given by

$$M_t = M_t^{xe} + \int_0^1 M_t^{C,e}(i) di + z_t^+ \phi^{M,C,e}. \tag{106}$$

It is also useful to define total imports of energy goods M_t^e as

$$M_t^e = \int_0^1 M_t^{C,e}(i) di + z_t^+ \phi^{M,C,e}. \tag{107}$$

2.7.4 Balance of payments and net foreign assets

Two types of international transactions occur between agents in Sweden and Foreign. Firms in the Swedish export and import sectors trade with Foreign firms, and Swedish households buy and sell in the Foreign (international) market for bonds. In the aggregate, the nominal value of these different transactions must balance. For the purpose of defining a balance of payments relationship for Sweden, let us start by adding up the different transactions that occur in the market for international bonds. Note that the value of this aggregate position in Swedish currency is $A_t = \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)}$. A_t may therefore be referred to as the period t nominal aggregate net foreign asset position of Sweden.[25] The value in Swedish currency of the settlement of bonds issued in the previous period is $\int_0^{1-s_{nr}} \left[S_t B_{k,t}^{FH}\right] dk = S_t B_t^{FH}$.

$\Phi(\cdot)$ represents an external risk premium on domestic (Swedish) holdings of Foreign bonds, and the choice of a specific functional form for this premium merits some discussion. Note that $s_t = \frac{S_t}{S_{t-1}}$. The value of $\Phi(\cdot)$ is determined by the two aggregate variables $\overline{a}_t = \frac{A_t}{z_t^+ P_t}$ and $E_t\left(\frac{S_{t+1}}{S_t}\frac{S_t}{S_{t-1}}\right) = E_t\left(s_{t+1}s_t\right)$, as well as by the (aggregate) shock $\widetilde{\phi}_t$. For notational convenience, we let s_t represent the second argument of the risk premium function and thus write $\Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)$.[26] $\overline{a}_t = \frac{A_t}{z_t^+ P_t} = \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right) z_t^+ P_t}$ denotes the real, stationarized (*per capita*) value of the net foreign asset position. $\Phi(\cdot)$ is assumed to be a negative function of $\overline{a}_t$, with the following interpretation. If $\overline{a}_t < 0$, so that Sweden is a net borrower on the international financial market in period t, then $\Phi(\cdot)$ is more likely to take on a positive value. In this case, $\Phi(\cdot)$ represents a premium that Swedish households will be charged over and above the international risk free, gross interest rate $R_{F,t}$. If $\overline{a}_t > 0$, then Sweden is a net lender and the claims on Foreign bonds owned by Swedish households are more likely to pay a return that is lower than the international rate. See Benigno (2009) for an early application of a similar functional form with this interpretation. The second argument of $\Phi(\cdot)$, $E_t\left(s_{t+1}s_t\right) = E_t\left(\frac{S_{t+1}}{S_t}\frac{S_t}{S_{t-1}}\right)$, is due to Adolfson et al. (2008). A positive value of $E_t\left(s_{t+1}s_t\right)$ implies a lower value of $\Phi(\cdot)$, *ceteris paribus*. The motivation for including this second argument is purely empirical, as it allows the model to reproduce the observed negative correlation between the risk premium and the expected exchange rate depreciation. Adolfson et al. (2008) offers a possible justification for this specification, namely that domestic investors are more likely to accept a lower expected return on their international bond portfolio if the exchange rate is easier to predict. $\widetilde{\phi}_t$ represents an exogenous shock that will absorb any residual movements in the external risk premium.

The purchase by Swedish households of Foreign bonds create a debit recording in Sweden's balance of payments, the total value of which is $\frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)}$. Part or all of this purchase may be financed by the settlement of international bonds that were acquired in the previous period $S_t B_t^{FH}$. The total or net debit recording arising from financial transactions is therefore:

$$\frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)} - S_t B_t^{FH}.$$

We now turn to the payments that arise from international trade in goods. The value of Sweden's net exports in period t is given by $S_t P_t^X X_t - S_t P_{F,t} M_t^{xe} - S_t P_{F,t}^{C,e} M_t^e$, and represents a credit recording in the current account of Sweden.[27] In equilibrium, the total value of the credit recording from Swedish net exports must be balanced by a debt recording of equal value, arising from the net value of all transactions in the international bond market:

$$S_t P_t^X X_t - S_t P_{F,t} M_t^{xe} - S_t P_{F,t}^{C,e} M_t^e = \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)} - S_t B_t^{FH}. \tag{108}$$

Using the definition of net foreign assets, A_t, to substitute for $S_t B_{t+1}^{FH}$ and $S_{t-1} B_t^{FH}$, this relationship may alternatively be written as:

$$A_t - A_{t-1} = S_t P_t^X X_t - S_t P_{F,t} M_t^{xe} - S_t P_{F,t}^{C,e} M_t^e + \left[\Phi\left(\overline{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right)\zeta_{t-1} R_{F,t-1}\frac{S_t}{S_{t-1}} - 1\right] A_{t-1}. \tag{109}$$

The right-hand-side terms in Equation (109) represent the value of Swedish net exports plus the net return between periods t and $(t-1)$, in Swedish currency, on the net foreign asset position of Sweden. The sum of receipts from net exports and net returns on the international investment position is equal to the change in the international investment position, i.e. the change in Sweden net foreign assets.

[25] Since all these bonds mature one period after they are issued, A_t will consist of the total value of all outstanding Foreign bonds at the end of period t. Note, however, that Swedish households can both save and borrow in Foreign bonds, implying that B_{t+1}^{FH} may be either a positive or a negative number. If $B_{t+1}^{FH} < 0$, the period t aggregate net foreign asset position of Sweden is negative.

[26] For an explicit statement of the functional form, see Section (2.10).

[27] Recall from Section (2.4.2) in the main text that the prices of all traded goods are assumed to be set in the currency of the importing country, so called local currency pricing.

2.8 Shock processes

In this section, the shock processes are defined. They are all written in log-linear form, where the variable $\hat{x}_t$ is the log-deviation of the variable x_t from it's steady-state value. The innovations are denoted $\epsilon_{x,t}$ where x is the process in question. In addition, some processes are assumed to also have MA-term. There are three types of shocks, global shocks that have an equal impact on both Sweden and Foreign, domestic shocks that affects only Sweden, and Foreign shocks that affect Foreign directly, but leads to spill-over effects on the Swedish economy.

2.9 International spillovers and correlated shocks

While it is well documented empirically in the literature that global consumption and investment co-moves, and also that Swedish economy is dependent on global economic developments, it is difficult to generate cross-country spillovers in a standard small open economy DSGE model, as shown in Justiniano and Preston (2010). To overcome this problem and let model be able to get co-movements between global variables and co-movements between Foreign and Swedish variables in the data, we allow correlated shock structure in the model. The private bond risk premium shock, ζ_t, the utility of consumption shock ζ_t^c, the investment efficiency shock Υ_t and the stationary technology shock ε_t are assumed to be correlated with the equivalent Foreign shock process, so that innovations to the Foreign shock process affects also the Swedish shock process. Finally the Foreign utility of consumption $\zeta_{F,t}^c$ and the Foreign marginal investment efficiency $\Upsilon_{F,t}$ are also allowed to be correlated.[28] Below, we illustrate simply how we model shock correlations, see Corbo and Strid (2020) for a thorough discussion about the interpretation of shock correlations.

Let $x_{1,t}$ a Foreign shock and $x_{2,t}$ a domestic shock. The correlation structure between these shocks is built into the shock processes as the following:

$$x_{1,t} = \rho_1 x_{1,t-1} + \epsilon_{1,t}$$
$$x_{2,t} = \rho_{2,1} x_{1,t} + \rho_2 x_{2,t-1} + \epsilon_{2,t} \tag{110}$$

where ρ_1 and ρ_2 are shock persistence parameters, and $\rho_{2,1}$ is the paramter that determines the correlation between shocks, and we estimate these $\rho_{2,1}$ parameters in the model. Also note that $\epsilon_{1,t} = s\sigma_1\tilde{\epsilon}_{1,t}$ and $\epsilon_{2,t} = s\sigma_2\tilde{\epsilon}_{2,t}$ are assumed to be independent, where s is the scaling parameter, sσ_1 and sσ_2 are the shock standard deviations, and $\tilde{\epsilon}$s represent the independent and standard normally distributed innovations.

The implied estimated correlation coefficient between shocks can be found by using the parameter $\rho_{2,1}$, shock standard deviations and shock persistence parameters.

2.9.1 Global exogenous shocks

The shock to the neutral interest rate is the only global shock process in the model. However, in the estimation we assume $z_t = z_{F,t}$, thus treating Foreign labor-augmenting shock as a global shock.

$$\hat{\mu}_{z_F,t} = \rho_{\mu_{z_F}}\, \hat{\mu}_{z_F,t-1} + \epsilon_{\mu_{z_F},t} \tag{111}$$

$$\hat{z}_t^R = \rho_{z^R}\, \hat{z}_{t-1}^R + \epsilon_{z^R,t} + \theta_{z^R}\epsilon_{z^R,t-1} \tag{112}$$

2.9.2 Swedish exogenous shocks

Except for the monetary policy shock and the fiscal shocks, which are defined in previous sections, the Swedish economy shocks are

$$\hat{\beta}_t^r = \rho_\beta \hat{\beta}_{t-1}^r + \epsilon_t^\beta \tag{113}$$
$$\hat{\zeta}_t = corr_\zeta \hat{\zeta}_{F,t} + \rho_\zeta\, \hat{\zeta}_{t-1} + \epsilon_t^\zeta \tag{114}$$
$$\hat{\zeta}_t^c = corr_{\zeta^c} \hat{\zeta}_{F,t}^c + \rho_{\zeta^c}\, \hat{\zeta}_{t-1}^c + \epsilon_t^{\zeta^c} \tag{115}$$
$$\hat{\tilde{\phi}}_t = \rho_{\tilde{\phi}}\, \hat{\tilde{\phi}}_{t-1} + \epsilon_t^{\tilde{\phi}} \tag{116}$$
$$\hat{\zeta}_t^n = \rho_{\zeta^n} \hat{\zeta}_{t-1}^n + \epsilon_t^{\zeta^n} \tag{117}$$
$$\hat{\lambda}_t^W = \rho_{\lambda^W} \hat{\lambda}_{t-1}^W + \epsilon_t^{\lambda^W} \tag{118}$$
$$\hat{\varepsilon}_t = corr_\varepsilon \hat{\varepsilon}_{F,t} \rho_\varepsilon \hat{\varepsilon}_{t-1} + \epsilon_t \tag{119}$$
$$\hat{\Upsilon}_t = corr_\Upsilon \hat{\Upsilon}_{F,t} + \rho_\Upsilon \hat{\Upsilon}_{t-1} + \epsilon_t^\Upsilon \tag{120}$$
$$\hat{\lambda}_t = \rho_\lambda \hat{\lambda}_{t-1} + \epsilon_t^\lambda \tag{121}$$
$$\hat{\lambda}_t^X = \rho_{\lambda^X} \hat{\lambda}_{t-1}^X + \epsilon_t^{\lambda^X} \tag{122}$$

[28] We start the estimation process by allowing a large set of correlated shocks but keep only the ones that have well identified and contribute to marginal likelihood.

$$\hat{\lambda}_t^{M,C,xe} = \rho_{\lambda^{M,C,xe}} \hat{\lambda}_{t-1}^{M,C,xe} + \epsilon_t^{\lambda^{M,C,xe}} \tag{123}$$
$$\hat{\lambda}_t^{M,I} = \rho_{\lambda^{M,I}} \hat{\lambda}_{t-1}^{M,I} + \epsilon_t^{\lambda^{M,I}} \tag{124}$$
$$\hat{\lambda}_t^{M,X} = \rho_{\lambda^{M,X}} \hat{\lambda}_{t-1}^{M,X} + \epsilon_t^{\lambda^{M,X}} \tag{125}$$

$$\hat{p}_t^{D,C,e} = \rho_{p^{D,C,e}} \hat{p}_{t-1}^{D,C,e} + \epsilon_t^{p^{D,C,e}} \tag{126}$$

$$\hat{\Pi}_t^{trend} = \rho_{\Pi^{trend}} \hat{\Pi}_{t-1}^{trend} + \epsilon_t^{\Pi^{trend}} \tag{127}$$

$$\hat{\mu}_{\gamma,t} = \rho_{\mu_\gamma} \, \hat{\mu}_{\gamma,t-1} + \epsilon_{\mu_\gamma,t} \tag{128}$$

2.9.3 Foreign exogenous shocks

The Foreign shocks, except for the monetary policy shock which is defined in a previous section, are

$$\hat{\beta}_{F,t}^r = \rho_{\beta_F} \hat{\beta}_{F,t-1}^r + \epsilon_{F,t}^{\beta} \tag{129}$$
$$\hat{\zeta}_{F,t} = \rho_{\zeta_F} \, \hat{\zeta}_{F,t-1} + \epsilon_{F,t}^{\zeta} \tag{130}$$
$$\hat{\zeta}_{F,t}^c = corr_{\zeta_F^c,\Upsilon} \hat{\Upsilon}_{F,t} + \rho_{\zeta_F} \, \hat{\zeta}_{F,t-1}^c + \epsilon_{F,t}^{\zeta^c} \tag{131}$$
$$\hat{\zeta}_{F,t}^n = \rho_{\zeta_F^n} \, \hat{\zeta}_{F,t-1}^n + \epsilon_{F,t}^{\zeta^n} \tag{132}$$
$$\hat{\varepsilon}_{F,t} = \rho_{\varepsilon_F} \, \hat{\varepsilon}_{F,t-1} + \epsilon_{F,t} \tag{133}$$
$$\hat{\Upsilon}_{F,t} = \rho_{\Upsilon_F} \, \hat{\Upsilon}_{F,t-1} + \epsilon_{F,t}^{\Upsilon} \tag{134}$$
$$\hat{\lambda}_{F,t} = \rho_{\lambda_F} \hat{\lambda}_{F,t-1} + \epsilon_{F,t}^{\lambda} \tag{135}$$
$$\hat{p}_{F,t}^{C,e} = \rho_{p_F^{D,C,e}} \hat{p}_{F,t-1}^{D,C,e} + \epsilon_{F,t}^{p^{D,C,e}} \tag{136}$$

$$\hat{\Pi}_{F,t}^{trend} = \rho_{\Pi_F^{trend}} \hat{\Pi}_{F,t-1}^{trend} + \epsilon_t^{\Pi_F^{trend}} \tag{137}$$

$$\hat{g}_{F,t} = \rho_{g_F} \hat{g}_{F,t-1} + \epsilon_t^{g_F} \tag{138}$$

$$\hat{\mu}_{\gamma_F,t} = \rho_{\mu_{\gamma_F}} \, \hat{\mu}_{\gamma_F,t-1} + \epsilon_{\mu_{\gamma_F},t} \tag{139}$$

2.10 Functional forms

The utility function is chosen so as to be consistent with a balanced growth path, and is given by

$$u(\tilde{C}_{h,t} - \rho_h \tilde{C}_{t-1}) = \ln(\tilde{C}_{h,t} - \rho_h \tilde{C}_{t-1}). \tag{140}$$

The disutility of labor function for households in Sweden including endogenous shifter and weighting parameter is given by

$$\nu(n_t) = \Theta_t^n A_n \frac{N_t^{1+\eta}}{1+\eta}. \tag{141}$$

The disutility of labor function in foreign economy is standard in the DSGE literature, and the same as in for example Adolfson et al. (2005). It is given by

$$\nu(n_{f,t}) = \frac{N_{f,t}^{1+\eta}}{1+\eta}. \tag{142}$$

The external risk premium function is as in Adolfson et al. (2008) and Corbo and Strid (2020), and is given by

$$\Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right) = e^{-\widetilde{\phi}_a(\overline{a}_t - \overline{a}) - \widetilde{\phi}_s E_t\left[s_{t+1}s_{t-1}\right] + \widetilde{\phi}_t}.$$

The investment adjustment cost function $F(I_t, I_{t-1})$ is taken from Christiano, Eichenbaum, and Evans (2005a) and is given by

$$F(I_t, I_{t-1}) = \left[1 - \widetilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.$$

where the functional form of $\widetilde{S}$ is defined as in Adolfson et al. (2005):

$$\widetilde{S}\left(\frac{I_t}{I_{t-1}}\right) = \frac{1}{2}\left[e^{\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - \mu_z + \mu_\gamma\right)} + e^{-\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - \mu_z + \mu_\gamma\right)} - 2\right].$$

3 Model parametrization

In this section, we describe how the model parameter values are set. For this, we aim to match the Swedish economy at a quarterly frequency between 1995Q1-2019Q4. Parameters are either calibrated or estimated. A general strategy for selecting which parameters to calibrate or estimate is to calibrate parameters that determine the steady state, while estimating parameters that don't affect the steady state and only determine the model dynamics, e.g. persistence of shocks, shock standard deviations, etc.[29] First, we calibrate the steady state and a number of model structural parameters and then we estimate the remaining parameters that are not calibrated.

3.1 Calibration of steady state and structural parameters

We calibrate the steady state by matching the sample average of series or aggregate great ratios for the period between 1995Q1-2019Q4, and by incorporating our assumptions into the model.

3.1.1 Great ratios and balanced growth path

The steady-state great ratios in SELMA are based on data from Statistic Sweden (SCB) for the period 1995Q1-2019Q4. Table 1 provides the average great ratios over the sample period and the corresponding ratios in the steady state calibration. The private investment-to-output ratio $p^I\overline{I}/\overline{y}$ is set to 0.184, and is calculated as the ratio of nominal private investment (including inventories) to nominal market price GDP. For exports-to-output ratio $p^X\overline{x}/\overline{y}$ and the imports-to-output ratio $p^M\overline{m}/\overline{y}$, we take nominal exports over nominal market price GDP. Note that we equalize the exports and imports-to-output ratio in order to match the assumption of balanced trade in the steady state. Thus, both the exports and imports-to-output ratios are calibrated to 0.432.

The government consumption-to-output ratio $\overline{g}/\overline{y}$ is calculated as the ratio of nominal government consumption-to-nominal market price GDP, which is 0.254. The government investment to output ratio $\overline{I}^G/\overline{y}$ is calibrated to match the ratio of nominal government investment to nominal market price GDP of 0.041.[30] Private consumption to output ratio $p^C\overline{c}/\overline{y}$, which equals 0.521, is a residual which can be obtained by using the following expression : $p^C\overline{c}/\overline{y} = 1 - p^X\overline{x}/\overline{y} + p^M\overline{m}/\overline{y} - p^I\overline{I}/\overline{y} - \overline{g}/\overline{y} - \overline{I}^G/\overline{y}$.[31] The aggregate transfers to output ratio $\overline{tr}^{agg}/\overline{y}$ is calibrated to match the ratio of nominal public transfers excluding pension payments-to-nominal market price GDP of 0.101.

Table 1 also shows the calibrated values of the parameters pertaining to the Swedish fiscal policy framework. Based on the Swedish fiscal policy framework, the surplus target is 1/3 of a percent of GDP. Thus, the surplus to output ratio target $\overline{surp}^{Target}/\overline{y}$ is set to 1/3*0.01.

As Table 3 shows, the growth rate of labor augmenting technology μ_z is set to 1.003 which implies that the annualized growth rate of labor augmenting technology is 1.3 percent per year, which matches the per capita growth rate of Foreign economy over the sample period. The average annualized per capita investment growth rate in Sweden over the sample period is 4.0 percent, which is significantly higher than the per capita GDP growth rate of 1.8 percent. Motivated by this large difference, the gross investment-specific technological growth rate in Sweden, μ_γ is calibrated to 1.005, which is consistent with the difference between investment growth and GDP growth. Note that the composite technological growth rate, μ_{z+} can be defined as a function of the growth rates of labor augmenting technology, μ_z and investment-specific technology, μ_γ:$\mu_{z+} = \mu_z(\mu_\gamma)^{\frac{\alpha}{1-\alpha}}$. Thus, the Swedish economy grows at 1.8 percent and Swedish investment grows at 4.0 percent per year in per capita terms along the balanced growth path.[32] On the other hand, the Foreign economy grows only with labor augmenting technology at 1.3 percent per year along the balance growth path. Finally, the steady state inflation rate is set to 2 percent per year in both Sweden and Foreign in line with the Riksbank's and the foreign central banks' inflation targets even though the average inflation rate is lower than 2 percent over the sample period.

3.1.2 Labor market aggregates

Table 2 shows steady state labor market aggregates. The rates of labor force participation and employment are based on their potential values in the NIER database over the period 1995Q1-2019Q4. The labor force participation rate l is calculated as the period average of potential labor force over population aged 15-74, which

[29]This does not necessarily mean that we estimate all parameters that solely affect the model dynamics. In some cases, although initially we aim to estimate certain parameters in the beginning of estimation process, we choose to calibrate them instead. This is mostly due to weak identification or due to their negligible effects on marginal likelihood. This criterion is fairly standard in estimating DSGE models.

[30]Government investment excludes military spending items.

[31]Current account balance is positive on average over the sample period, thus consumption to GDP ratio is overstated in the model.

[32]Investment-to-output ratio is constant in the balanced growth path due to decrease in relative prices of investment in the rate of technology. One can argue that investment is part of GDP, and in the far limit, GDP in real terms will consist of only investment. Although we find these critics of our assumptions fair, we still believe that our technological growth assumptions would be a good way of aligning theoretical assumptions with sample data properties, and also they are in line with earlier studies, e.g. Greenwood, Hercowitz, and Krusell (1997).

is 0.714. Similarly, employment rate n is calculated as the average of potential employment over population aged 15-74 for the same period, which is 0.665. We define the steady-state unemployment rate as the average potential unemployment rate calculated with the average potential labor force participation and the average potential employment over the sample period, by $(l-n)/l$ which is 0.069.

3.1.3 Household sector parameters

Table 4 shows the values of the calibrated household parameters. The household discount factors β and β_F in Sweden and in the Foreign economy are set to 0.999. This, given that the steady state inflation is 2 percent and the steady state composite technology growth rate is 1.8 percent annualized terms, implies that the steady state nominal net interest rate (or the monetary policy rate) is 4.3 percent in Sweden and 3.7 percent in Foreign in annualized terms, given by $R = \frac{\mu_z + \Pi^C}{\beta}$ and $R_F = \frac{\mu_{z_F} + \Pi_F^C}{\beta_F}$, in which all variables are represented in quarterly gross terms.[33] Our model's theoretical set up, specifically the UIP condition, requires equal policy rates for a steady state to exist. Therefore, we assume a constant negative steady state risk premium on Swedish bonds over Foreign bonds which allows for a model equilibrium, where households face the same level of steady state interest rate in Foreign and Sweden. The parameter associated with labor disutility A_n is a function of the marginal utility of consumption, real wage, labor supply and wage income tax in steady state. The exact formulation can be found in Appendix I.

We follow the calibration strategy in Coenen, Straub, and Trabandt (2013) by choosing the share of private consumption in the composite consumption α_G such that the marginal utility of private consumption equals the marginal utility of government consumption. Thus, α_G is calibrated to 0.66.

We set the share of aggregate transfers going to Non-Ricardians in steady state, ϖ_{ss} and the share of aggregate transfers going to Non-Ricardians off steady state ϖ_{dyn} to 0.5. Thus, we assume the aggregate transfers goes toward Ricardian and Non-Ricardian households are equal.[34]

3.1.4 Price and wage markups

Table 5 presents the values for the steady state markups of firms. For intermediate good firms and import goods firms to 1.2, a standard value in the DSGE literature. However, following Corbo and Strid (2020), Swedish export goods producing firms' markup is calibrated to a lower value to 1.05, in order to avoid double markup on these goods. The steady state wage markup can be defined as $\lambda^W = (\frac{l}{n})^\eta$. Given the steady state values of labor force participation l and employment n, the value of η determines the wage markup. Since we estimate the Frisch elasticity parameter η, wage markup is implicitly estimated. The implicit estimated wage markup is approximately 1.3.

3.1.5 Elasticities of substitution between imported and domestic good and other trade parameters

The elasticities of substitution between imported intermediate goods and domestically produced intermediate goods for different sectors are mostly estimated. Table 6 shows the values of calibrated parameters. The elasticity of substitution between domestic and Foreign energy, $\nu_{C,e}$, is set to 0.5 as in Corbo and Strid (2020), which implies a low substitutability between Foreign and Swedish energy goods.[35] The elasticities of substitution between energy and non-energy consumption in the creation of consumption goods, ν_c and $\nu_{F,c}$, are set to 0.5 in both Sweden and Foreign. The parameter governing the share of non-energy in total consumption in Sweden and Foreign, ϑ^C and $\vartheta^{C,F}$, are set to match the shares of energy consumption to total private consumption in data. These shares are 0.075 in Sweden and 0.09 in Foreign.

The Swedish elasticity of substitution between imported and domestically produced intermediate goods for export goods production ν_x is set to 1.53, which is the estimated value for Sweden in Corbo and Strid (2020).[36]

The Swedish elasticity of substitution between imported goods and domestically produced intermediate goods for non-energy consumption production, $\nu_{C,xe}$, the elasticity of substitution between imported and domestically produced intermediate goods for investment goods production, ν_I, and the elasticity of substitution between imported and domestic consumption goods in Foreign, ν_F which captures the sensitivity of Swedish exports to Foreign import (from Sweden) prices, are all estimated.[37]

The parameter determining the weight of consumption goods and investment goods in the export demand function, ω_C^X is also estimated.

[33] For example, annualized 2 percent is equivalent to 1.005 in quarterly gross terms.

[34] Model users can choose different values for ϖ_{dyn} to study different policy options.

[35] Sweden imports crude oil and petroleum products for fuel consumption but domestically produces electricity with nuclear and hydro power for heating. Thus, imported energy goods are not good substitutes for domestically produce energy goods.

[36] We attempted to estimate this parameter but chose to calibrate it due to poor identification.

[37] Since the Foreign economy behaves as a closed economy we do not need to specify value for the elasticity of substitution between imported and domestic inputs in Foreign, $\nu_{F,x}$.

3.1.6 Capital and investment parameters

Table 7 shows parameters that are associated with capital and investment. We calibrate the capital share in production α to be 0.24, and the private capital depreciation rate, δ to be 0.017, in order to match the average investment to output ratio in the sample. The public capital depreciation rate δ_G is also calibrated to be 0.017 to simplify the interpretation of the parameter for the share of public capital in composite capital under efficient allocation, α_K.[38] This value is solved numerically such that the steady state conditions of the composite capital function and the private capital accumulation function hold.[39] In SELMA, the government provides public capital to intermediate good producers. Hence, the elasticity of substitution between private and public capital υ_K determines the degree of complementary between private and government investment. The parameter α_K represents the weight of private capital in the composite capital function. We estimate υ_K and calibrate α_K. We follow the calibration strategy in Coenen, Straub, and Trabandt (2013) by choosing a value for α_K such that the marginal product of private capital equals the marginal product of public capital, which implies that α_K is calibrated to 0.82.

The Foreign capital share α_F in production is calibrated to be 0.21, and the depreciation rate δ_F is calibrated to be 0.016, in order to match the average investment-output ratio.

3.1.7 Tax rates

Table 8 reports the steady state tax rates. The tax rates are in general calculated by dividing the tax income by the tax base. All data series used are nominal, and the averages are taken for the sample period 1995Q1-2019Q4. For the labor income tax rate, τ^W, the tax revenue is calculated as the municipal and regional income tax plus the state income tax minus the earned income tax credit.[40] The municipal and regional income tax is however paid on both transfers and wages in Sweden. To estimate how much of the municipal and regional income tax is paid on wages, the tax income has been multiplied by the wage sum and divided by the sum of wages and transfers. The tax base for τ^W is given by the wage sum. For the transfer tax, τ^{TR}, the tax revenue is calculated as the municipal and regional income tax multiplied by the transfers divided by the sum of transfers and wages (to get the share of tax revenue paid on transfers). The tax base for τ^{TR} is given by total public transfers. For the capital income tax rate τ^K, we use paid corporate tax as the tax revenue, and use the private sector net surplus excluding the net surplus of small houses as the tax base. For the social security contributions, the tax revenue is given by the paid employee labor taxes plus the pension income contributions paid for employees. The tax base for τ^{SSC} is the wage sum. The consumption tax, τ^C, is calculated by dividing tax revenue from consumption by the difference between household consumption and consumption tax revenue.[41] The investment tax credit τ^I is set to zero. To close the government budget constraint, a steady-state value of the lump-sum tax needs to be introduced as well. This is set as a residual so that the government structural surplus is at its target level in steady state.

3.1.8 Average maturity of government bonds

In SELMA, we allow for the government bonds to have a stochastic maturity. The government issues bonds that mature with probability α_B in a given period. Note that if α_B is one, then we have a one-period government bond as in the standard DSGE framework. In our current framework, we allow α_B to be less than one, thus allowing for long-term government bonds. We set α_B to match the average maturity of Swedish government bonds of 4 years.[42] Thus, the probability that government debt will mature each period α_B is set to 0.0625.

[38]Efficient allocation of public and private capital requires $\frac{\alpha_K}{1-\alpha_K}\frac{\delta^{1-\upsilon_K}}{(\delta^G)^{1-\upsilon_K}} = \frac{I}{I^G}$, when we assume public and private capital depreciation rates to be equal, α_K will determine the equilibrium ratios of government-private capital and investment.

[39]This method is required due to the inclusion of public capital to the composite capital which is used for the production of intermediate goods.

[40]The earned income tax credit is calculated as total income tax credit minus the credit for public pensions.

[41]It is important to note here that these are all goods-related taxes, not only the ones paid for by households. This means that the average tax rate implied by this revenue is higher than the one households actually pays on average.

[42]According to the Swedish National Debt Office Statistics the average maturity of debt (nominal krona debt) is close to 5 years in between 2001-2019. But we target a lower value, which is 4, to incorporate the years with low maturity of debt, 1995-2000, into the sample average.

Table 1: Calibration: Great ratios

Symbol	Description	Data (or Target)	Steady state
$p^I \overline{I}/\overline{y}$	Private investment to gdp ratio	0.184	0.184
$p^C \overline{c}^{agg}/\overline{y}$	Private consumption to gdp ratio	0.466	0.521
$p^X \overline{x}/\overline{y}$	Exports to gdp ratio	0.432	0.432
$p^M \overline{m}/\overline{y}$	Imports to gdp ratio	0.382	0.432
$\overline{g}/\overline{y}$	Government consumption to gdp ratio	0.254	0.254
$\overline{I}^G/\overline{y}$	Government investment to gdp ratio	0.041	0.041
$\overline{tr}^{agg}/\overline{y}$	Government transfers to gdp ratio	0.101	0.101
$\overline{surp}^{Target}/\overline{y}$	Government surplus to gdp ratio target	1/3*0.01	1/3*0.01
$\overline{b}^{Target}/\overline{y}$	Government debt to gdp ratio target	0.35*4	0.35*4
$p_F^C \overline{c}_F/\overline{y}_F$	Foreign private consumption to gdp ratio	0.55 (EA)	0.55
$p_F^I \overline{I}_F/\overline{y}_F$	Foreign private investment to gdp ratio	0.22 (EA)	0.22
$\overline{g}_F/\overline{y}_F$	Foreign government consumption to gdp ratio	-	0.23

Note: EA is Euro Area. Foreign government consumption to gdp ratio is a residual in the model, the data related to this ratio is not used in the estimation or calibration.

Table 2: Calibration: Labor market aggregates

Symbol	Description	Data (or Target)	Steady state
l	Labor force participation rate	0.714	0.714
n	Employment rate	0.665	0.665
un	Unemployment rate	0.069	0.069

Table 3: Calibration: Technological growth and inflation

Symbol	Description	Data	Steady state
μ_z	Gross growth rate of labor augmenting technology	1.003	1.003
μ_γ	Gross growth rate of investment specific technology	1.005	1.005
μ_{z^+}	Gross composite technological growth rate	1.005	1.005
$\mu_{z_F^+}$	Foreign composite technological growth rate	1.003	1.003
Π	Gross inflation rate of intermediate goods	1.004	1.005
Π^{trend}	Gross inflation trend	-	1.005
Π^C	Gross inflation rate of consumption goods	1.004	1.005
$\Pi^{M,C}$	Gross inflation rate of imported consumption goods	1.001	1.005
Π^X	Gross inflation rate of export goods	-	1.005
Π^W	Gross inflation rate of wages	1.008	1.01
Π_F	Foreign gross inflation rate of intermediate goods	1.004	1.005
Π_F^C	Foreign gross inflation rate of consumption goods	1.004	1.005
Π_F^W	Foreign gross inflation rate of wages	1.006	1.008
Π_F^X	Foreign gross inflation rate of export goods	-	1.005

Table 4: Calibration: Household sector parameters

Symbol	Description	Value
β	Household discount factor	0.999
β_F	Foreign household discount factor	0.999
α_G	Share of private consumption in the composite consumption	0.66
ϖ_{ss}	Share of aggregate transfers going to Non-Ricardians in steady state	0.5
ϖ_{dyn}	Share of aggregate transfers going to Non-Ricardians off steady state	0.5

[p]

Table 5: Calibration: Steady state values of markups

Symbol	Description	Value
λ	Intermediate good price markup	1.2
λ^X	Export price markup	1.05
λ^M	Imported good price markup (consumption, investment and export)	1.2
λ^W	Wage markup	1.3
λ_F	Foreign intermediate good price markup	1.2
λ_F^W	Foreign wage markup	1.6

Note: Steady state wage markup for Sweden is implicitly estimated when Frisch elasticity parameter, η is estimated.

Table 6: Calibration: Elasticities of substitution in production sector

Symbol	Description	Value
ν_C	Elasticity between non-energy and energy goods in consumption goods	0.5
$\nu_{C,e}$	Elasticity between domestic and imported goods in energy consumption	0.5
$\nu_{F,c}$	Elasticity between non-energy and energy goods in consumption goods in Foreign	0.5
ν_x	Elasticity between domestic and imported goods in exports goods	1.53

Table 7: Calibration: Capital and investment parameters

Symbol	Description	Value
α	Capital share in production	0.24
α_K	Share of private capital in composite capital function	0.82
δ	Private capital depreciation rate	0.017
δ_G	Public capital depreciation rate	0.017
α_F	Foreign capital share in production	0.21
δ_F	Foreign capital depreciation rate	0.016

Table 8: Calibration: Steady state level of tax rates

Symbol	Description	Value
τ^C	Consumption tax rate	0.341
τ^W	Labor income tax rate	0.286
τ^{SSC}	Social security contribution rate	0.306
τ^K	Capital income tax rate	0.169
τ^{TR}	Tax rate on transfers to households	0.279
τ^I	Investment tax credit	0
τ_F^W	Foreign labor income tax rate	0.286

Table 9: Swedish data used for the steady state calibration

Data series	Description and transformation	Frequency	Source	NIER's database folder/code
GDP	Current prices, SA	Quarterly	SCB	Fb/nbnpmpls
Potential GDP	At market prices, constant prices	Quarterly	NIER	Gappr/bnppot
Consumption	Current prices, SA	Quarterly	SCB	Fb/nkohls
Investment	Current prices, SA	Quarterly	SCB	Inv/nfinl
Exports	Current prices, SA	Quarterly	SCB	Fb/nexls
Imports	Current prices, SA	Quarterly	SCB	Fb/nimls
Public consumption	Current prices, SA	Quarterly	SCB	Fb/nkools
Public investment	Current prices, SA	Quarterly	SCB	Inv/nfiomls
Unemployment rate	in percent	Quarterly	SCB	Am/arali1574s
Potential employment		Quarterly	NIER	Gappr/asypot
Labor income		Quarterly	SCB	Loner/nlsls
Working age population	Interpolated to quarterly	Annual	SCB and NIER	Befpr/bef1574to
Gross debt	Interpolated to quarterly	Annual	NIER	Offpr/skuldo
Municipal income tax revenue	SA	Quarterly	NIER	Offkv/dshk
State income tax revenue	Interpolated to quarterly	Annual	SCB, ESV and NIER	Skattpr/statsk
State income tax reductions	Interpolated to quarterly	Annual	SCB, ESV and NIER	Skattpr/skred
Tax deduction for pensioners	SA	Quarterly	SCB and NIER	Offkv/eaho
Firms' pension contributions	SA	Quarterly	SCB and NIER	Offkv/pstsivlon

SCB:Statistics Sweden; SNMO:Swedish National Mediation Office; ESV:The Swedish National Financial Management Authority.

3.2 Estimation

The remaining model parameters that are not calibrated are estimated using Bayesian methods as described in An and Schorfheide (2007) and Herbst and Schorfheide (2015). A brief overview of these methods, which are fairly standard in the literature, can be found in Appendix J. All computations are done by using the Dynare 4.6.4 toolbox. We estimate model parameters in two steps. First, we estimate Foreign parameters by using only Foreign data and by treating the Foreign economy block as a separate closed economy DSGE model. Second, we estimate Swedish parameters taking the estimated values of Foreign parameters as given. This is a technically easier as compared to the joint estimation of Foreign and Swedish parameters.[43] This two-step estimation strategy, in which Foreign estimated parameters are not affected by Swedish data, is consistent with a modelling framework in which Sweden is treated as a small open economy with negligible effects on Foreign.

3.2.1 Description of data used

The model is estimated using 24 Swedish data series and 9 Foreign aggregated data series. The estimation sample period is 1995Q1-2019Q4, which is based on data availability as well as so that to include inflation targeting regime periods and to exclude the COVID-19 crisis period.

In this section, we describe the data used in estimation for Foreign and Sweden, respectively.

Foreign data: The Foreign sector data series used for estimation are 9 trade-weighted quarterly data series for KIX-6 (krona index-6) countries, including the euro area (19 countries), the US, the UK, Denmark, Japan and Norway.[44] These are the most important trading partners of Sweden representing around 90 percent of the total Swedish international trade. As observable variables for the estimation, we use annualized quarterly growth of per capita gross domestic product (GDP), consumption, investment and hours worked, annualized quarterly inflation, inflation excluding energy and wage inflation, monetary policy rate and credit spread.[45] The data series necessary to construct the observable variables for Foreign along with their respective data sources are summarized in Table 10.[46] The transformation of raw data series into the observable variables can be found in Appendix K.

[43] In other words, we obtain the conditional posterior distribution of domestic parameters. Obviously, the uncertainty in the estimated domestic parameters is smaller than in the estimation of domestic parameters and Foreign parameters jointly.

[44] We use time-varying trade weights.

[45] The per capita hours worked series is non-stationary based on the ADF test, hence we decided to use its growth rate to make it stationary.

[46] In the table, we also provide NIER's database (Mbserier) codes for each series at the far right column.

Table 10: Foreign data for estimation

Data series	Data title	Source	Frequency	NIER code
Euro area				
GDP	Euro Area 19, Eurostat, ESA 2010, Main GDP Aggregates, GDP and Main Components (Output, Expenditure and Income), Volumes, Gross Domestic Product, 2010 Reference Year, Calendar Adjusted, Constant Prices, SA, Chained, Market Prices, EUR	Eurostat	Quarterly	ea19gdp
Investment	Euro Area 19, ECB, Gross Fixed Capital Formation, Total Fixed Assets (Gross), Total - All Activities, Reference Sector: Total Economy, Counterpart Area: World (All Entities, Including Reference Area and IO), Calendar Adjusted, Constant Prices, SA, Chained, EUR	ECB	Quarterly	ecb_00169530
Working age pop.	Euro Area 17, Working-Age Population, Age 15-74, OECD Economic Outlook, Estimate, Calendar Adjusted, SA	OECD	Annual	ea19popa
	Euro Area 19, OECD MEI, Labour Force Survey - Quarterly Levels, Working Age Population, Aged 15-74, All Persons	OECD	Quarterly	ea19popq
Hours worked	Euro Area 19, OECD QNA, Employment by Industry - Domestic Concept, Employment, Total, Total, Hours Worked, SA	OECD	Quarterly	hoursea
HICP	Euro Area 19, Eurostat, HICP, All-Items HICP, 2015=100, Index	Eurostat	Monthly	ea19hicp
HICP excl energy	Euro Area 19, Eurostat, HICP, Overall Index Excluding Energy, 2015=100, Index	Eurostat	Monthly	ea19hicpee
Wages	Euro Area 19, OECD MEI, Unit Labour Costs, Early Estimate of Quarterly ULC Indicators, Labour Compensation per Unit of Labour Input, Total, SA, Index	OECD	Quarterly	ea19ulc
Credit rates	Euro Area, ECB, MFI Interest Rates, Non-Financial Corporations, Loans Other than Revolving Loans and Overdrafts, Convenience and Extended Credit Card Debt [A20-A2Z], Annualised Agreed Rate (AAR) / Narrowly Defined Effective Rate (NDER), All Maturities, Total, New Business, Currency Denominator: Euro	ECB	Monthly	ecb_00015235
Mon. policy rate	Euro Area, ECB, Financial Market Provider: ECB, Money Market, EONIA Rate, Historical Close, Average of Period	ECB	Monthly	ecb_00858122
Personal consump. expen.	Euro Area 19, Eurostat, ESA 2010, Main GDP Aggregates, GDP and Main Components (Output, Expenditure and Income), Volumes, Final Consumption Expenditure of Household and NPISH, 2010 Reference Year, Calendar Adjusted, Constant Prices, SA, Chained, EUR	Eurostat	Quarterly	ea19cons
US				
GDP	United States, Gross Domestic Product, Total, Constant Prices, SA, Chained, AR, USD	BEA	Quarterly	usnaac0169
Investment	United States, OECD QNA, Gross Domestic Product, Gross Domestic Product - Expenditure Approach, Gross Capital Formation, Gross Fixed Capital Formation, Constant Prices, SA, Chained, USD	OECD	Quarterly	usgfcf
Working age pop.	United States, OECD MEI, Labour Force Survey - Quarterly Levels, Working Age Population, Aged 15-74, All Persons, SA	OECD	Monthly	uspop
Hours worked	United States, Productivity, Costs and Hours Worked, Hours Worked, Aggregate Quarterly Hours, Nonfarm, Total, SA	BLS	Quarterly	uslama7604
CPI	United States, Consumer Price Index, All Urban Consumers, U.S. City Average, All Items, SA, Index	BLS	Monthly	uspric2156
CPI excl energy	United States, Consumer Price Index, All Urban Consumers, U.S. City Average, All Items Less Energy, SA, Index	BLS	Monthly	uspric2376
Wages	United States, OECD MEI, Unit Labour Costs, Early Estimate of Quarterly ULC Indicators, Labour Compensation per Unit of Labour Input, Total, SA, Index	OECD	Quarterly	usulc
Credit rates	Weighted-Average Effective Loan Rate for All Commercial and Industry Loans, All Commercial Banks, Not Seasonally Adjusted	FRED	Quarterly	
Mon. policy rate	United States, Policy Rates, Target Rates, Federal Funds Target Rate	FRB	Daily	usrate0001
Personal consump. expen.	United States, Expenditure Approach, Personal Consumption Expenditures, Total, Constant Prices, SA, Index	BEA	Monthly	usnaac0591

Table 10: Foreign data for estimation (continued)

Data series	Data title	Source	Frequency	NIER code
UK				
GDP	United Kingdom, Gross Domestic Product, At Market Prices, Constant Prices, SA, GBP	ONS	Quarterly	gbnaac00072
Investment	United Kingdom, Expenditure Approach, Domestic Expenditure, Gross Capital Formation, Fixed, at Market Prices, Constant Prices, SA, GBP	ONS	Quarterly	gbnaac00462
Working age pop.	United Kingdom, Working-Age Population, Age 15-74, OECD Economic Outlook, Estimate, Calendar Adjusted, SA	OECD	Annual	ukpopa
	United Kingdom, OECD MEI, Labour Force Survey - Quarterly Levels, Working Age Population, Aged 15-74, All Persons	OECD	Quarterly	ukpopq
Hours worked	United Kingdom, Productivity, Costs and Hours Worked, Total, Weekly, SA	ONS	Monthly	gblama00361
HICP	United Kingdom, Eurostat, HICP, All-Items HICP, 2015=100, Index	Eurostat	Monthly	ukhicp
HICP excl energy	United Kingdom, Eurostat, HICP, Overall Index Excluding Energy, 2015=100, Index	Eurostat	Monthly	ukhicpee
Wages	United Kingdom, OECD MEI, Unit Labour Costs, Early Estimate of Quarterly ULC Indicators, Labour Compensation per Unit of Labour Input, Total, SA, Index	OECD	Quarterly	ukulc
Mon. policy rate	United Kingdom, Policy Rates, Bank Rate	BoE	Daily	gbrate0001
Personal consump. expen.	United Kingdom, OECD QNA, Gross Domestic Product, Gross Domestic Product - Expenditure Approach, Final Consumption Expenditure, Private Final Consumption Expenditure, Total, OECD Reference Year, Constant Prices, SA, AR, GBP	OECD	Quarterly	ukcons
Denmark				
GDP	Denmark, Eurostat, ESA 2010, Main GDP Aggregates, GDP and Main Components (Output, Expenditure and Income), Volumes, Gross Domestic Product, 2010 Reference Year, Calendar Adjusted, Constant Prices, SA, Chained, Market Prices, DKK	Eurostat	Quarterly	dkgdp
Investment	Expenditure Approach, Gross Fixed Capital Formation, Total, Chained, Constant Prices, SA, DKK	Statistics Denmark	Quarterly	dknaac0946
Working age pop.	Denmark, Working-Age Population, Age 15-74, OECD Economic Outlook, Estimate, Calendar Adjusted, SA	OECD	Annual	dkpopa
	Denmark, OECD MEI, Labour Force Survey - Quarterly Levels, Working Age Population, Aged 15-74, All Persons, SA	OECD	Quarterly	dkpopq
Hours worked	Productivity, Costs and Hours Worked, Hours Worked, Total, Total Industry (incl. Non-Residents), SA	Statistics Denmark	Quarterly	dklama0415
HICP	Denmark, Harmonized CPI, Total, Index	Statistics Denmark	Monthly	dkpric0358
HICP excl energy	Denmark, Eurostat, HICP, Overall Index Excluding Energy, 2005=100, Index	Eurostat	Monthly	dkhicpee
Wages	Denmark, OECD MEI, Unit Labour Costs, Early Estimate of Quarterly ULC Indicators, Labour Compensation per Unit of Labour Input, Total, SA, Index	OECD	Quarterly	dkulc
Mon. policy rate	Denmark, Policy Rates, Lending Rate	CB. of Denmark	Daily	dkrate0001
Personal consump. expen.	Denmark, OECD QNA, Gross Domestic Product, Gross Domestic Product - Expenditure Approach, Final Consumption Expenditure, Private Final Consumption Expenditure, Total, Constant Prices, SA, Chained, DKK	OECD	Quarterly	dkcons

Table 10: Foreign data for estimation (continued)

Data series	Data title	Source	Frequency	NIER code
Norway				
GDP	Norway, Gross Domestic Product, Total, Constant Prices, SA, Market Prices, NOK	Statistics Norway	Quarterly	nonaac0182
Investment	Expenditure Approach, Gross Fixed Capital Formation, Total, Constant Prices, SA, NOK	Statistics Norway	Quarterly	nonaac0157
Working age pop.	Norway, Working-Age Population, Age 15-74, OECD Economic Outlook, Estimate, Calendar Adjusted, SA	OECD	Annual	nopopa
	Norway, Norway, OECD MEI, Labour Force Survey - Quarterly Levels, Working Age Population, Aged 15-74, All Persons	OECD	Quarterly	nopopq
Hours worked	Norway, OECD QNA, Employment by Industry - Domestic Concept, Employees, Total, Total, Hours Worked, SA	OECD	Quarterly	nohours
	Norway, OECD QNA, Employment by Industry - Domestic Concept, Self-Employed, Total, Total, Hours Worked, SA	OECD	Quarterly	nosemp
HICP	Norway, Harmonized CPI, Total, Index	Statistics Norway	Monthly	nopric0044
HICP excl energy	Norway, Eurostat, HICP, Overall Index Excluding Energy, 2005=100, Index	Eurostat	Monthly	nohicpee
Wages	Norway, OECD MEI, Unit Labour Costs, Early Estimate of Quarterly ULC Indicators, Labour Compensation per Unit of Labour Input, Total, SA, Index	OECD	Quarterly	noulc
Mon. policy rate	Norway, Policy Rates, Sight Deposit (Folio) Rate	CB. of Norway	Daily	norate0001
Personal consump. expen.	Norway, OECD QNA, Gross Domestic Product, Gross Domestic Product - Expenditure Approach, Final Consumption Expenditure, Private Final Consumption Expenditure, Total, Constant Prices, SA, Chained, NOK	OECD	Quarterly	nocons
Japan				
GDP	Japan, Gross Domestic Product, Total, Constant Prices, SA, AR, JPY	Japanese Cabinet Office (CaO)	Quarterly	jpnaac0004
Investment	Japan, Expenditure Approach, Gross Capital Formation, Gross Fixed Capital Formation, Total, Constant Prices, SA, AR, JPY	Japanese Cabinet Office	Quarterly	jpnaac2404
Working age pop.	Japan, OECD MEI, Labour Force Survey - Quarterly Levels, Working Age Population, Aged 15-74, All Persons	OECD	Monthly	jppop
Hours worked	Japan, Productivity, Costs and Hours Worked, Hours Worked, Total, Industries Covered, Establishments with 5 or More Employees	Japanese Ministry of Health, Labour and Welfare	Monthly	jplama0246
	Japan, Employment, Employed Persons, Total, National, Males and Females, SA	Japanese Statistics Bureau	Monthly	jplama0402
HICP	Japan, Consumer Price Index, Total, All Japan, Index	Japanese Statistics Bureau	Monthly	jppric0513
HICP excl energy	Japan, Consumer Price Index, Total, Excluding Fresh Food and Energy, All Japan, Index	Japanese Statistics Bureau	Monthly	jppric5237
Wages	Japan, OECD MEI, Unit Labour Costs, Early Estimate of Quarterly ULC Indicators, Labour Compensation per Unit of Labour Input, Total, SA, Index	OECD	Quarterly	jpulc
Mon. policy rate		Bank of Japan		jpys
Personal consump. expen.	Japan, OECD QNA, Gross Domestic Product, Gross Domestic Product - Expenditure Approach, Final Consumption Expenditure, Private Final Consumption Expenditure, Total, Constant Prices, SA, Chained, JPY	OECD	Quarterly	jpcons

Swedish data: This section describes the Swedish data used in the estimation of SELMA. We use 24 data series for Sweden in the estimation, which are summarized along with their NIER database sources in Table 11. The data series were retrieved from the NIER's internal database in December 2021. The transformations of all the raw series to make them compatible with model observation equations are explained in Appendix K.

Table 11: Swedish data for estimation

Data series	Description and transformation	Frequency	Source	NIER's database folder/code
GDP	Constant prices, SA, annualized per capita growth, in percent	Quarterly	SCB	Fb/nbnpmpfs
GDP gap	Deviation from potential, in percent	Quarterly	NIER	Gappr/gapp
Consumption	Constant prices, SA, annualized per capita growth, in percent	Quarterly	SCB	Fb/nkohfs
Investment	Constant prices, SA, annualized per capita growth, in percent	Quarterly	SCB	Inv/nfinfs
Exports	Constant prices, SA, annualized per capita growth, in percent	Quarterly	SCB	Fb/nexfs
Imports	Constant prices, SA, annualized per capita growth, in percent	Quarterly	SCB	Fb/nimfs
HICP inflation	SA, annualized, in percent	Quarterly	SCB	Kpi/kpifs
HICP excl. energy	SA, annualized, in percent	Quarterly	SCB	Kpi/kpifees
CPI Imp. cons. goods excl. energy infl.	SA, annualized, in percent	Quarterly	SCB	Kpi/kpiimpees
Monetary policy rate	in percent	Quarterly	Riksbank	Rantor/repo
Real exchange rate	growth, in percent	Quarterly	Macrobond, Riksbank and NIER	Vx/realkix6s
Wages	Annual change, in percent	Quarterly	SNMO, NIER	Loner/kltot
Employment gap	Deviation from potential, in percent	Quarterly	NIER	Gappr/asypot and Am/asy1574s
Unemployment rate	in percent	Quarterly	SCB	Am/arali1574s
Capacity utilization	in percent	Quarterly	SCB	Barforq/btvi104s
Corporate spread	Difference between lending rates to non-Financial corp and the policy rate, in perc. points	Monthly	SCB	Mbserier/sebank0008 and Rantor/repo
Public consumption	Constant prices, SA, annualized per capita growth, in percent	Quarterly	SCB	Fb/nkoofs
Public investment	Constant prices, SA, annualized per capita growth, in percent	Quarterly	SCB and NIER	Inv/nfiomfs
Transfers to households	Current prices, SA, ratio to potential GDP, first difference	Quarterly	NIER	Offkv/troh and Offkv/trah
Consumption tax	first difference	Quarterly	SCB and NIER	Offkv/pstsv
Firms' social security contributions	first difference	Quarterly	SCB	Offkv/fahainkpens
Structural savings	Interpolated to quarterly	Annual	NIER	Offpr/bbhp
Labor income tax	first difference	Quarterly	SCB and NIER	Offkv/troh, dshk, eaho; loner/nlsls
Transfers tax	first difference	Quarterly	SCB and NIER	Offkv/troh, dshk, eaho

SCB:Statistics Sweden; SNMO:Swedish National Mediation Office.

3.2.2 Data properties

In this section, we provide some stylized facts that we can observe from the data. The properties that we highlight serve, among others, two main purposes. First, they help us to understand how the model's theoretical assumptions are in line with data. Second, these properties can be used to assess model fit.

Foreign data properties: Figure 3 shows the Foreign data used in the estimation, where real variables are expressed as annualized quarterly per capita growth rates, and inflation variables are expressed as quarterly annualized rates. In each graph, we note the sample mean and the standard deviation of the series.

In the Foreign economy, the average growth of consumption and GDP are nearly the same, while consumption is less volatile than GDP. On the other hand, both the sample mean and the standard deviation of Foreign investment growth is higher than Foreign GDP growth.[47] The average Foreign corporate spread, which is calculated as the difference between the interest rates on bank loans to non-financial institutions and the policy rate, is around 1.8 percent and tends to be higher in the sample period after the global financial crisis.[48] See Appendix L on how we calculate the equilibrium corporate spread from the data to define the observation equations. Headline inflation and inflation excluding energy over the sample period are below the major foreign central banks' inflation target of 2 percent. Also, Foreign headline inflation is more volatile than inflation excluding energy. An important observation from the Foreign data is that the monetary policy rate is downward-sloping over the sample period. Also, within the sample period, wage growth before the financial crisis is higher than the one that observed after the financial crisis. These consistently below-target inflation figures require some extra attention regarding the modelling of inflation.

Table 12 provides contemporaneous correlations between Foreign variables. Foreign GDP growth is positively correlated with consumption growth, investment growth, hours worked growth and wage growth. In addition, Foreign GDP demonstrates a positive correlation with headline inflation and a negative correlation with inflation excluding energy, though both correlations are relatively weak. Growth of investment and consumption, are also positively correlated with hours worked growth and wage growth. Furthermore, inflation and inflation excluding energy are positively correlated with the wage growth rate and the monetary policy rate. The corporate spread is negatively correlated with all the data series in our sample.

Table 12: Contemporaneous correlations between Foreign data variables

	$\Delta Y_{F,t}^{obs}$	$\Delta C_{F,t}^{obs}$	$\Delta I_{F,t}^{obs}$	$\Pi_{F,t}^{C,xe,obs}$	$\Pi_{F,t}^{C,obs}$	$\Delta N_{F,t}^{obs}$	$R_{F,t}^{obs}$	$\zeta_{F,t}^{obs}$	$\Delta w_{F,t}^{obs}$
$\Delta Y_{F,t}^{obs}$	1.00								
$\Delta C_{F,t}^{obs}$	0.81	1.00							
$\Delta I_{F,t}^{obs}$	0.78	0.63	1.00						
$\Pi_{F,t}^{C,xe,obs}$	-0.20	-0.30	-0.10	1.00					
$\Pi_{F,t}^{C,obs}$	0.25	0.00	0.23	0.60	1.00				
$\Delta N_{F,t}^{obs}$	0.80	0.59	0.79	-0.12	0.19	1.00			
$R_{F,t}^{obs}$	-0.04	0.01	-0.13	0.55	0.36	-0.08	1.00		
$\zeta_{F,t}^{obs}$	-0.17	-0.29	-0.12	-0.41	-0.29	-0.14	-0.80	1.00	
$\Delta w_{F,t}^{obs}$	0.47	0.32	0.33	0.28	0.41	0.39	0.49	-0.51	1.00

Notes: Δ refers to growth rate.

Swedish data properties: Figure 4 shows the Swedish data used in the estimation, where real variables are expressed as annualized quarterly per capita growth rates, and inflation variables are expressed as quarterly annualized rates. The average GDP growth in the sample period is 1.8 percent. While the average consumption growth is close to the average GDP growth, the average private investment growth is notably higher at 4.0 percent. This observation motivates introducing an investment-specific technology in Sweden as the second factor driving the economic growth, additional to the global labor augmenting technology. Similar to private investment, the average growth of exports and imports are significantly higher than GDP growth, 4.4 and 4.0 percent, respectively. However, to keep modelling trends simple, we handle the discrepancy between the data and the model assumptions for export and import growth by introducing excess parameters in the corresponding observation equations, see Appendix L.[49] The average government consumption growth and government invest-

[47]Investment having a higher growth rate than GDP is not consistent with the theoretical assumption that there is one single technology driving the economic growth together with the assumption of balanced growth. Since the difference is relatively small when compared to Swedish data, we hesitate to introduce investment-specific technology in the Foreign economy, which would otherwise solve the inconsistency to some extent.

[48]Recall that in the model there is only one equilibrium rate for the corporate spreads. Having different average (or trend) equilibrium corporate spreads in different time horizons is not consistent with the model's assumptions.

[49]See also Section 3.2.4 for more details about how we incorporate excess parameters.

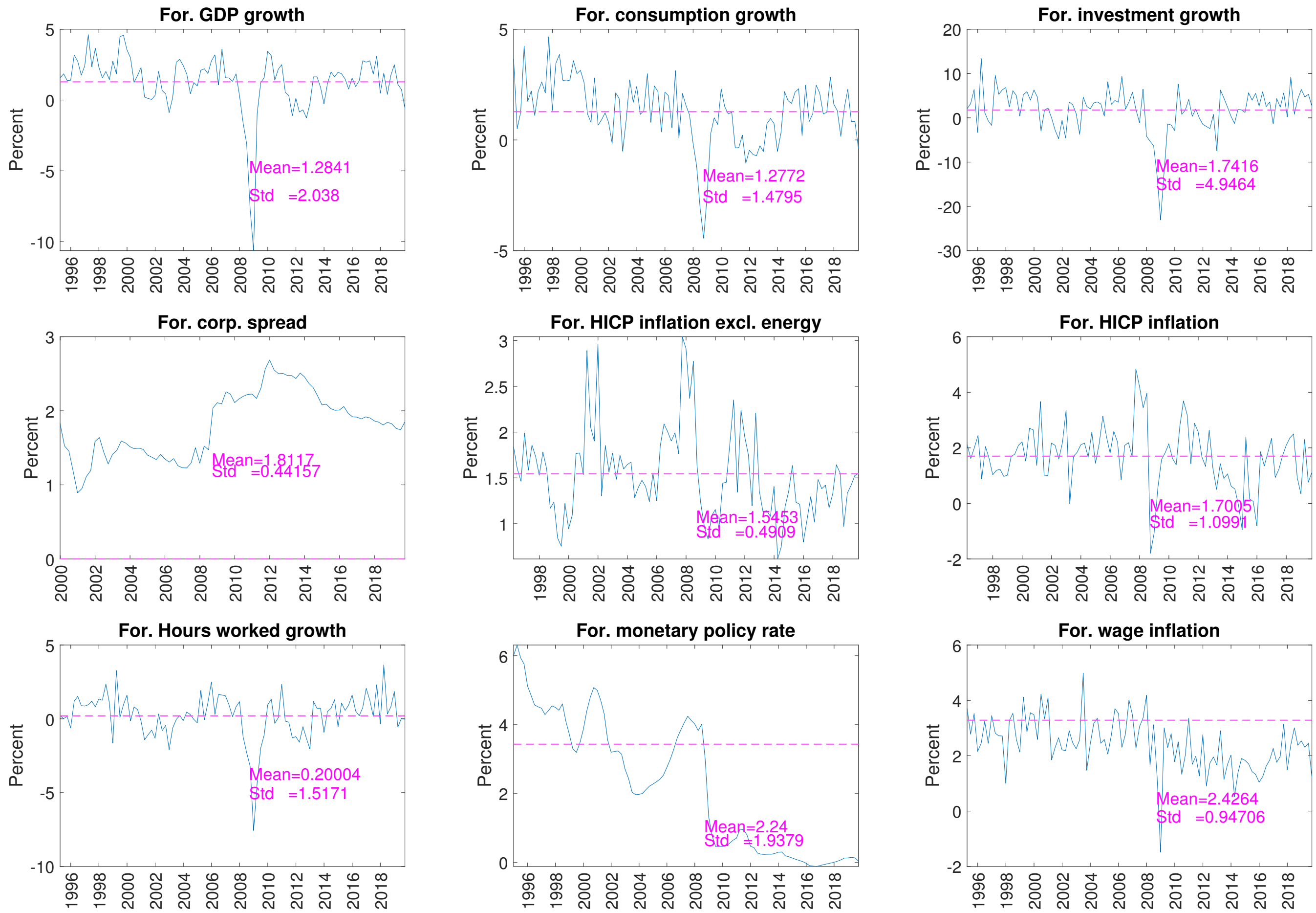


Figure 3: KIX-6 data

ment growth, 0.5 percent and 1.5 percent respectively, are both lower than GDP growth over the sample period. Again, it is difficult to reconcile data and model assumptions, where government and private investment have the same depreciation rates, and have constant shares to GDP ratios. For government investment, we introduce an excess parameter in the corresponding observation equation, see Appendix L. The average unemployment rate in the sample period is 7.6 percent. However, as the equilibrium unemployment rate we use the unemployment rate derived from NIER's potential labor force participation and potential employment data, which is on average 6.9 percent.[50] The average inflation and the average inflation excluding energy are below the Riksbank's 2 percent target. Both wage inflation and the monetary policy rate are downward sloping over the sample period. The average credit spread is 1.77 over the sample period, where the spread after financial crisis is higher than pre-crisis period. Transfers and all the tax rates except the consumption tax are downward sloping over the sample period, which probably reflect structural changes in fiscal policy to some extent. After 2006 elections in Sweden, significant changes in fiscal policy were implemented, among others, concerning transfers and social security contributions. To capture these structural changes we assume different equilibrium government transfers to GDP ratios and different social security contributions rates for pre-2006 vs. post-2010. In between these dates a smooth linear transition is assumed. For other taxes we use the first difference of the corresponding series as observables.

[50]Namely, the unemployment rate is above its equilibrium level on average over the sample period, which is in line with the negative sample mean of GDP gap, see Figure 4

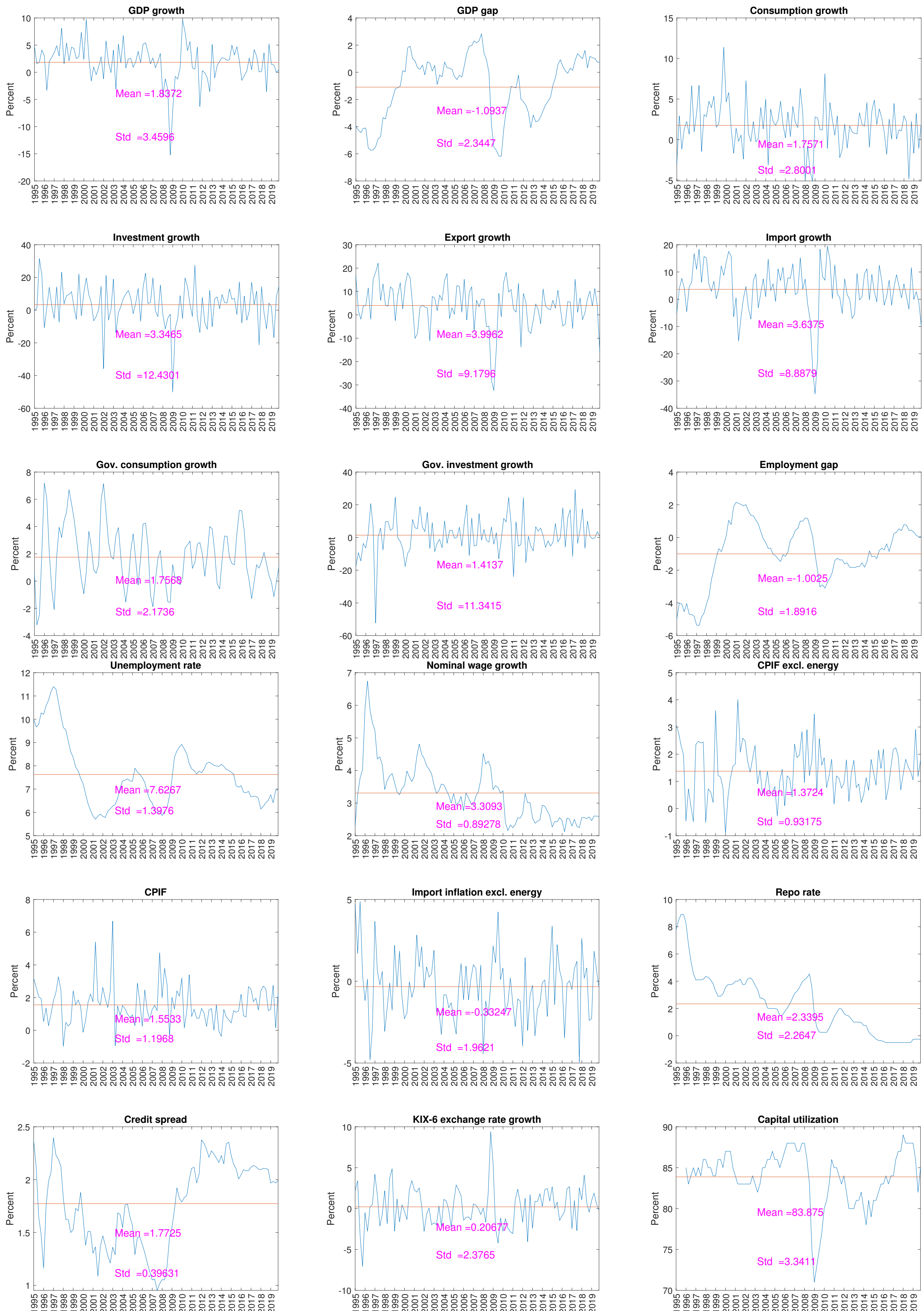


Figure 4: Swedish data used in estimation

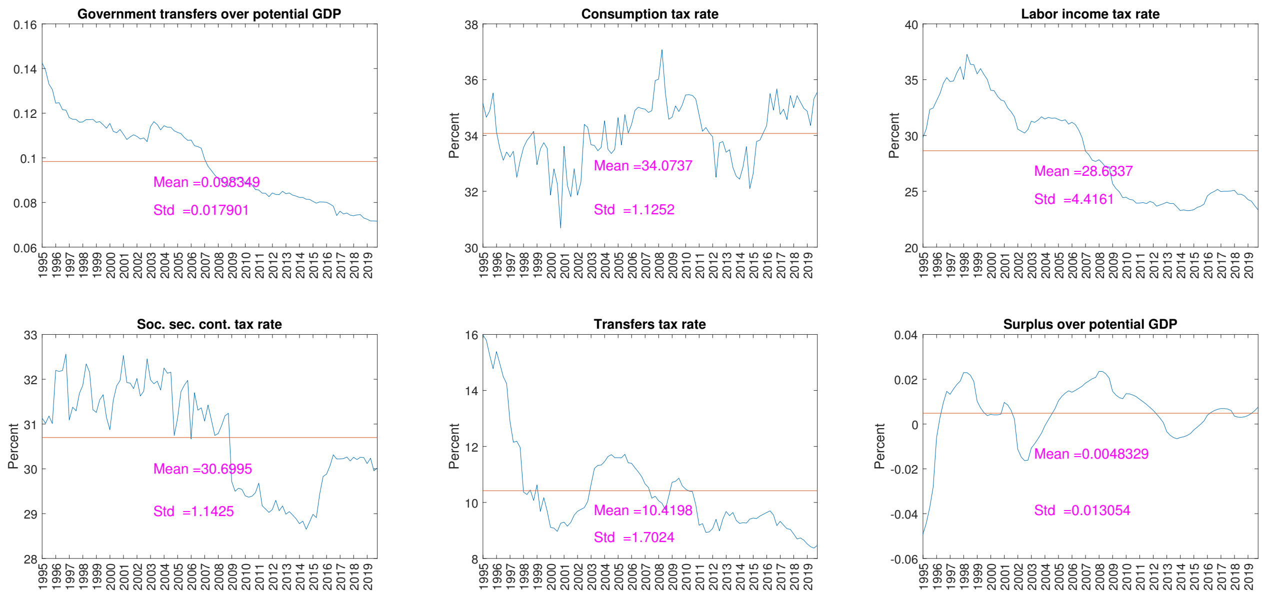


Figure 4: Swedish data used in estimation (continued)

Table 13 shows contemporaneous correlations for a subset of Swedish observable variables. GDP growth is positively correlated with the growth of its components, private consumption, private investment, exports and imports. Inflation doesn't have a strong correlation with any of the variables in the subset of the list of observables in the table. Wage inflation is strongly correlated with the monetary policy rate. Finally, the corporate spread is negatively correlated with the policy rate, wage inflation and the employment gap.[51]

We also provide a subset of cross-country contemporaneous correlations of variables in Sweden and Foreign in Table 14. Foreign GDP, consumption and investment are positively correlated with Swedish GDP, consumption and investment, respectively. These stylized facts, or in general, international spillovers to small open economy, are difficult to capture in small economy DSGE models as well documented in Justiniano and Preston (2010). We incorporate a number of correlated shocks in the same way as in Corbo and Strid (2020) in an attempt to capture international spillovers to Swedish economy and co-movements of aggregate variables both in Sweden and Foreign.[52] Moreover, Swedish exports is positively correlated with Foreign GDP, consumption and investment. Furthermore, the real exchange rate displays a negative correlation with Foreign GDP. Inflation, the monetary policy rate and corporate spreads are all positively correlated between Sweden and Foreign.

Table 13: Contemporaneous correlations between selected Swedish data variables

		ΔY_t^{obs}	ΔC_t^{obs}	ΔI_t^{obs}	ΔX_t^{obs}	ΔM_t^{obs}	$\Pi_t^{C,obs}$	$\Pi_t^{C,xe,obs}$	$\hat{n}_t^{obs}$	R_t^{obs}	ζ_t^{obs}	Δw_t^{obs}	ΔQ_t^{obs}
GDP growth	ΔY_t^{obs}	1.00											
Consumption growth	ΔC_t^{obs}	0.48	1.00										
Investment growth	ΔI_t^{obs}	0.42	0.09	1.00									
Export growth	ΔX_t^{obs}	0.62	0.18	0.31	1.00								
Import growth	ΔM_t^{obs}	0.53	0.34	0.44	0.74	1.00							
CPI inflation	$\Pi_t^{C,obs}$	0.10	-0.06	-0.02	-0.02	-0.12	1.00						
CPI inflation excl. energy	$\Pi_t^{C,xe,obs}$	-0.05	-0.12	-0.15	-0.19	-0.25	0.70	1.00					
Employment gap	$\hat{n}_t^{obs}$	-0.20	-0.16	-0.14	-0.24	-0.24	0.17	0.14	1.00				
Monetary policy rate	R_t^{obs}	0.02	-0.03	0.09	0.07	-0.00	0.12	0.09	-0.29	1.00			
Corporate Spread	ζ_t^{obs}	0.07	0.02	0.01	0.10	0.10	-0.15	-0.05	-0.45	-0.45	1.00		
Wage inflation	Δw_t^{obs}	-0.08	0.04	-0.06	0.05	-0.02	0.11	0.02	-0.21	0.72	-0.43	1.00	
R. Exch. rate	ΔQ_t^{obs}	-0.29	-0.12	-0.20	-0.14	-0.31	-0.13	0.01	0.13	-0.07	0.07	-0.02	1.00

Notes: Δ refers to growth rate.

[51] Several of the high correlations in the table are due to trends and structural changes, thus could be considered as "spurious". For example, the correlation between wage growth and the monetary policy rate is mostly driven by the downward sloping inflation (an important factor in determining nominal wage growth), and the downward-sloping neutral interest rate (an important factor in determining the nominal interest rate).

[52] See Section 2.8 for details.

Table 14: Cross-country contemporaneous correlation of variables over the sample period, 1995Q1-2019Q4

Foreign	Sweden	Corr
GDP growth	GDP growth	0.68
GDP growth	Exports growth	0.39
GDP growth	Imports growth	0.41
GDP growth	Change in real exchange rate	-0.32
Consumption growth	GDP growth	0.60
Consumption growth	Consumption growth	0.49
Consumption growth	Exports growth	0.51
Investment growth	GDP growth	0.43
Investment growth	Investment growth	0.44
Investment growth	Exports growth	0.51
Inflation	Inflation	0.51
Policy rate	Policy rate	0.92
Corporate spread	Corporate spread	0.84

3.2.3 Trend assumptions

In many instances, theoretically sound model trend assumptions are not in line with the data used for the estimation. We are not immune to these commonly seen contradictions between the theoretical assumptions of the model and the sample data. There are three main trends that requires extra attention: which are the trend in GDP components, the trend in price and wage inflation, and the trend in interest rates. To reconcile the model and the data, we follow a similar strategy as in Corbo and Strid (2020), explained below.

Modelling trend in GDP components and technology: In Section 2, where we describe the model, and in Section 3.1.1, where we discussed the balanced growth, we explained our trend assumptions for GDP components and underlying technologies to a large extent.

As mentioned earlier, the long-run balanced growth path of the Foreign economy is determined by a global labor-augmenting technology z_t^*, with a growth rate of $\mu_{z_t}^*$, and a global investment-specific technology γ_t^*, with a growth rate of $\mu_{\gamma_t}^*$.[53] Similarly, a labor-augmenting technology z_t, with a growth rate μ_{z_t}, and an investment-specific technology γ_t, with a growth rate of μ_{γ_t} are assumed to determine productivity growth in Sweden. We assume a common trend between Foreign and Sweden so that the growth rate of Swedish labor augmenting technology equals the global growth rate, $\mu_{z_t} = \mu_{z_t}^*$. The calibration of growth rates are in Section 3.1.1.

A balanced growth path assumption of the model given defined technological process implies that GDP and its components excluding private investment, and real wages have a common long-run trend both in Sweden and Foreign. However, in the data, the average growth rates vary across those variables for the same sample period.[54] To deal with this, we incorporate excess trend parameters in the corresponding observation equations. It is assumed that the trend component of the observable variable is partially explained by the model and that the remaining unexplained part of the trend component is captured by an exogenous component, which can be time-varying. We explain each observation equation and the respective trend assumption along with the excess parameter in detail in Appendix L.

Modelling trend in the interest rates: The Swedish and Foreign monetary policy rates have been trending down in the sample period and have been at record low levels since the euro-zone debt crisis until the end of the sample period, see Figure 3 and Figure 4. Since inflation is not trending down to the same degree, it will be assumed that it is the real interest rate which is responsible for the trend. To capture the trend in the policy rate empirically, we define the monetary policy rate by following Corbo and Strid (2020), where the monetary policy rate is changing not only in accordance with the Taylor rule, but also changing due to changes in the neutral real interest rate.

Recall the Foreign monetary policy expansion (gap):

$$\breve{i}_{F,t}^{gap} = \breve{i}_{F,t} - \breve{i}_{F,t}^{nat} \tag{143}$$

where $\breve{i}_{F,t}$ is the Foreign interest rate and $\breve{i}_{F,t}^{nat}$ is the Foreign neutral interest rate, both are in deviations from their long-run equilibrium levels. We assume that long-run inflation expectations doesn't change over time. Hence, the change in the nominal neutral interest rate is equal to the change in the neutral real interest rate, $\breve{i}_{F,t}^{nat} = \breve{r}_{F,t}^{nat}$.

The Foreign neutral interest rate is defined as:

$$\breve{i}_{F,t}^{nat} = \breve{r}_{F,t}^{nat} = r_{F,\mu}\hat{\mu}_{z_F^+,t} - r_{F,\varsigma}\hat{\zeta}_{F,t} + \hat{z}_t^R. \tag{144}$$

[53]Global investment-specific shock is inactive in the estimation, only the deterministic component of the trend exists in the model.

[54]For example, the average Swedish GDP per capita growth is 1.8 percent while the average export per capita growth rate is 4 percent.

where the real neutral interest rate fluctuates in response to permanent shift in Foreign productivity $\hat{\mu}_{z_F^+,t}$, change in Foreign risk premium $\hat{\zeta}_{F,t}$, and the neutral rate shock, $\hat{z}_t^R$, which captures the trend in the policy rate to a large extent with an estimated high degree of persistence.

Similarly recall Swedish monetary policy expansion (gap):

$$\breve{i}_t^{gap} = \breve{i}_t - \breve{i}_t^{nat} \tag{145}$$

where $\breve{i}_t^{nat}$ is the change in the Swedish neutral interest rate. The same assumption about long-run inflation expectations holds in Sweden as in Foreign which gives $\breve{i}_t^{nat} = \breve{r}_t^{nat}$. Finally the Swedish neutral interest rate is given by

$$\breve{i}_t^{nat} = \breve{r}_t^{nat} = r_\mu \hat{\mu}_{z^+,t} - r_\zeta \hat{\zeta}_t + \hat{z}_t^R. \tag{146}$$

where the real neutral interest rate fluctuates in response to permanent shift in Swedish productivity $\hat{\mu}_{z^+,t}$, change in Swedish risk premium $\hat{\zeta}_t$, and a global neutral rate shock, $\hat{z}_t^R$, which is common in Foreign and Sweden.

Trend in inflation: The sample means of CPIF inflation in Foreign and Sweden are 1.7 and 1.6 percent, respectively, which are significantly lower than the Foreign and the Riksbank's inflation targets of 2 percent. CPIF inflation excluding energy and import inflation for consumption goods have even lower sample means, 1.4 and -0.3, respectively. In the estimation, we attribute these deviations from the target rates partly to household and firm behaviour and partly to exogenous inflation trend shocks in Foreign and Sweden, $\hat{\Pi}_{F,t}^{C,trend}$ and $\hat{\Pi}_t^{trend}$. Moreover, for import inflation, we utilize an excess parameter of -1.5 percent attributing a role to factors driving the import inflation that can not be explained by the model.

3.2.4 Observation equations

Observation equations of the model in vector form are given by:[55]

$$Y_t^{obs} = c + SS_Y + AY_t + B_{s\sigma_Y} \varepsilon_{Y,t} \tag{147}$$

where Y_t^{obs} is the vector of observed variables, c is the excess parameter vector introduced to make the model steady state compatible with the data sample means of variables and also to capture trend and other special assumptions, SS_Y is the vector for the steady state of the model, Y_t is the vector for all model variables and A is the matrix mapping model variables to data series. $B_{s\sigma_Y} \varepsilon_{Y,t}$ is the vector for observation errors, where $B_{s\sigma_Y}$ is the diagonal matrix containing the scaled standard deviation of each observation error, $s_i \sigma_{Y,i}$, and $\varepsilon_{Y,t}$ is assumed to be independently and normally distributed as $\varepsilon_t \sim N(0,1)$. We calibrate each $\sigma_{Y,i}$ to the sample standard deviation of the corresponding observable variable. In the majority of observation equations, the scaling parameter s_i is set to 0.1, which implies that these errors account for a small fraction of the variation in the observed variables. And yet, we set $s_i = 0$ for some variables which are assumed to be measured without an error, i.e the monetary policy rate and the corporate spread.

When a data series used for an observed series defined in terms of growth rates, our model trend assumptions matter for the observation equation. If the trend in the sample, which could simply be a linear trend with a positive growth rate, and the model trend assumptions match, the excess parameter c would be set to zero and thus our model variable would be in line with observations. However, the model-implied steady state growth rate could be significantly different from the sample average growth rate. In that case, to make the model compatible with the data, we use the excess parameter to reduce the discrepancy between the model and the data. This strategy of handling trends allows us to incorporate some important features of the data into the model without imposing a complex trend structure. Excess parameters could also be viewed as components of the data that are not able to be explained by the model, and not necessarily constant over the whole sample period, which implies that c could be time-varying, and can be represented as c_t. We provide explanations in detail for observation equations for each observed variable of the model in Appendix L.

3.2.5 Estimated parameters

We estimate, mostly, those parameters that only affects the dynamics of the model, but have no effect on the steady state. This means that the list of parameters to be estimated consists, to a large extent, of those which are related to real and nominal frictions in the model, as well as monetary and fiscal policy parameters and shock processes. See Table 15 for estimated Foreign economy parameters and Table 16, 17 and 18 for estimated Swedish parameters.[56]

[55] The observation equations for each observable variable are illustrated in the Appendix L.

[56] We attempt to estimate more parameters than listed in Table 15, but we calibrate them if they exhibit poor identification.

3.2.6 Prior distributions and scaling

While assigning the prior distributions, the prior means and the standard deviations we follow the literature, e.g., Smets and Wouters (2007) and Justiniano and Preston (2010) and standard practices in estimating policy models, e.g., Kravik and Mimir (2019) and Corbo and Strid (2020). Parameters which are bounded by 0 and 1 are assigned a Beta distribution, parameters which are bounded only from below are assigned Gamma or inverse Gamma distribution, parameters which are not bounded are assigned Normal distribution.

Priors for non-fiscal structural parameters: The prior mean for the consumption habit persistence parameters $\rho_{h,F}$ and ρ_h are set to 0.75. Values between 0.5-0.75 is common practice and covers the range of estimated values in the literature to a large extent. Calvo price and wage parameters are set to 0.75, which implies that firms and unions are able to set their prices and wages once every fourth quarter. The labor disutility parameter η_F is given a prior mean of 3, which implies an inverse Frisch elasticity parameter of 0.33. The estimates of the Frisch elasticity with micro data is in the range between 0.1-0.7.[57] Investment adjustment cost parameters, S''_F for the Foreign economy and S'' for Sweden, have a prior mean of 5, which is in line with other studies, e.g., Smets and Wouters (2007). S''_F (or S'') is interpreted as the key parameter determining the elasticity of investment to a shock in the price of installed capital, p^K_F (p^K), see Christiano, Eichenbaum, and Evans (2005b). In SELMA, the prior mean implies an elasticity of approximately 0.19 for Sweden, which is given by $1/((S'')(ln(\mu_{z+})ln(\mu_\gamma))^2)$. The prior means for monetary policy rule are chosen with the prior information that monetary policy is persistent, could respond to inflation and resource utilization strong enough to change the real interest rate accordingly. The prior mean values we assign for monetary policy rules are not off the range of other studies including Corbo and Strid (2020), from which we borrow the policy monetary policy rule specification and the strategy of handling the interest rate trend. Moreover, the prior means for monetary policy parameters for Foreign and Sweden are chosen to be close to each other.[58] We set the prior mean of the coefficient parameter for the risk premium in the neutral interest rate equation, $r_{F,\zeta}$ to 0, and the prior standard deviation to 1 with normal distribution to give room for flexibility for the parameter to have a positive or negative sign in the estimation. For the correlation coefficient parameter between Foreign investment and Foreign consumption, $CorrC_{\zeta^c_F,\Upsilon_F}$, we choose a positive prior of 0.5, which is quite high, but we also estimate the model with zero correlation coefficient to reveal if a positive correlation coefficient is still being supported statistically.[59]

Priors for fiscal parameters: Regarding the elasticity of substitution parameters between public and private capital and consumption we choose a Gamma distribution, restricting them to be positive, with prior means of 0.5, and with a standard deviation of 0.15. These tight priors for public-private complementarity parameters are in line with estimates in Coenen, Straub, and Trabandt (2013). The priors for government transfers rule parameters, $\mathcal{F}_{tr,surp}$ and $\mathcal{F}_{tr,un}$ are set to 1 and 0, respectively. A positive prior (and a posterior) value is necessary for $\mathcal{F}_{tr,surp}$ for having a stationary model because the government transfers rule is the only channel in the model for fiscal stabilization. For economic stabilization parameter $\mathcal{F}_{tr,un}$ in the government transfers rule, we choose a normal distribution with zero mean to let the model consider negative values and allow historical data to decide on the sign of the response of government transfers to unemployment. However, we believe one of the prominent features of fiscal policy-making in Sweden is a positive response of government transfers to the unemployment rate. Priors for the AR parameters for government consumption, government investment and the tax rates are set to 0.75, and MA parameters are set to 0.5.

Priors for shock processes: We implement the standard practice and choose 0.75 as a prior mean for all shock processes, except the shock process for the neutral rate, which is set to 0.85. The reason for setting the latter higher is to allow using the neutral rate as a tool to capture the trend in the monetary policy rate. For the standard deviations of the innovations we use the prior mean of 0.2 for the majority of the parameters. However, for the innovations of the monetary policy rate and the neutral interest rate, we choose 0.1, following Corbo and Strid (2020). In the estimation, innovations are scaled mainly to prevent upward biases, which could result in those innovations to be over-represented in explaining the business cycle. Scaling of innovations is also displayed in Table 15. The implications of scaling for priors are illustrated below.

Let innovation $\epsilon_t \sim \mathrm{N}(0,1)$. A shock process with a scaling parameter s can be written as:

$$y_t = \beta y_{t-1} + s\sigma\epsilon_t, \tag{148}$$

[57] See the seminal estimates of Frisch elasticity with micro data, Altonji (1986) and MaCurdy (1981).

[58] Taylor rule for Foreign economy in Corbo and Strid (2020) has the unemployment rate as the resource utilization variable, but Foreign Taylor rule in SELMA has the GDP gap with very similar specification. For that reason, we set a higher prior mean than in Corbo and Strid (2020) for resource utilization variable given that GDP is more volatile than the unemployment rate in the data.

[59] When we set a zero prior for $CorrC_{\zeta^c_F,\Upsilon_F}$, it is estimated to be lower. By the posterior distribution, we can argue that they are not statistically different from zero. However, with a higher prior the model fit, in terms of marginal likelihood and model's ability to capture international spillovers, is better. For these reasons, we estimate the parameter with a high prior even though the situation shows some signs of weak identification.

where $\sigma \sim \text{Inv-Gamma}(a,b) \rightarrow E(\sigma) = 0.2,\ V(\sigma) = \infty$. A shock process without scaling is analogous to Equation 148 with $s = 1$.

To see the role of scaling we rewrite the shock process as the following:

$$y_t = \beta y_{t-1} + \tilde{\sigma}\epsilon_t,$$

where $\tilde{\sigma} = s\sigma$. The distribution of σ would imply:

$$\tilde{\sigma} \sim \text{Inv-Gamma}(\tilde{a}, \tilde{b}) \rightarrow E(\tilde{\sigma}) = 0.2s,\ V(\tilde{\sigma}) = \infty$$

where $\tilde{a}$ and $\tilde{b}$ are new shape and scaling parameters of the inverse Gamma distribution that would provide the calibrated mean and the variance.[60]

As a conclusion, scaling parameter shifts the prior mean of standard deviations up and down with its size, s.

3.2.7 Estimation results

Given our two-step strategy of estimating Foreign and Sweden sequentially the estimated parameters for Foreign and Sweden are reported separately below.

The posterior estimates of the Foreign parameters: Table 15 shows the posterior mode, the median and the 5% and 95% percentile estimates of the parameters we estimate. The posterior mode estimate of the Foreign consumption habit persistence parameter is $\rho_{h,F} = 0.70$ which is in the range of estimated values in the literature, e.g., Smets and Wouters (2007). Foreign Calvo pricing and wage setting parameters ξ_F and ξ_w^F are estimated to be 0.93 and 0.88, which are relatively high. For instance, $\xi_F = 0.93$ implies that firms set their prices in every 14 quarters. Nevertheless, the estimated values are very close to estimated values for the same parameters in Corbo and Strid (2020).[61] The Foreign labor disutility parameter, η_F, is estimated to be 4.42, and implies a Frisch labor supply elasticity parameter of 0.23, which is in the range of empirical results in micro studies.[62] The Foreign investment adjustment cost parameter, S''_F, is estimated to be 3.22, which is in line with other studies mentioned previously. Price indexation in the Foreign economy is estimated to be weak, which is also in line with other estimates, see Corbo and Strid (2020) and Smets and Wouters (2007). The estimates for all the Foreign monetary policy rule parameters are in line with the values in Corbo and Strid (2020). The coefficient for the risk premium in the neutral interest rate equation, $r_{F,\zeta}$ is estimated to be 0.65, which implies that the neutral rate decreases when spreads on borrowing, or the risk premium, increases. The Foreign shock persistence parameters are estimated to be high but in line with the studies mentioned earlier. The persistence parameter for the interest rate trend shock is estimated to be notably high at 0.99. This is because the interest rates shows a trend in the sample period which is captured by the estimation with a larger value of the shock persistence. Finally, the correlation coefficient parameter between the Foreign consumption preference shock and the Foreign stationary investment-specific shock, $CorrC_{\zeta_F^c,\Upsilon_F}$ is estimated to be 0.53, which is relatively high, helping to capture the comovement between Foreign consumption and Foreign investment.

The posterior estimates of the non-fiscal structural Swedish parameters: The posterior mode estimate of the Swedish consumption habit persistence parameter is $\rho_h = 0.91$, which is relatively high compared to other studies. Calvo pricing and wage setting parameters ξ, ξ_X, $\xi_{M,I}$, $\xi_{M,X}$, $\xi_{M,C,xe}$ and ξ_w are estimated to be in the high range of the earlier estimates in the literature but, still, slightly lower than the estimates of Corbo and Strid (2020). The parameter that measures the elasticity of substitution between government consumption and private consumption, υ_G is estimated to be 0.37, implying a strong complementarity. The elasticity of substitution between public capital and private capital, υ_K is estimated to be 0.52, also implying a strong complementarity. The estimates of these public-private complementarity parameters are close to the estimates in Coenen, Straub, and Trabandt (2013) for the euro area, although υ_K, is estimated to be somewhat higher. The elasticity of substitution between domestic and import goods for non-energy consumption goods, $\nu_{C,xe}$, investment goods ν_I and export goods, ν_X, are all estimated to be low, implying a low degree of substitution, especially for investment goods. The external risk premium parameter associated with the exchange rate fluctuations in the UIP condition, $\widetilde{\phi}_s$ is estimated to be 0.26. This low value implies that the exchange rate volatility has a

[60] As an example, given the shape parameter $a = 1.5$, the scale parameter b needs to be $0.2 * (a-1) = 0.1$ to achieve $E(\sigma) = 0.2$ and $V(\sigma) = \infty$. Similarly, when the theoretical mean of the distribution is scaled by 10, $E(\sigma) = 0.2 * 10$ keeping the theoretical variance infinite, and given the same value for the new shape parameter, $\tilde{a} = 1.5$, the new scaling parameter will be $\tilde{b} = 0.2 * 10(\tilde{a} - 1) = 1$.

[61] Recall that Corbo and Strid (2020) and SELMA have very similar Foreign blocks. Moreover, the list of observable variables used for the estimation and the sample period for the data in both models are very close 1995Q2-2018Q4 vs 1995Q1-2019Q4. Furthermore, estimated high persistence in price setting could be considered as plausible given persistent and low global inflation in the sample period, especially in the post-financial crisis period.

[62] For some macro economists labor disutility parameter in macro models and Frisch elasticity parameter in micro studies are distantly related concepts, thus according to those estimated value to be significantly different from micro estimates shouldn't be a concern. See useful discussion in Christiano, Trabandt, and Walentin (2010).

small effect on the risk premium. Thus, a more predictable exchange rate doesn't give very strong incentives to hold the Swedish krona, which is also the motivation for the risk premium specification in SELMA, which is in line with Adolfson et al. (2013) as illustrated in Section 2. The capital utilization parameter, σ_a is estimated to be 0.54, which implies a quite flexible capital utilization structure for Swedish industry; nevertheless, the estimate is larger than in Corbo and Strid (2020), which implies even more flexibility in capital utilization. The investment adjustment cost parameter, S'', is in line with the estimate in the Foreign block and other studies. The labor disutility parameter, η, is estimated to be 3.53, which is very close to the estimate in Corbo and Strid (2020), and implies a wage markup of 1.3. All the monetary policy rule parameters are close to estimates in Corbo and Strid (2020). The monetary policy rate is estimated to be highly persistent and respond strongly to the unemployment rate, both to deviation from its long run equilibrium rate and change in the unemployment rate. The risk premium parameter for the Swedish neutral rate, r_ζ is estimated to be 0.78. This implies a strong downward effect on the interest rates in Sweden after the financial crisis due to increasing demand for "safe asset" as risk premiums increased. The share of non-Ricardians is estimated to be 0.14, which is a relatively small value when compared to most of the earlier studies, e.g., Campbell and Mankiw (1991). However, it is well in line with Coenen, Straub, and Trabandt (2013), where low estimate for share of non-Ricardians is attributed to strong complementarity between government consumption and private consumption. This captures the comovement between those two variables in the data.

The posterior estimates of shock process parameters: Persistence parameters for stationary technology shock, energy price shock, labor supply and external risk premium (UIP) are estimated to be higher than the corresponding priors, and in line with estimates of their Foreign block counterparts and the estimates in Corbo and Strid (2020). The estimates of persistence parameters of the risk premium shock, ρ_ζ and the stationary investment shock, ρ_Υ are lower than their Foreign block counterparts. The reason can partially be attributed to the high estimates of correlation coefficient parameters for these shocks.[63] A low persistence parameter estimate in domestic shocks arise since the persistence is captured by the combination of high persistence in Foreign shocks and a strong correlation between Foreign and domestic shocks, thus there is no need for a high domestic persistence parameter. All the markup shock persistence parameters are estimated to be lower than the priors, which could be attributed to Calvo pricing parameters being estimated to be high and capturing the price stickiness to a large extent. The parameter that determines the correlation between Foreign and Swedish consumption preference shock is estimated to be significantly positive and high. Note that, the Swedish consumption preference shock is modelled with no persistence, thus any persistence in the consumption preference shock is driven by the Foreign consumption sentiment. The parameter that determines the correlation between Foreign and Swedish stationary technology is estimated to be not significant. In spite of this, we choose to keep this parameter in the list of estimated parameters due to its contribution to the marginal likelihood. The inflation trend persistence parameter is estimated to be high, which is, thus, able to capture a period of highly persistent below target inflation period.

The posterior estimates of the fiscal policy parameters: As noted earlier in Section 2, government transfers is the only policy instrument designed for economic stabilization in the model. Thus, the coefficient estimate for government transfers to unemployment is very critical to study fiscal policy with SELMA. The coefficient for the unemployment rate in the government transfers rule, $\mathcal{F}_{tr,un}$, is estimated to be 0.34. This implies that when the unemployment rate increase by one percentage point, government transfers to households increase by 0.34 percent of GDP. The coefficient for structural surplus, $\mathcal{F}_{tr,surp}$ is estimated to be 0.01. The relatively small value implies that budget stabilization through government transfers takes a long time. All fiscal policy persistence parameters (AR components) are estimated to be higher than the priors. The persistence parameter for government transfers is reflecting mainly the persistence of government transfers for the budget stabilization not for the economic stabilization because the government transfers rule is formulated in that way, see Equation 62 and 63. MA components of government investment, transfers tax and the labor income tax are small, but well identified and reported in the table.

[63] Note that the values reported in the tables are not correlation coefficient parameter values, but they are the estimated values of $\rho_{2,1}$ in Equation 110, which in turn determines the level of correlation coefficient between shocks. A value of $\rho_{2,1} = 0.25$ for risk premium shock corresponds to 0.82 for the correlation coefficient between shocks given the estimated values for shock persistence parameters and shock standard deviations.

Table 15: Estimation results: Foreign economy

Parameter		Prior Dist	Mean	Std	Scale	Posterior Mode	Median	Std	5%	95%
$\rho_{h,F}$	habit	B.	0.75	0.10		0.70	0.74	0.04	0.67	0.81
ξ_F	Calvo, price	B.	0.75	0.07		0.93	0.93	0.01	0.91	0.95
ξ_w^F	Calvo, wage	B.	0.75	0.07		0.88	0.88	0.02	0.84	0.92
η_F	Labor disutility	G.	3.00	1.50		4.42	4.94	1.92	2.34	8.68
S''_F	Inv adj. cost	N.	5.00	2.50		3.22	4.22	1.64	2.16	7.52
χ_F	Indexation, price	B.	0.50	0.20		0.19	0.22	0.09	0.09	0.37
ρ_F	Smoothing, MP rule	B.	0.85	0.05		0.92	0.92	0.03	0.87	0.96
$r_{F,\pi}$	Inflation, MP rule	N.	1.75	0.15		1.57	1.56	0.17	1.27	1.83
$r_{F,y}$	Output, MP rule	N.	0.12	0.12		0.05	0.05	0.03	-0.01	0.11
$r_{F,\triangle y}$	Output change	N.	0.30	0.07		0.15	0.15	0.02	0.11	0.18
$r_{\zeta,F}$	Risk prm, nat. rate	N.	0.00	1.00		0.65	0.65	0.15	0.39	0.89
ρ_{ε_F}	Temp. tech	B.	0.75	0.10		0.82	0.83	0.10	0.62	0.93
$\rho_{p_F^{Ce}}$	Energy price	B.	0.75	0.05		0.93	0.93	0.01	0.90	0.95
ρ_{ζ_F}	Risk prm	B.	0.75	0.10		0.95	0.94	0.02	0.90	0.98
ρ_{Υ_F}	Inv. efficiency	B.	0.75	0.10		0.64	0.62	0.08	0.48	0.75
$\rho_{\zeta_F^c}$	Cons. preference	B.	0.75	0.10		0.76	0.73	0.09	0.55	0.85
$\rho_{\zeta_F^n}$	Labor disutility	B.	0.75	0.10		0.74	0.70	0.11	0.47	0.83
ρ_{z^R}	trend, nat rate	B.	0.85	0.10		0.99	0.99	0.01	0.98	1.00
$\rho_{\mu_{zF}}$	Permanent, labor tech	B.	0.75	0.10		0.63	0.63	0.07	0.52	0.74
ρ_{g_F}	Gov, consumption	B.	0.75	0.10		0.95	0.95	0.02	0.90	0.97
$\rho_{\Pi_F^{C,trend}}$	Inflation trend	B.	0.75	0.10		0.89	0.87	0.06	0.75	0.94
$CorrC_{\zeta_F^c,\Upsilon_F}$	Corr, inv and cons	B.	0.50	0.20		0.60	0.49	0.14	0.25	0.71
σ_ε	Temp. tech	Inv. G	0.20	Inf	0.01	0.17	0.17	0.02	0.14	0.21
$\sigma_{p_F^{Ce}}$	Energy price	Inv. G	0.20	Inf	0.1	0.27	0.28	0.02	0.24	0.32
σ_{i_F}	Monetary policy	Inv. G	0.20	Inf	0.01	0.11	0.14	0.05	0.07	0.25
σ_{ζ_F}	Risk prm	Inv. G	0.20	Inf	0.001	0.32	0.32	0.03	0.28	0.37
σ_{λ_F}	Markup, price	Inv. G	0.20	Inf	0.01	0.06	0.06	0.01	0.04	0.08
$\sigma_{\mu_{zF}}$	Permanent, labor tech	Inv. G	0.20	Inf	0.01	0.28	0.28	0.03	0.23	0.34
σ_Υ	Inv. efficiency	Inv. G	0.20	Inf	0.1	0.22	0.27	0.09	0.16	0.46
$\sigma_{\zeta_F^c}$	Cons. preference	Inv. G	0.20	Inf	0.1	0.07	0.09	0.02	0.06	0.12
$\sigma_{\zeta_F^n}$	Labor disutility	Inv. G	0.20	Inf	10	0.07	0.08	0.03	0.05	0.14
σ_{z^R}	trend, nat rate	Inv. G	0.20	Inf	0.01	0.34	0.34	0.05	0.27	0.43
σ_{g_F}	Gov, consumption	Inv. G	0.20	Inf	0.1	0.10	0.11	0.01	0.09	0.12
$\sigma_{\Pi_F^{C,trend}}$	Inflation trend	Inv. G	0.20	Inf	0.01	0.10	0.10	0.02	0.08	0.14

Notes: Abbreviations at prior distribution column are used for the following distributions: B. is for the Beta distribution, G. is for the Gamma distribution, Inv. G. is for the inverse Gamma distribution and N is for Normal distribution. 5% and 95% are the percentiles of the posterior distribution of the corresponding parameter. Some other abbreviations are used as the following: Inv. adj. cost is for investment adjustment cost, MP rule is for monetary policy rule, Risk prm is for risk premium, nat rate is for the neutral rate, Cons. preference is for consumption preference, Temp. tech is for temporary technology, Perm. inv. is for permanent investment, Inv. efficiency is for investment efficiency.

Table 16: Estimation results: Swedish non-fiscal policy structural parameters

Parameter	Description	Prior Dist	Mean	Std	Posterior Mode	Median	Std	5%	95%
s_{nr}	Share of non-Ricardian	B.	0.30	0.10	0.14	0.15	0.05	0.08	0.24
ρ_h	Consumption habit	B.	0.75	0.10	0.91	0.91	0.02	0.88	0.94
ξ	Calvo, interm. goods price	B.	0.75	0.07	0.92	0.91	0.02	0.88	0.94
ξ_X	Calvo, exp. goods price	B.	0.75	0.07	0.93	0.93	0.02	0.88	0.96
$\xi_{m,I}$	Calvo, imp. for inv. goods price	B.	0.75	0.07	0.66	0.65	0.06	0.54	0.74
$\xi_{m,X}$	Calvo, imp. for exp. goods price	B.	0.75	0.07	0.88	0.87	0.04	0.80	0.92
$\xi_{m,Cxe}$	Calvo, imp. for non-E cons. goods price	B.	0.75	0.07	0.89	0.88	0.02	0.86	0.91
ξ_w	Calvo, wage	B.	0.75	0.07	0.83	0.84	0.02	0.80	0.88
χ_w	Wage indexation, wage setting	B.	0.50	0.20	0.17	0.20	0.10	0.07	0.39
υ_G	Public/private cons. complement.	G.	0.50	0.15	0.37	0.41	0.12	0.26	0.66
υ_K	Public/private inv. complement.	G.	0.50	0.15	0.52	0.55	0.14	0.36	0.82
$\nu_{C,xe}$	Imp. elasticity, non-E cons. goods	G.	1.01	0.50	0.51	0.60	0.30	0.23	1.22
ν_I	Imp. elasticity, inv. goods	G.	1.01	0.50	0.24	0.24	0.09	0.11	0.42
ν_F	Export price elasticity	G.	1.01	0.50	1.09	1.04	0.33	0.63	1.69
ω_C^X	For. consumption share in Swedish exports	G.	0.50	0.20	0.37	0.38	0.14	0.17	0.62
$\widetilde{\phi}_s$	External risk prm, exchange rate	B.	0.50	0.20	0.26	0.28	0.05	0.18	0.36
σ_a	Capital util coef., rental rate	B.	1.00	Inf	0.54	0.63	0.21	0.38	1.04
S''	Investment adj. cost	N.	5.00	2.50	8.16	8.61	1.12	6.94	10.62
η	Labor disutility	G.	3.00	0.20	3.53	3.55	0.20	3.24	3.90
ρ	Smoothing, MP rule	B.	0.75	0.10	0.91	0.90	0.02	0.87	0.93
r_π	Inflation, MP rule	N.	1.75	0.15	1.82	1.81	0.14	1.59	2.05
r_{un}	Unemployment, MP rule	N.	0.12	0.12	0.17	0.17	0.04	0.11	0.25
$r_{\triangle un}$	Change in unemployment, MP rule	N.	0.15	0.07	0.11	0.11	0.02	0.08	0.15
r_ζ	Risk prm, nat. rate	N.	0.00	1.00	0.78	0.78	0.16	0.51	1.05

Notes: Abbreviations at prior distribution column are used for the following distributions: B. is for the Beta distribution, G. is for the Gamma distribution, Inv. G. is for the inverse Gamma distribution and N is for Normal distribution. %5 and %95 are the percentiles of the posterior distribution of the corresponding parameter. Some other abbreviations are used as the following: Inv. adj. cost is for investment adjustment cost, MP rule is for monetary policy rule, Risk prm is for risk premium, nat rate is for the neutral rate, Cons. preference is for consumption preference, Temp. tech is for temporary technology, Perm. inv. is for permanent investment, Inv. efficiency is for investment efficiency.

Table 17: Estimation results: Swedish parameters for non-fiscal shock persistency and standard errors

Parameter		Prior Dist	Mean	Std	Scale	Posterior Mode	Median	Std	5%	95%
ρ_{ζ}	Risk prm, persistence	B.	0.50	0.20		0.74	0.73	0.05	0.65	0.81
ρ_{β}	Discount factor, persistence	B.	0.50	0.20		0.20	0.18	0.09	0.06	0.34
$\rho_{\widetilde{\phi}}$	UIP risk prm., persistence	B.	0.50	0.20		0.83	0.80	0.06	0.67	0.88
ρ_{ζ^n}	Labor disutility, persistence	B.	0.50	0.20		0.92	0.91	0.03	0.86	0.96
ρ_{λ^W}	Wage markup, persistence	B.	0.50	0.20		0.20	0.19	0.09	0.06	0.36
ρ_{λ}	Interm. goods price markup, persistence	B.	0.50	0.20		0.41	0.41	0.14	0.19	0.66
$\rho_{\lambda^{MC}}$	Imp. for non-E cons. goods price markup, persistence	B.	0.50	0.20		0.11	0.14	0.08	0.04	0.29
$\rho_{\lambda^{MI}}$	Imp. for inv. goods price markup, persistence	B.	0.50	0.20		0.40	0.41	0.09	0.25	0.56
$\rho_{\lambda^{MX}}$	Imp. for exp. goods price markup, persistence	B.	0.50	0.20		0.15	0.18	0.09	0.06	0.36
ρ_{ε}	Stationary technology, persistence	B.	0.50	0.20		0.85	0.84	0.06	0.73	0.92
ρ_{Υ}	Stationary inv.-specific, persistence	B.	0.50	0.20		0.14	0.16	0.08	0.05	0.33
$\rho_{p_F^{CeD}}$	Energy price, persistence	B.	0.50	0.20		0.93	0.92	0.02	0.88	0.96
ρ_{μ_γ}	Non-stationary, inv specific, persistence	B.	0.75	0.10		0.41	0.42	0.07	0.31	0.53
$\rho_{\Pi^{Tr}}$	Inflation trend, persistence	B.	0.50	0.20		0.94	0.93	0.04	0.84	0.97
σ_i	Monetary policy, std	Inv. G	0.20	Inf	0.01	0.05	0.06	0.00	0.05	0.07
σ_{ζ}	Risk prm, std	Inv. G	0.20	Inf	0.001	0.34	0.34	0.03	0.30	0.39
σ_{ζ^c}	Cons. preference, std	Inv. G	0.20	Inf	1	0.05	0.06	0.01	0.04	0.09
σ_{β}	Discount factor, std	Inv. G	0.20	Inf	1	0.08	0.09	0.08	0.06	0.20
$\sigma_{\widetilde{\phi}}$	UIP risk prm., std	Inv. G	0.20	Inf	0.01	0.31	0.37	0.09	0.24	0.55
σ_{ζ^n}	Labor disutility, std	Inv. G	0.20	Inf	0.1	0.11	0.12	0.01	0.10	0.14
σ_{λ^W}	Wage markup, std	Inv. G	0.20	Inf	10	0.11	0.12	0.05	0.07	0.21
σ_{λ}	Interm. goods price markup, std	Inv. G	0.20	Inf	0.01	0.19	0.19	0.03	0.13	0.25
σ_{λ^X}	Exp. goods price markup, std	Inv. G	0.20	Inf	0.1	0.19	0.20	0.07	0.12	0.35
$\sigma_{\lambda^{MC}}$	Imp. for non-E cons. goods price markup, std	Inv. G	0.20	Inf	0.01	0.42	0.42	0.04	0.35	0.49
$\sigma_{\lambda^{MI}}$	Imp. for inv. goods price markup, std	Inv. G	0.20	Inf	1	0.11	0.12	0.02	0.09	0.16
$\sigma_{\lambda^{MX}}$	Imp. for exp. goods price markup, std	Inv. G	0.20	Inf	0.1	0.35	0.36	0.04	0.30	0.42
σ_{ε}	Stationary technology, std	Inv. G	0.20	Inf	0.01	0.52	0.53	0.04	0.47	0.60
σ_{Υ}	Stationary inv.-specific, std	Inv. G	0.20	Inf	1	0.23	0.24	0.04	0.19	0.31
$\sigma_{p_F^{CeD}}$	Energy price, std	Inv. G	0.20	Inf	0.1	0.50	0.50	0.04	0.45	0.57
σ_{μ_γ}	Non-stationary, inv specific, std	Inv. G	0.20	Inf	0.1	0.12	0.12	0.01	0.10	0.13
$\sigma_{\Pi^{Tr}}$	Inflation trend, std	Inv. G	0.20	Inf	0.01	0.10	0.11	0.03	0.08	0.16
$corr_{\zeta}$	Correlation, risk prm	N.	0.00	0.20		0.25	0.25	0.05	0.17	0.33
$corr_{\varepsilon}$	Correlation, stationary technology	N.	0.00	0.20		0.12	0.12	0.14	-0.11	0.34
$corr_{inv}$	Correlation, private investment	N.	0.50	0.20		0.52	0.52	0.19	0.21	0.83
$corr_{con}$	Correlation, private consumption	N.	0.50	0.20		0.57	0.56	0.18	0.26	0.87

Table 18: Estimation results: Fiscal policy parameters

Parameter		Prior Dist	Mean	Std	Scale	Posterior Mode	Median	Std	5%	95%
$\mathcal{F}_{tr,surp}$	Gov. transfers, surplus target	N.	1.00	0.50		0.01	0.02	0.01	0.01	0.05
$\mathcal{F}_{tr,un}$	Gov. transfers, unemployment	N.	0.00	0.20		0.34	0.35	0.05	0.26	0.43
ρ_g	Persistence, government cons.	B.	0.75	0.10		0.95	0.95	0.02	0.91	0.98
ρ_{IG}	Persistence, government inv.	B.	0.75	0.10		0.94	0.93	0.03	0.88	0.97
ρ_{tr}	Persistence, government transfers	B.	0.75	0.10		0.96	0.96	0.02	0.93	0.98
ρ_{τ^C}	Persistence, cons. tax	B.	0.75	0.10		0.79	0.79	0.07	0.67	0.89
$\rho_{\tau^{TR}}$	Persistence, transfers tax	B.	0.75	0.10		0.96	0.95	0.02	0.91	0.98
ρ_{τ^W}	Persistence, labor income tax	B.	0.75	0.10		0.95	0.95	0.02	0.91	0.97
$\rho_{\tau^{SSC}}$	Persistence, social security contribution.	B.	0.75	0.10		0.75	0.75	0.06	0.65	0.84
σ_g	Std. errors, government cons.	Inv. G	0.20	Inf	0.1	0.06	0.06	0.00	0.05	0.06
σ_{IG}	Std. errors, government inv.	Inv. G	0.20	Inf	0.1	0.33	0.34	0.04	0.29	0.41
σ_{tr}	Std. errors, government transfers	Inv. G	0.20	Inf	0.01	0.34	0.35	0.03	0.30	0.41
σ_{τ^C}	Std. errors, cons. tax	Inv. G	0.20	Inf	0.1	0.07	0.07	0.01	0.07	0.08
$\sigma_{\tau^{TR}}$	Std. errors, transfers tax	Inv. G	0.20	Inf	0.01	0.50	0.51	0.04	0.45	0.58
σ_{τ^W}	Std. errors, labor income tax	Inv. G	0.20	Inf	0.01	0.50	0.51	0.04	0.45	0.58
$\sigma_{\tau^{SSC}}$	Std. errors, social security contribution.	Inv. G	0.20	Inf	0.01	0.33	0.33	0.03	0.29	0.38
η_{IG}	MA coefficient, government inv.	B.	0.50	0.20		0.14	0.15	0.06	0.06	0.25
$\eta_{\tau^{TR}}$	MA coefficient, transfers tax	B.	0.50	0.20		0.24	0.27	0.11	0.11	0.46
η_{τ^W}	MA coefficient, Labor income tax	B.	0.50	0.20		0.15	0.16	0.06	0.07	0.27

Notes: Abbreviations at prior distribution column are used for the following distributions: B. is for the Beta distribution, G. is for the Gamma distribution, Inv. G. is for the inverse Gamma distribution and N is for Normal distribution. 5% and 95% are the percentiles of the posterior distribution of the corresponding parameter.

3.2.8 Model properties

In this section, we assess the model fit by comparing data and empirical properties of the model using the posterior estimates. First, we report business cycle moments of Foreign economy and Sweden, respectively. Second, we present impulse response functions from selected shocks to explain transmission mechanisms in the model. Third, we examine historical shock decompositions for selected macro variables to show the model's explanation for the main drivers of the business cycles.

Business cycle moments: Table 19 and Table 20 show model-implied standard deviations for Foreign and Sweden by percentile of the posterior distribution as well as the sample standard deviations, respectively. Table 21 shows model-implied cross country correlations. Table 22 shows model-implied correlations between selected Swedish variables.[64] In Appendix M, we provide model-implied contemporaneous correlations between selected observable variables for Sweden and Foreign.

To calculate the posterior distributions of the Foreign statistics of interest, we obtain 3 chains with 500.000 draws from the posterior distribution of Foreign sector estimation. We drop the first 250.000 of those draws and then randomly obtain 1500 sub-draws from the chains. We simulate the model 300 quarters and take the last 100 (the same size as the data period, 1995Q1-2019Q4) observations of the simulations and calculate the statics from these simulations.

To calculate the posterior distributions of the Swedish statistics of interest, we obtain 5 chains with 1.500.000 draws from the posterior distribution. We drop the first 750.000 of those draws and then randomly obtain 1500 sub-draws from the chains. As in the Foreign sector case, we simulate the model 300 quarters and take the last 100 (the same size as the data period, 1995Q1-2019Q4) observations of the simulations and calculate the statics from these simulations.

It is important to note that the posterior distributions we calculate for Sweden doesn't take into account the parameter uncertainty in the Foreign sector, given that Swedish parameters are estimated conditional on the Foreign estimated parameters at the posterior mode. Thus, the posterior distributions intervals are probably narrower than the case where the Foreign parameters uncertainty is also taken into account. In that case, the model's fit to data most likely would look better than the results below.

The model-implied variations of observed variables for Foreign are mostly in line with the data. However, the model-implied variation in hours worked and wages are slightly larger than the variation in the data.

The model-implied variations of Swedish observed variables are also mostly in line with the data with small differences. For GDP components, the model-implied variations for GDP, private investment and household consumption are slightly larger than in the data, while variations in imports are smaller. The implied volatility in inflation and the the monetary policy rate are slightly larger in the model as compared to data. The model-implied standard deviation for employment gap is also in line with the data, but it is closer to the data at lower percentiles of the sample.

The model-implied cross-country (Foreign vs. Sweden) correlations show partial success in capturing international spillovers. While cross-country correlations in nominal variables, the monetary policy rate, the corporate spreads and inflation are close to the data, correlations in real variables are not fully in line with the data. The strong co-movement of global investment and Swedish investment (and household consumption) couldn't be achieved, even though shock correlations for private investment (and household consumption) were incorporated. Nevertheless, the correlations between Foreign investment and Swedish exports are well captured by the model.

The model-implied correlations of Swedish variables are also show partial success of being in line with the data. For example, while the correlations between GDP and exports, and GDP and private investment are well captured in the model, correlations between GDP and imports, and GDP and consumption; or correlations between exports and imports are not in the 90% interval of the posterior distribution. Nevertheless, the results in the table doesn't show large deviations from the data which would raise concerns about the estimation.

Table 19: Model implied standard deviations for Foreign variables (Sample)

Variable	Data	Post. dist. percentile		
		5	50	95
GDP	2.03	1.67	2.03	2.51
Consumption	1.48	1.24	1.55	1.98
Investment	4.94	4.48	5.58	6.95
CPI excl. energy	0.50	0.48	0.65	0.95
CPI	1.10	1.02	1.21	1.46
Hours worked	1.51	1.65	1.93	2.31
Monetary policy rate	1.94	0.72	1.17	1.95
Corporate spread	0.44	0.21	0.31	0.52
Wage	0.95	1.15	1.45	1.91

[64] Note that, in the tables below, GDP and its components and the real exchange rate are expressed in growth terms.

Table 20: Model implied standard deviations for Swedish variables (Sample)

Variable	Data	Post. dist. percentile		
		5	50	95
GDP	3.46	3.44	3.98	4.58
Consumption	2.80	2.92	3.40	3.95
Investment	12.43	13.51	16.53	20.04
Export	9.18	7.52	8.84	10.42
Import	8.89	6.03	7.01	8.13
CPIF	1.20	1.34	1.56	1.83
CPIF excl. energy	0.93	1.03	1.25	1.52
Employment gap	1.89	1.83	2.58	3.87
Monetary policy rate	2.26	1.00	1.43	2.17
Corporate Spread	0.40	0.22	0.32	0.50
Wage growth	0.89	0.76	1.05	1.46
R. Exch. rate	2.38	2.19	2.56	3.00

Table 21: Model implied cross-country (Foreign vs. Sweden) correlations (Sample)

Foreign	Sweden	Data	Post. dist. percentile		
			5	50	95
GDP	GDP	0.70	0.13	0.32	0.47
GDP	Export	0.70	0.17	0.34	0.51
GDP	Import	0.70	-0.01	0.18	0.35
GDP	R. Exch. rate	-0.39	-0.10	0.08	0.26
Consumption	GDP	0.61	0.05	0.25	0.41
Consumption	Consumption	0.46	-0.05	0.13	0.31
Consumption	Export	0.55	0.04	0.23	0.39
Investment	GDP	0.49	0.14	0.32	0.49
Investment	Investment	0.46	-0.18	0.04	0.26
Investment	Export	0.53	0.22	0.42	0.57
Inflation	CPI inflation	0.53	0.07	0.25	0.42
Monetary policy rate	Monetary policy rate	0.92	0.26	0.69	0.89
Corporate Spread	Corporate Spread	0.84	0.38	0.73	0.89

Table 22: Model implied correlations for Swedish variables (Sample)

Variable 1	Variable 2	Data	Posterior dist. percentile		
			5	50	95
GDP	Consumption	0.48	0.13	0.29	0.45
	Investment	0.42	0.24	0.42	0.56
	Exports	0.62	0.43	0.57	0.68
	Imports	0.53	-0.13	0.09	0.32
	CPIF	0.10	-0.37	-0.21	-0.02
	R. Exch. rate	-0.29	-0.32	-0.16	0.02
CPIF	Corporate Spread	-0.15	-0.28	-0.03	0.24
	R. Exch. rate	-0.13	0.01	0.20	0.38
	Monetary policy rate	0.12	-0.29	-0.02	0.26
Exports	Imports	0.74	0.28	0.44	0.57
	R. Exch. rate	-0.14	-0.26	-0.09	0.09

Impulse response functions: We discuss impulse response functions separately in Section 4.

Historical shock decompositions: In this section, we present historical shock decompositions for Swedish GDP growth and CPIF inflation to show the model's assessment of main drivers behind the movements in these variables. To interpret the results in a structured way, we categorize the shocks into 8 different groups as follows: foreign supply shocks, foreign demand shocks, domestic supply shocks, domestic demand shocks, monetary policy shock (Swedish), fiscal policy shocks, the exchange rate shocks, growth shocks and inflation trend shocks. Table 23 shows the categorization of each shock in the model.[65]

[65] The distinction between supply and demand shocks is made according to following criterion. If a shock leads to higher inflation and output, then it is classified as demand shock, and if a shock leads to higher inflation but lower output, then it is classified as a supply shock.

Table 23: Shock categorization

Group name	Shock name
Foreign supply shocks	Foreign labor disutility shock, Foreign stationary technology shock, Foreign price markup shock, Foreign energy price shock, Swedish exporters' price markup shock
Foreign demand shocks	Foreign consumption preference shock, Foreign stationary investment efficiency shock, Foreign risk premium shock, Foreign discount factor shock, Foreign monetary policy shock, Foreign fiscal policy shock
Domestic supply shocks	Labor disutility shock, Stationary technology shock, (non-exporters) price and wage markup shocks, energy price shock
Domestic demand shocks	Risk premium shock, consumption preference shock Investment efficiency shock, discount factor shock
Monetary policy shock	Monetary policy shock
Fiscal policy shocks	All fiscal policy shocks
Exchange rate shock	Exchange rate shock
Growth shocks	Global labor-augmenting technology shock, Swedish investment-specific technology shock
Inflation trend shock	Inflation trend shock

Figure 5 and 6 shows the historical shock decompositions, computed at the posterior mode, for the Swedish annual GDP growth and CPIF inflation between 1995Q1-2019Q4, respectively. The shock decomposition of GDP growth reveals that Foreign shocks are the main drivers of Swedish business cycles in the sample period. The shock decomposition of CPIF inflation shows that the low and persistently below-target inflation is largely captured by the inflation trend shock.

For the late 90s, the domestic supply side shocks explain high inflation and contribute negatively to GDP growth. In that period, an appreciated currency (mainly driven by the exchange rate shocks) lowered inflation and contributed positively to GDP growth. During the same period, which is the boom phase of dotcom boom-bust cycle, Foreign shocks and growth shocks contribute positively to GDP growth and negatively to inflation. In the bust phase of the cycle, starting in 2001, Foreign shocks contribute significantly negative to GDP growth but their effect on inflation is mixed and small. In this bust phase of the cycle, exchange rate shocks contribute the most to CPIF inflation, and its negative effect to GDP growth comes with a lag and continues to affect negatively during the recovery period until 2004. A positive contribution of domestic supply shocks to GDP growth in 2001 (the crisis year) reflects the fact that Swedish trade balance improves, while both imports and exports decrease. A larger decrease in imports compared to export is captured by domestic supply shocks, specifically the imported goods used for exports goods price markup shock.[66] The effect of this particular shock on inflation is relatively small. In the boom phase of the global financial crisis, while foreign shocks and growth shocks contributed positively to GDP growth, domestic demand shocks and fiscal policy shocks had a negative contribution. Beyond the inflation trend shock, the exchange rate shocks and growth shocks are the main drivers of low inflation up until the financial crisis. The Foreign shocks and the growth shocks (both global and domestic) have large contributions both in the downturn phase and the following recovery phase. The same shocks also have large effects in the euro area debt crisis, but these effects are relatively smaller compared to the financial crisis. The recovery phase of the euro area crisis is mainly driven by domestic demand shocks and fiscal policy shocks. Fiscal policy shocks contribute to GDP growth after the financial crisis until the end of 2017. The appreciation of the Swedish krona after the financial crisis is mostly captured by the exchange rate shocks, which positively contributed to GDP growth until the end of 2015. The exchange rate shocks and the domestic supply side shocks are the main drivers of low inflation from the financial crisis up until 2015. After 2015, GDP growth is negatively affected by domestic demand shocks and growth shocks and positively by Foreign shocks. The negative effects of a depreciated currency start to be significant in the latest part of the sample period.

[66] Recall that import price markup shocks are classified as "domestic", mostly because import firms are modelled as "domestic" in the model. It is important to keep in mind the classification of shocks in Table 23 while interpreting results.

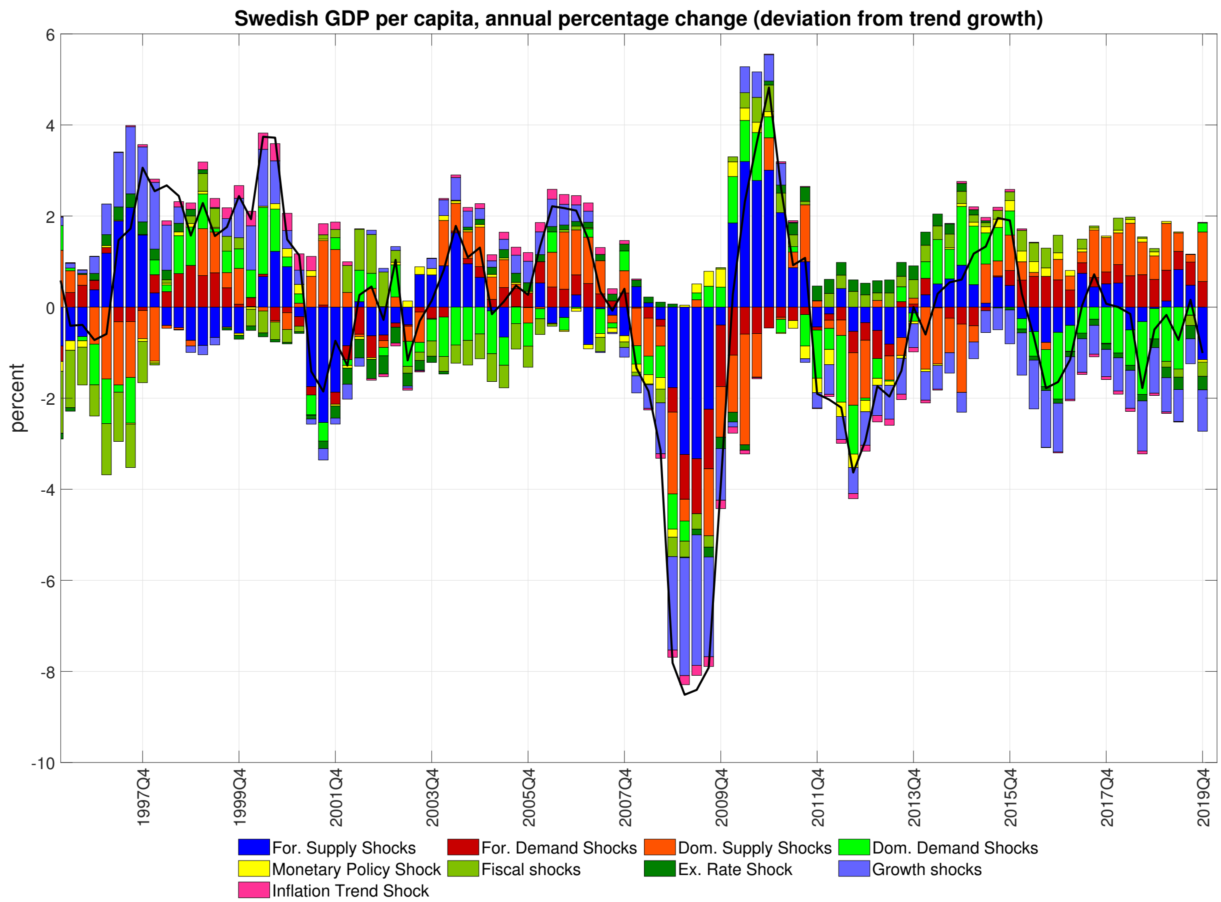


Figure 5: Historical shock decomposion, annual Swedish GDP growth

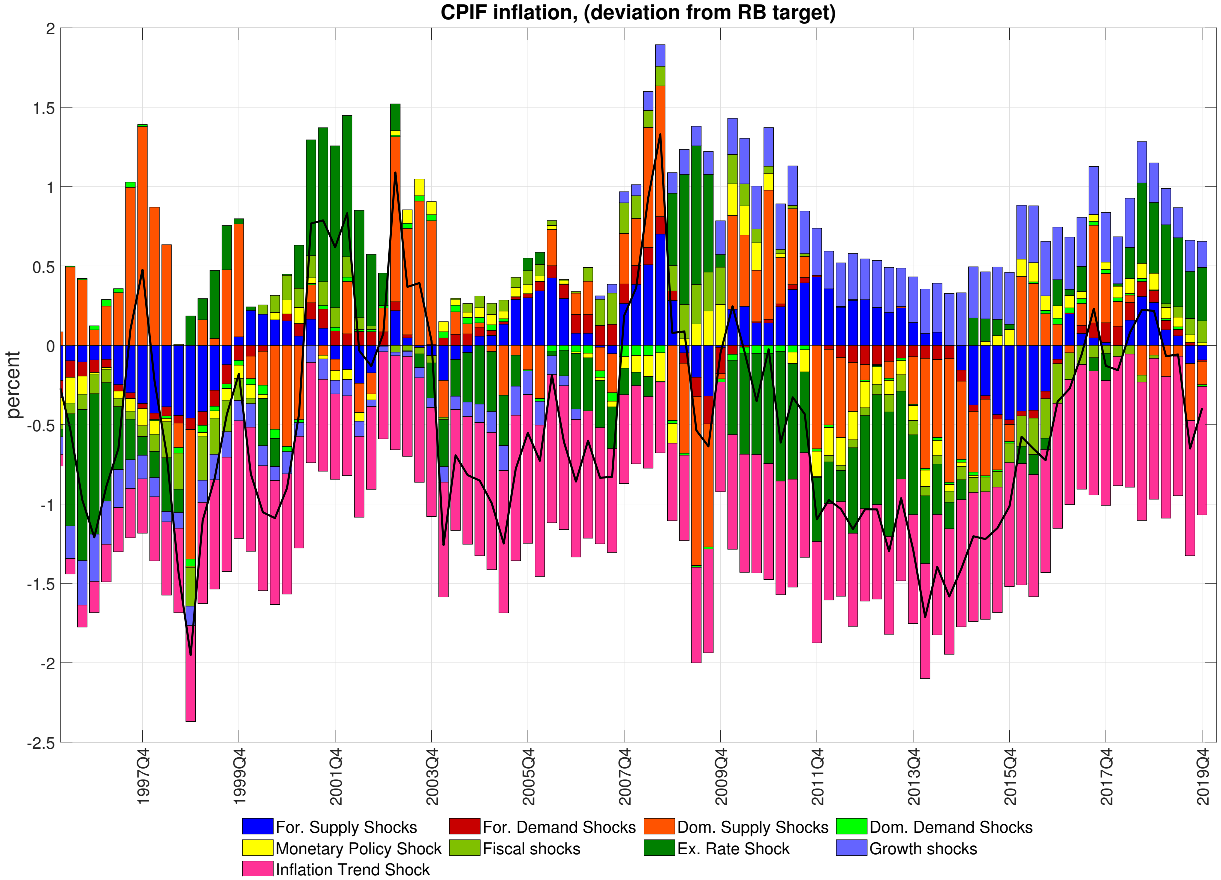


Figure 6: Historical shock decomposition, annual CPIF inflation

4 Impulse response analysis

In this section, we present some impulse response functions (IRFs) from SELMA to show the model's response to selected shocks.[67] Before showing the results from the simulations, we discuss two key equations, the consumption Euler equation and the modified UIP condition.[68].

The consumption Euler equation, which is a key equation to determine the Ricardian households' consumption, can be written in log-linearized form as:

$$\hat{\Omega}_t^C = E_t \left[\hat{\zeta}_t + \hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C + \frac{1}{R} \breve{i}_t - \hat{\Pi}_{t+1}^C - \hat{\mu}_{z^+,t+1} \right].$$

A variable with the *hat* notation represents the log-linear approximation of the variable in deviation from its steady-state value (which in turn can be interpreted as percent deviation of the variable from its steady state), and a variable with *breve* notation is interpreted as an absolute deviation of the variable from its steady state. For more details on notations, see Appendix A. $\hat{\Omega}_t^C$ is the marginal utility of consumption, $\hat{\zeta}_t$ is a risk premium shock to private bonds, $\breve{i}_t$ is the monetary policy rate set by the Riksbank, $\hat{\Pi}_{t+1}^C$ is the consumer price inflation (CPIF), and $\hat{\mu}_{z^+,t+1}$ is the compound effects of labor augmenting technological and investment-specific technological processes. $\hat{\beta}_{t+1}^r$ is a time-varying discount factor.

Remember that since $\hat{\Omega}_t^C$ is concave, an increase in the marginal utility of consumption means that consumption falls. On the balanced growth path, the marginal utility of consumption will decrease at a constant pace, so that consumption grows at the same rate as the rest of the economy. If, however, the economy is hit by shocks, the underlying economic conditions for the households will change, and the households might change how they choose to consume. Note, however, that households prefer to smooth their consumption. The Ricardian households, who are able to do so, would like the effects of temporary disturbances to affect their current consumption

[67]The impulse response functions for other selected shocks of the model are plotted in the Appendix M.2.
[68]For the derivation of the non-linear equations, see Appendix C

as little as possible, smoothing the burden of the shock over their whole lifespan.[69]

An increase in the monetary policy rate $\breve{i}_t$, will, *ceteris paribus*, increase the Ricardian households' incentives to save, since one extra unit of savings gives a higher level of consumption tomorrow than without a rate increase. As a result, $\hat{\Omega}_t^C$ will increase, meaning that current consumption will decrease. The effective interest rate on private bonds for the household is however given by $\frac{1}{R}\breve{i}_t + \hat{\zeta}_t$. Hence, a shock to the risk premium has a similar direct effect on the household as a change in the nominal interest rate. Now paying attention to the inflation rate, an increase in the inflation rate will, *ceteris paribus*, reduce the real interest rate that the household faces. It will therefore have a similar effect on consumption as a decrease in the nominal interest rate. Turning to the productivity parameter $\hat{\mu}_{z^+,t+1}$, an increase in productivity leads to an immediate consumption increase since the households want to smooth consumption.

Next we discuss the modified UIP condition that determines the exchange rate in the model, which in a log-linearized form is expressed as:

$$\frac{1}{R}\left(\breve{i}_t - \breve{i}_{F,t}\right) = \left(1 - \widetilde{\phi}_s\right) E_t\left[\hat{s}_{t+1}\right] - \widetilde{\phi}_s \hat{s}_t - \widetilde{\phi}_a \breve{a}_t + \hat{\widetilde{\phi}}_t.$$

The Swedish nominal interest rate, $\breve{i}_t$ represents the expected return on Swedish private bonds. Likewise, $\breve{i}_{F,t}$ is the Foreign nominal interest rate. $\hat{s}_t$ is the nominal exchange rate (a higher value means a depreciation of the Swedish krona relative to the Foreign currency). $\breve{a}_t$ is the real value of net foreign asset position of Sweden, which captures Foreign private bonds that are owned by Swedish Ricardian households, and $\hat{\widetilde{\phi}}_t$ is an exchange rate shock, also referred to as the external risk premium shock.

If the Swedish monetary policy rate, $\breve{i}_t$, is higher than the Foreign policy rate, we would expect a depreciation of the exchange rate between today and tomorrow, so that the returns on the two assets equalize. For this to occur, the exchange rate must appreciate today. This is captured by the first term in the right-hand side of the equation.

The next three terms on the right-hand-side of the equation captures an international bond market friction, called the external risk premium. The friction makes the return on Foreign assets held by Swedish Ricardian households decrease with the size of their Foreign bond portfolio. This means that it is more expensive for Swedish Ricardian households to borrow from Foreign households if they already have negative net Foreign assets. Similarly, the return on their Foreign bond portfolio is lower if they have positive net Foreign assets. The friction does also include the shock $\hat{\widetilde{\phi}}_t$.

Now, we continue by describing the model responses to different shocks in the economy. Note that in the IRF-diagrams, the monetary policy rate in Home and Foreign, all the inflation rates and the government bond interest rate are all presented in annualized quarter-on-quarter values.

4.1 A monetary policy shock in Sweden

In this section, we describe the model response to one standard deviation of monetary policy shock that increases the annualized quarter-on-quarter Swedish monetary policy rate, $\breve{i}_t^{notional}$, by approximately 0.2 percent. The economic outcome is illustrated in Figures 7 and 8. A positive shock to the policy rate leads to an increase in the returns to saving in private bonds for the Ricardian households in Sweden. Hence, *ceterus paribus*, they would like to decrease their consumption and save more of their income. As a result of decreased demand, the price level and consumption fall.

Furthermore, since it becomes more profitable to save in Swedish bonds than in Foreign bonds, the households would like to sell Foreign bonds and buy Swedish bonds, which leads to an appreciation of the exchange rate. The appreciated exchange rate does in turn lead to lower revenues for the export firms given the same set export price, since the price is set in the currency of Foreign. The firms respond to the decreased revenues by increasing the price of export goods. This leads to a fall in exports.

Private investment decreases following the shock. Due to the lower demand for consumption and exports, output decreases, which reduces the firm's demand for physical capital. Furthermore, the increased interest rate leads to an increased demand for bonds at the expense of other types of savings, in this case, physical capital. Both these channels lead to lower investment.

The demand for the four type of import goods (energy, non-energy, investment and export) are functions of the demand for their respective final good and of the price of the imported intermediate goods relative to the price of the respective final good. As the real exchange rate appreciates, the import firms decrease their prices. The final goods demand effect dominates for the non-energy consumption good and the investment good firms, and they decrease their output. However, for the energy good and export firms, the price effect dominates. Hence they increase their output. In sum, total imports first decrease, but then increase again above the steady-state level of imports in the medium term.

[69] We assume that Non-Ricardian households do not have access to capital markets. Therefore they consume all of their income in every period.

Due to the lower demand of domestic intermediate goods, output decreases. This leads to a reduced demand for labor, decreasing employment and nominal wages. However, real wages increase due to lower CPIF inflation. As a result, unemployment increases.[70] The wage income of the Non-Ricardian households decreases due to the reduced employment, which dominates the effect of the real wage on income. Therefore, Non-Ricardian consumption decreases.

Turning to the public sector, the reduced economic activity leads to a decrease in tax income, which in turn reduces the government surplus and increases government debt. In the short term aggregate transfers increase due to high unemployment, but in the medium term aggregate transfers are adjusted so that the structural surplus returns to its target level with a reasonable pace. Due to the definition of the structural surplus, it is affected by only discretionary fiscal policies, except the government transfers rule, and the debt payments, or permanent disturbances; hence the effects of a temporary monetary policy shock only affects the structural surplus via the debt payments. In order to correct the deviation of structural surplus from its long run target, the aggregate transfers fall, leading to a reduction in the Non-Ricardian households' consumption in the medium term.

4.2 A stationary technology shock

In this section, we describe the economic outcome after an increase in ϵ_t, which is the shock to the intermediate goods producers' technology ε_t, by one standard deviation. A temporary technology shock increases the productivity of Swedish intermediate goods producers; thus, they can produce more output for a given level of inputs. The economic outcome is illustrated in Figures 9 and 10.

A positive technology shock generates a temporary increase in total factor productivity, which directly reduces the marginal cost of production. As a consequence, this effect generates downward pressure on domestic inflation. The decrease in domestic inflation generates downward pressure on CPIF inflation. In addition, since firms are able to produce the same amount of output with lower input, the demand for labor, thus employment, decreases and unemployment increases. The increase in unemployment induces the Riksbank to reduce the nominal interest rate despite a modest increase in CPIF inflation, stemming from higher import inflation.

The fall in the nominal interest rate in Sweden leads to the return on savings abroad being higher than the return on Swedish bonds. This induces Swedish households to buy Foreign bonds, which leads to an exchange rate depreciation. The exchange rate depreciation increases the markups of the Swedish export firms, who respond by reducing their prices. The opposite holds for the Swedish import firms, increasing the price of Swedish import goods. These changes in prices increases Swedish exports and reduces Swedish imports.

The intermediate good firms substitute capital for labor input in their production, leading to an increase in the demand for capital. The increase in demand for capital, together with a lower interest rate path, leads to higher private investment. The real wage decreases as nominal wage inflation decreases while CPIF inflation increases. Real labor income drops since both employment and real wage decrease. Non-Ricardian households reduce their consumption as their real labor income drops.

A decline in the interest rate path induces Ricardian households to increase their consumption. Overall household consumption decrease in the initial periods, but afterwards the higher Ricardian consumption dominates. The lower wage income reduces the labor income tax revenue and lower household consumption reduces the consumption tax revenue. As a result, the government surplus decreases and the government debt increases.

Since the changes in the tax income is only temporary it does not affect the structural surplus. Thus, higher transfers to the households are mainly driven by higher unemployment.

4.3 A risk premium shock to Foreign private bonds

In this section, we describe the economic outcome after an increase of $\epsilon_t^{\zeta_F}$, which is the shock to the Foreign domestic risk premium $\zeta_{F,t}$, by one standard deviation. This can also be interpreted more generally as negative demand shock in Foreign, or as an increased demand for bond holdings by Foreign households. The economic outcome is illustrated in Figures 11 and 12.

The shock leads to Foreign households wanting to save more and consume less. The decreased demand leads to a decrease in demand for intermediate goods. As a response to the decreased demand, output and hours worked in Foreign are reduced. This puts a downward pressure on Foreign wages, reducing the costs for the Foreign intermediate good firms. As a response, they reduce their prices, leading to Foreign consumption good firms to reduce their prices. The central bank in Foreign responds to the decline in output and inflation by reducing the monetary policy rate.

Since the foreign risk premium shock is correlated with the risk premium shock to Swedish private bonds, a positive shock to the foreign risk premium leads to an increased demand for bond savings in Sweden as well. Because of increased savings, consumption of the Ricardian households in Sweden and hence output of

[70] Employment and output are highly correlated, since the main part of the costs of the production of intermediate goods consists of labor costs.

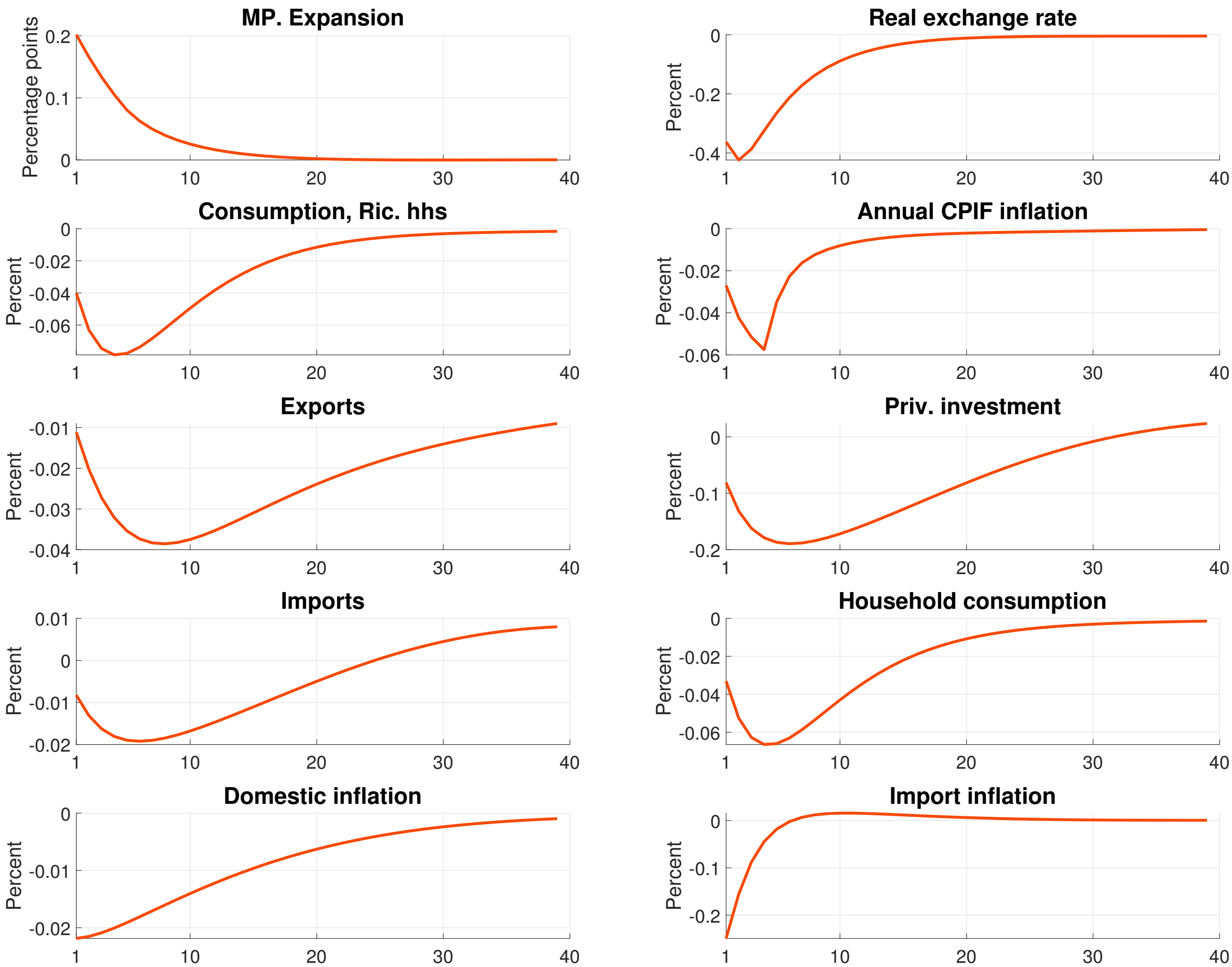


Figure 7: Economic outcome after a shock to the monetary policy rate $\breve{i}_t^{notational}$

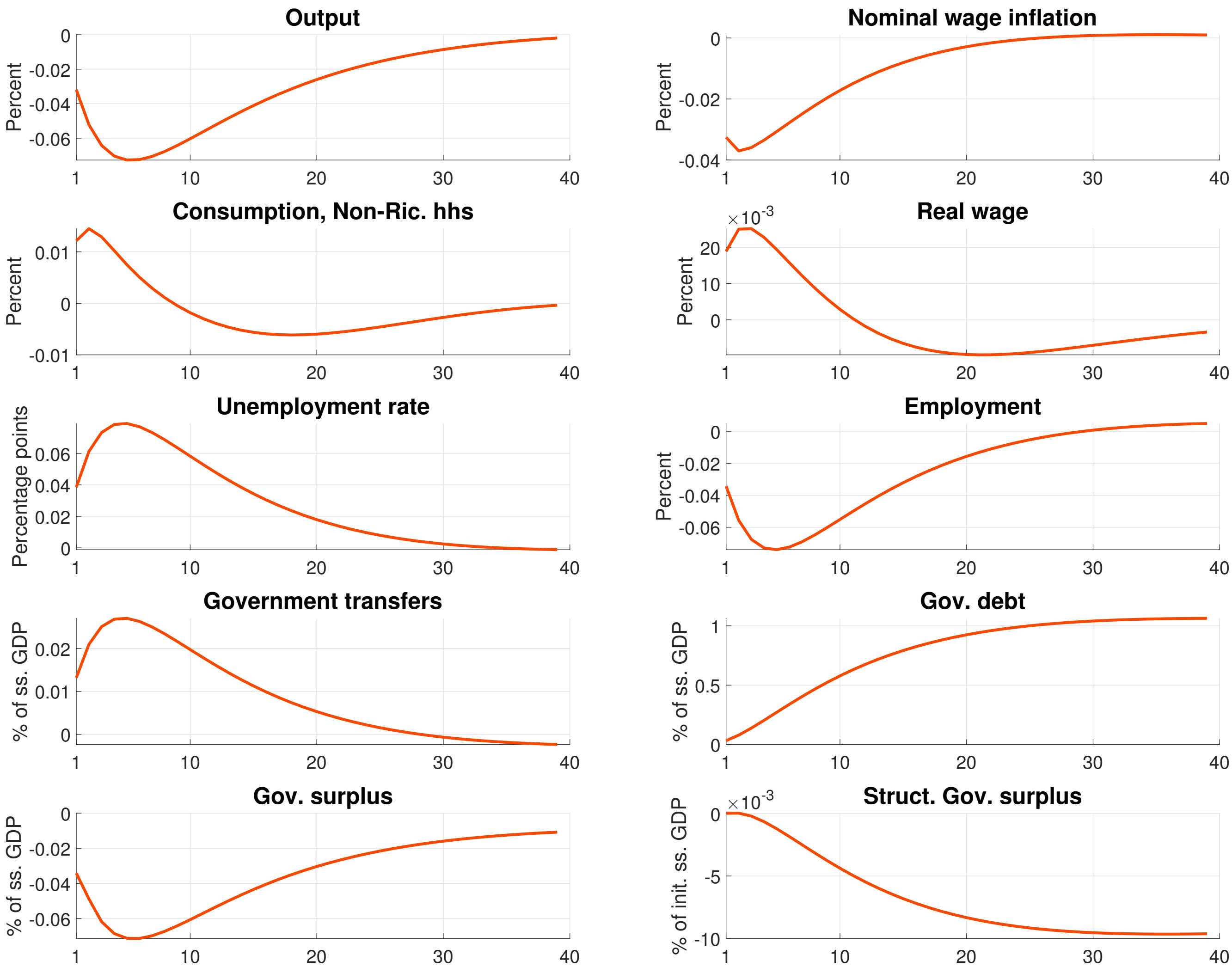


Figure 8: Economic outcome after a shock to the monetary policy rate $\breve{i}_t^{notational}$

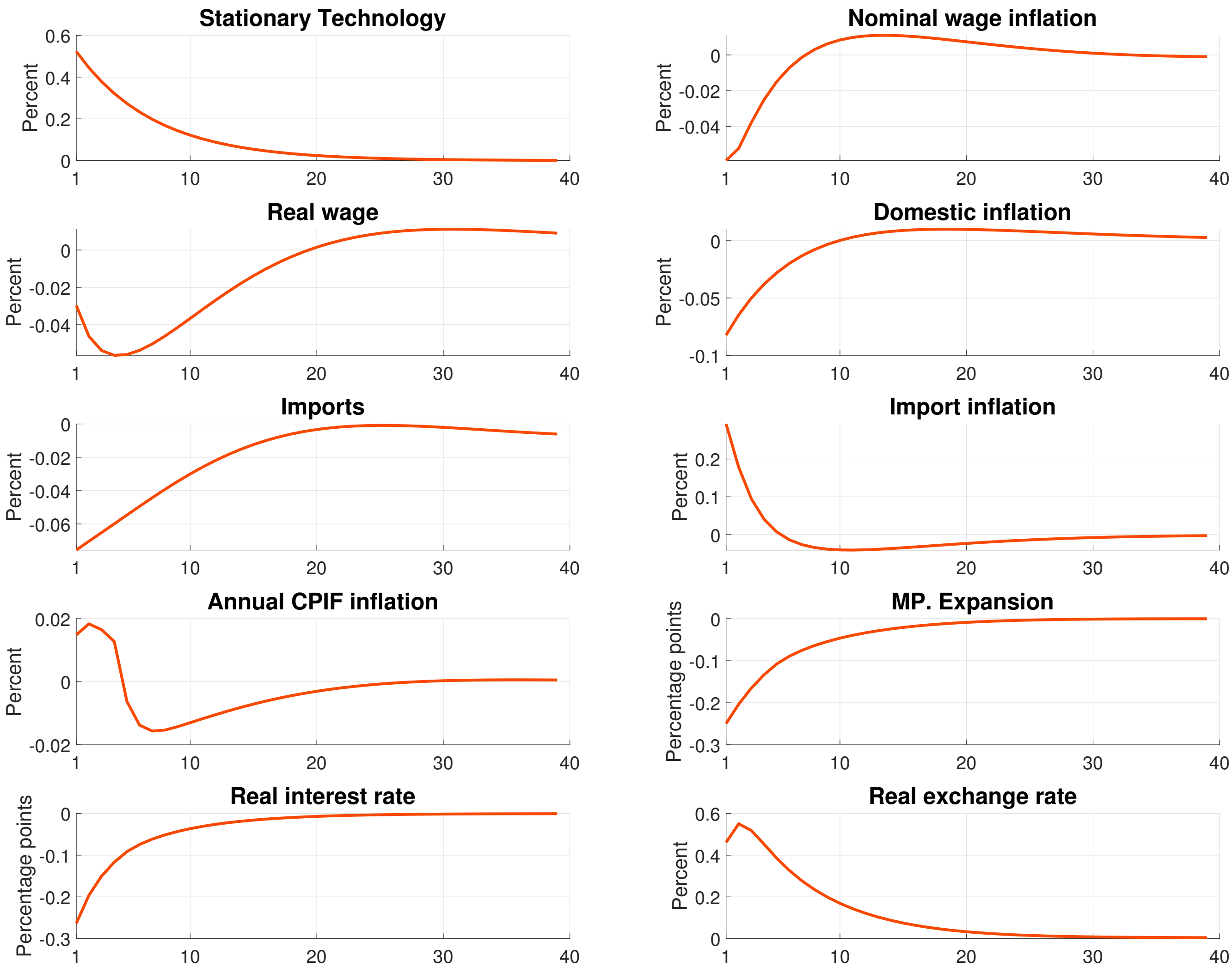


Figure 9: Economic outcome after a shock to productivity ε_t

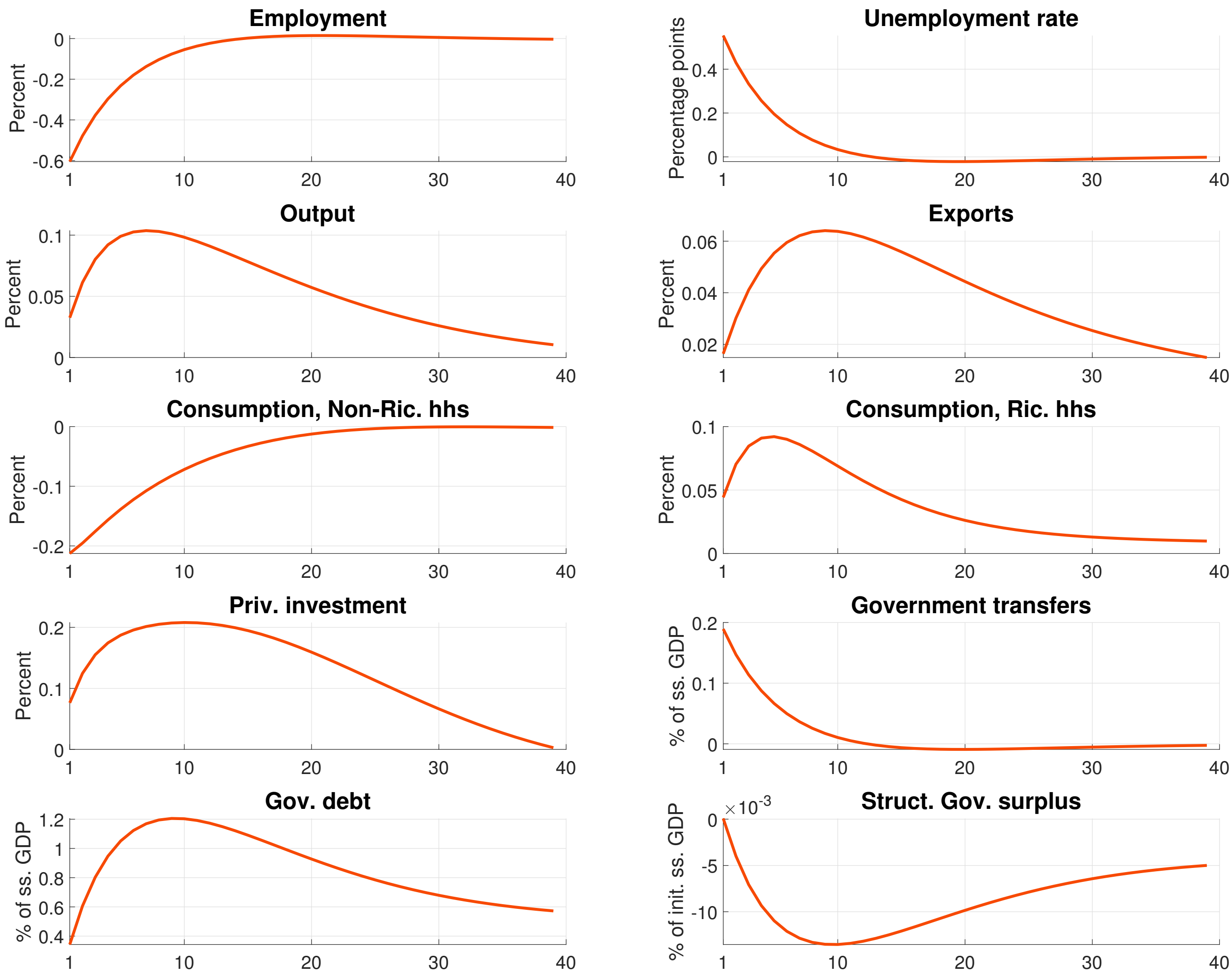


Figure 10: Economic outcome after a shock to productivity ε_t

consumption goods falls. This leads to a lower demand for domestically produced intermediate goods, which in turn leads to a decreased demand for labor and a higher unemployment rate. The latter induces the Swedish central bank to decrease the monetary policy rate. Initially the Foreign policy rate is lower than the Swedish rate. However, in the determination of the exchange rate the whole interest rate path is taken into account. After five years the foreign policy rate is higher than the Swedish one and the exchange rate is therefore depreciating.

Demand for Swedish exports falls due to the lower demand from abroad. The decline in Swedish exports leads to an additional decrease in demand for both domestically produced and imported intermediate goods. Due to lower demand in Sweden imports decrease as well.

Responding to the decline in wages following the lower demand, firms reduce their prices. Hence domestic inflation falls. In contrast, imported inflation doesn't change much due to mix effects from price stickiness and movements in the exchange rate. The effect on domestic inflation does thus dominate, leading CPIF inflation to fall.

An increase in the domestic risk premium leads to households requiring a higher return to capital and hence an increased rental cost of capital. This, together with the lower demand for domestically produced goods, reduces the demand for capital and hence private investments fall.

The decreased demand for domestically produced intermediate goods lowers the demand for labor and leads to a decline in wages, both in nominal and in real terms. Due to the lower labor income following the wage and employment decrease, Non-Ricardian households decrease their consumption.

Turning to the public sector, the tax on labor, consisting of both labor taxes levied on households and of social security contributions levied on firms, are the most important income source for the government. Tax income decreases due to the fall in output, leading to an increase in debt. Since the change in the tax income is only temporary it does not affect the structural surplus. The change in the structural surplus does primarily stem from the increase in debt and the resulting higher debt service cost. The transfers to the households are increased due to higher unemployment in the short and medium term but are decreased slightly to return the structural surplus to its target in the long term. Since the change in transfers is relatively small, there is only a small effect on Non-Ricardian consumption.

4.4 An external risk premium shock

In this section, we describe the economic outcome after an increase of $\epsilon_t^{\tilde{\phi}_t}$, which is the shock to the external risk premium $\tilde{\phi}_t$, by one standard deviation. The positive external risk premium shock makes holding of domestic currency bonds less attractive relative to holding bonds in foreign currency. As a result the exchange rate depreciates. The economic outcome is illustrated in Figures 13 and 14.

The depreciation of the Krona leads to a higher marginal cost for the Swedish import firms, leading to a lower markup. To restore the markup, the firms increase their prices, leading to a decrease in Swedish imports. Furthermore, the exchange rate depreciation leads to a higher markup for Swedish export firms. As a response, they reduce their prices, leading to higher exports.

The increased price of imported goods leads to higher costs for the consumption and investment good producers, who therefore increase their prices. The increased CPIF inflation leads to lower Ricardian consumption and private investment. Due to the lower demand for investment and consumption, demand for domestically produced inputs to investment and consumption also decrease. Since the decrease in consumption and investment is stronger than increase in exports, output decreases. Due to the decreased output, labor demand also decreases. This leads employment to be lower in the short term.

The increased demand for domestic goods leads to an increase in labor demand, putting a slight upwards pressure on nominal wages. The CPIF inflation does however increase more than the nominal wages, which means that real wages fall. This does in turn leads to a lower consumption for Non-Ricardian households in the short term. However, in the medium term, employment and real wages increase and thus the Non-Ricardian consumption increases in the medium term as well. Labor force participation decreases in the short term due to lower real wages, but increases in the medium term. Moreover, the decrease in employment is not strong thus unemployment is moving around the steady state with mixed sign.

Turning to the public sector, consumption tax revenue increases but other taxes decrease. Simultaneously, change in transfers are mixed due to mixed unemployment response. Therefore, overall, change in government surplus and change in government debt are mixed.

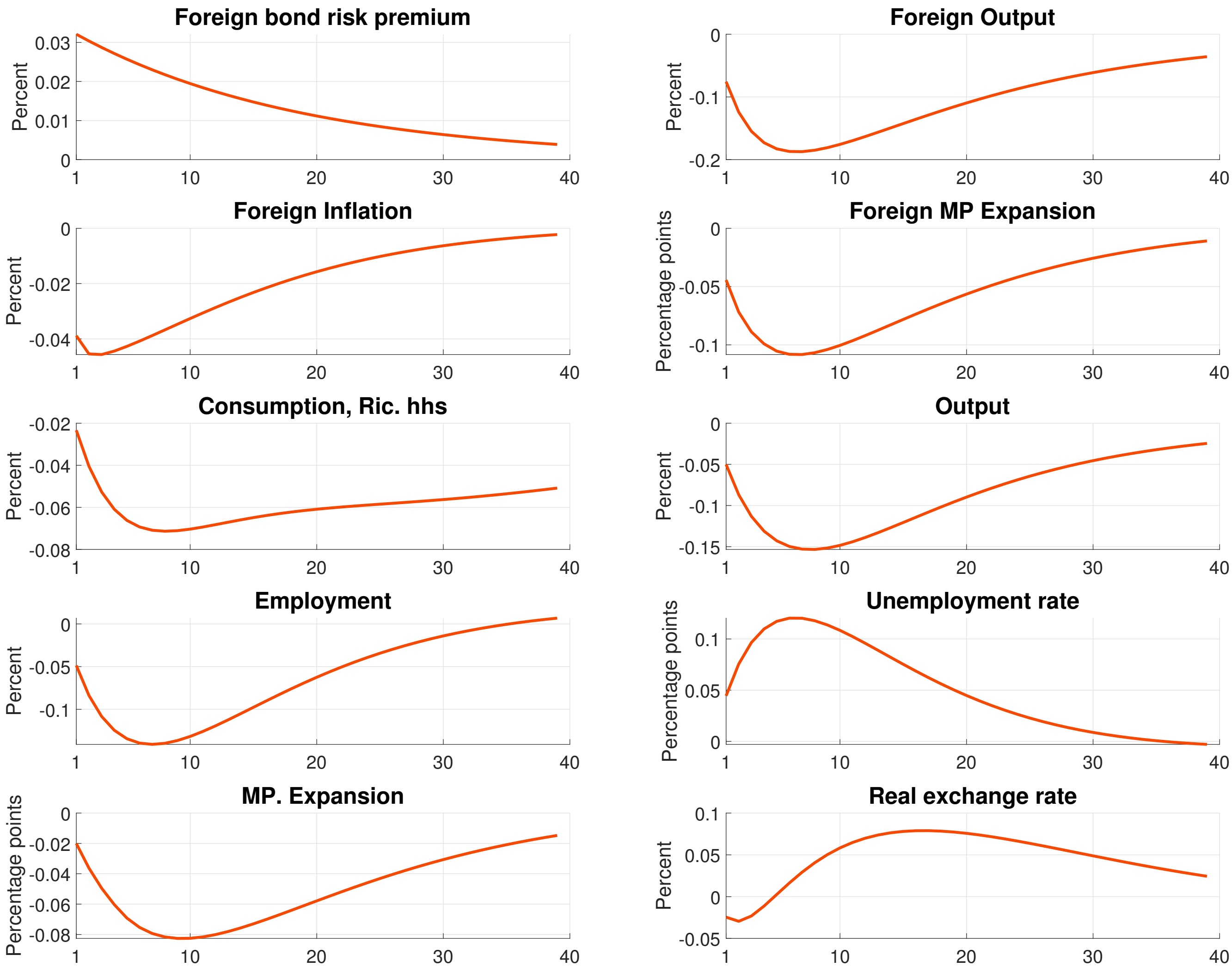


Figure 11: Economic outcome after a shock to Foreign bond risk premium $\zeta_{F,t}$

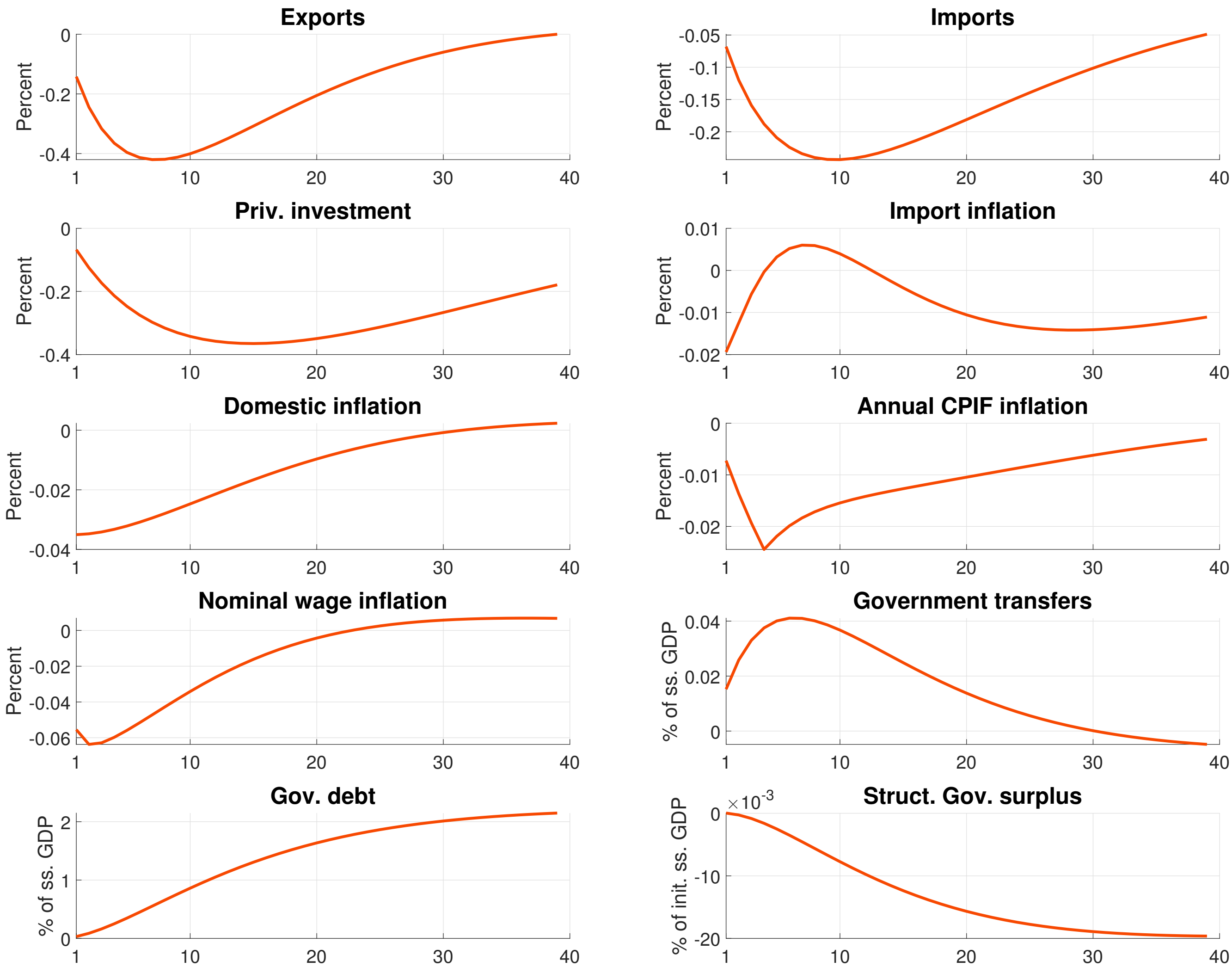


Figure 12: Economic outcome after a shock to Foreign bond risk premium $\zeta_{F,t}$

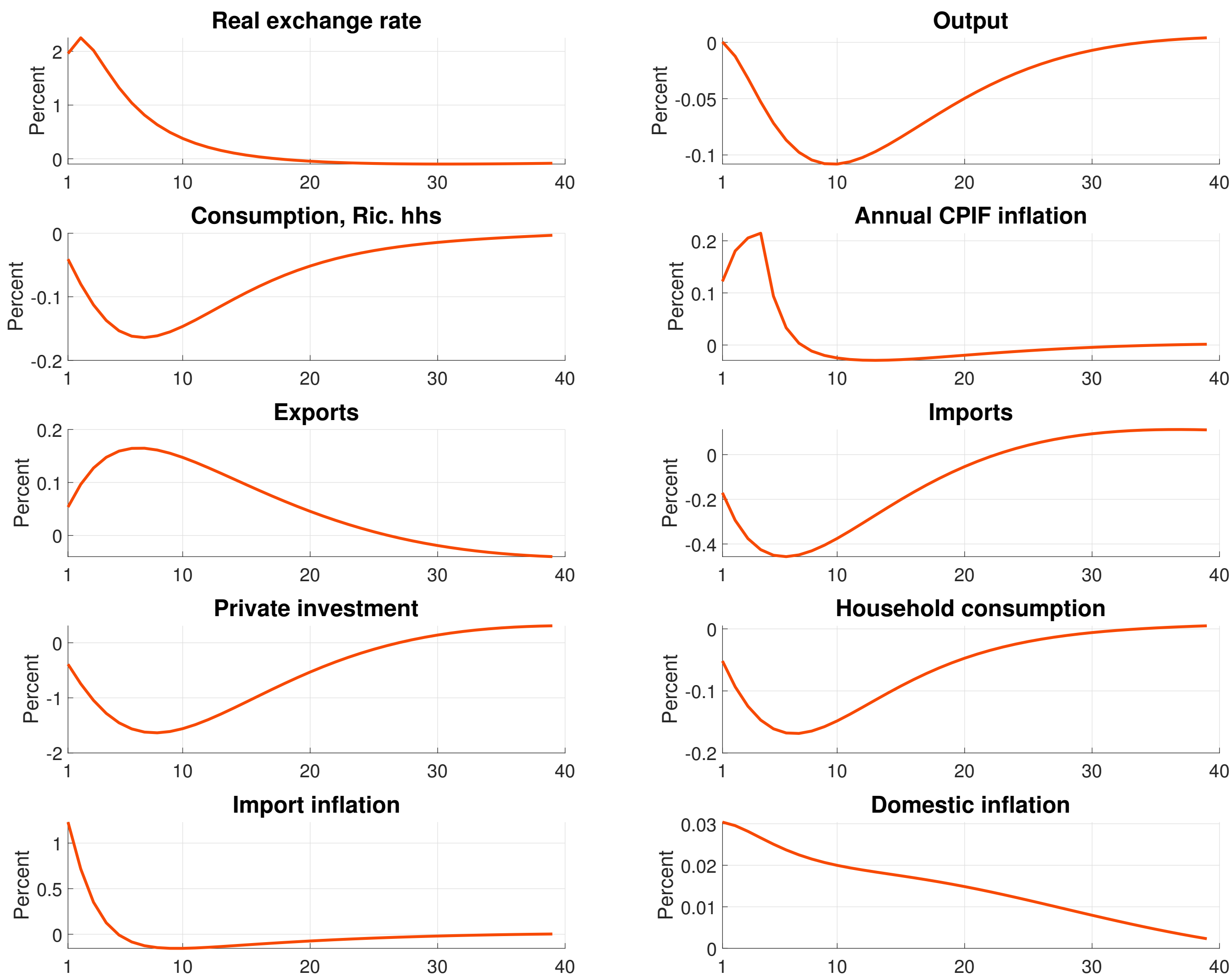


Figure 13: Economic outcome after a shock to the external risk premium $\hat{\tilde{\phi}}_t$

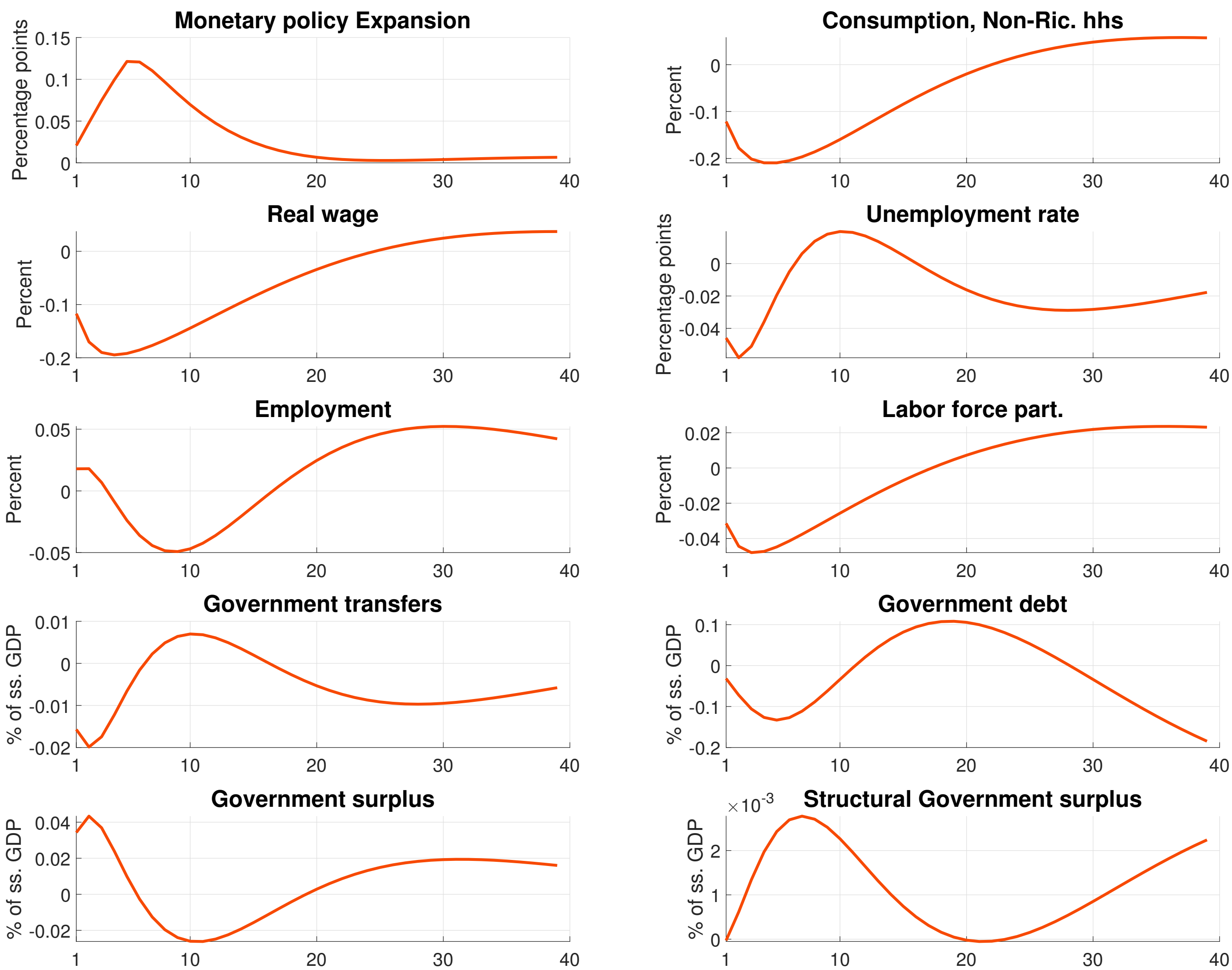


Figure 14: Economic outcome after a shock to the external risk premium $\hat{\tilde{\phi}}_t$

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Appendix

A Appendix: Model equations

In this appendix, we present the non-linear model equations and the corresponding log-linearized model equations.

Before we present the model equations, we clarify some notations. Variables that are trending along the balanced growth path have been stationarized. In most cases, we use the *bar* notation to distinguish stationarized variables from the non-stationary variables. For example, $\overline{C}_t^{agg} = \frac{C_t^{agg}}{z_t^+}$ denotes the stationarized level of aggregate household consumption in period t. K_t denotes the non-stationarized level of aggregate capital. Thus, $\overline{K}_t = \frac{K_t}{z_{t-1}(\gamma_{t-1})^{\frac{1}{1-\alpha}}}$ denotes the stationarized level of aggregate capital. $K_t^s = u_t K_t$, in turn, denotes the non-stationarized level of aggregate capital services, and $\overline{K}_t^s$ is the stationarized level of aggregate capital services which is defined as $\overline{K}_t^s = u_t \overline{K}_t$. The different indexation variables, such as the gross inflation of intermediate good prices $\overline{\Pi}_t$, constitutes an exception to the *bar* notation. The different gross inflation rates do not need to be stationarized. For these different gross inflation rates, the *bar* instead denotes the corresponding indexation variable.

When applicable, variables that appear in the model equations are expressed in *per capita* terms. For example, $\overline{C}_t^{agg}$ is the stationarized level of aggregate household consumption in the Swedish economy, and $\overline{c}_t^{agg}$ is the stationarized *per capita* level of aggregate household consumption in the Swedish economy. In the Swedish economy, where the size of the population is normalized to unity, the distinction between aggregate and *per capita* terms is trivial. For the Foreign economy, however, it is essential to distinguish, for example, between the stationarized level of aggregate household consumption $\overline{C}_{F,t}$, and the stationarized level of aggregate consumption per inhabitant, which is denoted by $\overline{c}_{F,t} = \frac{C_{F,t}}{z_t^+ \omega}$, where ω is the size of the population in Foreign.[71]

In addition to the non-linear form of model equations, we also present the corresponding log-linearized model equations. In this documentation, a variable with the *hat* notation can be interpreted as a log-linear approximation of the variable around its steady state (percent deviations). Two examples may help to clarify the use of the *hat* notation. The first example is for a variable such as $\overline{c}_t^{agg}$, which has been stationarized. Thus, we have: $\hat{c}_t^{agg} = ln\left(\frac{\overline{c}_t^{agg}}{\overline{c}^{agg}}\right)$, where $\overline{c}^{agg}$ denotes the steady state level of aggregate consumption *per capita* in the Swedish economy. $\hat{c}_t^{agg}$ can be interpreted as a log-linear approximation of the stationarized level of aggregate consumption *per capita* around its steady state level. The second example is for a variable such as Π_t, which does not need to be stationarized. Hence, we have: $\hat{\Pi}_t = ln\left(\frac{\Pi_t}{\Pi}\right)$, where Π denotes the steady state level of gross inflation of intermediate good prices. $\hat{\Pi}_t$ can be interpreted as a log-linear approximation of the gross inflation rate of intermediate goods around its steady state level.

Finally, a variable with *breve* notation is interpreted as an absolute deviation of the variable from its steady state. The first example is for a variable such as the real government debt $\overline{b}_t$, which has been stationarized. Thus, we have: $\breve{b}_t = \overline{b}_t - \overline{b}$, where $\overline{b}$ is the steady state level of the real government debt. $\breve{b}_t$ can be interpreted as an absolute deviation of the stationarized level of the real government debt from its steady state level. The second example for a variable such as $\breve{\tau}_t^C$, is the time-varying consumption tax rate, which does not need to be stationarized. Thus, we have: $\breve{\tau}_t^C = \tau_t^C - \tau^C$, where τ^C is the consumption tax rate in steady state. $\breve{\tau}_t^C$ is interpreted as an absolute deviation of the consumption tax rate from its steady state (percentage point deviations).

Our model equations include both equilibrium conditions and various definitions that are used to solve and simulate the model. In the subsequent sections, we present these equations.

A.1 Sweden: Household sector

Consumption Euler equation:

$$\overline{\Omega}_t^C = R_t \zeta_t E_t \left[\beta_{t+1}^r \frac{1}{\mu_{z^+,t+1}\Pi_{t+1}^C} \overline{\Omega}_{t+1}^C\right] \tag{A.1a}$$

$$\hat{\Omega}_t^C = E_t \left[\hat{\zeta}_t + \hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C + \frac{1}{R}\breve{i}_t - \hat{\Pi}_{t+1}^C - \hat{\mu}_{z^+,t+1}\right] \tag{A.1b}$$

[71] One exception to the *per capita* notation is investment: because we use i_t to denote the net nominal interest rate, $\bar{I}_t$ denotes both the stationarized level of aggregate investment and the stationarized level of aggregate investment per inhabitant in the Swedish economy.

Definition of nominal gross interest rate on private bonds:

$$R_t = 1 + i_t \tag{A.2a}$$

$$\hat{R}_t = \frac{1}{R}\breve{i}_t \tag{A.2b}$$

Lagrange multiplier, marginal utility of consumption equation:

$$\overline{\Omega}_t^C = \frac{\zeta_t^c}{(1+\tau_t^C)\left(\bar{\bar{c}}_t - \rho_h \frac{1}{\mu_{z^+,t}}\bar{\bar{c}}_{t-1}\right)}\left(\alpha_G \frac{\bar{\bar{c}}_t}{\bar{c}_t}\right)^{\frac{1}{\upsilon_G}} \tag{A.3a}$$

$$\hat{\Omega}_t^C = \hat{\zeta}_t^c + \left(1 - \frac{\rho_h}{\mu_{z^+,}}\right)^{-1}\left[-\hat{\bar{c}}_t + \frac{\rho_h}{\mu_{z^+,}}\hat{\bar{c}}_{t-1} - \frac{\rho_h}{\mu_{z^+}}\hat{\mu}_{z^+,t}\right] + \frac{1}{\upsilon_G}\left(\hat{\bar{c}}_t - \hat{c}_t\right) - \frac{1}{1+\tau^C}\breve{\tau}_t^C \tag{A.3b}$$

Marginal utility of consumption equation:

$$\bar{U}_{c,t} = \frac{\zeta_t^c}{\left(\bar{\bar{c}}_t - \rho_h \frac{1}{\mu_{z^+,t}}\bar{\bar{c}}_{t-1}\right)}\left(\alpha_G \frac{\bar{\bar{c}}_t}{\bar{c}_t}\right)^{\frac{1}{\upsilon_G}} \tag{A.4a}$$

$$\hat{U}_{c,t} = \hat{\zeta}_t^c + \left(1 - \frac{\rho_h}{\mu_{z^+,}}\right)^{-1}\left[-\hat{\bar{c}}_t + \frac{\rho_h}{\mu_{z^+,}}\hat{\bar{c}}_{t-1} - \frac{\rho_h}{\mu_{z^+}}\hat{\mu}_{z^+,t}\right] + \frac{1}{\upsilon_G}\left(\hat{\bar{c}}_t - \hat{c}_t\right) \tag{A.4b}$$

Composite consumption function:

$$\bar{\bar{c}}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}}\, \bar{c}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}}\, \bar{g}_t^{\frac{\upsilon_G - 1}{\upsilon_G}}\right)^{\frac{\upsilon_G}{\upsilon_G - 1}} \tag{A.5a}$$

$$\left(\frac{\bar{\bar{c}}}{\bar{c}}\right)^{\frac{\upsilon_G - 1}{\upsilon_G}}\hat{\bar{c}}_t = \alpha_G^{\frac{1}{\upsilon_G}}\hat{c}_t + (1-\alpha_G)^{\frac{1}{\upsilon_G}}\left(\frac{\bar{g}}{\bar{c}}\right)^{\frac{\upsilon_G - 1}{\upsilon_G}}\hat{g}_t \tag{A.5b}$$

Average interest rate on government bonds:

$$\bar{\Omega}_t^R = E_t \beta_{t+1}^r \frac{\bar{\Omega}_{t+1}^C}{\bar{\Omega}_t^C \Pi_{t+1}^C \mu_{z^+,t+1}}\left[1 + \bar{\Omega}_{t+1}^R(1-\alpha_B)\right] \tag{A.6a}$$

$$\hat{\Omega}_t^R = E_t\left[\hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C + \frac{\bar{\Omega}^R(1-\alpha_B)}{1+\bar{\Omega}^R(1-\alpha_B)}\hat{\Omega}_{t+1}^R - \hat{\Omega}_t^C - \hat{\Pi}_{t+1}^C - \hat{\mu}_{z^+,t+1}\right] \tag{A.6b}$$

Euler equation for government bond holdings:

$$1 = E_t \beta_{t+1}^r \frac{\bar{\Omega}_{t+1}^C}{\bar{\Omega}_t^C \Pi_{t+1}^C \mu_{z^+,t+1}}\left[R_t^{B,n} - (1-\alpha_B)\bar{\Omega}_{t+1}^R\left(R_{t+1}^{B,n} - R_t^{B,n}\right)\right] \tag{A.7a}$$

$$0 = E_t\hat{\beta}_{t+1}^r + E_t\hat{\Omega}_{t+1}^C - \hat{\Omega}_t^C - E_t\hat{\Pi}_{t+1}^C - E_t\hat{\mu}_{z^+,t+1} + \frac{1}{R^{B,n}}\breve{R}_t^{B,n} - (1-\alpha_B)\bar{\Omega}^R\left(E_t\breve{R}_{t+1}^{B,n} - \breve{R}_t^{B,n}\right) \tag{A.7b}$$

Capital utilization decision equation:

$$r_t^K = p_t^I a'(u_t) \tag{A.8a}$$

$$\hat{r}_t^K = \hat{p}_t^I + \sigma_a \hat{u}_t \tag{A.8b}$$

Household purchases of installed capital equation:

$$p_t^K = E_t \beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C \Pi_{t+1}}{\overline{\Omega}_t^C \Pi_{t+1}^C \mu_{z^+,t+1}\mu_{\gamma,t+1}}\left[\left(1-\tau_{t+1}^K\right)\left(r_{t+1}^K u_{t+1} - p_{t+1}^I a(u_{t+1})\right) + \iota^K \delta \tau_{t+1}^K \frac{\mu_{\gamma,t+1}}{\Pi_{t+1}}p_t^K + p_{t+1}^K(1-\delta)\right] \tag{A.9a}$$

$$\left(1-\iota^K\frac{1}{H}\tau^K\delta\frac{\mu_\gamma}{\Pi}p^K\right)\left(\hat{p}_t^K-\hat{\Pi}_{t+1}+\hat{\mu}_{\gamma,t+1}\right)=$$

$$E_t\hat{\beta}^r_{t+1}+E_t\hat{\Omega}^C_{t+1}-\hat{\Omega}^C_t-E_t\hat{\Pi}^C_{t+1}-E_t\hat{\mu}_{z^+,t+1}+\frac{1}{H}r^K\left(1-\tau^K\right)E_t\hat{r}^K_{t+1}-\frac{1}{H}\left(r^K-\iota^K\delta\frac{\mu_\gamma}{\Pi}p^K\right)E_t\breve{\tau}^K_{t+1}+\frac{1}{H}p^K(1-\delta)E_t\hat{p}^K_{t+1} \tag{A.9b}$$

Household investment decision equation:

$$p_t^I\left(1-\tau_t^I\right)=p_t^K\Upsilon_tF_1(\overline{I}_t,\overline{I}_{t-1},\mu_{z^+,t},\mu_{\gamma,t})+E_t\left[\beta^r_{t+1}\frac{\overline{\Omega}^C_{t+1}\Pi_{t+1}}{\overline{\Omega}^C_t\Pi^C_{t+1}}\frac{p^K_{t+1}}{\mu_{z^+,t+1}\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(\overline{I}_{t+1},\overline{I}_t,\mu_{z^+,t+1},\mu_{\gamma,t+1})\right] \tag{A.10a}$$

$$(1-\tau^I)\frac{p^I}{p^K\Upsilon}\hat{p}^I_t-\breve{\tau}^I_t=\hat{p}^K_t+\hat{\Upsilon}_t-S''\left(\mu_{z^+}\mu_\gamma\right)^2E_t\left[\triangle\hat{I}_t+\hat{\mu}_{z^+,t}+\hat{\mu}_{\gamma,t}-\beta\triangle\hat{I}_{t+1}-\beta\hat{\mu}_{z^+,t+1}-\beta\hat{\mu}_{\gamma,t+1}\right] \tag{A.10b}$$

Definition of capital services:

$$\overline{k}^s_t=u_t\bar{k}_t \tag{A.11a}$$

$$\hat{k}^s_t=\hat{u}_t+\hat{k}_t \tag{A.11b}$$

Capital accumulation equation:

$$\overline{k}_{t+1}=(1-\delta)\overline{k}_t\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}+\Upsilon_t\left[1-\widetilde{S}\left(\frac{\overline{I}_t\mu_{z^+}\mu_\gamma}{\overline{I}_{t-1}}\right)\right]\overline{I}_t+\overline{\triangle}^K_t \tag{A.12a}$$

$$\hat{k}_{t+1}=\frac{(1-\delta)}{\mu_{z^+}\mu_\gamma}\left(\hat{k}_t-\hat{\mu}_{z^+,t}-\hat{\mu}_{\gamma,t}\right)+\frac{\overline{I}}{\overline{k}}\Upsilon\left(\hat{I}_t+\hat{\Upsilon}_t\right) \tag{A.12b}$$

Optimal wage setting equation:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^{k}\beta^r_{t+i}\right)n_{t+k|t}\overline{\Omega}^C_{t+k}\frac{1}{(1-\lambda^W_{t+k})}\left[\left(1-\tau^W_{t+k}\right)\overline{w}_{t+k|t}-\lambda^W_{t+k}\zeta^n_{t+k}\frac{\nu'(n_{t+k|t})}{\overline{\Omega}^C_{t+k}}\right]=0 \tag{A.13a}$$

$$\triangle\hat{w}_t=\beta E_t\left[\triangle\hat{w}_{t+1}\right]-\kappa_W(\hat{\Psi}^W_t-\hat{\lambda}^W_t)+\hat{\overline{\Pi}}^W_t-\hat{\mu}_{z^+,t}-\hat{\Pi}^C_t-\beta E_t\left[\hat{\overline{\Pi}}^W_{t+1}-\hat{\mu}_{z^+,t+1}-\hat{\Pi}^C_{t+1}\right] \tag{A.13b}$$

Labor force participation equation:

$$\overline{\Omega}^c_t(1-\tau^W_t)\overline{w}_t = \zeta^n_t\Theta^n_tA_nl^\eta_t \tag{A.14a}$$

$$\hat{w}_t = \hat{\zeta}^n_t+\hat{\Theta}^n_t+\eta\hat{l}_t-\hat{\Omega}^C_t+\frac{1}{1-\tau^W}\breve{\tau}^W_t \tag{A.14b}$$

Definition of endogenous shifter equation:

$$\Theta^n_t = \bar{Z}^n_t\bar{U}_{c,t} \tag{A.15a}$$

$$\hat{\Theta}^n_t = \hat{Z}^n_t+\hat{U}_{c,t} \tag{A.15b}$$

Trend of wealth effect in endogenous shifter:

$$\bar{Z}^n_t = \left(\frac{\bar{Z}^n_{t-1}}{\mu_{z^+,t}}\right)^{1-\chi_n}(\bar{U}_{c,t})^{-\chi_n} \tag{A.16a}$$

$$\hat{Z}^n_t = (1-\chi_n)\hat{Z}^n_{t-1}-(1-\chi_n)\hat{\mu}_{z^+,t}-\chi_n\hat{U}_{c,t} \tag{A.16b}$$

Unemployment definition:

$$un_t = \frac{L_t-N_t}{L_t} \tag{A.17a}$$

$$\breve{un}_t = \frac{n}{l}\left(\hat{l}_t-\hat{n}_t\right) \tag{A.17b}$$

Real wage markup equation:

$$\overline{\Psi}_t^W = \frac{\left(1-\tau_t^W\right)\overline{w}_t}{\zeta_t^n \frac{\nu'(n_t)}{\overline{\Omega}_t^C}} \tag{A.18a}$$

$$\hat{\Psi}_t^W = \eta\left(\hat{l}_t - \hat{n}_t\right) \tag{A.18b}$$

Definition of wage inflation:

$$\Pi_t^W = \frac{\overline{w}_t}{\overline{w}_{t-1}}\mu_{z^+,t}\Pi_t^C \tag{A.19a}$$

$$\hat{\Pi}_t^W = \triangle\hat{w}_t + \hat{\mu}_{z^+,t} + \hat{\Pi}_t^C \tag{A.19b}$$

Definition of wage inflation indexation:

$$\overline{\Pi}_t^W = (\Pi_{t-1}^W)^{\chi_w}(\Pi_t^{trend})^{1-\chi_w} \tag{A.20a}$$

$$\hat{\overline{\Pi}}_t^W = \chi_w\hat{\Pi}_{t-1}^W + (1-\chi_W)\hat{\Pi}_t^{trend} \tag{A.20b}$$

Real wage relevant to employers:

$$\overline{w}_t^e = \overline{w}_t p_t^C \tag{A.21a}$$

$$\hat{w}_t^e = \hat{w}_t + \hat{p}_t^C \tag{A.21b}$$

Modified uncovered interest rate parity equation:

$$R_t E_t\left[\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C}\right] = R_{F,t}\Phi\left(\overline{a}_t, s_t, \widetilde{\phi}_t\right)E_t\left[\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C}s_{t+1}\right] \tag{A.22a}$$

$$\frac{1}{R}\left(\breve{i}_t - \breve{i}_{F,t}\right) = \left(1-\widetilde{\phi}_s\right)E_t\left[\hat{s}_{t+1}\right] - \widetilde{\phi}_s\hat{s}_t - \widetilde{\phi}_a\breve{a}_t + \hat{\widetilde{\phi}}_t \tag{A.22b}$$

Aggregate consumption:

$$\bar{c}_t^{agg} = (1-s_{nr})\bar{c}_t + s_{nr}\bar{c}_t^{nr} \tag{A.23a}$$

$$\bar{c}^{agg}\hat{c}_t^{agg} = (1-s_{nr})\bar{c}\hat{c}_t + s_{nr}\bar{c}^{nr}\hat{c}_t^{nr} \tag{A.23b}$$

Non-Ricardian budget constraint:

$$(1+\tau_t^C)p_t^C\bar{c}_t^{nr} = (1-\tau_t^W)\bar{w}_t^e n_t + \left(1-\tau_t^{TR}\right)\overline{tr}_t^{nr} \tag{A.24a}$$

$$\left(1+\tau^C\right)p^C\bar{c}^{nr}\left(\hat{c}_t^{nr} + \hat{p}_t^C\right) + p^C\bar{c}^{nr}\breve{\tau}_t^C = (1-\tau^W)\bar{w}^e n\left(\hat{w}_t^e + \hat{n}_t\right) - \bar{w}^e n\breve{\tau}_t^W + (1-\tau^{TR})\breve{tr}_t^{nr} - \overline{tr}^{nr}\breve{\tau}_t^{TR} \tag{A.24b}$$

A.2 Sweden: Firm sector

A.2.1 Sweden: Intermediate good producers

Definition of composite technological growth rate:

$$\mu_{z^+,t} = \mu_{z,t}\left(\mu_{\gamma,t}\right)^{\frac{\alpha}{1-\alpha}} \tag{A.25a}$$

$$\hat{\mu}_{z^+,t} = \hat{\mu}_{z,t} + \frac{\alpha}{1-\alpha}\hat{\mu}_{\gamma,t} \tag{A.25b}$$

Real marginal cost of production for intermediate good producers equation:

$$\overline{mc}_t = \frac{\left(\left(1+\tau_t^{SSC}\right)\overline{w}_t^e\right)^{1-\alpha}(r_t^K)^\alpha}{\varepsilon_t \alpha^\alpha (1-\alpha)^{1-\alpha}\bar{\Gamma}_{G,t}} \tag{A.26a}$$

$$\hat{mc}_t = (1-\alpha)\left(\hat{w}_t^e + \frac{1}{1+\tau^{SSC}}\breve{\tau}_t^{SSC}\right) + \alpha\hat{r}_t^K - \hat{\varepsilon}_t - \hat{\Gamma}_{G,t} \tag{A.26b}$$

Simplifying expression variable Gamma:

$$\bar{\Gamma}_{G,t} = \alpha_K^{\frac{\alpha}{\upsilon_K}}\left(\frac{\bar{\bar{k}}_t^s}{\bar{\bar{k}}_t^s}\right)^{\frac{\alpha}{\upsilon_K}} \tag{A.27a}$$

$$\hat{\Gamma}_{G,t} = -\frac{\alpha}{\upsilon_K}\left(\hat{\bar{k}}^s{}_t - \hat{k}_t^s\right) \tag{A.27b}$$

Real rental rate for capital services equation:

$$r_t^K = \alpha\varepsilon_t\left(\frac{\bar{\bar{k}}_t^s}{n_t}\frac{1}{\mu_{z+,t}\mu_{\gamma,t}}\right)^{\alpha-1}\overline{mc}_t\bar{\Gamma}_{G,t}^{\frac{1}{\alpha}} \tag{A.28a}$$

$$\hat{r}_t^K = \hat{\varepsilon}_t + (\alpha-1)\left(\hat{\bar{k}}_t^s - \hat{n}_t - \hat{\mu}_{z+,t} - \hat{\mu}_{\gamma,t}\right) + \hat{mc}_t + \frac{1}{\alpha}\hat{\Gamma}_{G,t} \tag{A.28b}$$

Composite capital function:

$$\bar{\bar{k}}_t^s = \left(\alpha_K{}^{\frac{1}{\upsilon_K}}\left(\bar{k}_t^s\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}}\left(\bar{k}_{G,t}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K-1}} \tag{A.29a}$$

$$\hat{\bar{k}}_t^s = \alpha_K{}^{\frac{1}{\upsilon_K}}\left(\frac{\bar{k}^s}{\bar{\bar{k}}^s}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\hat{k}_t^s + (1-\alpha_K)^{\frac{1}{\upsilon_K}}\left(\frac{\bar{k}_G}{\bar{\bar{k}}^s}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\hat{k}_{G,t} \tag{A.29b}$$

Public capital accumulation equation:

$$\bar{k}_{G,t+1} = (1-\delta_G)\bar{k}_{G,t}\frac{1}{\mu_{z+,t}\mu_{\gamma,t}} + \bar{I}_t^G \tag{A.30a}$$

$$\hat{k}_{G,t+1} = \frac{(1-\delta_G)}{\mu_{z+}\mu_\gamma}\left(\hat{k}_{G,t} - \hat{\mu}_{z+,t} - \hat{\mu}_{\gamma,t}\right) + \frac{\bar{I}^G}{\bar{k}_G}\hat{I}_t^G \tag{A.30b}$$

Optimal price of intermediate goods equation:[72]

$$E_t\sum_{k=0}^{\infty}(\xi)^k\left(\prod_{j=1}^{k}\beta_{t+j}^r\right)\frac{\overline{\Omega}_{t+k}^C}{\overline{\Omega}_t^C}\left(\prod_{j=1}^{k}\frac{\Pi_{t+j}}{\Pi_{t+j}^C}\right)\frac{\overline{y}_{t+k|t}}{(\lambda_{t+k}-1)}\left[\left(\prod_{j=1}^{k}\frac{\overline{\Pi}_{t+j}}{\Pi_{t+j}}\right)\frac{p_t^{opt}}{\Pi_t} - \lambda_{t+k}\overline{mc}_{t+k}\right] = 0 \tag{A.31a}$$

$$\hat{\Pi}_t = \beta E_t\left[\hat{\Pi}_{t+1} - \hat{\overline{\Pi}}_{t+1}\right] + \kappa\left(\frac{1}{\kappa}\hat{\lambda}_t + \hat{mc}_t\right) + \hat{\overline{\Pi}}_t \tag{A.31b}$$

Definition of intermediate good price inflation indexation:

$$\overline{\Pi}_t = (\Pi_{t-1})^\chi(\Pi_t^{trend})^{1-\chi} \tag{A.32a}$$

$$\hat{\overline{\Pi}}_t = \chi\hat{\Pi}_{t-1} + (1-\chi)\hat{\Pi}_t^{trend} \tag{A.32b}$$

[72]We scale the markup shock $\hat{\lambda}_t$ by $\frac{1}{\kappa}$.

A.2.2 Sweden: Consumption good producers

Relative price of consumption goods equation:

$$p_t^C = \left[\vartheta^C \left(p_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(p_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}} \tag{A.33a}$$

$$\hat{p_t}^C = \vartheta^C \left(\frac{p^{C,xe}}{p^C}\right)^{1-\nu_C} \hat{p_t}^{C,xe} + \left(1-\vartheta^C\right)\left(\frac{p^{C,e}}{p^C}\right)^{1-\nu_{c,xe}} \hat{p_t}^{C,e} \tag{A.33b}$$

Definition of consumption good price inflation:

$$\Pi_t^C = \frac{p_t^C}{p_{t-1}^C}\Pi_t \tag{A.34a}$$

$$\hat{\Pi}_t^C = \hat{p}_t^C - \hat{p}_{t-1}^C + \hat{\Pi}_t \tag{A.34b}$$

Demand for non-energy consumption goods equation:

$$\bar{c}_t^{xe} = \vartheta^C \left(\frac{p_t^{C,xe}}{p_t^C}\right)^{-\nu_C} \bar{c}_t^{agg} \tag{A.35a}$$

$$\hat{c}_t^{xe} = \nu_C\left(\hat{p}_t^C - \hat{p}_t^{C,xe}\right) + \hat{c}_t^{agg} \tag{A.35b}$$

Demand for energy consumption goods equation:

$$\bar{c}_t^{e} = \left(1-\vartheta^C\right)\left(\frac{p_t^{C,e}}{p_t^C}\right)^{-\nu_C} \bar{c}_t^{agg} \tag{A.36a}$$

$$\hat{c}_t^{e} = \nu_C\left(\hat{p}_t^C - \hat{p}_t^{C,e}\right) + \hat{c}_t^{agg} \tag{A.36b}$$

Relative price of non-energy consumption goods equation:

$$p_t^{C,xe} = \left[\vartheta^{C,xe} + \left(1-\vartheta^{C,xe}\right)\left(p_t^{M,C,xe}\right)^{1-\nu_{c,xe}}\right]^{\frac{1}{1-\nu_{c,xe}}} \tag{A.37a}$$

$$\hat{p_t}^{C,xe} = \left(1-\vartheta^{C,xe}\right)\left(\frac{p^{M,Cxe}}{p^{C,xe}}\right)^{1-\nu_{c,xe}} \hat{p_t}^{M,C,xe} \tag{A.37b}$$

Definition of non-energy consumption good price inflation:

$$\Pi_t^{C,xe} = \frac{p_t^{C,xe}}{p_{t-1}^{C,xe}}\Pi_t \tag{A.38a}$$

$$\hat{\Pi}_t^{C,xe} = \hat{p}_t^{C,xe} - \hat{p}_{t-1}^{C,xe} + \hat{\Pi}_t \tag{A.38b}$$

Relative price of energy consumption goods equation:

$$p_t^{C,e} = \left[\vartheta^{C,e}\left(p_t^{D,C,e}\right)^{1-\nu_{c,e}} + \left(1-\vartheta^{C,e}\right)\left(p_t^{M,C,e}\right)^{1-\nu_{c,e}}\right]^{\frac{1}{1-\nu_{c,e}}} \tag{A.39a}$$

$$\hat{p_t}^{C,e} = \vartheta^{C,e}\left(\frac{p^{D,C,e}}{p^{C,e}}\right)^{1-\nu_{c,e}} \hat{p_t}^{D,C,e} + \left(1-\vartheta^{C,e}\right)\left(\frac{p^{M,C,e}}{p^{C,e}}\right)^{1-\nu_{c,e}} \hat{p_t}^{M,C,e} \tag{A.39b}$$

Definition of energy consumption good price inflation:

$$\Pi_t^{C,e} = \frac{p_t^{C,e}}{p_{t-1}^{C,e}}\Pi_t \tag{A.40a}$$

$$\hat{\Pi}_t^{C,e} = \hat{p}_t^{C,e} - \hat{p}_{t-1}^{C,e} + \hat{\Pi}_t \tag{A.40b}$$

Demand for domestic energy equation:

$$\bar{d}_t^{e} = \vartheta^{C,e}\left(\frac{p_t^{D,C,e}}{p_t^{C,e}}\right)^{-\nu_{C,e}} \bar{c}_t^{e} \tag{A.41a}$$

$$\hat{d}_t^{e} = \nu_{C,e}\left(\hat{p}_t^{C,e} - \hat{p}_t^{D,C,e}\right) + \hat{c}_t^{e} \tag{A.41b}$$

Demand for imported energy equation:

$$\bar{m}_t^{C,e} = \left(1 - \vartheta^{C,e}\right)\left(\frac{p_t^{M,C,e}}{p_t^{C,e}}\right)^{-\nu_{C,e}} \bar{c}_t^e \tag{A.42a}$$

$$\hat{c}_t^e = \nu_{C,e}\left(\hat{p}_t^{C,e} - \hat{p}_t^{M,C,e}\right) + \hat{c}_t^e \tag{A.42b}$$

Definition of domestic energy inflation:

$$\Pi_t^{D,C,e} = \frac{p_t^{D,C,e}}{p_{t-1}^{D,C,e}}\Pi_t \tag{A.43a}$$

$$\hat{\Pi}_t^{D,C,e} = \hat{p}_t^{D,C,e} - \hat{p}_{t-1}^{D,C,e} + \hat{\Pi}_t \tag{A.43b}$$

A.2.3 Sweden: Investment good producers

Relative price of investment goods equation:

$$p_t^I = \left[\vartheta^I + \left(1 - \vartheta^I\right)\left(p_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}} \tag{A.44a}$$

$$\hat{p}_t^I = \left(1 - \vartheta^I\right)\left(\frac{p^{M,I}}{p^I}\right)^{1-\nu_I} \hat{p}_t^{M,I} \tag{A.44b}$$

Definition of investment good price inflation:

$$\Pi_t^I = \frac{p_t^I}{p_{t-1}^I}\Pi_t \tag{A.45a}$$

$$\hat{\Pi}_t^I = \hat{p}_t^I - \hat{p}_{t-1}^I + \hat{\Pi}_t \tag{A.45b}$$

A.2.4 Sweden: Export good producers

Real marginal cost of production for export good producers equation:

$$\overline{mc}_t^X = \left[\vartheta^X + \left(1 - \vartheta^X\right)\left(p_t^{M,X}\right)^{1-\nu_x}\right]^{\frac{1}{1-\nu_x}} \tag{A.46a}$$

$$\hat{mc}_t^X = \left(1 - \vartheta^X\right)\left(\frac{p^{M,X}}{mc^X}\right)^{1-\nu_x} \hat{p}_t^{M,X} \tag{A.46b}$$

Optimal price of export goods equation:[73]

$$E_t \sum_{k=0}^{\infty} (\xi_x)^k \left(\prod_{j=1}^{k} \beta_{t+j}^r\right) \frac{\overline{\Omega}_{t+k}^C}{\overline{\Omega}_t^C} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi_{t+j}^C}\right) \frac{\overline{x}_{t+k|t}}{(\lambda_{t+k}^X - 1)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}_{t+j}^X s_{t+j}}{\Pi_{t+j}}\right) p_t^{X,opt} - \lambda_{t+k}^X \overline{mc}_{t+k}^X\right] = 0 \tag{A.47a}$$

$$\hat{\Pi}_t^X = \beta E_t\left[\hat{\Pi}_{t+1}^X - \hat{\overline{\Pi}}_{t+1}^X\right] + \kappa_X\left(\frac{1}{\kappa_X}\hat{\lambda}_t^X + \hat{mc}_t^X - \hat{p}_t^X\right) + \hat{\overline{\Pi}}_t^X \tag{A.47b}$$

Definition of export good price inflation indexation:

$$\overline{\Pi}_t^X = \left(\Pi_{t-1}^X\right)^{\chi_x}\left(\Pi_F^{trend}\right)^{1-\chi_x} \tag{A.48a}$$

$$\hat{\overline{\Pi}}_t^X = \chi_x \hat{\Pi}_{t-1}^X + (1 - \chi_x)\hat{\Pi}_t^{trend} \tag{A.48b}$$

Definition of export good price inflation:

$$\frac{p_t^X}{p_{t-1}^X} = \frac{\Pi_t^X s_t}{\Pi_t} \tag{A.49a}$$

$$\hat{p}_t^X = \hat{p}_{t-1}^X + \hat{\Pi}_t^X - \hat{\Pi}_t + \hat{s}_t \tag{A.49b}$$

[73] We scale the markup shock $\hat{\lambda}_t^X$ by $\frac{1}{\kappa_X}$.

A.2.5 Sweden: Import good producers

Optimal price for import firms specializing in non-energy consumption goods equation:[74]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,C,xe})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^{C,xe}_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^{C,xe}_{t+j}} \right) \frac{\overline{m}^{C,xe}_{t+k|t}}{\left(\lambda^{M,C}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,C,xe}_{t+j}}{\Pi_{t+j}} \right) p^{M,C,xe}_{t,opt} - \lambda^{M,C,xe}_t \overline{mc}^{M,C,xe}_{F,t+k} \right] = 0 \tag{A.50a}$$

$$\hat{\Pi}^{M,C,xe}_t = \beta E_t \left[\hat{\Pi}^{M,C,xe}_{t+1} - \hat{\overline{\Pi}}^{M,C,xe}_{t+1} \right] + \kappa_{M,C,xe} \left(\frac{1}{\kappa_{M,C,xe}} \hat{\lambda}^{M,C,xe}_t + \hat{mc}^{M,C,xe}_{F,t} - \hat{p}^{M,C,xe}_t \right) + \hat{\overline{\Pi}}^{M,C,xe}_t \tag{A.50b}$$

Definition of import price inflation indexation, import firms specializing in non-energy consumption goods:

$$\overline{\Pi}^{M,C,xe}_t = \left(\Pi^{M,C,xe}_{t-1}\right)^{\chi_{m,C,xe}} (\Pi^{trend}_t)^{1-\chi_{m,C,xe}} \tag{A.51a}$$

$$\hat{\overline{\Pi}}^{M,C,xe}_t = \chi_{m,C,xe} \hat{\Pi}^{M,C,xe}_{t-1} + (1-\chi_{m,C,xe}) \hat{\Pi}^{trend}_t \tag{A.51b}$$

Definition of import price inflation, import firms specializing in non-energy consumption goods:

$$\frac{p^{M,C,xe}_t}{p^{M,C,xe}_{t-1}} = \frac{\Pi^{M,C,xe}_t}{\Pi_t} \tag{A.52a}$$

$$\hat{p}^{M,C,xe}_t = \hat{p}^{M,C,xe}_{t-1} + \hat{\Pi}^{M,C,xe}_t - \hat{\Pi}_t \tag{A.52b}$$

Optimal price for import firms specializing in investment goods equation:[75]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,I})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^{C}_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^{C}_{t+j}} \right) \frac{\overline{m}^{I}_{t+k|t}}{\left(\lambda^{M,I}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,I}_{t+j}}{\Pi_{t+j}} \right) p^{M,I}_{t,opt} - \lambda^{M,I}_t \overline{mc}^{M,I}_{F,t+k} \right] = 0 \tag{A.53a}$$

$$\hat{\Pi}^{M,I}_t = \beta E_t \left[\hat{\Pi}^{M,I}_{t+1} - \hat{\overline{\Pi}}^{M,I}_{t+1} \right] + \kappa_{M,I} \left(\frac{1}{\kappa_{M,I}} \hat{\lambda}^{M,I}_t + \hat{mc}^{M,I}_{F,t} - \hat{p}^{M,I}_t \right) + \hat{\overline{\Pi}}^{M,I}_t \tag{A.53b}$$

Definition of import price inflation indexation, import firms specializing in investment goods:

$$\overline{\Pi}^{M,I}_t = \left(\Pi^{M,I}_{t-1}\right)^{\chi_{m,I}} (\Pi^{trend}_t)^{1-\chi_{m,I}} \tag{A.54a}$$

$$\hat{\overline{\Pi}}^{M,I}_t = \chi_{m,I} \hat{\Pi}^{M,I}_{t-1} + (1-\chi_{m,I}) \hat{\Pi}^{trend}_t \tag{A.54b}$$

Definition of import price inflation, import firms specializing in investment goods:

$$\frac{p^{M,I}_t}{p^{M,I}_{t-1}} = \frac{\Pi^{M,I}_t}{\Pi_t} \tag{A.55a}$$

$$\hat{p}^{M,I}_t = \hat{p}^{M,I}_{t-1} + \hat{\Pi}^{M,I}_t - \hat{\Pi}_t \tag{A.55b}$$

Optimal price for import firms specializing in export goods equation:[76]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,X})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^{C}_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^{C}_{t+j}} \right) \frac{\overline{m}^{X}_{t+k|t}}{\left(\lambda^{M,X}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,X}_{t+j}}{\Pi_{t+j}} \right) p^{M,X}_{t,opt} - \lambda^{M,X}_t \overline{mc}^{M,X}_{F,t+k} \right] = 0 \tag{A.56a}$$

$$\hat{\Pi}^{M,X}_t = \beta E_t \left[\hat{\Pi}^{M,X}_{t+1} - \hat{\overline{\Pi}}^{M,X}_{t+1} \right] + \kappa_{M,X} \left(\frac{1}{\kappa_{M,X}} \hat{\lambda}^{M,X}_t + \hat{mc}^{M,X}_{F,t} - \hat{p}^{M,X}_t \right) + \hat{\overline{\Pi}}^{M,X}_t \tag{A.56b}$$

Definition of import price inflation indexation, import firms specializing in export goods:

$$\overline{\Pi}^{M,X}_t = \left(\Pi^{M,X}_{t-1}\right)^{\chi_{m,X}} (\Pi^{trend}_t)^{1-\chi_{m,X}} \tag{A.57a}$$

$$\hat{\overline{\Pi}}^{M,X}_t = \chi_{m,X} \hat{\Pi}^{M,X}_{t-1} + (1-\chi_{m,X}) \hat{\Pi}^{trend}_t \tag{A.57b}$$

[74] We scale the markup shock $\hat{\lambda}^{M,C,xe}_t$ by $\frac{1}{\kappa_{M,C,xe}}$.

[75] We scale the markup shock $\hat{\lambda}^{M,I}_t$ by $\frac{1}{\kappa_{M,I}}$.

[76] We scale the markup shock $\hat{\lambda}^{M,X}_t$ by $\frac{1}{\kappa_{M,X}}$.

Definition of import price inflation, import firms specializing in export goods:

$$\frac{p_t^{M,X}}{p_{t-1}^{M,X}} = \frac{\Pi_t^{M,X}}{\Pi_t} \tag{A.58a}$$

$$\hat{p}_t^{M,X} = \hat{p}_{t-1}^{M,X} + \hat{\Pi}_t^{M,X} - \hat{\Pi}_t \tag{A.58b}$$

Optimal price for import firms specializing in energy consumption goods equation:[77]

$$E_t \sum_{k=0}^{\infty} (\xi_{m,C,e})^k \left(\prod_{j=1}^{k} \beta_{t+j}^r \right) \frac{\overline{\Omega}_{t+k}^C}{\overline{\Omega}_t^C} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi_{t+j}^{C,e}} \right) \frac{\overline{m}_{t+k|t}^{C,e}}{\left(\lambda_{t+k}^{M,C,e} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}_{t+j}^{M,C,e}}{\Pi_{t+j}} \right) p_{t,opt}^{M,C,e} - \lambda_t^{M,C,e} \overline{mc}_{t+k}^{M,C,e} \right] = 0 \tag{A.59a}$$

$$\hat{\Pi}_t^{M,C,e} = \beta E_t \left[\hat{\Pi}_{t+1}^{M,C,e} - \hat{\overline{\Pi}}_{t+1}^{M,C,e} \right] + \kappa_{M,C,e} \left(\frac{1}{\kappa_{M,C,e}} \hat{\lambda}_t^{M,C,e} + \hat{mc}_t^{M,C,e} - \hat{p}_t^{M,C,e} \right) + \hat{\overline{\Pi}}_t^{M,C,e} \tag{A.59b}$$

Definition of import price inflation indexation, import firms specializing in energy consumption goods:

$$\overline{\Pi}_t^{M,C,e} = \left(\Pi_{t-1}^{M,C,e} \right)^{\chi_{m,C,e}} \left(\Pi_t^{trend} \right)^{1-\chi_{m,C,e}} \tag{A.60a}$$

$$\hat{\overline{\Pi}}_t^{M,C,e} = \chi_{m,C,e} \hat{\Pi}_{t-1}^{M,C,e} + (1 - \chi_{m,C,e}) \hat{\Pi}_t^{trend} \tag{A.60b}$$

Definition of import price inflation, import firms specializing in energy consumption goods:

$$\frac{p_t^{M,C,e}}{p_{t-1}^{M,C,e}} = \frac{\Pi_t^{M,C,e}}{\Pi_t} \tag{A.61a}$$

$$\hat{p}_t^{M,C,e} = \hat{p}_{t-1}^{M,C,e} + \hat{\Pi}_t^{M,C,e} - \hat{\Pi}_t \tag{A.61b}$$

Marginal cost of energy importer:

$$\overline{mc}_t^{M,C,e} = p_{F,t}^{C,e} Q_t \frac{p_t^C}{p_{F,t}^C} \tag{A.62a}$$

$$\hat{mc}_t^{M,C,e} = \hat{p}_{F,t}^{C,e} + \hat{Q}_t + \hat{p}_t^C - \hat{p}_{F,t}^C \tag{A.62b}$$

Marginal cost of non-energy importer:

$$\overline{mc}_t^{M,xe} = Q_t \frac{p_t^C}{p_{F,t}^C} \tag{A.63a}$$

$$\hat{mc}_t^{M,xe} = \hat{Q}_t + \hat{p}_t^C - \hat{p}_{F,t}^C \tag{A.63b}$$

Definition of real exchange rate:

$$\frac{Q_t}{Q_{t-1}} = s_t \frac{\Pi_{F,t}^C}{\Pi_t^C} \tag{A.64a}$$

$$\hat{Q}_t - \hat{Q}_{t-1} = \hat{s}_t + \hat{\Pi}_{F,t}^C - \hat{\Pi}_t^C \tag{A.64b}$$

A.3 Swedish monetary policy rule

Monetary policy rule:

$$\check{i}_t^{notional} = \rho \check{i}_{t-1}^{notional} + (1 - \rho) \left(r_\pi \hat{\Pi}_{t-1}^{a,C} + r_{un} \check{un}_{t-1} \right) + r_{\triangle\pi} \left(\hat{\Pi}_t^C - \hat{\Pi}_{t-1}^C \right) + r_{\triangle un} \left(\check{un}_t - \check{un}_{t-1} \right) + \epsilon_t^i, \tag{A.65}$$

$$\hat{\Pi}_t^{a,C} = \frac{1}{4} \left(\hat{\Pi}_t^C + \hat{\Pi}_{t-1}^C + \hat{\Pi}_{t-2}^C + \hat{\Pi}_{t-3}^C \right)$$

[77] We scale the markup shock $\hat{\lambda}_t^{M,C,e}$ by $\frac{1}{\kappa_{M,C,e}}$.

Nominal interest rate with and without the zero lower bound:

$$\breve{i}_t^{ss} = \max\left\{\breve{\underline{i}}, \breve{i}_t^{notional} + \breve{i}_t^{nat}\right\} \tag{A.66}$$

Real interest rate:

$$\breve{r}_t = \breve{i}_t - \hat{\Pi}_{t+1}^c \tag{A.67}$$

Monetary policy expansion, definition:

$$\breve{i}_t = \breve{i}_t^{ss} - \breve{i}_t^{nat} \tag{A.68}$$

The neutral interest rate:

$$\breve{i}_t^{nat} = r_\mu \hat{\mu}_{z+,t} - r_\zeta \hat{\zeta}_t + \hat{z}_t^r \tag{A.69}$$

A.4 Swedish fiscal authority

Government budget constraint:

$$\tau_t^C p_t^C \bar{c}_t^{agg} + (\tau_t^{SSC} + \tau_t^W) p_t^C \bar{w}_t n_t + \bar{\Upsilon}_t^K + \bar{b}_t^n + \bar{t}_t = \left(\alpha_B + (R_{t-1}^B - 1)\right) \frac{\bar{b}_t}{\mu_{z+,t} \Pi_t} + \bar{g}_t + \tau_t^I p_t^I \bar{I}_t + \bar{I}_t^G + (1 - \tau_t^{TR}) \bar{tr}_t^{agg} \tag{A.70a}$$

$$\begin{aligned} p^C \bar{c}^{agg} \breve{\tau}_t^C + \tau^C p^C \bar{c}^{agg} \left(\hat{p}_t^C + \hat{c}_t^{agg}\right) + p^C \bar{w} n \breve{\tau}_t^W + p^C \bar{w} n \breve{\tau}_t^{SSC} + (\tau^{SSC} + \tau^W) p^C \bar{w} n \left(\hat{p}_t^C + \hat{w}_t + \hat{n}_t\right) + \breve{\bar{\Upsilon}}_t^K + \breve{b}_t^n + \breve{t}_t \\ = (\alpha_B + R^B - 1) \left(\frac{1}{\mu_{z+} \Pi} \breve{b}_t - \frac{\bar{b}}{\mu_{z+} \Pi} \left(\hat{\mu}_{z+,t} + \hat{\Pi}_t\right)\right) + \frac{\bar{b}}{\mu_{z+} \Pi} \breve{R}_{t-1}^B + \bar{I}^G \hat{I}_t^G + \bar{g} \hat{g}_t + (1 - \tau^{TR}) \breve{tr}_t^{agg} \\ - \bar{tr}^{agg} \breve{\tau}_t^{TR} + p^I \bar{I} \breve{\tau}_t^I + \tau^I p^I \bar{I} \left(\hat{p}_t^I + \hat{I}_t\right) \end{aligned} \tag{A.70b}$$

Law of motion for aggregate total government debt stock:

$$\bar{b}_{t+1} = (1 - \alpha_B) \bar{b}_t \frac{1}{\mu_{z+,t} \Pi_t} + \bar{b}_t^n \tag{A.71a}$$

$$\breve{b}_{t+1} = \frac{1 - \alpha_B}{\mu_{z+} \Pi} \breve{b}_t - \frac{(1 - \alpha_B) \bar{b}}{\mu_{z+} \Pi} \left(\hat{\mu}_{z+,t} + \hat{\Pi}_t\right) + \breve{b}_t^n \tag{A.71b}$$

Definition of average interest rate on all outstanding government debt:

$$\left(R_t^B - 1\right) \bar{b}_{t+1} = (1 - \alpha_B) \left(R_{t-1}^B - 1\right) \bar{b}_t \frac{1}{\mu_{z+,t} \Pi_t} + \left(R_t^{B,n} - 1\right) \bar{b}_t^n \tag{A.72a}$$

$$\bar{b} \breve{R}_t^B + \left(R^B - 1\right) \breve{b}_{t+1} = \frac{(1 - \alpha_B)\left(R^B - 1\right)}{\mu_{z+} \Pi} \left[\frac{\bar{b}}{R^B - 1} \breve{R}_{t-1}^B + \breve{b}_t - \bar{b}\left(\hat{\mu}_{z+,t} + \hat{\Pi}_t\right)\right] + \bar{b}^n \breve{R}_t^{B,n} + \left(R^{B,n} - 1\right) \breve{b}_t^n \tag{A.72b}$$

Capital income tax revenues:

$$\bar{\Upsilon}_t^K = \frac{\bar{k}_t}{\mu_{z+,t} \mu_{\gamma,t}} \tau_t^K \left(r_t^K u_t - p_t^I a(u_t) - \iota^K \delta \frac{\mu_{\gamma,t} p_{t-1}^K}{\Pi_t}\right) \tag{A.73a}$$

$$\begin{aligned} \breve{\bar{\Upsilon}}_t^K = \frac{\tau^K \bar{k}}{\mu_{z+} \mu_\gamma} \left[\left(r^K - \iota^K \delta \frac{p^K \mu_\gamma}{\Pi}\right) \left(\hat{k}_t - \hat{\mu}_{z+,t}\right) + r^K \left(\hat{r}_t^K - \hat{\mu}_{\gamma,t}\right) + \iota^K \delta \frac{p^K \mu_\gamma}{\Pi} \left(\hat{\Pi}_t - \hat{p}_{t-1}^K\right)\right] \\ + \frac{\bar{k}}{\mu_{z+} \mu_\gamma} \left(r^K - \iota^K \delta \frac{p^K \mu_\gamma}{\Pi}\right) \breve{\tau}_t^K \end{aligned} \tag{A.73b}$$

Aggregate transfers:

$$\overline{tr}_t^{agg} = (1 - s_{nr}) \overline{tr}_t + s_{nr} \overline{tr}_t^{nr} \tag{A.74a}$$

$$\breve{tr}_t^{agg} = (1 - s_{nr})\breve{tr}_t + s_{nr}\breve{tr}_t^{nr} \tag{A.74b}$$

Transfer allocation:

$$\varpi_{dyn}\left(\overline{tr}_t - \overline{tr}\right) = (1 - \varpi_{dyn})(\overline{tr}_t^{nr} - \overline{tr}^{nr}) \tag{A.75a}$$

$$\varpi_{dyn}\breve{tr}_t = (1 - \varpi_{dyn})\breve{tr}_t^{nr} \tag{A.75b}$$

Government surplus:

$$\overline{surp}_t = \alpha_B \frac{\overline{b}_t}{\mu_{z^+t}\Pi_t} - \overline{b}_t^n \tag{A.76a}$$

$$su\breve{r}p_t = \frac{\alpha_B}{\mu_{z^+}\Pi}\left(\breve{b}_t - \overline{b}\left(\hat{\mu}_{z^+t} + \hat{\Pi}_t\right)\right) - \breve{b}_t^n \tag{A.76b}$$

Fiscal policy rule for aggregate transfers:

$$\begin{aligned}\breve{tr}_t^{agg} =& \rho_{tr}(\breve{tr}_{t-1}^{agg} - \mathcal{F}_{tr,un}\breve{un}_t)\\ &+\overline{y}\left(\mathcal{F}_{tr,b}\left(\breve{b}_{\overline{y},t} - \breve{b}_{\overline{y},t}^{Target}\right) + \mathcal{F}_{tr,surp}\left(Sts\breve{urp}_{\overline{y},t} - St\breve{surp}_{\overline{y},t}^{Target}\right) + \mathcal{F}_{tr,un}\breve{un}_t\right)\\ &+\epsilon_t^{tr^{agg}}\end{aligned} \tag{A.77}$$

Fiscal policy rule for government consumption:

$$\begin{aligned}\hat{g}_t =& \rho_g\hat{g}_{t-1}\\ &+\mathcal{F}_{g,b}\left(\breve{b}_{\overline{y},t} - \breve{b}_{\overline{y},t}^{Target}\right) + \mathcal{F}_{g,surp}\left(St\breve{surp}_{\overline{y},t} - St\breve{surp}_{\overline{y},t}^{Target}\right) + \mathcal{F}_{g,y}\hat{y}_t\\ &+\epsilon_t^g\end{aligned} \tag{A.78}$$

Fiscal policy rule for government investment:

$$\begin{aligned}\hat{I}_t^G =& \rho_{IG}\hat{I}_{t-1}^G\\ &+\mathcal{F}_{IG,b}\left(\breve{b}_{\overline{y},t} - \breve{b}_{\overline{y},t}^{Target}\right) + \mathcal{F}_{IG,surp}\left(St\breve{surp}_{\overline{y},t} - St\breve{surp}_{\overline{y},t}^{Target}\right) + \mathcal{F}_{IG,y}\hat{y}_t\\ &+\epsilon_t^{IG}\end{aligned} \tag{A.79}$$

Fiscal policy rule for consumption tax, labor tax, social security contribution, capital tax and transfer tax:

$$\begin{aligned}\breve{\tau}_t^x =& \rho_{\tau^x}\breve{\tau}_{t-1}^x\\ &+\mathcal{F}_{\tau^x,b}\left(\breve{b}_{\overline{y},t} - \breve{b}_{\overline{y},t}^{Target}\right) + \mathcal{F}_{\tau^x,surp}\left(St\breve{surp}_{\overline{y},t} - St\breve{surp}_{\overline{y},t}^{Target}\right)\\ &+\epsilon_t^{\tau^x}, \quad x \in \{C, W, SSC, K, TR\}\end{aligned} \tag{A.80}$$

Investment tax credits:

$$\tau_t^I = \rho_{\tau^I}\tau_{t-1}^I + \epsilon_t^{\tau^I} \tag{A.81}$$

Debt target equation:

$$\breve{b}_{\overline{y},t}^{Target} = \left(\rho_{1,b^T} + \rho_{2,b^T}\right)\breve{b}_{\overline{y},t-1}^{Target} - \rho_{1,b^T}\rho_{2,b^T}\left(\breve{b}_{\overline{y},t-2}^{Target}\right) + \epsilon_t^{b^{Target}} \tag{A.82}$$

Structural government surplus:

$$\overline{Stsurp}_t = \overline{Stprev}_t - \overline{Stpexp}_t - \frac{R_{t-1}^B - 1}{\Pi_t \mu_{z^+,t}} \bar{b}_t \tag{A.83a}$$

$$St\breve{surp}_t = St\breve{prev}_t - St\breve{pexp}_t - \left(\frac{R^B - 1}{\Pi \mu_{z^+}} \breve{b}_t + \bar{b} \left(\frac{1}{R^B - 1} \breve{R}_{t-1}^B - \hat{\mu}_{z^+,t} - \hat{\pi}_t \right) \right) \tag{A.83b}$$

Structural primary expenditures:

$$\overline{Stpexp}_t = ((\bar{tr}_t^{agg} - \bar{y} \mathcal{F}_{tr,un} \breve{un}_t)) + \left(\bar{I}^G - \mathcal{F}_{IG,y} \bar{I}^G \frac{(\bar{y}_t - \bar{y})}{\bar{y}} \right) + \left(\bar{g} - \mathcal{F}_{g,y} \bar{g} \frac{(\bar{y}_t - \bar{y})}{\bar{y}} \right) + \tau_t^I p^I \bar{I} \tag{A.84a}$$

$$St\breve{pexp}_t = \left(\breve{tr}_t^{agg} - \bar{y} \mathcal{F}_{tr,un} \breve{un}_t \right) + \bar{I}^G \left(\hat{I}_t^G - \mathcal{F}_{IG,y} \hat{y}_t \right) + \bar{g} \left(\hat{g}_t - \mathcal{F}_{g,y} \hat{y}_t \right) + p^I \bar{I} \breve{\tau}_t^I \tag{A.84b}$$

Structural primary revenues:

$$\overline{Stprev}_t = \tau_t^C p^C \bar{c}^{agg} + (\tau_t^{SSC} + \tau_t^W) \overline{w} n + \tau_t^K \frac{\overline{k}_t}{\mu_{z^+} \mu_{\gamma,}} \left(r_t^K - \iota^K \delta \frac{\mu_{\gamma,t} p^K}{\Pi} \right) + \tau_t^{TR} \left(\overline{tr}_t^{agg} - \bar{y} \mathcal{F}_{tr,un} \breve{un}_t \right) + \bar{t} \tag{A.85a}$$

$$\begin{aligned} St\breve{prev}_t = & \breve{\tau}_t^C p^C \bar{c}^{agg} + (\breve{\tau}_t^{SSC} + \breve{\tau}_t^W) \overline{w} n + \breve{\tau}_t^K \frac{\overline{k}}{\mu_{z^+} \mu_\gamma} \left(r^K - \iota^K \delta \frac{p^K \mu_\gamma}{\Pi} \right) \\ & + \breve{\tau}_t^{TR} (\breve{tr}_t^{agg} - \mathcal{F}_{tr,un} \bar{y} \breve{un}_t) + \breve{t}_t \end{aligned} \tag{A.85b}$$

Relation between debt target and surplus target:

$$Stsurp_{\bar{y},t}^{Target} = \left(\frac{1}{\mu_{z^+} \Pi} - 1 \right) b_{\bar{y},t}^{Target} \tag{A.86a}$$

$$St\breve{surp}_{\bar{y},t}^{Target} = \left(\frac{1}{\mu_{z^+} \Pi} - 1 \right) \breve{b}_{\bar{y},t}^{Target} \tag{A.86b}$$

A.5 Auxiliary variables

There are some variables which do not affect the simulations of the model, but which are used for the purpose of illustration and comparison with data. These are called auxiliary variables, and are stated below.

Aggregate investment:

$$\bar{I}_t^{agg} = \bar{I}_t + \bar{I}_t^G \tag{A.87a}$$

$$\hat{I}_t = \frac{\bar{I}}{\bar{I} + \bar{I}^G} \hat{I}_t + \frac{\bar{I}^G}{\bar{I} + \bar{I}^G} \hat{I}_t^G \tag{A.87b}$$

Price of aggregate investment:

$$p_t^{Iagg} = \frac{\bar{I}_t}{\bar{I}_t^{agg}} p_t^I + \frac{\bar{I}_t^G}{\bar{I}_t^{agg}} \tag{A.88a}$$

$$\hat{p}_t^{Iagg} = \frac{\bar{I}}{\bar{I}^{agg}} \frac{p^I}{p^{Iagg}} (\hat{p}_t^I + \hat{I}_t) + \frac{\bar{I}}{\bar{I}^{agg}} \frac{1}{p^{Iagg}} \hat{I}_t^G - \hat{I}_t^{agg} \tag{A.88b}$$

Aggregate investment inflation:

$$p_t^{Iagg} = p_{t-1}^{Iagg} \frac{\Pi_t^{Iagg}}{\Pi_t} \tag{A.89a}$$

$$\hat{p}_t^{Iagg} = \hat{p}_{t-1}^{Iagg} + \hat{\Pi}_t^{Iagg} - \hat{\Pi}_t \tag{A.89b}$$

Aggregate import prices:

$$p_t^M \overline{m}_t^D = \overline{m}_t^{C,xe} p_t^{MC,xe} + \overline{m}_t^I p_t^{MI} + \overline{m}_t^X p_t^{MX} + \overline{m}_t^{C,e} p_t^{MC,e} \tag{A.90a}$$

$$\hat{p}_t^M = \frac{\overline{m}^{C,xe}}{\overline{m}^D}\hat{p}_t^{MC,xe} + \frac{\overline{m}^I}{\overline{m}^D}\hat{p}_t^{MI} + \frac{\overline{m}^X}{\overline{m}^D}\hat{p}_t^{MX} + \frac{\overline{m}^{C,e}}{\overline{m}^D}\hat{p}_t^{MC,e} \tag{A.90b}$$

Aggregate import inflation:

$$\frac{p_t^M}{p_{t-1}^M} = \frac{\Pi_t^M}{\Pi_t} \tag{A.91a}$$

$$\hat{p}_t^M = \hat{p}_{t-1}^M + \hat{\Pi}_t^M - \hat{\Pi}_t \tag{A.91b}$$

Consumption tax revenues:

$$\overline{Rev}_t^{\tau^C} = \tau_t^C p_t^C \bar{c}_t^{agg} \tag{A.92a}$$

$$\breve{Rev}_t^{\tau^C} = p^C \bar{c}^{agg} \breve{\tau}_t^C + \tau^C p^C \bar{c}^{agg} \left(\hat{p}_t^C + \hat{c}_t{}^{agg}\right) \tag{A.92b}$$

Labor tax revenues:

$$\overline{Rev}_t^{\tau^W} = \tau_t^W p_t^C \bar{w}_t n_t \tag{A.93a}$$

$$\breve{Rev}_t^{\tau^W} = p^C \bar{w} n \breve{\tau} + \tau^W p^C \bar{w} n \left(\hat{p}_t^C + \hat{w}_t + \hat{n}_t\right) \tag{A.93b}$$

Social security contribution revenues:

$$\overline{Rev}_t^{\tau^{SSC}} = \tau_t^{SSC} p_t^C \bar{w}_t n_t \tag{A.94a}$$

$$\breve{Rev}_t^{\tau^{SSC}} = p^C \bar{w} n \breve{\tau} + \tau^{SSC} p^C \bar{w} n \left(\hat{p}_t^C + \hat{w}_t + \hat{n}_t\right) \tag{A.94b}$$

Transfer tax revenues:

$$\overline{Rev}_t^{\tau^{TR}} = \tau_t^{TR} \bar{tr}_t^{agg} \tag{A.95a}$$

$$\breve{Rev}_t^{\tau^{TR}} = \tau^{TR} \breve{tr}_t^{agg} + \bar{tr}^{agg} \breve{\tau}_t^{TR} \tag{A.95b}$$

Primary revenues:

$$\overline{PRev}_t = \overline{Rev}_t^{\tau^C} + \overline{Rev}_t^{\tau^W} + \overline{Rev}_t^{\tau^{SSC}} + \overline{Rev}_t^{\tau^{TR}} + \bar{\Upsilon}_t^K \tag{A.96a}$$

$$P\breve{rev}_t = R\breve{ev}_t^{\tau^C} + R\breve{ev}_t^{\tau^W} + R\breve{ev}_t^{\tau^{SSC}} + R\breve{ev}_t^{\tau^{TR}} + \breve{\Upsilon}_t^K \tag{A.96b}$$

Investment tax credit expenditures:

$$\overline{Exp}_t^{\tau^I} = \tau_t^I p_t^I \bar{I}_t \tag{A.97a}$$

$$\breve{Exp}_t^{\tau^I} = p^I \bar{I} \breve{\tau}_t^I + \tau^I p^I \bar{I} \left(\hat{p}_t^I + \hat{I}_t\right) \tag{A.97b}$$

Primary expenditure:

$$\overline{Pexp}_t = \tau_t^I p_t^I \bar{I}_t + \overline{g}_t + \bar{I}_t^G + \bar{tr}_t^{agg} \tag{A.98a}$$

$$Pe\breve{x}p_t = p^I \bar{I} \breve{\tau}_t^I + \tau^I p^I \bar{I} \left(\hat{p}_t^I + \hat{I}_t\right) + \bar{g}\hat{g}_t + \bar{I}_G \hat{I}_{G,t} + \breve{tr}_t^{agg} \tag{A.98b}$$

Primary surplus:

$$\overline{Psurp}_t = \overline{PRev}_t - \overline{Pexp}_t \tag{A.99a}$$

$$Ps\breve{u}rp_t = Pr\breve{e}v_t - Pe\breve{x}p_t \tag{A.99b}$$

Aggregate transfers, percent of GDP:

$$tr^{agg}oy_t = \frac{\bar{tr}_t^{agg}}{\overline{y}_t^m} \tag{A.100a}$$

$$tr^{a\breve{g}g}oy_t = \frac{1}{\overline{y}}\breve{tr}_t^{agg} - \frac{\bar{tr}^{agg}}{\overline{y}}\hat{y}_t^m \tag{A.100b}$$

Government debt to GDP:

$$boy_t = \frac{\bar{b}_t}{\overline{y}_t^m} \tag{A.101a}$$

$$bo\breve{y}_t = \frac{1}{\overline{y}}\breve{b}_t - \frac{\bar{b}}{\overline{y}}\hat{y}_t^m \tag{A.101b}$$

Surplus to GDP:

$$surpoy_t = \frac{\overline{surp_t}}{\overline{y}_t^m} \tag{A.102a}$$

$$sur\breve{p}oy_t = \frac{1}{\overline{y}}su\breve{r}p_t - \frac{\overline{surp}}{\overline{y}}\hat{y}_t^m \tag{A.102b}$$

Net exports:

$$\overline{nx}_t = \bar{x}_t - \bar{m}_t \tag{A.103a}$$

$$n\breve{x}_t = \bar{x}\hat{x}_t - \bar{m}\hat{m}_t \tag{A.103b}$$

A.6 Foreign: Household sector

Foreign consumption Euler equation:

$$\overline{\Omega}_{F,t}^C = R_{F,t}\zeta_{F,t}E_t\left[\beta_{F,t+1}^r \frac{\overline{\Omega}_{F,t+1}^C}{\mu_{z_F^+,t+1}\Pi_{F,t+1}^C}\right] \tag{A.104a}$$

$$\hat{\Omega}_{F,t}^C = E_t\left[\hat{\zeta}_{F,t} + \hat{\beta}_{F,t+1}^r + \hat{\Omega}_{F,t+1}^C + \frac{1}{R_F}\breve{i}_{F,t} - \hat{\Pi}_{F,t+1}^C - \hat{\mu}_{z_F^+,t+1}\right] \tag{A.104b}$$

Foreign marginal utility of consumption equation:

$$\overline{\Omega}_{F,t}^C = \frac{\zeta_{F,t}^c}{\left(\overline{c}_{F,t} - \rho_{h,F}\frac{1}{\mu_{z_F^+,t}}\overline{c}_{F,t-1}\right)} \tag{A.105a}$$

$$\hat{\Omega}_{F,t}^C = \hat{\zeta}_{F,t}^c\left(1 - \frac{\rho_{h,F}}{\mu_{z_F^+}}\right)^{-1}\left[-\hat{c}_{F,t} + \frac{\rho_{h,F}}{\mu_{z_F^+}}\left(\hat{c}_{F,t-1} - \hat{\mu}_{z_F^+,t}\right)\right] \tag{A.105b}$$

Foreign capital utilization decision equation:

$$r_{F,t}^K = p_{F,t}^I a'(u_{F,t}) \tag{A.106a}$$

$$\hat{r}_{F,t}^K = \hat{p}_{F,t}^I + \sigma_a\hat{u}_{F,t} \tag{A.106b}$$

Foreign household purchases of installed capital equation:

$$p_{F,t}^K = E_t\beta_{F,t+1}^r\frac{\overline{\Omega}_{F,t+1}^C}{\overline{\Omega}_{F,t}^C}\frac{\Pi_{F,t+1}}{\Pi_{F,t+1}^C}\frac{1}{\mu_{z_F^+,t+1}\mu_{\gamma,t+1}}\left[r_{F,t+1}^K u_{F,t+1} - p_{F,t+1}^I a(u_{F,t+1}) + p_{F,t+1}^K(1-\delta_F)\right] \tag{A.107a}$$

$$\left(\hat{p}^K_{F,t} - \hat{\Pi}_{F,t+1} + \hat{\mu}_{\gamma,t+1}\right) =$$

$$E_t\hat{\beta}^r_{F,t+1} + E_t\hat{\Omega}^C_{F,t+1} - \hat{\Omega}^C_{F,t} - E_t\hat{\Pi}^C_{F,t+1} - E_t\hat{\mu}_{z^+_F,t+1} + \frac{1}{H_F}r^K_F E_t\hat{r}^K_{F,t+1} + \frac{1}{H_F}p^K_F(1-\delta_F)E_t\hat{p}^K_{F,t+1} \quad \text{(A.107b)}$$

Foreign household investment decision equation:

$$p^I_{F,t} = p^K_{F,t}\Upsilon_{F,t}F_1(\overline{I}_{F,t},\overline{I}_{F,t-1},\mu_{z^+_F,t},\mu_{\gamma,t}) + E_t\left[\beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\overline{\Omega}^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{p^K_{F,t+1}}{\mu_{z^+_F,t+1}\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(\overline{I}_{F,t+1},\overline{I}_{F,t},\mu_{z^+_F,t+1}\mu_{\gamma,t+1})\right] \quad \text{(A.108a)}$$

$$\frac{p^I_F}{p^K_F\Upsilon_F}\hat{p}^I_{F,t} = \hat{p}^K_{F,t} + \hat{\Upsilon}_{F,t} - S''\left(\mu_{z^+_F}\mu_\gamma\right)^2 E_t\left[\triangle\hat{I}_{F,t} + \hat{\mu}_{z^+_F,t} + \hat{\mu}_{\gamma,t} - \beta_F\triangle\hat{I}_{F,t+1} - \beta_F\hat{\mu}_{z^+_F,t+1} - \beta_F\hat{\mu}_{\gamma,t+1}\right] \quad \text{(A.108b)}$$

Foreign definition of capital services:

$$\overline{k}^s_{F,t} = u_{F,t}\bar{k}_{F,t} \quad \text{(A.109a)}$$

$$\hat{k}^s_{F,t} = \hat{u}_{F,t} + \hat{k}_{F,t} \quad \text{(A.109b)}$$

Foreign capital accumulation equation:

$$\overline{k}_{F,t+1} = (1-\delta)\overline{k}_{F,t}\frac{1}{\mu_{z^+_F,t}\mu_{\gamma,t}} + \Upsilon_{F,t}\left[1 - \widetilde{S}\left(\frac{\overline{I}_{F,t}\mu_{z^+_F}\mu_\gamma}{\overline{I}_{F,t-1}}\right)\right]\overline{I}_{F,} + \overline{\triangle}^K_{F,t} \quad \text{(A.110a)}$$

$$\hat{k}_{F,t+1} = \frac{(1-\delta_F)}{\mu_{z^+_F}\mu_\gamma}\left(\hat{k}_{F,t} - \hat{\mu}_{z^+_F,t} - \hat{\mu}_{\gamma,t}\right) + \frac{\overline{I}_F}{\bar{k}_F}\Upsilon_F\left(\hat{I}_{F,t} + \Upsilon_{F,t}\right) \quad \text{(A.110b)}$$

Foreign optimal wage setting equation:

$$E_t\sum_{k=0}^{\infty}\left(\xi^F_w\right)^k\left(\prod_{j=1}^{k}\beta^r_{F,t+j}\right)n_{F,t+k|t}\overline{\Omega}^C_{F,t+k}\left[(1-\tau^w_F)\,\overline{w}_{F,t+k|t} - \lambda^W_F\,\zeta^n_{F,t+k}\frac{\nu'(n_{F,t+k|t})}{\overline{\Omega}^C_{F,t+k}}\right] = 0 \quad \text{(A.111a)}$$

$$\triangle\hat{w}_{F,t} = \beta_F E_t\left[\triangle\hat{w}_{F,t+1}\right] - \kappa_{F,W}\hat{\Psi}^W_{F,t} + \hat{\overline{\Pi}}^W_{F,t} - \hat{\mu}_{z^+_F,t} - \hat{\Pi}^C_{F,t} - \beta_F E_t[\hat{\overline{\Pi}}^W_{F,t+1} - \hat{\mu}_{z^+_F,t+1} - \hat{\Pi}^C_{F,t+1}] \quad \text{(A.111b)}$$

Foreign real wage markup equation:

$$\overline{\Psi}^W_{F,t} = \frac{(1-\tau^w_F)\,\overline{w}_{F,t}}{\zeta^n_{F,t}\frac{\nu'_F(n_{F,t})}{\overline{\Omega}^C_{F,t}}} \quad \text{(A.112a)}$$

$$\hat{\Psi}^W_{F,t} = \hat{w}_{F,t} - \hat{\zeta}^n_{F,t} - \eta_F\hat{n}_{F,t} + \hat{\Omega}^C_{F,t} \quad \text{(A.112b)}$$

Definition of Foreign wage inflation:

$$\Pi^W_{F,t} = \frac{\overline{w}_{F,t}}{\overline{w}_{F,t-1}}\mu_{z^+_F,t}\Pi^C_{F,t} \quad \text{(A.113a)}$$

$$\hat{\Pi}^W_{F,t} = \triangle\hat{w}_{F,t} + \hat{\mu}_{z^+_F,t} + \hat{\Pi}^C_{F,t} \quad \text{(A.113b)}$$

Definition of Foreign wage inflation indexation:

$$\overline{\Pi}^W_{F,t} = (\Pi^W_{F,t-1})^{\chi_{F,w}}(\Pi^{trend}_{F,t})^{1-\chi_{F,w}} \quad \text{(A.114a)}$$

$$\hat{\overline{\Pi}}^W_{F,t} = \chi_{F,w}\hat{\Pi}^W_{F,t-1} + (1-\chi_{F,w})\,\hat{\Pi}^{trend}_{F,t} \quad \text{(A.114b)}$$

Real wage relevant to Foreign employers:

$$\overline{w}^e_{F,t} = \overline{w}_{F,t}p^C_{F,t} \quad \text{(A.115a)}$$

$$\hat{w}^e_{F,t} = \hat{w}_{F,t} + \hat{p}^C_{F,t} \quad \text{(A.115b)}$$

A.7 Foreign: Firm sector

A.7.1 Foreign: Intermediate good producers

Definition of Foreign composite technological growth rate

$$\mu_{z_F^+,t} = \mu_{z,t} \left(\mu_{\gamma,t}\right)^{\frac{\alpha_F}{1-\alpha_F}} \tag{A.116a}$$

$$\hat{\mu}_{z_F^+,t} = \hat{\mu}_{z,t} + \frac{\alpha_F}{1-\alpha_F}\hat{\mu}_{\gamma,t} \tag{A.116b}$$

Real marginal cost of production for Foreign intermediate good producers equation:

$$\overline{mc}_{F,t} = \frac{\left(\overline{w}^e_{F,t}\right)^{1-\alpha_F} (r^K_{F,t})^{\alpha_F}}{\varepsilon_{F,t}\alpha_F^{\alpha_F}(1-\alpha_F)^{1-\alpha_F}} \tag{A.117a}$$

$$\hat{mc}_{F,t} = (1-\alpha_F)\,\hat{w}^e_{F,t} + \alpha_F\hat{r}^K_{F,t} - \hat{\varepsilon}_{F,t} \tag{A.117b}$$

Real rental rate for capital services equation:

$$r^K_{F,t} = \alpha_F\varepsilon_{F,t}\left(\frac{\bar{k}^s_{F,t}}{n_{F,t}}\frac{1}{\mu_{z_F^+,t}\mu_{\gamma,t}}\right)^{\alpha_F-1}\overline{mc}_{F,t} \tag{A.118a}$$

$$\hat{r}^K_{F,t} = \hat{\varepsilon}_{F,t} + (\alpha_F-1)\left(\hat{k}^s_{F,t} - \hat{n}_{F,t} - \hat{\mu}_{z_F^+,t} - \hat{\mu}_{\gamma,t}\right) + \hat{mc}_{F,t} \tag{A.118b}$$

Optimal price of Foreign intermediate goods equation:[78]

$$E_t\sum_{k=0}^{\infty}\left(\xi^F\right)^k\left(\prod_{j=1}^{k}\beta^r_{F,t+j}\right)\frac{\overline{\Omega}^C_{F,t+k}}{\overline{\Omega}^C_{F,t}}\left(\prod_{j=1}^{k}\frac{\Pi_{F,t+j}}{\Pi^C_{F,t+j}}\right)\frac{\overline{y}_{F,t+k|t}}{(\lambda_{F,t+k}-1)}$$
$$\left[\left(\prod_{j=1}^{k}\frac{\overline{\Pi}_{F,t+j}}{\Pi_{F,t+j}}\right)\frac{p^{opt}_{F,t}}{\Pi_{F,t}} - \lambda_{F,t+k}\overline{mc}_{F,t+k}\right] = 0 \tag{A.119a}$$

$$\hat{\Pi}_{F,t} = \beta_F E_t\left[\hat{\Pi}_{F,t+1} - \hat{\overline{\Pi}}_{F,t+1}\right] + \kappa_F\left(\frac{1}{\kappa_F}\hat{\lambda}_{F,t} + \hat{mc}_{F,t}\right) + \hat{\overline{\Pi}}_{F,t} \tag{A.119b}$$

Definition of Foreign intermediate good price inflation indexation:

$$\overline{\Pi}_{F,t} = (\Pi_{F,t-1})^{\chi_F}(\Pi^{trend}_{F,t})^{1-\chi_F} \tag{A.120a}$$

$$\hat{\overline{\Pi}}_{F,t} = \chi_F\hat{\Pi}_{F,t-1} + (1-\chi_F)\,\hat{\Pi}^{trend}_{F,t} \tag{A.120b}$$

A.7.2 Foreign: Consumption good producers

Relative price of Foreign consumption goods equation:

$$p^C_{F,t} = \left[\vartheta^C_F\left(p^{C,xe}_{F,t}\right)^{1-\nu_{F,C}} + \left(1-\vartheta^C_F\right)\left(p^{C,e}_{F,t}\right)^{1-\nu_{F,C}}\right]^{\frac{1}{1-\nu_{F,C}}} \tag{A.121a}$$

$$\hat{p}^C_{F,t} = \vartheta^C_F\left(\frac{p^{C,xe}_F}{p^C_F}\right)^{1-\nu_{F,C}}\hat{p}^{C,xe}_{F,t} + \left(1-\vartheta^C_F\right)\left(\frac{p^{C,e}_F}{p^C_F}\right)^{1-\nu_{F,C}}\hat{p}^{C,e}_{F,t} \tag{A.121b}$$

Definition of Foreign consumption good price inflation:

$$\Pi^C_{F,t} = \frac{p^C_{F,t}}{p^C_{F,t-1}}\Pi_{F,t} \tag{A.122a}$$

$$\hat{\Pi}^C_{F,t} = \hat{p}^C_{F,t} - \hat{p}^C_{F,t-1} + \hat{\Pi}_{F,t} \tag{A.122b}$$

[78]We scale the markup shock $\hat{\lambda}_{F,t}$ by $\frac{1}{\kappa_F}$.

Demand for non-energy consumption goods equation:

$$\bar{c}_{F,t}^{xe} = \vartheta_F^C \left(\frac{p_{F,t}^{C,xe}}{p_{F,t}^C} \right)^{-\nu_{F,C}} \bar{c}_{F,t} \tag{A.123a}$$

$$\hat{c}_{F,t}^{xe} = \nu_{F,C} \left(\hat{p}_{F,t}^C - \hat{p}_{F,t}^{C,xe} \right) + \hat{c}_{F,t} \tag{A.123b}$$

Demand for energy consumption goods equation:

$$\bar{c}_{F,t}^{e} = \left(1 - \vartheta_F^C\right) \left(\frac{p_{F,t}^{C,e}}{p_{F,t}^C} \right)^{-\nu_{F,C}} \bar{c}_{F,t} \tag{A.124a}$$

$$\hat{c}_{F,t}^{e} = \nu_{F,C} \left(\hat{p}_{F,t}^C - \hat{p}_{F,t}^{C,e} \right) + \hat{c}_{F,t} \tag{A.124b}$$

Relative price of non-energy consumption good:

$$p_{F,t}^{C,xe} = 1 \tag{A.125a}$$

$$\hat{p}_{F,t}^{C,xe} = 0 \tag{A.125b}$$

Definition of Foreign non-energy consumption good price inflation:

$$\Pi_{F,t}^{C,xe} = \Pi_{F,t} \tag{A.126a}$$

$$\hat{\Pi}_{F,t}^{C,xe} = \hat{\Pi}_{F,t} \tag{A.126b}$$

Definition of Foreign energy consumption good price inflation:

$$\Pi_{F,t}^{C,e} = \frac{p_{F,t}^{C,e}}{p_{F,t-1}^{C,e}} \Pi_{F,t} \tag{A.127a}$$

$$\hat{\Pi}_{F,t}^{C,e} = \hat{p}_{F,t}^{C,e} - \hat{p}_{F,t-1}^{C,e} + \hat{\Pi}_{F,t} \tag{A.127b}$$

A.7.3 Foreign: Investment good producers

Relative price of Foreign investment:

$$p_{F,t}^{I} = 1 \tag{A.128a}$$

$$\hat{p}_{F,t}^{I} = 0 \tag{A.128b}$$

Foreign investment inflation:

$$\Pi_{F,t}^{I} = \frac{p_{F,t}^{I}}{p_{F,t-1}^{I}} \Pi_{F,t} \tag{A.129a}$$

$$\hat{\Pi}_{F,t}^{I} = \hat{p}_{F,t}^{I} - \hat{p}_{F,t-1}^{I} + \hat{\Pi}_{F,t-1}^{I} \tag{A.129b}$$

A.7.4 Price of Swedish exports in terms of Foreign intermediate goods

Definition of relative price of Swedish export goods in terms of Foreign intermediate goods:

$$\widetilde{p}_t^X = \frac{p_t^X p_{F,t}^C}{Q_t p_t^C} \tag{A.130a}$$

$$\hat{\widetilde{p}}_t^X = \hat{p}_t^X + \hat{p}_{F,t}^C - \hat{Q}_t - \hat{p}_t^C \tag{A.130b}$$

A.8 Foreign monetary policy rule

Foreign monetary policy rule:

$$\breve{i}_{F,t}^{notational} = \rho_F \breve{i}_{F,t-1}^{notational} + (1-\rho_F)\left(r_{F,\pi}\hat{\Pi}_{F,t-1}^{a,C} + r_{F,y}\hat{y}_{F,t-1}\right) + r_{F,\triangle\pi}\left(\hat{\Pi}_{F,t}^{C} - \hat{\Pi}_{F,t-1}^{C}\right) + r_{F,\triangle y}\left(\hat{y}_{F,t} - \hat{y}_{F,t-1}\right) + \epsilon_t^{i_F}, \tag{A.131}$$

$$\hat{\Pi}_{F,t}^{a,C} = \frac{1}{4}\left(\hat{\Pi}_{F,t}^{C} + \hat{\Pi}_{F,t-1}^{C} + \hat{\Pi}_{F,t-2}^{C} + \hat{\Pi}_{F,t-3}^{C}\right)$$

Foreign nominal interest rate with and without the zero lower bound:

$$\breve{i}_{F,t}^{ss} = max(\underline{i_F}, \breve{i}_{F,t}^{notional} + \breve{i}_{F,t}^{nat}) \tag{A.132}$$

Definition of monetary policy expansion

$$\breve{i}_{F,t} = \breve{i}_{F,t}^{ss} - \breve{i}_{F,t}^{nat} \tag{A.133}$$

Foreign nominal interest rate with and without the zero lower bound:

$$\breve{i}_{F,t}^{nat} = r_{F,\mu}\hat{\mu}_{z_F^+,t} - r_{F,\zeta}\hat{\zeta}_{F,t} + \hat{z}_t^r \tag{A.134}$$

Foreign real interest rate:

$$\breve{r}_{F,t} = \breve{i}_t^F - \hat{\Pi}_{F,t+1}^c \tag{A.135}$$

A.9 Market clearing

A.9.1 Swedish aggregate resource constraint

$$\bar{y}_t = \vartheta^{C,xe}\left(p_t^{C,xe}\right)^{\nu_{c,xe}}\bar{c}_t^{xe} + \bar{d}_t^{C,e} + \vartheta^I\left(p_t^I\right)^{\nu_I}\left[\bar{I}_t + a\left(u_t\right)\frac{\bar{k}_t}{\mu_{z_F^+,t}\mu_{\gamma,t}}\right]$$
$$+ \vartheta^X\left(\overline{mc}_t^X\right)^{\nu_x}\left[\bar{x}_t\overleftrightarrow{P}_t^X + \phi^X\right] + \bar{g}_t + \bar{I}_t^G \tag{A.136a}$$

$$\hat{y}_t = \vartheta^{C,xe}(p^{C,xe})^{\nu_c}\frac{\bar{c}^{xe}}{\bar{y}}\left(\nu_{c,xe}\hat{p}_t^{C,xe} + \hat{c}_t^{xe}\right) + \frac{\bar{d}^{C,e}}{\bar{y}}\hat{d}_t^e + \vartheta^I(p^I)^{\nu_I}\frac{\bar{I}}{\bar{y}}\left(\nu_I\hat{p}_t^I + \hat{I}_t + \frac{a'\bar{k}}{\mu_{z_F^+}\mu_\gamma\bar{I}}\hat{u}_t\right)$$
$$+ \vartheta^X\left(\overline{mc}^X\right)^{\nu_x}\frac{\left(\bar{x}+\phi^X\right)}{\bar{y}}\left(\nu_x\hat{mc}_t^X + \frac{\bar{x}}{\left(\bar{x}+\phi^X\right)}\hat{x}_t\right) + \frac{\bar{g}}{\bar{y}}\hat{g}_t + \frac{\bar{I}^G}{\bar{y}}\hat{I}_t^G \tag{A.136b}$$

A.9.2 Foreign aggregate resource constraint

$$\bar{y}_{F,t} = \bar{c}_{F,t}^{xe} + \bar{c}_{F,t}^{e} + \bar{I}_{F,t} + a\left(u_{F,t}\right)\bar{k}_{F,t}\frac{1}{\mu_{z_F^+,t}\mu_{\gamma,t}} + \bar{g}_t \tag{A.137a}$$

$$\hat{y}_{F,t} = \frac{\bar{c}^{xe}}{\bar{y}_F}\hat{c}_t^{xe} + \frac{\bar{c}^{e}}{\bar{y}_F}\hat{c}_t^{e} + \frac{\bar{I}_F}{\bar{y}_F}\left(\hat{I}_{F,t} + \frac{a'\bar{k}_F}{\mu_{z_F^+}\mu_\gamma\bar{I}_F}\hat{u}_{F,t}\right) + \frac{\bar{g}_F}{\bar{y}_F}\hat{g}_{F,t} \tag{A.137b}$$

A.9.3 Balance of payments

$$\bar{a}_t = p_t^X\bar{x}_t - \bar{mc}_t^{M,xe}\,\overline{m}_t^{xe} - \bar{mc}_t^{M,C,e}\overline{m}_t^{e} + \Phi\left(\bar{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right)R_{F,t-1}\zeta_{t-1}\,s_t\,\bar{a}_{t-1}\frac{1}{\mu_{z_F^+,t}\Pi_t} \tag{A.138a}$$

$$\breve{a}_t = p^X\bar{x}\left(\hat{p}_t^X + \hat{x}_t\right) - \bar{mc}^{M,xe}\overline{m}^{xe}\left(\hat{mc}_t^{M,xe} + \hat{m}_t^{xe}\right) - \bar{mc}^{M,C,e}\overline{m}^{e}\left(\hat{mc}_t^{M,C,e} + \hat{m}_t^{e}\right)$$
$$+ \frac{\bar{a}}{\beta}\left[-\widetilde{\phi}_a\breve{a}_{t-1} - \widetilde{\phi}_s(\hat{s}_t + \hat{s}_{t-1}) + \hat{\widetilde{\phi}}_{t-1} + \frac{1}{R_F}\breve{i}_{F,t-1} + \hat{\zeta}_{t-1} + \hat{s}_t - \hat{\mu}_{z_F^+,t} - \hat{\Pi}_t\right] + \frac{1}{\beta}\breve{a}_{t-1} \tag{A.138b}$$

A.9.4 Swedish exports

$$\bar{x}_t = \left(1 - \vartheta_F^{C,xe}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe} + \left(1 - \vartheta_F^I\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \bar{I}_{F,t} \tag{A.139a}$$

$$\hat{x}_t = -\nu_F \hat{\widetilde{p}}_t^X + \omega_C^X \hat{c}_{F,t}^{xe} + \left(1 - \omega_C^X\right) \hat{I}_{F,t} \tag{A.139b}$$

A.9.5 Swedish imports for non-energy consumption

$$\overleftrightarrow{P}_t^{M,C,xe} \overline{m}_t^{C,xe} = \left(1 - \vartheta^{C,xe}\right) \left[\vartheta^{C,xe} \left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1 - \vartheta^{C,xe}\right]^{\frac{\nu_{C,xe}}{1-\nu_{C,xe}}} \overline{c}_t^{xe} \tag{A.140a}$$

$$\hat{m}_t^{C,xe} = (1 - \vartheta^{C,xe}) \left(\frac{p^{C,xe}}{p^{M,C,xe}}\right)^{\nu_{C,xe}} \frac{\overline{c}^{xe}}{\overline{m}^{C,xe}} \left[\hat{c}_t^{xe} - \nu_{C,xe} \vartheta^{C,xe} \left(p^{C,xe}\right)^{\nu_{C,xe}-1} \hat{p}_t^{M,C,xe}\right] \tag{A.140b}$$

A.9.6 Swedish imports for investment

$$\overleftrightarrow{P}_t^{M,I} \overline{m}_t^I = (1 - \vartheta^I) \left[\vartheta^I \left(p_t^{M,I}\right)^{\nu_I - 1} + 1 - \vartheta^I\right]^{\frac{\nu_I}{1-\nu_I}} \left[\overline{I}_t + a(u_t) \overline{k}_t \frac{1}{\mu_{z^+,t}, \mu_{\gamma,t}}\right] \tag{A.141a}$$

$$\hat{m}_t^I = (1 - \vartheta^I) \left(\frac{p^I}{p^{M,I}}\right)^{\nu_I} \frac{\overline{I}}{\overline{m}^I} \left[\hat{I}_t + \left(\frac{a'}{\mu_{z^+} \mu_\gamma - 1 + \delta}\right) \hat{u}_t - \nu_I \vartheta^I \left(p^I\right)^{\nu_I - 1} \hat{p}_t^{M,I}\right] \tag{A.141b}$$

A.9.7 Swedish imports for export

$$\overleftrightarrow{P}_t^{M,X} \overline{m}_t^X = \left(1 - \vartheta^X\right) \left[\vartheta^X \left(p_t^{M,X}\right)^{\nu_x - 1} + 1 - \vartheta^X\right]^{\frac{\nu_x}{1-\nu_x}} \left[\overline{x}_t \overleftrightarrow{P}_t^X + \phi^X\right] \tag{A.142a}$$

$$\hat{m}_t^X = (1 - \vartheta^X) \left(\frac{\overline{mc}^X}{p^{M,X}}\right)^{\nu_x} \frac{\overline{x}}{\overline{m}^X} \left[\hat{x}_t - \nu_x \vartheta^X \lambda^X \left(\overline{mc}^X\right)^{\nu_x - 1} \hat{p}_t^{M,X}\right] \tag{A.142b}$$

A.9.8 Imports of non-energy goods including fixed costs

$$\overline{m}_t^{xe} = \overleftrightarrow{P}_t^{M,C,xe} \overline{m}_t^{C,xe} + \overleftrightarrow{P}_t^{M,I} \overline{m}_t^I + \overleftrightarrow{P}_t^{M,X} \overline{m}_t^X + \phi^{M,C,xe} + \phi^{M,I} + \phi^{M,X} \tag{A.143a}$$

$$\hat{m}_t^{xe} = \frac{\overline{m}^{C,xe}}{\overline{m}^{xe}} \hat{m}_t^{C,xe} + \frac{\overline{m}^I}{\overline{m}^{xe}} \hat{m}_t^I + \frac{\overline{m}^X}{\overline{m}^{xe}} \hat{m}_t^X \tag{A.143b}$$

A.9.9 Imports of non-energy goods excluding fixed costs

$$\overline{m}_t^{D,xe} = \overleftrightarrow{P}_t^{M,C,xe} \overline{m}_t^{C,xe} + \overleftrightarrow{P}_t^{M,I} \overline{m}_t^I + \overleftrightarrow{P}_t^{M,X} \overline{m}_t^X \tag{A.144a}$$

$$\hat{m}_t^{D,xe} = \frac{\overline{m}^{C,xe}}{\overline{m}^{D,xe}} \hat{m}_t^{C,xe} + \frac{\overline{m}^I}{\overline{m}^{D,xe}} \hat{m}_t^I + \frac{\overline{m}^X}{\overline{m}^{D,xe}} \hat{m}_t^X \tag{A.144b}$$

A.9.10 Imports of energy goods including fixed cost

$$\overline{m}_t^e = \overleftrightarrow{P}_t^{M,C,e} \overline{m}_t^{C,e} + \phi^{M,C,e} \tag{A.145a}$$

$$\hat{m}_t^e = \frac{\overline{m}^{C,e}}{\overline{m}^e} \hat{m}_t^{C,e} \tag{A.145b}$$

A.9.11 Aggregate imports excluding fixed costs

$$\overline{m}_t^D = \overline{m}_t^{D,xe} + \overleftrightarrow{P}_t^{M,C,e} \overline{m}_t^{C,e} \tag{A.146a}$$

$$\hat{m}_t^D = \frac{\overline{m}^{D,xe}}{\overline{m}^D} \hat{m}_t^{D,xe} + \frac{\overline{m}^{C,e}}{\overline{m}^D} \hat{m}_t^{C,e} \tag{A.146b}$$

A.9.12 Aggregate imports including fixed costs

$$\overline{m}_t = \overline{m}_t^{xe} + \overleftrightarrow{P}_t^{M,C,e} \overline{m}_t^{C,e} + \phi^{M,C,e} \tag{A.147a}$$

$$\hat{m}_t = \frac{\overline{m}^{xe}}{\overline{m}} \hat{m}_t^{xe} + \frac{\overline{m}^{C,e}}{\overline{m}} \hat{m}_t^{C,e} \tag{A.147b}$$

A.9.13 Swedish aggregate output

$$\overline{y}_t \overleftrightarrow{P}_t = \left(\varepsilon_t \left[\frac{\bar{\tilde{k}}_t^s}{\mu_{z^+,t} \mu_{\gamma,t}} \right]^\alpha n_t^{1-\alpha} \right) - \phi \tag{A.148a}$$

$$\hat{y}_t = \frac{\lambda}{F} \left(\hat{\varepsilon}_t + \alpha \left(\hat{\tilde{k}}_t^s - \hat{\mu}_{z^+,t} - \hat{\mu}_{\gamma,t} \right) + (1-\alpha) \hat{n}_t \right) \tag{A.148b}$$

A.9.14 Measured Swedish aggregate output

$$\overline{y}_t^m = \overline{y}_t - \vartheta^I (p_t^I)^{\nu_I} a(u_t) \frac{\overline{k}_t}{\mu_{z^+,t} \mu_{\gamma,t}} \tag{A.149a}$$

$$\hat{y}_t^m = \hat{y}_t - \frac{\vartheta^I (p^I)^{\nu_I}}{\overline{y}} \left(\frac{r^K}{p^I} \frac{\overline{k}}{\mu_{z^+} \mu_\gamma} \right) \hat{u}_t \tag{A.149b}$$

A.9.15 Foreign aggregate output

$$\overline{y}_{F,t} \overleftrightarrow{P}_{F,t} = \left(\varepsilon_{F,t} \left[\frac{\bar{\tilde{k}}_{F,t}^s}{\mu_{z_F^+,t} \mu_{\gamma,t}} \right]^{\alpha_F} n_{F,t}^{1-\alpha_F} \right) - \phi_F \tag{A.150a}$$

$$\hat{y}_t = \lambda_F \left(\hat{\varepsilon}_t + \alpha_F \left(\hat{\tilde{k}}_{F,t}^s - \hat{\mu}_{z_F^+,t} - \hat{\mu}_{\gamma,t} \right) + (1-\alpha_F) \hat{n}_{F,t} \right) \tag{A.150b}$$

A.9.16 Measured Foreign aggregate output

$$\overline{y}_{F,t}^m = \overline{y}_{F,t} - a(u_{F,t}) \frac{\overline{k}_{F,t}}{\mu_{z_F^+,t} \mu_{\gamma,t}} \tag{A.151a}$$

$$\hat{y}_{F,t}^m = \hat{y}_{F,t} - \frac{1}{\overline{y}_F} \left(\frac{r_F^K}{p_F^I} \frac{\bar{k}_F}{\mu_{z_F^+} \mu_\gamma} \right) \hat{u}_{F,t} \tag{A.151b}$$

A.10 Stochastic exogenous shocks

A.10.1 Global exogenous shocks

Labor augmenting technology shock:

$$\hat{\mu}_{z,t} = \rho_{\mu_z}\,\hat{\mu}_{z,t-1} + \epsilon_{\mu_z,t} \tag{A.152}$$

Investment-specific technology shock:

$$\hat{\mu}_{\gamma,t} = \rho_{\mu_\gamma}\,\hat{\mu}_{\gamma,t-1} + \epsilon_{\mu_\gamma,t} \tag{A.153}$$

Neutral rate shock:

$$\hat{z}_t^R = \rho_{z^R}\,\hat{z}_{t-1}^R + \epsilon_{z^R,t} + \theta_{z^R}\epsilon_{z^R,t-1} \tag{A.154}$$

A.10.2 Swedish exogenous shocks

Discount factor shock:

$$\hat{\beta}_t^r = \rho_\beta \hat{\beta}_{t-1}^r + \epsilon_t^\beta \tag{A.155}$$

Monetary policy shock

$$\epsilon_{i,t} \tag{A.156}$$

Private bond risk premium shock:

$$\hat{\zeta}_t = corr_\zeta \hat{\zeta}_{F,t} + \rho_\zeta\,\hat{\zeta}_{t-1} + \epsilon_t^\zeta \tag{A.157}$$

Consumption preference shock:

$$\hat{\zeta}_t^c = corr_{\zeta^c} \hat{\zeta}_{F,t}^c + \rho_{\zeta^c}\,\hat{\zeta}_{t-1}^c + \epsilon_t^{\zeta^c} \tag{A.158}$$

Exchange rate shock (external risk premium shock):

$$\hat{\tilde{\phi}}_t = \rho_{\tilde{\phi}}\,\hat{\tilde{\phi}}_{t-1} + \epsilon_t^{\tilde{\phi}} \tag{A.159}$$

Labor disutility shock:

$$\hat{\zeta}_t^n = \rho_{\zeta^n} \hat{\zeta}_{t-1}^n + \epsilon_t^{\zeta^n} \tag{A.160}$$

Wage markup shock:

$$\hat{\lambda}_t^W = \rho_{\lambda^W} \hat{\lambda}_{t-1}^W + \epsilon_t^{\lambda^W} \tag{A.161}$$

Productivity shock (stationary technology shock):

$$\hat{\varepsilon}_t = \rho_\varepsilon \hat{\varepsilon}_{t-1} + \epsilon_t \tag{A.162}$$

Stationary investment-specific shock:

$$\hat{\Upsilon}_t = corr_\Upsilon \hat{\Upsilon}_{F,t} + \rho_\Upsilon \hat{\Upsilon}_{t-1} + \epsilon_t^\Upsilon \tag{A.163}$$

Intermediate good price markup shock:

$$\hat{\lambda}_t = \rho_\lambda \hat{\lambda}_{t-1} + \epsilon_t^\lambda \tag{A.164}$$

Export price markup shock:

$$\hat{\lambda}_t^X = \rho_{\lambda^X} \hat{\lambda}_{t-1}^X + \epsilon_t^{\lambda^X} \tag{A.165}$$

Markup shock to import firms specializing in non-energy consumption goods:

$$\hat{\lambda}_t^{M,C,xe} = \rho_{\lambda^{M,C,xe}} \hat{\lambda}_{t-1}^{M,C,xe} + \epsilon_t^{\lambda^{M,C,xe}} \tag{A.166}$$

Markup shock to import firms specializing in investment goods:

$$\hat{\lambda}_t^{M,I} = \rho_{\lambda^{M,I}} \hat{\lambda}_{t-1}^{M,I} + \epsilon_t^{\lambda^{M,I}} \tag{A.167}$$

Markup shock to import firms specializing in export goods:

$$\hat{\lambda}_t^{M,X} = \rho_{\lambda^{M,X}} \hat{\lambda}_{t-1}^{M,X} + \epsilon_t^{\lambda^{M,X}} \tag{A.168}$$

Markup shock to import firms specializing in energy consumption goods:

$$\hat{\lambda}_t^{M,C,e} = \rho_{\lambda^{M,C,xe}} \hat{\lambda}_{t-1}^{M,C,e} + \epsilon_t^{\lambda^{M,C,e}} \tag{A.169}$$

Domestic energy price shock:

$$\hat{p}_t^{D,C,e} = \rho_{p^{D,C,e}} \hat{p}_{t-1}^{D,C,e} + \epsilon_t^{p^{D,C,e}} \tag{A.170}$$

Inflation trend shock:

$$\hat{\Pi}_t^{trend} = \rho_{\Pi^{trend}} \hat{\Pi}_{t-1}^{trend} + \epsilon_t^{\Pi^{trend}} \tag{A.171}$$

A.10.3 Foreign exogenous shocks

Discount factor shock:

$$\hat{\beta}_{F,t}^r = \rho_{\beta_F} \hat{\beta}_{F,t-1}^r + \epsilon_{F,t}^{\beta} \tag{A.172}$$

Monetary policy shock

$$\epsilon_{i,t}^F \tag{A.173}$$

Private bond risk premium shock:

$$\hat{\zeta}_{F,t} = \rho_{\zeta_F} \hat{\zeta}_{F,t-1} + \epsilon_{F,t}^{\zeta} \tag{A.174}$$

Consumption preference shock:

$$\hat{\zeta}_{F,t}^c = corr_{\zeta_{F,\Upsilon_F}^c} \hat{\Upsilon}_{F,t} + \rho_{\zeta_F^c} \hat{\zeta}_{F,t-1}^c + \epsilon_{F,t}^{\zeta^c} \tag{A.175}$$

Labor disutility shock:

$$\hat{\zeta}_{F,t}^n = \rho_{\zeta_F^n} \hat{\zeta}_{F,t-1}^n + \epsilon_{F,t}^{\zeta^n} \tag{A.176}$$

Productivity shock (stationary technology shock):

$$\hat{\varepsilon}_{F,t} = \rho_{\varepsilon_F} \hat{\varepsilon}_{F,t-1} + \epsilon_{F,t} \tag{A.177}$$

Stationary investment-specific shock:

$$\hat{\Upsilon}_{F,t} = \rho_{\Upsilon_F} \hat{\Upsilon}_{F,t-1} + \epsilon_{F,t}^{\Upsilon} \tag{A.178}$$

Intermediate good price markup shock:

$$\hat{\lambda}_{F,t} = \rho_{\lambda_F} \hat{\lambda}_{F,t-1} + \epsilon_{F,t}^{\lambda} \tag{A.179}$$

Foreign domestic energy price shock:

$$\hat{p}_{F,t}^{C,e} = \rho_{p_F^{D,C,e}} \hat{p}_{F,t-1}^{D,C,e} + \epsilon_{F,t}^{p^{D,C,e}} \tag{A.180}$$

Foreign inflation trend shock:

$$\hat{\Pi}_{F,t}^{C,trend} = \rho_{\Pi_F^{C,trend}} \hat{\Pi}_{F,t-1}^{C,trend} + \epsilon_t^{\Pi_F^{C,trend}} \tag{A.181}$$

Foreign government consumption shock:

$$\hat{g}_{F,t} = \rho_{g_F} \hat{g}_{F,t-1} + \epsilon_t^{g_F} \tag{A.182}$$

B Appendix: Steady state

B.1 The Swedish economy

B.1.1 Sweden: Household sector

Consumption Euler equation:

$$R = \frac{\mu_{z+}\Pi^C}{\beta} \tag{B.1}$$

Definition of nominal gross interest rate on private bonds:

$$R = 1 + i \tag{B.2}$$

Lagrange multiplier, Marginal utility of consumption equation:

$$\overline{\Omega}^C = \frac{(\alpha_G)^{\frac{1}{\upsilon_G}}}{(1+\tau^C)\,\bar{c}\left(1-\frac{\rho_h}{\mu_{z+}}\right)}\left(\frac{\bar{\bar{c}}}{\bar{c}}\right)^{\frac{1}{\upsilon_G}-1} \tag{B.3}$$

Marginal utility of consumption equation:

$$\bar{U}_c = \frac{(\alpha_G)^{\frac{1}{\upsilon_G}}}{\bar{c}\left(1-\frac{\rho_h}{\mu_{z+}}\right)}\left(\frac{\bar{\bar{c}}}{\bar{c}}\right)^{\frac{1}{\upsilon_G}-1} \tag{B.4}$$

Composite consumption function:

$$\frac{\bar{\bar{c}}}{\bar{c}} = \left(\alpha_G^{\frac{1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}}\left(\frac{\bar{g}}{\bar{c}}\right)^{\frac{\upsilon_G-1}{\upsilon_G}}\right)^{\frac{\upsilon_G}{\upsilon_G-1}} \tag{B.5}$$

Average interest rate on government bonds:

$$\bar{\Omega}^R = \frac{\beta}{\Pi^C\mu_{z+} - \beta(1-\alpha_B)} \tag{B.6}$$

Euler equation for government bond holdings:

$$R^{B,n} = \frac{\Pi^C\mu_{z+}}{\beta} \tag{B.7}$$

Capital utilization decision equation:

$$r^K = p^I a' \tag{B.8}$$

Household purchases of installed capital equation:

$$p^K = \frac{\beta\left(1-\tau^K\right)r^K}{\mu_{z+}\mu_\gamma - \beta(1-\delta) - \beta\tau^K\delta\frac{\mu_\gamma}{\Pi}} \tag{B.9}$$

Household investment decision equation:

$$p^I\left(1-\tau^I\right) = p^K \tag{B.10}$$

Definition of capital services:

$$\bar{k}^s = \overline{k} \tag{B.11}$$

Capital accumulation equation:

$$1 = (1-\delta)\frac{1}{\mu_{z+}\mu_\gamma} + \frac{\overline{I}}{\bar{\bar{k}}} \tag{B.12}$$

Optimal wage setting equation:

$$(1-\tau_w)\,\overline{w} = \lambda^W \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \tag{B.13}$$

Labor force participation equation:

$$\overline{\Omega}^C\,(1-\tau_w)\,\overline{w} = \zeta^n \Theta^n A_n l^\eta \tag{B.14}$$

Definition of endogenous shifter:

$$\Theta^n = \bar{Z}^n \bar{U}_c \tag{B.15}$$

Trend of wealth effect in endogenous shifter:

$$\bar{Z}^n = (\mu_{z+})^{(\frac{\chi_n-1}{\chi_n})}(\bar{U}_c)^{-1} \tag{B.16}$$

Unemployment rate definition:

$$un = \frac{l-n}{l} \tag{B.17}$$

Real wage markup equation:

$$\overline{\Psi}^W = (\tfrac{l}{n})^\eta \tag{B.18}$$

Definition of wage inflation:

$$\Pi^W = \mu_{z+}\Pi^C \tag{B.19}$$

Definition of wage inflation indexation:

$$\overline{\Pi}^W = \Pi^W \tag{B.20}$$

Real wage relevant to employers:

$$\overline{w}^e = \overline{w}p^C \tag{B.21}$$

Modified uncovered interest rate parity equation:

$$R = sR_F \tag{B.22}$$

Aggregate consumption:

$$\bar{c}^{agg} = (1-s_{nr})\bar{c} + s_{nr}\bar{c}^{nr} \tag{B.23}$$

Non-Ricardian budget constraint:

$$(1+\tau^C)p^C\bar{c}^{nr} = (1-\tau^W)\,\bar{w}^e n + \left(1-\tau^{TR}\right)\overline{tr}^{nr} \tag{B.24}$$

B.1.2 Sweden: Firm sector

Definition of composite technological growth rate:

$$\mu_{z+} = \mu_z\,(\mu_\gamma)^{\frac{\alpha}{1-\alpha}} \tag{B.25}$$

Real marginal cost of production for intermediate good producers equation:

$$\overline{mc} = \frac{\left(\left(1+\tau^{SSC}\right)\overline{w}^e\right)^{1-\alpha}(r^K)^\alpha}{\varepsilon\alpha^\alpha(1-\alpha)^{1-\alpha}\bar{\Gamma}_G} \tag{B.26}$$

Simplifying expression variable Gamma:

$$\bar{\Gamma}_{G,t} = \alpha_K^{\frac{\alpha}{\upsilon_K}} \left(\frac{\bar{\tilde{k}}_t^s}{\bar{\bar{k}}_t^s} \right)^{\frac{\alpha}{\upsilon_K}} \tag{B.27}$$

Real rental rate for capital services equation:

$$r^K = \alpha \varepsilon \left(\frac{\bar{\tilde{k}}^s}{n} \frac{1}{\mu_{z+}\mu_\gamma} \right)^{\alpha-1} \overline{mc} \bar{\Gamma}_{G,t}^{\frac{1}{\alpha}} \tag{B.28}$$

Composite capital function:

$$\bar{\tilde{k}}^s = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(\bar{k}^s\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}} \left(\bar{k}_G\right)^{\frac{\upsilon_K-1}{\upsilon_K}} \right)^{\frac{\upsilon_K}{\upsilon_K-1}} \tag{B.29}$$

Public capital accumulation equation:

$$1 = (1-\delta_G) \frac{1}{\mu_{z+}\mu_\gamma} + \frac{\bar{I}^G}{\bar{k}_G} \tag{B.30}$$

Optimal price of intermediate goods equation:

$$\overline{mc} = \frac{1}{\lambda} \tag{B.31}$$

Definition of intermediate good price inflation indexation:

$$\overline{\Pi} = \Pi^C \tag{B.32}$$

Relative price of consumption goods equation:

$$p^C = \left[\vartheta^C \left(p^{C,xe}\right)^{1-\nu_c} + \left(1-\vartheta^C\right) \left(p^{C,e}\right)^{1-\nu_c} \right]^{\frac{1}{1-\nu_c}} \tag{B.33}$$

Definition of consumption good price inflation:

$$\Pi^C = \Pi \tag{B.34}$$

Demand for non-energy consumption goods equation:

$$\bar{c}^{xe} = \vartheta^C \left(\frac{p^{C,xe}}{p^C} \right)^{-\nu_C} \bar{c}^{agg} \tag{B.35}$$

Demand for energy consumption goods equation:

$$\bar{c}^{e} = \left(1-\vartheta^C\right) \left(\frac{p^{C,e}}{p^C} \right)^{-\nu_C} \bar{c}^{agg} \tag{B.36}$$

Relative price of consumption goods equation:

$$p^{C,xe} = \left[\vartheta^{C,xe} + \left(1-\vartheta^{C,xe}\right) \left(p^{M,C,xe}\right)^{1-\nu_{c,xe}} \right]^{\frac{1}{1-\nu_{c,xe}}} \tag{B.37}$$

Definition of non-energy consumption good price inflation:

$$\Pi^{C,xe} = \Pi \tag{B.38}$$

Relative price of energy consumption goods equation:

$$p^{C,e} = \left[\vartheta^{C,e}\left(p^{D,C,e}\right)^{1-\nu_{c,e}} + \left(1-\vartheta^{C,e}\right)\left(p^{M,C,e}\right)^{1-\nu_{c,e}}\right]^{\frac{1}{1-\nu_{c,e}}} \tag{B.39}$$

Definition of consumption good price inflation:

$$\Pi^{C,e} = \Pi \tag{B.40}$$

Demand for non-energy consumption goods equation:

$$\bar{d}^{e} = \vartheta^{C,e}\left(\frac{p^{D,C,e}}{p^{C,e}}\right)^{-\nu_{C,e}}\bar{c}^{e} \tag{B.41}$$

Import demand for energy consumption goods equation:

$$\bar{m}^{C,e} = \left(1-\vartheta^{C,e}\right)\left(\frac{p^{M,C,e}}{p^{C,e}}\right)^{-\nu_{C,e}}\bar{c}^{e} \tag{B.42}$$

Definition of consumption good price inflation:

$$\Pi^{D,C,e} = \Pi \tag{B.43}$$

Relative price of investment goods equation:

$$p^{I} = \left[\vartheta^{I} + \left(1-\vartheta^{I}\right)\left(p^{M,I}\right)^{1-\nu_{I}}\right]^{\frac{1}{1-\nu_{I}}} \tag{B.44}$$

Definition of investment good price inflation:

$$\Pi^{I} = \Pi \tag{B.45}$$

Real marginal cost for export good producers equation:

$$\overline{mc}^{X} = \left[\vartheta^{X} + \left(1-\vartheta^{X}\right)\left(p^{M,X}\right)^{1-\nu_{x}}\right]^{\frac{1}{1-\nu_{x}}} \tag{B.46}$$

Optimal price of export goods equation:

$$\overline{mc}^{X} = \frac{p^{X}}{\lambda^{X}} \tag{B.47}$$

Definition of export good price inflation indexation:

$$\overline{\Pi}^{X} = \Pi^{C} \tag{B.48}$$

Definition of export good price inflation:

$$\Pi^{X} = \Pi \tag{B.49}$$

Optimal price for import firms specializing in non-energy consumption goods equation:

$$p^{M,C,xe} = \lambda^{M}\overline{mc}^{M,xe} \tag{B.50}$$

Definition of import price inflation indexation, import firms specializing in non-energy consumption goods:

$$\overline{\Pi}^{M,C,xe} = \Pi^{C,xe} \tag{B.51}$$

Definition of import price inflation, import firms specializing in non-energy consumption goods:

$$\Pi^{M,C,xe} = \Pi \tag{B.52}$$

Optimal price for import firms specializing in investment goods equation:

$$p^{M,I} = \lambda^M \overline{mc}^{M,xe} \tag{B.53}$$

Definition of import price inflation indexation, import firms specializing in investment goods:

$$\overline{\Pi}^{M,I} = \Pi^C \tag{B.54}$$

Definition of import price inflation, import firms specializing in investment goods:

$$\Pi^{M,I} = \Pi \tag{B.55}$$

Optimal price for import firms specializing in export goods equation:

$$p^{M,X} = \lambda^M \overline{mc}^{M,xe} \tag{B.56}$$

Definition of import price inflation indexation, import firms specializing in export goods:

$$\overline{\Pi}^{M,X} = \Pi^C \tag{B.57}$$

Definition of import price inflation, import firms specializing in export goods:

$$\Pi^{M,X} = \Pi \tag{B.58}$$

Optimal price for import firms specializing in energy consumption goods equation:

$$p^{M,C,e} = \lambda^M \overline{mc}^{M,C,e} \tag{B.59}$$

Definition of import price inflation indexation, import firms specializing in energy consumption goods:

$$\overline{\Pi}^{M,C,e} = \Pi^{C,e} \tag{B.60}$$

Definition of import price inflation, import firms specializing in energy consumption goods:

$$\Pi^{M,C,e} = \Pi \tag{B.61}$$

Marginal cost of energy importer:

$$\overline{mc}^{M,C,e} = p_F^e Q \frac{p^C}{p_F^C} \tag{B.62}$$

Marginal cost of non-energy importer:

$$\overline{mc}^{M,xe} = Q \frac{p^C}{p_F^C} \tag{B.63}$$

Definition of real exchange rate:

$$s = 1 \tag{B.64}$$

B.1.3 Swedish monetary policy rule

Monetary policy rule:

$$i = R - 1 \tag{B.65}$$

B.1.4 Swedish fiscal authority equations

Government budget constraint:

$$\tau^C p^C \bar{c}^{agg} + (\tau^{SSC} + \tau^W) p^C \bar{w} n + \bar{\Upsilon}^K + \bar{b}^n + \bar{t} = \Big(\alpha_B + (R^B - 1)\Big) \bar{b} \frac{1}{\mu_{z^+} \Pi} + \bar{g} + \tau^I p^I \bar{I} + \bar{I}^G + (1 - \tau^{TR}) \bar{tr}^{agg}$$

Law of motion for aggregate total government debt stock:

$$\bar{b}^n = \left(1 - \frac{1 - \alpha_B}{\mu_{z^+} \Pi}\right) \bar{b}$$

Definition of average interest rate on all outstanding government debt:

$$R^B = R^{B,n}$$

Capital income tax revenues:

$$\bar{\Upsilon}^K = \frac{\bar{k}}{\mu_{z^+} \mu_\gamma} \tau^K \left(r^K - \delta \frac{\mu_\gamma p^K}{\Pi}\right)$$

Aggregate transfers:

$$\bar{tr}^{agg} = (1 - s_{nr}) \bar{tr} + s_{nr} \bar{tr}^{nr}$$

Transfer allocation:

$$\varpi_{ss} \bar{tr} = (1 - \varpi_{ss}) \bar{tr}^{nr}$$

Government surplus:

$$\overline{surp} = \alpha_B \frac{\bar{b}}{\mu_{z^+} \Pi} - \bar{b}^n \tag{B.66}$$

B.1.5 Auxiliary variables

Aggregate investment:

$$\bar{I}^{agg} = \bar{I} + \bar{I}^G \tag{B.67}$$

Price of aggregate investment:

$$p^{Iagg} = \frac{\bar{I}}{\bar{I}^{agg}} p^I + \frac{\bar{I}^G}{\bar{I}^{agg}} \tag{B.68}$$

Aggregate investment inflation:

$$\Pi^{Iagg} = \Pi \tag{B.69}$$

Aggregate import prices:

$$p^M = \frac{\overline{m}^{C,xe}}{\overline{m}^D} p^{MC,xe} + \frac{\overline{m}^I}{\overline{m}^D} p^{MI} + \frac{\overline{m}^X}{\overline{m}^D} p^{MX} + \frac{\overline{m}^{C,e}}{\overline{m}^D} p^{MC,e} \tag{B.70}$$

Aggregate import inflation:

$$\Pi^M = \Pi \tag{B.71}$$

Consumption tax revenues:

$$\overline{Rev}^{\tau^C} = \tau^C p^C \bar{c}^{agg} \tag{B.72}$$

Labor tax revenues:

$$\overline{Rev}^{\tau^W} = \tau^W p^C \bar{w} n \tag{B.73}$$

Social security contribution revenues:

$$\overline{Rev}^{\tau^{SSC}} = \tau^{SSC} p^C \bar{w} n \tag{B.74}$$

Transfer tax revenues:

$$\overline{Rev}^{\tau^{TR}} = \tau^{TR} \bar{tr}^{agg} \tag{B.75}$$

Primary revenues:

$$\overline{Prev} = \overline{Rev}^{\tau^C} + \overline{Rev}^{\tau^W} + \overline{Rev}^{\tau^{SSC}} + \overline{Rev}^{\tau^{TR}} + \bar{\Upsilon}^K \tag{B.76}$$

Investment tax credit expenditures:

$$\overline{Exp}^{\tau^I} = \tau^I p^I \bar{I} \tag{B.77}$$

Primary Expenditure:

$$\overline{Pexp} = \tau^I p^I \bar{I} + \overline{g} + \bar{I}^G + \bar{tr}^{agg} \tag{B.78}$$

Primary surplus:

$$\overline{Psurp} = \overline{Prev} - \overline{Pexp} \tag{B.79}$$

Aggregate transfers, percent of GDP:

$$tr^{agg}oy = \frac{\bar{tr}^{agg}}{\overline{y}^m} \tag{B.80}$$

Government debt to GDP:

$$boy = \frac{\bar{b}}{\overline{y}^m} \tag{B.81}$$

Surplus to GDP:

$$surpoy = \frac{\overline{surp}}{\overline{y}^m} \tag{B.82}$$

Net exports:

$$\overline{nx} = \bar{x} - \bar{m} \tag{B.83}$$

B.2 Foreign economy

B.2.1 Foreign: Household sector

Foreign consumption Euler equation:

$$R_F = \frac{\mu_{z_F^+} \Pi_F^C}{\beta_F} \tag{B.84}$$

Foreign marginal utility of consumption equation:

$$\overline{\Omega}_F^C = \frac{1}{\bar{c}_F (1 - \frac{\rho_{h,F}}{\mu_{z_F^+}})} \tag{B.85}$$

Foreign capital utilization decision equation:

$$r_F^K = p_F^I a'{}_F \tag{B.86}$$

Foreign household purchases of installed capital equation:

$$p_F^K = \frac{\beta_F r_F^K}{\mu_{z_F^+} \mu_\gamma - \beta_F (1 - \delta_F)} \tag{B.87}$$

Foreign household investment decision equation:

$$p_F^I = p_F^K \tag{B.88}$$

Foreign definition of capital services:

$$\bar{k}_F^s = \bar{k}_F \tag{B.89}$$

Foreign capital accumulation equation:

$$1 = (1 - \delta_F) \frac{1}{\mu_{z_F^+} \mu_\gamma} + \frac{\overline{I}_F}{\bar{k}_F} \tag{B.90}$$

Foreign optimal wage setting equation:

$$(1 - \tau_F^w)\, \overline{w}_F = \lambda_F^W \zeta_F^n \frac{\nu'(n_F)}{\overline{\Omega}_F^C} \tag{B.91}$$

Foreign real wage markup equation:

$$\overline{\Psi}_F^W = \lambda_F^W \tag{B.92}$$

Definition of Foreign wage inflation:

$$\Pi_F^W = \mu_{z_F^+} \Pi_F^C \tag{B.93}$$

Definition of Foreign wage inflation indexation equation:

$$\overline{\Pi}_F^W = \Pi_F^W \tag{B.94}$$

Real wage relevant to Foreign employers:

$$\overline{w}_F^e = \overline{w}_F p_F^C \tag{B.95}$$

B.2.2 Foreign: Firm sector

Definition of Foreign composite technological growth rate:

$$\mu_{z_F^+} = \mu_z \left(\mu_\gamma\right)^{\frac{\alpha_F}{1-\alpha_F}} \tag{B.96}$$

Real marginal cost of production for Foreign intermediate good producers equation:

$$\overline{mc}_F = \frac{\left(\overline{w}_F^e\right)^{1-\alpha_F} \left(r_F^K\right)^{\alpha_F}}{\varepsilon_F \alpha_F^{\alpha_F} \left(1-\alpha_F\right)^{1-\alpha_F}} \tag{B.97}$$

Foreign rental rate for capital services equation:

$$r_F^K = \alpha_F \varepsilon_F \left(\frac{\bar{k}_F^s}{n_F} \frac{1}{\mu_{z_F^+}\mu_\gamma}\right)^{\alpha_F - 1} \overline{mc}_F \tag{B.98}$$

Optimal price of Foreign intermediate goods equation:

$$\overline{mc}_F = \frac{1}{\lambda_F} \tag{B.99}$$

Foreign Intermediate good inflation indexation:

$$\overline{\Pi}_F = \Pi_F^C \tag{B.100}$$

Relative price of Foreign consumption goods equation:

$$p_F^C = \left[\vartheta_F^C \left(p_F^{C,xe}\right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right)\left(p_F^{C,e}\right)^{1-\nu_{F,C}}\right]^{\frac{1}{1-\nu_{F,C}}} \tag{B.101}$$

Definition of Foreign consumption good price inflation:

$$\Pi_F^C = \Pi_F \tag{B.102}$$

Demand for non-energy consumption:

$$\bar{c}_F^{xe} = \vartheta_F^C \left(\frac{p_F^{C,xe}}{p_F^C}\right)^{-\nu_{F,C}} \bar{c}_F \tag{B.103}$$

Demand for energy consumption:

$$\bar{c}_F^{e} = \left(1-\vartheta_F^C\right) \left(\frac{p_F^{C,e}}{p_F^C}\right)^{-\nu_{F,C}} \bar{c}_F \tag{B.104}$$

Relative price of non-energy consumption good:

$$p_F^{C,xe} = 1 \tag{B.105}$$

Definition of Foreign non-energy consumption good price inflation:

$$\Pi_F^{C,xe} = \Pi_F \tag{B.106}$$

Definition of Foreign energy consumption good price inflation:

$$\Pi_F^{C,e} = \Pi_F \tag{B.107}$$

Definition of Foreign investment good price inflation:

$$\Pi_F^I = \Pi_F \tag{B.108}$$

Definition of relative price of Swedish export goods in terms of Foreign intermediate goods:

$$\widetilde{p}^X = \frac{p^X p_F^C}{Q p^C} \tag{B.109}$$

B.2.3 Foreign monetary policy rule

Foreign monetary policy rule:

$$i_F = R_F - 1 \tag{B.110}$$

B.3 Market clearing

B.3.1 Swedish aggregate resource constraint

$$\begin{aligned}\bar{y} = {} & \vartheta^{C,xe}(p^{C,xe})^{\nu_{c,xe}}\bar{c}^{xe} + \bar{d}^{C,e} + \vartheta^{I}(p^{I})^{\nu_I}\bar{I} \\ & + \vartheta^{X}(\overline{mc}^{X})^{\nu_x}\left(\bar{x} + \phi^{X}\right) + \bar{g} + \bar{I}^{G}\end{aligned} \tag{B.111}$$

B.3.2 Foreign aggregate resource constraint

$$\bar{y}_F = \bar{c}_F^{xe} + \bar{c}_F^{e} + \bar{I}_F + +\bar{g}_F \tag{B.112}$$

B.3.3 Balance of payments

$$\bar{a} = \frac{\beta}{(1-\beta)}(\bar{mc}^{M,xe}\overline{m}^{xe} - \bar{mc}^{M,C,e}\overline{m}^{e} - p^{X}\bar{x}) \tag{B.113}$$

B.3.4 Swedish exports

$$\bar{x} = \left(1 - \vartheta_F^{C,xe}\right)\left(\frac{\widetilde{p}^{X}}{p_F^{C,xe}}\right)^{-\nu_{F,C,xe}}\bar{c}_F^{xe} + \left(1 - \vartheta_F^{I}\right)\left(\frac{\widetilde{p}^{X}}{p_F^{I}}\right)^{-\nu_{F,I}}\bar{I}_F \tag{B.114}$$

B.3.5 Swedish imports for consumption

$$\overline{m}^{C,xe} = (1 - \vartheta^{C,xe})\left[\vartheta^{C,xe}(p^{M,C,xe})^{\nu_{c,xe}-1} + 1 - \vartheta^{C,xe}\right]^{\frac{\nu_{c,xe}}{1-\nu_{c,xe}}}\bar{c}^{xe} \tag{B.115}$$

B.3.6 Swedish imports for investment

$$\overline{m}^{I} = (1 - \vartheta^{I})\left[\vartheta^{I}(p^{M,I})^{\nu_I - 1} + 1 - \vartheta^{I}\right]^{\frac{\nu_I}{1-\nu_I}}\bar{I} \tag{B.116}$$

B.3.7 Swedish imports for export

$$\overline{m}^{X} = (1 - \vartheta^{X})\left[\vartheta^{X}(p^{M,X})^{\nu_x - 1} + 1 - \vartheta^{X}\right]^{\frac{\nu_x}{1-\nu_x}}(\bar{x} + \phi^{X}) \tag{B.117}$$

B.3.8 Import of non-energy goods including fixed costs

$$\overline{m}^{xe} = \overline{m}^{C,xe} + \overline{m}^{I} + \overline{m}^{X} + \phi^{M,C,xe} + \phi^{M,I} + \phi^{M,X} \tag{B.118}$$

B.3.9 Import of non-energy goods excluding fixed costs

$$\overline{m}^{D,xe} = \overline{m}^{C,xe} + \overline{m}^{I} + \overline{m}^{X} \tag{B.119}$$

B.3.10 Import of energy goods including fixed costs

$$\overline{m}^e = \overline{m}^{C,e} + \phi^{M,e} \tag{B.120}$$

B.3.11 Aggregate imports excluding fixed costs

$$\overline{m}^D = \overline{m}^{D,xe} + \overline{m}^{C,e} \tag{B.121}$$

B.3.12 Aggregate imports including fixed costs

$$\overline{m} = \overline{m}^{xe} + \overline{m}^{C,e} + \phi^{M,e} \tag{B.122}$$

B.3.13 Swedish aggregate output

$$\overline{y} = \varepsilon \left[\frac{\bar{\tilde{k}}^s}{\mu_{z+}\mu_\gamma} \right]^\alpha n^{1-\alpha} - \phi \tag{B.123}$$

B.3.14 Measured Swedish aggregate output

$$\overline{y}^m = \overline{y} \tag{B.124}$$

B.3.15 Foreign aggregate output

$$\overline{y}_F = \varepsilon \left[\frac{\bar{\tilde{k}}^s_F}{\mu_{z^+_F}\mu_\gamma} \right]^\alpha n_F^{1-\alpha} - \phi_F \tag{B.125}$$

B.3.16 Measured Foreign aggregate output

$$\overline{y}^m_F = \overline{y}_F \tag{B.126}$$

C Technical appendix: The Swedish economy

In this technical appendix, we derive the key equilibrium conditions and model equations for the Swedish economy.

C.1 Household sector

There are two types of households, Ricardian households and Non-Ricardian households. The problem of Ricardian household is described in Section C.1.1 and the problem of Non-Ricardian household is described in Section C.1.10.

C.1.1 Ricardian household

There is a continuum of household members who are represented by the unit square $(h, j) \in [0, 1] \times [0, 1]$, where each member is indexed by h according to their type of labor service they are specialized in and indexed by j according to their degree of disutility of work. The utility function of househol memeber (h, j) is defined as:

$$E_0^{h,j} \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_{h,j,t} - \rho_h \tilde{C}_{t-1}) - 1(h, j)\zeta_t^n \Theta_t^n A_n j^\eta \right]. \tag{C.1}$$

where β is the household's factor, $\tilde{C}_{h,j,t}$ is composite consumption of household member (h, j), $1(h, j)$ is an indicator that is equal to one if the household member works and zero otherwise. ζ_t^c is the consumption preference shock, ρ_h is the consumption habit formation parameter. We assume external habit formation and in line with that $\tilde{C}_{t-1}$ is aggregate consumption. ζ_t^n is a labor disutility preference shock,$\Theta_t^n$is the endogenous shifter, $A_n$is a parameter that determines the weight of disutility of work.

Under symmetric equilibrium and full consumption risk sharing $\tilde{C}_{h,j,t} = \tilde{C}_t$ for all (h, j) and integrating over all household members' utilities gives

$$E_0 \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \int_0^{N_{h,t}} j^\eta \, dj dh \right], \tag{C.2}$$

$$= E_0 \sum_{t=0}^{\infty} \beta_t \left[\zeta_t^c u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \frac{N_{h,t}^{1+\eta}}{1+\eta} dh \right]. \tag{C.3}$$

The composite consumption $\tilde{C}_t$ is consist of C_t and public consumption G_t. The weight on private consumption is α_G and υ_G is the elasticity of substitution between private and public consumption. The composite consumption function is given by:

$$\tilde{C}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} C_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} G_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}. \tag{C.4}$$

The Ricardian household chooses private consumption C_t, investment I_t, capacity utilization u_t, capital K_{t+1}, transacted capital between households $\triangle_t^K$, domestic nominal private bonds B_{t+1}^{priv}, domestic nominal government bonds B_{t+1}, newly issued domestic nominal government bonds B_t^n and Foreign nominal private bonds B_{t+1}^{FH}. The aggregate nominal wage W_t is described later. The household's budget constraint is given by:

$$\begin{aligned}
&(1+\tau_t^C)P_t^C C_t + (1-\tau_t^I)\frac{P_t^I}{\gamma_t} I_t + P_t^K \triangle_t^K + \frac{B_{t+1}^{priv}}{R_t \zeta_t} + B_t^n + \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)} + T_t = \\
&(1-\tau_t^W) \int_0^1 W_{h,t} N_{h,t} dh + (1-\tau_t^K) \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t \right) + \iota^K \tau_t^K \delta P_{t-1}^K K_t \\
&+ B_t^{priv} + \left(\alpha_B + (R_{t-1}^B - 1) \right) B_t + S_t B_t^{FH} + (1-\tau_t^{TR}) TR_t + \Xi_{B,t} + \Xi_{B^{FH},t} + \Psi_t.
\end{aligned} \tag{C.5}$$

Now, we explain the budget constraint. The right-hand side of the budget constraint represents the Ricardian household's income sources. $(1-\tau_t^W) \int_0^1 W_{h,t} N_{h,t} dh$ captures the after-tax labor income, where τ_t^W is the labor income tax rate. $\left(1-\tau_t^K\right) R_t^K u_t K_t$ represents the return from renting capital services to intermediate good firms and taxes on the return from renting capital services to intermediate good firms. The term $\iota^K \tau_t^K \delta P_{t-1}^K K_t$ captures the notion that the capital depreciation can be exempted from taxation, and ι^K is an indicator variable, where $\iota^K \in \{0, 1\}$. If ι^K is set to 1, the capital depreciation can be exempted from taxation. $\left(1-\tau_t^K\right) \frac{P_t^I}{\gamma_t} a(u_t) K_t$ captures that the maintenance cost of capital can be deducted from the capital tax bill. The stock of private bonds from the previous period is B_t^{priv}. $S_t B_t^{FH}$ is the return from owning Foreign bonds and the return is affected by nominal exchange rate S_t. $\left(1-\tau_t^{TR}\right) TR_t$ represents transfers from the government and τ_t^{TR} is the tax rate on transfers. Ψ_t is a lump-sum profit from owning Swedish firms. The Ricardian household owns a representative portfolio of government bonds B_t. The government issues bonds that mature with a probability α_B in a given period. Until stochastic maturity, the government pays a non-state contingent interest rate R_{t-1}^B on the government bonds. $\Xi_{B,t}$ and $\Xi_{B^{BF},t}$, t are financial intermediation premia associated with Swedish and Foreign bonds that are rebated in form of lump-sum payments.

Now, we explain the left-hand side of the budget constraint which represents the Ricardian household's expenditures. This term $\left(1+\tau_t^C\right) P_t^C C_t$ captures the consumption expenditure, where τ_t^C is the consumption tax rate and P_t^C is the price of consumption goods. The Ricardian household uses some of her\his income for purchasing investment goods which are captured by the following term $\left(1-\tau_t^I\right) \frac{P_t^I}{\gamma_t} I_t$, where τ_t^I represents the investment tax credit, and P_t^I is the price of investment goods subjected to investment-specific technological process γ_t. The Ricardian household can trade capital in the capital market which is captured by the following term $P_t^K \triangle_t^K$, where P_t^K is the price of capital. The Ricardian household buys Swedish private bonds B_{t+1}^{priv} and the effective price of Swedish private bonds is $\frac{1}{R_t \zeta_t}$, where R_t is the nominal gross interest rate and ζ_t is a risk premium shock to private bonds. The Ricardian household can also invest in newly issued government bonds B_t^n. Finally, the Ricardian household can buy Foreign bonds B_{t+1}^{FH} and the effective price of Foreign bonds is $\frac{S_t}{R_{F,t}\zeta_t \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)}$. $R_{F,t}$ is Foreign nominal gross interest rate and $\Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)$ is the external risk premium term. For the exact functional form of $\Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)$, please see Section 3.1. Finally, the Ricardian household pays the lump-sum taxes T_t.

Now, we present law of motion equations. First, the stock of government bonds that the Ricardian household holds evolves as:

$$B_{t+1} = (1 - \alpha_B) B_t + B_t^n, \tag{C.6}$$

where B_t^n denotes the newly issued debt by the government in period t. Following Krause and Moyen, 2016, Ricardian households are assumed to buy a representative portfolio of government bonds with all possible stochastic maturities.

Second, the average interest rate R_t^B on outstanding government debt bought by Ricardian household h is given by:

$$\left(R_t^B - 1\right) B_{t+1} = (1 - \alpha_B) \left(R_{t-1}^B - 1\right) B_t + \left(R_t^{B,n} - 1\right) B_t^n \tag{C.7}$$

where the interest rate on newly issued government debt is denoted by $R_t^{B,n}$.

Finally, the capital accumulation equation for private capital is given by:

$$K_{t+1} = (1 - \delta) K_t + \Upsilon_t F(I_t, I_{t-1}) + \triangle_t^K. \tag{C.8}$$

C.1.2 Ricardian household's first-order conditions

Ricardian household chooses C_t, I_t, u_t, $\triangle_t^K$, K_{t+1}, B_{t+1}^{priv}, B_{t+1}, B_t^n, and B_{t+1}^{FH} to maximize its expected utility (C.1) subject to the composite consumption equation (C.4), the budget constraint (C.5), the capital accumulation equation (C.8), the government bond equation (C.6) and the average interest rate on long-term government debt equation (C.7) .

We derive the FOC:s by setting up the Lagrangian $\mathscr{L}_t$. We denote θ_t^b as the Lagrange multiplier associated with the budget constraint (C.5), θ_t^k as the Lagrange multiplier associated with the capital accumulation equation (C.8), θ_t^S as the Lagrange multiplier associated with the stock of long-term government bond accumulation equation (C.6), and θ_t^R as the Lagrange multiplier associated with the average interest rate on outstanding government debt equation (C.7). The Lagrangian for the household's optimization problem is expressed as:

$$\begin{aligned}
\mathscr{L}_t = {} & E_0 \sum_{t=0}^{\infty} \beta_t \{ \left[\zeta_t^c u(\tilde{C}_t, \tilde{C}_{t-1}) - \zeta_t^n \Theta_t^n A_n \int_0^1 \frac{N_{h,t}^{1+\eta}}{1+\eta} dh \right] \\
& + \theta_t^b \left[(1 - \tau_t^W) \int_0^1 W_{h,t} N_{h,t} dh + \left(1 - \tau_t^K\right) \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t \right) \right] \\
& + \theta_t^b \left[\iota^K \tau_t^K \delta P_{t-1}^K K_t + B_t^{priv} + \left(\alpha_B + \left(R_{t-1}^B - 1 \right) \right) B_t + S_t B_t^{FH} + \left(1 - \tau_t^{TR}\right) TR_t + \Xi_{B,t} + \Xi_{B^{FH},t} + \Psi_t \right] \\
& - \theta_t^b \left[\left(1 + \tau_t^C\right) P_t^C C_t + \left(1 - \tau_t^I\right) \frac{P_t^I}{\gamma_t} I_t + P_t^K \triangle_t^K + \frac{B_{t+1}^{priv}}{R_t \zeta_t} + B_{h,t}^n + \frac{S_t B_{t+1}^{FH}}{R_{F,t} \zeta_t \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)} + T_t \right] \\
& + \theta_t^S \left[(1 - \alpha_B) B_t + B_t^n - B_{t+1} \right] \\
& + \theta_t^R \left[(1 - \alpha_B) \left(R_{t-1}^B - 1 \right) B_t + \left(R_t^{B,n} - 1 \right) B_t^n - \left(R_t^B - 1 \right) B_{t+1} \right] \\
& + \theta_t^k \left[(1 - \delta) K_t + \Upsilon_t F(I_t, I_{t-1}) + \triangle_t^K - K_{t+1} \right] \}.
\end{aligned} \tag{C.9}$$

When one is solving this optimization problem, one has to keep in mind that the utility function is a function of C_t via the following composite consumption function:

$$\tilde{C}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} C_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1 - \alpha_G)^{\frac{1}{\upsilon_G}} G_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}.$$

First, we derive the FOC for C_t. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to C_t, and we obtain the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial C_t} = \beta_t \left[\zeta_t^c u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) - \theta_t^b P_t^C \left(1 + \tau_t^C\right) \right] = 0.$$

Rearranging the first order condition above equation, we have the following equation:

$$\theta_t^b P_t^C \left(1 + \tau_t^C\right) = \zeta_t^c u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1}). \tag{C.10}$$

We define Ω_t^C as the marginal utility of consumption:

$$\Omega_t^C = \frac{\zeta_t^c u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1})}{1 + \tau_t^C} = \frac{U_{c,t}}{1+\tau_t^C}. \tag{C.11}$$

Note that the definition of Ω_t^C includes consumption taxes to simplify the derivations below.

We use Equation (C.11) to rewrite Equation (C.10) as

$$\theta_t^b P_t^C = \Omega_t^C. \tag{C.12}$$

Equation (C.12), which is the same as Equation (12) in Section 2.1.5, represents the FOC for C_t.

Second, we derive the FOC with respect to I_t. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to I_t, and we have the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial I_t} = \beta_t \left[-\theta_t^b \frac{P_t^I}{\gamma_t} \left(1 - \tau_t^I\right) + \theta_t^k \; F_1(I_t, I_{t-1}) \right] + E_t \left[\beta_{t+1} \theta_{t+1}^k \; F_2(I_{t+1}, I_t) \right] = 0.$$

Rearranging the above equation and using the following definition: $\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$, we have the following FOC for I_t:

$$\theta_{h,t}^b \frac{P_t^I}{\gamma_t} \left(1 - \tau_t^I\right) = \theta_t^k \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta_{t+1}^r \theta_{t+1}^k \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right]. \tag{C.13}$$

Equation (C.13), which is the same as Equation (13) in Section 2.1.5, captures the FOC for I_t.

Third, we derive the FOC for u_t. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to u_t, and we have the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial u_t} = \left(1 - \tau_t^K\right) \beta_t \theta_t^b \; R_t^K K_t - \left(1 - \tau_t^K\right) \beta_t \theta_t^b \; \frac{P_t^I}{\gamma_t} a'(u_t) K_t = 0.$$

Rewriting the above equation, we obtain the following FOC for u_t:

$$R_t^K K_t = \frac{P_t^I}{\gamma_t} a'(u_t) K_t. \tag{C.14}$$

Equation (C.14), which is the same as Equation (14) in Section 2.1.5, represents the FOC for u_t.

Fourth, we find the FOC for Δ_t^K. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to Δ_t^K, and we obtain the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial \Delta_t^K} = -\theta_t^b \beta_t P_t^K + \theta_t^k \beta_t = 0.$$

We rewrite the above equation, and we have the following FOC for Δ_t^K:

$$\theta_t^b P_t^K = \theta_t^k. \tag{C.15}$$

Equation (C.15), which is the same as Equation (15) in Section 2.1.5, represents the FOC for Δ_t^K.

Fifth, we find the FOC for K_{t+1}. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to K_{t+1}, and we have the following equation:

$$\begin{aligned} \frac{\partial \mathscr{L}_t}{\partial K_{t+1}} &= -\beta_t \theta_t^k + E_t \beta_{t+1} \left[\left(1 - \tau_{t+1}^K\right) \left(\theta_{t+1}^b R_{t+1}^K u_{t+1} - \theta_{t+1}^b \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) \right) \right] \\ &+ E_t \beta_{t+1} \left[\iota^K \theta_{t+1}^b \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^k (1 - \delta) \right] = 0. \end{aligned}$$

Rearranging the above equation and using the following definition: $\beta^r_{t+1} = \frac{\beta_{t+1}}{\beta_t}$, we obtain the following FOC for K_{t+1}:

$$\theta^k_t = E_t\beta^r_{t+1}\left[\left(1-\tau^K_{t+1}\right)\theta^b_{t+1}\left(R^K_{t+1}u_{t+1} - \frac{P^I_{t+1}}{\gamma_{t+1}}a(u_{t+1})\right) + \theta^b_{t+1}\iota^K\tau^K_{t+1}\delta P^K_t + \theta^k_{t+1}(1-\delta)\right]. \tag{C.16}$$

Equation (C.16), which is the same as Equation (16) in Section 2.1.5, represents the FOC for K_{t+1}.

Sixth, we derive the FOC for B^{priv}_{t+1}. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to B^{priv}_{t+1}, and this gives us the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial B^{priv}_{t+1}} = -\beta_t\theta^b_t\frac{1}{R_t\zeta_t} + E_t\beta_{t+1}\theta^b_{t+1} = 0.$$

We rearrange the above equation, and then we use the following definition: $\beta^r_{t+1} = \frac{\beta_{t+1}}{\beta_t}$ and multiply both sides by P^C_t. Hence, we have the following FOC for B^{priv}_{t+1}:

$$\theta^b_t P^C_t = E_t\beta^r_{t+1}\theta^b_{t+1}P^C_t R_t\zeta_t. \tag{C.17}$$

Equation (C.17), which is the same as Equation (17) in Section 2.1.5, captures the FOC for B^{priv}_{t+1}.

Seventh, we take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to B_{t+1}. We have the following FOC for B_{t+1}:

$$\begin{aligned}\frac{\partial \mathscr{L}_t}{\partial B_{h,t+1}} &= E_t\beta_{t+1}\theta^b_{t+1}\left(\alpha_B + \left(R^B_t - 1\right)\right) + E_t\beta_{t+1}\theta^S_{t+1}\left(1-\alpha_B\right) - \beta_t\theta^S_t \\ &+\beta_{t+1}\theta^R_{t+1}\left(1-\alpha_B\right)\left(R^B_t - 1\right) - \beta_t\theta^R_t\left(R^B_t - 1\right) = 0.\end{aligned}$$

We rearrange the above equation, and then we use the following definition: $\beta^r_{t+1} = \frac{\beta_{t+1}}{\beta_t}$. We have the following equation:

$$\begin{aligned}\frac{\partial \mathscr{L}_t}{\partial B_{t+1}} &= E_t\beta^r_{t+1}\theta^b_{t+1}\left(\alpha_B + \left(R^B_t - 1\right)\right) + E_t\beta^r_{t+1}\theta^S_{t+1}\left(1-\alpha_B\right) - \theta^S_t \\ &+ E_t\left(\beta^r_{t+1}\theta^R_{t+1}\left(1-\alpha_B\right) - \theta^R_t\right)\left(R^B_t - 1\right) = 0.\end{aligned} \tag{C.18}$$

The above equation can be rewritten as follows:

$$E_t\beta^r_{t+1}\theta^b_{t+1}\left(\alpha_B + \left(R^B_t - 1\right)\right) = \theta^S_t - E_t\beta^r_{t+1}\theta^S_{t+1}\left(1-\alpha_B\right) + \left(\theta^R_t - \left(1-\alpha_B\right)E_t\beta^r_{t+1}\theta^R_{t+1}\right)\left(R^B_t - 1\right). \tag{C.19}$$

Equation (C.19), which is the same as Equation (18) in Section 2.1.5, which captures the FOC of government bond holdings.

Eighth, we take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to B^n_t. We have the following FOC for B^n_t:

$$\frac{\partial \mathscr{L}_t}{\partial B^n_t} = -\theta^b_t\beta_t + \theta^S_t\beta_t + \beta_t\theta^R_t\left(R^{B,n}_t - 1\right) = 0. \tag{C.20}$$

Rearranging the above equation, we have the following FOC for B^n_t:

$$\theta^b_t\beta_t = \theta^S_t\beta_t + \beta_t\theta^R_t\left(R^{B,n}_t - 1\right). \tag{C.21}$$

Equation (C.21), which is the same as Equation (19) in Section 2.1.5, captures the FOC of newly issued government bonds.

Ninth, we take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to R^B_t. We have the following FOC for R^B_t:

$$\frac{\partial \mathscr{L}_t}{\partial R^B_t} = E_t\beta_{t+1}\theta^b_{t+1}B_{t+1} + E_t\beta_{t+1}\theta^R_{t+1}\left(1-\alpha_B\right)B_{t+1} - \beta_t\theta^R_t B_{t+1} = 0. \tag{C.22}$$

We use the following definition: $\beta^r_{t+1} = \frac{\beta_{t+1}}{\beta_t}$, and the above equation can be rewritten as follows:

$$\theta^R_t B_{t+1} = E_t\beta^r_{t+1}\theta^b_{t+1}B_{t+1} + E_t\beta^r_{t+1}\theta^R_{t+1}\left(1-\alpha_B\right)B_{t+1}. \tag{C.23}$$

Equation (C.23), which is the same as Equation (20) in Section 2.1.5, captures the FOC for average interest rate on outstanding government debt (or the price of government bonds that the household is willing to pay).

Finally, we find the FOC for B_{t+1}^{FH}. We take the first derivative of the Lagrangian $\mathscr{L}_t$ with respect to B_{t+1}^{FH}, and we have the following equation:

$$\frac{\partial \mathscr{L}_t}{\partial B_{t+1}^{FH}} = -\frac{\beta_t \theta_t^b S_t}{\Phi(\overline{a}_t, s_t, \widetilde{\phi}_t) R_{F,t}} + E_t \left[\beta_{t+1} \theta_{t+1}^b S_{t+1} \right] = 0.$$

Rearranging the above equation and using the following definition: $\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$, we have the following FOC for $B_{h,t+1}^{FH}$:

$$\theta_t^b S_t = E_t \left[\beta_{t+1}^r \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t) R_{F,t} \zeta_t S_{t+1} \theta_{t+1}^b \right]. \tag{C.24}$$

Equation (C.24), which is the same as Equation (21) in Section 2.1.5, captures the FOC for B_{t+1}^{FH}.

C.1.3 Consumption Euler equation

In this section, we derive the stationarized version of consumption Euler equation (A.1a).

We use equation (C.12), which shows $\theta_t^b P_t^C = \Omega_t^C$ and the following definitions: $p_t^C = \frac{P_t^C}{P_t}$, and $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$. Thus, we can rewrite Equation (C.17) as follows:

$$\theta_t^b P_t^C = E_t \left[\beta_{t+1}^r \theta_{t+1}^b P_t^C \frac{P_{t+1}^C}{P_{t+1}^C} R_t \zeta_t \right],$$

$$\Omega_t^C = E_t \left[\beta_{t+1}^r \frac{R_t \zeta_t}{\Pi_{t+1}^C} \Omega_{t+1}^C \right].$$

Given our assumptions about the possibility of households to diversify the idiosyncratic risk component associated with their wage income, all households in Sweden will choose the same level of consumption in every period (see Section 2.1.2 in the main text). We may drop the subscript h from the above equation. We have the following non-stationarized version of the consumption Euler equation:

$$\Omega_t^C = E_t \left[\beta_{t+1}^r \frac{R_t \zeta_t}{\Pi_{t+1}^C} \Omega_{t+1}^C \right]. \tag{C.25}$$

We now stationarize the consumption Euler equation. In particular, we stationarize Equation (C.25) by using the following definitions: $\mu_{z^+,t+1} = \frac{z_{t+1}^+}{z_t^+}$, $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$. Equation (C.25) becomes:

$$z_t^+ \Omega_t^C = E_t \left[\beta_{t+1}^r z_{t+1}^+ \frac{z_t^+}{z_{t+1}^+} \frac{R_t \zeta_t}{\Pi_{t+1}^C} \Omega_{t+1}^C \right],$$

and we obtain the following stationarized version of consumption Euler equation:

$$\overline{\Omega}_t^C = R_t \zeta_t E_t \left[\beta_{t+1}^r \frac{1}{\mu_{z^+,t+1} \Pi_{t+1}^C} \overline{\Omega}_{t+1}^C \right]. \tag{C.26}$$

Equation (C.26), which represents the stationarized version of consumption Euler equation, is the same as Equation (A.1a).

C.1.4 Marginal utility of consumption

In this section, first we explicitly define the functional form of the household utility function. Second, we derive the stationarized version of marginal utility of consumption equation (A.3a).

Recall from Section 2.10, we have the following functional form for the utility function:

$$u(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) = ln(\tilde{C}_t - \rho_h \tilde{C}_{t-1}).$$

Recall, Equation (C.11), which shows the definition of marginal utility of consumption including the consumption tax, is expressed as:

$$\Omega_t^C = \frac{\zeta_t^c u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1})}{1+\tau_t^C}.$$

Recall, the composite consumption function is expressed as:

$$\tilde{C}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} C_{h,t}^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} G_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}.$$

Using the above utility functional form and the above composite consumption function and taking the first derivative of the utility function with respect to C_t, we can obtain the following marginal utility of consumption equation $U_{c,t}$:

$$U_{c,t} = u_{C_t}(\tilde{C}_t - \rho_h \tilde{C}_{t-1}) = \frac{\zeta_t^c}{\tilde{C}_t - \rho_h \tilde{C}_{t-1}} \left(\alpha_G \frac{\tilde{C}_t}{C_t} \right)^{\frac{1}{\upsilon_G}}. \tag{C.27}$$

Using Equation (C.27), we can rewrite Equation (C.11) as:

$$\Omega_t^C = U_{c,t} \frac{1}{1+\tau_t^C} = \frac{\zeta_t^c}{\tilde{C}_t - \rho_h \tilde{C}_{t-1}} \left(\alpha_G \frac{\tilde{C}_t}{C_t} \right)^{\frac{1}{\upsilon_G}} \frac{1}{1+\tau_t^C}. \tag{C.28}$$

We stationarize Equation (C.28) by using the following definitions: $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$, $\overline{C}_t = \frac{C_t}{z_t^+}$ and $\overline{\overline{C}}_t = \frac{\tilde{C}_t}{z_t^+}$. Equation (C.28) becomes:

$$z_t^+ \Omega_t^C = \frac{\zeta_t^c}{(1+\tau_t^C)\left(\frac{1}{z_t^+}\tilde{C}_t - \rho_h \frac{1}{z_t^+} \frac{z_{t-1}^+}{z_{t-1}^+} \tilde{C}_{t-1} \right)} \left(\alpha_G \frac{z_t^+ \tilde{C}_t}{z_t^+ C_t} \right)^{\frac{1}{\upsilon_G}},$$

and we obtain the following equation:

$$\overline{\Omega}_t^C = \frac{\zeta_t^c}{(1+\tau_t^C)\left(\overline{\overline{C}}_t - \rho_h \frac{1}{\mu_{z^+,t}} \overline{\overline{C}}_{t-1} \right)} \left(\alpha_G \frac{\overline{\overline{C}}_t}{\overline{C}_t} \right)^{\frac{1}{\upsilon_G}}.$$

We define $\bar{G}_t$ as $\frac{G_t}{z_t^+}$, and the composite consumption function can be written in stationarized form as follows:

$$\overline{\overline{C}}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} \bar{C}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} \bar{G}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}. \tag{C.29}$$

In the Swedish economy, where the size of the population is normalized to unity, the distinction between aggregate and *per capita* variables are trivial. Nonetheless, we express the above equation in *per capita* terms, so we denote $\bar{c}_t$ as the stationarized aggregate consumption of Ricardian households *per capita* terms, and $\bar{\bar{c}}_t$ as the stationarized composite consumption in *per capita* terms. Hence, the stationarized version of marginal utility of consumption equation can be written as:

$$\overline{\Omega}_t^C = \frac{\zeta_t^c}{(1+\tau_t^C)\left(\bar{\bar{c}}_t - \rho_h \frac{1}{\mu_{z^+,t}} \bar{\bar{c}}_{t-1} \right)} \left(\alpha_G \frac{\bar{\bar{c}}_t}{\bar{c}_t} \right)^{\frac{1}{\upsilon_G}}. \tag{C.30}$$

Equation (C.30), which represents the stationarized version of marginal utility of consumption equation, is the same as Equation (A.3a).

We can rewrite Equation (C.29) in *per capita* terms. We denote $\bar{g}_t$ as the stationarized government consumption in *per capita* terms. Thus, the stationarized composite consumption equation in *per capita* terms can be expressed as:

$$\bar{\bar{c}}_t = \left(\alpha_G^{\frac{1}{\upsilon_G}} \bar{c}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} + (1-\alpha_G)^{\frac{1}{\upsilon_G}} \bar{g}_t^{\frac{\upsilon_G - 1}{\upsilon_G}} \right)^{\frac{\upsilon_G}{\upsilon_G - 1}}.$$

The above equation is the same as Equation (A.5a).

C.1.5 Capital utilization and household purchases of installed capital

This section derives the capital utilization decision equation (A.8a) and the household purchases of installed capital equation (A.9a) respectively.

First, we derive the capital utilization decision equation. Recall, Equation (C.14), which shows the FOC for $u_{h,t}$, is written as:

$$R_t^K K_t = \frac{P_t^I}{\gamma_t} a'(u_t) K_t.$$

Using the following definitions: $r_t^K = \frac{\gamma_t R_t^K}{P_t}$, and $p_t^I = \frac{P_t^I}{P_t}$, the above equation can be rewritten as follows:

$$\frac{\gamma_t R_t^K}{P_t} = \frac{P_t^I}{P_t} a'(u_t),$$

$$r_t^K = p_t^I a'(u_t).$$

Note that the optimal rate of utilization is a function of the two aggregate relative prices r_t^K and p_t^I. All households in Sweden will then choose the same utilization rate, and the subscript h may be dropped from the above equation. Thus, we have the following capital utilization decision equation:

$$r_t^K = p_t^I a'(u_t). \tag{C.31}$$

Equation (C.31), which captures the capital utilization decision, is the same as Equation (A.8a).

Next, we derive the household purchases of installed capital equation (A.9a). Recall, Equation (C.16), which represents the FOC for $K_{h,t+1}$, is expressed as:

$$\theta_t^k = E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \theta_{t+1}^b \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) \right) + \theta_{t+1}^b \iota^K \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^k (1-\delta) \right].$$

Using Equation (C.15) that shows $\theta_t^b P_t^K = \theta_t^k$, we can rewrite the above equation as:

$$\theta_t^b P_t^K = E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \theta_{t+1}^b \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) \right) + \theta_{t+1}^b \iota^K \tau_{t+1}^K \delta P_t^K + \theta_{t+1}^b P_{t+1}^K (1-\delta) \right].$$

We use Equation (C.12) that shows $\theta_t^b P_t^C = \Omega_t^C$ and use the following definition: $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$. Thus, we can rewrite the above equation as follows:

$$\begin{aligned} P_t^C \theta_t^b P_t^K = {} & E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \theta_{t+1}^b P_{t+1}^C \frac{1}{\Pi_{t+1}^C} \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) + \iota^K \frac{\tau_{t+1}^K}{\left(1 - \tau_{t+1}^K\right)} \delta P_t^K \right) \right] \\ & + E_t \beta_{t+1}^r \left[\theta_{t+1}^b P_{t+1}^C \frac{1}{\Pi_{t+1}^C} P_{t+1}^K (1-\delta) \right], \end{aligned}$$

and

$$\Omega_t^C P_t^K = E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \Omega_{t+1}^C \frac{1}{\Pi_{t+1}^C} \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) + \iota^K \frac{\tau_{t+1}^K}{\left(1 - \tau_{t+1}^K\right)} \delta P_t^K \right) + \Omega_{t+1}^C \frac{1}{\Pi_{t+1}^C} P_{t+1}^K (1-\delta) \right].$$

We multiply both sides of the above equation by $\frac{\gamma_t}{P_t}$, and then we rewrite the above equation as follows:

$$\begin{aligned} \frac{\gamma_t P_t^K}{P_t} = {} & E_t \beta_{t+1}^r \left[\left(1 - \tau_{t+1}^K\right) \frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} \frac{\gamma_t}{P_t} \left(R_{t+1}^K u_{t+1} - \frac{P_{t+1}^I}{\gamma_{t+1}} a(u_{t+1}) + \iota^K \frac{\tau_{t+1}^K}{\left(1 - \tau_{t+1}^K\right)} \delta P_t^K \right) \right] \\ & + E_t \beta_{t+1}^r \left[\frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} \frac{\gamma_t}{P_t} P_{t+1}^K (1-\delta) \right]. \end{aligned}$$

In order to stationarize the above equation, we use the following definitions: $r^K_{t+1} = \frac{\gamma_{t+1} R^K_{t+1}}{P_{t+1}}$, $p^I_{t+1} = \frac{P^I_{t+1}}{P_{t+1}}$, $\mu_{\gamma,t+1} = \frac{\gamma_{t+1}}{\gamma_t}$, $p^K_t = \frac{\gamma_t P^K_t}{P_t}$, and $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. Thus, we have the following equation for the household purchases of installed capital:

$$\frac{\gamma_t P^K_t}{P_t} = E_t \beta^r_{t+1} \left[\left(1 - \tau^K_{t+1}\right) \frac{\Omega^C_{t+1}}{\Omega^C_t} \frac{1}{\Pi^C_{t+1}} \frac{\gamma_t}{P_t} \frac{P_{t+1}}{\gamma_{t+1}} \left(r^K_{t+1} u_{t+1} - p^I_{t+1} a(u_{t+1}) + \iota^K \frac{\tau^K_{t+1}}{\left(1 - \tau^K_{t+1}\right)} \delta \frac{\mu_{\gamma,t+1}}{\Pi_{t+1}} p^K_t \right) \right]$$
$$+ E_t \beta^r_{t+1} \left[\frac{\Omega^C_{t+1}}{\Omega^C_t} \frac{1}{\Pi^C_{t+1}} \frac{\gamma_t}{P_t} P^K_{t+1} (1 - \delta) \right].$$

We use the following definition: $p^K_t = \frac{\gamma_t P^K_t}{P_t}$, and the above equation can be written as follows:

$$p^K_t = E_t \beta^r_{t+1} \frac{\Omega^C_{t+1}}{\Omega^C_t} \frac{\Pi_{t+1}}{\Pi^C_{t+1}} \frac{1}{\mu_{\gamma,t+1}} \left[\left(1 - \tau^K_{t+1}\right) \left(r^K_{t+1} u_{t+1} - p^I_{t+1} a(u_{t+1}) \right) + \iota^K \tau^K_{t+1} \delta \frac{\mu_{\gamma,t+1}}{\Pi_{t+1}} p^K_t + p^K_{t+1} (1 - \delta) \right]. \tag{C.32}$$

Using the following definitions: $\overline{\Omega}^C_t = z^+_t \Omega^C_t$ and $\mu_{z^+,t+1} = \frac{z^+_{t+1}}{z^+_t}$, Equation (C.32) can be written as:

$$p^K_t = E_t \beta^r_{t+1} \frac{\overline{\Omega}^C_{t+1}}{\overline{\Omega}^C_t} \frac{\Pi_{t+1}}{\Pi^C_{t+1}} \frac{1}{\mu_{z^+,t+1} \mu_{\gamma,t+1}} \left[\left(1 - \tau^K_{t+1}\right) \left(r^K_{t+1} u_{t+1} - p^I_{t+1} a(u_{t+1}) \right) + \iota^K \delta \tau^K_{t+1} \frac{\mu_{\gamma,t+1}}{\Pi_{t+1}} p^K_t + p^K_{t+1} (1 - \delta) \right]. \tag{C.33}$$

Equation (C.33) is the same as Equation (A.9a), which shows the stationarized version of the household purchase of installed capital.

C.1.6 Investment decision

This section derives the household investment decision equation (A.10a). Recall that we have Equation (C.13) that shows the following FOC for $I_{h,t}$:

$$\theta^b_t \frac{P^I_t}{\gamma_t} \left(1 - \tau^I_t\right) = \theta^k_t \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta^r_{t+1} \theta^k_{t+1} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

The above equation can be expressed as:

$$P^I_t \left(1 - \tau^I_t\right) = \frac{\gamma_t \theta^k_t}{\theta^b_t} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta^r_{t+1} \frac{\gamma_t \theta^k_{t+1}}{\theta^b_t} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

We use Equation (C.15), which shows $\theta^b_t P^K_t = \theta^k_t$. We can rewrite the above equation as follows:

$$P^I_t \left(1 - \tau^I_t\right) = \frac{\gamma_t \theta^b_t P^K_t}{\theta^b_t} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta^r_{t+1} \frac{\gamma_t \theta^b_{t+1} P^K_{t+1}}{\theta^b_t} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right],$$
$$P^I_t \left(1 - \tau^I_t\right) = \gamma_t P^K_t \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta^r_{t+1} \frac{\gamma_t \theta^b_{t+1} P^K_{t+1}}{\theta^b_t} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

We use the following definition: $p^I_t = P^I_t / P_t$, and the above equation becomes:

$$p^I_t \left(1 - \tau^I_t\right) = \frac{\gamma_t P^K_t}{P_t} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta^r_{t+1} \frac{\gamma_t \theta^b_{t+1} P^K_{t+1}}{P_t \theta^b_t} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

We multiply the second term on the right hand side of the above equation by $\frac{P_{t+1}\gamma_{t+1}}{P_{t+1}\gamma_{t+1}}$. We use the following definitions: $\mu_{\gamma,t+1} = \frac{\gamma_{t+1}}{\gamma_t}$ and $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. The above equation can then be rewritten as follows:

$$p^I_t \left(1 - \tau^I_t\right) = \frac{\gamma_t P^K_t}{P_t} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta^r_{t+1} \frac{\theta^b_{t+1}}{\theta^b_t} \frac{\gamma_{t+1} P^K_{t+1}}{P_{t+1}} \frac{P_{t+1}}{P_t} \frac{\gamma_t}{\gamma_{t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right],$$
$$p^I_t \left(1 - \tau^I_t\right) = \frac{\gamma_t P^K_t}{P_t} \Upsilon_t F_1(I_t, I_{t-1}) + E_t \left[\beta^r_{t+1} \frac{\theta^b_{t+1}}{\theta^b_t} \frac{\gamma_{t+1} P^K_{t+1}}{P_{t+1}} \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t) \right].$$

Using the following definition: $p_t^K = \frac{\gamma_t P_t^K}{P_t}$, this gives us the following equation:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right].$$

Using the following definitions: $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$ and $\Omega_{h,t}^C = \theta_{h,t}^b P_t^C$, we can rewrite the above equation as follows:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\theta_{t+1}^b P_{t+1}^C}{\theta_t^b P_t^C} \frac{P_t^C}{P_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right],$$

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\theta_{t+1}^b P_{t+1}^C}{\theta_t^b P_t^C} \frac{1}{\Pi_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right],$$

and we can obtain the following equation:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right].$$

Hence, we have the following equation for the household investment decision:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\Omega_{t+1}^C}{\Omega_t^C} \frac{1}{\Pi_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right]. \tag{C.34}$$

Now, we continue the effort to stationarize Equation (C.34). Using the following definitions: $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$ and $\mu_{z^+,t+1} = \frac{z_{t+1}^+}{z_t^+}$, Equation (C.34) can be written as follows:

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{z_{t+1}^+ \Omega_{t+1}^C}{z_t^+ \Omega_t^C} \frac{z_t^+}{z_{t+1}^+} \frac{1}{\Pi_{t+1}^C} p_{t+1}^K \Pi_{t+1} \frac{1}{\mu_{t+1}^\gamma} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right],$$

$$p_t^I\left(1-\tau_t^I\right) = p_t^K \Upsilon_t F_1(I_t, I_{t-1}) + E_t\left[\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\overline{\Omega}_t^C} \frac{\Pi_{t+1}}{\Pi_{t+1}^C} \frac{p_{t+1}^K}{\mu_{z^+,t+1}\mu_{\gamma,t+1}} \Upsilon_{t+1} F_2(I_{t+1}, I_t)\right]. \tag{C.35}$$

Furthermore, we need to express $F_1(I_t, I_{t-1})$ and $F_2(I_{t+1}, I_t)$ as functions of stationary variables. Recall from Section 2.10, we have the following investment adjustment cost function $F(I_t, I_{t-1})$:

$$F(I_t, I_{t-1}) = \left[1 - \widetilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t.$$

We take the first derivative of $F(I_t, I_{t-1})$ with respect to I_t, and we can find $F_1(I_t, I_{t-1})$. We then take the first derivative of $F(I_{t+1}, I_t)$ with respect to I_t, and we can find $F_2(I_{t+1}, I_t)$. We have the following results:

$$F_1(I_t, I_{t-1}) = -\widetilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} + \left[1 - \widetilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right], \tag{C.36}$$

and

$$F_2(I_{t+1}, I_t) = \widetilde{S}'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2. \tag{C.37}$$

We express Equation (C.36) and Equation (C.37) by applying the following definition: $\overline{I}_t = \frac{I_t}{z_t^+ \gamma_t}$. Using this definition, together with $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$ and $\mu_{\gamma,t} = \frac{\gamma_t^+}{\gamma_{t-1}^+}$, the ratio $\frac{I_t}{I_{t-1}}$ can be written as: $\mu_{z^+,t}\,\mu_{\gamma,t}\,\frac{\overline{I}_t}{\overline{I}_{t-1}}$. We use the notation $F_1(\overline{I}_t, \overline{I}_{t-1}, \mu_{z^+,t}, \mu_{\gamma,t})$ to express $F_1(I_t, I_{t-1})$ as a function of the stationary variables $\overline{I}_t$, $\overline{I}_{t-1}$, $\mu_{z^+,t}$ and $\mu_{\gamma,t}$. Moreover, $F_2(\overline{I}_{t+1}, \overline{I}_t, \mu_{z^+,t+1}, \mu_{\gamma,t+1})$ represents $F_2(I_{t+1}, I_t)$ expressed as a function of stationary variables. Hence, Equation (C.36) and Equation (C.37) become:

$$F_1(\overline{I}_t, \overline{I}_{t-1}, \mu_{z^+,t}, \mu_{\gamma,t}) = -\widetilde{S}'\left(\frac{\mu_{z^+,t}\;\mu_{\gamma,t}\overline{I}_t}{\overline{I}_{t-1}}\right)\frac{\mu_{z^+,t}\;\mu_{\gamma,t}\overline{I}_t}{\overline{I}_{t-1}} + \left[1 - \widetilde{S}\left(\frac{\mu_{z^+,t}}{\overline{I}_{t-1}}\right)\right], \tag{C.38}$$

and

$$F_2(\overline{I}_{t+1}, \overline{I}_t, \mu_{z^+,t+1}, \mu_{\gamma,t+1}) = \widetilde{S}'\left(\frac{\mu_{z^+,t+1}\;\mu_{\gamma,t+1}\overline{I}_{t+1}}{\overline{I}_t}\right)\left(\frac{\mu_{z^+,t+1}\;\mu_{\gamma,t+1}\overline{I}_{t+1}}{\overline{I}_t}\right)^2. \tag{C.39}$$

With these notations, we can rewrite Equation (C.35) as:

$$p_t^I\left(1-\tau_t^I\right)=p_t^K\Upsilon_t F_1(\overline{I}_t,\overline{I}_{t-1},\mu_{z^+,t},\mu_{\gamma,t})+E_t\left[\beta_{t+1}^r\frac{\overline{\Omega}_{t+1}^C}{\overline{\Omega}_t^C}\frac{\Pi_{t+1}}{\Pi_{t+1}^C}\frac{p_{t+1}^K}{\mu_{z^+,t+1}\mu_{\gamma,t+1}}\Upsilon_{t+1}F_2(\overline{I}_{t+1},\overline{I}_t,\mu_{z^+,t+1},\mu_{\gamma,t+1})\right]. \tag{C.40}$$

Equation (C.40), which represents the stationarized version of the household investment decision equation, is the same as Equation (A.10a).

C.1.7 Modified uncovered interest rate parity

This section derives the stationarized version of uncovered interest rate parity equation (A.22a).

Recall, Equation (C.24), which shows the FOC for B_{t+1}^{FH}, is written as:

$$\theta_t^b S_t=E_t\left[\beta_{t+1}^r\Phi(\overline{a}_t,s_t,\widetilde{\phi}_t)R_{F,t}\zeta_t S_{t+1}\theta_{t+1}^b\right].$$

Using the following definitions: $\theta_t^b P_t^C=\Omega_t^C$ and $\Pi_{t+1}^C=\frac{P_{t+1}^C}{P_t^C}$, the above equation can be written as follows.

$$\theta_t^b P_t^C=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\frac{P_t^C}{P_{t+1}^C}\theta_{t+1}^b P_{t+1}^C R_{F,t}\zeta_t\frac{S_{t+1}}{S_t}\right], \tag{C.41}$$

$$\Omega_t^C=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\frac{1}{\Pi_{t+1}^C}\Omega_{t+1}^C R_{F,t}\zeta_t\frac{S_{t+1}}{S_t}\right].$$

Using the following definition: $s_{t+1}=\frac{S_{t+1}}{S_t}$, the above equation can be expressed as:

$$\Omega_t^C=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\frac{1}{\Pi_{t+1}^C}\Omega_{t+1}^C R_{F,t}\zeta_t s_{t+1}\right]. \tag{C.42}$$

Recall, we have the following non-stationarized version of consumption Euler equation (C.25), which is expressed as:

$$\Omega_t^C=E_t\left[\beta_{t+1}^r\frac{R_t\zeta_t}{\Pi_{t+1}^C}\Omega_{t+1}^C\right].$$

Using the above non-stationarized version of consumption Euler equation and Equation (C.42), we can obtain the following non-stationarized version of the uncovered interest parity equation:

$$E_t\left[\beta_{t+1}^r\frac{R_t\zeta_t}{\Pi_{t+1}^C}\Omega_{t+1}^C\right]=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\zeta_t\frac{R_{F,t}}{\Pi_{t+1}^C}\Omega_{t+1}^C s_{t+1}\right].$$

Now, we stationarize the above equation. Using the following definitions: $\overline{\Omega}_{t+1}^C=z_{t+1}^+\Omega_{t+1}^C$ and $\mu_{z^+,t+1}=\frac{z_{t+1}^+}{z_t^+}$, the above equation becomes:

$$E_t\left[\beta_{t+1}^r\frac{R_t\zeta_t}{\Pi_{t+1}^C}z_t^+\frac{z_{t+1}^+}{z_{t+1}^+}\Omega_{t+1}^C\right]=E_t\left[\beta_{t+1}^r\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)\frac{R_{F,t}\zeta_t}{\Pi_{t+1}^C}z_t^+\frac{z_{t+1}^+}{z_{t+1}^+}\Omega_{t+1}^C s_{t+1}\right].$$

We have the following stationarized version of the uncovered interest rate parity equation:

$$R_t E_t\left[\beta_{t+1}^r\frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C}\right]=R_{F,t}\Phi\left(\overline{a}_t,s_t,\widetilde{\phi}_t\right)E_t\left[\beta_{t+1}^r\frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1}\Pi_{t+1}^C}s_{t+1}\right]. \tag{C.43}$$

Equation (C.43) is the same as Equation (A.22a), which shows the stationarized version of the modified uncovered interest rate parity equation.

C.1.8 Average interest rate on government bonds and Euler equation for government bonds

In this section, we derive the optimal condition for average interest rate on government bonds, Equation (A.6a) and Euler equation for government bond holdings, Equation (A.7a).

The FOC for average interest rate on outstanding government debt, Equation (C.23) can be written as:

$$\frac{\theta_t^R}{\theta_t^b} = E_t \beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \left[1 + \frac{\theta_{t+1}^R}{\theta_{t+1}^b} (1 - \alpha_B)\right]. \tag{C.44}$$

The FOC of newly issued government bonds, Equation (C.21) can be written as:

$$\frac{\theta_t^S}{\theta_t^b} = 1 - \frac{\theta_t^R}{\theta_t^b} \left(R_t^{B,n} - 1\right). \tag{C.45}$$

The FOC of government bond holdings, Equation (C.19) can be written as:

$$\frac{\theta_t^S}{\theta_t^b} + \frac{\theta_t^R}{\theta_t^b} \left(R_t^B - 1\right) = E_t \beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \left[\left(\alpha_B + \left(R_t^B - 1\right)\right) + \frac{\theta_{t+1}^S}{\theta_{t+1}^b} (1 - \alpha_B) + \frac{\theta_{t+1}^R}{\theta_{t+1}^b} (1 - \alpha_B) \left(R_t^B - 1\right)\right]. \tag{C.46}$$

Using Equation (C.45), we can rewrite Equation (C.46) as:

$$1 + \frac{\theta_t^R}{\theta_t^b} \left(R_t^B - R_t^{B,n}\right) = E_t \beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \left[R_t^B - (1 - \alpha_B) \frac{\theta_{t+1}^R}{\theta_{t+1}^b} \left(R_{t+1}^{B,n} - R_t^B\right)\right].$$

Using Equation (C.44), we can rewrite the above equation as:

$$1 = E_t \beta_{t+1}^r \frac{\theta_{t+1}^b}{\theta_t^b} \left[R_t^{B,n} - (1 - \alpha_B) \frac{\theta_{t+1}^R}{\theta_{t+1}^b} \left(R_{t+1}^{B,n} - R_t^{B,n}\right)\right]. \tag{C.47}$$

We use the following definitions: $\theta_t^b P_t^C = \Omega_t^C$, $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$, $\bar{\Omega}_t^R = \frac{\theta_t^R}{\theta_t^b}$, and $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$. Thus, we can rewrite Equation (C.44) as:

$$\bar{\Omega}_t^R = E_t \beta_{t+1}^r \frac{\bar{\Omega}_{t+1}^C}{\bar{\Omega}_t^C \Pi_{t+1}^C \mu_{z^+,t+1}} \left[1 + \bar{\Omega}_{t+1}^R (1 - \alpha_B)\right]. \tag{C.48}$$

Equation (C.48), which represents the stationarized version of the optimal condition for average interest rate on government bonds, is the same as Equation (A.6a).

We use the following definitions: $\theta_t^b P_t^C = \Omega_t^C$, $\overline{\Omega}_t^C = z_t^+ \Omega_t^C$, $\bar{\Omega}_t^R = \frac{\theta_t^R}{\theta_t^b}$, and $\Pi_{t+1}^C = \frac{P_{t+1}^C}{P_t^C}$ as well as we drop the subscript h. Thus, we can rewrite Equation (C.47) as:

$$1 = E_t \beta_{t+1}^r \frac{\bar{\Omega}_{t+1}^C}{\bar{\Omega}_t^C \Pi_{t+1}^C \mu_{z^+,t+1}} \left[R_t^{B,n} - (1 - \alpha_B) \bar{\Omega}_{t+1}^R \left(R_{t+1}^{B,n} - R_t^{B,n}\right)\right]. \tag{C.49}$$

Equation (C.49), which represents the stationarized version of Euler equation for government bond holdings, is the same as Equation (A.7a).

C.1.9 Wage setting

This section derives Equation (A.13a), which represents the stationarized version of the optimal wage setting equation. Ricardian household member labor type h choose the optimal wage rate $W_{h,t}^{opt}$ that maximizes the expected utility of household (C.1) rather than its individual utility, subject to the household budget constraint (C.5), the labor demand schedule (C.50), and the Calvo wage contract (C.51). In each period, the individual labor type resets its wage with probability $(1 - \xi_w)$. With probability ξ_w, the household member cannot reset its wage, in which case the wage rate evolves according to: $W_{h,t+k|t} = W_{h,t}^{opt} \overline{\Pi}_{t1}^W \overline{\Pi}_{t+2}^W \ldots \overline{\Pi}_{t+k}^W$. Note $\overline{\Pi}_t^W = \left(\Pi_{t-1}^W\right)^{\chi_w} \left(\Pi_t^{C,trend}\right)^{1-\chi_w}$.

The demand for labor is given by

$$N_{h,t+k|t} = \left(\frac{W_{h,t+k|t}}{W_{t+k}}\right)^{-\varepsilon_{w,t}} N_{t+k} \tag{C.50}$$

and the Calvo wage contract is given by

$$W_{h,t+k} = \begin{cases} \overline{\Pi}^W_{t+k} W_{h,t+k-1} & \text{with probability } \xi_w, \\ W^{\text{opt}}_{h,t+k} & \text{with probability } (1-\xi_w). \end{cases} \tag{C.51}$$

We let θ^b_t denote the Lagrange multiplier associated with the budget constraint (C.5). To solve the optimization problem, we set up the following Lagrangian:

$$\begin{aligned}
\mathscr{L}^W_t &= \beta_t \left[\zeta^c_t u(\tilde{C}_t, \tilde{C}_{t-1}) - \zeta^n_t \Theta^n_t A_n \int_0^1 \frac{N^{1+\eta}_{h,t}}{1+\eta} dh\right] \\
&+ \theta^b_t \left[(1-\tau^W_t) \int_0^1 W_{h,t} N_{h,t} dh + (1-\tau^K_t)\left(R^K_t u_t K_{h,t} - \frac{P^I_t}{\gamma_t} a(u_t) K_t\right) + \iota^K \tau^K_t \delta P^K_{t-1} K_t + \right. \\
&\left. + B^{priv}_t + \left(\alpha_B + \left(R^B_{t-1} - 1\right)\right) B_t + S_t B^{FH}_t + (1-\tau^{TR}_t) TR_t + \Xi_{B,t} + \Xi_{B^{FH},t} + \Psi_t\right] \\
&- \theta^b_t \left[(1+\tau^C_t) P^C_t C_t + (1-\tau^I_t) \frac{P^I_t}{\gamma_t} I_t + P^K_t \triangle^K_t + \frac{B^{priv}_{t+1}}{R_t \zeta_t} + B^n_t + \frac{S_t B^{FH}_{t+1}}{R_{F,t} \Phi(\overline{a}_t, s_t, \widetilde{\phi}_t)} + T_t\right] \\
&+ \xi_w \beta_{t+1} \left[\zeta^c_{t+1} u(\tilde{C}_{t+1}, \tilde{C}_t) - \zeta^n_{t+1} \Theta^n_{t+1} A_n \int_0^1 \frac{N^{1+\eta}_{h,t+1}}{1+\eta} dh\right] \\
&+ \theta^b_{t+1} \left[(1-\tau^W_{t+1}) \int_0^1 W_{h,t+1} N_{h,t+1} dh + \left(1-\tau^K_{t+1}\right)\left(R^K_{t+1} u_{t+1} K_{t+1} - \frac{P^I_{t+1}}{\gamma_{t+1}} a(u_{t+1}) K_{t+1}\right) + \iota^K \tau^K_{t+1} \delta P^K_t K_{t+1} + \right. \\
&\left. + B^{priv}_{t+1} + \left(\alpha_B + \left(R^B_t - 1\right)\right) B_{t+1} + S_{t+1} B^{FH}_{t+1} + (1-\tau^{TR}_{t+1}) TR_{t+1} + \Xi_{B,t+1} + \Xi_{B^{FH},t+1} + \Psi_{t+1}\right] \\
&- \theta^b_{t+1} \left[(1+\tau^C_{t+1}) P^C_{t+1} C_{t+1} + (1-\tau^I_{t+1}) \frac{P^I_{t+1}}{\gamma_{t+1}} I_{t+1} + P^K_{t+1} \triangle^K_{t+1} + \frac{B^{priv}_{t+2}}{R_{t+1} \zeta_{t+1}} + \frac{S_{t+1} B^{FH}_{t+2}}{R_{F,t+1} \Phi(\overline{a}_{t+1}, s_{t+1}, \widetilde{\phi}_{t+1})} + T_{t+1}\right] \\
&+ (\xi_w)^2 \beta_{t+2} \left[\zeta^c_{t+2} u(\tilde{C}_{t+2}, \tilde{C}_{t+1}) - \zeta^n_{t+2} \Theta^n_{t+2} A_n \int_0^1 \frac{N^{1+\eta}_{h,t+2}}{1+\eta} dh\right] \\
&+ \theta^b_{t+2} \left[(1-\tau^W_{t+2}) \int_0^1 W_{h,t+2} N_{h,t+2} dh + \left(1-\tau^K_{t+2}\right)\left(R^K_{t+2} u_{t+2} K_{h,t+2} - \frac{P^I_{t+2}}{\gamma_{t+2}} a(u_{t+2}) K_{t+2}\right) + \iota^K \tau^K_{t+2} \delta P^K_{t+1} K_{t+2} + \right. \\
&\left. + B^{priv}_{t+2} + \left(\alpha_B + \left(R^B_{t+1} - 1\right)\right) B_{t+2} + S_{t+2} B^{FH}_{t+2} + (1-\tau^{TR}_{t+2}) TR_{t+2} + \Xi_{B,t+2} + \Xi_{B^{FH},t+2} + \Psi_{t+2}\right] \\
&- \theta^b_{t+2} \left[(1+\tau^C_{t+2}) P^C_{t+2} C_{t+2} + (1-\tau^I_{t+2}) \frac{P^I_{t+2}}{\gamma_{t+2}} I_{t+2} + P^K_{t+2} \triangle^K_{t+2} + \frac{B^{priv}_{t+3}}{R_{t+2} \zeta_{t+2}} + \frac{S_{t+2} B^{FH}_{t+3}}{R_{F,t+2} \Phi(\overline{a}_{t+2}, s_{t+2}, \widetilde{\phi}_{t+2})} + T_{t+2}\right] \\
&+ \dots
\end{aligned} \tag{C.52}$$

We take the first derivative of $\mathscr{L}_t^W$ with respect to $W_{h,t}^{opt}$, and we obtain the following equation:

$$\begin{aligned}
\frac{\partial \mathscr{L}_{h,t}^W}{\partial W_{h,t}^{opt}} &= \beta_t E_t[-\zeta_t^n \nu'(N_{h,t|t}) \frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}} + \theta_t^b (1-\tau_t^W) \left(N_{h,t|t} + W_{h,t}^{opt} \frac{\partial N_{h,t|t}}{\partial W_{h,t|t}^{opt}} \right)] \\
&+ \xi_w \ E_t \beta_{t+1}[-\zeta_{t+1}^n \nu'(N_{h,t+1|t}) \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}} \\
&+ \theta_{t+1}^b (1-\tau_{t+1}^W) \left(N_{h,t+1|t} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}} + W_{h,t+1|t} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}} \right)] \\
&+ (\xi_w)^2 E_t \beta_{t+2}[-\zeta_{t+2}^n \nu'(N_{h,t+2|t}) \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}} \\
&+ \theta_{t+2}^b (1-\tau_{t+2}^W) \left(N_{h,t+2|t} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}} + W_{h,t+2|t} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}} \right)] + \ldots = 0.
\end{aligned} \tag{C.53}$$

We use the following definition: $\beta_{t+k}^r = \frac{\beta_{t+k}}{\beta_{t+k-1}}$, and then we rearrange the above equation. Note that $\frac{\beta_{t+2}}{\beta_t} = \beta_{t+1}^r \beta_{t+2}^r$. We have the following equation:

$$\begin{aligned}
0 &= \theta_t^b E_t[W_{h,t}^{opt} \frac{\partial N_{h,t}}{\partial W_{h,t}^{opt}} (1-\tau_t^W) \left(\frac{N_{h,t|t}}{W_{h,t}^{opt}} \left(\frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}} \right)^{-1} + 1 \right) - \zeta_t^n \frac{\nu'(N_{h,t|t})}{\theta_t^b} \frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}}] \\
&+ \xi_w E_t \beta_{t+1}^r \theta_{t+1}^b [W_{h,t+1|t} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}} (1-\tau_{t+1}^W) \left(\frac{N_{h,t+1|t}}{W_{h,t+1|t}} \left(\frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \right)^{-1} + 1 \right) \\
&- \zeta_{t+1}^n \frac{\nu'(N_{h,t+1|t})}{\theta_{t+1}^b} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \frac{\partial W_{h,t+1|t}}{\partial W_{h,t}^{opt}}] \\
&+ (\xi_w)^2 E_t \beta_{t+1}^r \beta_{t+2}^r \theta_{t+2}^b [W_{h,t+2|t} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}} (1-\tau_{t+2}^W) \left(\frac{N_{h,t+2|t}}{W_{h,t+2|t}} \left(\frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \right)^{-1} + 1 \right) \\
&- \zeta_{t+2}^n \frac{\nu'(N_{h,t+2|t})}{\theta_{t+2}^b} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \frac{\partial W_{h,t+2|t}}{\partial W_{h,t}^{opt}}] + \ldots
\end{aligned} \tag{C.54}$$

Recall, we have the following definition: $W_{h,t+k|t} = W_{h,t}^{opt} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W \ldots \overline{\Pi}_{t+k}^W$. Thus, the partial derivative of $W_{h,t+k|t}$ with respect to $W_{h,t}^{opt}$ is:

$$\frac{\partial W_{h,t+k|t}}{\partial W_{h,t}^{opt}} = \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W \ldots \overline{\Pi}_{t+k}^W. \tag{C.55}$$

Using Equation (C.55), Equation (C.54) can be written as:

$$\begin{aligned}
0 &= \theta_t^b E_t[W_{h,t}^{opt} \frac{\partial N_{h,t}}{\partial W_{h,t}^{opt}} (1-\tau_t^W) \left(\frac{N_{h,t|t}}{W_{h,t}^{opt}} \left(\frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}} \right)^{-1} + 1 \right) - \zeta_t^n \frac{\nu'(N_{h,t|t})}{\theta_t^b} \frac{\partial N_{h,t|t}}{\partial W_{h,t}^{opt}}] \\
&+ \xi_w E_t \beta_{t+1}^r \theta_{t+1}^b [W_{h,t+1|t} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \overline{\Pi}_{t+1}^W (1-\tau_{t+1}^W) \left(\frac{N_{h,t+1|t}}{W_{h,t+1|t}} \left(\frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \right)^{-1} + 1 \right) \\
&- \zeta_{t+1}^n \frac{\nu'(N_{h,t+1|t})}{\theta_{t+1}^b} \frac{\partial N_{h,t+1|t}}{\partial W_{h,t+1|t}} \overline{\Pi}_{t+1}^W] \\
&+ (\xi_w)^2 E_t \beta_{t+1}^r \beta_{t+2}^r \theta_{t+2}^b [W_{h,t+2|t} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W (1-\tau_{t+2}^W) \left(\frac{N_{h,t+2|t}}{W_{h,t+2|t}} \left(\frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \right)^{-1} + 1 \right) \\
&- \zeta_{t+2}^n \frac{\nu'(N_{h,t+2|t})}{\theta_{t+2}^b} \frac{\partial N_{h,t+2|t}}{\partial W_{h,t+2|t}} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W] + \ldots
\end{aligned} \tag{C.56}$$

Using the labor demand schedule, which is captured by Equation (C.50), we can find the following wage-elasticity of labor demand:

$$-\frac{\partial N_{h,t+k|t}}{\partial W_{h,t+k|t}} \frac{W_{h,t+k|t}}{N_{h,t+k|t}} = \varepsilon_{w,t}. \tag{C.57}$$

Using the following definition: $\varepsilon_{w,t} = \frac{\lambda_t^W}{\lambda_t^W - 1}$ and the result from Equation (C.57), we have the following equation:

$$\frac{\partial N_{h,t+k|t}}{\partial W_{h,t+k|t}} \frac{W_{h,t+k|t}}{N_{h,t+k|t}} = \frac{\lambda_t^W}{1-\lambda_t^W}. \tag{C.58}$$

Using the result from Equation (C.58), the derivative of $N_{h,t+k|t}$ with respect to $W_{h,t+k|t}$ is:

$$\frac{\partial N_{h,t+k|t}}{\partial W_{h,t+k|t}} = \frac{\lambda_t^W}{1-\lambda_t^W}\frac{N_{h,t+k|t}}{W_{h,t+k|t}}. \tag{C.59}$$

We use Equation (C.58) and Equation (C.59); hence, Equation (C.56) can be expressed as:

$$\begin{aligned}0 &= \theta_t^b E_t[\frac{\lambda_t^W}{(1-\lambda_t^W)}\frac{N_{h,t|t}}{W_{h,t}^{opt}}\left[(1-\tau_t^W)W_{h,t}^{opt}\frac{1}{\lambda_t^W} - \zeta_t^n\frac{\nu'(N_{h,t|t})}{\theta_t^b}\right]] \\ &+ \xi_w E_t\ \beta_{t+1}^r\theta_{t+1}^b[\frac{\lambda_{t+1}^W}{(1-\lambda_{t+1}^W)}\frac{N_{h,t+1|t}}{W_{h,t+1|t}}\overline{\Pi}_{t+1}^W\left[(1-\tau_{t+1}^W)W_{h,t+1|t}\frac{1}{\lambda_{t+1}^W} - \zeta_{t+1}^n\frac{\nu'(N_{h,t+1|t})}{\theta_{t+1}^b}\right]] \\ &+ (\xi_w)^2 E_t\ \beta_{t+1}^r\beta_{t+2}^r\theta_{t+2}^b[\frac{\lambda_{t+2}^W}{(1-\lambda_{t+2}^W)}\frac{N_{h,t+2|t}}{W_{h,t+2|t}}\overline{\Pi}_{t+1}^W\overline{\Pi}_{t+2}^W\left[(1-\tau_{t+2}^W)W_{h,t+2|t}\frac{1}{\lambda_{t+2}^W} - \zeta_{t+2}^n\frac{\nu'(N_{h,t+2|t})}{\theta_{t+2}^b}\right]] + \ldots\end{aligned}$$

We use the following definition: $W_{h,t+k|t} = W_{h,t}^{opt}\overline{\Pi}_{t+1}^W\overline{\Pi}_{t+2}^W\ldots\overline{\Pi}_{t+k}^W$, and we multiply both sides of the above equation by $W_{h,t}^{opt}$. We have following equation:

$$\begin{aligned}0 &= \theta_t^b\frac{1}{1-\lambda_t^W}E_t[N_{h,t|t}\left[(1-\tau_t^W)W_{h,t}^{opt} - \lambda_t^W\zeta_t^n\frac{\nu'(N_{h,t|t})}{\theta_t^b}\right]] \\ &+ \xi_w E_t\ \beta_{t+1}^r\theta_{t+1}^b\frac{1}{1-\lambda_{t+1}^W}[N_{h,t+1|t}\left[(1-\tau_{t+1}^W)W_{h,t+1|t} - \lambda_{t+1}^W\zeta_{t+1}^n\frac{\nu'(N_{h,t+1|t})}{\theta_{t+1}^b}\right]] \\ &+ (\xi_w)^2 E_t\ \beta_{t+1}^r\beta_{t+2}^r\theta_{t+2}^b\frac{1}{1-\lambda_{t+2}^W}[N_{h,t+2|t}\left[(1-\tau_{t+2}^W)W_{h,t+2|t} - \lambda_{t+2}^W\zeta_{t+2}^n\frac{\nu'(N_{h,t+2|t})}{\theta_{t+2}^b}\right]] + \ldots\end{aligned}$$

Using the following definitions: $\prod_{i=1}^k \beta_{t+i}^r = \beta_{t+1}^r\beta_{t+2}^r\ldots\beta_{t+k}^r$ and $\prod_{i=1}^0 \beta_{t+i}^r = 1$, the above equation can be written as:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)N_{h,t+k|t}\theta_{t+k}^b\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)W_{h,t+k|t} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(N_{h,t+k|t})}{\theta_{t+k}^b}\right] = 0. \tag{C.60}$$

Equation (C.60) is the FOC for $W_{h,t}^{opt}$, which is the optimal wage decision by a household member with labor type h. Equation (C.60) is the same as Equation (22) in Section 2.1.5.

Since all labor types in the household face the same optimization problem, we can drop the subscript h from the above equation. Thus, the optimal wage setting condition can be rewritten as:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}\theta_{t+k}^b\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)W_{t+k|t} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{\theta_{t+k}^b}\right] = 0. \tag{C.61}$$

Using the following definition: $\theta_{t+k}^b\ P_{t+k}^C = \Omega_{t+k}^C$, the above equation can be written as follows:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}\theta_{t+k}^b P_{t+k}^C\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)\frac{W_{t+k|t}}{P_{t+k}^C} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{\theta_{t+k}^b P_{t+k}^C}\right] = 0,$$

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}\Omega_{t+k}^C\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)\frac{W_{t+k|t}}{P_{t+k}^C} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{\Omega_{t+k}^C}\right] = 0. \tag{C.62}$$

We continue the stationarization of Equation (C.62) by using the following definitions: $\overline{w}_{t+k|t} = \frac{W_{t+k|t}}{z_{t+k}^+ P_{t+k}^C}$ and $\overline{\Omega}_{t+k}^C = z_{t+k}^+\Omega_{t+k}^C$. The above equation can be expressed as:

$$E_t\sum_{k=1}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}z_{t+k}^+\Omega_{t+k}^C\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)\frac{W_{t+k|t}}{z_{t+k}^+ P_{t+k}^C} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{z_{t+k}^+\Omega_{t+k}^C}\right] = 0,$$

and we have the following stationarized version of the optimal wage setting equation:

$$E_t\sum_{k=0}^{\infty}(\xi_w)^k\left(\prod_{i=1}^k\beta_{t+i}^r\right)n_{t+k|t}\overline{\Omega}_{t+k}^C\frac{1}{1-\lambda_{t+k}^W}\left[\left(1-\tau_{t+k}^W\right)\overline{w}_{t+k|t} - \lambda_{t+k}^W\zeta_{t+k}^n\frac{\nu'(n_{t+k|t})}{\overline{\Omega}_{t+k}^C}\right] = 0. \tag{C.63}$$

Equation (C.63) is the same as Equation (A.13a), which shows the stationarized version of the optimal wage setting equation. We assume that Non-Ricardian households set their wage rate equal to the average wage rate of Ricardian households and face identical labor demand, this assumption implies that Ricardian and Non-Ricardian households will have the same wage rate and supply the same amount of labor.

C.1.10 Non-Ricardian Household

We assume that Non-Ricardian households has the same wage, employment and labor supply as Ricardian households. Hence, we have the following results:

$$\begin{aligned} W_{m,t} &= W_t \\ n_{m,t} &= n_t \\ l_{m,t} &= l_t \end{aligned}$$

Since Non-Ricardian households are not able to save, each Non-Ricardian household m sets her nominal consumption expenditure equal to after-tax disposable wage income plus transfers. We have the following the nominal consumption expenditure for Non-Ricardian household m:

$$\left(1+\tau_t^C\right) P_t^C C_{m,t} = \left(1-\tau_t^W\right) W_{m,t} N_{m,t} + \left(1-\tau_t^{TR}\right) TR_{m,t}.$$

We can drop subscript m from the above equation since we assume all Non-Ricardian households face the same budget constraint and will choose the same level of consumption. We denote C_t^{nr} as aggregate Non-Ricardian household consumption, and TR_t^{nr} as aggregate transfers to Non-Ricardian households. The above equation can be written as:

$$\left(1+\tau_t^C\right) P_t^C C_t^{nr} = \left(1-\tau_t^W\right) W_t N_t + \left(1-\tau_t^{TR}\right) TR_t^{nr}.$$

We can express the above equation in *per capita* terms. Especially, we define $\overline{c}_t^{nr}$ as the stationarized aggregate Non-Ricardian household consumption in *per capita* terms, $\overline{tr}_t^{nr}$ is transfers to Non-Ricardian households in *per capita* terms, and n_t is aggregate employment per capita (employment rate). We use the following definitions to stationarize the above equation: $\overline{c}_t^{nr} = \frac{c_t^{nr}}{z_t^+}$, $\overline{tr}_t^{nr} = \frac{tr_t^{nr}}{P_t z_t^+}$, $p_t^C = \frac{P_t^C}{P_t}$, $\overline{w}_t^e = \frac{W_t}{z_t^+ P_t}$. The above equation can be rewritten as:

$$(1+\tau_t^C) p_t^C \overline{c}_t^{nr} = \left(1-\tau_t^W\right) \overline{w}_t^e n_t + \left(1-\tau_t^{TR}\right) \overline{tr}_t^{nr}. \tag{C.64}$$

Equation (C.64) is the same as Equation (A.24a), which captures the stationarized aggregate Non-Ricardian household consumption.

C.1.11 Aggregation of households

Recall, snr is a share of Non-Ricardian households over total population, and we denote C_t^{agg} as aggregate household consumption. Aggregate private consumption C_t^{agg} is a sum of aggregate Ricardian household consumption and aggregate Non-Ricardian household consumption, which is written as:

$$C_t^{agg} = \int_0^{1-s_{nr}} C_{k,t} dh + \int_{1-s_{nr}}^{1} C_{m,t} dm.$$

C_t is aggregate Ricardian household consumption and C_t^{nr} as aggregate Non-Ricardian household consumption. Aggregate private consumption can be written as:

$$C_t^{agg} = (1-s_{nr}) C_t + s_{nr} C_t^{nr}.$$

We can express the above equation in *per capita* terms. Especially, we define $\overline{c}_t^{nr}$ as the stationarized aggregate Non-Ricardian household consumption in *per capita* terms, $\overline{c}_t$ as the stationarized aggregate Ricardian household consumption in *per capita* terms, and $\overline{c}_t^{agg}$ as the stationarized aggregate household consumption in *per capita* terms. As in Section C.1.10, we can stationarize the above equation by using the following definitions: $\overline{c}_t^{agg} = \frac{c_t^{agg}}{z_t^+}$, $\overline{c}_t = \frac{c_t}{z_t^+}$, and $\overline{c}_t^{nr} = \frac{c_t^{nr}}{z_t^+}$. Thus, we have the following equation:

$$\overline{c}_t^{agg} = (1-s_{nr}) \overline{c}_t + s_{nr} \overline{c}_t^{nr}. \tag{C.65}$$

Equation (C.65) is the same as Equation (A.23a), which captures the stationarized aggregate private consumption equation.

Aggregate transfers TR_t is a sum of transfers to Ricardian and Non-Ricardian households:

$$TR_t^{agg} = \int_0^{1-s_{nr}} TR_{k,t}dk + \int_{1-s_{nr}}^{1} TR_{m,t}dm.$$

We denote TR_t^{agg} as aggregate transfers, TR_t^{nr} as aggregate transfers to Non-Ricardian households, and TR_t as aggregate transfers to Ricardian households. Thus, aggregate transfer equation TR_t^{agg} can be expressed as:

$$TR_t^{agg} = (1 - s_{nr})TR_t + s_{nr}TR_t^{nr}.$$

We can express the above equation in *per capita* terms. Especially, $\overline{tr}_t^{nr}$ is transfers to Non-Ricardian households in *per capita* terms, $\overline{tr}_t$ is transfers to Ricardian households in *per capita* terms, and $\overline{tr}_t^{agg}$ is aggregate transfers in *per capita* terms. We stationarize the above equation by using the following definitions: $\overline{tr}_t = \frac{tr_t}{P_t z_t^+}$, $\overline{tr}_t^{nr} = \frac{tr_t^{nr}}{P_t z_t^+}$ and $\overline{tr}_t^{agg} = \frac{tr_t^{agg}}{P_t z_t^+}$. Hence, we have the following equation:

$$\overline{tr}_t^{agg} = (1 - s_{nr})\overline{tr}_t + s_{nr}\overline{tr}_t^{nr}. \tag{C.66}$$

Equation (C.66) is the same as Equation (A.74a), which is the stationarized version of aggregate transfer equation.

The stationarized version of aggregate transfer distribution off steady state equation is given by:

$$\varpi_{dyn}\left(\overline{tr}_t - \overline{tr}\right) = (1 - \varpi_{dyn})(\overline{tr}_t^{nr} - \overline{tr}^{nr}). \tag{C.67}$$

Equation (C.67) is the same as Equation (A.75a).

Similarly, the transfer distribution in steady state equation is expressed as:

$$\varpi_{ss}\overline{tr} = (1 - \varpi_{ss})\overline{tr}^{nr}.$$

C.2 Intermediate good producers

In this section, first we derive the stationarized version of the real marginal cost of production for intermediate good producers, Equation (A.26a). Second, we derive the stationarized version of the real rental rate for capital services, Equation (A.28a). There is a continuum of intermediate good producers of mass one, and i denotes the individual firm in the Swedish economy. Now, we present the optimization problem of intermediate good producers in the Swedish economy.

Firm i chooses capital services $K_t^s(i)$ and labor input $N_t(i)$ to minimize the following cost function:

$$TC_t(i) = R_t^K K_t^s(i) + \left(1 + \tau_t^{SSC}\right) W_t N_t(i) \tag{C.68}$$

subject to the production constraint:

$$Y_t(i) = \varepsilon_t \left[\tilde{K}_t^s(i)\right]^{\alpha} [z_t N_t(i)]^{1-\alpha} - z_t^+ \phi. \tag{C.69}$$

where τ_t^{SSC} denotes the social security – or payroll – tax paid by firms.

$\tilde{K}_t^s(i)$ denotes a composite capital input made up by private capital services $K_t^s(i)$ and public capital $K_{G,t}$. We assume the following constant elasticity of substitution (CES) aggregator of private capital services $K_t^s(i)$ and public capital stock $K_{G,t}(i)$:

$$\tilde{K}_t^s(i) = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(K_t^s(i)\right)^{\frac{\upsilon_K - 1}{\upsilon_K}} + (1 - \alpha_K)^{\frac{1}{\upsilon_K}} \left(K_{G,t}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K - 1}}.$$

Hence, we assume that each intermediate-good firm i has access to the same public capital stock. We also assume that public capital grows at the same speed as private capital services along the balanced growth path. The parameter υ_K is the elasticity of substitution between private capital services and the public capital stock, and α_K is a share parameter. For $\alpha_K = 1$ we obtain the standard production function without public capital stock. For $\upsilon_K \to 1$ the production function converges to a Cobb-Douglas specification.

We denote $\theta_t(i)$ as the Lagrange multiplier associated with the production constraint (C.69). To solve the optimization problem, we set up the following Lagrangian $\mathscr{L}_t(i)$:

$$\mathscr{L}_t(i) = R_t^K K_t^s(i) + \left(1 + \tau_t^{SSC}\right) W_t N_t(i) - \theta_t(i) \left[\varepsilon_t \left[\tilde{K}_t^s(i)\right]^{\alpha} [z_t N_t(i)]^{1-\alpha} - z_t^{+}\phi - Y_t(i)\right], \tag{C.70}$$

where

$$\tilde{K}_t^s(i) = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(K_t^s(i)\right)^{\frac{\upsilon_K - 1}{\upsilon_K}} + (1 - \alpha_K)^{\frac{1}{\upsilon_K}} \left(K_{G,t}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K - 1}}.$$

We take the partial derivative of $\mathscr{L}_t(i)$ with respect to $K_t^s(i)$ and $N_t(i)$ respectively, and we can find the FOC for $K_t^s(i)$ and $N_t(i)$.

The FOC for $K_t(i)$ is:

$$R_t^K - \alpha\theta_t(i)\varepsilon_t \frac{\tilde{K}_t^s(i)^{\alpha}}{K_t^s(i)} [z_t N_t(i)]^{1-\alpha} \alpha_K{}^{\frac{1}{\upsilon_K}} \left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}} = 0. \tag{C.71}$$

The FOC for $N_t(i)$ is:

$$\left(1 + \tau_t^{SSC}\right) W_t - \theta_t(i) (1 - \alpha) \varepsilon_t \left[\tilde{K}_t^s(i)\right]^{\alpha} z_t^{1-\alpha} [N_t(i)]^{-\alpha} = 0. \tag{C.72}$$

Using Equation (C.71) and Equation (C.72), we obtain the following capital-labor input efficiency condition:

$$K_t^s(i) = \frac{\alpha}{1 - \alpha} \frac{\left(1 + \tau_t^{SSC}\right) W_t}{R_t^K} N_t(i) \alpha_K{}^{\frac{1}{\upsilon_K}} \left(\frac{K_t^s(i)}{\tilde{K^s}_t(i)}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}}. \tag{C.73}$$

Note that Equation (C.69) can be written as:

$$\left[Y_t(i) + z_t^{+}\phi\right] = \varepsilon_t \left[\tilde{K}_t^s(i)\right]^{\alpha} [z_t N_t(i)]^{1-\alpha}. \tag{C.74}$$

Now, we find the total cost of production equation. We substitute Equation (C.71) and Equation (C.72) into Equation (C.68), and we have the following equation:

$$TC_t(i) = \theta_t(i) \left[\alpha\varepsilon_t \tilde{K}_t^s(i)^{\alpha} [z_t N_t(i)]^{1-\alpha} \alpha_K{}^{\frac{1}{\upsilon_K}} \left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}} + (1 - \alpha) \varepsilon_t \left[\tilde{K}_t{}^s(i)\right]^{\alpha} z_t^{1-\alpha} [N_t(i)]^{1-\alpha}\right]. \tag{C.75}$$

Using Equation (C.74), we can rewrite Equation (C.75) as follows:

$$TC_t(i) = \theta_t(i) \left[(1 - \alpha) + \alpha\alpha_K{}^{\frac{1}{\upsilon_K}} \left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}}\right] \left(Y_t(i) + z_t^{+}\phi\right). \tag{C.76}$$

We use Equation (C.76), and we take the partial derivative of $TC_t(i)$ with respect to $Y_t(i)$. Hence, the lagrangian multiplier, $\theta_t(i)$, can be defined as the marginal cost of production $MC_t(i)$:

$$\frac{\partial TC_t(i)}{\partial Y_t(i)} = MC_t(i) = \frac{1}{\alpha^{\alpha}(1 - \alpha)^{1-\alpha}\varepsilon_t} \left(\frac{(1 + \tau_t^{SSC}) W_t}{z_t}\right)^{\alpha} \left(\frac{K_t^s(i)}{\alpha_K \tilde{K}_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}}. \tag{C.77}$$

There are three equations that summarize optimal input decisions by firms: 1) the capital-labor input efficiency condition, 2) the optimal capital inputs in terms of marginal cost and 3) the composite capital equation. First, we find the capital-labor input efficiency equation. In particular, we rewrite Equation (C.73) as follows:

$$\frac{K_t^s(i)}{L_t(i)} = \frac{\alpha}{1 - \alpha} \frac{\left(1 + \tau_t^{SSC}\right) W_t}{R_t^K} \alpha_K{}^{\frac{1}{\upsilon_K}} \left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K - 1}{\upsilon_K}}. \tag{C.78}$$

Equation (C.78) is the capital-labor input efficiency condition.

Second, we find the equation for the optimal capital input in terms of marginal cost. Using Equation (C.77) and Equation (C.78), Equation (C.71). can be written as

$$R_t^K = \alpha MC_t(i) \left[z_t\right]^{1-\alpha} \varepsilon_t \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} \left(\frac{\left(1+\tau_t^{SSC}\right) W_t}{R_t^K}\right)^{\alpha-1} \varGamma_{G,t} \quad (C.79)$$

$$\varGamma_{G,t} = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}},$$

$$R_t^K = \alpha MC_t(i) \left[z_t\right]^{1-\alpha} \varepsilon_t \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} \left(\frac{\left(1+\tau_t^{SSC}\right) W_t}{R_t^K}\right)^{\alpha-1} \varGamma_{G,t}$$

The above equation is the optimal capital input in terms of marginal cost.

Finally, we have the following composite capital function:

$$\tilde{K}_t^s(i) = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(K_t^s(i)\right)^{\frac{\upsilon_K-1}{\upsilon_K}} + \left(1-\alpha_K\right)^{\frac{1}{\upsilon_K}} \left(K_{G,t}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right)^{\frac{\upsilon_K}{\upsilon_K-1}}.$$

We can simplify Equation (C.79) by letting: $\varGamma_{G,t}(i) = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}}$. Thus, Equation (C.79) can be rewritten as:

$$MC_t(i) = \frac{R_t^K}{\alpha z_t^{1-\alpha} \varepsilon_t \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} \left(\left(1+\tau_t^{SSC}\right) W_t\right)^{\alpha-1} (R_t^K)^{1-\alpha} \varGamma_{G,t}}.$$

The above equation can be expressed as:

$$MC_t(i) = \frac{\left(\frac{\left(1+\tau_t^{SSC}\right) W_t}{z_t}\right)^{1-\alpha} \left(R_t^K\right)^{\alpha}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} \varepsilon_t \varGamma_{G,t}(i)}. \quad (C.80)$$

Equation (C.80), which is the same as Equation (27) in Section 2.4.1, is the nominal marginal cost of production for the intermediate good firm i.

Firms hiring from homogeneous labor and private markets, i.e. face the same wage and rental rates. In the standard model without public capital, this implies that marginal costs are identical across firms. With added public capital, the expression $\varGamma_{G,t}(i)$ in principle would make marginal costs different across firms. For simplicity, we assume that each firm uses a constant private to public capital ratio in its production. This means that the amount of private capital services operated is proportional and constant in relation to the amount of public capital used. For example, the number of plants operated by a firm requires the same number of roads to get to the plants. With this assumption, marginal costs are identical across firms, and thus we can drop the subscript i. Equation (C.80) can be written as:

$$MC_t = \frac{\left(\frac{\left(1+\tau_t^{SSC}\right) W_t}{z_t}\right)^{1-\alpha} \left(R_t^K\right)^{\alpha}}{\alpha^{\alpha} \left(1-\alpha\right)^{1-\alpha} \varepsilon_t \varGamma_{G,t}}, \quad (C.81)$$

where

$$\varGamma_{G,t} = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}},$$

Equation (C.81) is the non-stationarized version of the nominal marginal cost of production for intermediate good firm.

Next, we derive the non-stationarized version of the rental rate for capital services. Using Equation (C.71) and that the lagrange multiplier $\theta_t(i)$ equals the marginal cost $MC_t(i)$, we obtain the following equation:

$$R_t^K = \alpha \varepsilon_t z_t^{1-\alpha} MC_t(i) \left(\frac{\tilde{K}_t^s(i)}{N_t(i)}\right)^{\alpha-1} \left(\varGamma_{G,t}(i)\right)^{\frac{1}{\alpha}}, \quad (C.82)$$

where

$$\varGamma_{G,t} = \left(\frac{\alpha_K \tilde{K}_t^s(i)}{K_t^s(i)}\right)^{\frac{\alpha}{\upsilon_K}},$$

Equation (C.82), which captures the nominal rental rate for capital services, is the same as Equation (28) in Section 2.4.1.

We assume identical capital labor ratios, identical marginal costs and identical private to public capital ratios. This means that we can drop the subscript i and rewrite equation (C.82) as:

$$R_t^K = \alpha \varepsilon_t z_t^{1-\alpha} MC_t \left(\frac{\tilde{K}_t^s}{N_t} \right)^{\alpha-1} (\Gamma_{G,t})^{\frac{1}{\alpha}} . \tag{C.83}$$

Equation (C.83), captures the non-stationarized version of rental rate for capital services.

Now, we find the stationarized version of the marginal cost of production for intermediate good producers. We stationarize Equation (C.81) by applying the following definitions: $r_t^K = \frac{\gamma_t R_t^K}{P_t}$, $\overline{w}_t^e = \frac{W_t}{z_t^+ P_t}$, $z_t^+ = z_t (\gamma_t)^{\frac{\alpha}{1-\alpha}}$, and $\overline{mc}_t = \frac{MC_t}{P_t}$. Stationarizing $\Gamma_{G,t}$ is trivial as private and public capital services have the same growth rate along a balanced growth path. Equation (C.81) can be written as follows:

$$\frac{MC_t}{P_t} = \frac{\left(\frac{(1+\tau_t^{SSC}) W_t}{z_t} \right)^{1-\alpha} \left(\frac{1}{P_t} \right)^{1-\alpha} \left(\frac{1}{P_t} \right)^{\alpha} \left(R_t^K \right)^{\alpha} \frac{\gamma_t^{\alpha}}{\gamma_t^{\alpha}}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} \varepsilon_t \Gamma_{G,t}},$$

$$\overline{mc}_t = \frac{\left(\frac{(1+\tau_t^{SSC}) W_t}{z_t (\gamma_t)^{\alpha/(1-\alpha)} P_t} \right)^{1-\alpha} \left(\frac{\gamma_t R_t^K}{P_t} \right)^{\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} \varepsilon_t \bar{\Gamma}_{G,t}},$$

$$\overline{mc}_t = \frac{\left(\frac{(1+\tau_t^{SSC}) W_t}{z_t^+ P_t} \right)^{1-\alpha} \left(\frac{\gamma_t R_t^K}{P_t} \right)^{\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} \varepsilon_t \bar{\Gamma}_{G,t}}.$$

Thus, the stationarized version of the marginal cost of production for intermediate good producers (the real marginal cost of production) equation can be expressed as:

$$\overline{mc}_t = \frac{\left((1+\tau_t^{SSC}) \overline{w}_t^e \right)^{1-\alpha} (r_t^K)^{\alpha}}{\varepsilon_t \alpha^{\alpha} (1-\alpha)^{1-\alpha} \bar{\Gamma}_{G,t}}. \tag{C.84}$$

Equation (C.84), which represents the real marginal cost of production for intermediate good producers, is the same as Equation (A.26a) in Section A.2.

Lastly, we find the stationarized version of the rental rate for capital services. We stationarize Equation (C.83) by applying the following definitions: $r_t^K = \frac{\gamma_t R_t^K}{P_t}$, $z_t^+ \gamma_t = z_t \gamma_t^{1/1-\alpha}$, $\bar{K}_t^s = \frac{K_t^s}{z_{t-1}^+ \gamma_{t-1}}$, and $\overline{mc}_t = \frac{MC_t}{P_t}$. We follow the similar steps when deriving the stationarized version of the marginal cost for intermediate good producers equation. Hence, Equation (C.83) can be written as:

$$r_t^K = \alpha \varepsilon_t \left(\frac{\bar{\tilde{K}}_t^s}{N_t} \frac{1}{\mu_{z^+,t} \mu_{\gamma,t}} \right)^{\alpha-1} \overline{mc}_t (\Gamma_{G,t})^{\frac{1}{\alpha}}$$

Furthermore, we can rewrite the above equation in terms of *per capita*, so we denote $\bar{k}_t^s$ as stationarized capital services *per capita*, and n_t as aggregate labor input *per capita*. Hence, we can rewrite the above equation as:

$$r_t^K = \alpha \varepsilon_t \left(\frac{\bar{\tilde{k}}_t^s}{n_t} \frac{1}{\mu_{z^+,t} \mu_{\gamma,t}} \right)^{\alpha-1} \overline{mc}_t (\Gamma_{G,t})^{\frac{1}{\alpha}} \tag{C.85}$$

Equation (C.85), which is the real rental rate for capital services equation, is the same as Equation (A.28a) in Section A.2.

Finally, the equation for composite capital in stationary form is written as:

$$\bar{\tilde{k}}_t^s = \left(\alpha_K{}^{\frac{1}{\upsilon_K}} \left(\bar{k}_t^s \right)^{\frac{\upsilon_K - 1}{\upsilon_K}} + (1-\alpha_K)^{\frac{1}{\upsilon_K}} \left(\bar{k}_{G,t} \right)^{\frac{\upsilon_K - 1}{\upsilon_K}} \right)^{\frac{\upsilon_K}{\upsilon_K - 1}} \tag{C.86}$$

Equation (C.86) is the same as Equation (A.29a) in Section A.2.

Note that, the law of motion for public capital is given by:

$$K_{G,t+1} = (1-\delta_G)K_{G,t} + I_t^G.$$

We stationarize the law of motion for public capital by dividing the non-stationarized function by $\gamma_t z_t^+$, and we have the following equation:

$$\frac{K_{G,t+1}}{\gamma_t z_t^+} = (1-\delta_G)\frac{K_{G,t}}{\gamma_t z_t^+} + \frac{I_t^G}{\gamma_t z_t^+}$$

The above equation can be written in *per capita* terms:

$$\bar{k}_{G,t+1} = (1-\delta_G)\bar{k}_{G,t}\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}} + \bar{I}_t^G. \tag{C.87}$$

Equation (C.87) is the same as Equation (A.30a) in Section A.2.

C.2.1 Optimal price of intermediate goods

In this section, we derive the stationarized version of the optimal price of intermediate goods equation (A.31a). In this section, firm i chooses the optimal price $P_t^{opt}(i)$ that maximizes its profit, subject to its demand schedule and the Calvo price contract. In each period, the individual firm i resets its price with probability $(1-\xi)$. With probability ξ, the firm cannot reset its price, and then it faces the following price evolution: $P_{t+k|t}(i) = P_t^{opt}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}$. We define the stochastic discount factor as $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega_{t+k}^C}{\Omega_t^C}\frac{P_t^C}{P_{t+k}^C}$.

Firm i chooses the optimal price of intermediate goods $P_t^{opt}(i)$ to maximize the following profit function:

$$\underset{P_t^{opt}(i)}{max}\ E_t\sum_{k=0}^{\infty}(\xi)^k\Lambda_{t,t+k}\left\{P_{t+k|t}(i)Y_{t+k|t}(i) - TC_{t+k|t}\left[Y_{t+k|t}(i)\right]\right\}\upsilon_K \tag{C.88}$$

subject to the demand function:

$$Y_{t+k|t}(i) = \left(\frac{P_{t+k|t}(i)}{P_{t+k}}\right)^{\frac{\lambda_{t+k}}{1-\lambda_{t+k}}} Y_{t+k}, \tag{C.89}$$

and the Calvo price setting contract:

$$P_{t+k}(i) = \begin{cases}\overline{\Pi}_{t+k}P_{t+k-1}(i) & \text{with probability } \xi\\ P_{t+k}^{opt}(i) & \text{with probability } (1-\xi).\end{cases} \tag{C.90}$$

The FOC for $P_t^{opt}(i)$ is:

$$\begin{aligned}
&E_t\{Y_{t|t}(i) + P_t^{opt}(i)\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)} - MC_t(i)\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)}\\
&+\xi\Lambda_{t,t+1}\left[\frac{\partial P_{t+1|t}(i)}{\partial P_t^{opt}(i)}Y_{t+1|t}(i) + P_{t+1|t}(i)\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\frac{\partial P_{t+1|t}(i)}{\partial P_t^{opt}(i)} - MC_{t+1}(i)\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\frac{\partial P_{t+1|t}(i)}{\partial P_t^{opt}(i)}\right]\\
&+(\xi)^2\Lambda_{t,t+2}\\
&\left[\frac{\partial P_{t+2|t}(i)}{\partial P_t^{opt}(i)}Y_{t+2|t}(i) + P_{t+2|t}(i)\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\frac{\partial P_{t+2|t}(i)}{\partial P_t^{opt}(i)} - MC_{t+2}(i)\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\frac{\partial P_{t+2|t}(i)}{\partial P_t^{opt}(i)}\right]\\
&+\ldots\} = 0.
\end{aligned} \tag{C.91}$$

Recall, we have the following definition: $P_{t+k|t}(i) = P_t^{opt}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}$. Hence, the partial derivative of $P_{t+k|t}(i)$ with respect to $P_t^{opt}(i)$ is:

$$\frac{\partial P_{t+k|t}(i)}{\partial P_t^{opt}(i)} = \overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}. \tag{C.92}$$

Using Equation (C.92), Equation (C.91) can be rewritten as:

$$
\begin{aligned}
&E_t\{Y_{t|t}(i) + P_t^{opt}(i)\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)} - MC_t(i)\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)} \\
&+ \xi\Lambda_{t,t+1}\left[\overline{\Pi}_{t+1}Y_{t+1|t}(i) + P_{t+1|t}(i)\overline{\Pi}_{t+1}\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)} - MC_{t+1}(i)\overline{\Pi}_{t+1}\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\right] \\
&+ (\xi)^2\Lambda_{t,t+2} \\
&\left[\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}Y_{t+2|t}(i) + P_{t+2|t}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)} - MC_{t+2}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\right] \\
&+ \ldots\} = 0.
\end{aligned}
$$

Based on Equation (C.81), we have the following result: $MC_{t+k}(i) = MC_{t+k}$. We rearrange the above equation, and we obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)}\left[P_t^{opt}(i)\left(\frac{Y_{t|t}(i)}{P_t^{opt}(i)}\left(\frac{\partial Y_{t|t}(i)}{\partial P_t^{opt}(i)}\right)^{-1} + 1\right) - MC_t\right] \\
&+ \xi\Lambda_{t,t+1}\overline{\Pi}_t\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\left[P_{t+1|t}(i)\left(\frac{Y_{t+1|t}(i)}{P_{t+1|t}(i)}\left(\frac{\partial Y_{t+1|t}(i)}{\partial P_{t+1|t}(i)}\right)^{-1} + 1\right) - MC_{t+1}\right] \\
&+ (\xi)^2\Lambda_{t,t+2}\overline{\Pi}_t\overline{\Pi}_{t+1}\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\left[P_{t+2|t}(i)\left(\frac{Y_{t+2|t}(i)}{P_{t+2|t}(i)}\left(\frac{\partial Y_{t+2|t}(i)}{\partial P_{t+2|t}(i)}\right)^{-1} + 1\right) - MC_{t+2}\right] + \ldots\} = 0.
\end{aligned} \tag{C.93}
$$

Given the demand schedule for intermediate goods, which is captured by Equation (C.89), we can find the following price elasticity of demand for intermediate goods:

$$
-\frac{\partial Y_{t+k|t}(i)}{\partial P_{t+k|t}(i)}\frac{P_{t+k|t}(i)}{Y_{t+k|t}(i)} = \frac{\lambda_{t+k}}{\lambda_{t+k} - 1}. \tag{C.94}
$$

Using the result from Equation (C.94), the derivative of $Y_{t+k|t}(i)$ with respect to $P_{t+k|t}(i)$ is:

$$
\frac{\partial Y_{t+k|t}(i)}{\partial P_{t+k|t}(i)} = \frac{\lambda_{t+k}}{1 - \lambda_{t+k}}\frac{Y_{t+k|t}(i)}{P_{t+k|t}(i)}. \tag{C.95}
$$

Using Equation (C.94) and Equation (C.95), we can rewrite Equation (C.93) as follows:

$$
\begin{aligned}
&E_t\{\frac{Y_{t|t}(i)}{P_t^{opt}(i)}\frac{\lambda_t}{1-\lambda_t}\left[P_t^{opt}(i)\frac{1}{\lambda_t} - MC_t\right] \\
&+ \xi\Lambda_{t,t+1}\overline{\Pi}_{t+1}\frac{Y_{t+1|t}(i)}{P_{t+1|t}(i)}\frac{\lambda_{t+1}}{1-\lambda_{t+1}}\left[P_{t+1|t}(i)\frac{1}{\lambda_{t+1}} - MC_{t+1}\right] \\
&+ (\xi)^2\Lambda_{t,t+2}\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\frac{Y_{t+2|t}(i)}{P_{t+2|t}(i)}\frac{\lambda_{t+2}}{1-\lambda_{t+2}}\left[P_{t+2|t}(i)\frac{1}{\lambda_{t+2}} - MC_{t+2}\right] + \ldots\} = 0.
\end{aligned}
$$

We use the following definition: $P_{t+k|t}(i) = P_t^{opt}(i)\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}$. We multiply both sides of the above equation by $P_t^{opt}(i)$ and -1. We can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{Y_{t|t}(i)}{\lambda_t - 1}\left[P_t^{opt}(i) - \lambda_t MC_t\right] \\
&+ \xi\Lambda_{t,t+1}\frac{Y_{t+1|t}(i)}{\lambda_{t+1} - 1}\left[P_{t+1|t}(i) - \lambda_{t+1}MC_{t+1}\right] \\
&+ (\xi)^2\Lambda_{t,t+2}\frac{Y_{t+2|t}(i)}{\lambda_{t+2} - 1}\left[P_{t+2|t}(i) - \lambda_{t+2}MC_{t+2}\right] + \ldots\} = 0.
\end{aligned}
$$

We can drop the subscript i from the above equation because when firms can reset their price, they will set the same price. As a result, all firms will choose the same quantity of output. We rewrite the above equation, and the optimal price of intermediate goods equation can be expressed as:

$$
E_t\sum_{k=0}^{\infty}(\xi)^k\Lambda_{t,t+k}\frac{Y_{t+k|t}}{(\lambda_{t+k} - 1)}\left[P_{t+k|t} - \lambda_{t+k}MC_{t+k}\right] = 0. \tag{C.96}
$$

Equation (C.96), which is the non-stationarized version of the optimal price of intermediate goods equation, is the same as Equation (31) in Section 2.4.1.

Now, we would like to derive the stationarized version of the optimal price of intermediate goods equation. We use the following definition: $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega^C_{t+k}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+k}}$, and then we expand Equation (C.96). Hence, we have the following equation:

$$
\begin{aligned}
E_t\{ & \frac{Y_{t|t}}{(\lambda_t - 1)}\left[P^{opt}_t - \lambda_t MC_t\right] \\
& + \xi \frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{Y_{t+1|t}}{(\lambda_{t+1}-1)}\left[P_{t+1|t} - \lambda_{t+1} MC_{t+1}\right] \\
& + (\xi)^2 \frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{Y_{t+2|t}}{(\lambda_{t+2}-1)}\left[P_{t+2|t} - \lambda_{t+2} MC_{t+2}\right] + \ldots\} = 0.
\end{aligned}
$$

We multiply the third term of the above equation by $\frac{P^C_{t+1}}{P^C_{t+1}}$, and we obtain the following equation:

$$
\begin{aligned}
E_t\{ & \frac{Y_{t|t}}{(\lambda_t - 1)}\left[P^{opt}_t - \lambda_t MC_t\right] \\
& + \xi \frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{Y_{t+1|t}}{(\lambda_{t+1}-1)}\left[P_{t+1|t} - \lambda_{t+1} MC_{t+1}\right] \\
& + (\xi)^2 \frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{Y_{t+2|t}}{(\lambda_{t+2}-1)}\left[P_{t+2|t} - \lambda_{t+2} MC_{t+2}\right] + \ldots\} = 0.
\end{aligned}
$$

We use the following definition: $P_{t+k|t} = P^{opt}_t \overline{\Pi}_{t+1}\overline{\Pi}_{t+2}\ldots\overline{\Pi}_{t+k}$. We multiply the optimal firm price P^{opt}_t by $\frac{1}{P_{t-1}}\frac{P_{t-1}}{P_t}$, multiply the marginal utility of consumption Ω^C_{t+k} by z^+_{t+k}, and divide the output of firm $Y_{t+k|t}$ by z^+_{t+k}. We multiply the nominal marginal cost MC_t by $\frac{1}{P}_t$, multiply MC_{t+1} by $\frac{P_{t+1}}{P_t}\frac{1}{P_{t+1}}$, and multiply MC_{t+2} by $\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\frac{1}{P_{t+2}}$. Thus, the above equation can be rewritten as:

$$
\begin{aligned}
E_t\{ & \frac{Y_{t|t}}{(\lambda_t - 1)\, z^+_t}\left[\frac{P^{opt}_t}{P_{t-1}}\frac{P_{t-1}}{P_t} - \lambda_t \frac{MC_t}{P_t}\right] \\
& + \xi \frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1} z^+_{t+1}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{Y_{t+1|t}}{(\lambda_{t+1}-1)\, z^+_{t+1}}\left[\frac{\overline{\Pi}_{t+1} P^{opt}_t}{P_t}\frac{P_{t-1}}{P_{t-1}} - \lambda_{t+1}\frac{P_{t+1}}{P_t}\frac{MC_{t+1}}{P_{t+1}}\right] \\
& + (\xi)^2 \frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2} z^+_{t+2}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{Y_{t+2|t}}{(\lambda_{t+2}-1)\, z^+_{t+2}}\left[\frac{\overline{\Pi}_{t+1}\overline{\Pi}_{t+2} P^{opt}_t P_{t-1}}{P_{t-1} P_t} - \lambda_{t+2}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\frac{MC_{t+2}}{P_{t+2}}\right] \\
& + \ldots\} = 0.
\end{aligned}
$$

Using the following definitions: $p^{opt}_t = \frac{P^{opt}_t}{P_{t-1}}$, $\Pi_{t+k} = \frac{P_{t+k}}{P_{t+k-1}}$, $\Pi^C_{t+k} = \frac{P^C_{t+k}}{P^C_{t+k-1}}$, and $\overline{mc}_{t+k} = \frac{MC_{t+k}}{P_{t+k}}$, we can obtain the following equation:

$$
\begin{aligned}
E_t\{ & \frac{Y_{t|t}}{(\lambda_t - 1)\, z^+_t}\left[\frac{p^{opt}_t}{\Pi_t} - \lambda_t \overline{mc}_t\right] \\
& + \xi \frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1} z^+_{t+1}}{\Omega^C_t z^+_t}\frac{1}{\Pi^C_{t+1}}\frac{Y_{t+1|t}}{(\lambda_{t+1}-1)\, z^+_{t+1}}\left[\frac{\overline{\Pi}_{t+1} p^{opt}_t}{\Pi_t} - \lambda_{t+1}\Pi_{t+1}\overline{mc}_{t+1}\right] \\
& + (\xi)^2 \frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2} z^+_{t+2}}{\Omega^C_t z^+_t}\frac{1}{\Pi^C_{t+2}\Pi^C_{t+1}}\frac{Y_{t+2|t}}{(\lambda_{t+2}-1)\, z^+_{t+2}}\left[\frac{\overline{\Pi}_{t+1}\overline{\Pi}_{t+2} p^{opt}_t}{\Pi_t} - \lambda_{t+2}\Pi_{t+2}\Pi_{t+1}\overline{mc}_{t+2}\right] \\
& + \ldots\} = 0.
\end{aligned}
$$

We express the above equation in terms of *per capita*, so we denote $y_{t+k|t}$ as output *per capita*, and we rearrange the above equation. Thus, we have the following equation:

$$
\begin{aligned}
&E_t\{\frac{y_{t|t}}{(\lambda_t-1)\,z_t^+}\left[\frac{p_t^{opt}}{\Pi_t}-\lambda_t\overline{mc}_t\right] \\
&+\xi\frac{\beta_{t+1}}{\beta_t}\frac{\Omega_{t+1}^C z_{t+1}^+}{\Omega_t^C z_t^+}\frac{\Pi_{t+1}}{\Pi_{t+1}^C}\frac{y_{t+1|t}}{(\lambda_{t+1}-1)\,z_{t+1}^+}\left[\frac{\overline{\Pi}_{t+1}p_t^{opt}}{\Pi_{t+1}\Pi_t}-\lambda_{t+1}\overline{mc}_{t+1}\right] \\
&+(\xi)^2\,\frac{\beta_{t+2}}{\beta_t}\frac{\Omega_{t+2}^C z_{t+2}^+}{\Omega_t^C z_t^+}\frac{\Pi_{t+2}\Pi_{t+1}}{\Pi_{t+2}^C\Pi_{t+1}^C}\frac{y_{t+2|t}}{(\lambda_{t+2}-1)\,z_{t+2}^+}\left[\frac{\overline{\Pi}_{t+1}\overline{\Pi}_{t+2}p_t^{opt}}{\Pi_{t+2}\Pi_{t+1}\Pi_t}-\lambda_{t+2}\overline{mc}_{t+2}\right] \\
&+\ldots\}=0.
\end{aligned}
$$

Using the following definitions: $\beta_{t+j}^r=\frac{\beta_{t+j}}{\beta_{t+j-1}}$, $\overline{\Omega}_{t+k}^C=\Omega_{t+k}^C z_{t+k}^+$, and $\overline{y}_{t+k|t}=\frac{y_{t+k}}{z_{t+k}^+}$, we rewrite the above equation. We have the following stationarized version of the optimal price of intermediate goods equation under the sticky price assumption:

$$
E_t\sum_{k=0}^{\infty}(\xi)^k\left(\prod_{j=1}^{k}\beta_{t+j}^r\right)\frac{\overline{\Omega}_{t+k}^C}{\overline{\Omega}_t^C}\left(\prod_{j=1}^{k}\frac{\Pi_{t+j}}{\Pi_{t+j}^C}\right)\frac{\overline{y}_{t+k|t}}{(\lambda_{t+k}-1)}\left[\left(\prod_{j=1}^{k}\frac{\overline{\Pi}_{t+j}}{\Pi_{t+j}}\right)\frac{p_t^{opt}}{\Pi_t}-\lambda_{t+k}\overline{mc}_{t+k}\right]=0. \tag{C.97}
$$

Equation (C.97), which is the stationarized version of the optimal price of intermediate goods, is the same as Equation (A.31a).

C.3 Private consumption good producers

C.3.1 Consumption good producers

This section presents the optimization problem of the consumption good producers in the Swedish economy and derives the demand functions of non-energy and energy consumption, and derives the relative price of the consumption goods equation.

The optimization problem of the representative non-energy consumption good producer can be defined as follows:

$$
\max_{C_t^{agg},C_t^{xe},C_t^e} P_t^C C_t^{agg}-P_t^{C,xe}C_t^{xe}-P_t^{C,e}C_t^e
$$

subject to the CES aggregate consumption good function

$$
C_t^{agg}=\left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}}+\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^e\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}}. \tag{C.98}
$$

By substituting the CES aggregate consumption good equation (C.98) into the above profit function, we can rewrite the profit function as:

$$
P_t^C\left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}}+\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^e\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}}-P_t^{C,xe}C_t^{xe}-P_t^{C,e}C_t^e.
$$

Taking the derivatives of C_t^{xe} and C_t^e respectively gives us the two following first-order-conditions:

$$
\begin{aligned}
\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}-1}P_t^C\left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}}+\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^e\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}-1}-P_t^{C,xe}=0 \\
\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^e\right)^{\frac{\nu_C-1}{\nu_C}-1}P_t^C\left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}}+\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^e\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}-1}-P_t^{C,e}=0
\end{aligned}
$$

If we substitute the expression in square brackets by the production function, and put the prices of non-energy and energy respectively on the right-hand-side of the equation, the two equations can be written as

$$
\begin{aligned}
P_t^{C,xe}&=\left(\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^{xe}\right)^{-\frac{1}{\nu_C}}P_t^C\left(C_t^{agg}\right)^{\frac{1}{\nu_C}} \\
P_t^{C,e}&=\left(1-\vartheta^C\right)^{\frac{1}{\nu_C}}\left(C_t^e\right)^{-\frac{1}{\nu_C}}P_t^C\left(C_t^{agg}\right)^{\frac{1}{\nu_C}}.
\end{aligned}
$$

Rearrange and multiply through with ν_C in the two equations above to get the demand function for the non-energy and energy consumption goods:

$$C_t^{xe} = \vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C} C_t^{agg} \tag{C.99}$$

$$C_t^{e} = \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C} C_t^{agg} \tag{C.100}$$

which are the same equations that are presented in Equation (43) and Equation (44). Next, we create the price index. This is done by substituting the two demand functions into the production function:

$$\begin{aligned}
C_t^{agg} &= \left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}} \left(C_t^{xe}\right)^{\frac{\nu_C-1}{\nu_C}} + \left(1-\vartheta^C\right)^{\frac{1}{\nu_C}} \left(C_t^{e}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}} \\
C_t^{agg} &= \left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}} \left(\vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C} C_t^{agg}\right)^{\frac{\nu_C-1}{\nu_C}} + \left(1-\vartheta^C\right)^{\frac{1}{\nu_C}} \left(\left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C} C_t^{agg}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}} \\
C_t^{agg} &= \left[\left(\vartheta^C\right)^{\frac{1}{\nu_C}+\frac{\nu_C-1}{\nu_C}} \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C-1} \left(C_t^{agg}\right)^{\frac{\nu_C-1}{\nu_C}} + \left(1-\vartheta^C\right)^{\frac{1}{\nu_c}+\frac{\nu_C-1}{\nu_C}} \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C-1} \left(C_t^{agg}\right)^{\frac{\nu_C-1}{\nu_C}}\right]^{\frac{\nu_C}{\nu_C-1}} \\
C_t^{agg} &= C_t^{agg} \left[\left(\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C-1} + \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C-1}\right]^{\frac{\nu_C}{\nu_C-1}} \\
1 &= \left[\left(\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C-1} + \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C-1}\right]^{\frac{\nu_C}{\nu_C-1}} \\
1 = \left(P_t^C\right)^{(\nu_C-1)\frac{\nu_C}{\nu_C-1}} &\left[\vartheta^C \left(\frac{1}{P_t^{C,xe}}\right)^{\nu_C-1} + \left(1-\vartheta^C\right) \left(\frac{1}{P_t^{C,e}}\right)^{\nu_C-1}\right]^{\frac{\nu_C}{\nu_C-1}} \\
\left(P_t^C\right)^{-\nu_C} &= \left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right) \left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{\nu_C}{\nu_C-1}} \\
P_t^C &= \left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right) \left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}}
\end{aligned}$$

which is the same function as is presented in Equation (45). Now, we are going to stationarize all equations and write the equations in per capita form (using the fact the the population is normalized to 1 for convenience). We use the definitions $p_t^C = P_t^C/P_t$, $p_t^{C,xe} = P_t^{C,xe}/P_t$, $p_t^{C,e} = P_t^{C,e}/P_t$, $\bar{c}_t^{agg} = C_t^{agg}/z_t^+$, $\bar{c}_t^{xe} = C_t^{xe}/z_t^+$, $\bar{c}_t^{e} = C_t^{e}/z_t^+$. The non-energy consumption demand function can be written as

$$\begin{aligned}
C_t^{xe} &= \vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\right)^{\nu_C} C_t^{agg} \\
\frac{C_t^{xe}}{z_t^+} &= \vartheta^C \left(\frac{P_t^C}{P_t^{C,xe}}\frac{P_t}{P_t}\right)^{\nu_C} \frac{C_t^{agg}}{z_t^+} \\
\bar{c}_t^{xe} &= \vartheta^C \left(\frac{p_t^C}{p_t^{C,xe}}\right)^{\nu_C} \bar{c}_t^{agg} \qquad \text{(C.101)}
\end{aligned}$$

Equation (C.101), which captures the demand for non-energy consumption goods, is the same as Equation (A.35a).

Next, we stationarize the demand for energy goods:

$$\begin{aligned}
C_t^{e} &= \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\right)^{\nu_C} C_t^{agg} \\
\frac{C_t^{e}}{z_t^+} &= \left(1-\vartheta^C\right) \left(\frac{P_t^C}{P_t^{C,e}}\frac{P_t}{P_t}\right)^{\nu_C} \frac{C_t^{agg}}{z_t^+} \\
\bar{c}_t^{e} &= \left(1-\vartheta^C\right) \left(\frac{p_t^C}{p_t^{C,e}}\right)^{\nu_C} \bar{c}_t^{agg} \qquad \text{(C.102)}
\end{aligned}$$

Equation (C.102), which captures the demand for energy consumption goods, is the same as Equation (A.36a).

Finally, we stationarize the price index:

$$
\begin{aligned}
P_t^C &= \left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}} \\
\frac{P_t^C}{P_t} &= \frac{1}{P_t}\left[\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(P_t^{C,e}\right)^{1-\nu_C}\right]^{\frac{1}{1-\nu_C}} \\
\frac{P_t^C}{P_t} &= \left[\left(\vartheta^C \left(P_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(P_t^{C,e}\right)^{1-\nu_C}\right)P_t^{\nu_C-1}\right]^{\frac{1}{1-\nu_C}} \\
p_t^C &= \left[\left(\vartheta^C \left(\frac{P_t^{C,xe}}{P_t}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(\frac{P_t^{C,e}}{P_t}\right)^{1-\nu_C}\right)\right]^{\frac{1}{1-\nu_C}} \\
p_t^C &= \left[\left(\vartheta^C \left(p_t^{C,xe}\right)^{1-\nu_C} + \left(1-\vartheta^C\right)\left(p_t^{C,e}\right)^{1-\nu_C}\right)\right]^{\frac{1}{1-\nu_C}}. \qquad \text{(C.103)}
\end{aligned}
$$

Equation (C.103), which captures the demand for energy consumption goods, is the same as Equation (REF).

C.3.2 Non-energy consumption good producers

This section presents the optimization problem of non-energy consumption good producers in the Swedish economy and derives the relative price of the non-energy consumption goods equation.

The optimization problem of the representative non-energy consumption good producer can be defined as follows:

$$
\max_{D_t^{C,xe},M_t^{C,xe}} P_t^{C,xe}C_t^{xe} - P_t D_t^{C,xe} - P_t^{M,C,xe}M_t^{C,xe}
$$

subject to the CES aggregate consumption good function

$$
C_t^{xe} = \left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_c}}\left(D_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}. \qquad \text{(C.104)}
$$

By substituting the CES aggregate consumption good equation (D.49) into the above profit function, we can rewrite the profit function as:

$$
P_t^{C,xe}\left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(D_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}} - P_t D_t^{C,xe} - P_t^{M,C,xe}M_t^{C,xe}.
$$

First, we derive the demand function for the domestically produced intermediate goods used as inputs by the representative firm in the consumption good sector. The FOC for the domestically produced intermediate goods $D_t^{C,xe}$ is:

$$
\frac{\nu_{C,xe}}{\nu_{C,xe}-1}P_t^{C,xe}\left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(D_t^{C}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}-1} \frac{\nu_{C,xe}-1}{\nu_{C,xe}}\left(D_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}-1}\left(\psi^{C,xe}\right)^{\frac{1}{\nu
$$

We can rewrite the above FOC for $D_t^{C,xe}$ as:

$$
P_t^{C,xe}\left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(D_t^{C}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}-1}\left(D_t^{C,xe}\right)^{\frac{-1}{\nu_{C,xe}}}\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} - P_t = 0.
$$

Note that the CES aggregate consumption good equation (D.49) can be written as:

$$
\left(C_t^{xe}\right)^{\frac{1}{\nu_{C,xe}}} = \left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(D_t^{C}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}}\right]^{\frac{1}{\nu_{C,xe}-1}}. \qquad \text{(C.105)}
$$

Using Equation (C.105), we can rewrite the FOC for $D_t^{C,xe}$ as:

$$
P_t^{C,xe}\left(C_t^{xe}\right)^{\frac{1}{\nu_{C,xe}}}\left(D_t^{C,xe}\right)^{\frac{-1}{\nu_{C,xe}}}\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} - P_t = 0.
$$

The demand function for the domestically produced intermediate goods can be written as:

$$D_t^{C,xe} = \psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_{C,xe}} C_t^{xe}. \tag{C.106}$$

Next, we find the demand function for imported goods used as inputs by the representative firm in the consumption good sector. The FOC for the imported good M_t^C is:

$$P_t^{C,xe} \left[\left(\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(D_t^C\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} + \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{\nu_{C,xe}-1}{\nu_{C,xe}}} \right]^{\frac{1}{\nu_{C,xe}-1}} \left(M_t^{C,xe}\right)^{\frac{-1}{\nu_{C,xe}}} \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} - P_t^{M,C,xe} = 0.$$

Using Equation (C.105), the FOC for $M_t^{C,xe}$ can be rewritten as:

$$P_t^{C,xe} \left(C_t^{xe}\right)^{\frac{1}{\nu_{C,xe}}} \left(M_t^{C,xe}\right)^{\frac{-1}{\nu_{C,xe}}} \left(1-\psi^{C,xe}\right)^{\frac{1}{\nu_{C,xe}}} - P_t^{M,C,xe} = 0.$$

The demand function for the imported goods can be expressed as:

$$M_t^{C,xe} = \left(1-\psi^{C,xe}\right) \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}} C_t^{xe}. \tag{C.107}$$

Substituting the above demand functions (C.106) and (C.107) into the CES aggregate consumption equation (D.49), and we have the following equation:

$$C_t^{xe} = \left[\psi^{C,xe} \left(C_t^{xe}\right)^{\frac{\nu_{C,xe}-1}{C,xe}} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(C_t^{xe}\right)^{\frac{\nu_{C,xe}-1}{C,xe}} \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}} \tag{C.108}$$

$$\begin{aligned}
C_t^{xe} &= C_t^{xe} \left[\psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}, \\
1 = \left(P_t^{C,xe}\right)^{\nu_{C,xe}} &\left[\psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}, \\
\left(P_t^{C,xe}\right)^{\nu_{C,xe}} &= \left[\psi^{C,xe} \left(\frac{1}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(\frac{1}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{\nu_{C,xe}}{\nu_{C,xe}-1}}, \\
P_t^{C,x} &= \left[\psi^{C,xe} \left(\frac{1}{P_t} \right)^{\nu_c - 1} + \left(1-\psi^{C,xe}\right) \left(\frac{1}{P_t^{M,C,xe}} \right)^{\nu_{C,xe}-1} \right]^{\frac{1}{\nu_{C,xe}-1}}.
\end{aligned}$$

The above equation can be rewritten as:

$$P_t^{C,xe} = \left[\psi^{C,xe} \left(P_t\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C,xe}\right) \left(P_t^{M,C,xe}\right)^{1-\nu_{C,xe}} \right]^{\frac{1}{1-\nu_{C,xe}}}. \tag{C.109}$$

Equation (C.109) captures the aggregate consumption price index. Equation (C.109) is the same as Equation (48) in Section 2.4.5.

Using the following definitions: $p_t^{C,xe} = P_t^{C,xe}/P_t$ and $p_t^{M,C,xe} = P_t^{M,C,xe}/P_t$, Equation (C.109) becomes:

$$\begin{aligned}
\frac{P_t^{C,xe}}{P_t} &= \left[\left(P_t\right)^{\nu_{C,xe}-1} \left(\psi^{C,xe} \left(P_t\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C,xe}\right) \left(P_t^{M,C,xe}\right)^{1-\nu_{C,xe}} \right) \right]^{\frac{1}{1-\nu_{C,xe}}}, \\
\frac{P_t^{C,xe}}{P_t} &= \left[\psi^{C,xe} + \left(1-\psi^{C,xe}\right) \left(\frac{P_t^{M,C,xe}}{P_t} \right)^{1-\nu_{C,xe}} \right]^{\frac{1}{1-\nu_{C,xe}}}.
\end{aligned}$$

Thus, the relative price of consumption goods equation is expressed as:

$$p_t^{C,xe} = \left[\psi^{C,xe} + \left(1-\psi^{C,xe}\right) \left(p_t^{M,C,xe}\right)^{1-\nu_{C,xe}} \right]^{\frac{1}{1-\nu_{C,xe}}}. \tag{C.110}$$

Note that: $\psi^{C,xe} = \vartheta^{C,xe} + \frac{1}{1+\omega}(1-\vartheta^{C,xe})$, where $\frac{1}{1+\omega}$ is the relative size of the Swedish economy, and $\vartheta^{C,xe} \in [0,1]$ is a measure of home bias in the production of consumption goods in Sweden. Since the size of the Foreign economy ω is infinitely larger than the Swedish economy, we have: $\psi^{C,xe} = \vartheta^{C,xe}$. Equation (C.110) becomes:

$$p_t^{C,xe} = \left[\vartheta^{C,xe} + \left(1-\vartheta^{C,xe}\right)\left(p_t^{M,C,xe}\right)^{1-\nu_{C,xe}}\right]^{\frac{1}{1-\nu_{C,xe}}}. \tag{C.111}$$

Equation (C.111), which captures the relative price of consumption goods, is the same as Equation (A.37a).

C.3.3 Energy good producers

This section presents the optimization problem of the energy consumption good producers in the Swedish economy, derives the demand functions of domestic and imported energy consumption, and derives the relative price of the energy consumption goods equation.

The optimization problem of the representative non-energy consumption good producer can be defined as follows:

$$\max_{C_t^e, D_t^{C,e}, M_t^{C,e}} P_t^C C_t^e - P_t^{D,C,e} D_t^{C,e} - P_t^{M,C,e} M_t^{C,e}$$

subject to the CES aggregate consumption good function

$$C_t^e = \left[\left(\psi^{C,e}\right)^{\frac{1}{\nu_{c,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\psi^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}}. \tag{C.112}$$

First note, that the definition of $\psi^{C,e} = \vartheta^{C,e} + \frac{1}{1+\omega}(1-\vartheta^{C,xe})$ and since $\omega \to \infty$, $\psi^{C,e} = \vartheta^{C,e}$. By substituting the CES aggregate consumption good equation (C.112) into the above profit function, we can rewrite the profit function as:

$$P_t^{C,e}\left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{c,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} - P_t^{D,C,e}D_t^{C,e} - P_t^{M,C,e}M_t^{C,e}.$$

Taking the derivatives of $D_t^{C,e}$ and $M_t^{C,e}$ respectively gives us the two following first-order-conditions:

$$\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}-1} P_t^{C,e}\left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}-1} - P_t^{D,C,e} = 0$$

$$\left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}-1} P_t^{C,e}\left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}}\right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}-1} - P_t^{M,C,e} = 0$$

If we substitute the expression in square brackets by the production function, and put the prices of non-energy and energy respectively on the right-hand-side of the equation, the two equations can be written as

$$P_t^{D,C,e} = \left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(D_t^{C,e}\right)^{-\frac{1}{\nu_{C,e}}} P_t^{C,e}\left(C_t^e\right)^{\frac{1}{\nu_{C,e}}}$$
$$P_t^{M,C,e} = \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}}\left(M_t^{C,e}\right)^{-\frac{1}{\nu_{C,e}}} P_t^{C,e}\left(C_t^e\right)^{\frac{1}{\nu_{C,e}}}.$$

Rearrange and multiply through with $\nu_{C,e}$ in the two equations above to get the demand function for the non-energy and energy consumption goods:

$$D_t^{C,e} = \vartheta^{C,e}\left(\frac{P_t^{C,e}}{P_t^{D,C,e}}\right)^{\nu_{C,e}} C_t^e \tag{C.113}$$
$$M_t^{C,e} = \left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{C,e}}{P_t^{M,C,e}}\right)^{\nu_{C,e}} C_t^e \tag{C.114}$$

which are the same equations that are presented in section 2.4.5. Next, we create the price index. This is done by substituting the two demand functions into the production function:

$$
\begin{aligned}
C_t^e &= \left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{c,e}}} \left(D_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}} \left(M_t^{C,e}\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} \\
C_t^e &= \left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{c,e}}} \left(\vartheta^{C,e} \left(\frac{P_t^{C,e}}{P_t^{D,C,e}} \right)^{\nu_{C,e}} C_t^e \right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}} \left(\left(1-\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{M,C,e}} \right)^{\nu_{C,e}} C_t^e \right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} \\
C_t^e &= \left[\left(\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}+\frac{\nu_{C,e}-1}{\nu_{C,e}}} \left(\frac{P_t^{C,e}}{P_t^{D,C,e}} \right)^{\nu_{C,e}-1} \left(C_t^e\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} + \left(1-\vartheta^{C,e}\right)^{\frac{1}{\nu_{C,e}}+\frac{\nu_{C,e}-1}{\nu_{C,e}}} \left(\frac{P_t^{C,e}}{P_t^{M,C,e}} \right)^{\nu_{C,e}-1} \left(C_t^e\right)^{\frac{\nu_{C,e}-1}{\nu_{C,e}}} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} \\
C_t^e &= C_t^e \left[\left(\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{D,C,e}} \right)^{\nu_{C,e}-1} + \left(1-\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{M,C,e}} \right)^{\nu_{C,e}-1} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} \\
1 &= \left[\left(\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{D,C,e}} \right)^{\nu_{C,e}-1} + \left(1-\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{M,C,e}} \right)^{\nu_{C,e}-1} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} \\
1 = \left(P_t^{C,e}\right)^{(\nu_C-1)\frac{\nu_C}{\nu_C-1}} & \left[\left(\vartheta^{C,e}\right) \left(\frac{1}{P_t^{D,C,e}} \right)^{\nu_{C,e}-1} + \left(1-\vartheta^{C,e}\right) \left(\frac{1}{P_t^{M,C,e}} \right)^{\nu_{C,e}-1} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} \\
\left(P_t^{C,e}\right)^{-\nu_C} &= \left[\vartheta^{C,e} \left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right) \left(P_t^{M,C,e}\right)^{1-\nu_{C,e}} \right]^{\frac{\nu_{C,e}}{\nu_{C,e}-1}} \\
P_t^{C,e} &= \left[\vartheta^{C,e} \left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right) \left(P_t^{M,C,e}\right)^{1-\nu_{C,e}} \right]^{\frac{1}{1-\nu_{C,e}}}
\end{aligned}
$$

which is the same function as is presented in section 2.4.5. Now, we are going to stationarize all equations and write the equations in per capita form (using the fact the the population is normalized to 1 for convenience). We use the definitions $p_t^{C,e} = P_t^{C,e}/P_t$, $p_t^{D,C,e} = P_t^{D,C,e}/P_t$, $p_t^{M,C,e} = P_t^{M,C,e}/P_t$, $\bar{c}_t^e = C_t^e/z_t^+$, $\bar{d}_t^{C,e} = D_t^{C,e}/z_t^+$, $\bar{m}_t^{C,e} = M_t^{C,e}/z_t^+$. The non-energy consumption demand function can be written as

$$
\begin{aligned}
D_t^{C,e} &= \vartheta^{C,e} \left(\frac{P_t^{C,e}}{P_t^{D,C,e}} \right)^{\nu_{C,e}} C_t^e \\
\frac{D_t^{C,e}}{z_t^+} &= \vartheta^{C,e} \left(\frac{P_t^{C,e}}{P_t^{D,C,e}} \frac{P_t}{P_t} \right)^{\nu_{C,e}} \frac{C_t^e}{z_t^+} \\
\bar{d}_t^{C,e} &= \vartheta^{C,e} \left(\frac{p_t^{C,e}}{p_t^{D,C,e}} \right)^{\nu_{C,e}} \bar{c}_t^e \qquad \text{(C.115)}
\end{aligned}
$$

Equation (C.115), which captures the demand for non-energy consumption goods, is the same as Equation (A.41a).

Next, we stationarize the demand for energy goods:

$$
\begin{aligned}
M_t^{C,e} &= \left(1-\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{M,C,e}} \right)^{\nu_{C,e}} C_t^e \\
\frac{M_t^{C,e}}{z_t^+} &= \left(1-\vartheta^{C,e}\right) \left(\frac{P_t^{C,e}}{P_t^{M,C,e}} \frac{P_t}{P_t} \right)^{\nu_{C,e}} \frac{C_t^e}{z_t^+} \\
\bar{m}_t^{C,e} &= \left(1-\vartheta^{C,e}\right) \left(\frac{p_t^{C,e}}{p_t^{M,C,e}} \right)^{\nu_{Ce}} \bar{c}_t^e \qquad \text{(C.116)}
\end{aligned}
$$

Equation (C.116), which captures the demand for energy consumption goods, is the same as Equation (A.42a).

Finally, we stationarize the price index:

$$P_t^{C,e} = \left[\vartheta^{C,e}\left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(P_t^{M,C,e}\right)^{1-\nu_{C,e}}\right]^{\frac{1}{1-\nu_{C,e}}}$$

$$\frac{P_t^{C,e}}{P_t} = \frac{1}{P_t}\left[\vartheta^{C,e}\left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(P_t^{M,C,e}\right)^{1-\nu_{C,e}}\right]^{\frac{1}{1-\nu_{C,e}}}$$

$$\frac{P_t^{C,e}}{P_t} = \left[\left(\vartheta^{C,e}\left(P_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(P_t^{M,C,e}\right)^{1-\nu_{C,e}}\right)(P_t)^{\nu_{C,e}-1}\right]^{\frac{1}{1-\nu_{C,e}}}$$

$$p_t^{C,e} = \left[\left(\vartheta^{C,e}\left(\frac{P_t^{D,C,e}}{P_t}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(\frac{P_t^{M,C,e}}{P_t}\right)^{1-\nu_{C,e}}\right)\right]^{\frac{1}{1-\nu_{C,e}}}$$

$$p_t^{C,e} = \left[\left(\vartheta^{C,e}\left(p_t^{D,C,e}\right)^{1-\nu_{C,e}} + \left(1-\vartheta^{C,e}\right)\left(p_t^{M,C,e}\right)^{1-\nu_{C,e}}\right)\right]^{\frac{1}{1-\nu_{C,e}}}. \tag{C.117}$$

Equation (C.117), which captures the demand for energy consumption goods, is the same as Equation (A.39a).

C.4 Private investment good producers

This section presents the optimization problem of the investment good producers, and derives the relative price of investment goods equation (A.44a). We define V_t^I to be the output of a representative investment firm. We define V_t^I as $\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]$.

A profit function of the representative investment good producer is defined as:

$$P_t^I V_t^I - P_t D_t^I - P_t^{M,I} M_t^I.$$

The investment good producer faces the following CES investment function:

$$V_t^I = \left[\left(\psi^I\right)^{\frac{1}{\nu_I}}\left(D_t^I\right)^{\frac{\nu_I-1}{\nu_I}} + \left(1-\psi^I\right)^{\frac{1}{\nu_I}}\left(M_t^I\right)^{\frac{\nu_I-1}{\nu_I}}\right]^{\frac{\nu_I}{\nu_I-1}}. \tag{C.118}$$

The optimization problem can be defined as follows:

$$\max_{D_t^I, M_t^I} P_t^I V_t^I - P_t D_t^I - P_t^{M,I} M_t^I$$

subject to

$$V_t^I = \left[\left(\psi^I\right)^{\frac{1}{\nu_I}}\left(D_t^I\right)^{\frac{\nu_I-1}{\nu_I}} + \left(1-\psi^I\right)^{\frac{1}{\nu_I}}\left(M_t^I\right)^{\frac{\nu_I-1}{\nu_I}}\right]^{\frac{\nu_I}{\nu_I-1}}.$$

We follow the similar steps as described in Section C.3 to find the demand functions for the domestically produced intermediate goods and for the imported goods used in the production of investment goods. The demand function for the domestically produced intermediate goods used in the production of investment goods D_t^I is:

$$D_t^I = \psi^I\left(\frac{P_t^I}{P_t}\right)^{\nu_I} V_t^I.$$

This demand function is the same the demand function that is presented in Section 2.4.4.

Using the following definition: $V_t^I = \frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]$, the above demand function for the domestically produced intermediate goods used in the production of investment goods can be rewritten as:

$$D_t^I = \psi^I\left(\frac{P_t^I}{P_t}\right)^{\nu_I}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{C.119}$$

The demand function for the imported goods used in the production of investment goods is:

$$M_t^I = (1-\psi^I)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I} V_t^I.$$

This demand function is the same the demand function that is presented in Section 2.4.4.

Using the following definition: $V_t^I = \frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]$, the above demand function for the imported goods used in the production of investment goods can be rewritten as:

$$M_t^I = (1-\psi^I)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{C.120}$$

Substituting the above demand functions (C.119) and (C.120) into the CES investment equation (C.118), this gives us the following aggregate investment price index equation:

$$P_t^I = \left[\psi^I\left(P_t\right)^{1-\nu_I} + \left(1-\psi^I\right)\left(P_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{C.121}$$

Equation (C.121) captures the aggregate investment price index. Equation (C.121) is the same as Equation (42) in Section 2.4.4.

Equation (C.121) can be rewritten as:

$$\frac{P_t^I}{P_t} = \left[\psi^I + \left(1-\psi^I\right)\left(\frac{P_t^{M,I}}{P_t}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{C.122}$$

Using the following definitions: $p_t^I = P_t^I/P_t$ and $p_t^{M,I} = P_t^{M,I}/P_t$. The relative price of investment goods equation can be expressed as:

$$p_t^I = \left[\psi^I + \left(1-\psi^I\right)\left(p_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{C.123}$$

Note that: $\psi^I = \vartheta^I + \frac{1}{1+\omega}(1-\vartheta^I)$. $\frac{1}{1+\omega}$ is the relative size of the Swedish economy, and $\vartheta^I \in [0,1]$ is a measure of home bias in the production of investment goods in the Swedish economy. Since the size of the Foreign economy ω is infinitely larger than the Swedish economy, we have: $\psi^I = \vartheta^I$. Thus, the relative price of investment goods equation can be rewritten as:

$$p_t^I = \left[\vartheta^I + \left(1-\vartheta^I\right)\left(p_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}}. \tag{C.124}$$

Equation (C.124), which represents the relative price of investment goods equation, is the same as Equation (A.44a).

C.5 Export good producers

This section presents the optimization problem of export good producers in the Swedish economy. First, we derive the real marginal cost of production for export good producers, Equation (A.46a). Second, we derive the optimal price for export good producers, Equation (A.47a). There is a continuum of export good producers. Each firm i produces export goods $X_t(i)$ by using domestically produced intermediate goods $D_t^X(i)$ and imported goods $M_t^X(i)$ as inputs.

The export good firm i faces the following cost minimization problem:

$$\min_{D_t^X(i),\, M_t^X(i)} \quad TC_t^X(i) = P_t D_t^X(i) + P_t^{M,X} M_t^X(i), \tag{C.125}$$

subject to the following production function:

$$X_t(i) = \left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}} - z_t^+\phi^X. \tag{C.126}$$

We let $\theta_t^X(i)$ to be the Lagrange multiplier associated with the production constraint for export goods (C.126). To solve the optimization problem, we set up the following Lagrangian $\mathscr{L}_t^X(i)$:

$$\begin{aligned}\mathscr{L}_t^X(i) &= \left[P_t D_t^X(i) + P_t^{M,X} M_t^X(i)\right] \\ &+ \theta_t^X(i)\left\{X_t(i) - \left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}} + z_t^+\phi^X\right\}.\end{aligned}$$

We take the partial derivative of $\mathscr{L}_t^X(i)$ with respect to $D_t^X(i)$ and $M_t^X(i)$ respectively, and we can find the FOC for $D_t^X(i)$ and $M_t^X(i)$.

The FOC for $D_t^X(i)$ is:

$$P_t = \theta_t^X(i)\left(\psi^X\right)^{\frac{1}{\nu_x}}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}} D_t^X(i)^{\frac{-1}{\nu_x}}. \tag{C.127}$$

The FOC for $M_t^X(i)$ is:

$$P_t^{M,X} = \theta_t^X(i)\left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}} M_t^X(i)^{\frac{-1}{\nu_x}}. \tag{C.128}$$

We rewrite Equation (C.126) as:

$$\left[X_t(i) + z_t^+\phi^X\right] = \left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}}. \tag{C.129}$$

We use (C.127) and Equation (C.128) to find the following total cost function:

$$\begin{aligned}TC_t^X(i) = {} & \theta_t^X(i)\left(\psi^X\right)^{\frac{1}{\nu_x}}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}} D_t^X(i)^{\frac{-1}{\nu_x}} D_t^X(i)\\ & + \theta_t^X(i)\left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}} M_t^X(i)^{\frac{-1}{\nu_x}} M_t^X(i),\end{aligned}$$

$$\begin{aligned}& TC_t^X(i) = \\ & \theta_t^X(i)\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]\left[\left(\psi^X\right)^{\frac{1}{\nu_x}} D_t^X(i)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}} M_t^X(i)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{1}{\nu_x-1}}.\end{aligned} \tag{C.130}$$

Using Equation (C.129), Equation (C.130) can be rewritten as follows:

$$TC_t^X(i) = \theta_t^X(i)\left[X_t(i) + z_t^+\phi^X\right]^{\frac{\nu_x-1}{\nu_x}}\left[X_t(i) + z_t^+\phi^X\right]^{\frac{1}{\nu_x}},$$

$$TC_t^X(i) = \theta_t^X(i)\left[X_t(i) + z_t^+\phi^X\right]. \tag{C.131}$$

We take the derivative of the function of $TC_t^X(i)$ with respect to $X_t(i)$, and we have the following definition:

$$MC_t^X(i) = \frac{\partial TC_t^X(i)}{\partial X_t(i)} = \theta_t^X(i). \tag{C.132}$$

Equation (C.132) implies that $\theta_t^X(i) = MC_t^X(i)$. We can now establish that $\theta_t^X(i)$ is the nominal marginal cost of production for export good producers $MC_t^X(i)$.

We use the result from Equation (C.132), and then we rearrange Equation (C.127) and Equation (C.128). We have the following equations:

$$D_t^X(i) = \psi^X\left(\frac{MC_t^X(i)}{P_t}\right)^{\nu_x}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}}\left(D_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left(M_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}}, \tag{C.133}$$

$$M_t^X(i) = \left(1-\psi^X\right)\left(\frac{MC_t^X(i)}{P_t^{M,X}}\right)^{\nu_x}\left[\left(\psi^X\right)^{\frac{1}{\nu_x}}\left(D_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}} + \left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left(M_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}}. \tag{C.134}$$

We substitute Equation (C.133) and Equation (C.134) into Equation (C.125), and Equation (C.125) can be expressed as:

$$TC_t^X(i) =$$
$$\left[\left(\psi^X\right)^{\frac{1}{\nu_x}}\left(D_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}}+\left(1-\psi^X\right)^{\frac{1}{\nu_x}}\left(M_t^X(i)\right)^{\frac{\nu_x-1}{\nu_x}}\right]^{\frac{\nu_x}{\nu_x-1}}$$
$$\left[\psi^X\left(MC_t^X(i)\right)^{\nu_x}(P_t)^{(1-\nu_x)}+\left(1-\psi^X\right)\left(MC_t^X(i)\right)^{\nu_x}\left(P_t^{M,X}\right)^{(1-\nu_x)}\right].$$

Using Equation (C.129), the above equation can be simplified to the following equation:

$$TC_t^X(i)=\left[X_t(i)+z_t^+\phi^X\right]\left[\psi^X\left(MC_t^X(i)\right)^{\nu_x}(P_t)^{(1-\nu_x)}+\left(1-\psi^X\right)\left(MC_t^X(i)\right)^{\nu_x}\left(P_t^{M,X}\right)^{(1-\nu_x)}\right]. \quad (C.135)$$

We use Equation (C.135) and Equation (C.132), and we take the partial derivative of $TC_t^X(i)$ with respect to $X_t(i)$. We have the following equation:

$$MC_t^X(i)=\frac{\partial TC_t^X(i)}{\partial X_t(i)}=\left[\psi^X\left(MC_t^X(i)\right)^{\nu_x}(P_t)^{(1-\nu_x)}+\left(1-\psi^X\right)\left(MC_t^X(i)\right)^{\nu_x}\left(P_t^{M,X}\right)^{(1-\nu_x)}\right]. \quad (C.136)$$

We rearrange the above equation, and we can obtain the nominal marginal cost of production for Swedish exports, which depends on the domestic intermediate good and imported good prices. The nominal marginal cost of Swedish export goods can be expressed as:

$$MC_t^X=\left[\psi^X(P_t)^{(1-\nu_x)}+\left(1-\psi^X\right)\left(P_t^{M,X}\right)^{(1-\nu_x)}\right]^{\frac{1}{1-\nu_x}}. \quad (C.137)$$

Equation (C.137) is the same as Equation (38) in Section 2.4.3. The marginal cost of firm i is independent to the firm-specific variables, so there is no subscript$i$ in the RHS of Equation (C.137). As a consequence, we have the following result: $MC_t^X(i)=MC_t^X$.

Using the following definition: $p_t^{M,X}=\frac{P_t^{M,X}}{P_t}$ and $\overline{mc}_t=\frac{MC_t^X}{P_t}$, Equation (C.137) can be rewritten as:

$$\overline{mc}_t^X=\left[\psi^X+\left(1-\psi^X\right)\left(p_t^{M,X}\right)^{1-\nu_x}\right]^{\frac{1}{1-\nu_x}}. \quad (C.138)$$

Note that: $\psi^X=\vartheta^X+\frac{1}{1+\omega}(1-\vartheta^X)$, where $\frac{1}{1+\omega}$ is the relative size of the Swedish economy, and $\vartheta^X\in[0,1]$ is a measure of home bias in the production of export goods in Sweden. We thus have $\psi^X\to\vartheta^X$ as $\omega\to\infty$, and Equation (C.138) becomes:

$$\overline{mc}_t^X=\left[\vartheta^X+\left(1-\vartheta^X\right)\left(p_t^{M,X}\right)^{1-\nu_x}\right]^{\frac{1}{1-\nu_x}}. \quad (C.139)$$

Equation (C.139), which represents the real marginal cost of production for export good producers, is the same as Equation (A.46a).

Furthermore, we can find the demand function for domestically produced intermediate goods that are used in the production of export goods by firm i and the demand function for imported goods that are used in the production of export goods by firm i. We apply Equation (C.129) and the result that the marginal cost across firms are equal: $MC_t^X(i)=MC_t^X$. Thus, we can rewrite Equation (C.133) and Equation (C.134) as follows.

The demand function for domestically produced intermediate goods that are used in the production of export goods $D_t^X(i)$ is:

$$D_t^X(i)=\psi^X\left(\frac{MC_t^X}{P_t}\right)^{\nu_x}\left[X_t(i)+z_t^+\phi^X\right]. \quad (C.140)$$

The demand function for imported goods that are used in the production of export goods $M_t^X(i)$ is:

$$M_t^X(i)=\left(1-\psi^X\right)\left(\frac{MC_t^X}{P_t^{M,X}}\right)^{\nu_x}\left[X_t(i)+z_t^+\phi^X\right]. \quad (C.141)$$

These demand functions are the same as the demand functions in the footnote of Section 2.4.3.

Now, we derive the optimal price of export goods equation (A.47a). Export firm i chooses the optimal price of export goods $P_t^{X,opt}(i)$ that maximizes its profit, subject to its demand schedule for export goods and the Calvo price contract. In each period, the individual firm resets its price with probability $(1-\xi_x)$. With probability ξ_x, the firm cannot reset its price, and then it faces the following price evolution: $P_{t+k|t}^X(i) = P_t^{X,opt}(i)\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X \dots \overline{\Pi}_{t+k}^X$. Recall that the prices of Swedish export goods are set in the currency of Foreign, so called local currency pricing. We define the stochastic discount factor as $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega_{t+k}^C}{\Omega_t^C}\frac{P_t^C}{P_{t+k}^C}$.

The profit function of firm i is written as:

$$\max_{P_t^{X,opt}(i)} E_t \sum_{k=0}^{\infty}(\xi_x)^k \Lambda_{t,t+k}\left\{P_{t+k|t}^X(i)S_{t+k}X_{t+k|t}(i) - TC_{t+k|t}^X\left[X_{t+k|t}(i)\right]\right\}, \tag{C.142}$$

subject to the demand function:

$$X_{t+k|t}(i) = \left(\frac{P_{t+k|t}^X(i)}{P_{t+k}^X}\right)^{\frac{\lambda_{t+k}^X}{1-\lambda_{t+k}^X}} X_{t+k}, \tag{C.143}$$

and the Calvo price setting contract:

$$P_{t+k}^X(i) = \begin{cases} \overline{\Pi}_{t+k}^X P_{t+k-1}^X(i) & \text{with probability } \xi_x \\ P_{t+k}^{X,opt}(i) & \text{with probability } (1-\xi_x). \end{cases} \tag{C.144}$$

The FOC of $P_t^{X,opt}(i)$ is:

$$\begin{aligned}
&E_t\{S_tX_{t|t}(i) + S_tP_t^{X,opt}(i)\frac{\partial X_{t|t}(i)}{\partial P_t^{X,opt}(i)} - MC_t^X(i)\frac{\partial X_{t|t}(i)}{\partial P_t^{X,opt}(i)} \\
&\quad + \xi_x\Lambda_{t,t+1}\left[S_{t+1}\frac{\partial P_{t+1|t}^X(i)}{\partial P_t^{X,opt}(i)}X_{t+1|t}(i) + S_{t+1}P_{t+1|t}^X(i)\frac{\partial X_{t+1|t}(i)}{\partial P_{t+1|t}^X(i)}\frac{\partial P_{t+1|t}^X(i)}{\partial P_t^{X,opt}(i)} - MC_{t+1}^X(i)\frac{\partial X_{t+1|t}(i)}{\partial P_{t+1|t}^X(i)}\frac{\partial P_{t+1|t}^X(i)}{\partial P_t^{X,opt}(i)}\right] \\
&\quad + (\xi_x)^2\Lambda_{t,t+2} \\
&\left[S_{t+2}\frac{\partial P_{t+2|t}^X(i)}{\partial P_t^{X,opt}(i)}X_{t+2|t}(i) + S_{t+2}P_{t+2|t}^X(i)\frac{\partial X_{t+2|t}(i)}{\partial P_{t+2|t}^X(i)}\frac{\partial P_{t+2|t}^X(i)}{\partial P_t^{X,opt}(i)} - MC_{t+2}^X(i)\frac{\partial X_{t+2|t}(i)}{\partial P_{t+2|t}^X(i)}\frac{\partial P_{t+2|t}^X(i)}{\partial P_t^{X,opt}(i)}\right] \\
&\quad + \dots\} = 0.
\end{aligned} \tag{C.145}$$

Recall, we have the following definition: $P_{t+k|t}^X(i) = P_t^{X,opt}(i)\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X \dots \overline{\Pi}_{t+k}^X$. Hence, the partial derivative of $P_{t+k|t}^X(i)$ with respect to $P_t^{X,opt}(i)$ is:

$$\frac{\partial P_{t+k|t}^X(i)}{\partial P_t^{X,opt}(i)} = \overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X \dots \overline{\Pi}_{t+k}^X. \tag{C.146}$$

Using Equation (C.146), Equation (C.145) can be rewritten as:

$$\begin{aligned}
&E_t\{S_tX_{t|t}(i) + S_tP_t^{X,opt}(i)\frac{\partial X_{t|t}(i)}{\partial P_t^{X,opt}(i)} - MC_t^X(i)\frac{\partial X_{t|t}(i)}{\partial P_t^{X,opt}(i)} \\
&\quad + \xi_x\Lambda_{t,t+1}\left[S_{t+1}\overline{\Pi}_{t+1}^X X_{t+1|t}(i) + S_{t+1}P_{t+1|t}^X(i)\overline{\Pi}_{t+1}^X\frac{\partial X_{t+1|t}^X(i)}{\partial P_{t+1|t}^X(i)} - MC_{t+1}^X(i)\overline{\Pi}_{t+1}^X\frac{\partial X_{t+1|t}(i)}{\partial P_{t+1|t}^X(i)}\right] \\
&\quad + (\xi_x)^2\Lambda_{t,t+2} \\
&\left[S_{t+2}\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X X_{t+2|t}(i) + S_{t+2}P_{t+2|t}^X(i)\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X\frac{\partial X_{t+2|t}(i)}{\partial P_{t+2|t}^X(i)} - MC_{t+2}^X(i)\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X\frac{\partial X_{t+2|t}(i)}{\partial P_{t+2|t}^X(i)}\right] \\
&\quad + \dots\} = 0.
\end{aligned}$$

From Equation (C.137), we have the following result: $MC_{t+k}^X(i) = MC_{t+k}^X$. We rearrange the above equation, and we can obtain the following equation:

$$E_t\{\frac{\partial X_{t|t}(i)}{\partial P_t^{X,opt}(i)}\left[S_t P_t^{X,opt}(i)\left(\frac{X_{t|t}(i)}{P_t^{X,opt}(i)}\left(\frac{\partial X_{t|t}(i)}{\partial P_t^{X,opt}(i)}\right)^{-1}+1\right)-MC_t^X\right]$$
$$+\xi_x\Lambda_{t,t+1}\overline{\Pi}_{t+1}^X\frac{\partial X_{t+1|t}(i)}{\partial P_{t+1|t}^X(i)}\left[S_{t+1}P_{t+1|t}^X(i)\left(\frac{X_{t+1|t}(i)}{P_{t+1|t}^X(i)}\left(\frac{\partial X_{t+1|t}(i)}{\partial P_{t+1|t}^X(i)}\right)^{-1}+1\right)-MC_{t+1}^X\right]$$
$$+\left(\xi_x\right)^2\Lambda_{t,t+2}\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X\frac{\partial X_{t+2|t}(i)}{\partial P_{t+2|t}^X(i)}\left[S_{t+2}P_{t+2|t}^X(i)\left(\frac{X_{t+2|t}(i)}{P_{t+2|t}^X(i)}\left(\frac{\partial X_{t+2|t}(i)}{\partial P_{t+2|t}^X(i)}\right)^{-1}+1\right)-MC_{t+2}^X\right]+\ldots\}=0. \tag{C.147}$$

Given the demand schedule for export goods, which is captured by Equation (C.143), we can find the following price elasticity of demand for export goods:

$$-\frac{\partial X_{t+k|t}(i)}{\partial P_{t+k|t}^X(i)}\frac{P_{t+k|t}^X(i)}{X_{t+k|t}(i)}=\frac{\lambda_{t+k}^X}{\lambda_{t+k}^X-1}. \tag{C.148}$$

Using the result from Equation (C.148), the derivative of $X_{t+k|t}(i)$ with respect to $P_{t+k|t}^X(i)$ is:

$$\frac{\partial X_{t+k|t}(i)}{\partial P_{t+k|t}^X(i)}=\frac{\lambda_{t+k}^X}{1-\lambda_{t+k}^X}\frac{X_{t+k|t}(i)}{P_{t+k|t}^X(i)}. \tag{C.149}$$

Using Equation (C.148) and Equation (C.149), we can rewrite Equation (C.147) as follows:

$$E_t\{\frac{X_{t|t}(i)}{P_t^{X,opt}(i)}\frac{\lambda_t^X}{1-\lambda_t^X}\left[S_tP_t^{X,opt}(i)\frac{1}{\lambda_t^X}-MC_t^X\right]$$
$$+\xi_x\Lambda_{t,t+1}\overline{\Pi}_{t+1}^X\frac{X_{t+1|t}(i)}{P_{t+1|t}^X(i)}\frac{\lambda_{t+1}^X}{1-\lambda_{t+1}^X}\left[S_{t+1}P_{t+1|t}^X(i)\frac{1}{\lambda_{t+1}^X}-MC_{t+1}^X\right]$$
$$+\left(\xi_x\right)^2\Lambda_{t,t+2}\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X\frac{X_{t+2|t}(i)}{P_{t+2|t}^X(i)}\frac{\lambda_{t+2}^X}{1-\lambda_{t+2}^X}\left[S_{t+2}P_{t+2|t}^X(i)\frac{1}{\lambda_{t+2}^X}-MC_{t+2}^X\right]+\ldots\}=0.$$

We use the following definition: $P_{t+k|t}^X(i)=P_t^{X,opt}(i)\overline{\Pi}_{t+1}^X\overline{\Pi}_{t+2}^X\ldots\overline{\Pi}_{t+k}^X$. We multiply both sides of the above equation by $P_t^{X,opt}(i)$ and -1. We can obtain the following equation:

$$E_t\{\frac{X_{t|t}(i)}{\lambda_t^X-1}\left[S_tP_t^{X,opt}(i)-\lambda_t^XMC_t^X\right]$$
$$+\xi_x\Lambda_{t,t+1}\frac{X_{t+1|t}(i)}{\lambda_{t+1}^X-1}\left[S_{t+1}P_{t+1|t}^X(i)-\lambda_{t+1}^XMC_{t+1}^X\right]$$
$$+\left(\xi_x\right)^2\Lambda_{t,t+2}\frac{X_{t+2|t}(i)}{\lambda_{t+2}^X-1}\left[S_{t+2}P_{t+2|t}^X(i)-\lambda_{t+2}^XMC_{t+2}^X\right]+\ldots\}=0.$$

We can drop the subscript i from the above equation because when firms can reset their price, they will set the same price. As a result, all firms that reset their price in period t will choose the same quantity of output. We rewrite the above equation, and the optimal price of export goods equation can be expressed as:

$$E_t\sum_{k=0}^{\infty}\left(\xi_x\right)^k\Lambda_{t,t+k}\frac{X_{t+k|t}}{\left(\lambda_{t+k}^X-1\right)}\left[S_{t+k}P_{t+k|t}^X-\lambda_{t+k}^XMC_{t+k}^X\right]=0. \tag{C.150}$$

Equation (C.150), which is the non-stationarized version of the optimal price of export goods, is the same as Equation (41) in Section 2.4.3.

The above equation can be rewritten in terms of *per capita* quantities. To this end, we let $x_{t+k|t}$ denote the per capita quantity of output of export firms that reset their price in period t. The optimal price for export goods equation can be written as:

$$E_t\sum_{k=0}^{\infty}\left(\xi_x\right)^k\Lambda_{t,t+k}\frac{x_{t+k|t}}{\left(\lambda_{t+k}^X-1\right)}\left[S_{t+k}P_{t+k|t}^X-\lambda_{t+k}^XMC_{t+k}^X\right]=0. \tag{C.151}$$

Now, we stationarize Equation (C.151). We use the following definition: $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega^C_{t+k}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+k}}$, and we expand Equation (C.151). Thus, we have the following equation:

$$E_t\{\frac{x_{t|t}}{(\lambda^X_t-1)}\left[S_tP^{X,opt}_t-\lambda^X_tMC^X_t\right]$$
$$+\xi_x\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{x_{t+1|t}}{(\lambda^X_{t+1}-1)}\left[S_{t+1}P^X_{t+1|t}-\lambda^X_{t+1}MC^X_{t+1}\right]$$
$$+(\xi_x)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{x_{t+2|t}}{(\lambda^X_{t+2}-1)}\left[S_{t+2}P^X_{t+2|t}-\lambda^X_{t+2}MC^X_{t+2}\right]+\ldots\}=0.$$

We expand further and multiply through by $\frac{1}{P_t}$ to get:

$$E_t\{\frac{x_{t|t}}{(\lambda^X_t-1)}\frac{P_t}{P_t}\left[\frac{S_tP^{X,opt}_t}{P_t}-\frac{\lambda^X_tMC^X_t}{P_t}\right]$$
$$+\xi_x\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{x_{t+1|t}}{(\lambda^X_{t+1}-1)}\frac{P_{t+1}}{P_t}\left[\frac{S_{t+1}P^X_{t+1|t}}{P_{t+1}}-\frac{\lambda^X_{t+1}MC^X_{t+1}}{P_{t+1}}\right]$$
$$+(\xi_x)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{x_{t+2|t}}{(\lambda^X_{t+2}-1)}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\left[\frac{S_{t+2}P^X_{t+2|t}}{P_{t+2}}-\frac{\lambda^X_{t+2}MC^X_{t+2}}{P_{t+2}}\right]+\ldots\}=0.$$

We use the following definition: $P^X_{t+k|t}=P^{X,opt}_t\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}\ldots\overline{\Pi}^X_{t+k}$. We multiply the marginal utility of consumption Ω^C_{t+k} by z^+_{t+k}, and divide export goods $x_{t+k|t}$ by z^+_{t+k} and expand further to get:

$$E_t\{\frac{x_{t|t}}{(\lambda^X_t-1)\,z^+_t}\left[\frac{S_tP^{X,opt}_t}{P_t}-\lambda^X_t\frac{MC^X_t}{P_t}\right]$$
$$+\xi_x\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_tz^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{x_{t+1|t}}{\left(\lambda^X_{t+1}-1\right)z^+_{t+1}}\frac{P_{t+1}}{P_t}\left[\frac{S_{t+1}\overline{\Pi}^X_{t+1}P^{X,opt}_t}{P_{t+1}}-\lambda^X_{t+1}\frac{MC^X_{t+1}}{P_{t+1}}\right]$$
$$+(\xi_x)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_tz^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{x_{t+2|t}}{\left(\lambda^X_{t+2}-1\right)z^+_{t+2}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}$$
$$\left[\frac{S_{t+2}\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}P^{X,opt}_t}{P_{t+2}}-\lambda^X_{t+2}\frac{MC^X_{t+2}}{P_{t+2}}\right]+\ldots\}=0.$$

We multiply the second term of the above equation by $\frac{S_t}{S_t}$, and $\frac{P_t}{P_t}$, whereas we multiply the third term of the above equation by $\frac{S_{t+1}S_t}{S_{t+1}S_t}$ and $\frac{P_tP_{t+1}}{P_tP_{t+1}}$. We have the following equation:

$$E_t\{\frac{x_{t|t}}{(\lambda^X_t-1)\,z^+_t}\left[\frac{S_tP^{X,opt}_t}{P_t}-\lambda^X_t\frac{MC^X_t}{P_t}\right]$$
$$+\xi_x\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_tz^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{x_{t+1|t}}{\left(\lambda^X_{t+1}-1\right)z^+_{t+1}}\frac{P_{t+1}}{P_t}\left[\frac{P_t}{P_t}\frac{S_t}{S_t}\frac{S_{t+1}\overline{\Pi}^X_{t+1}P^{X,opt}_t}{P_{t+1}}-\lambda^X_{t+1}\frac{MC^X_{t+1}}{P_{t+1}}\right]$$
$$+(\xi_x)^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_tz^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{x_{t+2|t}}{\left(\lambda^X_{t+2}-1\right)z^+_{t+2}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}$$
$$\left[\frac{S_{t+1}S_t}{S_{t+1}S_t}\frac{P_tP_{t+1}}{P_tP_{t+1}}\frac{S_{t+2}\overline{\Pi}^X_{t+1}\overline{\Pi}^X_{t+2}P^{X,opt}_t}{P_{t+2}}-\lambda^X_{t+2}\frac{MC^X_{t+2}}{P_{t+2}}\right]+\ldots\}=0.$$

We use the following definitions: $p^{X,opt}_t=\frac{S_tP^{X,opt}_t}{P_t}$, $\overline{mc}^X_{t+k}=\frac{MC^X_{t+k}}{P_{t+k}}$, $s_{t+k}=\frac{S_{t+k}}{S_{t+k-1}}$, $\Pi_{t+k}=\frac{P_{t+k}}{P_{t+k-1}}$, and

$\Pi_{t+k}^C = \frac{P_{t+k}^C}{P_{t+k-1}^C}$. We can obtain the following equation:

$$E_t\{\frac{x_{t|t}}{(\lambda_t^X - 1)\, z_t^+}\left[p_t^{X,opt} - \lambda_t^X \overline{mc}_t^X\right]$$
$$+ \xi_x \frac{\beta_{t+1}}{\beta_t}\frac{\Omega_{t+1}^C z_{t+1}^+}{\Omega_t^C z_t^+}\frac{\Pi_{t+1}}{\Pi_{t+1}^C}\frac{x_{t+1|t}}{(\lambda_{t+1}^X - 1)\, z_{t+1}^+}\left[\frac{\overline{\Pi}_{t+1}^X s_{t+1} p_t^{X,opt}}{\Pi_{t+1}} - \lambda_{t+1}^X \overline{mc}_{t+1}^X\right]$$
$$+ (\xi_x)^2 \frac{\beta_{t+2}}{\beta_t}\frac{\Omega_{t+2}^C z_{t+2}^+}{\Omega_t^C z_t^+}\frac{\Pi_{t+2}\Pi_{t+1}}{\Pi_{t+2}^C \Pi_{t+1}^C}\frac{x_{t+2|t}}{(\lambda_{t+2}^X - 1)\, z_{t+2}^+}$$
$$\left[\frac{\overline{\Pi}_{t+1}^X \overline{\Pi}_{t+2}^X s_{t+1} s_{t+2} p_t^{X,opt}}{\Pi_{t+2}\Pi_{t+1}} - \lambda_{t+2}^X \overline{mc}_{t+2}^X\right] + \ldots\} = 0.$$

Using the following definitions: $\beta_{t+j}^r = \frac{\beta_{t+j}}{\beta_{t+j-1}}$, $\overline{\Omega}_{t+k}^C = \Omega_{t+k}^C z_{t+k}^+$ and $\overline{x}_{t+k} = \frac{x_{t+k}}{z_{t+k}^+}$, the stationarized version of the optimal price of export goods equation can be written as:

$$E_t \sum_{k=0}^{\infty} (\xi_x)^k \left(\prod_{j=1}^{k} \beta_{t+j}^r\right) \frac{\overline{\Omega}_{t+k}^C}{\overline{\Omega}_t^C} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi_{t+j}^C}\right) \frac{\overline{x}_{t+k|t}}{(\lambda_{t+k}^X - 1)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}_{t+j}^X s_{t+j}}{\Pi_{t+j}}\right) p_t^{X,opt} - \lambda_{t+k}^X \overline{mc}_{t+k}^X\right] = 0. \quad \text{(C.152)}$$

Equation (C.152), which is the stationarized version of the optimal price of export goods, is the same as Equation (A.47a).

C.6 Import good producers

This section presents the optimization problem of import good producers in the Swedish economy and derives optimal prices. There are four different types of import firms in the Swedish economy. Each type operates in a separate and monopolistically competitive market. The first type of import firm is denoted by index Cxe and provides imported inputs to the non-energy consumption good producers. The second type of import firm is denoted by index I and provides imported goods to investment good producers. The third type of import firm is denoted by index X and provides imported goods to export firms. The fourth type of firm is denoted by index Ce and provides imported goods to the energy consumption producers. We derive the optimal price for import firms specializing in consumption goods, Equation (A.50a), investment goods, Equation (A.53a), export goods Equation (A.56a) and energy consumption goods, Equation (REF).

Let $n\epsilon\{Cxe, I, X, Ce\}$ be the index for different types of import firm, and let $M_t^n(i)$ represents the quantity produced by the individual firm i of type n. The individual import firm i of type n chooses the optimal price of imported goods $P_{t,opt}^{M,n}(i)$ that maximizes its profit, subject to its demand schedule and the Calvo sticky price friction. In each period, the individual firm i resets its price with probability $(1-\xi_{m,n})$. With probability $\xi_{m,n}$, the firm cannot reset its price, and then it faces the following price evolution: $P_{t+k|t}^{M,n}(i) = P_{t,opt}^{M,n}(i)\overline{\Pi}_{t+1}^{M,n}\overline{\Pi}_{t+2}^{M,n}\ldots\overline{\Pi}_{t+k}^{M,n}$. We define the stochastic discount factor as $\Lambda_{t,t+k} = \frac{\beta_{t+k}}{\beta_t}\frac{\Omega_{t+k}^C}{\Omega_t^C}\frac{P_t^C}{P_{t+k}^C}$.

The firms have to purchace the import goods from abroad, which differs between the three non-energy good firms, and the energy good firms. The non-energy good firms purchace Foreign domestic intermediate goods for the price $S_t P_{F,t}$ and transforms the goods to import good suitable for the respective input good purchaser. The energy good firms purchace foreign energy goods for the price $S_t P_{F,t}^{C,e}$ and transform them into goods suitable for the Swedish energy producer. This means that we can define the marginal cost for the different types of firms as

$$MC_t^{M,n} = S_t P_{F,t} = \frac{Q_t P_t^C}{p_{F,t}^C}, n \in \{Cxe, I, X\}$$
$$MC_t^{M,n} = S_t P_{F,t}^{C,e} = \frac{Q_t P_t^C}{p_{F,t}^C} p_{F,t}^{C,e}, n \in \{Ce\}$$

using the definition that $Q_t = S_t P_{F,t}^C / P_t^C$, $p_{F,t}^C = P_{F,t}^C / P_{F,t}$ and $p_{F,t}^{C,e} = P_{F,t}^{C,e} / P_{F,t}$.
The optimization problem of import good producers under sticky prices can then be defined as follows:

$$\max_{P_{t,opt}^{M,n}(i)} E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \Lambda_{t,t+k} \left\{P_{t+k|t}^{M,n}(i) M_{t+k|t}^n(i) - MC_{F,t+k}^{M,n} M_{t+k|t}^n(i) - MC_{t+k}^{M,n} z_{t+k}^+ \phi^{M,n}\right\} \quad \text{(C.153)}$$

subject to the demand function:

$$M_{t+k|t}^{n}(i)=\left(\frac{P_{t+k|t}^{M,n}(i)}{P_{t+k}^{M,n}}\right)^{\frac{\lambda_{t+k}^{M,n}}{1-\lambda_{t+k}^{M,n}}}M_{t+k}^{n}, \tag{C.154}$$

and the Calvo price setting contract:

$$P_{t+k}^{M,n}(i)=\begin{cases}\overline{\Pi}_{t+k}^{M,n}P_{t+k-1}^{M,n}(i) & \text{with probability } \xi_{m,n}\\ P_{t+k,opt}^{M,n}(i) & \text{with probability } (1-\xi_{m,n}).\end{cases} \tag{C.155}$$

The FOC of $P_{t,opt}^{M,n}(i)$ is:

$$\begin{aligned}&E_t\{M_{t|t}^{n}(i)+P_{t,opt}^{M,n}(i)\frac{\partial M_{t|t}^{n}(i)}{\partial P_{t,opt}^{M,n}(i)}-MC_{t+k}^{M,n}\frac{\partial M_{t|t}^{n}(i)}{\partial P_{t,opt}^{M,n}(i)}\\&+\xi_{m,n}\Lambda_{t,t+1}\left[\frac{\partial P_{t+1|t}^{M,n}(i)}{\partial P_{t,opt}^{M,n}(i)}M_{t+1|t}^{n}(i)+P_{t+1|t}^{M,n}(i)\frac{\partial M_{t+1|t}^{n}(i)}{\partial P_{t+1|t}^{M,n}(i)}\frac{\partial P_{t+1|t}^{M,n}(i)}{\partial P_{t,opt}^{M,n}(i)}-MC_{t+1}^{M,n}\frac{\partial M_{t+1|t}^{n}(i)}{\partial P_{t+1|t}^{M,n}(i)}\frac{\partial P_{t+1|t}^{M,n}(i)}{\partial P_{t,opt}^{M,n}(i)}\right]\\&+\left(\xi_{m,n}\right)^{2}\Lambda_{t,t+2}\\&\left[\frac{\partial P_{t+2|t}^{M,n}(i)}{\partial P_{t,opt}^{M,n}(i)}M_{t+2|t}^{n}(i)+P_{t+2|t}^{M,n}(i)\frac{\partial M_{t+2|t}^{n}(i)}{\partial P_{t+2|t}^{M,n}(i)}\frac{\partial P_{t+2|t}^{M,n}(i)}{\partial P_{t,opt}^{M,n}(i)}-MC_{t+2}^{M,n}\frac{\partial M_{t+2|t}^{n}(i)}{\partial P_{t+2|t}^{M,n}(i)}\frac{\partial P_{t+2|t}^{M,n}(i)}{\partial P_{t,opt}^{M,n}(i)}\right]\\&+\ldots\}=0.\end{aligned} \tag{C.156}$$

Recall, we have the following definition: $P_{t+k|t}^{M,n}(i)=P_{t,opt}^{M,n}(i)\overline{\Pi}_{t+1}^{M,n}\overline{\Pi}_{t+2}^{M,n}\ldots\overline{\Pi}_{t+k}^{M,n}$. Hence, the partial derivative of $P_{t+k|t}^{M,n}(i)$ with respect to $P_{t,opt}^{M,n}(i)$ is:

$$\frac{\partial P_{t+k|t}^{M,n}(i)}{\partial P_{t,opt}^{M,n}(i)}=\overline{\Pi}_{t+1}^{M,n}\overline{\Pi}_{t+2}^{M,n}\ldots\overline{\Pi}_{t+k}^{M,n}. \tag{C.157}$$

Using Equation (C.157), Equation (C.156) can be rewritten as:

$$\begin{aligned}&E_t\{M_{t|t}^{n}(i)+P_{t,opt}^{M,n}(i)\frac{\partial M_{t|t}^{n}(i)}{\partial P_{t,opt}^{M,n}(i)}-MC_{t}^{M,n}\frac{\partial M_{t|t}^{n}(i)}{\partial P_{t,opt}^{M,n}(i)}\\&+\xi_{m,n}\Lambda_{t,t+1}\left[\overline{\Pi}_{t+1}^{M,n}M_{t+1|t}^{n}(i)+P_{t+1|t}^{M,n}(i)\overline{\Pi}_{t+1}^{M,n}\frac{\partial M_{t+1|t}^{M,n}(i)}{\partial P_{t+1|t}^{M,n}(i)}-MC_{t+1}^{M,n}\overline{\Pi}_{t+1}^{M,n}\frac{\partial M_{t+1|t}^{n}(i)}{\partial P_{t+1|t}^{M,n}(i)}\right]\\&+\left(\xi_{m,n}\right)^{2}\Lambda_{t,t+2}\\&\left[\overline{\Pi}_{t+1}^{M,n}\overline{\Pi}_{t+2}^{M,n}M_{t+2|t}^{n}(i)+P_{t+2|t}^{M,n}(i)\overline{\Pi}_{t+1}^{M,n}\overline{\Pi}_{t+2}^{M,n}\frac{\partial M_{t+2|t}^{n}(i)}{\partial P_{t+2|t}^{M,n}(i)}-MC_{t+2}^{M,n}\overline{\Pi}_{t+1}^{M,n}\overline{\Pi}_{t+2}^{M,n}\frac{\partial M_{t+2|t}^{n}(i)}{\partial P_{t+2|t}^{M,n}(i)}\right]\\&+\ldots\}=0.\end{aligned}$$

We rearrange the above equation, and we can obtain the following equation:

$$\begin{aligned}&E_t\{\frac{\partial M_{t|t}^{n}(i)}{\partial P_{t,opt}^{M,n}(i)}\left[P_{t,opt}^{M,n}(i)\left(\frac{M_{t|t}^{n}(i)}{P_{t,opt}^{M,n}(i)}\left(\frac{\partial M_{t|t}^{n}(i)}{\partial P_{t,opt}^{M,n}(i)}\right)^{-1}+1\right)-MC_{t}^{M,n}\right]\\&+\xi_{m,n}\Lambda_{t,t+1}\overline{\Pi}_{t+1}^{M,n}\frac{\partial M_{t+1|t}^{n}(i)}{\partial P_{t+1|t}^{M,n}(i)}\left[P_{t+1|t}^{M,n}(i)\left(\frac{M_{t+1|t}^{n}(i)}{P_{t+1|t}^{M,n}(i)}\left(\frac{\partial M_{t+1|t}^{n}(i)}{\partial P_{t+1|t}^{M,n}(i)}\right)^{-1}+1\right)-MC_{t+1}^{M,n}\right]\\&+\left(\xi_{m,n}\right)^{2}\Lambda_{t,t+2}\overline{\Pi}_{t+1}^{M,n}\overline{\Pi}_{t+2}^{M,n}\frac{\partial M_{t+2|t}^{n}(i)}{\partial P_{t+2|t}^{M,n}(i)}\left[P_{t+2|t}^{M,n}(i)\left(\frac{M_{t+2|t}^{n}(i)}{P_{t+2|t}^{M,n}(i)}\left(\frac{\partial M_{t+2|t}^{n}(i)}{\partial P_{t+2|t}^{M,n}(i)}\right)^{-1}+1\right)-MC_{t+2}^{M,n}\right]+\ldots\}=0.\end{aligned} \tag{C.158}$$

Given the demand schedule for imported goods, which is captured by Equation (C.154), we can find the following price elasticity of demand for imported goods:

$$-\frac{\partial M^{n}_{t+k|t}(i)}{\partial P^{M,n}_{t+k|t}(i)}\frac{P^{M,n}_{t+k|t}(i)}{M^{n}_{t+k|t}(i)}=\frac{\lambda^{M,n}_{t+k}}{\lambda^{M,n}_{t+k}-1}. \tag{C.159}$$

Using the result from Equation (C.159), the derivative of $M^{n}_{t+k|t}(i)$ with respect to $P^{M,n}_{t+k|t}(i)$ is:

$$\frac{\partial M^{n}_{t+k|t}(i)}{\partial P^{M,n}_{t+k|t}(i)}=\frac{\lambda^{M,n}_{t+k}}{1-\lambda^{M,n}_{t+k}}\frac{M^{n}_{t+k|t}(i)}{P^{M,n}_{t+k|t}(i)}. \tag{C.160}$$

Using Equation (C.159) and Equation (C.160), we can rewrite Equation (C.158) as follows:

$$\begin{aligned}
&E_t\{\frac{M^{n}_{t}(i)}{P^{M,n}_{t,opt}(i)}\frac{\lambda^{M,n}_{t}}{1-\lambda^{M,n}_{t}}\left[P^{M,n}_{t,opt}(i)\frac{1}{\lambda^{M,n}_{t}}-MC^{M,n}_{t+k}\right]\\
&+\xi_{m,n}\Lambda_{t,t+1}\overline{\Pi}^{M,n}_{t+1}\frac{M^{n}_{t+1|t}(i)}{P^{M,n}_{t+1|t}(i)}\frac{\lambda^{M,n}_{t+1}}{1-\lambda^{M,n}_{t+1}}\left[P^{M,n}_{t+1|t}(i)\frac{1}{\lambda^{M,n}_{t+1}}-MC^{M,n}_{t+1}\right]\\
&+(\xi_{m,n})^2\Lambda_{t,t+2}\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\frac{M^{n}_{t+2|t}(i)}{P^{M,n}_{t+2|t}(i)}\frac{\lambda^{M,n}_{t+2}}{1-\lambda^{M,n}_{t+2}}\left[P^{M,n}_{t+2|t}(i)\frac{1}{\lambda^{M,n}_{t+2}}-MC^{M,n}_{t+2}\right]+\ldots\}=0.
\end{aligned}$$

We use the following definition: $P^{M,n}_{t+k|t}(i)=P^{M,n}_{t,opt}(i)\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\ldots\overline{\Pi}^{M,n}_{t+k}$. We multiply both sides of the above equation by $P^{M,n}_{t,opt}(i)$ and -1. We can obtain the following equation:

$$\begin{aligned}
&E_t\{\frac{M^{n}_{t|t}(i)}{\lambda^{M,n}_{t}-1}\left[P^{M,n}_{t,opt}(i)-\lambda^{M,n}_{t}MC^{M,n}_{t}\right]\\
&+\xi_{m,n}\Lambda_{t,t+1}\frac{M^{n}_{t+1|t}(i)}{\lambda^{M,n}_{t+1}-1}\left[P^{M,n}_{t+1|t}(i)-\lambda^{M,n}_{t+1}MC^{M,n}_{t+1}\right]\\
&+(\xi_{m,n})^2\Lambda_{t,t+2}\frac{M^{n}_{t+2|t}(i)}{\lambda^{M,n}_{t+2}-1}\left[P^{M,n}_{t+2|t}(i)-\lambda^{M,n}_{t+2}MC^{M,n}_{t+2}\right]+\ldots\}=0.
\end{aligned}$$

We can drop the subscript i from the above equation because when firms can reset their price, they will set the same price. As a result, all firms that reset their price in period t will choose the same quantity of output. We rewrite the above equation, and the optimal price of imported goods equation can be expressed as:

$$E_t\sum_{k=0}^{\infty}(\xi_{m,n})^k\Lambda_{t,t+k}\frac{M^{n}_{t+k|t}}{\left(\lambda^{M,n}_{t+k}-1\right)}\left[P^{M,n}_{t+k|t}-\lambda^{M,n}_{t+k}MC^{M,n}_{t+k}\right]=0. \tag{C.161}$$

Equation (C.161), which is the non-stationarized version of the optimal price of imported goods, is the same as Equation (37) in Section 2.4.2.

We can rewrite the above equation in *per capita* terms, so we let m^{n}_{t} denote *per capita* output of import firms of type n who reset their price in period t. The optimal price of imported goods equation can be written as:

$$E_t\sum_{k=0}^{\infty}(\xi_{m,n})^k\Lambda_{t,t+k}\frac{m^{n}_{t+k|t}}{\left(\lambda^{M,n}_{t+k}-1\right)}\left[P^{M,n}_{t+k|t}-\lambda^{M,n}_{t+k}MC^{M,n}_{t+k}\right]=0. \tag{C.162}$$

Now, we stationarize Equation (C.162). We use the following definition: $\Lambda_{t,t+k}=\frac{\beta_{t+k}}{\beta_t}\frac{\Omega^{C}_{t+k}}{\Omega^{C}_{t}}\frac{P^{C}_{t}}{P^{C}_{t+k}}$, and we expand Equation (C.162). Thus, we have the following equation:

$$\begin{aligned}
&E_t\{\frac{m^{n}_{t|t}}{(\lambda^{M,n}_{t}-1)}\left[P^{M,n}_{t,opt}-\lambda^{M,n}_{t}MC^{M,n}_{t}\right]\\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^{C}_{t+1}}{\Omega^{C}_{t}}\frac{P^{C}_{t}}{P^{C}_{t+1}}\frac{m^{n}_{t+1|t}}{(\lambda^{M,n}_{t+1}-1)}\left[P^{M,n}_{t+1|t}-\lambda^{M,n}_{t+1}MC^{M,n}_{t+1}\right]\\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^{C}_{t+2}}{\Omega^{C}_{t}}\frac{P^{C}_{t}}{P^{C}_{t+2}}\frac{m^{n}_{t+2|t}}{(\lambda^{M,n}_{t+2}-1)}\left[P^{M,n}_{t+2|t}-\lambda^{M,n}_{t+2}MC^{M,n}_{t+2}\right]+\ldots\}=0.
\end{aligned}$$

We multiply all terms of the above equation by $\frac{1}{P_t}$ and expand. We can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{(\lambda^{M,n}_t-1)}\left[\frac{P^{M,n}_{t,opt}}{P_t}-\frac{\lambda^{M,n}_t MC^{M,n}_t}{P_t}\right] \\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+1}}\frac{m^n_{t+1|t}}{(\lambda^{M,n}_{t+1}-1)}\frac{P_{t+1}}{P_t}\left[\frac{P^{M,n}_{t+1|t}}{P_{t+1}}-\frac{\lambda^{M,n}_{t+1}MC^{M,n}_{t+2}}{P_{t+1}}\right] \\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}}{\Omega^C_t}\frac{P^C_t}{P^C_{t+2}}\frac{m^n_{t+2|t}}{(\lambda^{M,n}_{t+2}-1)}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\left[\frac{P^{M,n}_{t+2|t}}{P_{t+2}}-\frac{\lambda^{M,n}_{t+2}MC^{M,n}_{t+3}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We use the following definition: $P^{M,n}_{t+k|t}=P^{M,n}_{t,opt}\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}\ldots\overline{\Pi}^{M,n}_{t+k}$. We multiply the marginal utility of consumption Ω^C_{t+k} by z^+_{t+k}, and divide imported goods $m^n_{t+k|t}$ by z^+_{t+k}, and expand further to get:

$$
\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{\left(\lambda^{M,n}_t-1\right)z^+_t}\left[\frac{P^{M,n}_{t,opt}}{P_t}-\lambda^{M,n}_t\frac{MC^{M,n}_t}{P_t}\right] \\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{m^n_{t+1|t}}{\left(\lambda^{M,n}_{t+1}-1\right)z^+_{t+1}}\frac{P_{t+1}}{P_t}\left[\frac{\overline{\Pi}^{M,n}_{t+1}P^{M,n}_{t,opt}}{P_{t+1}}-\lambda^{M,n}_{t+1}\frac{MC^{M,n}_{t+1}}{P_{t+1}}\right] \\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{m^n_{t+2|t}}{\left(\lambda^{M,n}_{t+2}-1\right)z^+_{t+2}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}} \\
&\left[\frac{\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}P^{M,n}_{t,opt}}{P_{t+2}}-\lambda^{M,n}_{t+2}\frac{MC^{M,n}_{t+2}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We multiply the second term of the above equation by $\frac{P_t}{P_t}$, whereas we multiply the third term of the above equation by $\frac{P_tP_{t+1}}{P_tP_{t+1}}$. We have the following equation:

$$
\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{\left(\lambda^{M,n}_t-1\right)z^+_t}\left[\frac{P^{M,n}_{t,opt}}{P_t}-\lambda^{M,n}_t\frac{MC^{M,n}_t}{P_t}\right] \\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+1}}\frac{m^n_{t+1|t}}{\left(\lambda^{M,n}_{t+1}-1\right)z^+_{t+1}}\frac{P_{t+1}}{P_t}\left[\frac{P_t}{P_t}\frac{\overline{\Pi}^{M,n}_{t+1}P^{M,n}_{t,opt}}{P_{t+1}}-\lambda^{M,n}_{t+1}\frac{MC^{M,n}_{t+1}}{P_{t+1}}\right] \\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_t z^+_t}\frac{P^C_t}{P^C_{t+2}}\frac{P^C_{t+1}}{P^C_{t+1}}\frac{m^n_{t+2|t}}{\left(\lambda^{M,n}_{t+2}-1\right)z^+_{t+2}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}} \\
&\left[\frac{P_tP_{t+1}}{P_tP_{t+1}}\frac{\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}P^{M,n}_{t,opt}}{P_{t+2}}-\lambda^{M,n}_{t+2}\frac{MC^{M,n}_{t+2}}{P_{t+2}}\right]+\ldots\}=0.
\end{aligned}
$$

We use the following definitions: $p^{M,n}_{t,opt}=\frac{P^{M,n}_{t,opt}}{P_t}$, $\overline{mc}^{M,n}_{F,t+k}=\frac{MC^{M,n}_{t+k}}{P_{t+k}}$, $\Pi_{t+k}=\frac{P_{t+k}}{P_{t+k-1}}$, and $\Pi^C_{t+k}=\frac{P^C_{t+k}}{P^C_{t+k-1}}$. We can obtain the following equation:

$$
\begin{aligned}
&E_t\{\frac{m^n_{t|t}}{\left(\lambda^{M,n}_t-1\right)z^+_t}\left[p^{M,n}_{t,opt}-\lambda^{M,n}_t\overline{mc}^{M,n}_{F,t}\right] \\
&+\xi_{m,n}\frac{\beta_{t+1}}{\beta_t}\frac{\Omega^C_{t+1}z^+_{t+1}}{\Omega^C_t z^+_t}\frac{\Pi_{t+1}}{\Pi^C_{t+1}}\frac{m^n_{t+1|t}}{\left(\lambda^{M,n}_{t+1}-1\right)z^+_{t+1}}\left[\frac{\overline{\Pi}^{M,n}_{t+1}p^{M,n}_{t,opt}}{\Pi_{t+1}}-\lambda^{M,n}_{t+1}\overline{mc}^{M,n}_{F,t+1}\right] \\
&+(\xi_{m,n})^2\frac{\beta_{t+2}}{\beta_t}\frac{\Omega^C_{t+2}z^+_{t+2}}{\Omega^C_t z^+_t}\frac{\Pi_{t+2}\Pi_{t+1}}{\Pi^C_{t+2}\Pi^C_{t+1}}\frac{m^n_{t+2|t}}{\left(\lambda^{M,n}_{t+2}-1\right)z^+_{t+2}} \\
&\left[\frac{\overline{\Pi}^{M,n}_{t+1}\overline{\Pi}^{M,n}_{t+2}p^{M,n}_{t,opt}}{\Pi_{t+2}\Pi_{t+1}}-\lambda^{M,n}_{t+2}\overline{mc}^{M,n}_{F,t+2}\right]+\ldots\}=0.
\end{aligned}
$$

Using the following definitions: $\beta^r_{t+j}=\frac{\beta_{t+j}}{\beta_{t+j-1}}$, $\overline{\Omega}^C_{t+k}=\Omega^C_{t+k}z^+_{t+k}$ and $\overline{m}^n_{t+k}=\frac{m^n_{t+k}}{z^+_t}$, the stationarized version of optimal price of imported goods equation can be written as:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,n})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^C_{t+j}} \right) \frac{\overline{m}^n_{t+k|t}}{\left(\lambda^{M,n}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,n}_{t+j}}{\Pi_{t+j}} \right) p^{M,n}_{t,opt} - \lambda^{M,n}_{t+k} \overline{mc}^{M,n}_{F,t+k} \right] = 0. \quad \text{(C.163)}$$

Equation (C.163), which is the stationarized version of optimal price of imported goods. Recall that, $n\epsilon\{Cxe, I, X, Ce\}$ represents different types of import firms in the Swedish economy.

Replacing the index n with C, we have the following optimal price for import firms specializing in consumption goods:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,C})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^C_{t+j}} \right) \frac{\overline{m}^C_{t+k|t}}{\left(\lambda^{M,C}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,C}_{t+j}}{\Pi_{t+j}} \right) p^{M,C}_{t,opt} - \lambda^{M,C}_{t+k} \overline{mc}^{M,n}_{F,t+k} \right] = 0. \quad \text{(C.164)}$$

Equation (C.164), which is the stationarized version of optimal price for import firms specializing in consumption goods is the same as Equation (A.50a).

Replacing the index n with I, we have the following optimal price for import firms specializing in investment goods:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,I})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^C_{t+j}} \right) \frac{\overline{m}^I_{t+k|t}}{\left(\lambda^{M,I}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,I}_{t+j}}{\Pi_{t+j}} \right) p^{M,I}_{t,opt} - \lambda^{M,I}_{t+k} \overline{mc}^{M,n}_{F,t+k} \right] = 0. \quad \text{(C.165)}$$

Equation (C.166), which is the stationarized version of optimal price for import firms specializing in investment goods is the same as Equation (A.53a).

Replacing the index n with X, we have the following optimal price for import firms specializing in export goods:

$$E_t \sum_{k=0}^{\infty} (\xi_{m,X})^k \left(\prod_{j=1}^{k} \beta^r_{t+j} \right) \frac{\overline{\Omega}^C_{t+k}}{\overline{\Omega}^C_t} \left(\prod_{j=1}^{k} \frac{\Pi_{t+j}}{\Pi^C_{t+j}} \right) \frac{\overline{m}^X_{t+k|t}}{\left(\lambda^{M,X}_{t+k} - 1\right)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}^{M,X}_{t+j}}{\Pi_{t+j}} \right) p^{M,X}_{t,opt} - \lambda^{M,X}_{t+k} \overline{mc}^{M,n}_{F,t+k} \right] = 0. \quad \text{(C.166)}$$

Equation (C.166) which is the stationarized version of optimal price for import firms specializing in export goods is the same as Equation (A.56a).
Finally, note that the stationarized marginal costs of the firms can be written as

$$\overline{mc}^{M,n}_t = \frac{S_t P_{F,t}}{P_t} = \frac{Q_t p^C_t}{p^C_{F,t}}, n \in \{Cxe, I, X\} \quad \text{(C.167)}$$

$$\overline{mc}^{M,n}_t = \frac{S_t P^{C,e}_{F,t}}{P_t} = \frac{Q_t p^C_t}{p^C_{F,t}} p^{C,e}_{F,t}, n \in \{Ce\} \quad \text{(C.168)}$$

Equation (C.168), which is the stationarized marginal cost for the energy importer, is the same as Equation (A.62a).

C.7 Fiscal authority

Given the fiscal authority budget constraint described in section 2.5, we can derive the stationarized version of the government budget constraint. The government budget constraint is given by:

$$\tau^C_t P^C_t C^{agg}_t + (\tau^{SSC}_t + \tau^W_t) W_t N_t + \Upsilon^K_t + B^n_t + T_t = \left(\alpha_B + (R^B_{t-1} - 1)\right) B_t + \tau^I_t \frac{P^I_t}{\gamma_t} I_t + P_t G_t + P_t \frac{I^G_t}{\gamma_t} + (1 - \tau^{TR}_t) TR^{agg}_t. \quad \text{(C.169)}$$

We stationarize the above equation by dividing the above equation by $z^+_t P_t$:

$$\tau^C_t \frac{P^C_t C^{agg}_t}{z^+_t P_t} + (\tau^{SSC}_t + \tau^W_t) \frac{W_t N_t}{z^+_t P_t} + \frac{\Upsilon^K_t}{z^+_t P_t} + \frac{B^n_t}{z^+_t P_t} + \frac{T_t}{z^+_t P_t} = \left(\alpha_B + (R^B_{t-1} - 1)\right) \frac{B_t}{z^+_t P_t} + \tau^I_t \frac{P^I_t}{z^+_t P_t \gamma_t} I_t + \frac{G_t}{z^+_t} + \frac{I^G_t}{\gamma_t z^+_t} + (1 - \tau^{TR}_t) \frac{TR^{agg}_t}{z^+_t P_t}.$$

In the Swedish economy, where the size of the population is normalized to unity, the distinction between aggregate and *per capita* variables are trivial. Nonetheless, we can express the above equation in terms of *per capita*. We denote $\bar{c}_t^{agg}$ as the stationarized aggregate consumption in *per capita*, $\bar{g}_t$ is the stationarized government consumption in *per capita*, $\bar{t}_t$ is lump-sum tax in *per capita*, and $\overline{tr}_t^{agg}$ is aggregate transfers in *per capita*. We use the following definitions: $\bar{c}_t^{agg} = \frac{c_t^{agg}}{z_t^+}$, $\bar{w}_t = \frac{W_t}{z_t^+ P_t^C}$, $p_t^C = \frac{P_t^C}{P_t}$, $\bar{t}_t = \frac{T_t}{z_t^+ P_t}$, $\bar{\Upsilon}_t^K = \frac{\Upsilon_t^K}{z_t^+ P_t}$, $\bar{b}_t^n = \frac{B_t^n}{z_t^+ P_t}$, $\bar{b}_t = \frac{B_t}{z_{t-1}^+ P_{t-1}}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, $\mu_{\gamma,t} = \frac{\gamma_t^+}{\gamma_{t-1}^+}$, $p_t^I = \frac{P_t^I}{P_t}$, $\bar{I}_t = \frac{I_t}{z_t^+ \gamma_t}$, $\bar{g}_t = \frac{g_t}{z_t^+}$, $\bar{I}_t^G = \frac{I_t^G}{\gamma_t z_t^+}$, and $\overline{tr}_t^{agg} = \frac{tr_t^{agg}}{z_t^+ P_t}$.[79]

The above government budget constraint can be written as:

$$\tau_t^C p_t^C \bar{c}_t^{agg} + (\tau_t^{SSC} + \tau_t^W) p_t^C \bar{w}_t n_t + \bar{\Upsilon}_t^K + \bar{b}_t^n + \bar{t}_t = \Big(\alpha_B + (R_{t-1}^B - 1)\Big) \frac{\bar{b}_t}{\mu_{z^+,t} \Pi_t} + \bar{g}_t + \tau_t^I p_t^I \bar{I}_t + \bar{I}_t^G + (1 - \tau_t^{TR}) \bar{tr}_t^{agg}. \tag{C.170}$$

Note that equation (C.170) is the same as equation (A.70a) in Section A.4.

Capital income tax revenues are given by:

$$\Upsilon_t^K = \tau_t^K \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t} a(u_t) K_t \right) - \iota^K \tau_t^K \delta P_{t-1}^K K_t. \tag{C.171}$$

We stationarize the above equation by dividing the above equation by $z_t^+ P_t$:

$$\frac{\Upsilon_t^K}{z_t^+ P_t} = \tau_t^K \frac{1}{z_t^+ P_t} \left(R_t^K u_t K_t - \frac{P_t^I}{\gamma_t z_t^+ P_t} a(u_t) K_t \right) - \iota^K \tau_t^K \delta P_{t-1}^K K_t \frac{1}{z_t^+ P_t}.$$

Using the following definitions: $\bar{\Upsilon}_t^K = \frac{\Upsilon_t^K}{z_t^+ P_t}$, $\bar{K}_t = \frac{K_t}{z_{t-1}(\gamma_{t-1})^{\frac{1}{1-\alpha}}}$, $r_{t+1}^K = \frac{\gamma_{t+1} R_{t+1}^K}{P_{t+1}}$, $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, $\mu_{\gamma,t} = \frac{\gamma_t^+}{\gamma_{t-1}^+}$, $p_t^I = \frac{P_t^I}{P_t}$, $p_t^K = \frac{\gamma_t P_t^K}{P_t}$, the above equation becomes:

$$\bar{\Upsilon}_t^K = \tau_t^K \left(\frac{1}{\mu_{z^+,t} \mu_{\gamma,t}} r_t^K u_t \bar{K}_t - \frac{1}{\mu_{z^+,t} \mu_{\gamma,t}} p_t^I a(u_t) \bar{K}_t \right) - \iota^K \tau_t^K \delta \frac{p_{t-1}^K}{\mu_{z^+,t} \Pi_t} \bar{K}_t.$$

We express the above equation in *per capita* terms as follows:

$$\bar{\Upsilon}_t^K = \tau_t^K \left(\frac{1}{\mu_{z^+,t} \mu_{\gamma,t}} r_t^K u_t \bar{k}_t - \frac{1}{\mu_{z^+,t} \mu_{\gamma,t}} p_t^I a(u_t) \bar{k}_t \right) - \iota^K \tau_t^K \delta \frac{p_{t-1}^K}{\mu_{z^+,t} \Pi_t} \bar{k}_t,$$

$$\bar{\Upsilon}_t^K = \frac{\bar{k}_t}{\mu_{z^+,t} \mu_{\gamma,t}} \tau_t^K \left(r_t^K u_t - p_t^I a(u_t) - \iota^K \delta \frac{\mu_{\gamma,t} p_{t-1}^K}{\Pi_t} \right). \tag{C.172}$$

Equation (C.172) is the same as Equation (A.73a) in Section A.4.

From Section 2.5, the government surplus is given by:

$$SURP_t = \alpha_B B_t - B_t^n.$$

We stationarize the above equation by dividing the above equation by $z_t^+ P_t$:

$$\frac{SURP_t}{P_t z_t^+} = \alpha_B \frac{B_t}{P_t z_t^+} - \frac{B_t^n}{P_t z_t^+}.$$

Using the following definitions: $\overline{surp}_t = \frac{SURP_t}{P_t z_t^+}$, $\bar{b}_t = \frac{B_t}{z_{t-1}^+ P_{t-1}}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, and $\bar{b}_t^n = \frac{B_t^n}{z_t^+ P_t}$, the above equation can be written as:

$$\overline{surp}_t = \alpha_B \frac{\bar{b}_t}{\mu_{z^+t} \Pi_t} - \bar{b}_t^n. \tag{C.173}$$

Note that equation (C.173) is the same as equation (A.76a) in Section A.4.

[79] Two exceptions to the *per capita* notation are private and government investment: because we use i_t to denote the net nominal interest rate, $\bar{I}_t$ denotes both the stationarized level of aggregate investment and the stationarized level of aggregate investment per inhabitant in the Swedish economy. $\bar{I}_t^G$ is the stationarized level of government investment and the stationarized level of government investment *per capita*.

C.7.1 The structural surplus

The structural surplus is defined as the difference between the structural primary revenue, $Stprev_t$ and the structural primary expenditure, $Stpexp_t$, net of the interest payments on the current debt:

$$Stsurp_t = Stprev_t - Stpexp_t - (R^B_{t-1} - 1)B_t \tag{C.174}$$

where the structural primary expenditure is given by the cyclically adjusted government expenditure:

$$\frac{Stpexp_t}{P_t} = \left(\frac{TR^{agg}_t}{P_t} - F_{tr,un}Y\hat{un}_t\right) + \left(\frac{I^G_t}{\gamma_t} - \mathcal{F}_{IG,y}\frac{I^G(Y_t - Y)}{Y}\right) + \left(G_t - \mathcal{F}_{g,y}\frac{G(Y_t - Y)}{Y}\right) + \tau^I_t\frac{P^I}{\gamma P_t}I \tag{C.175}$$

and the structural primary revenues are given by the structural tax bases times the tax rates:

$$Stprev_t = \tau^C_t P^C C^{agg} + \left(\tau^{SSC}_t + \tau^W_t\right)WN + \tau^K_t K\left(R^K - \iota^K\delta\frac{P^K}{\Pi}\right) + \tau^{TR}_t(TR^{agg}_t - F_{tr,un}P_t\hat{un}_t) + T \tag{C.176}$$

where the variables without any time notation are the steady-state values of the stationarized per-capita equivalents of the variables.

The structural surplus and its right-hand-side variables can be stationarized by dividing all variables with $P_t z^+_t$ except for B_t, which is stationarized via $\bar{b}_t = B_t/(P_{t-1}z^+_{t-1})$, which can be written as $\bar{b} = B_t/(P_{t-1}z^+_{t-1}) = B_t\Pi_t\mu_{z^+,t}/(P_t z^+_t)$. Hence we can write the stationarized structural surplus as

$$\overline{Stsurp}_t = \overline{Stprev}_t - \overline{Stpexp}_t - \frac{(R^B_{t-1} - 1)}{\Pi_t\mu_{z^+,t}}\bar{b}_t \tag{C.177}$$

Equation C.177 is equivalent to equation A.83a in Section A.4.

The equation for structural primary expenditures (equation C.176) is stationarized by dividing through the equation by $P_t z^+_t$ to get

$$\frac{Stpexp_t}{P_t z^+_t} = \left(\frac{TR^{agg}_t}{P_t z^+_t} - \frac{F_{tr,un}Y\hat{un}_t}{z^+_t}\right) + \left(\frac{I^G_t}{\gamma_t z^+_t} - \mathcal{F}_{IG,y}\frac{I^G(Y_t - Y)}{Y z^+_t}\right) + \left(\frac{G_t}{z^+_t} - \mathcal{F}_{g,y}\frac{G(Y_t - Y)}{Y z^+_t}\right) + \tau^I_t\frac{P^I}{\gamma P_t z^+_t}I$$

Define $\bar{g}_t = \frac{g_t}{z^+_t}$, $\bar{I}^G_t = \frac{I^G_t}{\gamma_t z^+_t}$, and $\overline{tr}^{agg}_t = \frac{tr^{agg}_t}{z^+_t P_t}$

$$\overline{Stpexp}_t = (\bar{tr}^{agg}_t - \mathcal{F}_{tr,un}\bar{y}\hat{un}_t) + \left(\bar{I}^G - \mathcal{F}_{IG,y}\bar{I}^G\frac{(\bar{y}_t - \bar{y})}{\bar{y}}\right) + \left(\bar{g} - \mathcal{F}_{g,y}\bar{g}\frac{(\bar{y}_t - \bar{y})}{\bar{y}}\right) + \tau^I_t p^I\bar{I} \tag{C.178}$$

Equation C.178 is equivalent to equation A.84a in Section A.4.

The equation for structural primary revenues (equation C.175) is stationarized by dividing through the equation by $P_t z^+_t$ to get:

$$\frac{Stprev_t}{P_t z^+_t} = \frac{\tau^C_t P^C C^{agg}}{P_t z^+_t} + \frac{\left(\tau^{SSC}_t + \tau^W_t\right)WN}{P_t z^+_t} + \frac{\tau^K_t K(R^K - \iota^K\delta\frac{P^K}{\Pi})}{P_t z^+_t} + \frac{\tau^{TR}_t(TR^{agg}_t - F_{tr,un}P_t\hat{un}_t)}{P_t z^+_t} + \frac{T}{P_t z^+_t}$$

by using the definitions $\bar{c}^{agg}_t = \frac{c^{agg}_t}{z^+_t}$, $p^C_t = \frac{P^C_t}{P_t}$, $\overline{w}_t = \frac{W_t}{z^+_t P^C_t}$, $\bar{t}_t = \frac{t_t}{z^+_t P_t}$, $\overline{\Upsilon}^K_t = \frac{\Upsilon^K_t}{z^+_t P_t}$, $\overline{tr}^{agg}_t = \frac{tr^{agg}_t}{z^+_t P_t}$, $r^K_{t+1} = \frac{\gamma_{t+1}R^K_{t+1}}{P_{t+1}}$, $\mu_{z^+,t} = \frac{z^+_t}{z^+_{t-1}}$, $\mu_{\gamma,t} = \frac{\gamma^+_t}{\gamma^+_{t-1}}$, $p^K_t = \frac{\gamma_t P^K_t}{P_t}$,

$$\overline{Stprev}_t = \tau^C_t p^C\bar{c}^{agg} + (\tau^{SSC}_t + \tau^W_t)\overline{w}n + \tau^K_t\frac{\overline{k}}{\mu_{z^+}\mu_\gamma}\left(r^K - \iota^K\delta\frac{\mu_\gamma p^K}{\Pi}\right) + \tau^{TR}_t(\overline{tr}^{agg}_t - \mathcal{F}_{tr,un}\bar{y}\hat{un}_t) + \bar{t} \tag{C.179}$$

Equation C.179 is equivalent to equation A.85a in Section A.4.

D Technical appendix: Foreign economy

In this technical appendix, first we present the optimization problems for households and firms in the Foreign economy. Second, we present the key equilibrium conditions and model equations for the Foreign economy. We denote ω as the size of Foreign economy. We denote the subscript f as the individual household in the Foreign economy, and we denote the subscript j as the individual firm in the Foreign economy. We use the subscript F to denote the aggregate variables and the economy-wide average variables for the Foreign economy.

D.1 Foreign: Household sector

In this section, first we present the optimization problem of households in the Foreign economy. Second, we present the first-order conditions (FOCs) for households in the Foreign economy. The optimization problem of households in the Foreign economy are similar to those of the Swedish economy. However, households in the Foreign economy do not have an access to the market for bonds denominated in the Swedish currency. Moreover, there are no Non-Ricardian consumers and no fiscal sector. Households in the Foreign economy can only buy bonds that are denominated in the currency of Foreign. We let $\theta^b_{f,t}$ denote the Lagrangian multiplier associated with the budget constraint and $\theta^k_{f,t}$ denote Lagrangian multiplier associated with capital accumulation equation for the household f .

The utility function of individual household f is defined as:

$$E_0 \sum_{t=0}^{\infty} \beta_{F,t} \left[u(C_{f,t} - \rho_{F,h} C_{F,t-1}) - \zeta^n_{F,t} \nu(N_{f,t}) \right]. \tag{D.1}$$

where $\rho_{F,h}$ is the consumption habit formation parameter.
The individual household f chooses consumption $C_{f,t}$, physical capital $K_{f,t+1}$, investment $I_{f,t}$, capital utilization $u_{f,t}$, the change in capital stock by trading in the market $\triangle^K_{f,t}$, domestic nominal bonds that are denominated in the Foreign currency $B^{FF}_{f,t+1}$ and the nominal wage $W_{f,t}$ to maximize its expected utility subject to,

the budget constraint:

$$\begin{aligned} P^C_{F,t} C_{f,t} + \frac{P^I_{F,t}}{\gamma_t} I_{f,t} + P^K_{F,t} \triangle^K_{f,t} + \frac{B^{FF}_{f,t+1}}{R_{F,t}\zeta_{F,t}} &= (1-\tau^w_F) W_{f,t} N_{f,t} + R^K_{F,t} u_{f,t} K_{f,t} - \frac{P^I_{F,t}}{\gamma_t} a(u_{f,t}) K_{f,t} \\ &+ B^{FF}_{f,t} + \Xi_{B^{FF},t} + \Psi_{f,t} + TR_{f,t}, \end{aligned} \tag{D.2}$$

the labor demand schedule:

$$N_{f,t} = \frac{1}{\omega} \left(\frac{W_{f,t}}{W_{F,t}} \right)^{-\varepsilon^F_w} N_{F,t}, \tag{D.3}$$

the capital accumulation:

$$K_{f,t+1} = (1-\delta) K_{f,t} + \Upsilon_{F,t} F\left(I_{f,t}, I_{f,t-1}\right) + \triangle^K_{f,t}, \tag{D.4}$$

the Calvo wage contract:

$$W_{f,t+k} = \begin{cases} \overline{\Pi}^W_{F,t+k} W_{f,t+k-1} & \text{with probability } \xi^F_w \\ W^{\text{opt}}_{f,t+k} & \text{with probability } (1-\xi^F_w). \end{cases} \tag{D.5}$$

We make the use of the following definition: $\beta^r_{F,t+1} = \frac{\beta_{F,t+1}}{\beta_{F,t}}$, and we follow the similar steps to those in Section C.1.2, we can obtain the FOC for $C_{f,t}$, $B^{FF}_{f,t+1}$, $K_{f,t+1}$, $I_{f,t}$, $u_{f,t}$ and $\triangle^K_{f,t}$.

The FOC for $C_{f,t}$ is:

$$\theta^b_{f,t} P^C_{F,t} = \Omega^C_{f,t}. \tag{D.6}$$

Equation (D.6) is the same as Equation (73) in Section 2.6.1.

The FOC for $B^{FF}_{f,t+1}$ is:

$$\theta^b_{f,t} P^C_{F,t} = E_t \left[\beta^r_{F,t+1} \theta^b_{f,t+1} P^C_{F,t} R_{F,t} \zeta_{F,t} \right]. \tag{D.7}$$

Equation (D.7) is the same as Equation (74) in Section 2.6.1.

The FOC for $K_{f,t+1}$ is:

$$\theta^k_{f,t} = E_t \beta^r_{F,t+1} \left[\theta^b_{f,t+1} \left(R^K_{F,t+1} u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}} a(u_{f,t+1}) \right) + \theta^k_{f,t+1}(1-\delta_F) \right]. \tag{D.8}$$

Equation (D.8) is the same as Equation (75) in Section 2.6.1.

The FOC for $I_{f,t}$ is:

$$\theta^b_{f,t}\frac{P^I_{F,t}}{\gamma_{F,t}} = \theta^k_{f,t}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\theta^k_{f,t+1}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right]. \tag{D.9}$$

Equation (D.9) is the same as Equation (76) in Section 2.6.1.

The FOC for $u_{f,t}$ is:

$$R^K_{F,t}K_{f,t} = \frac{P^I_{F,t}}{\gamma_t}a'(u_{f,t})K_{f,t}. \tag{D.10}$$

Equation (D.10) is the same as Equation (77) in Section 2.6.1.

The FOC for $\triangle^K_{f,t}$. is:

$$\theta^b_{f,t}P^K_{F,t} = \theta^k_{f,t}. \tag{D.11}$$

Equation (D.11) is the same as Equation (78) in Section 2.6.1.

D.1.1 Foreign: Consumption Euler equation

This section presents the stationarized version of consumption Euler equation. We use Equation (D.6) and Equation (D.7), and apply the following definitions: $p^C_{F,t} = \frac{P^C_{F,t}}{P_{F,t}}$, and $\Pi^C_{F,t+1} = \frac{P^C_{F,t+1}}{P^C_{F,t}}$ to find the non-stationarized version of consumption Euler equation for the Foreign economy.

Following the similar steps that are used to derive the non-stationarized version of consumption Euler equation for the Swedish economy in Section C.1.3, we can obtain the following non-stationarized version of consumption Euler equation for the Foreign economy:

$$\Omega^C_{F,t} = E_t\left[\beta^r_{F,t+1}\frac{R_{F,t}\zeta_{F,t}}{\Pi^C_{F,t+1}}\Omega^C_{F,t+1}\right]. \tag{D.12}$$

We use the following definitions: $\mu_{z^+_F,t+1} = \frac{z^+_{F,t+1}}{z^+_{F,t}}$, $\overline{\Omega}^C_{F,t} = z^+_{F,t}\Omega^C_{F,t}$. To find the stationarized version of consumption Euler equation for the Foreign economy. Following the similar steps as in Section C.1.3, we can find the stationarized version of consumption Euler equation for the Foreign economy. Thus, Equation (D.12) becomes:

$$\overline{\Omega}^C_{F,t} = R_{F,t}\zeta_{F,t}E_t\left[\beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\mu_{z^+_F,t+1}\Pi^C_{F,t+1}}\right]. \tag{D.13}$$

Equation (D.13), which represents the stationarized version of consumption Euler equation for the Foreign economy, is the same as Equation (A.104a).

D.1.2 Foreign: Marginal utility of consumption

In this section, we present the stationarized version of marginal utility of consumption for the Foreign economy equation.

The Foreign utility function is the same as the Swedish utility function, and the functional form of the Foreign utility function is:

$$u(C_{f,t} - \rho_{F,h}C_{F,t-1}) = ln(C_{f,t} - \rho_{F,h}C_{F,t-1}).$$

Note that we abstract from government consumption in the foreign economy and thereby we can abstract from potential non-separability between foreign private consumption and government consumption.
We use the above utility functional form and follow the similar steps that are used to derive the non-stationarized version of the Swedish marginal utility of consumption equation in Section C.1.4. Thus, we can obtain the following non-stationarized version of the Foreign marginal utility of consumption:

$$\Omega^C_{F,t} = \frac{1}{(C_{F,t} - \rho_{F,h}C_{F,t-1})}. \tag{D.14}$$

Equation (D.14) can be written in *per capita* terms by applying the following definition: $c_{F,t} = \frac{C_{F,t}}{\omega}$. We define

$c_{F,t}$ as Foreign consumption *per capita* and ω is the size of the Foreign economy. We stationarize Equation (D.14) by using the following definitions: $\mu_{z_F^+,t+1} = \frac{z^+_{F,t+1}}{z^+_{F,t}}$, $\overline{\Omega}^C_{F,t} = z^+_{F,t}\Omega^C_{F,t}$, and $\overline{c}_{F,t} = \frac{c_{F,t}}{z^+_{F,t}}$. Equation (D.14) becomes:

$$\overline{\Omega}^C_{F,t} = \frac{1}{\left(\overline{c}_{F,t} - \rho_{F,h}\frac{1}{\mu_{z_F^+,t}}\overline{c}_{F,t-1}\right)}. \tag{D.15}$$

Equation (D.15), which represents the stationarized version of the Foreign marginal utility of consumption, is the same as Equation (A.105a).

D.1.3 Foreign: Capital utilization and household purchase of installed capital

This section derives stationarized capital utilization decision equation and the household purchases of installed capital equation respectively.

First, we derive the capital utilization decision equation. Recall, Equation (D.10), which shows the FOC for $u_{f,t}$, is written as:

$$R^K_{F,t}K_{f,t} = \frac{P^I_{F,t}}{\gamma_t}a'(u_{f,t})K_{f,t}.$$

Using the following definitions: $r^K_{F,t} = \frac{\gamma_t R^K_{F,t}}{P_{F,t}}$, and $p^I_{F,t} = \frac{P^I_{F,t}}{P_{F,t}}$, the above equation can be rewritten as follows:

$$\frac{\gamma_t R^K_{F,t}}{P_{F,t}} = \frac{P^I_{F,t}}{P_{F,t}}a'(u_{f,t}),$$

$$r^K_{F,t} = p^I_{F,t}a'(u_{f,t}).$$

Note that the optimal rate of utilization is a function of the two aggregate relative prices $r^K_{F,t}$ and $p^I_{F,t}$. All households in Foreign will then choose the same utilization rate, and the subscript f may be dropped from the above equation. Thus, we have the following capital utilization decision equation:

$$r^K_{F,t} = p^I_{F,t}a'(u_{F,t}). \tag{D.16}$$

Equation (D.16), which captures the capital utilization decision, is the same as Equation (A.106a).

Next, we derive the household purchases of installed capital equation (A.107a). Recall, Equation (D.8), which represents the FOC for $K_{f,t+1}$, is expressed as:

$$\theta^k_{f,t} = E_t\beta^r_{F,t+1}\left[\theta^b_{f,t+1}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right) + \theta^k_{f,t+1}(1-\delta_F)\right].$$

Using Equation (D.11) that shows $\theta^b_{f,t}P^K_{F,t} = \theta^k_{f,t}$, we can rewrite the above equation as:

$$\theta^b_{f,t}P^K_{F,t} = E_t\beta^r_{F,t+1}\left[\theta^b_{f,t+1}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right) + \theta^b_{f,t+1}P^K_{F,t+1}(1-\delta_F)\right].$$

We use Equation (D.6) that shows $\theta^b_{f,t}P^C_{F,t} = \Omega^C_{f,t}$ and use the following definition: $\Pi^C_{F,t+1} = \frac{P^C_{F,t+1}}{P^C_{F,t}}$. Thus, we can rewrite the above equation as follows:

$$\begin{aligned} P^C_{F,t}\theta^b_{f,t}P^K_{F,t} = {} & E_t\beta^r_{F,t+1}\left[\theta^b_{f,t+1}P^C_{F,t+1}\frac{1}{\Pi^C_{F,t+1}}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right)\right] \\ & + E_t\beta^r_{F,t+1}\left[\theta^b_{f,t+1}P^C_{F,t+1}\frac{1}{\Pi^C_{F,t+1}}P^K_{F,t+1}(1-\delta_F)\right], \end{aligned}$$

and

$$\Omega^C_{f,t}P^K_{F,t} = E_t\beta^r_{F,t+1}\left[\Omega^C_{f,t+1}\frac{1}{\Pi^C_{F,t+1}}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right) + \Omega^C_{f,t+1}\frac{1}{\Pi^C_{F,t+1}}P^K_{F,t+1}(1-\delta_F)\right].$$

We multiply both sides of the above equation by $\frac{\gamma_t}{P_{F,t}}$, and then we rewrite the above equation as follows:

$$\begin{aligned}\frac{\gamma_t P^K_{F,t}}{P_{F,t}} &= E_t \beta^r_{F,t+1}\left[\frac{\Omega^C_{f,t+1}}{\Omega^C_{f,t}}\frac{1}{\Pi^C_{F,t+1}}\frac{\gamma_t}{P_{F,t}}\left(R^K_{F,t+1}u_{f,t+1} - \frac{P^I_{F,t+1}}{\gamma_{t+1}}a(u_{f,t+1})\right)\right] \\ &+ E_t \beta^r_{F,t+1}\left[\frac{\Omega^C_{f,t+1}}{\Omega^C_{f,t}}\frac{1}{\Pi^C_{F,t+1}}\frac{\gamma_t}{P_{F,t}}P^K_{F,t+1}(1-\delta_F)\right].\end{aligned}$$

In order to stationarize the above equation, we use the following definitions: $r^K_{F,t+1} = \frac{\gamma_{t+1}R^K_{t+1}}{P_{F,t+1}}$, $p^I_{F,t+1} = \frac{P^I_{F,t+1}}{P_{F,t+1}}$, $\mu_{\gamma,t+1} = \frac{\gamma_{t+1}}{\gamma_t}$, $p^K_{F,t} = \frac{\gamma_t P^K_{F,t}}{P_{F,t}}$, and $\Pi_{F,t+1} = \frac{P_{F,t+1}}{P_{F,t}}$. Also, since all households choose the same level of consumption and the same utilization rate, the subscript f may be dropped from the above equation. Thus, we have the following equation for the household purchases of installed capital:

$$\begin{aligned}\frac{\gamma_t P^K_{F,t}}{P_{F,t}} &= E_t \beta^r_{F,t+1}\left[\frac{\Omega^C_{F,t+1}}{\Omega^C_{F,t}}\frac{1}{\Pi^C_{F,t+1}}\frac{\gamma_t}{P_{F,t}}\frac{P_{F,t+1}}{\gamma_{t+1}}\left(r^K_{F,t+1}u_{F,t+1} - p^I_{F,t+1}a(u_{F,t+1})\right)\right] \\ &+ E_t \beta^r_{F,t+1}\left[\frac{\Omega^C_{F,t+1}}{\Omega^C_{F,t}}\frac{1}{\Pi^C_{F,t+1}}\frac{\gamma_t}{P_{F,t}}P^K_{F,t+1}(1-\delta_F)\right].\end{aligned}$$

We use the following definition: $p^K_{F,t} = \frac{\gamma_t P^K_{F,t}}{P_{F,t}}$, and the above equation can be written as follows:

$$p^K_{F,t} = E_t \beta^r_{F,t+1}\frac{\Omega^C_{F,t+1}}{\Omega^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{1}{\mu_{\gamma,t+1}}\left[r^K_{F,t+1}u_{F,t+1} - p^I_{F,t+1}a(u_{F,t+1}) + p^K_{F,t+1}(1-\delta_F)\right]. \quad \text{(D.17)}$$

Using the following definitions: $\overline{\Omega}^C_{F,t} = z^+_{F,t}\Omega^C_{F,t}$ and $\mu_{z^+_F,t+1} = \frac{z^+_{F,t+1}}{z^+_{F,t}}$, Equation (D.17) can be written as:

$$p^K_{F,t} = E_t \beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\overline{\Omega}^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{1}{\mu_{z^+_F,t+1}\mu_{\gamma,t+1}}\left[r^K_{F,t+1}u_{F,t+1} - p^I_{F,t+1}a(u_{F,t+1}) + p^K_{F,t+1}(1-\delta_F)\right]. \quad \text{(D.18)}$$

Equation (D.18) is the same as Equation (A.107a), which shows the stationarized version of the household purchase of installed capital.

D.1.4 Foreign: Investment

This section derives the household investment decision equation (A.108a). Recall that we have Equation (D.9) that shows the following FOC for $I_{f,t}$:

$$\theta^b_{f,t}\frac{P^I_{F,t}}{\gamma_{F,t}} = \theta^k_{f,t}\ \Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\theta^k_{f,t+1}\ \Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

The above equation can be expressed as:

$$P^I_{F,t} = \frac{\gamma_t\theta^k_{f,t}}{\theta^b_{f,t}}\ \Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\gamma_t\theta^k_{f,t+1}}{\theta^b_{f,t}}\ \Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

We use Equation (D.11), which shows $\theta^b_{f,t}P^K_{F,t} = \theta^k_{f,t}$. We can rewrite the above equation as follows:

$$\begin{aligned}P^I_{F,t} &= \frac{\gamma_t\theta^b_{f,t}P^K_{F,t}}{\theta^b_{f,t}}\ \Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\gamma_t\theta^b_{f,t+1}P^K_{F,t+1}}{\theta^b_{f,t}}\ \Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right], \\ P^I_{F,t} &= \gamma_t P^K_{F,t}\ \Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\gamma_t\theta^b_{f,t+1}P^K_{F,t+1}}{\theta^b_{f,t}}\ \Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].\end{aligned}$$

We use the following definition: $p^I_{F,t} = P^I_{F,t}/P_{F,t}$, and the above equation becomes:

$$p^I_{F,t} = \frac{\gamma_t P^K_{F,t}}{P_{F,t}}\Upsilon_{F,t}F_1(I_{f,t}, I_{f,t-1}) + E_t\left[\beta^r_{F,t+1}\frac{\gamma_t\theta^b_{f,t+1}P^K_{F,t+1}}{P_{F,t}\theta^b_{f,t}}\Upsilon_{F,t+1}F_2(I_{f,t+1}, I_{f,t})\right].$$

We multiply the second term on the right hand side of the above equation by $\frac{P_{F,t+1}\gamma_{t+1}}{P_{F,t+1}\gamma_{t+1}}$. We use the following definitions: $\mu_{\gamma,t+1}=\frac{\gamma_{t+1}}{\gamma_t}$ and $\Pi_{F,t+1}=\frac{P_{F,t+1}}{P_{F,t}}$. The above equation can then be rewritten as follows:

$$p^I_{F,t}=\frac{\gamma_t P^K_{F,t}}{P_{F,t}}\Upsilon_{F,t}F_1(I_{f,t},I_{f,t-1})+E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}}{\theta^b_{f,t}}\frac{\gamma_{t+1}P^K_{F,t+1}}{P_{F,t+1}}\frac{P_{F,t+1}}{P_{F,t}}\frac{\gamma_t}{\gamma_{t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1},I_{f,t})\right],$$

$$p^I_{F,t}=\frac{\gamma_t P^K_{F,t}}{P_{F,t}}\Upsilon_{F,t}F_1(I_{f,t},I_{f,t-1})+E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}}{\theta^b_{f,t}}\frac{\gamma_{t+1}P^K_{F,t+1}}{P_{F,t+1}}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1},I_{f,t})\right].$$

Using the following definition: $p^K_{F,t}=\frac{\gamma_{F,t}P^K_{F,t}}{P_{F,t}}$, this gives us the following equation:

$$p^I_{F,t}=p^K_{F,t}\Upsilon_{F,t}F_1(I_{f,t},I_{f,t-1})+E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}}{\theta^b_{f,t}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1},I_{f,t})\right].$$

Using the following definitions: $\Pi^C_{F,t+1}=\frac{P^C_{F,t+1}}{P^C_{F,t}}$ and $\Omega^C_{f,t}=\theta^b_{f,t}P^C_{F,t}$, we can rewrite the above equation as follows:

$$p^I_{F,t}=p^K_{F,t}\Upsilon_{F,t}F_1(I_{f,t},I_{f,t-1})+E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}P^C_{F,t+1}}{\theta^b_{f,t}P^C_{F,t}}\frac{P^C_{F,t}}{P^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1},I_{f,t})\right]$$

$$p^I_{F,t}=p^K_{F,t}\Upsilon_{F,t}F_1(I_{f,t},I_{f,t-1})+E_t\left[\beta^r_{F,t+1}\frac{\theta^b_{f,t+1}P^C_{F,t+1}}{\theta^b_{f,t}P^C_{F,t}}\frac{1}{\Pi^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1},I_{f,t})\right],$$

and we can obtain the following equation:

$$p^I_{F,t}=p^K_{F,t}\Upsilon_{F,t}F_1(I_{f,t},I_{f,t-1})+E_t\left[\beta^r_{F,t+1}\frac{\Omega^C_{f,t+1}}{\Omega^C_{f,t}}\frac{1}{\Pi^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{f,t+1},I_{f,t})\right].$$

Since all households choose the same level of investment we replace the subscript f with F and write the equation in aggregate variables. Hence, we have the following equation for the household investment decision:

$$p^I_{F,t}=p^K_{F,t}\Upsilon_{F,t}F_1(I_{F,t},I_{F,t-1})+E_t\left[\beta^r_{F,t+1}\frac{\Omega^C_{F,t+1}}{\Omega^C_{F,t}}\frac{1}{\Pi^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{F,t+1},I_{F,t})\right]. \quad \text{(D.19)}$$

Now, we continue the effort to stationarize Equation (D.19). Using the following definitions: $\overline{\Omega}^C_{F,t}=z^+_{F,t}\Omega^C_{F,t}$ and $\mu_{z^+_F,t+1}=\frac{z^+_{F,t+1}}{z^+_{F,t}}$, Equation (D.19) can be written as follows:

$$p^I_{F,t}=p^K_{F,t}\Upsilon_{F,t}F_1(I_{F,t},I_{F,t-1})+E_t\left[\beta^r_{F,t+1}\frac{z^+_{F,t+1}\Omega^C_{F,t+1}}{z^+_{F,t}\Omega^C_{F,t}}\frac{z^+_{F,t}}{z^+_{F,t+1}}\frac{1}{\Pi^C_{F,t+1}}p^K_{F,t+1}\Pi_{F,t+1}\frac{1}{\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{F,t+1},I_{F,t})\right],$$

$$p^I_{F,t}=p^K_{F,t}\Upsilon_{F,t}F_1(I_{F,t},I_{F,t-1})+E_t\left[\beta^r_{F,t+1}\frac{\overline{\Omega}^C_{F,t+1}}{\overline{\Omega}^C_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^C_{F,t+1}}\frac{p^K_{F,t+1}}{\mu_{z^+_F,t+1}\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(I_{F,t+1},I_{F,t})\right]. \quad \text{(D.20)}$$

Furthermore, we need to express $F_1(I_{F,t},I_{F,t-1})$ and $F_2(I_{F,t+1},I_{F,t})$ as functions of stationary variables. Recall from Section 2.10 , we have the following investment adjustment cost function $F(I_{F,t},I_{F,t-1})$:

$$F(I_{F,t},I_{F,t-1})=\left[1-\widetilde{S}\left(\frac{I_{F,t}}{I_{F,t-1}}\right)\right]I_{F,t}.$$

We take the first derivative of $F(I_{F,t},I_{F,t-1})$ with respect to $I_{F,t}$, and we can find $F_1(I_{F,t},I_{F,t-1})$. We then take the first derivative of $F(I_{F,t+1},I_{F,t})$ with respect to $I_{F,t}$, and we can find $F_2(I_{F,t+1},I_{F,t})$. We have the following results:

$$F_1(I_{F,t},I_{F,t-1})=-\widetilde{S}'\left(\frac{I_{F,t}}{I_{F,t-1}}\right)\frac{I_{F,t}}{I_{F,t-1}}+\left[1-\widetilde{S}\left(\frac{I_{F,t}}{I_{F,t-1}}\right)\right], \quad \text{(D.21)}$$

and

$$F_2(I_{F,t+1},I_{F,t})=\widetilde{S}'\left(\frac{I_{F,t+1}}{I_{F,t}}\right)\left(\frac{I_{F,t+1}}{I_{F,t}}\right)^2. \quad \text{(D.22)}$$

We express Equation (D.21) and Equation (D.22) by applying the following definition: $\overline{I}_{F,t} = \frac{I_{F,t}}{z^{+}_{F,t}\gamma_t}$. Using this definition, together with $\mu_{z^{+}_{F},t} = \frac{z^{+}_{F,t}}{z^{+}_{F,t-1}}$ and $\mu_{\gamma,t} = \frac{\gamma^{+}_{t}}{\gamma^{+}_{t-1}}$, the ratio $\frac{I_{F,t}}{I_{F,t-1}}$ can be written as: $\mu_{z^{+}_{F},t}\,\mu_{\gamma,t}\,\frac{\overline{I}_{F,t}}{\overline{I}_{F,t-1}}$. We use the notation $F_1(\overline{I}_{F,t},\overline{I}_{F,t-1},\mu_{z^{+},t},\mu_{\gamma,t})$ to express $F_1(I_{F,t},I_{F,t-1})$ as a function of the stationary variables $\overline{I}_{F,t}$, $\overline{I}_{F,t-1}$, $\mu^{F}_{z^{+}_{F},t}$ and $\mu^{F}_{\gamma,t}$. Moreover, $F_2(\overline{I}_{F,t+1},\overline{I}_{F,t},\mu_{z^{+}_{F},t+1},\mu_{\gamma,t+1})$ represents $F_2(I_{F,t+1},I_{F,t})$ expressed as a function of stationary variables. Hence, Equation (D.21) and Equation (D.22) become:

$$F_1(\overline{I}_{F,t},\overline{I}_{F,t-1},\mu_{z^{+}_{F},t},\mu_{\gamma,t}) = -\widetilde{S}'\left(\frac{\mu_{z^{+}_{F},t}\,\mu_{\gamma,t}\overline{I}_{t}}{\overline{I}_{F,t-1}}\right)\frac{\mu_{z^{+}_{F},t}\,\mu_{\gamma,t}\overline{I}_{F,t}}{\overline{I}_{F,t-1}} + \left[1-\widetilde{S}\left(\frac{\mu_{z^{+}_{F},t}}{\overline{I}_{F,t-1}}\right)\right],$$

and

$$F_2(\overline{I}_{F,t+1},\overline{I}_{F,t},\mu_{z^{+}_{F},t+1},\mu_{\gamma,t+1}) = \widetilde{S}'\left(\frac{\mu_{z^{+}_{F},t+1}\,\mu_{\gamma,t+1}\overline{I}_{F,t+1}}{\overline{I}_{F,t}}\right)\left(\frac{\mu_{z^{+}_{F},t+1}\,\mu_{\gamma,t+1}\overline{I}_{F,t+1}}{\overline{I}_{F,t}}\right)^{2}.$$

With these notations, we can rewrite Equation (D.20) as:

$$p^{I}_{F,t} = p^{K}_{F,t}\Upsilon_{F,t}F_1(\overline{I}_{F,t},\overline{I}_{F,t-1},\mu_{z^{+}_{F},t},\mu_{\gamma,t}) + E_t\left[\beta^{r}_{F,t+1}\frac{\overline{\Omega}^{C}_{F,t+1}}{\overline{\Omega}^{C}_{F,t}}\frac{\Pi_{F,t+1}}{\Pi^{C}_{F,t+1}}\frac{p^{K}_{F,t+1}}{\mu_{z^{+}_{F},t+1}\mu_{\gamma,t+1}}\Upsilon_{F,t+1}F_2(\overline{I}_{F,t+1},\overline{I}_{F,t},\mu_{z^{+}_{F},t+1},\mu_{\gamma,t+1})\right] \tag{D.23}$$

Equation (D.23), which represents the stationarized version of the household investment decision equation, is the same as Equation (A.108a).

D.1.5 Foreign: Wage setting

This section presents the stationarized version of the optimal wage setting equation. In this section, the household f chooses the optimal wage rate $W^{opt}_{f,t}$ that maximizes the expected utility (D.1), subject to the budget constraint (D.2), the labor demand schedule (D.3), and the Calvo wage contract (D.5). In each period, the individual household resets its wage with probability $(1-\xi^{F}_{w})$. With probability ξ^{F}_{w}, the household cannot reset its wage, in which case the wage rate evolves according to: $W_{f,t+k|t} = W^{opt}_{f,t}\overline{\Pi}^{W}_{F,t+1}\overline{\Pi}^{W}_{F,t+2}\ldots\overline{\Pi}^{W}_{F,t+k}$. Note that $\overline{\Pi}^{W}_{F,t} = (\Pi^{W}_{F,t-1})^{\chi_{F,w}}\left(\Pi^{C,trend}_{F,t}\right)^{1-\chi_{F,w}}$.

We apply the following definitions: $\varepsilon^{F}_{w} = \frac{\lambda^{W}_{F}}{\lambda^{W}_{F}-1}$ and $\frac{\partial N_{f,t+k|t}}{\partial W_{f,t+k|t}}\frac{W_{f,t+k|t}}{N_{f,t+k|t}} = \frac{\lambda^{W}_{F}}{1-\lambda^{W}_{F}}$. We also use the following definition: $W_{f,t+k|t} = W^{opt}_{f,t}\overline{\Pi}^{W}_{F,t+1}\overline{\Pi}^{W}_{F,t+2}\ldots\overline{\Pi}^{W}_{F,t+k}$. We follow the same steps to those in Section C.1.9; hence, we can find the following non-stationarized version of the optimal wage setting equation for the Foreign economy:

$$E_t\sum_{k=0}^{\infty}\left(\xi^{F}_{w}\right)^{k}\left(\prod_{i=1}^{k}\beta^{r}_{F,t+i}\right)N_{f,t+k|t}\,\theta^{b}_{f,t+k}\left[\left(1-\tau^{w}_{F}\right)W_{f,t+k|t} - \lambda^{W}_{F}\,\zeta^{n}_{F,t+k}\frac{v'\left(N_{f,t+k|t}\right)}{\theta^{b}_{f,t+k}}\right] = 0. \tag{D.24}$$

Equation (D.24) is the same as Equation (79) in Section 2.6.1.

Equation (D.24) can be expressed in terms of *per capita* quantities by applying the following definition: $n_{F,t} = \frac{N_{F,t}}{\omega}$. We define $n_{F,t}$ as Foreign aggregate hours *per capita* and ω is the size of the Foreign economy. We stationarize Equation (D.24) by using the following definitions: $\overline{w}_{F,t+k|t} = \frac{W_{F,t+k|t}}{z^{+}_{F,t+k}P^{C}_{F,t+k}}$ and $\overline{\Omega}^{C}_{F,t+k} = z^{+}_{F,t+k}\Omega^{C}_{F,t+k}$.

We follow the similar steps as in Section C.1.9, and we can obtain the following stationarized version of the optimal wage setting equation for the Foreign economy:

$$E_t\sum_{k=0}^{\infty}\left(\xi^{F}_{w}\right)^{k}\left(\prod_{j=1}^{k}\beta^{r}_{F,t+j}\right)n_{F,t+k|t}\overline{\Omega}^{C}_{F,t+k}\left[\left(1-\tau^{w}_{F}\right)\overline{w}_{F,t+k|t} - \lambda^{W}_{F}\,\zeta^{n}_{F,t+k}\frac{\nu'(n_{F,t+k|t})}{\overline{\Omega}^{C}_{F,t+k}}\right] = 0. \tag{D.25}$$

Equation (D.25), which represents the stationarized version of the optimal wage setting for the Foreign economy, is the same as Equation (A.111a).

D.2 Foreign: Intermediate good producers

In this section, we present the stationarized version of the expression for the real marginal cost of production of Foreign intermediate good producers. Intermediate good producers in the Foreign economy use labor and private capital as inputs, so different from the Swedish economy they do not use public capital. As in Swedish economy, $z_{F,t}^{+}$ combines a global labor augmenting technological process z_t and a technological process specific to the production of investment goods γ_t. There is a continuum of intermediate good producers of mass ω. The individual firm in the Foreign economy is denoted by j. Firm j uses capital services $K_{f,t}(j)$ and labor $L_{F,t}(j)$ to minimize the following cost function:

$$TC_t(j) = R_{F,t}^{K} K_{F,t}^{s}(j) + W_{F,t} N_{F,t}(j) \tag{D.26}$$

subject to the production constraint:

$$Y_{F,t}(j) = \varepsilon_{F,t} \left[K_{F,t}^{s}(j)\right]^{\alpha_F} \left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z_{F,t}^{+} \phi_F. \tag{D.27}$$

We denote $\theta_{F,t}(j)$ as the Lagrange multiplier associated with the production constraint D.27. To solve the optimization problem, we set up the following Lagrangian $\mathscr{L}_{F,t}(j)$:

$$\mathscr{L}_t(i) = R_{F,t}^{K} K_{F,t}^{s}(j) + W_{F,t} N_{F,t}(j) - \theta_{F,t}(j) \left[\varepsilon_{F,t} \left[K_{F,t}^{s}(j)\right]^{\alpha_F} \left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z_{F,t}^{+} \phi_F - Y_{F,t}(j)\right].$$

We take the partial derivative of $\mathscr{L}_{F,t}(j)$ with respect to $K_{F,t}^{s}(j)$ and $N_{F,t}(j)$ respectively, and we can find the FOCs.

The FOC for $K_{F,t}(j)$ is:

$$R_{F,t}^{K} - \alpha_F \theta_{F,t}(j) \varepsilon_{F,t} \left[K_{F,t}^{s}(j)\right]^{\alpha_F - 1} \left[z_t N_{F,t}(j)\right]^{1-\alpha_F} = 0. \tag{D.28}$$

The FOC for $N_{F,t}(j)$ is:

$$W_{F,t} - \theta_{F,t}(j) \left(1 - \alpha_F\right) \varepsilon_{F,t} \left[K_{F,t}^{s}(j)\right]^{\alpha_F} z_t^{1-\alpha} \left[N_{F,t}(j)\right]^{-\alpha_F} = 0. \tag{D.29}$$

Using Equation (D.28) and Equation (D.29), we obtain the following capital-labor input efficiency condition:

$$K_{F,t}^{s}(j) = \frac{\alpha_F}{1-\alpha_F} \frac{W_{F,t}}{R_{F,t}^{K}} N_{F,t}(j).$$

Note that Equation (D.27) can be written as:

$$\left[Y_{F,t}(j) + z_t^{+} \phi^{F}\right] = \varepsilon_{F,t} \left[K_{F,t}^{s}(j)\right]^{\alpha_F} \left[z_t N_{F,t}(j)\right]^{1-\alpha_F}. \tag{D.30}$$

Now, we find the total cost of production equation. We substitute Equation (D.28) and Equation (D.29) into Equation (D.26) , and we have the following equation:

$$TC_{F,t}(j) = \theta_{F,t}(j) \left[\alpha_F \varepsilon_{F,t} K_{F,t}^{s}(j)^{\alpha_F} \left[z_t N_{F,t}(j)\right]^{1-\alpha_F} + \left(1-\alpha_F\right) \varepsilon_{F,t} \left[K_{F,t}^{s}(j)\right]^{\alpha_F} z_t^{1-\alpha_F} \left[N_{F,t}(j)\right]^{1-\alpha_F}\right]. \tag{D.31}$$

Using Equation (D.30), we can rewrite Equation (D.31) as follows:

$$TC_{F,t}(j) = \theta_{F,t}(j) \left[\left(1-\alpha_F\right) + \alpha_F\right] \left(Y_{F,t}(j) + z_{F,t}^{+} \phi^{F}\right). \tag{D.32}$$

We use Equation (D.32), and we take the partial derivative of $TC_{F,t}(j)$ with respect to $Y_{F,t}(j)$. Hence, the lagrangian multiplier, $\theta_{F,t}(j)$, can be defined as the marginal cost of production $MC_{F,t}(j)$:

$$\frac{\partial TC_{F,t}(j)}{\partial Y_{F,t}(j)} = MC_{F,t}(j) = \theta_{F,t}(j).$$

Combining the two first-order conditions and solving for $N_{F,t}(j)$ yields

$$N_{F,t}(j) = \frac{(1-\alpha_F) R_{F,t}^{K} K_{F,t}^{S}(j)}{\alpha_F W_{F,t}^{K}}. \tag{D.33}$$

Substituting this expression back into the first-order condition with respect to for $N_{F,t}(j)$ gives

$$\theta_{F,t}(j) = \frac{W_{F,t}}{(1-\alpha_F)\varepsilon_{F,t}\left[K^S_{F,t}(j)\right]^{\alpha_F} z^{1-\alpha_F}_{F,t}} \left[N_{F,t}(j)\right]^{\alpha_F} = \frac{\left[R^K_{F,t}\right]^{\alpha_F} W^{1-\alpha_F}_{F,t}}{\alpha^{\alpha_F}_F (1-\alpha_F)^{1-\alpha_F} \varepsilon_{F,t} z^{1-\alpha_F}_{F,t}},$$

and hence

$$\frac{\partial TC_{F,t}(j)}{\partial Y_{F,t}(j)} = MC_{F,t}(j) = \theta_{F,t}(j) = \frac{W^{1-\alpha_F}_{F,t}\left(R^K_{F,t}\right)^{\alpha_F}}{\alpha^{\alpha_F}_F (1-\alpha_F)^{1-\alpha_F} \varepsilon_{F,t} z^{1-\alpha_F}_t} \tag{D.34}$$

Equation (D.34), which is the same as Equation (81) in Section 2.6.2, is the nominal marginal cost of production for the intermediate good firm j.

There are two equations that summarize optimal input decisions by firms: 1) the capital-labor input efficiency condition and 2) the optimal capital inputs in terms of marginal cost First, we find the capital-labor input efficiency equation. In particular, we rewrite Equation (D.33) as follows:

$$\frac{K^s_{F,t}(j)}{N_{F,t}(j)} = \frac{\alpha_F}{1-\alpha_F} \frac{W_{F,t}}{R^K_{F,t}}. \tag{D.35}$$

Equation (D.35) is the capital-labor input efficiency condition.

Firms hiring from homogeneous labor and private markets, i.e. face the same wage and rental rates. This implies that marginal costs are identical across firms. Equation (D.34) can be written as:

$$MC_{F,t} = \frac{\left(\frac{W_{F,t}}{z_t}\right)^{1-\alpha_F}\left(R^K_{F,t}\right)^{\alpha_F}}{\alpha^{\alpha_F}_F \left(1-\alpha_F\right)^{1-\alpha_F} \varepsilon_{F,t}}. \tag{D.36}$$

Equation (D.36) is the non-stationarized version of the nominal marginal cost of production for intermediate good firm.

Next, we derive the non-stationarized version of the rental rate for capital services. Using Equation (D.35) and Equation (D.36), we obtain the following equation:

$$R^K_{F,t} = \alpha_F \varepsilon_{F,t} MC_{F,t}(j)\left[K^s_{F,t}(j)\right]^{\alpha_F - 1}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F}. \tag{D.37}$$

Since we have identical capital labor ratios and identical marginal costs, we can drop the subscript j and rewrite Equation (D.37) as:

$$R^K_{F,t} = \alpha_F \varepsilon_{F,t} z^{1-\alpha_F}_t MC_{F,t}\left(\frac{K^s_{F,t}}{N_{F,t}}\right)^{\alpha_F - 1}. \tag{D.38}$$

Equation (D.38) is the same as Equation (82) in Section 2.6.2 and captures the non-stationarized version of rental rate for capital services.

Now, we find the stationarized version of the marginal cost of production for intermediate good producers. We stationarize Equation (D.36) by applying the following definitions: $r^K_{F,t} = \frac{\gamma_t R^K_{F,t}}{P_{F,t}}$, $\overline{w}^e_{F,t} = \frac{W_{F,t}}{z^+_{F,t} P_{F,t}}$, $z^+_{F,t} = z_t\left(\gamma_t\right)^{\frac{\alpha_F}{1-\alpha_F}}$, and $\overline{mc}_{F,t} = \frac{MC_{F,t}}{P_{F,t}}$. Equation (D.36) can be written as follows:

$$\frac{MC_{F,t}}{P_{F,t}} = \frac{\left(\frac{W_{F,t}}{z_t}\right)^{1-\alpha_F}\left(\frac{1}{P_{F,t}}\right)^{1-\alpha_F}\left(\frac{1}{P_{F,t}}\right)^{\alpha_F}\left(R^K_{F,t}\right)^{\alpha_F}\frac{\gamma^{\alpha_F}_t}{\gamma^{\alpha_F}_t}}{\alpha^{\alpha_F}_F\left(1-\alpha_F\right)^{1-\alpha_F}\varepsilon_{F,t}},$$

$$\overline{mc}_{F,t} = \frac{\left(\frac{W_{F,t}}{z_t(\gamma_t)^{\alpha_F/(1-\alpha_F)}\; P_{F,t}}\right)^{1-\alpha_F}\left(\frac{\gamma_t R^K_{F,t}}{P_{F,t}}\right)^{\alpha_F}}{\alpha^{\alpha_F}_F\left(1-\alpha_F\right)^{1-\alpha_F}\varepsilon_{F,t}},$$

$$\overline{mc}_{F,t} = \frac{\left(\frac{W_{F,t}}{z^+_{F,t} P_{F,t}}\right)^{1-\alpha_F}\left(\frac{\gamma_t R^K_{F,t}}{P_{F,t}}\right)^{\alpha_F}}{\alpha^{\alpha_F}_F\left(1-\alpha_F\right)^{1-\alpha_F}\varepsilon_{F,t}}.$$

Thus, the stationarized version of the marginal cost of production for intermediate good producers (the real marginal cost of production) equation can be expressed as:

$$\overline{mc}_{F,t} = \frac{\left(\overline{w}^e_{F,t}\right)^{1-\alpha_F}\left(r^K_{F,t}\right)^{\alpha_F}}{\varepsilon_{F,t}\alpha^{\alpha_F}_F(1-\alpha_F)^{1-\alpha_F}}. \tag{D.39}$$

Equation (D.39), which represents the real marginal cost of production for intermediate good producers, is the same as Equation (A.117a) in Section A.

Lastly, we find the stationarized version of the rental rate for capital services. We stationarize Equation (D.38) by applying the following definitions: $r^K_{F,t} = \frac{\gamma_t R^K_{F,t}}{P_{F,t}}$, $z^+_{F,t}\gamma_t = z_t \gamma_t^{1/(1-\alpha_F)}$, $\bar{K}^s_{F,t} = \frac{K^s_{F,t}}{z^+_{F,t-1}\gamma_{t-1}}$, and $\overline{mc}_{F,t} = \frac{MC_{F,t}}{P_{F,t}}$. We follow the similar steps when deriving the stationarized version of the marginal cost for intermediate good producers equation. Hence, Equation (D.38) can be written as:

$$r^K_{F,t} = \alpha_F \varepsilon_{F,t} \left(\frac{\bar{K}^s_{F,t}}{N_{F,t}} \frac{1}{\mu_{z^+_F,t}\mu_{\gamma,t}} \right)^{\alpha_F - 1} \overline{mc}_{F,t}$$

Furthermore, we can rewrite the above equation in terms of *per capita*, so we denote $\bar{k}^s_{F,t}$ as stationarized capital services *per capita*, and $n_{F,t}$ as aggregate labor input *per capita* by using the following definitions: $\bar{k}^s_{F,t} = \frac{\bar{K}^s_{F,t}}{\omega}$ and $n_{F,t} = \frac{N_{F,t}}{\omega}$. Hence, we can rewrite the above equation as:

$$r^K_{F,t} = \alpha_F \varepsilon_{F,t} \left(\frac{\bar{k}^s_{F,t}}{n_{F,t}} \frac{1}{\mu_{z^+_F,t}\mu_{\gamma,t}} \right)^{\alpha_F - 1} \overline{mc}_{F,t} \tag{D.40}$$

Equation (D.40), which is the real rental rate for capital services equation, is the same as Equation (A.118a) in Section A.

D.2.1 Foreign: Optimal price of intermediate goods

In this section, we present the stationarized version of the expression for the optimal price of intermediate goods for Foreign intermediate good producers. The firm j chooses the optimal price $P^{opt}_{F,t}(j)$ that maximizes its profit, subject to its demand schedule and the Calvo price contract. In each period, the individual firm j resets its price with probability $(1-\xi^F)$. With probability ξ^F, the firm cannot reset its price, and then it faces the following price evolution: $P_{F,t+k|t}(j) = P^{opt}_{F,t}(j)\overline{\Pi}_{F,t+1}\overline{\Pi}_{F,t+2}\ldots\overline{\Pi}_{F,t+k}$. We define the stochastic discount factor as $\Lambda^F_{t,t+k} = \frac{\beta_{F,t+k}}{\beta_{F,t}} \frac{\Omega^C_{F,t+k}}{\Omega^C_{F,t}} \frac{P^C_{F,t}}{P^C_{F,t+k}}$.

Firm j chooses the optimal price of intermediate goods $P^{opt}_{F,t}(j)$ to maximize the following profit function:

$$\max_{P^{opt}_{F,t}(j)} E_t \sum_{k=0}^{\infty} (\xi^F)^k \Lambda^F_{t,t+k} \left\{ P_{F,t+k|t}(j) Y_{F,t+k|t}(j) - TC_{F,t+k|t}\left[Y_{F,t+k|t}(j) \right] \right\}$$

subject to the demand function:

$$Y_{F,t+k|t}(j) = \frac{1}{\omega} \left(\frac{P_{F,t+k|t}(j)}{P_{F,t+k}} \right)^{\frac{\lambda_{F,t+k}}{1-\lambda_{F,t+k}}} Y_{F,t+k},$$

and the Calvo price setting contract:

$$P_{F,t+k}(j) = \begin{cases} \overline{\Pi}_{F,t+k} P_{F,t+k-1}(j) & \text{with probability } \xi^F \\ P^{opt}_{F,t+k}(j) & \text{with probability } (1-\xi^F). \end{cases}$$

We apply the following definitions: $-\frac{\partial Y_{F,t+k|t}(j)}{\partial P_{F,t+k|t}(j)} \frac{P_{F,t+k|t}(j)}{Y_{F,t+k|t}(j)} = \frac{\lambda_{F,t+k}}{\lambda_{F,t+k}-1}$, and $P_{F,t+k|t}(j) = P^{opt}_{F,t}(j)\overline{\Pi}_{F,t+1}\overline{\Pi}_{F,t+2}\ldots\overline{\Pi}_{F,t+k}$, and we follow the same steps to those in Section C.2.1. Hence, we can find the following non-stationarized version of the optimal price of intermediate goods equation for the Foreign economy:

$$E_t \sum_{k=0}^{\infty} (\xi^F)^k \Lambda^F_{t,t+k} \frac{Y_{F,t+k|t}}{(\lambda_{F,t+k}-1)} \left(P_{F,t+k|t} - \lambda_{F,t+k}\, MC_{F,t+k} \right) = 0. \tag{D.41}$$

Equation (D.41), which is the non-stationarized version of the optimal price of intermediate goods equation, is the same as Equation (83) in Section 2.6.2.

Equation (D.41) can be written in terms of *per capita* quantities by using the following definition: $y_{F,t} = \frac{Y_{F,t}}{\omega}$. We define $y_{F,t}$ as Foreign aggregate output *per capita* and ω is the size of the Foreign economy. We stationarize Equation (D.41) by using the following definitions: $p^{opt}_{F,t} = \frac{P^{opt}_{F,t}}{P_{F,t-1}}$, $\Pi_{F,t+k} = \frac{P_{F,t+k}}{P_{F,t+k-1}}$, $\Pi^C_{F,t+k} = \frac{P^C_{F,t+k}}{P^C_{F,t+k-1}}$, and $\overline{mc}_{F,t+k} = \frac{MC_{F,t+k}}{P_{F,t+k}}$. We also use the following definitions: $\overline{y}_{F,t} = \frac{y_{F,t}}{z^+_{F,t}}$ and $\overline{\Omega}^C_{F,t+k} = z^+_{F,t+k}\Omega^C_{F,t+k}$ when we stationarize Equation (D.41).

We follow the similar steps as in Section C.2.1, and we can obtain the following stationarized version of the optimal price for intermediate good producers in the Foreign economy:

$$E_t \sum_{k=0}^{\infty} \left(\xi^F\right)^k \left(\prod_{j=1}^{k} \beta^r_{F,t+j}\right) \frac{\overline{\Omega}^C_{F,t+k}}{\overline{\Omega}^C_{F,t}} \left(\prod_{j=1}^{k} \frac{\Pi_{F,t+j}}{\Pi^C_{F,t+j}}\right) \frac{\overline{y}_{F,t+k|t}}{(\lambda_{F,t+k}-1)} \left[\left(\prod_{j=1}^{k} \frac{\overline{\Pi}_{F,t+j}}{\Pi_{F,t+j}}\right) \frac{p^{opt}_{F,t}}{\Pi_{F,t}} - \lambda_{F,t+k}\overline{mc}_{F,t+k}\right] = 0. \tag{D.42}$$

Equation (D.42), which captures the stationarized version of the optimal price for Foreign intermediate good producers, is the same as Equation (A.119a).

D.3 Foreign: Consumption good producers

This section presents the optimization problem of the consumption good producers in the Foreign economy and derives the demand functions of non-energy and energy consumption and derives the relative price of the consumption goods equation.

The optimization problem of the representative consumption good producer can be defined as follows:

$$\max_{C_{F,t}, C^{xe}_{F,t}, C^e_{F,t}} P^C_{F,t} C_{F,t} - P^{C,xe}_{F,t} C^{xe}_{F,t} - P^{C,e}_{F,t} C^e_{F,t}$$

subject to the CES aggregate consumption good function

$$C_{F,t} = \left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_t\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^e_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}. \tag{D.43}$$

By substituting the CES aggregate consumption good equation (D.43) into the above profit function, we can rewrite the profit function as:

$$P^C_{F,t}\left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_t\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^e_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}} - P^{C,xe}_{F,t} C^{xe}_{F,t} - P^{C,e}_{F,t} C^e_{F,t}.$$

Taking the derivatives of $C^{xe}_{F,t}$ and $C^e_{F,t}$ respectively gives us the two following first-order-conditions:

$$\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}-1} P^C_{F,t}\left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_t\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^e_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}-1} - P^{C,xe}_{F,t} = 0$$

$$\left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^e_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}-1} P^C_{F,t}\left[\left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_t\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^e_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}}\right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}-1} - P^{C,e}_{F,t} = 0$$

If we substitute the expression in square brackets by the production function, and put the prices of non-energy and energy respectively on the right-hand-side of the equation, the two equations can be written as

$$\begin{aligned} P^{C,xe}_{F,t} &= \left(\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^{xe}_{F,t}\right)^{-\frac{1}{\nu_{F,C}}} P^C_{F,t} \left(C_{F,t}\right)^{\frac{1}{\nu_{F,C}}} \\ P^{C,e}_{F,t} &= \left(1-\vartheta^C_F\right)^{\frac{1}{\nu_{F,C}}} \left(C^e_{F,t}\right)^{-\frac{1}{\nu_{F,C}}} P^C_{F,t} \left(C_{F,t}\right)^{\frac{1}{\nu_{F,C}}}. \end{aligned}$$

Rearrange and multiply through with $\nu_{F,C}$ in the two equations above to get the demand function for the non-energy and energy consumption goods:

$$C^{xe}_{F,t} = \vartheta^C_F \left(\frac{P^C_{F,t}}{P^{C,xe}_{F,t}}\right)^{\nu_C} C_{F,t} \tag{D.44}$$

$$C^e_{F,t} = \left(1-\vartheta^C_F\right) \left(\frac{P^C_{F,t}}{P^{C,e}_{F,t}}\right)^{\nu_C} C_{F,t} \tag{D.45}$$

which are the same equations that are presented in Equation (84) and Equation (85). Next, we create the price index. This is done by substituting the two demand functions into the production function:

$$C_{F,t} = \left[\left(\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}} \left(C_t^{xe}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}} \left(C_{F,t}^e\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$C_{F,t} = \left[\left(\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}} \left(\vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_C} C_{F,t} \right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}} \left(\left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_C} C_{F,t} \right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$C_{F,t} = \left[\left(\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}+\frac{\nu_{F,C}-1}{\nu_{F,C}}} \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}-1} \left(C_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} + \left(1-\vartheta_F^C\right)^{\frac{1}{\nu_{F,C}}+\frac{\nu_{F,C}-1}{\nu_{F,C}}} \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}-1} \left(C_{F,t}\right)^{\frac{\nu_{F,C}-1}{\nu_{F,C}}} \right]$$

$$C_{F,t} = \left[\vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}-1} + \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}-1} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$1 = \left[\vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}-1} + \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}-1} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$1 = \left(P_{F,t}^C\right)^{\left(\nu_{F,C}-1\right)\frac{\nu_{F,C}}{\nu_{F,C}-1}} \left[\vartheta_F^C \left(\frac{1}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}-1} + \left(1-\vartheta_F^C\right) \left(\frac{1}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}-1} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$\left(P_{F,t}^C\right)^{-\nu_{F,C}} = \left[\vartheta_F^C \left(P_{F,t}^{C,xe}\right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(P_{F,t}^{C,e}\right)^{1-\nu_{F,C}} \right]^{\frac{\nu_{F,C}}{\nu_{F,C}-1}}$$

$$P_{F,t}^C = \left[\vartheta_F^C \left(P_{F,t}^{C,xe}\right)^{1-\nu_{F,C}} + \left(1-\vartheta_F^C\right) \left(P_{F,t}^{C,e}\right)^{1-\nu_{F,C}} \right]^{\frac{1}{1-\nu_{F,C}}}$$

which is the same function as is presented in . We use the definitions $p_{F,t}^C = P_{F,t}^C / P_{F,t}$, $p_{F,t}^{C,xe} = P_{F,t}^{C,xe}/P_{F,t}$, $p_{F,t}^{C,e} = P_{F,t}^{C,e}/P_{F,t}$, $\bar{c}_{F,t} = C_{F,t}/z_{F,t}^+$, $\bar{c}_{F,t}^{xe} = C_{F,t}^{xe}/z_{F,t}^+$, $\bar{c}_{F,t}^e = C_{F,t}^e/z_{F,t}^+$. The non-energy consumption demand function can be written as

$$\begin{aligned} C_{F,t}^{xe} &= \vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \right)^{\nu_{F,C}} C_{F,t} \\ \frac{C_{F,t}^{xe}}{z_t^+} &= \vartheta_F^C \left(\frac{P_{F,t}^C}{P_{F,t}^{C,xe}} \frac{P_{F,t}}{P_{F,t}} \right)^{\nu_{F,C}} \frac{C_{F,t}}{z_t^+} \\ \bar{c}_{F,t}^{xe} &= \vartheta_F^C \left(\frac{p_{F,t}^C}{p_{F,t}^{C,xe}} \right)^{\nu_{F,C}} \bar{c}_{F,t} \end{aligned} \tag{D.46}$$

Equation (D.46), which captures the demand for non-energy consumption goods, is the same as Equation (A.123a).

Next, we stationarize the demand for energy goods:

$$\begin{aligned} C_{F,t}^e &= \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \right)^{\nu_{F,C}} C_{F,t} \\ \frac{C_{F,t}^e}{z_t^+} &= \left(1-\vartheta_F^C\right) \left(\frac{P_{F,t}^C}{P_{F,t}^{C,e}} \frac{P_{F,t}}{P_{F,t}} \right)^{\nu_{F,C}} \frac{C_{F,t}}{z_t^+} \\ \bar{c}_{F,t}^e &= \left(1-\vartheta_F^C\right) \left(\frac{p_{F,t}^C}{p_{F,t}^{C,e}} \right)^{\nu_{F,C}} \bar{c}_{F,t} \end{aligned} \tag{D.47}$$

Equation (D.47), which captures the demand for energy consumption goods, is the same as Equation (A.124a).

Finally, we stationarize the price index:

$$P^C_{F,t} = \left[\vartheta^C_F \left(P^{C,xe}_{F,t}\right)^{1-\nu_{F,C}} + \left(1-\vartheta^C_F\right)\left(P^{C,e}_{F,t}\right)^{1-\nu_{F,C}}\right]^{\frac{1}{1-\nu_{F,C}}}$$

$$\frac{P^C_{F,t}}{P_{F,t}} = \frac{1}{P_{F,t}}\left[\vartheta^C_F \left(P^{C,xe}_{F,t}\right)^{1-\nu_{F,C}} + \left(1-\vartheta^C_F\right)\left(P^{C,e}_{F,t}\right)^{1-\nu_{F,C}}\right]^{\frac{1}{1-\nu_{F,C}}}$$

$$\frac{P^C_{F,t}}{P_{F,t}} = \left[\left(\vartheta^C_F \left(P^{C,xe}_{F,t}\right)^{1-\nu_{F,C}} + \left(1-\vartheta^C_F\right)\left(P^{C,e}_{F,t}\right)^{1-\nu_{F,C}}\right)P_{F,t}^{\nu_{F,C}-1}\right]^{\frac{1}{1-\nu_{F,C}}}$$

$$p^C_{F,t} = \left[\left(\vartheta^C_F \left(\frac{P^{C,xe}_{F,t}}{P_{F,t}}\right)^{1-\nu_{F,C}} + \left(1-\vartheta^C_F\right)\left(\frac{P^{C,e}_{F,t}}{P_{F,t}}\right)^{1-\nu_{F,C}}\right)\right]^{\frac{1}{1-\nu_{F,C}}}$$

$$p^C_{F,t} = \left[\left(\vartheta^C_F \left(p^{C,xe}_{F,t}\right)^{1-\nu_{F,C}} + \left(1-\vartheta^C_F\right)\left(p^{C,e}_{F,t}\right)^{1-\nu_{F,C}}\right)\right]^{\frac{1}{1-\nu_{F,C}}}. \tag{D.48}$$

Equation (D.48), which captures the demand for energy consumption goods, is the same as Equation (A.121a).

D.3.1 Foreign: Non-energy consumption good producers

This section presents the optimization problem of non-energy consumption good producers in the Foreign economy and derives the relative price of the non-energy consumption goods equation (A.37a).

The optimization problem of the representative non-energy consumption good producer can be defined as follows:

$$\max_{D^{C,xe}_{F,t},M^{C,xe}_{F,t}} P^{C,xe}_{F,t}C^{xe}_{F,t} - P_{F,t}D^{C,xe}_{F,t} - P^{M,C,xe}_{F,t}M^{C,xe}_{F,t}$$

subject to the CES aggregate consumption good function

$$C^{xe}_{F,t} = \left[\left(\psi^{C,xe}_F\right)^{\frac{1}{\nu_{F,C.xe}}}\left(D^{C,xe}_{F,t}\right)^{\frac{\nu_{F,C,xe}-1}{\nu_{F,C,xe}}} + \left(1-\psi^{C,xe}_F\right)^{\frac{1}{\nu_{F,C,xe}}}\left(M^{C,xe}_{F,t}\right)^{\frac{\nu_{F,C,xe}-1}{\nu_{F,C,xe}}}\right]^{\frac{\nu_{F,C,xe}}{\nu_{F,C,xe}-1}}. \tag{D.49}$$

Following the same procedures as in Section (C.3), we can obtain the demand function for intermediate goods used in the production of non-energy consumption goods in the Foreign economy, $D^{C,xe}_{F,t}$, and the demand function for imported goods used in the production of non-energy consumption goods in the Foreign economy, $M^{C,xe}_{F,t}$. The demand for intermediate goods used in the production of non-energy consumption goods in Forein economy, $D^{C,xe}_{F,t}$ is:

$$D^{C,xe}_{F,t} = \psi^{C,xe}_F\left(\frac{P^{C,xe}_{F,t}}{P_{F,t}}\right)^{\nu_{F,C,xe}} C^{xe}_{F,t}. \tag{D.50}$$

The demand for import goods used in the production of non-energy consumption goods in Forein economy, $M^{C,xe}_{F,t}$ is:

$$M^{C,xe}_{F,t} = (1-\psi^{C,xe}_F)\left(\frac{P^{C,xe}_{F,t}}{P^X_t}\right)^{\nu_{F,C,xe}} C^{xe}_{F,t}.$$

Note that $\psi^{C,xe}_F = \vartheta^{C,xe}_F + \frac{\omega}{1+\omega}\left(1-\vartheta^{C,xe}_F\right)$, and that, Foreign being infinitely large compared to Sweden. This means that $\psi^{C,xe}_F \to 1$, and that the share of imports become arbitrtarily small. Hence, the production function reduces to:

$$C^{xe}_{F,t} = D^{C,xe}_{F,t} \tag{D.51}$$

and the profit function then reduces to

$$P^{C,xe}_{F,t}D^{C,xe}_{F,t} - P_{F,t}D^{C,xe}_{F,t}.$$

This means that we get the following relationship between the price of Foreign domestic goods and Forein non-energy goods:

$$P^{C,xe}_t = P_{F,t} \tag{D.52}$$

We stationarize this equation by using the definition $p^{C.xe}_t = P^{C,xe}_t/P_{F,t}$ which simply means that

$$p^{C.xe}_t = 1 \tag{D.53}$$

Equation (D.53), which captures the relative price of consumption goods, is the same as Equation (A.125a).

D.4 Foreign: Investment good producers

This section presents optimization problem of investment good producers and derives the relative price of investment goods equation for the Foreign economy. Note that if the Swedish economy is infinitely small relative to the Foreign economy, then the investment goods and the intermedite goods will have the same price. Then the intermediate goods and investment goods will essentially be the same. We define $V^I_{F,t}$ to be the output of a representative investment firm. We define $V^I_{F,t}$ as $\frac{1}{\gamma_{F,t}}\left[I_{F,t}+a(u_t)K_{F,t}\right]$.

The representative investment good producer maximizes the following profit function:

$$\max_{D^I_{F,t},M^I_{F,t}} P^I_{F,t}V^I_{F,t} - P_{F,t}D^I_{F,t} - P^M_{F,t}M^I_{F,t},$$

subject to the following CES aggregate investment good function:

$$V^I_{F,t} = \left[\left(\psi^I_F\right)^{\frac{1}{\nu_{F,c}}}\left(D^I_{F,t}\right)^{\frac{\nu_{F,I}-1}{\nu_{F,I}}} + \left(1-\psi^I_F\right)^{\frac{1}{\nu_{F,I}}}\left(M^I_{F,t}\right)^{\frac{\nu_{F,I}-1}{\nu_{F,I}}}\right]^{\frac{\nu_{F,I}}{\nu_{F,I}-1}}. \tag{D.54}$$

Following the same procedures as in Section (D.3), we can obtain the demand function for intermediate goods used in the production of investment goods in the Foreign economy, $D^I_{F,t}$, and the demand function for imported goods used in the production of investment goods in the Foreign economy, $M^I_{F,t}$.
The demand for intermediate goods used in the production of investment goods in Forein economy, $D^I_{F,t}$ is:

$$D^I_{F,t} = \psi^I_F\left(\frac{P^I_{F,t}}{P_{F,t}}\right)^{\nu_I} V^I_{F,t}. \tag{D.55}$$

The demand for import goods used in the production of investment goods in Foreign economy, $M^I_{F,t}$ is:

$$M^I_{F,t} = (1-\psi^I_F)\left(\frac{P^I_{F,t}}{P^X_t}\right)^{\nu_I} V^I_{F,t}.$$

The assumption that Sweden is small economy implies that Foreign imports have a negligible share in the production of foreign investment goods. Thus, $\psi^I_F \to 1$ and the demand for the domestically produced intermediate good for investment good production is:

$$D^I_{F,t} = V^I_{F,t} \tag{D.56}$$

and the price index for foreign investment goods is:

$$P^I_{F,t} = P_{F,t}.$$

Using definition of relative foreign investment price $p^I_{F,t} = \frac{P^I_{F,t}}{P_{F,t}}$, we have

$$p^I_{F,t} = 1. \tag{D.57}$$

E Technical appendix: Market clearing

This section shows how to derive the Swedish aggregate resource constraint, the expressions for Swedish exports and imports, the Foreign aggregate resource constraint, as well as the balance of payments equation. These equilibrium conditions are stated and discussed in Section (2.7) in the main text. In addition, this section also includes a discussion of the value of the different fixed cost of production that exist in the Swedish and Foreign firm sectors.

Several of the equilibrium conditions to be discussed here are stated in two versions: one version that applies to the general case when the size of the Foreign economy, ω, can take on any non-negative value; and a second version that applies to the limiting case when ω tends to infinity. In all derivations that apply to the second case, we assume that all relative prices and all stationarized real quantities, expressed in *per capita* terms, take on non-negative, finite values in the limit as $\omega \to \infty$. We thus assume, for example, that the stationarized (*per capita*) level of Swedish exports converges to a non-negative, finite number as $\omega \to \infty$, i.e. that $\lim_{\omega\to\infty}\left(\frac{X_t}{z^+_t}\right) = \lim_{\omega\to\infty}\left(\overline{x}_t\right)$ is a non-negative, finite number. Concerning these derivations and the associated notation, a note of caution is warranted. In order to keep the notation relatively simple, we do not stringently distinguish between, on the hand, relative prices and stationarized (*per capita*) quantities that apply to any equilibrium where ω take on any positive, finite value and, on the hand, the corresponding relative prices and stationarized (*per capita*) quantities that apply to the limiting equilibrium. In the end, our focus is on the limiting equilibrium that obtains when $\omega \to \infty$ and when we state equilibrium conditions in terms of relative prices and stationarized (*per capita*) quantities, the final aim is always to make statements about this limiting equilibrium.

E.1 Swedish aggregate resource constraint

This subsection shows how to derive the aggregate resource constraint of the Swedish economy. In the first part of the subsection, different market clearing conditions are combined to derive the non-stationarized version of the Swedish aggregate resource constraint. This is Equation (95) in the main text. In a second part of the section, we derive a stationarized version of the constraint that applies to the limiting case when $\omega \to \infty$ and that can be used to solve the model with numerical methods. The result is Equation (A.136a).

E.1.1 Market clearing in Sweden

Y_t represents total demand for the homogeneous intermediate good, which in turn is the sum of demand from consumption and investment good producers, from export good producers and from the government. We denote these different demand components $D_t^{C,xe}$,$D_t^{C,e}$, D_t^I, D_t^X and G_t and $D_t^{I^G}$, respectively, and thus write:

$$Y_t = D_t^{C,xe} + D_t^{C,e} + D_t^I + D_t^X + G_t + D_t^{I^G} \tag{E.1}$$

Based on Equation (C.140), we have: $D_t^X(i) = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t(i) + z_t^+ \phi^X\right]$. $D_t^X(i)$ represents demand from the individual export good producer i and where $X_t(i)$ denotes production of the same firm. Let $X_t^P = \int_0^1 X_t(i)di$ denote total production of Swedish export goods. Thus, total demand for the homogeneous, intermediate good from Swedish export good producers can be written as:

$$\begin{aligned} D_t^X &= \int_0^1 D_t^X(i)di = \int_0^1 \left\{\psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t(i) + z_t^+ \phi^X\right]\right\} di, \\ &= \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[\int_0^1 X_t(i)di + z_t^+ \phi^X\right] = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t^P + z_t^+ \phi^X\right]. \end{aligned} \tag{E.2}$$

The market for differentiated Swedish export goods clears when production of each individual export good firm i equals demand for the export goods produced by the same firm. For the individual firm i, this implies $X_t(i) = \left[\frac{P_t^X(i)}{P_t^X}\right]^{\frac{\lambda_t^X}{1-\lambda_t^X}} X_t$, where X_t denotes total demand for the homogeneous Swedish export good (see Section 2.4.3 in the main text). Aggregating over all firms in the export good sector, and defining $\overset{\leftrightarrow}{P}{}_t^X = \int_0^1 \left[\frac{P_t^X(i)}{P_t^X}\right]^{\frac{\lambda_t^X}{1-\lambda_t^X}} di$ as a measure of price dispersion in the export good sector, we have:

$$X_t^P = \int_0^1 X_t(i)di = \int_0^1 \left\{\left[\frac{P_t^X(i)}{P_t^X}\right]^{\frac{\lambda_t^X}{1-\lambda_t^X}} X_t\right\} di = X_t \overset{\leftrightarrow}{P}{}_t^X. \tag{E.3}$$

Substituting $X_t \overset{\leftrightarrow}{P}{}_t^X$ into Equation (E.2), we have the following equation:

$$D_t^X = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t \overset{\leftrightarrow}{P}{}_t^X + z_t^+ \phi^X\right].$$

Note we have the following demand functions: Equation (C.106), $D_t^{C,xe} = \psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t}\right)^{\nu_{C,xe}} C_t^{xe}$; Equation (C.119), $D_t^I = \psi^I \left(\frac{P_t^I}{P_t}\right)^{\nu_I} \left[\frac{I_t}{\gamma_t} + a(u_t)\frac{K_t}{\gamma_t}\right]$; $D_t^X = \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t \overset{\leftrightarrow}{P}{}_t^X + z_t^+ \phi^X\right]$ and $D_t^{I^G} = \frac{I_t^G}{\gamma_t}$.

Substituting D_t^C, D_t^I, D_t^X and $D_t^{I^G}$ into Equation (E.1), we have the following equation:

$$Y_t = \psi^{C,xe} \left(\frac{P_t^{C,xe}}{P_t}\right)^{\nu_{C,xe}} C_t^{agg} + D_t^{C,e} + \psi^I \left(\frac{P_t^I}{P_t}\right)^{\nu_I} \left[\frac{I_t}{\gamma_t} + a(u_t)\frac{K_t}{\gamma_t}\right] + \psi^X \left(\frac{MC_t^X}{P_t}\right)^{\nu_x} \left[X_t \overset{\leftrightarrow}{P}{}_t^X + z_t^+ \phi^X\right] + G_t + \frac{I_t^G}{\gamma_t}. \tag{E.4}$$

Equation (E.4) is the same as Equation (95) in the main text.

E.1.2 Stationarizing the Swedish aggregate resource constraint

In the stationarization of the Swedish aggregate resource constraint, we make use of the following definitions of relative prices and of the real marginal cost of export good producers: $p_t^{C,xe} = \frac{P_t^{C,xe}}{P_t}$, $p_t^I = \frac{P_t^I}{P_t}$, $\overline{mc}_t^X = \frac{MC_t^X}{P_t}$. Furthermore, real variables are stationarized as follows: $\overline{C}_t^{xe} = \frac{C_t^{xe}}{z_t^+}$, $\overline{D}_t^e = \frac{D_t^e}{z_t^+}$, $\overline{I}_t = \frac{I_t}{z_t \gamma_t^{\frac{1}{1-\alpha}}}$, $\overline{K}_t = \frac{K_t}{z_{t-1}(\gamma_{t-1})^{\frac{1}{1-\alpha}}}$, $\overline{X}_t = \frac{X_t}{z_t^+}$, and $\overline{G}_t = \frac{G_t}{z_t^+}$. With these definitions, Equation (E.4) can be rewritten as follows:

$$\frac{Y_t}{z_t^+} = \psi^{C,xe}\left(\frac{P_t^{C,xe}}{P_t}\right)^{\nu_{C,xe}}\frac{C_t^{xe}}{z_t^+}+\frac{D_t^e}{z_t^+}+\psi^I\left(\frac{P_t^I}{P_t}\right)^{\nu_I}\left[\frac{I_t}{z_t^+\gamma_t}+a\left(u_t\right)\frac{K_t}{z_t^+\gamma_t}\right]+\psi^X\left(\frac{MC_t^X}{P_t}\right)^{\nu_x}\left[\frac{\overleftrightarrow{P}_t^X X_t}{z_t^+}+\phi^X\right]+\frac{G_t}{z_t^+}+\frac{I_t^G}{z_t^+\gamma_t}.$$

Expressing the above equation in *stationarized per capita* terms, we have the following equation:

$$\bar{y}_t = \psi^{C,xe}\left(p_t^{C,xe}\right)^{\nu_{C,xe}}\bar{c}_t^{xe}+\bar{d}_t^{C,e}+\psi^I\left(p_t^I\right)^{\nu_I}\left[\overline{I}_t+a\left(u_t\right)\bar{k}_t\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}\right]+\psi^X\left(\overline{mc}_t^X\right)^{\nu_x}\left[\overline{x}_t\overleftrightarrow{P}_t^X+\phi^X\right]+\bar{g}_t+\overline{I}_t^G \tag{E.5}$$

The last step to take, in order to arrive at Equation (A.136a), is to consider the implications of letting the size of the Foreign economy, ω, tend to infinity. Consider $\psi^C = \vartheta^C + \frac{1}{1+\omega}\left(1-\vartheta^C\right)$, where $\vartheta^C \in [0,1]$. Note that $\lim_{\omega\to\infty}\psi^C = \vartheta^C$. By analogous arguments, we have $\lim_{\omega\to\infty}\psi^I = \vartheta^I$ and $\lim_{\omega\to\infty}\psi^X = \vartheta^X$. Substituting for ψ^C, ψ^I and ψ^X in Equation (E.5), we have:

$$\bar{y}_t = \vartheta^{C,xe}\left(p_t^{C,xe}\right)^{\nu_{C,xe}}\bar{c}_t^{xe}+\bar{d}_t^{C,e}+\vartheta^I\left(p_t^I\right)^{\nu_I}\left[\overline{I}_t+a\left(u_t\right)\bar{k}_t\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}\right]+\vartheta^X\left(\overline{mc}_t^X\right)^{\nu_x}\left[\overline{x}_t\overleftrightarrow{P}_t^X+\phi^X\right]+\bar{g}_t+\overline{I}_t^G. \tag{E.6}$$

Equation (E.6) is the same as Equation (A.136a).

E.2 Fixed costs

In the Swedish economy, $z_t^+\phi$, $z_t^+\phi^X$ and $z_t^+\phi^{M,n}$ for $n \in \{Cxe, I, X, Ce\}$ represent real, fixed costs associated with the production, respectively, of intermediate goods, export goods and the three different types of import goods. $z_t^+\phi_F$, $z_t^+\phi_F^M$ and $z_t^+\phi_F^X$ represent corresponding fixed costs in the Foreign economy. For all of these different fixed costs, it is assumed that their value is such that along the balanced growth path, *ex post* profits are zero. In this subsection, we discuss the implication of this assumption for the value of ϕ, the stationarized fixed cost associated with the production of intermediate goods in Sweden. ϕ is chosen as an example and it should be noted that the same reasoning that applies to the value of ϕ also applies to the values of the other fixed costs mentioned here.

$Y_t(i)$ denotes the supply of good i from firm i in the Swedish intermediate good sector. $P_t(i)$ and $TC_t(i)$ represents, respectively, the price charged by firm i and the total cost of production of the same firm. Profits in period t may thus be written $P_t(i)Y_t(i) - TC_t(i)$, and real profits are

$$Y_t(i) - \frac{1}{P_t(i)}TC_t(i). \tag{E.7}$$

We assume that along a balanced growth path, enough time has elapsed that all firms charge the same price. This price will be the firms' desired price, i.e. the one that maximizes profits. We now focus on an equilibrium associated with such a balanced growth path, and we thus drop the subscript i from $P_t(i)$. Note that it follows, if profits are maximized, that $P_t = \lambda MC_t$, where λ is the (steady state) value of the desired markup and where MC_t denotes the nominal, marginal cost. Using equations (C.76) and (C.77) from Section (C.2), total costs may be written: $TC_t(i) = MC_t\left[(1-\alpha)+\alpha\alpha_K^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right]\left[Y_t(i)+z_t^+\phi\right]$. Use this expression to substitute for $TC_t(i)$ in (E.7):

$$Y_t(i) - \frac{1}{P_t}MC_t F_t(i)\left[Y_t(i)+z_t^+\phi\right]. \tag{E.8}$$

where

$$F_t(i) = \left[(1-\alpha)+\alpha\alpha_K^{\frac{1}{\upsilon_K}}\left(\frac{K_t^s(i)}{\tilde{K}_t^s(i)}\right)^{\frac{\upsilon_K-1}{\upsilon_K}}\right]$$

Using $P_t = \lambda MC_t$ to substitute for $\frac{MC_t}{P_t}$ in the above equation, we have the following equation:

$$Y_t(i) - \frac{F_t(i)}{\lambda}\left[Y_t(i) + z_t^+\phi\right] = \left(1 - \frac{F_t(i)}{\lambda}\right)Y_t(i) - \frac{F_t(i)}{\lambda}z_t^+\phi. \tag{E.9}$$

Impose zero profits and rearrange:

$$\left(\frac{\lambda - F_t(i)}{\lambda}\right)Y_t(i) - \frac{F_t(i)}{\lambda}z_t^+\phi = 0, \tag{E.10}$$

$$\phi = \left(\frac{\lambda}{F_t(i)} - 1\right)\frac{Y_t(i)}{z_t^+}. \tag{E.11}$$

Aggregate over all firms, using the definition $Y_t^P = \int_0^1 Y_t(i)di$ for total (aggregate) production of intermediate goods:

$$\int_0^1 \phi di = \phi = \int_0^1 \left(\frac{\lambda}{F_t(i)} - 1\right)\frac{Y_t(i)}{z_t^+}di = \left(\frac{\lambda}{F_t} - 1\right)\frac{Y_t^P}{z_t^+}. \tag{E.12}$$

From the previous subsection we have $Y_t^P = Y_t \overset{\leftrightarrow}{P}_t$, where $\overset{\leftrightarrow}{P}_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{\lambda_t}{1-\lambda_t}} di$. Given, however, that we consider the case (along a balanced growth path) where all intermediate good firms charge the same price P_t, we have $Y_t^P = Y_t$ and thus $\phi = \left(\frac{\lambda}{F_t} - 1\right)\frac{Y_t}{z_t^+}$. Using our notation for stationarized variables expressed in *per capita* terms, this may be stated: $\phi = \left(\frac{\lambda}{F_t} - 1\right)\overline{y}_t$. This expression indicates that $\overline{y}_t$ is a constant, and this is indeed the case along a balanced growth path, where Swedish output (GDP) grows at the constant rate $\mu_{z^+} = \frac{z_t^+}{z_{t-1}^+}$. We may therefore write $\phi = \left(\frac{\lambda}{F} - 1\right)\overline{y}$. By analogous reasoning, the following results may be obtained: $\phi^X = \left(\lambda^X - 1\right)\overline{x}$, $\phi^{M,n} = \left(\lambda^{M,n} - 1\right)\overline{m}^n$ for $n \in \{\{C, xe\}, I, X, \{C, e\}\}$, $\phi_F = \left(\lambda_F - 1\right)\overline{y}_F$, [80]

E.3 Imports and exports

This section contains derivations of the expressions for Swedish imports and exports. Because one country's imports is the other countries exports, this is also a treatment of Foreign exports and imports. The first part of this section focuses on Swedish imports, while a second part contains the derivations of an expression for Swedish exports.

E.3.1 Swedish imports of consumption goods

The total demand for the homogeneous imported intermediate good used in the production of non-energy consumption goods is denoted $M_t^{P,C,xe}$. This must equal the production of these goods minus the fixed costs:

$$M_t^{P,Cxe} = \int_0^1 \left[M_t^{C,xe}(i)\right] di \tag{E.13}$$

From Section 2.4.2 we have $M_t^{C,xe}(i) = \left[\frac{P_t^{M,C,xe}(i)}{P_t^{M,C,xe}}\right]^{\frac{\lambda_t^{M,C,xe}}{1-\lambda_t^{M,Cxe}}} M_t^{C,xe}$, where $P_t^{M,C,xe}(i)$ is the price charged by firm i and where $P_t^{M,C,xe}$ is the price index of the homogeneous Swedish import good. Now let $\overset{\leftrightarrow}{P}_t^{M,C,xe} = \int_0^1 \left(\frac{P_t^{M,C,xe}(i)}{P_t^{M,C,xe}}\right)^{\frac{\lambda_t^{M,C,xe}}{1-\lambda_t^{M,C,xe}}} di$ be a measure of price dispersion in the sector for Swedish import goods. Thus, the total demand for the homogeneous imported intermediate good used in the production of consumption goods can be written as:

$$M_t^{P,Cxe} = \int_0^1 M_t^{C,xe}(i)di = \int_0^1 \left[\frac{P_t(i)^{M,C,xe}}{P_t^{M,C,xe}}\right]^{\frac{\lambda_t^{M,C,xe}}{1-\lambda_t^{M,C,xe}}} M_t^{C,xe}di = \overset{\leftrightarrow}{P}_t^{M,C,xe}M_t^{C,xe}. \tag{E.14}$$

Recall from Equation (C.107), we have the following demand function for imported consumption goods:

[80] There are four different types of import firms in the Swedish economy, as described in Section (2.4.2) in the main text. For each of these three types, there exists an exogenous markup $\lambda_t^{M,n}$, that fluctuates stochastically around its long-run (unconditional) mean. Three markup shocks in the Swedish import sector are assumed to share the same unconditional mean. Also, note that $\overline{m}^n$ refers to the total demand for Swedish import good of type n, and that $\overline{m}_F^D$ denotes total demand (from Foreign consumption good producers and Foreign export firms) of the homogenous Foreign import good.

$$M_t^{Cxe} = \left(1-\psi^{C,xe}\right)\left(\frac{P_t^{C,xe}}{P_t^{M,Cxe}}\right)^{\nu_{C,xe}} C_t^{xe}.$$

Substituting the above demand function for imported consumption goods into Equation (E.14), we have the following equation:

$$\overleftrightarrow{P}_t^{M,C,xe} M_t^{C,xe} = \overleftrightarrow{P}_t^{M,C,xe}\left(1-\psi^{C,xe}\right)\left(\frac{P_t^{C,xe}}{P_t^{M,C,xe}}\right)^{\nu_{C,xe}} C_t^{xe}. \tag{E.15}$$

Using the following definition $p_t^{M,C,xe} = \frac{P_t^{M,C,xe}}{P_t}$ and Equation (C.109) which captures the consumption good price index, we can obtain the following equation for price ratio:

$$\begin{aligned}
\frac{P_t^{C,xe}}{P_t^{M,C,xe}} &= \frac{1}{P_t^{M,C,xe}}\left[\psi^{C,xe}\left(P_t\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C}\right)\left(P_t^{M,C,xe}\right)^{1-\nu_{C,xe}}\right]^{\frac{1}{1-\nu_{C,xe}}}, \\
\frac{P_t^{C,xe}}{P_t^{M,C,xe}} &= \left[\psi^{C,xe}\left(\frac{P_t}{P_t^{M,C,xe}}\right)^{1-\nu_{C,xe}} + \left(1-\psi^{C,xe}\right)\left(\frac{P_t^{M,C,xe}}{P_t^{M,C,xe}}\right)^{1-\nu_{C,xe}}\right]^{\frac{1}{1-\nu_{C,xe}}}, \\
\frac{P_t^{C,xe}}{P_t^{M,C,xe}} &= \left[\psi^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1-\psi^{C,xe}\right]^{\frac{1}{1-\nu_{C,xe}}}.
\end{aligned} \tag{E.16}$$

Using Equation (E.16), Equation (E.15) can be written as follows:

$$\overleftrightarrow{P}_t^{M,C,xe} M_t^{C,xe} = \left(1-\psi^{C,xe}\right)\left[\psi^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1-\psi^{C,xe}\right]^{\frac{\nu_{C,xe}}{1-\nu_{C,xe}}} C_t^{xe} \tag{E.17}$$

We stationarize this expression by dividing both sides by z_t^+ and express the above equation in *per capita* terms. Thus, we have the following equation:

$$\overleftrightarrow{P}_t^{M,C,xe} \overline{m}_t^{C,xe} = \left(1-\psi^{C,xe}\right)\left[\psi^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1-\psi^{C,xe}\right]^{\frac{\nu_{C,xe}}{1-\nu_{C,xe}}} \overline{c}_t^{xe}. \tag{E.18}$$

Next, consider the limit as ω tends to infinity. Recall from Section (E.1.2) that $\lim_{\omega\to\infty}\psi^C = \vartheta^C$. Then we can rewrite the above equation as:

$$\overleftrightarrow{P}_t^{M,C.xe} \overline{m}_t^{C,xe} = \left(1-\vartheta^{C,xe}\right)\left[\vartheta^{C,xe}\left(p_t^{M,C,xe}\right)^{\nu_{C,xe}-1} + 1-\vartheta^{C,xe}\right]^{\frac{\nu_{C,xe}}{1-\nu_{C,xe}}} \overline{c}_t^{xe}. \tag{E.19}$$

Note that Equation (E.19) is the same as Equation (A.140a) in Section A.9.

E.3.2 Swedish imports of investment goods

The total demand for the homogeneous imported intermediate good used in the production of investment goods is denoted $M_t^{P,I}$. This must equal the production of these goods minus the fixed costs:

$$M_t^{P,I} = \int_0^1 \left[M_t^I(i)\right] di \tag{E.20}$$

From Section 2.4.2 we have $M_t^I(i) = \left[\frac{P_t^{M,I}(i)}{P_t^{M,I}}\right]^{\frac{\lambda_t^{M,I}}{1-\lambda_t^{M,I}}} M_t^I$, where $P_t^{M,I}(i)$ is the price charged by firm i and where $P_t^{M,I}$ is the price index of the homogeneous Swedish import good. Now let $\overleftrightarrow{P}_t^{M,I} = \int_0^1 \left(\frac{P_t^{M,I}(i)}{P_t^{M,I}}\right)^{\frac{\lambda_t^{M,I}}{1-\lambda_t^{M,I}}} di$ be a measure of price dispersion in the sector for Swedish import goods. Thus, the total demand for the homogeneous imported intermediate good used in the production of investment goods can be written as:

$$M_t^{P,I} = \int_0^1 M_t^I(i) di = \int_0^1 \left[\frac{P_t(i)^{M,I}}{P_t^{M,I}}\right]^{\frac{\lambda_t^{M,I}}{1-\lambda_t^{M,I}}} M_t^I di = \overleftrightarrow{P}_t^{M,I} M_t^I. \tag{E.21}$$

Recall from Equation(C.120), we have the following demand function for imported investment goods:

$$M_t^I = (1-\psi^I)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{E.22}$$

Substituting the above demand function into Equation (E.21), we have the following equation:

$$\overset{\leftrightarrow}{P}_t^{M,I} M_t^I = \overset{\leftrightarrow}{P}_t^{M,I}\left(1-\psi^I\right)\left(\frac{P_t^I}{P_t^{M,I}}\right)^{\nu_I}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{E.23}$$

Using the definition $p_t^{M,I} = \frac{P_t^{M,I}}{P_t}$ and Equation (C.121) which shows the investment good price index, we can obtain the following price ratio:

$$\frac{P_t^I}{P_t^{M,I}} = \frac{1}{P_t^{M,I}}\left[\psi^I (P_t)^{1-\nu_I} + \left(1-\psi^I\right)\left(P_t^{M,I}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}},$$

$$\frac{P_t^I}{P_t^{M,I}} = \left[\psi^I\left(\frac{P_t}{P_t^{M,I}}\right)^{1-\nu_I} + \left(1-\psi^I\right)\left(\frac{P_t^{M,I}}{P_t^{M,I}}\right)^{1-\nu_I}\right]^{\frac{1}{1-\nu_I}},$$

$$\frac{P_t^I}{P_t^{M,I}} = \left[\psi^I\left(p_t^{M,I}\right)^{\nu_I-1} + 1 - \psi^I\right]^{\frac{1}{1-\nu_I}}.$$

We substitute the above price ratio equation into Equation (E.23). Thus, we have the following equation:

$$\overset{\leftrightarrow}{P}_t^{M,I} M_t^I = \left(1-\psi^I\right)\left[\psi^I\left(p_t^{M,I}\right)^{\nu_I-1} + 1 - \psi^I\right]^{\frac{\nu_I}{1-\nu_I}}\frac{1}{\gamma_t}\left[I_t + a(u_t)K_t\right]. \tag{E.24}$$

We stationarize the above expression by dividing both sides by z_t^+ and then express the above equation in *per capita* terms. Thus, we have the following equation:

$$\overset{\leftrightarrow}{P}_t^{M,I}\overline{m}_t^I = \left(1-\psi^I\right)\left[\psi^I\left(p_t^{M,I}\right)^{\nu_I-1} + 1 - \psi^I\right]^{\frac{\nu_I}{1-\nu_I}}\left[\bar{I}_t + a(u_t)\bar{k}_t\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}\right] \tag{E.25}$$

Next, consider the limit as ω tends to infinity. Recall from Section (E.1.2) that $\lim_{\omega\to\infty}\psi^I = \vartheta^I$. Then we can write as:

$$\overset{\leftrightarrow}{P}_t^{M,I}\overline{m}_t^I = \left(1-\vartheta^I\right)\left[\vartheta^I\left(p_t^{M,I}\right)^{\nu_I-1} + 1 - \vartheta^I\right]^{\frac{\nu_I}{1-\nu_I}}\left[\bar{I}_t + a(u_t)\bar{k}_t\frac{1}{\mu_{z^+,t}\mu_{\gamma,t}}\right] \tag{E.26}$$

Note that Equation (E.26) is the same as Equation (A.141a) in Section A.9.

E.3.3 Swedish imports of export goods

The total demand for the homogeneous imported intermediate good used in the production of export goods is denoted $M_t^{P,X}$. This must equal the production of these goods minus the fixed costs:

$$M_t^{P,X} = \int_0^1 \left[M_t^X(i)\right] di \tag{E.27}$$

From Section 2.4.2 we have $M_t^X(i) = \left[\frac{P_t^{M,X}(i)}{P_t^{M,X}}\right]^{\frac{\lambda_t^{M,X}}{1-\lambda_t^{M,X}}} M_t^X$, where $P_t^{M,X}(i)$ is the price charged by firm i and where $P_t^{M,X}$ is the price index of the homogeneous Swedish import good. Now let $\overset{\leftrightarrow}{P}_t^{M,X} = \int_0^1\left(\frac{P_t^X(i)}{P_t^{M,X}}\right)^{\frac{\lambda_t^{M,X}}{1-\lambda_t^{M,X}}} di$ be a measure of price dispersion in the sector for Swedish import goods. Thus, the total demand for the homogeneous imported intermediate good used in the production of export goods can be written as:

$$M_t^{P,X} = \int_0^1 M_t^X(i)di = \int_0^1\left[\frac{P_t(i)^{M,X}}{P_t^{M,X}}\right]^{\frac{\lambda_t^{M,I}}{1-\lambda_t^{M,I}}} M_t^X di = \overset{\leftrightarrow}{P}_t^{M,X} M_t^X. \tag{E.28}$$

Recall from Equation (C.141), we have the following demand function for imported goods for export production:

$$M_t^X(i) = \left(1-\psi^X\right)\left(\frac{MC_t^X}{P_t^{M,X}}\right)^{\nu_x}\left[X_t(i) + z_t^+\phi^X\right]. \tag{E.29}$$

Substituting the above demand function into Equation (E.28), we have the following equation:

$$\overleftrightarrow{P}_t^{M,X} M_t^X = \overleftrightarrow{P}_t^{M,X} \left(1-\psi^X\right) \left(\frac{MC_t^X}{P_t^{M,X}}\right)^{\nu_X} \int_0^1 \left[X_t(i) + z_t^+ \phi^X\right] di. \tag{E.30}$$

Using the definition $p_t^{M,X} = \frac{P_t^{M,X}}{P_t}$ and Equation (C.137) which captures nominal marginal cost of export production , we can obtain the following marginal cost of export production ratio:

$$\begin{aligned}
\frac{MC_t^X}{P_t^{M,X}} &= \frac{1}{P_t^{M,X}} \left[\psi^X \left(P_t\right)^{(1-\nu_x)} + \left(1-\psi^X\right)\left(P_t^{M,X}\right)^{(1-\nu_x)}\right]^{\frac{1}{1-\nu_x}}, \\
\frac{MC_t^X}{P_t^{M,X}} &= \left[\psi^X \left(\frac{P_t}{P_t^{M,X}}\right)^{1-\nu_x} + \left(1-\psi^X\right)\left(\frac{P_t^{M,X}}{P_t^{M,X}}\right)^{1-\nu_x}\right]^{\frac{1}{1-\nu_x}}, \\
\frac{MC_t^X}{P_t^{M,X}} &= \left[\psi^X \left(p_t^{M,X}\right)^{\nu_x-1} + 1-\psi^X\right]^{\frac{1}{1-\nu_x}}.
\end{aligned}$$

Defining the price dispersion of export goods as $\overleftrightarrow{P}_t^X$, and using the above marginal cost of export production ratio, we can rewrite Equation (E.30) as follows:

$$\overleftrightarrow{P}_t^{M,X} M_t^X = \left(1-\psi^X\right) \left[\psi^X \left(p_t^{M,X}\right)^{\nu_x-1} + 1-\psi^X\right]^{\frac{1}{1-\nu_x}} \left[\overleftrightarrow{P}_t^X X_t + z_t^+ \phi^X\right]. \tag{E.31}$$

We stationarize this expression by dividing both sides by z_t^+ and express the above equation in *per capita* terms. Thus, we have the following equation:

$$\overleftrightarrow{P}_t^{M,X} \overline{m}_t^X = \left(1-\psi^X\right) \left[\psi^X \left(p_t^{M,X}\right)^{\nu_x-1} + 1-\psi^X\right]^{\frac{1}{1-\nu_x}} \left[\overleftrightarrow{P}_t^X \overline{x}_t + \phi^X\right]. \tag{E.32}$$

Next, consider the limit as ω tends to infinity. Recall from Section (E.1.2) that $\lim_{\omega\to\infty} \psi^X = \vartheta^X$. Then we can write as follows:

$$\overleftrightarrow{P}_t^{M,X} \overline{m}_t^X = \left(1-\vartheta^X\right) \left[\psi^X \left(p_t^{M,X}\right)^{\nu_x-1} + 1-\psi^X\right]^{\frac{1}{1-\nu_x}} \left[\overleftrightarrow{P}_t^X \overline{x}_t + \phi^X\right] \tag{E.33}$$

Note that Equation (E.33) is the same as Equation (A.142a) in Section A.9.

E.3.4 Total Swedish non-energy imports

Total demand for Swedish imports from Foreign is given by

$$M_t^{xe} = \overleftrightarrow{P}_t^{M,C,xe} M_t^{C,xe} + \overleftrightarrow{P}_t^{M,I} M_t^I + \overleftrightarrow{P}_t^{M,X} M_t^X + z_t^+ \phi^{M,C,xe} + z_t^+ \phi^{M,I} + z_t^+ \phi^{M,X}. \tag{E.34}$$

We stationarize the above expression by dividing both sides by z_t^+ and express the above equation in *per capita* terms.

$$\overline{m}_t^{xe} = \overleftrightarrow{P}_t^{M,C} \overline{m}_t^C + \overleftrightarrow{P}_t^{M,I} \overline{m}_t^I + \overleftrightarrow{P}_t^{M,X} \overline{m}_t^X + \phi^{C,xe} + \phi^{M,I} + \phi^{M,X} \tag{E.35}$$

Note that Equation (E.35) is the same is Equation (A.143a) in Section A.9.
It is also useful to have an equation of total import demand $\overline{m}_t^{D,xe}$, that is, the amount of import goods which are used as intermediate goods in the other sectors in the economy. This expression is given by removing the fixed cost:

$$\overline{m}_t^{D,xe} = \overleftrightarrow{P}_t^{M,C} \overline{m}_t^C + \overleftrightarrow{P}_t^{M,I} \overline{m}_t^I + \overleftrightarrow{P}_t^{M,X} \overline{m}_t^X. \tag{E.36}$$

Note that Equation (E.36) is the same is Equation (A.144a) in Section A.9.

E.3.5 Swedish exports

We turn now to the discussion of Swedish exports, and the demand for Swedish export goods in Foreign.

Since we allow the two economies, Sweden and Foreign, to potentially grow at different paces via $z_t^+$and $z_{F,t}^+$, we also need to make some additional assumptions about the weights of Swedish export goods in the production of Foreign, to assure that Swedish exports grow at the same rate as output on the balanced growth path. More specifically, we need to let the weights $\psi_{F,t}^I$ and $\psi_{F,t}^C$ vary over time. We abstract from the time varying weights in the main text to make it more easy for the reader to follow, since $\lim_{\omega\to\infty}\psi_{F,t}^I = 1$ and $\lim_{\omega\to\infty}\psi_{F,t}^{C,xe} = 1$. The demand for Swedish exports that goes to Foreign investment is given by

$$X_t^I = \left(1-\psi_{F,t}^I\right)\left(\frac{P_t^X}{P_{F,t}^I}\right)^{-\nu_{F,I}} I_{F,t} \tag{E.37}$$

where ψ_F^I is the share of Swedish exports in Foreign investment good production. P_t^X is the price of Swedish export goods in Foreign currency, $P_{F,t}^I$ is the price of the Foreign investment good and $I_{F,t}$ is Foreign investment.

Similarly, the demand for Swedish exports that goes to Foreign consumption is given by

$$X_t^C = \left(1-\psi_{F,t}^{C,xe}\right)\left(\frac{P_t^X}{P_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} C_{F,t}^{xe} \tag{E.38}$$

where $\psi_{F,t}^{C,xe}$ is the share of Swedish exports in Foreign non-energy consumption good production. P_t^X is the price of Swedish export goods in Foreign currency, $P_{F,t}^{C,xe}$ is the price of the Foreign non-energy good and $C_{F,t}^{xe}$ is Foreign non-energy consumption.

This means that total demand for exports is given by

$$X_t = \left(1-\psi_{F,t}^{C,xe}\right)\left(\frac{P_t^X}{P_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} C_{F,t}^{xe} + \left(1-\psi_{F,t}^I\right)\left(\frac{P_t^X}{P_{F,t}^I}\right)^{-\nu_{F,I}} I_{F,t} \tag{E.39}$$

We stationarize this equation by dividing through with $z_t^+$and divide the prices with $P_{F,t}$, using the definitions $\widetilde{p}_t^X = \frac{P_t^X}{P_{F,t}}$, $p_{F,t}^{C,xe} = \frac{P_{F,t}^{C,xe}}{P_{F,t}}$ and $p_{F,t}^I = \frac{P_{F,t}^I}{P_{F,t}}$ to get

$$\frac{X_t}{z_t^+} = \left(1-\psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \frac{C_{F,t}^{xe}}{z_t^+} + \left(1-\psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \frac{I_{F,t}}{z_t^+}$$

$$\frac{X_t}{z_t^+} = \left(1-\psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \frac{C_{F,t}^{xe}}{z_{F,t}^+}\frac{z_{F,t}^+}{z_t^+} + \left(1-\psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \frac{I_{F,t}}{z_{F,t}^+}\frac{z_{F,t}^+}{z_t^+} \tag{E.40}$$

$$\frac{X_t}{z_t^+} = \frac{\omega}{\omega}\left(1-\psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \frac{C_{F,t}^{xe}}{z_{F,t}^+}\frac{z_{F,t}^+}{z_t^+} + \frac{\omega}{\omega}\left(1-\psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \frac{I_{F,t}}{z_{F,t}^+}\frac{z_{F,t}^+}{z_t^+} \tag{E.41}$$

$$\frac{X_t}{z_t^+} = \frac{\omega}{\omega}\left(1-\psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \frac{C_{F,t}^{xe}}{z_{F,t}^+}\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} + \frac{\omega}{\omega}\left(1-\psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \frac{I_{F,t}}{z_{F,t}^+}\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} \tag{E.42}$$

Write it in per-capita form (using the fact that the size of the population in Sweden is 1 and the size of the population in Foreign is ω):

$$\bar{x}_t = \omega\left(1-\psi_{F,t}^{C,xe}\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe}\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} + \omega\left(1-\psi_{F,t}^I\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}} \bar{I}_{F,t}\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} \tag{E.43}$$

Now, we let $\psi_{F,t}^{C,xe} = \tilde{\vartheta}_{F,t}^{C,xe} + \frac{\omega}{1+\omega}(1-\tilde{\vartheta}_{F,t}^{C,xe})$ and $\psi_{F,t}^I = \tilde{\vartheta}_{F,t}^I + \frac{\omega}{1+\omega}(1-\tilde{\vartheta}_{F,t}^I)$ where $\tilde{\vartheta}_{F,t}^{C,xe}$ and $\tilde{\vartheta}_{F,t}^I$ denotes the home-bias in the production functions of Foreign exports and investment. Using these expressions we can write

$$\begin{aligned}\bar{x}_t =& \gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)}\left[\omega\left(1-\tilde{\vartheta}_{F,t}^{C,xe}-\frac{\omega}{1+\omega}(1-\tilde{\vartheta}_{F,t}^{C,xe})\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}}\right)^{-\nu_{F,C,xe}}\bar{c}_{F,t}^{xe}\right] \\ &+\gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)}\left[\omega\left(1-\tilde{\vartheta}_{F,t}^I-\frac{\omega}{1+\omega}(1-\tilde{\vartheta}_{F,t}^I)\right)\left(\frac{\widetilde{p}_t^X}{p_{F,t}^I}\right)^{-\nu_{F,I}}\bar{I}_{F,t}\right]\end{aligned} \tag{E.44}$$

or

$$\bar{x}_t = \gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} \left[\left(\frac{\omega(1+\omega)-\omega^2}{1+\omega}(1-\tilde{\vartheta}_{F,t}^{C,xe}) \right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}} \right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe} + \left(\frac{\omega(1+\omega)-\omega^2}{1+\omega}(1-\tilde{\vartheta}_{F,t}^{I}) \right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{I}} \right)^{-\nu_{F,I}} \bar{I}_{F,t} \right]$$

Note that since $\omega \to \infty$,we can use l'hôspital's rule, which means that

$$lim_{\omega\to\infty} \frac{\omega(1+\omega)-\omega^2}{1+\omega} = \frac{1+2\omega-2\omega}{1} = 1$$

Hence, we can write the above expression as

$$\bar{x}_t = \gamma_t^{\alpha_F/(1-\alpha_F)-\alpha/(1-\alpha)} \left[\left(1-\tilde{\vartheta}_{F,t}^{C,xe}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}} \right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe} + \left(1-\tilde{\vartheta}_{F,t}^{I}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{I}} \right)^{-\nu_{F,I}} \bar{I}_{F,t} \right].$$

Now, we make the two following restrictive assumptions above the home bias processes:

$$\tilde{\vartheta}_{F,t}^{C,xe} = 1-(1-\vartheta_{F,t}^{C,xe})\gamma_t^{-\alpha_F/(1-\alpha_F)+\alpha/(1-\alpha)}$$
$$\tilde{\vartheta}_{F,t}^{I} = 1-(1-\vartheta_{F,t}^{I})\gamma_t^{-\alpha_F/(1-\alpha_F)+\alpha/(1-\alpha)}$$

These imply that if the global economy grows faster than the Swedish economy due to the investment technology process, then the Swedish market share of the global economy will shrink over time and vice versa. Inserting these two equations into the demand for export gives us the following stationarized export demand function:

$$\bar{x}_t = \left(1-\vartheta_{F}^{C,xe}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{C,xe}} \right)^{-\nu_{F,C,xe}} \bar{c}_{F,t}^{xe} + \left(1-\vartheta_{F}^{I}\right) \left(\frac{\widetilde{p}_t^X}{p_{F,t}^{I}} \right)^{-\nu_{F,I}} \bar{I}_{F,t}. \tag{E.45}$$

Equation (E.45) is the same as Equation (A.139a) in Appendix A.

E.4 Swedish aggregate output

E.4.1 Swedish aggregate output

Swedish aggregate output is given by:

$$\overleftrightarrow{P}_t Y_t = \int_0^1 \left(\varepsilon_t \left[\tilde{K}_t^s(i) \right]^\alpha \left[z_t N_t(i) \right]^{1-\alpha} - z_t^+ \phi \right) di. \tag{E.46}$$

We stationarize the above equation by dividing the equation with z_t^+. Note that $z_t^+ = z_t \gamma_t^{\frac{\alpha}{1-\alpha}}$, and the definition of stationarized capital services is given by: $\bar{\tilde{k}}_t^s(i) = \tilde{K}_t^s(i)/(z_{t-1}\gamma_{t-1}^{\frac{1}{1-\alpha}}) = \tilde{K}_t^s(i)/\left[(z_t\gamma_t^{\frac{1}{1-\alpha}})/(\mu_{z,t}\mu_{\gamma,t}^{\frac{1}{1-\alpha}})\right]$. This means that if we divide both sides of the Swedish aggregate output expression with z_t^+ , we have the following equation:

$$\frac{Y_t}{z_t^+} \overleftrightarrow{P}_t = \int_0^1 \left(\varepsilon_t \left[\frac{\tilde{K}_t^s(i)}{(z_t\gamma_t^{\frac{1}{1-\alpha}})/(\mu_{z,t}\mu_{\gamma,t}^{\frac{1}{1-\alpha}})} \right]^\alpha \left[\frac{N_t(i)}{z_t^+} \right]^{1-\alpha} \right) di - \phi,$$

$$\overline{Y}_t \overleftrightarrow{P}_t = \int_0^1 \left(\varepsilon_t \left[\frac{\bar{\tilde{K}}_t^s(i)}{\mu_{z^+,t}\mu_{\gamma,t}} \right]^\alpha L_t(i)^{1-\alpha} \right) di - \phi$$

We can remove the subscript i since it is shown above that the firms in Sweden choose the same capital stock. We rewrite the above equation in terms of stationarized variables in per-capita:

$$\overline{y}_t \overleftrightarrow{P}_t = \left(\varepsilon_t \left[\frac{\bar{\tilde{k}}_t^s}{\mu_{z^+,t}\mu_{\gamma,t}} \right]^\alpha n_t^{1-\alpha} \right) - \phi \tag{E.47}$$

Note that Equation (E.47) is the same as Equation (A.148a) in Section A.

E.4.2 Measured Swedish aggregate output

The measured Swedish aggregate input is given by

$$Y_t^m = Y_t - \psi^I \left(\frac{P_t^I}{P_t}\right)^{\nu_I} a\left(u_t\right) \frac{K_t}{\gamma_t}. \tag{E.48}$$

We follow Section E.4.1. In particular, we divide both sides of the above equation by z_t^+ and express the equation in per-capita terms. This yields:

$$\overline{y}_t^m = \overline{y}_t - \vartheta^I (p_t^I)^{\nu_I} a(u_t) \frac{\overline{k}_t}{\mu_{z^+,t}\mu_{\gamma,t}}. \tag{E.49}$$

Note that since $\omega \to \infty$ and $\psi^I = \vartheta^I + \frac{1}{1+\omega}(1-\vartheta^I)$,we get that $\psi^I = \vartheta^I$. Note that Equation (E.49) is the same as Equation (A.149a) in Section A.

E.5 Foreign aggregate resource constraint

The derivation of the aggregate resource constraint for Foreign proceeds in much the same way as the corresponding derivations for Sweden, which were laid out in Section (E.1) above. In the first part of this section, we derive the non-stationary version of the Foreign aggregate resource constraint, which corresponds to Equation (96) in the main text. In the second part of the section, a stationarized version is derived (Equation A.137a).

E.5.1 Market clearing in Foreign

The market for intermediate goods in Foreign clears when the production of each individual firm j, $Y_{F,t}(j)$, equals the demand for the output of the same firm: $Y_{F,t}(j) = \frac{1}{\omega}\left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} Y_{F,t}$. $Y_{F,t}$ represents aggregate demand for the homogeneous, intermediate good in Foreign, and $P_{F,t}$ is the associated aggregate price index. $P_{F,t}(j)$ denotes the price charged by the individual firm j and $\lambda_{F,t}$ is a time varying markup. After having defined $Y_{F,t}^P = \int_0^\omega Y_{F,t}(j)dj$ as aggregate production of intermediate goods and $\overleftrightarrow{P}_{F,t} = \int_0^\omega \frac{1}{\omega}\left(\frac{P_{F,t}(j)}{P_{F,t}}\right)^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} dj$ as a measure of price dispersion, we proceed by aggregating over all firms in the sector:

$$Y_{F,t}^P = \int_0^\omega \left\{\frac{1}{\omega}\left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} Y_{F,t}\right\} dj = Y_{F,t}\int_0^\omega \frac{1}{\omega}\left[\frac{P_{F,t}(j)}{P_{F,t}}\right]^{\frac{\lambda_{F,t}}{1-\lambda_{F,t}}} dj = Y_{F,t}\overleftrightarrow{P}_{F,t}. \tag{E.50}$$

Recall from Section (2.6.2) that $Y_{F,t}(j) = \varepsilon_{F,t}\left[\tilde{K}_{F,t}^s(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z_{F,t}^+\phi_F$. Once again aggregating over all firms in the sector, write:

$$Y_{F,t}^P = \int_0^\omega \left(\varepsilon_{F,t}\left[\tilde{K}_{F,t}^s(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} - z_{F,t}^+\phi_F\right)dj = Y_{F,t}\overleftrightarrow{P}_{F,t}. \tag{E.51}$$

Turn now to the demand side of the economy. In Foreign, demand for the homogeneous intermediate good comes from non-energy consumption good producers, energy consumption good producers and investment good producers:

$$Y_{F,t} = D_{F,t}^{C,xe} + D_{F,t}^{C,e} + D_{F,t}^I + G_{F,t} \tag{E.52}$$

From Equation (D.51), we have $D_{F,t}^{C,xe} = C_{F,t}^{xe}$ represents demand from non-energy foreign consumption good production. Similarly, demand for intermediate goods for foreign energy consumption production is $D_{F,t}^{C,e} = C_{F,t}^e$ and from Equation (D.56), we have $D_{F,t}^I = V_{F,t}^I$ represents demand from foreign investment good production where $V_{F,t}^I = \frac{1}{\gamma_{F,t}}\left[I_{F,t} + a(u_t)K_{F,t}\right]$.

Continue by substituting for $D_{F,t}^{C,xe}$ and $D_{F,t}^I$ in Equation (E.52). We have:

$$Y_{F,t} = C_{F,t}^{xe} + C_{F,t}^e + V_{F,t}^I + G_{F,t}. \tag{E.53}$$

Now use Equation (E.53) to substitute for $Y_{F,t}$ in (E.50), and then combine this expression with (E.51) to get:

$$Y_{F,t}^P = \int_0^\omega \varepsilon_{F,t}\left[\tilde{K}_{F,t}^s(j)\right]^{\alpha_F}\left[z_t N_{F,t}(j)\right]^{1-\alpha_F} dj - z_{F,t}^+\omega\phi_F =$$

$$\overleftrightarrow{P}_{F,t}C_{F,t}^{xe} + \overleftrightarrow{P}_{F,t}C_{F,t}^e + \overleftrightarrow{P}_{F,t}\frac{1}{\gamma_{F,t}}\left[I_{F,t} + a(u_t)K_{F,t}\right] + \overleftrightarrow{P}_{F,t}G_{F,t}.$$

Rearrange slightly to get the aggregate resource constraint for Foreign, the same as Equation (96) in the main text:[81]

$$\varepsilon_{F,t}\left[K_{F,t}^{s}\right]^{\alpha_F}\left[z_t N_{F,t}\right]^{1-\alpha_F} = \overleftrightarrow{P}_{F,t} C_{F,t}^{xe} + \overleftrightarrow{P}_{F,t} C_{F,t}^{e} \tag{E.54}$$

$$+ \overleftrightarrow{P}_{F,t} \frac{1}{\gamma_{F,t}}\left[I_{F,t} + a(u_t) K_{F,t}\right] + \overleftrightarrow{P}_{F,t} G_{F,t} + z_{F,t}^{+}\omega\phi_F.$$

E.5.2 Stationarizing and simplifying the Foreign aggregate resource constraint

The following definitions are needed to stationarize Equation (E.54), and to express it in per capita values: $\bar{y}_{F,t} = \frac{Y_{F,t}}{z_{F,t}^{+}\omega}$, $\bar{c}_{F,t}^{xe} = \frac{C_{F,t}^{xe}}{z_{F,t}^{+}\omega}$, $\bar{c}_{F,t}^{e} = \frac{C_{F,t}^{e}}{z_{F,t}^{+}\omega}$, $\bar{g}_{F,t} = \frac{G_{F,t}}{z_{F,t}^{+}\omega}$, and $\bar{I}_{F,t} = \frac{I_{F,t}}{\omega z_t \gamma_t^{\frac{1}{1-\alpha_F}}}$. Also, let $\bar{k}_{F,t}^{s} = \frac{K_{F,t}^{s}}{z_{t-1}\gamma_{t-1}^{\frac{1}{1-\alpha_F}}\omega}$ and $n_{F,t} = \frac{N_{f,t}}{\omega}$. Moreover, use $Y_{F,t}^{P} = Y_{F,t}\overleftrightarrow{P}_{F,t}$ Using these definitions, and then dividing through by $z_{F,t}^{+}\omega$, we get:

$$\bar{y}_{F,t} = \bar{c}_{F,t}^{xe} + \bar{c}_{F,t}^{e} + \bar{I}_{F,t} + a\left(u_{F,t}\right)\bar{k}_{F,t}\frac{1}{\mu_{z_F^{+},t}\mu_{\gamma,t}} + \bar{g}_{F,t} \tag{E.55}$$

E.6 Balance of payments and net foreign assets

Section (2.7.4) in the main text established the balance of payments identity of Sweden (Equation 108), which we reproduce here for convenience:

$$S_t P_t^X X_t - S_t P_{Ft} M_t^{xe} - S_t P_{Ft}^{C,e} M_t^e = \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t\Phi(\bar{a}_t, s_t, \widetilde{\phi}_t)} - S_t B_t^{FH}. \tag{E.56}$$

Note that the prices of imported goods are the same as the marginal costs of the import firms, $S_t P_{Ft}^{C,e} = MC_t^{M,C,e}$ and $S_t P_{Ft} = MC_t^{M,xe}$. Therefore, we can write the expression as the following:

$$S_t P_t^X X_t - MC_t^{M,xe} M_t^{xe} - MC_t^{M,C,e} M_t^e = \frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t\Phi(\bar{a}_t, s_t, \widetilde{\phi}_t)} - \frac{S_t}{S_{t-1}} S_{t-1} B_t^{FH}.$$

Now substitute A_t for $\frac{S_t B_{t+1}^{FH}}{R_{F,t}\zeta_t\Phi(\bar{a}_t, s_t, \widetilde{\phi}_t)}$, as well as A_{t-1} for $\frac{S_{t-1} B_t^{FH}}{R_{F,t-1}\zeta_{t-1}\Phi(\bar{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1})}$. Then subtract A_{t-1} from both sides to get:

$$A_t - A_{t-1} = S_t P_t^X X_t - MC_t^{M,xe} M_t^{xe} - MC_t^{M,C,e} M_t^e + \Phi\left(\bar{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right) R_{F,t-1}\zeta_{t-1}\frac{S_t}{S_{t-1}} A_{t-1} - A_{t-1}.$$

Simplify the last term to arrive at:

$$A_t - A_{t-1} = S_t P_t^X X_t - MC_t^{M,xe} M_t^{xe} - MC_t^{M,C,e} M_t^e + \left[\Phi\left(\bar{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right) R_{F,t-1}\zeta_{t-1}\frac{S_t}{S_{t-1}} - 1\right] A_{t-1},$$

which is the same as Equation (109) in the main text.

Let $\bar{a}_t = \frac{A_t}{z_t^{+} P_t}$ denote the stationarized net foreign asset position of the Swedish economy, and use the following definitions to rewrite the above condition: $\widetilde{p}_{F,t}^{X} = \frac{P_{F,t}^{X}}{P_t}$, $\bar{mc}_t^{M,C,xe} = \frac{MC_t^{M,xe}}{P_t}$, $\overline{m}_t^{xe} = \frac{M_t}{z_t^{+}}$, $\bar{mc}_t^{M,C,e} = \frac{MC_t^{M,C,e}}{P_t}$, $\bar{m}_t^{e} = \frac{M_t^{;e}}{z_t^{+}}$, $p_t^X = \frac{S_t P_t^X}{P_t}$, $\bar{x}_t = \frac{X_t}{z_t^{+}}$, $s_t = \frac{S_t}{S_{t-1}}$, $\mu_{z^+,t} = \frac{z_t^{+}}{z_{t-1}^{+}}$ and $\Pi_t = \frac{P_t}{P_{t-1}}$. The above equation can be written as:

$$\bar{mc}_t^{M,C,xe}\,\overline{m}_t^{xe} + \bar{mc}_t^{M,C,e}\bar{m}_t^{e} + \bar{a}_t = p_t^X\,\bar{x}_t + \Phi\left(\bar{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right) R_{F,t-1}\zeta_{t-1}\, s_t\,\bar{a}_{t-1}\frac{1}{\mu_{z^+,t}\Pi_t}.$$

[81]Note that $\psi_F^{C.xe}\left(\frac{P_{F,t}}{P_{F,t}^{C,xe}}\right)^{-\nu_{F,C}} C_{F,t}^{xe} = C_{F,t}^{xe}$ and $\psi_F^{I}\left(\frac{P_{F,t}}{P_{F,t}^{I}}\right)^{-\nu_{F,I}}\left[\frac{I_{F,t}}{\gamma_t} + a\left(u_{F,t}\right)\frac{K_{F,t}}{\gamma_t}\right] = \frac{1}{\gamma_{F,t}}\left[I_{F,t} + a(u_t)K_{F,t}\right]$.

Note that the demand function for individual goods imported energy goods are given by $\bar{m}_t^{C,e}(i) = \left(\frac{p_t^{M,C,e}(i)}{p_t^{M,C,e}}\right)^{\frac{\lambda_t^{M,C,e}}{1-\lambda_t^{M,C,e}}} \bar{m}_t^{C,e}$.

Denote $\overleftrightarrow{p}_t^{M,C,e} = \int_0^1 \left(\frac{p_t^{M,C,e}(i)}{p_t^{M,C,e}}\right)^{\frac{\lambda_t^{M,C,e}}{1-\lambda_t^{M,C,e}}} di$ and rearrange slightly to get an expression that is identical to Equation (A.138a) in Appendix A:

$$\overline{a}_t = p_t^X \, \overline{x}_t - \bar{m}c_t^{M,xe} \, \overline{m}_t^{xe} - \bar{m}c_t^{M,C,e} \bar{m}_t^e + \Phi\left(\overline{a}_{t-1}, s_{t-1}, \widetilde{\phi}_{t-1}\right) R_{F,t-1}\zeta_{t-1} \, s_t \, \overline{a}_{t-1} \frac{1}{\mu_{z^+,t} \, \Pi_t}. \tag{E.57}$$

E.7 Total energy imports

Total energy imports is given by

$$M_t^e = \int_0^1 M_t^{C,e}(i)di + z_t^+ \phi^{M,C,e} \tag{E.58}$$

Note that the demand function for individual goods imported energy goods are given by $M_t^{C,e}(i) = \left(\frac{P_t^{M,C,e}(i)}{P_t^{M,C,e}}\right)^{\frac{\lambda_t^{M,C,e}}{1-\lambda_t^{M,C,e}}} P_t^{C,e}$.

Denote $\overleftrightarrow{P}_t^{M,C,e} = \int_0^1 \left(\frac{p_t^{M,C,e}(i)}{p_t^{M,C,e}}\right)^{\frac{\lambda_t^{M,C,e}}{1-\lambda_t^{M,C,e}}} di$ to write the import function as

$$M_t^e = \overleftrightarrow{P}_t^{M,C,e} M_t^{C,e} + z_t^+ \phi^{M,C,e} \tag{E.59}$$

To stationarize, divide through with z_t^+ to get

$$\bar{m}_t^e = \overleftrightarrow{P}_t^{M,C,e} \bar{m}_t^{C,e} + \phi^{M,C,e} \tag{E.60}$$

which is the same equation as Equation (A.147a) in Appendix A.

F Appendix: Log-linearization

F.1 Log-linearization method

Suppose, we have the following function: $F(X_t, Z_t)$. We take the natural log of the function $F(X_t, Z_t)$, and then we take a first-order Taylor approximation of the function $F(X_t, Z_t)$ around the steady state X and the steady state Z respectively.

A log-linear approximation of $F(X_t, Z_t)$ around the steady state X and the steady state Z is:

$$ln\left[F(X_t, Z_t)\right] \approx ln\left[F(X, Z)\right] + \frac{F_X(X,Z)}{F(X,Z)} X \left[\frac{X_t - X}{X}\right] + \frac{F_Z(X,Z)}{F(X,Z)} Z \left[\frac{Z_t - Z}{Z}\right].$$

Note that: $\hat{X}_t = ln(X_t) - ln(X) \approx \frac{X_t - X}{X}$. We can interpret $\hat{X}_t$ as a log-linear approximation of the variable X_t around its steady state value X.

F.2 Example of log-linearization method

In this section, we demonstrate how to implement a first-order log-linear approximation. For an illustrative purpose, we will take a first-order log-linear approximation of the stationarized version of consumption Euler equation (C.26).

Recall, the stationarized version of consumption Euler equation is written as:

$$\overline{\Omega}_t^C = R_t \zeta_t E_t \left[\beta_{t+1}^r \frac{\overline{\Omega}_{t+1}^C}{\mu_{z^+,t+1} \Pi_{t+1}^C}\right].$$

Note that $\beta^r = \beta$ and $\zeta = 1$. Thus, the steady state of consumption Euler equation can be expressed as:

$$\begin{aligned} \overline{\Omega}^C &= R\beta \frac{1}{\mu_{z+}\Pi^C}\overline{\Omega}^C \\ R &= \frac{\mu_{z+}\Pi^C}{\beta}. \end{aligned} \tag{F.1}$$

The first-order log-linear approximation of the LHS of Equation (C.26) is:

$$ln\overline{\Omega}_t^C \approx ln\overline{\Omega}^C + \frac{1}{\overline{\Omega}^C}\overline{\Omega}^C\left(\frac{\overline{\Omega}_t^C - \overline{\Omega}^C}{\overline{\Omega}^C}\right) = ln\overline{\Omega}^C + \widehat{\Omega}_t^C. \tag{F.2}$$

We have the following definitions:

$$\begin{aligned} \hat{\zeta}_t &= \frac{\zeta_t - \zeta}{\zeta} \\ \zeta &= 1 \\ \breve{i}_t &= R_t - R. \end{aligned}$$

We use the above definitions, and the first-order log-linear approximation of the RHS of Equation (C.26) can be written as follows:

$$\begin{aligned} & ln\overline{\Omega}^C + \frac{1}{\overline{\Omega}^C}\beta\overline{\Omega}^C\frac{1}{\mu_{z+}\Pi^C}(R_t - R) \\ & + \frac{1}{\overline{\Omega}^C}\beta\overline{\Omega}^C\frac{R\zeta}{\mu_{z+}\Pi^C}\left(\frac{\zeta_t - \zeta}{\zeta}\right) \\ & + \frac{1}{\overline{\Omega}^C}R\beta\overline{\Omega}^C\frac{1}{\mu_{z+}\Pi^C}\left(\frac{\beta_{t+1}^r - \beta}{\beta}\right) \\ & + \frac{1}{\overline{\Omega}^C}R\beta\overline{\Omega}^C\frac{1}{\mu_{z+}\Pi^C}\left(\frac{\overline{\Omega}_{t+1}^C - \overline{\Omega}^C}{\overline{\Omega}^C}\right) \\ & - \frac{1}{\overline{\Omega}^C}R\beta\overline{\Omega}^C\frac{1}{\Pi^C}\left(\frac{1}{\mu_{z+}}\right)^2\mu_{z+}\left(\frac{\mu_{z+,t+1} - \mu_{z+}}{\mu_{z+}}\right) \\ & - \frac{1}{\overline{\Omega}^C}R\beta\overline{\Omega}^C\frac{1}{\mu_{z+}}\left(\frac{1}{\Pi^C}\right)^2\Pi^C\left(\frac{\Pi_{t+1}^C - \Pi^C}{\Pi^C}\right). \end{aligned} \tag{F.3}$$

Using Equation (F.1) and the above definitions, Equation (F.3) can be written as:

$$ln\overline{\Omega}^C + \hat{\zeta}_t + \frac{1}{R}(R_t - R) + \hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C. \tag{F.4}$$

Combing Equation (F.2) and Equation (F.4), this gives us the following log-linearized version of consumption of Euler equation:

$$\hat{\Omega}_t^C = E_t\left[\hat{\zeta}_t + \hat{\beta}_{t+1}^r + \hat{\Omega}_{t+1}^C + \frac{1}{R}\breve{i}_t - \hat{\Pi}_{t+1}^C - \hat{\mu}_{z^+,t+1}\right]. \tag{F.5}$$

Equation (F.5) is the same as Equation (A.1b).

G Appendix: Derivation of log-linear wage equation

This section derives the log-linearized version of the optimal wage condition, Equation (A.13b). First, we present the key equations that will be used to derive the final log-linearized version of the optimal wage equation. The key equations are the real wage markup, aggregate wage index and labor demand equation. Second, we log-linearize Equation (C.63) which is the nonlinear version of the optimal wage equation and then use the key equations to obtain the final log-linearized version of the optimal wage equation.

G.1 Real wage markup

In this section, we present Equation (A.18b) that captures the log-linearized version of the real wage markup.

Recall, Equation (A.18a), which shows the non-linearized version of the real wage markup, is expressed as:

$$\overline{\Psi}_t^W = \frac{\left(1-\tau_t^W\right)\overline{w}_t}{\zeta_t^n \frac{\nu'(n_t)}{\overline{\Omega}_t^C}}. \tag{G.1}$$

Recall from Section 2.10, the labor disutility function is specified as:

$$\nu(N_{h,t}) = \Theta_t^n A_n \frac{N_{h,t}^{1+\eta}}{1+\eta}. \tag{G.2}$$

We can drop the subscript h since all household members choose the same optimal wage when they have chance to update their wage. As a result those labor types will have the same employment in this model. We can also rewrite the labor disutility function in terms of per *capita*, so n_t is denoted as employment per *capita* (employment rate). Thus, we have the following labor disutility function:

$$\nu(n_t) = \Theta_t^n A_n \frac{n_t^{1+\eta}}{1+\eta}. \tag{G.3}$$

The first derivative of $\nu(n_t)$ with respect to n_t is:

$$\nu'(n_t) = \Theta_t^n A_n n_t^{\eta}. \tag{G.4}$$

The second derivative of $\nu(n_t)$ with respect to n_t is:

$$\nu''(n_t) = \eta\Theta_t^n A_n n_t^{\eta-1}. \tag{G.5}$$

We can rewrite Equation (G.1) as follows:

$$\overline{\Psi}_t^W = \frac{\left(1-\tau_t^W\right)\overline{w}_t}{\zeta_t^n \frac{\Theta_t^n A_n n_t^{\eta}}{\overline{\Omega}_t^C}}. \tag{G.6}$$

We apply the log-linearization method from Section F.1 to Equation (G.6), and we can obtain the following log-linearized version of the real wage markup:

$$\hat{\Psi}_t^W = \hat{w}_t - \frac{1}{1-\tau^W}\breve{\tau}_t^W - \hat{\zeta}_t^n - \eta\hat{n}_t + \hat{\Omega}_t^C. \tag{G.7}$$

Furthermore, the labor force participation condition, Equation (A.14a) can be written as:

$$\hat{w}_t = \hat{\zeta}_t^n + \hat{\Theta}_t^n + \eta\hat{l}_t - \hat{\Omega}_t^C + \frac{1}{1-\tau^W}\breve{\tau}_t^W \tag{G.8}$$

When we combine Equation (G.8) and Equation (G.7) we obtain the following relationship between the wage markup and unemployment:

$$\hat{\Psi}_t^W = \eta\hat{un}_t \tag{G.9}$$

Equation (G.9) is the same as Equation (A.18b).

G.2 Aggregate wage index

In this section, we present the log-linearized version of the aggregate wage index. Recall from the main text in Section 2.1.3, the aggregate wage index in non-linear terms is specified as follows:

$$W_t = \left[\int_0^1 W_{h,t}^{\left(1-\varepsilon_{w,t}\right)} dh\right]^{\frac{1}{1-\varepsilon_{w,t}}}. \tag{G.10}$$

Recall from Section 2.1.3, we have the following Calvo wage contract:

$$W_{h,t} = \begin{cases} \overline{\Pi}_t^W W_{h,t-1} & \text{with probability } \xi_w \\ W_{h,t}^{\text{opt}} & \text{with probability } (1-\xi_w). \end{cases} \tag{G.11}$$

We apply the above Calvo wage contract and the following definition: $\varepsilon_{w,t} = \frac{\lambda_t^W}{\lambda_t^W - 1}$. Thus, we can rewrite Equation (G.10) as follows:

$$W_t = \left[\int_0^1 W_{h,t}^{\frac{1}{1-\lambda_t^W}} dh\right]^{1-\lambda_t^W}, \tag{G.12}$$

$$W_t^{\frac{1}{1-\lambda_t^W}} = \int_0^1 (W_{h,t})^{\frac{1}{1-\lambda_t^W}} dh, \tag{G.13}$$

$$W_t^{\frac{1}{1-\lambda_t^W}} = \int_0^{\xi_w} \left(\overline{\Pi}_t^W W_{h,t-1}\right)^{\frac{1}{1-\lambda_t^W}} dh + \int_{\xi_w}^1 \left(W_{h,t}^{opt}\right)^{\frac{1}{1-\lambda_t^W}} dh, \tag{G.14}$$

$$W_t^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W\right)^{\frac{1}{1-\lambda_t^W}} \int_0^{\xi_w} (W_{h,t-1})^{\frac{1}{1-\lambda_t^W}} dh + \int_{\xi_w}^1 \left(W_{h,t}^{opt}\right)^{\frac{1}{1-\lambda_t^W}} dh. \tag{G.15}$$

Now, we evaluate the integrals in Equation (G.15). First, we evaluate the first term of the RHS of Equation (G.15). Recall that the opportunity to reset the wage in any given period is governed by a random variable, and that this variable is identically and independently distributed across individual labor types and across different time periods. From this assumption, it follows that the subset of labor types that do not have the opportunity to reset their wage in period t will constitute a representative sample of all labor types. The wages posted by those same labor types in period $(t-1)$ will, by the same argument, constitute a representative sample of all the individual wages that were posted in that period. By the law of large numbers, the term $\int_0^{\xi_w} (W_{h,t-1})^{\frac{1}{1-\lambda_t^W}} dh$ may therefore be evaluated as follows:

$$\int_0^{\xi_w} (W_{h,t-1})^{\frac{1}{1-\lambda_t^W}} dh = \xi_w W_{t-1}^{\frac{1}{1-\lambda_t^W}}, \tag{G.16}$$

where $W_{t-1} = \left[\int_0^1 W_{h,t-1}^{\frac{1}{1-\lambda_t^W}} dh\right]^{1-\lambda_t^W}$. Using Equation (G.16), Equation (G.15) can be written as:

$$W_t^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W\right)^{\frac{1}{1-\lambda_t^W}} \xi_w (W_{t-1})^{\frac{1}{1-\lambda_t^W}} + \int_{\xi_w}^1 \left(W_{h,t}^{opt}\right)^{\frac{1}{1-\lambda_t^W}} dh. \tag{G.17}$$

Now, we evaluate the second term of the RHS of Equation (G.15). All labor types that get the opportunity to reset their wage in period t will face the same maximization problem. This follows from our assumption concerning the existence of contingent claims that allow individual household members to diversify the risk associated with the nominal wage friction (see Section 2.1.2 in the main text). As a consequence, all labor types that have the opportunity to reset their wage in period t will choose the same optimal wage, and we may therefore write $W_{h,t}^{opt} = W_t^{opt}$. Hence, Equation (G.17) can be written as follows:

$$W_t^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W\right)^{\frac{1}{1-\lambda_t^W}} \xi_w (W_{t-1})^{\frac{1}{1-\lambda_t^W}} + \left(W_t^{opt}\right)^{\frac{1}{1-\lambda_t^W}} \int_{\xi_w}^1 dh. \tag{G.18}$$

Using the following result: $\int_{\xi_w}^1 dh = (1-\xi_w)$, Equation (G.18) can be expressed as:

$$W_t^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W\right)^{\frac{1}{1-\lambda_t^W}} \xi_w (W_{t-1})^{\frac{1}{1-\lambda_t^W}} + (1-\xi_w)\left(W_t^{opt}\right)^{\frac{1}{1-\lambda_t^W}}. \tag{G.19}$$

We stationarize the above equation. Using the following definitions: $\overline{w}_t = \frac{W_t}{z_t^+ P_t^C}$, $\overline{w}_t^{opt} = \frac{W_t^{opt}}{z_t^+ P_t^C}$, $\Pi_t^C = \frac{P_t^C}{P_{t-1}^C}$ and $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, Equation (G.19) becomes:

$$\left(\frac{W_t}{z_t^+ P_t^C}\right)^{\frac{1}{1-\lambda_t^W}} = \left(\overline{\Pi}_t^W\right)^{\frac{1}{1-\lambda_t^W}} \xi_w \left(\frac{1}{\mu_{z^+,t}\Pi_t^C} \frac{W_{t-1}}{z_{t-1}^+ P_{t-1}^C}\right)^{\frac{1}{1-\lambda_t^W}} + (1-\xi_w)\left(\frac{W_t^{opt}}{z_t^+ P_t^C}\right)^{\frac{1}{1-\lambda_t^W}}.$$

The above equation can be simplified to the following equation:

$$(\overline{w}_t)^{\frac{1}{1-\lambda_t^W}} = \xi_w \left(\frac{\overline{\Pi}_t^W}{\mu_{z^+,t}\Pi_t^C}\right)^{\frac{1}{1-\lambda_t^W}} (\overline{w}_{t-1})^{\frac{1}{1-\lambda_t^W}} + (1-\xi_w)\left(\overline{w}_t^{opt}\right)^{\frac{1}{1-\lambda_t^W}}. \tag{G.20}$$

Equation (G.20) captures the stationarized version of the aggregate wage index.

We apply the log-linearization method from Section F.1 to Equation (G.20), and we can obtain the following log-linearized version of the aggregate wage index:

$$\hat{w}_t = \xi_w \left(\hat{\overline{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C + \hat{w}_{t-1} \right) + (1 - \xi_w)\hat{w}_t^{opt}. \tag{G.21}$$

Equation (G.21) captures the log-linearized version the aggregate wage index.

G.3 Labor demand

In this section, we present the log-linearized version of labor demand equation.

Recall from Section C.1.9, we have the following labor demand schedule:

$$N_{h,t+k|t} = \left(\frac{W_{h,t+k|t}}{W_{t+k}} \right)^{-\varepsilon_{w,t+k}} N_{t+k}.$$

Recall, $N_{h,t+k|t}$ denotes demand for labor type h, whereas N_{t+k} is aggregate labor demand. First, we can drop the subscript h from the labor demand schedule since all labor types choose the same optimal wage as a result the same amount of labor supply. Second, we stationarize the above demand schedule by using the following definitions: $\overline{w}_{t+k|t} = \frac{W_{t+k|t}}{z^+_{t+k} P^C_{t+k}}$ and $\overline{w}_{t+k} = \frac{W_{t+k}}{z^+_{t+k} P^C_{t+k}}$. We also rewrite the above equation in per *capita* terms. The stationarized version of the labor demand schedule is:

$$n_{t+k|t} = \left(\frac{\overline{w}_{t+k|t}}{\overline{w}_{t+k}} \right)^{-\varepsilon_{w,t+k}} n_{t+k}. \tag{G.22}$$

Recall from Section C.1.9, we have the following definition:

$$W_{t+k|t} = W_t^{opt} \overline{\Pi}^W_{t+1} \overline{\Pi}^W_{t+2} \dots \overline{\Pi}^W_{t+k}. \tag{G.23}$$

We stationarize Equation (G.23) by applying the following definitions: $\overline{w}_{t+k} = \frac{W_{t+k}}{z^+_{t+k} P^C_{t+k}}$, $\overline{w}_t^{opt} = \frac{W_t^{opt}}{z^+_t P^C_t}$, $\Pi^C_{t+k} = \frac{P^C_{t+k}}{P^C_{t+k-1}}$, and $\mu_{z^+,t+k} = \frac{z^+_{t+k}}{z^+_{t+k-1}}$. Thus, Equation (G.23) becomes:

$$\overline{w}_{t+k|t} = \frac{\overline{w}_t^{opt} \overline{\Pi}^W_{t+1} \overline{\Pi}^W_{t+2} \dots \overline{\Pi}^W_{t+k}}{\mu_{z^+,t+1} \mu_{z^+,t+2} \dots \mu_{z^+,t+k} \Pi^C_{t+1} \Pi^C_{t+2} \dots \Pi^C_{t+k}}. \tag{G.24}$$

Recall, Equation (A.19a), which shows the definition of wage inflation, is expressed as:

$$\Pi_t^W = \frac{\overline{w}_t}{\overline{w}_{t-1}} \mu_{z^+,t} \Pi_t^C.$$

Note that along the balanced growth path, the definition of wage inflation is:

$$\Pi_t^W = \overline{\Pi}^W = \mu_{z^+} \Pi^C. \tag{G.25}$$

Along the balanced growth path, Equation (G.24) can be expressed as:

$$\overline{w} = \left(\frac{\overline{\Pi}^W}{\mu_{z^+} \Pi^C} \right)^k \overline{w}^{opt}. \tag{G.26}$$

We apply the log-linearization method from Section F.1 to Equation (G.22) and also take into account Equation

(G.24), (G.25) and (G.26) when log-linearizing Equation (G.22). Hence, we can obtain the following log-linearized version of the labor demand equation:

$$\begin{aligned}
&\hat{n}_{t+k|t} = \varepsilon_w \hat{w}_{t+k} + \hat{n}_{t+k} - \varepsilon_w \hat{w}_t^{opt} \\
&- \varepsilon_w \hat{\overline{\Pi}}_{t+1}^W - \varepsilon_w \hat{\overline{\Pi}}_{t+2}^W - \ldots - \varepsilon_w \hat{\overline{\Pi}}_{t+k}^W \\
&+ \varepsilon_w \hat{\mu}_{z^+,t+1} + \varepsilon_w \hat{\mu}_{z^+,t+2} + \ldots + \varepsilon_w \hat{\mu}_{z^+,t+k} \\
&+ \varepsilon_w \hat{\Pi}_{t+1}^C + \varepsilon_w \hat{\Pi}_{t+2}^C + \ldots + \varepsilon_w \hat{\Pi}_{t+k}^C \\
&+ \frac{\lambda^W}{(1-\lambda^W)^2} log(\frac{\overline{w}^{opt}}{\overline{w}}) \hat{\lambda}_{t+1}^W + \frac{\lambda^W}{(1-\lambda^W)^2} log\left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^{-2} \hat{\lambda}_{t+2}^W \ldots + \frac{\lambda^W}{(1-\lambda^W)^2} log\left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^{-k} \hat{\lambda}_{t+k}^W
\end{aligned} \tag{G.27}$$

Equation (G.27) is the log-linearized version of labor demand equation.

G.4 Optimal wage equation

In this section, we derive the log-linearized version of the optimal wage equation, Equation (A.13b). The first step is to log-linearize Equation (C.63), which captures the nonlinear version of the optimal wage condition. In the second step, we use the following key equations: Equation (G.7), Equation (G.21) and Equation (G.27) to obtain the final log-linearized version of the optimal wage condition, Equation (A.13b).

Recall, we have the nonlinear version of the optimal wage equation:

$$E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^{k} \beta_{t+i}^r\right) n_{t+k|t} \overline{\Omega}_{t+k}^C \frac{1}{1-\lambda_{t+k}^W} \left[\left(1-\tau_{t+k}^W\right) \overline{w}_{t+k|t} - \lambda_{t+k}^W \zeta_{t+k}^n \frac{\nu'(n_{t+k|t})}{\overline{\Omega}_{t+k}^C}\right] = 0 \tag{G.28}$$

Equation (G.28) is the same as Equation (C.63) which shows the stationarized version of the optimal wage setting equation.

We rewrite Equation (G.28) as follows:

$$E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^{k} \beta_{t+i}^r\right) n_{t+k|t} \overline{\Omega}_{t+k}^C \frac{1}{1-\lambda_{t+k}^W} \left[\left(1-\tau_{t+k}^W\right) \overline{w}_{t+k|t}\right] = E_t \sum_{k=0}^{\infty} (\xi_w)^k \left(\prod_{i=1}^{k} \beta_{t+i}^r\right) n_{t+k|t} \overline{\Omega}_{t+k}^C \frac{\lambda_{t+k}^W}{1-\lambda_{t+k}^W} \zeta_{t+k}^n \frac{\nu'(n_{t+k|t})}{\overline{\Omega}_{t+k}^C}. \tag{G.29}$$

We expand Equation (G.29), and we can obtain the following equation:

$$\begin{aligned}
& n_{t|t} \overline{\Omega}_t^C \frac{1}{1-\lambda_t^W} \left(1-\tau_t^W\right) \overline{w}_t^{opt} + E_t \left[\xi_w \beta_{t+1}^r n_{t+1|t} \overline{\Omega}_{t+1}^C \frac{1}{1-\lambda_{t+1}^W} \left(1-\tau_{t+1}^W\right) \overline{w}_{t+1|t}\right] \\
&+ E_t \left[(\xi_w)^2 \beta_{t+1}^r \beta_{t+2}^r n_{t+2|t} \overline{\Omega}_{t+2}^C \frac{1}{1-\lambda_{t+2}^W} \left(1-\tau_{t+2}^W\right) \overline{w}_{t+2|t} + \ldots\right] \\
&= \\
& n_{t|t} \overline{\Omega}_t^C \frac{\lambda_t^W}{1-\lambda_t^W} \zeta_t^n \frac{\nu'(n_{t|t})}{\overline{\Omega}_t^C} \\
&+ E_t \left[\xi_w \beta_{t+1}^r n_{t+1|t} \overline{\Omega}_{t+1}^C \frac{\lambda_{t+1}^W}{1-\lambda_{t+1}^W} \zeta_{t+1}^n \frac{\nu'(n_{t+1|t})}{\overline{\Omega}_{t+1}^C}\right] \\
&+ E_t \left[(\xi_w)^2 \beta_{t+1}^r \beta_{t+2}^r n_{t+2|t} \overline{\Omega}_{t+2}^C \frac{\lambda_{t+2}^W}{1-\lambda_{t+2}^W} \zeta_{t+2}^n \frac{\nu'(n_{t+2|t})}{\overline{\Omega}_{t+2}^C} + \ldots\right].
\end{aligned} \tag{G.30}$$

Recall from Section G.3, we have the following definition:

$$\overline{w}_{t+k|t} = \frac{\overline{w}_t^{opt} \overline{\Pi}_{t+1}^W \overline{\Pi}_{t+2}^W \ldots \overline{\Pi}_{t+k}^W}{\mu_{z^+,t+1} \mu_{z^+,t+2} \ldots \mu_{z^+,t+k} \Pi_{t+1}^C \Pi_{t+2}^C \ldots \Pi_{t+k}^C}. \tag{G.31}$$

Recall from Section G.3, along the balanced growth path, the definition of wage inflation is written as:

$$\overline{\Pi}^W = \mu_{z^+} \Pi^C. \tag{G.32}$$

Along the balanced growth path, Equation (G.31) can be expressed as:

$$\overline{w} = \left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^k \overline{w}^{opt}. \tag{G.33}$$

When log-linearizing Equation (G.30), we take account of Equation (G.31), (G.32) and (G.33). We also let $H_1 = \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\overline{w}$. Note that variables such as $n_{t+k|t}$, β_t^r, and $\overline{\Omega}_{t+k}^C$ are that common to both sides of Equation (G.30) will cancel out after we have log-linearized. We therefore ignore them when log-linearizing, but we still have to log-linearize $\frac{1}{\Omega_{t+k}^C}$ as this particular variable only appears in the RHS of Equation (G.30). Note that in equilibrium, we have $\beta^r = \beta$ and define $\breve{\tau}_t^W$ as $\tau_t^W - \tau^W$.

Now, we log-linearize the LHS of Equation (G.30) and we can obtain the following equation:

$$
\begin{aligned}
&\ln H_1 + \frac{1}{H_1} n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\overline{w}\hat{w}_t^{opt} \\
&+ \frac{1}{H_1}\ \xi_w \beta n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\hat{w}_t^{opt} \\
&+ \frac{1}{H_1}\ \left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{w}_t^{opt} + \dots \\
&+ \frac{1}{H_1}\ \xi_w \beta n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\hat{\overline{\Pi}}_t^W \\
&+ \frac{1}{H_1}\ \left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}E\left[\hat{\overline{\Pi}}_{t+1}^W\right] + \dots \\
&+ \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\overline{\Pi}}_{t+2}^W + \dots\right] \\
&- \frac{1}{H_1}\ E_t\left[\xi_w \beta n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\hat{\mu}_{z^+,t+1}\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\mu}_{z^+,t+1} - \dots\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\mu}_{z^+,t+2} - \dots\right] \\
&- \frac{1}{H_1}\ E_t\left[\xi_w \beta n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\hat{\Pi}_{t+1}^C\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\Pi}_{t+1}^C - \dots\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\hat{\Pi}_{t+2}^C - \dots\right] \\
&- \frac{1}{H_1}\ n\overline{\Omega}^C \frac{1}{1-\lambda^W}\overline{w}\breve{\tau}_t^W - \frac{1}{H_1}E_t\left[\xi_w\beta n\overline{\Omega}^C\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\overline{w}\breve{\tau}_{t+1}^W\right] \\
&- \frac{1}{H_1}\ E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \frac{1}{1-\lambda^W}\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\overline{w}\breve{\tau}_{t+2}^W - \dots\right] \\
&+ \frac{1}{H_1}\ n\overline{\Omega}^C \frac{1}{(1-\lambda^W)^2}\overline{w}\left(1-\tau^W\right)\lambda^W\hat{\lambda}_t^W + \frac{1}{H_1}E_t\left[\xi_w\beta n\overline{\Omega}^C\frac{\lambda^W}{(1-\lambda^W)^2}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)\bar{w}\hat{\lambda}_{t+1}^W\right] \\
&+ \frac{1}{H_1}E_t\left[\xi_w\beta n\overline{\Omega}^C\frac{\lambda^W}{(1-\lambda^W)^2}\left(1-\tau^W\right)\left(\frac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2\bar{w}\hat{\lambda}_{t+2}^W - \dots\right].
\end{aligned}
\tag{G.34}
$$

We let $H_2 = \frac{\lambda^W}{1-\lambda^W}\zeta^n\frac{\nu'(n)}{\overline{\Omega}^C}$. Also recall Equation G.4 below:

$$\nu'(n_t) = \Theta_t^n A_n n_t^\eta, \tag{G.35}$$

and denote first derivative of $\nu'(n_t)$ with respect to Θ_t^n as $\nu'_{\Theta^n}(n_t)$, which is $\nu'_{\Theta^n}(n_t) = \frac{\partial\nu'(n_t)}{\partial\Theta_t^n} = A_n n_t^\eta = \frac{\nu'(n_t)}{\Theta_t^n}$. In steady state $\nu'_{\Theta^n}(n) = \frac{\nu'(n)}{\Theta^n}$ and we use it below while we log-linearize the RHS of Equation (G.30), and we have the following equation:

$$
\begin{aligned}
&\ln H_2 + \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}_t^n \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n \overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}_{t+1}^n \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n \overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}_{t+2}^n + \ldots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \nu^{''}(n) n \hat{n}_{t|t} \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n \overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \nu^{''}(n) n \hat{n}_{t+1|t} \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n \overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \nu^{''}(n) n \hat{n}_{t+2|t} + \ldots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \hat{\Theta}_t^n \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n \overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \Theta_{t+1}^{\hat{n}} \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n \overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \Theta_{t+2}^{\hat{n}} + \ldots \right] \\
&- \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}_t^C \\
&- \frac{1}{H_2}\ E_t \left[\xi_w \beta n \overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}_{t+1}^C \right] \\
&- \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n \overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}_{t+2}^C + \ldots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{1}{(1-\lambda^W)^2} \lambda^W \hat{\lambda}_t^W \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n \overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{\lambda^W}{(1-\lambda^W)^2} \hat{\lambda}_{t+1}^W \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n \overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{\lambda^W}{(1-\lambda^W)^2} \hat{\lambda}_{t+2}^W + \ldots \right].
\end{aligned}
\tag{G.36}
$$

Note that in steady state, Equation (G.5) is: $\nu''(n) = \eta \Theta^n A_n n^{\eta-1}$. We use the following results: $\nu''(n) = \eta \Theta^n A_n n^{\eta-1}$ and $\nu'(n) = \Theta^n A_n n^{\eta}$. Thus, $\nu''(n)n$ can be defined as:

$$
\nu''(n) n = \eta \Theta^n A_n n^{\eta} = \eta \nu'(n). \tag{G.37}
$$

Using Equation (G.37), Equation (G.36) can be rewritten as follows:

$$
\begin{aligned}
&\ln H_2 + \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}^n_t \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}^n_{t+1} \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \frac{\nu'(n)}{\overline{\Omega}^C} \zeta^n \hat{\zeta}^n_{t+2} + \dots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \eta \nu'(n) \hat{n}_{t|t} \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \eta \nu'(n) \hat{n}_{t+1|t} \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \eta \nu'(n) \hat{n}_{t+2|t} + \dots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \hat{\Theta^n}_{t|t} \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \hat{\Theta^n}_{t+1|t} \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \frac{1}{\overline{\Omega}^C} \frac{\nu'(n)}{\Theta^n} \Theta^n \hat{\Theta^n}_{t+2|t} + \dots \right] \\
&- \frac{1}{H_2}\ n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}^C_t \\
&- \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}^C_{t+1} \right] \\
&- \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n\overline{\Omega}^C \frac{\lambda^W}{1-\lambda^W} \zeta^n \nu'(n) \left(\frac{1}{\overline{\Omega}^C} \right)^2 \overline{\Omega}^C \hat{\Omega}^C_{t+2} + \dots \right] \\
&+ \frac{1}{H_2}\ n\overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{1}{(1-\lambda^W)^2} \lambda^W \hat{\lambda}^W_t \\
&+ \frac{1}{H_2}\ E_t \left[\xi_w \beta n\overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{\lambda^W}{(1-\lambda^W)^2} \hat{\lambda}^W_{t+1} \right] \\
&+ \frac{1}{H_2}\ E_t \left[(\xi_w \beta)^2 n\overline{\Omega}^C \zeta^n \frac{\nu'(n)}{\overline{\Omega}^C} \frac{\lambda^W}{(1-\lambda^W)^2} \hat{\lambda}^W_{t+2} + \dots \right].
\end{aligned}
\tag{G.38}
$$

We use the following result: $H_1 = H_2$, and multiply the terms with $\check{\tau}^W_{t+k}$ by $\frac{(1-\tau^W)}{(1-\tau^W)}$. We then combine Equation

(G.34) and (G.38). Thus, we have the following equation:

$$
\begin{aligned}
& n\overline{\Omega}^C \left(1-\tau^W\right) \overline{w}\hat{w}_t^{opt} \\
& +\xi_w \beta n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right) \overline{w}\hat{w}_t^{opt} \\
& +\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \overline{w}\hat{w}_t^{opt} + \ldots \\
& +E_t\left[\xi_w \beta n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right) \overline{w}\hat{\overline{\Pi}}_{t+1}^W\right] \\
& +E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \overline{w}\hat{\overline{\Pi}}_{t+1}^W\right] + \ldots \\
& +E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \overline{w}\hat{\overline{\Pi}}_{t+2}^W + \ldots\right] \\
& -E_t\left[\xi_w \beta n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right) \overline{w}\hat{\mu}_{z^+,t+1}\right] \\
& -E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \overline{w}\hat{\mu}_{z^+,t+1} - \ldots\right] \\
& -E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \overline{w}\hat{\mu}_{z^+,t+2} - \ldots\right] \\
& -E_t\left[\xi_w \beta n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right) \overline{w}\hat{\Pi}_{t+1}^C\right] \\
& -E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \overline{w}\hat{\Pi}_{t+1}^C - \ldots\right] \\
& -E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \overline{w}\hat{\Pi}_{t+2}^C - \ldots\right] \\
& -n\overline{\Omega}^C \left(1-\tau^W\right) \overline{w}\tfrac{1}{\left(1-\tau^W\right)}\breve{\tau}_t^W \\
& -E_t\left[\xi_w \beta n\overline{\Omega}^C \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right) \left(1-\tau^W\right) \overline{w}\tfrac{1}{\left(1-\tau^W\right)}\breve{\tau}_{t+1}^W\right] \\
& -E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \left(1-\tau^W\right) \overline{w}\tfrac{1}{\left(1-\tau^W\right)}\breve{\tau}_{t+2}^W - \ldots\right] \\
& +n\overline{\Omega}^C \tfrac{1}{\left(1-\lambda^W\right)}\overline{w}\left(1-\tau^W\right)\lambda^W\hat{\lambda}_t^W \\
& +E_t\left[\xi_w \beta n\overline{\Omega}^C \tfrac{\lambda^W}{\left(1-\lambda^W\right)} \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right) \bar{w}\hat{\lambda}_{t+1}^W\right] \\
& +E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega} \tfrac{\lambda^W}{\left(1-\lambda^W\right)} \left(1-\tau^W\right) \left(\tfrac{\overline{\Pi}^W}{\mu_{z+}\Pi^C}\right)^2 \bar{w}\hat{\lambda}_{t+2}^W + \ldots\right]. \\
= \; & n\overline{\Omega}^C \hat{\zeta}_t^n \\
& +E_t\left[\xi_w \beta n\overline{\Omega}^C \hat{\zeta}_{t+1}^n\right] \\
& +E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \hat{\zeta}_{t+2}^n + \ldots\right] \\
& +n\overline{\Omega}^C \eta\hat{n}_{t|t} \\
& +E_t\left[\xi_w \beta n\overline{\Omega}^C \eta\hat{n}_{t+1|t}\right] \\
& +E_t\left[\left(\xi_w\beta\right)^2 n\overline{\Omega}^C \eta\hat{n}_{t+2|t} + \ldots\right]
\end{aligned}
\tag{G.39}
$$

$$
\begin{aligned}
&+\ n\overline{\Omega}^C\hat{\Theta}^n{}_{t|t}\\
&+\ E_t\left[\xi_w\beta n\overline{\Omega}^C\hat{\Theta}^n{}_{t+1|t}\right]\\
&+\ E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C\hat{\Theta}^n{}_{t+2|t}+\ldots\right]\\
&-n\overline{\Omega}^C\hat{\Omega}^C_t\\
&-E_t\left[\xi_w\beta n\overline{\Omega}^C\hat{\Omega}^C_{t+1}\right]\\
&-E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C\hat{\Omega}^C_{t+2}+\ldots\right]\\
&+n\overline{\Omega}^C\frac{1}{(1-\lambda^W)}\hat{\lambda}^W_t\\
&+E_t\left[\xi_w\beta n\overline{\Omega}^C\frac{1}{(1-\lambda^W)}\hat{\lambda}^W_{t+1}\right]\\
&+E_t\left[(\xi_w\beta)^2\, n\overline{\Omega}^C\frac{1}{(1-\lambda^W)}\hat{\lambda}^W_{t+2}+\ldots\right].
\end{aligned}
$$

We have the following summation formula for an infinite geometric series:

$$
b+bz+bz^2+\ldots+bz^{n-1}+\ldots=\frac{b}{1-z}.
$$

We assume that $|z|<1$.
We make use of following definitions $(1-\tau^W)\overline{w}=\lambda^W\zeta^n\frac{\nu'(n)}{\overline{\Omega}^C}$ and $\frac{\overline{\Pi}^W}{\mu_{z^+,t}\Pi^C}=1$. We also gather all $\hat{\lambda}^W_{t+k}$ terms on the right hand side by using the following for all $t+k$:

$$
(\xi_w\beta)^k\, n\overline{\Omega}^C\frac{\nu'(n)}{\overline{\Omega}^C}\frac{\lambda^W}{(1-\lambda^W)}\hat{\lambda}^W_{t+k}-(\xi_w\beta)^k\, n\overline{\Omega}\frac{\lambda^W}{(1-\lambda^W)}\left(1-\tau^W\right)\overline{w}\hat{\lambda}^W_{t+k}=(1-\lambda^W)\,(\xi_w\beta)^k\, n\overline{\Omega}^C\frac{\nu'(n)}{\overline{\Omega}^C}\frac{\lambda^W}{(1-\lambda^W)}\hat{\lambda}^W_{t+k}
$$

We apply the summation formula for an infinite geometric series to Equation (G.39), and then we simplify Equation (G.39). This gives us the following equation:

$$
\begin{aligned}
&\frac{\hat{w}^{opt}_t}{(1-\xi_w\beta)}+\frac{\xi_w\beta}{(1-\xi_w\beta)}\hat{\overline{\Pi}}^W_{t+1}+\frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}E_t\hat{\overline{\Pi}}^W_{t+2}+\ldots\\
&-\frac{\xi_w\beta}{(1-\xi_w\beta)}E_t\hat{\mu}_{z^+,t+1}-\frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}E_t\hat{\mu}_{z^+,t+2}-\ldots\\
&-\frac{\xi_w\beta}{(1-\xi_w\beta)}E_t\hat{\Pi}^C_{t+1}-\frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}E_t\hat{\Pi}^C_{t+2}-\ldots\\
&-\frac{1}{(1-\tau^W)}\breve{\tau}^W_t-\xi_w\beta\frac{1}{(1-\tau^W)}E_t\breve{\tau}^W_{t+1}-(\xi_w\beta)^2\frac{1}{(1-\tau^W)}E_t\breve{\tau}^W_{t+2}-\ldots\\
&=E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}^n_{t+k}+\eta\hat{n}_{t+k|t}+\hat{\Theta}^n_{t+k}-\hat{\Omega}^C_{t+k}+\hat{\lambda}^W_{t+k}\right].
\end{aligned}
\tag{G.40}
$$

Substituting the labor demand equation (G.27) into Equation (G.40), Equation (G.40) becomes

$$\begin{aligned}
&\frac{\hat{w}_t^{opt}}{(1-\xi_w\beta)} + \frac{\xi_w\beta}{(1-\xi_w\beta)}E_t\hat{\overline{\Pi}}^W_{t+1} + \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}E_t\hat{\overline{\Pi}}^W_{t+2} + \ldots \\
&- \frac{\xi_w\beta}{(1-\xi_w\beta)}E_t\hat{\mu}_{z^+,t+1} - \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}E_t\hat{\mu}_{z^+,t+2} - \ldots \\
&- \frac{\xi_w\beta}{(1-\xi_w\beta)}E_t\hat{\Pi}^C_{t+1} - \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}E_t\hat{\Pi}^C_{t+2} - \ldots \\
&= E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}^n_{t+k} + \hat{\Theta}^n_{t+k} - \hat{\Omega}^C_{t+k} + \hat{\lambda}^W_{t+k} + \frac{1}{(1-\tau^W)}\breve{\tau}^W_{t+k}\right] \\
&+ \eta\left[\varepsilon_w\hat{w}_t + \hat{n}_t - \varepsilon_w\hat{w}_t^{opt}\right] \\
&+ (\xi_w\beta)\eta E_t\left[\varepsilon_w\hat{w}_{t+1} + \hat{n}_{t+1} - \varepsilon_w\hat{w}_t^{opt} - \varepsilon_w\hat{\overline{\Pi}}^W_{t+1} + \varepsilon_w\hat{\mu}_{z^+,t+1} + \varepsilon_w\hat{\Pi}^C_{t+1}\right] \\
&+ (\xi_w\beta)^2\eta E_t\left[\varepsilon_w\hat{w}_{t+2} + \hat{n}_{t+2} - \varepsilon_w\hat{w}_t^{opt} - \varepsilon_w\hat{\overline{\Pi}}^W_{t+1} - \varepsilon_w\hat{\overline{\Pi}}^W_{t+2}\right] \\
&+ (\xi_w\beta)^2\eta E_t\left[\varepsilon_w\hat{\mu}_{z^+,t+1} + \varepsilon_w\hat{\mu}_{z^+,t+2} + \varepsilon_w\hat{\Pi}^C_{t+1} + \varepsilon_w\hat{\Pi}^C_{t+2}\right] + \ldots \\
&+ (\xi_w\beta)\frac{\lambda^W}{(1-\lambda^W)^2}log(\frac{\bar{w}^{opt}}{\bar{w}})\hat{\lambda}^W_{t+1} + (\xi_w\beta)^2\frac{\lambda^W}{(1-\lambda^W)^2}log\left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^{-2}\hat{\lambda}^W_{t+2}\ldots + (\xi_w\beta)^k\frac{\lambda^W}{(1-\lambda^W)^2}log\left(\frac{\overline{\Pi}^W}{\mu_{z^+}\Pi^C}\right)^{-k}\hat{\lambda}^W_{t+k}.
\end{aligned}$$

Since the all the terms of last row of the above equation cancels out in the steps below, we continue without that part of the equation. With this simplification, the above equation can be written as:

$$\begin{aligned}
&\frac{\hat{w}_t^{opt}}{(1-\xi_w\beta)} + \frac{\xi_w\beta}{(1-\xi_w\beta)}E_t\left[\hat{\overline{\Pi}}^W_{t+1} - \hat{\mu}_{z^+,t+1} - \hat{\Pi}^C_{t+1}\right] \\
&+ \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}E_t\left[\hat{\overline{\Pi}}^W_{t+2} - \hat{\mu}_{z^+,t+2} - \hat{\Pi}^C_{t+2}\right] + \ldots \\
&= E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\left[\hat{\zeta}^n_{t+k} + \hat{\Theta}^n_{t+k} - \hat{\Omega}^C_{t+k} + \hat{\lambda}^W_{t+k} + \frac{1}{(1-\tau^W)}\breve{\tau}^W_{t+k}\right] \\
&+ E_t\sum_{k=0}^{\infty}(\xi_w\beta)^k\eta\ E_t\left[\varepsilon_w\hat{w}_{t+k} + \hat{n}_{t+k} - \varepsilon_w\hat{w}_t^{opt}\right] \\
&- (\xi_w\beta)\eta\varepsilon_w\ E_t\left[\hat{\overline{\Pi}}^W_{t+1} + (\xi_w\beta)\hat{\overline{\Pi}}^W_{t+1} + (\xi_w\beta)^2\hat{\overline{\Pi}}^W_{t+1} + \ldots\right] \\
&+ (\xi_w\beta)\eta\varepsilon_w\ E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}^C_{t+1}\right] \\
&+ (\xi_w\beta)\eta\varepsilon_w\ E_t\left[(\xi_w\beta)\left(\hat{\mu}_{z^+,t+1} + \hat{\Pi}^C_{t+1}\right) + (\xi_w\beta)^2\left(\hat{\mu}_{z^+,t+1} + \hat{\Pi}^C_{t+1}\right) + \ldots\right] \\
&- (\xi_w\beta)^2\eta\varepsilon_w\ E_t\left[\hat{\overline{\Pi}}^W_{t+2} + (\xi_w\beta)\hat{\overline{\Pi}}^W_{t+2} + (\xi_w\beta)^2\hat{\overline{\Pi}}^W_{t+2} + \ldots\right] \\
&+ (\xi_w\beta)^2\eta\varepsilon_w\ E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}^C_{t+2}\right] \\
&+ (\xi_w\beta)^2\eta\varepsilon_w\ E_t\left[(\xi_w\beta)\left(\hat{\mu}_{z^+,t+2} + \hat{\Pi}^C_{t+2}\right) + (\xi_w\beta)^2\left(\hat{\mu}_{z^+,t+2} + \hat{\Pi}^C_{t+2}\right) + \ldots\right] \\
&+ \ldots.
\end{aligned} \tag{G.41}$$

We apply the above summation formula for an infinite geometric series to Equation (G.41), and we have the following equation:

$$
\begin{aligned}
&\frac{\hat{w}_t^{opt}}{(1-\xi_w\beta)} + \frac{\xi_w\beta}{(1-\xi_w\beta)} E_t\left[\hat{\bar{\Pi}}^W_{t+1} - \hat{\mu}_{z^+,t+1} - \hat{\Pi}^C_{t+1}\right] \\
&+ \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)} E_t\left[\hat{\bar{\Pi}}^W_{t+2} - \hat{\mu}_{z^+,t+2} - \hat{\Pi}^C_{t+2}\right] + \ldots \\
&= \\
&E_t \sum_{k=0}^{\infty} (\xi_w\beta)^k \left[\hat{\zeta}^n_{t+k} + \hat{\Theta}^n_{t+k} - \hat{\Omega}^C_{t+k} + \hat{\lambda}^W_{t+k} + \frac{1}{(1-\tau^W)}\check{\tau}^W_{t+k} + \eta\varepsilon_w \hat{w}_{t+k} + \eta\hat{n}_{t+k}\right] \\
&- \eta\varepsilon_w \frac{\hat{w}_t^{opt}}{(1-\xi_w\beta)} - \frac{\xi_w\beta}{(1-\xi_w\beta)}\eta\varepsilon_w E_t\left[\hat{\bar{\Pi}}^W_{t+1}\right] \\
&- \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}\eta\varepsilon_w E_t\left[\hat{\bar{\Pi}}^W_{t+2}\right] - \ldots \\
&+ \frac{\xi_w\beta}{(1-\xi_w\beta)}\eta\varepsilon_w E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}^C_{t+1}\right] + \frac{(\xi_w\beta)^2}{(1-\xi_w\beta)}\eta\varepsilon_w E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}^C_{t+2}\right] + \ldots
\end{aligned}
\tag{G.42}
$$

We multiply both sides of Equation (G.42) by $(1-\xi_w\beta)$ and rearrange the equation. Hence, Equation (G.42) can be rewritten as:

$$
\begin{aligned}
&(1+\eta\varepsilon_w)\hat{w}_t^{opt} = \\
&(1-\xi_w\beta)E_t \sum_{k=0}^{\infty} (\xi_w\beta)^k \left[\hat{\zeta}^n_{t+k} + \hat{\Theta}^n_{t+k} - \hat{\Omega}^C_{t+k} + \hat{\lambda}^W_{t+k} + \frac{1}{(1-\tau^W)}\check{\tau}^W_{t+k} + \eta\varepsilon_w \hat{w}_{t+k} + \eta\hat{n}_{t+k}\right] \\
&- (1+\eta\varepsilon_w)(\xi_w\beta)\hat{\bar{\Pi}}^W_{t+1} - (1+\eta\varepsilon_w)(\xi_w\beta)^2 E_t\left[\hat{\bar{\Pi}}^W_{t+2}\right] - \ldots \\
&+ (1+\eta\varepsilon_w)(\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}^C_{t+1}\right] + (1+\eta\varepsilon_w)(\xi_w\beta)^2 E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}^C_{t+2}\right] + \ldots
\end{aligned}
\tag{G.43}
$$

Dividing both sides of Equation (G.43) by $(1+\eta\varepsilon_w)$, we have the following equation:

$$
\begin{aligned}
&\hat{w}_t^{opt} = \\
&\frac{(1-\xi_w\beta)}{(1+\eta\varepsilon_w)} E_t \sum_{k=0}^{\infty} (\xi_w\beta)^k \left[\hat{\zeta}^n_{t+k} + \hat{\Theta}^n_{t+k} - \hat{\Omega}^C_{t+k} + \hat{\lambda}^W_{t+k} + \frac{1}{(1-\tau^W)}\check{\tau}^W_{t+k} + \eta\varepsilon_w \hat{w}_{t+k} + \eta\hat{n}_{t+k}\right] \\
&- (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}^W_{t+1}\right] - (\xi_w\beta)^2 E_t\left[\hat{\bar{\Pi}}^W_{t+2}\right] - \ldots \\
&+ (\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}^C_{t+1}\right] + (\xi_w\beta)^2 E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}^C_{t+2}\right] + \ldots
\end{aligned}
\tag{G.44}
$$

Now, we iterate Equation (G.44) one period forward and multiply both sides of the equation by $(\xi_w\beta)$. Thus, we have the following equation:

$$
\begin{aligned}
&(\xi_w\beta)\hat{w}_{t+1}^{opt} = \\
&\frac{(1-\xi_w\beta)}{(1+\eta\varepsilon_w)} E_t \sum_{k=1}^{\infty} (\xi_w\beta)^k \left[\hat{\zeta}^n_{t+k} + \hat{\Theta}^n_{t+k} - \hat{\Omega}^C_{t+k} + \hat{\lambda}^W_{t+k} + \frac{1}{(1-\tau^W)}\check{\tau}^W_{t+k} + \eta\varepsilon_w \hat{w}_{t+k} + \eta\hat{n}_{t+k}\right] \\
&- (\xi_w\beta)^2 E_t\hat{\bar{\Pi}}^W_{t+2} - (\xi_w\beta)^3 E_t\left[\hat{\bar{\Pi}}^W_{t+3}\right] - \ldots \\
&+ (\xi_w\beta)^2 E_t\left[\hat{\mu}_{z^+,t+2} + \hat{\Pi}^C_{t+2}\right] + (\xi_w\beta)^3 E_t\left[\hat{\mu}_{z^+,t+3} + \hat{\Pi}^C_{t+3}\right] + \ldots
\end{aligned}
\tag{G.45}
$$

Subtracting Equation (G.45) from (G.44), we have the following equation:

$$
\begin{aligned}
&\hat{w}_t^{opt} - (\xi_w\beta)E_t\hat{w}_{t+1}^{opt} = \\
&\frac{(1-\xi_w\beta)}{(1+\eta\varepsilon_w)} \left[\hat{\zeta}^n_t + \hat{\Theta}^n_t - \hat{\Omega}^C_t + \hat{\lambda}^W_t + \frac{1}{(1-\tau^W)}\check{\tau}^W_t + \eta\varepsilon_w \hat{w}_t + \eta\hat{n}_t\right] \\
&+ (\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}^C_{t+1} - \hat{\bar{\Pi}}^W_{t+1}\right].
\end{aligned}
$$

The above equation can be written as:

$$\begin{aligned}\hat{w}_t^{opt} &= (\xi_w\beta)E_t\hat{w}_{t+1}^{opt} \\ &+ \frac{(1-\xi_w\beta)}{(1+\eta\varepsilon_w)}\left[\hat{\zeta}_t^n + \hat{\Theta}_t^n - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\breve{\tau}_t^W + \eta\varepsilon_w\hat{w}_t + \eta\hat{n}_t\right] \\ &+ (\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C - \hat{\bar{\Pi}}_{t+1}^W\right].\end{aligned} \tag{G.46}$$

Recall from Equation (G.21) in Section G.2, we have the following log-linearized version of the aggregate wage index, which is expressed as:

$$\hat{w}_t = \xi_w\left(\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C + \hat{w}_{t-1}\right) + (1-\xi_w)\hat{w}_t^{opt}.$$

We rewrite the above aggregate wage index as:

$$\hat{w}_t^{opt} = \frac{1}{(1-\xi_w)}\hat{w}_t - \frac{\xi_w}{(1-\xi_w)}\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C + \hat{w}_{t-1}\right]. \tag{G.47}$$

We iterate Equation (G.47) one period forward and multiply the equation by $(\xi_w\beta)$, and we have the following equation:

$$(\xi_w\beta)E_t\hat{w}_{t+1}^{opt} = \frac{(\xi_w\beta)}{(1-\xi_w)}E_t\hat{w}_{t+1} - \frac{(\xi_w\beta)\xi_w}{(1-\xi_w)}E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C + \hat{w}_t\right]. \tag{G.48}$$

Substituting Equation (G.47) and (G.48) into Equation (G.46), Equation (G.46) becomes:

$$\begin{aligned}\hat{w}_t &= \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C + \hat{w}_{t-1}\right] \\ &+ \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[\hat{\zeta}_t^n + \hat{\Theta}_t^n - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\breve{\tau}_t^W + \eta\varepsilon_w\hat{w}_t + \eta\hat{n}_t\right] \\ &+ (1-\xi_w)(\xi_w\beta)E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C - \hat{\bar{\Pi}}_{t+1}^W\right] + (\xi_w\beta)E_t\left[\hat{w}_{t+1}\right] \\ &- (\xi_w)^2\beta\ E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C + \hat{w}_t\right].\end{aligned}$$

We add $\hat{w}_t - \hat{w}_t$ to the second term of the RHS of the above equation, and then we rearrange the above equation. Hence, we have the following equation:

$$\begin{aligned}\hat{w}_t &= \xi_w\hat{w}_{t-1} + (\xi_w\beta)E_t\left[\hat{w}_{t+1}\right] - \beta(\xi_w)^2\hat{w}_t \\ &+ \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[\hat{w}_t - \hat{w}_t + \hat{\zeta}_t^n + \hat{\Theta}_t^n - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\breve{\tau}_t^W + \eta\varepsilon_w\hat{w}_t + \eta\hat{n}_t\right] \\ &+ \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] \\ &+ \left((\xi_w)^2\beta + \xi_w\beta - (\xi_w)^2\beta\right)\ E_t\left[\hat{\mu}_{z^+,t+1} + \hat{\Pi}_{t+1}^C - \hat{\bar{\Pi}}_{t+1}^W\right].\end{aligned} \tag{G.49}$$

We add $\xi_w\hat{w}_t - \xi_w\hat{w}_t$ to the LHS and $\beta\xi_w\hat{w}_t - \beta\xi_w\hat{w}_t$ to the RHS of Equation (G.49). Thus, we have the following equation:

$$\begin{aligned}\hat{w}_t + (\xi_w\hat{w}_t - \xi_w\hat{w}_t) - \xi_w\hat{w}_{t-1} &= (\xi_w\beta)E_t\left[\hat{w}_{t+1}\right] + (\beta\xi_w\hat{w}_t - \beta\xi_w\hat{w}_t) - \beta(\xi_w)^2\hat{w}_t \\ &+ \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[-\hat{w}_t + \hat{\zeta}_t^n + \eta\hat{n}_t - \hat{\Omega}_t^C + \hat{\lambda}_t^W + \frac{1}{(1-\tau^W)}\breve{\tau}_t^W + (1+\eta\varepsilon_w)\hat{w}_t\right] \\ &+ \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] - (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right].\end{aligned}$$

Note that: $\triangle\hat{w}_t = (\hat{w}_t - \hat{w}_{t-1})$ and the above equation can be written as:

$$\begin{aligned}\xi_w\triangle\hat{w}_t &= (\xi_w\beta)E_t\left[\triangle\hat{w}_{t+1}\right] + (1-\xi_w)(\xi_w\beta)\hat{w}_t - (1-\xi_w)\hat{w}_t \\ &- \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[\hat{w}_t - \hat{\zeta}_t^n - \hat{\Theta}_t^n - \eta\hat{n}_t + \hat{\Omega}_t^C - \hat{\lambda}_t^W - \frac{1}{(1-\tau^W)}\breve{\tau}_t^W - (1+\eta\varepsilon_w)\hat{w}_t\right] \\ &+ \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] - (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right].\end{aligned} \tag{G.50}$$

Recall from Equation (G.7) in Section G.1, we have the following log-linearized version of the real wage markup equation:

$$\hat{\Psi}_t^W = \hat{w}_t - \frac{1}{(1-\tau^W)}\breve{\tau}_t^W - \hat{\zeta}_t^n - \eta\hat{n}_t + \hat{\Omega}_t^C.$$

Using the above real wage markup equation, Equation (G.50) can be written as:

$$\begin{aligned}\xi_w \triangle\hat{w}_t &= (\xi_w\beta)E_t\left[\triangle\hat{w}_{t+1}\right] + (1-\xi_w)(\xi_w\beta)\hat{w}_t - (1-\xi_w)\hat{w}_t \\ &- \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}\left[\hat{\Psi}_t^W - \hat{\lambda}_t^W - (1+\eta\varepsilon_w)\hat{w}_t\right] + \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] \\ &- (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right].\end{aligned} \tag{G.51}$$

Equation (G.51) can be simplified as follows:

$$\xi_w\triangle\hat{w}_t = (\xi_w\beta)E_t\left[\triangle\hat{w}_{t+1}\right] + (1-\xi_w)\hat{w}_t - (1-\xi_w)\hat{w}_t - \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)}(\hat{\Psi}_t^W - \hat{\lambda}_t^W)$$
$$+ \xi_w\left[\hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C\right] - (\xi_w\beta)E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right],$$

$$\begin{aligned}\triangle\hat{w}_t &= \beta E_t\left[\triangle\hat{w}_{t+1}\right] - \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)\xi_w}(\hat{\Psi}_t^W - \hat{\lambda}_t^W) \\ &+ \hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C - \beta E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right].\end{aligned} \tag{G.52}$$

We let $\kappa_W = \frac{(1-\xi_w\beta)(1-\xi_w)}{(1+\eta\varepsilon_w)\xi_w}$. Thus, Equation (G.52) can be written as:

$$\triangle\hat{w}_t = \beta E_t\left[\triangle\hat{w}_{t+1}\right] - \kappa_W(\hat{\Psi}_t^W - \hat{\lambda}_t^W) + \hat{\bar{\Pi}}_t^W - \hat{\mu}_{z^+,t} - \hat{\Pi}_t^C - \beta E_t\left[\hat{\bar{\Pi}}_{t+1}^W - \hat{\mu}_{z^+,t+1} - \hat{\Pi}_{t+1}^C\right]. \tag{G.53}$$

Equation (G.53), which represents the log-linearized version of the optimal wage equation, is the same as Equation (A.13b).

H Appendix: List of variables, relative prices and definitions

H.1 List of global variables

Table 24: Global variables

Symbol	Description
z_t	State of labor augmenting technology
γ_t	State of investment-specific technology
$z_t^+ = z_t\left(\gamma_t\right)^{\frac{\alpha}{1-\alpha}}$	Composite state of technology
$z_{Ft}^+ = z_t\left(\gamma_t\right)^{\frac{\alpha_F}{1-\alpha_F}}$	Composite state of technology
$\mu_{z,t} = \frac{z_t}{z_{t-1}}$	Growth rate of labor augmenting technology
$\mu_{\gamma,t} = \frac{\gamma_t}{\gamma_{t-1}}$	Growth rate of investment-specific technology
$\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$	Composite technological growth rate
$\mu_{z_F^+,t} = \frac{z_{Ft}^+}{z_{Ft-1}^+}$	Composite technological growth rate

As mentioned earlier in Section 2, the long-run path for productivity is affected by two global, stochastic processes, z_t and γ_t. z_t^+ and z_{Ft}^+ may be interpreted as the compound effect of the labor augmenting technological process z_t and the investment-specific technological process γ_t.

Using the following definitions from this section: $z_t^+ = z_t \left(\gamma_t\right)^{\frac{\alpha}{1-\alpha}}$, $\mu_{\gamma,t} = \frac{\gamma_t}{\gamma_{t-1}}$ and $\mu_{z^+,t} = \frac{z_t^+}{z_{t-1}^+}$, we can rewrite the definition of $\mu_{z^+,t}$ as:

$$\mu_{z^+,t} = \frac{z_t \left(\gamma_t\right)^{\frac{\alpha}{1-\alpha}}}{z_{t-1} \left(\gamma_{t-1}\right)^{\frac{\alpha}{1-\alpha}}}.$$

We rewrite the above expression, and we have the following alternative expression for the definition of $\mu_{z^+,t}$:

$$\mu_{z^+,t} = \mu_{z,t} (\mu_{\gamma,t})^{\frac{\alpha}{1-\alpha}}.$$

H.2 List of Swedish variables

In this section, we present the list of variables that are specific to the Swedish economy, with a focus on aggregate variables.

Table 25: Swedish variables

Symbol	Description
	Continues on next page
C_t^{agg}	Aggregate household consumption
C_t	Aggregate Ricardian consumption
C_t^{nr}	Aggregate Non-Ricardian consumption
G_t	Government consumption
$\widetilde{C}_t$	Aggregate composite consumption
B_{t+1}^{priv}	Domestic nominal private bonds held by Ricardian households in the Swedish economy
B_{t+1}^{FH}	Foreign nominal bonds held by Ricardian households in the Swedish economy
B_t^n	Newly issued domestic nominal government bonds held by Ricardian households in the Swedish economy
B_{t+1}	Domestic nominal government bonds held by Ricardian households in the Swedish economy
R_t^B	Average interest rate on outstanding government debt
$R_t^{B,n}$	Interest rate on newly issued government debt
Ω_t^C	Average marginal utility of consumption
β_t	Discount factor
$\beta_{t+1}^r = \frac{\beta_{t+1}}{\beta_t}$	Discount factor ratio
R_t	Nominal gross interest rate
i_t	Nominal net interest rate
R_t^K	Nominal rental rate of capital services
$r_t^K = \frac{\gamma_t R_t^K}{P_t}$	Real rental rate of capital services
P_t^I	Price of private investment
u_t	Average rate of capital utilization
I_t	Aggregate private investment
I_t^G	Government investment
Ψ_t	Lump-sum profits to Ricardian households
P_t^K	Price of capital
K_t^s	Aggregate capital services
K_t	Aggregate capital
$\triangle_t^K$	Traded capital
$K_{G,t}$	Public capital
$\widetilde{K}_t^s$	Composite capital services
N_t	Aggregate labor labor demand
L_t	Aggregate labor force participation
un_t	Unemployment rate
Θ_t^n	Endogenous shifter of disutility or work
Z_t^n	Trend for marginal utility of consumption
W_t	Nominal wage index
W_t^{opt}	Optimal nominal wage
$\overline{w}_t = \frac{W_t}{z_t^+ P_t^C}$	Stationarized real wage
$\overline{w}_t^e = \frac{W_t}{z_t^+ P_t}$	Stationarized real wage relevant to employers
Π_t^W	Gross rate of aggregate wage inflation
$\overline{\Pi}_t^W$	Wage indexation factor
Ψ_t^W	Real wage markup
θ_t^b	Lagrange multiplier associated with the Ricardian household budget constraint
θ_t^k	Lagrange multiplier associated with the capital accumulation equation

Table 25 – continued from previous page	
Symbol	**Description**
θ_t^R	Lagrange multiplier associated with the average rate of return on government bonds
θ_t^{S}	Lagrange multiplier associated with the government bond equation
$\bar{\Omega}_t^R$	$\theta_t^{\mathrm{R}}/\theta_t^b$
$A_t = S_t B_{t+1}^{FH}$	Net foreign assets of the Swedish economy
$\overline{a}_t = \frac{A_t}{z_t^+ P_t}$	Stationarized net foreign assets of the Swedish economy
$\Lambda_{t,t+1}$	Stochastic discount factor
P_t^{opt}	Optimal price of intermediate goods
$P_{t,opt}^{M,n}$	Optimal price of imported goods of type n used as inputs in the production of final good $n \in \{C, I, X\}$
$P_t^{X,opt}$	Optimal price of export goods
P_t	Price of intermediate goods
P_t^C	Price of consumption goods
C_t^{xe}	Consumption of non-energy goods
C_t^e	Consumption of energy goods
$P_t^{C,xe}$	Price of non-energy consumption goods
$P_t^{C,e}$	Price of energy consumption goods
$P_t^{M,n}$	Price of imported goods of type n used as inputs in the production of final good $n \in \{Cxe, I, X, Ce\}$
P_t^X	Price of export goods
$P_t^{C,D,e}$	Price of domestically produced energy goods
Π_t	Gross inflation rate of intermediate goods
Π_t^{trend}	Inflation trend
Π_t^C	Gross inflation rate of consumption goods
$\Pi_t^{C,xe}$	Gross inflation rate of non-energy consumption goods
$\Pi_t^{C,e}$	Gross inflation rate of energy consumption goods
$\Pi_t^{M,n}$	Gross inflation of imported goods of type $n \in \{Cxe, I, X, Ce\}$
Π_t^X	Gross inflation rate of export goods
$\overline{\Pi}_t$	Indexation factor, intermediate good prices
$\overline{\Pi}_t^X$	Indexation factor, export good prices
$\overline{\Pi}_t^{M,n}$	Indexation factor, prices of import goods of type $n \in \{Cxe, I, X, Ce\}$
$D_t^{C,xe}$	Quantity of domestically produced intermediate goods used by consumption good producers
$M_t^{C,xe}$	Quantity of imported goods used by consumption good producers
D_t^I	Quantity of domestically produced intermediate goods used by investment good producers
M_t^I	Quantity of imported goods used by investment good producers
D_t^X	Quantity of domestically produced intermediate goods used by export good producers
M_t^X	Quantity of imported goods used by export good producers
$D_t^{C,e}$	Quantity of domestic goods used by energy good producers
$M_t^{C,e}$	Quantity of imported goods used by energy good producers
$M_t^{D,e}$	Swedish energy import goods excluding fixed costs (Total energy imports excluding fixed costs)
M_t	Swedish import goods taking into account fixed costs (Total imports with fixed costs)
M_t^D	Swedish import goods excluding fixed costs (Total imports excluding fixed costs)
M_t^e	Swedish imports of energy goods including fixed costs
X_t	Swedish exports
TC_t	Total cost of producing intermediate goods
TC_t^X	Total cost of producing export goods
MC_t	Nominal marginal cost of intermediate good firms
MC_t^X	Nominal marginal cost for export good firms
$\overline{mc}_t$	Real marginal cost for intermediate good firms

Table 25 – continued from previous page	
Symbol	**Description**
$\overline{mc}_t^X$	Real marginal cost for export good firms
MC_t^n	Nominal marginal cost of import good firms, $n \in \{\{C, xe\}, I, X, \{C, e\}\}$
$\overline{mc}_t^n$	Real marginal cost of import good firms, $n \in \{\{C, xe\}, I, X, \{C, e\}\}$
Y_t	Aggregate output
Y_t^m	Measured aggregate output
S_t	Nominal exchange rate: the Swedish currency price of a unit of Foreign currency
$s_t = \frac{S_t}{S_{t-1}}$	Rate of change in nominal exchange rate
$Q_t = \frac{S_t P_{F,t}^C}{P_t^C}$	Real exchange rate
$\overleftrightarrow{P}_t$	Intermediate good price dispersion
$\overleftrightarrow{P}_t^X$	Export price dispersion
$\overleftrightarrow{P}_t^{M,n}$	Import price dispersion of type n used as inputs in the production of final good $n \in \{\{C, xe\}, I, X, \{C, e\}\}$
λ_t	Intermediate good price markup
λ_t^X	Export price markup
$\lambda_t^{M,C,xe}$	Import price markup, import firms specializing in non-energy consumption goods
$\lambda_t^{M,I}$	Import price markup, import firms specializing in investment goods
$\lambda_t^{M,X}$	Import price markup, import firms specializing in export goods
$\lambda_t^{M,C,e}$	Import price markup, import firms specializing in energy consumption goods
ζ_t^n	Labor disutility shock
ζ_t	Private bond risk premium shock
$\widetilde{\phi}_t$	External risk premium shock (exchange rate shock)
ε_t	Productivity shock (stationary technology shock)
$\epsilon_{i,t}$	Monetary policy shock
τ_t^C	Consumption tax rate
τ_t^W	Labor income tax rate
τ_t^{SSC}	Social security contribution tax rate
τ_t^K	Capital income tax rate
τ_t^{TR}	Transfer tax rate
τ_t^I	Investment tax credit
TR_t^{agg}	Government transfers
TR_t	Government transfers to Ricardian households
TR_t^{nr}	Government transfers to Non-Ricardian households
T_t	Lump-sum tax on Ricardian households
B_t	Government debt
$SURP_t$	Government budget surplus

H.3 List of Swedish relative prices

In this section, we present the list of Swedish relative prices.

Table 26: Swedish relative prices

Symbol	Description
$p_t^{opt} = \frac{P_t^{opt}}{P_{t-1}}$	Relative optimal price of intermediate goods
$p_t^{C} = \frac{P_t^{C}}{P_t}$	Relative price of consumption goods
$p_t^{C,xe} = \frac{P_t^{C,xe}}{P_t}$	Relative price of non-energy consumption goods
$p_t^{C,e} = \frac{P_t^{C,e}}{P_t}$	Relative price of energy consumption goods
$p_t^{C,D,e} = \frac{P_t^{C,D,e}}{P_t}$	Relative price of domestic energy goods
$p_t^{I} = \frac{P_t^{I}}{P_t}$	Relative price of investment goods
$p_t^{K} = \frac{\gamma_t P_t^{K}}{P_t}$	Relative price of capital
$p_t^{X} = \frac{S_t P_t^{X}}{P_t}$	Relative price of export goods
$p_t^{X,opt} = \frac{S_t P_t^{X,opt}}{P_t}$	Relative optimal price of export goods
$p_t^{M,n} = \frac{P_t^{M,n}}{P_t}$	Relative price of import goods of type $n \in \{\{C,xe\},I,X,\{C,e\}\}$
$p_{t,opt}^{M,n} = \frac{P_{t,opt}^{M,n}}{P_t}$	Relative optimal price of import goods of type $n \in \{\{C,xe\},I,X,\{C,e\}\}$

Note that: the relative price of Swedish export goods in terms of Foreign intermediate goods $\frac{P_t^X}{P_{F,t}}$ can be expressed as $\frac{p_t^X p_{F,t}^C}{Q_t p_t^C}$.

H.4 List of Foreign variables

In this section, we present the list of variables that are specific to the Foreign economy, with a focus on the aggregate variables. We use the subscript F to denote the aggregate variables and the economy-wide average variables for the Foreign economy.

Table 27: Foreign variables

Symbol	Description
	Continues on next page
$C_{F,t}$	Aggregate household consumption
$C^{xe}_{F,t}$	Aggregate non-energy consumption
$C^{e}_{F,t}$	Aggregate energy consumption
B^{FF}_{t+1}	Domestic nominal bonds held by households in the Foreign economy
$\Omega^{C}_{F,t}$	Average marginal utility of consumption
$\beta_{F,t}$	Discount factor
$\beta^{r}_{F,t} = \frac{\beta_{F,t+1}}{\beta_{F,t}}$	Discount factor ratio
$R_{F,t}$	Nominal gross interest rate
$i_{F,t}$	Nominal net interest rate
$\Psi_{F,t}$	Lump-sum transfers from firms to households
$TR_{F,t}$	Lump-sum transfers from government to households
$N_{F,t}$	Aggregate labor supply
$L_{F,t}$	Aggregate labor demand
$W_{F,t}$	Aggregate nominal wage index
$\theta^{b}_{F,t}$	Lagrange multiplier associated with the household budget constraint
$\overline{w}_{F,t} = \frac{W_{F,t}}{z^{+}_{t} P^{C}_{F,t}}$	Stationarized real wage
$\overline{w}^{e}_{F,t} = \frac{W_{F,t}}{z^{+}_{t} P_{F,t}}$	Stationarized real wage relevant to employers
$\Pi^{W}_{F,t}$	Gross rate of aggregate wage inflation
$\overline{\Pi}^{W}_{F,t}$	Wage indexation factor
$\Psi^{W}_{F,t}$	Real wage markup
$\Lambda^{F}_{t,t+1}$	Stochastic discount factor
$P^{opt}_{F,t}$	Optimal price of intermediate goods
$P_{F,t}$	Price of intermediate goods
$P^{C}_{F,t}$	Price of consumption goods
$P^{C,xe}_{F,t}$	Price of non-energy consumption goods
$P^{C,e}_{F,t}$	Price of energy consumption goods
$\Pi_{F,t}$	Gross inflation rate of intermediate goods
$\Pi^{C}_{F,t}$	Gross inflation rate of consumption goods
$\Pi^{trend}_{F,t}$	Inflation trend
$\Pi^{C,xe}_{F,t}$	Gross inflation rate of non-energy consumption goods
$\Pi^{C,e}_{F,t}$	Gross inflation rate of energy consumption goods
$\overline{\Pi}_{F,t}$	Indexation factor, intermediate good prices
$TC_{F,t}$	Total cost of producing intermediate goods
$MC_{F,t}$	Nominal marginal cost for intermediate good firms
$\overline{mc}_{F,t} = \frac{MC_{F,t}}{P_{F,t}}$	Real marginal cost of intermediate good firms
$Y_{F,t}$	Aggregate output
$G_{F,t}$	Government Consumption
$\lambda_{F,t}$	Intermediate price markup
$\zeta_{F,t}$	Private bond risk premium shock
$\zeta^{n}_{F,t}$	Labor disutility shock
$\varepsilon_{F,t}$	Productivity shock
$\epsilon^{F}_{i,t}$	Monetary policy shock

H.5 List of Foreign relative prices

In this section, we present the list of Foreign relative prices.

Table 28: Foreign relative prices

Symbol	Description
$p_{F,t}^{opt} = \frac{P_{F,t}^{opt}}{P_{F,t-1}}$	Relative optimal price of intermediate goods
$p_{F,t}^{C} = \frac{P_{F,t}^{C}}{P_{F,t}}$	Relative price of consumption goods
$p_{F,t}^{C,xe} = \frac{P_{F,t}^{C,xe}}{P_{F,t}}$	Relative price of non-energy consumption goods
$p_{F,t}^{C,e} = \frac{P_{F,t}^{C,e}}{P_{F,t}}$	Relative price of energy consumption goods

Continued on next page

I Appendix: Model parameters and functional forms

I.1 Model parameters

In this section, we present the list of parameters that are used in the model equations that are listed in Appendix A.

Table 29: Model parameters

Symbol	Description
ω	Size of Foreign economy relative to the Swedish economy
μ_z	Gross growth rate of labor augmenting technology
μ_γ	Gross growth rate of investment-specific technology
μ_{z^+}	Composite technological growth rate
β	Discount factor
β_F	Foreign discount factor
ρ_h	Consumption habit
$\rho_{F,h}$	Foreign consumption habit
α_G	Share of private consumption in the composite consumption
υ_G	Elasticity of substitution between private and public consumption
snr	Share of Non-Ricardian households over total population
ϖ_{ss}	Share of aggregate transfers going to Non-Ricardians in steady state
ϖ_{dyn}	Share of aggregate transfers going to Non-Ricardians off steady state
S''	Investment adjustment cost
χ	Indexation to previous inflation, intermediate goods
χ_F	Foreign indexation to previous inflation, intermediate goods
$\chi_{m,C,xe}$	Indexation to previous inflation, import firms specializing in non-energy consumption goods
$\chi_{m,C,e}$	Indexation to previous inflation, import firms specializing in energy consumption goods
$\chi_{m,I}$	Indexation to previous inflation, import firms specializing in investment goods
$\chi_{m,X}$	Indexation to previous inflation, import firms specializing in export goods
$\chi_{F,m}$	Foreign indexation to previous inflation, imported goods
χ_x	Indexation to previous inflation, export goods

Continued on next page

Table 29 – continued from previous page	
Symbol	**Description**
$\chi_{F,x}$	Foreign indexation to previous inflation, export goods
χ_w	Indexation to previous wage inflation
$\chi_{F,w}$	Foreign indexation to previous wage inflation
Π^C	Gross inflation target
Π_F^C	Foreign gross inflation target
$\widetilde{\phi}_a$	External risk premium parameter associated with net foreign asset
$\widetilde{\phi}_s$	External risk premium parameter associated with exchange rate
λ^W	Wage markup
λ_F^W	Foreign wage markup
λ	Intermediate good price markup
λ_F	Foreign intermediate good price markup
$\lambda^{M,C,xe}$	Import price markup, import firms specializing in non-energy consumption goods
$\lambda^{M,I}$	Import price markup, import firms specializing in investment goods
$\lambda^{M,X}$	Import price markup, import firms specializing in export goods
$\lambda^{M,C,e}$	Import price markup, import firms specializing in energy consumption goods
λ_F^M	Foreign import price markup
λ^X	Export price markup
λ_F^X	Foreign export price markup
τ_F^w	Tax on labor in Foreign
ν_C	Elasticity of substitution between non-energy and energy goods used for consumption goods production
$\nu_{C,xe}$	Elasticity of substitution between domestic and imported goods used for non-energy consumption goods production
$\nu_{C,e}$	Elasticity of substitution between domestic and imported goods used for energy consumption goods production
ν_I	Elasticity of substitution between domestic and imported goods used for investment goods production
ν_x	Elasticity of substitution between domestic and imported goods used for export goods production
$\nu_{F,C}$	Elasticity of substitution between imported and foreign consumption goods in Foreign
ν_K	Elasticity of substitution between private and public capital
α_K	Share of private capital in composite capital
ϑ^C	Weight of non-energy in the production of consumption goods
$\vartheta^{C,xe}$	Home bias in the production of non-energy consumption goods
$\vartheta^{C,e}$	Home bias in the production of energy consumption goods
ϑ^I	Home bias in the production of investment goods
ϑ^X	Home bias in the production of export goods
ϑ_F^C	Foreign home bias in the production of consumption
A_n	Labor disutility
$A_{F,n}$	Foreign labor disutility
A_F	Foreign production parameter
η	Inverse of Frisch elasticity
χ_n	Parameter associated with persistency of trend component of endogenous shifter in labor disutility
η_F	Foreign inverse of Frisch elasticity
α	Capital share in production
δ	Private capital depreciation rate
δ_G	Public capital depreciation rate
σ_a	Capital utilization cost, $\sigma_a = a''/a'$
a'	Parameter associated with capital utilization cost
a''	Parameter associated with capital utilization cost
ι^K	Indicator parameter for tax deduction of depreciation of capital
ξ	Calvo domestic prices
ξ_x	Calvo export prices

Continued on next page

Table 29 – continued from previous page	
Symbol	**Description**
$\xi_{m,C,xe}$	Calvo import prices, import firms specializing in non-energy consumption goods
$\xi_{m,C,e}$	Calvo import prices, import firms specializing in energy consumption goods
$\xi_{m,I}$	Calvo import prices, import firms specializing in investment goods
$\xi_{m,X}$	Calvo import prices, import firms specializing in export goods
ξ_w	Calvo wages
ξ^F	Foreign Calvo domestic prices
ξ_x^F	Foreign Calvo export prices
ξ_m^F	Foreign Calvo import prices
ξ_w^F	Foreign Calvo wages
ω_C^X	Weight on consumption in investment demand
ν_F	Price elasticity of export demand
ρ	Interest rate smoothing, Taylor rule
r_π	Inflation response, Taylor rule
r_{un}	Unemployment response, Taylor rule
$r_{\triangle\pi}$	Difference in inflation response, Taylor rule
$r_{\triangle un}$	Difference in unemployment response, Taylor rule
r_ζ	Neutral rate response to risk premium
$r_{F,\zeta}$	Foreign neutral rate response to risk premium
ρ_F	Foreign interest rate smoothing, Taylor rule
$r_{F,\pi}$	Foreign inflation response, Taylor rule
$r_{F,y}$	Foreign output response, Taylor rule
$r_{F,\triangle\pi}$	Foreign difference in inflation response, Taylor rule
$r_{F,\triangle y}$	Foreign difference in output response, Taylor rule
ρ_ζ	Persistence, private bond risk premium shock
ρ_β	Persistence, discount factor shock
ρ_{μ^z}	Persistence, labor augmenting technology shock
ρ_γ	Persistence, investment-specific technology shock
ρ_{ζ^c}	Persistence, consumption shock
$\rho_{\zeta_F^c}$	Persistence, Foreign consumption shock
ρ_Υ	Persistence, stationary investment-specific shock
ρ_{Υ_F}	Persistence, Foreign stationary investment-specific shock
$\rho_{\tilde{\phi}}$	Persistence, exchange rate shock (external risk premium shock)
ρ_{ζ^n}	Persistence, labor disutility preference shock
$\rho_{p^{D,C,e}}$	Persistence, domestic energy price
$\rho_{p_F^{D,C,e}}$	Persistence, Foreign energy price
ρ_ε	Persistence, productivity shock
ρ_{ε_F}	Persistence, Foreign productivity shock
$\rho_{\zeta_F^n}$	Persistence, Foreign labor disutility preference shock
ρ_{ζ_F}	Persistence, Foreign private bond risk premium shock
ρ_{I^G}	Persistence, government investment shock
ρ_g	Persistence, government consumption shock
ρ_{τ^C}	Persistence, consumption tax shock
$\rho_{\tau^{SSC}}$	Persistence, social security contribution shock
ρ_{τ^W}	Persistence, labor income tax shock
ρ_{τ^K}	Persistence, capital income tax shock
ρ_{τ^I}	Persistence, investment tax credit shock
$\rho_{\tau^{TR}}$	Persistence, transfer tax shock
$\rho_{tr^{agg}}$	Persistence, aggregate transfer shock
ρ_{1,b^T}	Persistence, debt target shock AR(1)
ρ_{2,b^T}	Persistence, debt target shock AR(2)
ρ_{λ^W}	Persistence, wage markup shock to intermediate good producers
ρ_λ	Persistence, markup shock to intermediate good producers
ρ_{λ_F}	Persistence, markup shock to Foreign intermediate good producers
$\rho_{\lambda^{M,C}}$	Persistence, markup shock to import firms specializing in consumption goods
	Continued on next page

Table 29 – continued from previous page	
Symbol	**Description**
$\rho_{\lambda^{M,I}}$	Persistence, markup shock to import firms specializing in investment goods
$\rho_{\lambda^{M,X}}$	Persistence, markup shock to import firms specializing in export goods
$\rho_{\lambda^{X}}$	Persistence, markup shock to exporting good firms
$\rho_{\Pi^{trend}}$	Persistence, inflation trend shock
$corr_{\varepsilon}$	Parameter governing correlation, stationary technology
$corr_{\zeta}$	Parameter governing correlation, risk premium
$corr_{\Upsilon}$	Parameter governing correlation, investment efficiency
$corr_{\zeta^{c}}$	Parameter governing correlation, consumption preference
$corr_{\zeta^{c}_{F},\Upsilon_{F}}$	Parameter governing correlation between consumption and investment in Foreign
α_{B}	Probability of debt maturing in every period (i.e. average maturity)
$\mathcal{F}_{tr,surp}$	Surplus gap coefficient in aggregate transfer policy rule
$\mathcal{F}_{tr,un}$	Unemployment coefficient in aggregate transfer policy rule
$\mathcal{F}_{g,b}$	Debt gap coefficient in government consumption policy rule
$\mathcal{F}_{g,surp}$	Surplus gap coefficient in government consumption policy rule
$\mathcal{F}_{g,y}$	Output gap coefficient in government consumption policy rule

I.2 Auxiliary parameters

In this section, we present the list of auxiliary parameters that are used in our model equations, which are shown in Appendix A. The auxiliary model parameters are functions of structural parameters, which are calibrated and can be found in Section I.1.

Table 30: Auxiliary model parameters

Symbol	Description
$\varepsilon_w^F = \frac{\lambda_F^W}{\lambda_F^W - 1}$	Foreign wage-elasticity of labor demand
$\kappa_W = \frac{(1-\xi_w)(1-\xi_w\beta)}{\xi_w(1+\eta\varepsilon_w)}$	Slope of wage Phillips curve
$\kappa_{F,W} = \frac{\left(1-\xi_w^F\right)\left(1-\xi_w^F\beta_F\right)}{\xi_w\left(1+\eta_F\varepsilon_w^F\right)}$	Slope of Foreign wage Phillips curve
$\kappa = \frac{(1-\xi\beta)(1-\xi)}{\xi}$	Slope of Phillips curve, intermediate goods
$\kappa_F = \frac{\left(1-\xi^F\beta_F\right)\left(1-\xi^F\right)}{\xi^F}$	Slope of Foreign Phillips curve, intermediate goods
$\kappa_X = \frac{(1-\xi_x)(1-\xi_x\beta)}{\xi_x}$	Slope of Phillips curve, export goods
$\kappa_{F,X} = \frac{\left(1-\xi_x^F\right)\left(1-\xi_x^F\beta\right)}{\xi_x^F}$	Slope of Foreign Phillips curve, export goods
$\kappa_{M,C,xe} = \frac{(1-\xi_{m,C,xe})(1-\beta\xi_{m,C,xe})}{\xi_{m,C,xe}}$	Slope of Phillips curve, import firms specializing in non-energy consumption goods
$\kappa_{M,C,e} = \frac{(1-\xi_{m,C,e})(1-\beta\xi_{m,C,e})}{\xi_{m,C,e}}$	Slope of Phillips curve, import firms specializing in energy consumption goods
$\kappa_{M,I} = \frac{(1-\xi_{m,I})(1-\beta\xi_{m,I})}{\xi_{m,I}}$	Slope of Phillips curve, import firms specializing in investment goods
$\kappa_{M,X} = \frac{(1-\xi_{m,X})(1-\beta\xi_{m,X})}{\xi_{m,X}}$	Slope of Phillips curve, import firms specializing in export goods
$\kappa_{F,M} = \frac{(1-\xi_m^F)(1-\beta_F\xi_m^F)}{\xi_m^F}$	Slope of Foreign Phillips curve, imported goods
$\vartheta^{C,xe} = \frac{\left(1-\frac{\overline{m}^{D,Cxe}}{\overline{c}}\right)}{\left(\frac{\overline{m}^{D,Cxe}}{\overline{c}}\left[\left(p^M\right)^{\nu_c-1}-1\right]\right)+1}$	Home bias for non-energy consumption goods
$\vartheta^{C,e} = \frac{\left(1-\frac{\overline{m}^{D,Ce}}{\overline{c}}\right)}{\left(\frac{\overline{m}^{D,Ce}}{\overline{c}}\left[\left(p^M\right)^{\nu_c-1}-1\right]\right)+1}$	Home bias for energy consumption goods
$\vartheta^{I} = \frac{\left(1-\frac{\overline{m}^{D,I}}{\overline{I}}\right)}{\left(\frac{\overline{m}^{D,I}}{\overline{I}}\left[\left(p^M\right)^{\nu_I-1}-1\right]\right)+1}$	Home bias for investment goods
$\vartheta^{X} = \frac{\left(1-\frac{\overline{m}^{D,X}}{\overline{x}}\right)}{\left(\frac{\overline{m}^{D,X}}{\overline{x}}\left[\left(p^M\right)^{\nu_x-1}-1\right]\right)+1}$	Home bias for export goods
$\psi^{C,xe} = \vartheta^{C,xe} + \frac{1}{1+\omega}(1-\vartheta^{C,xe})$	Weight of the domestically produced intermediate goods in the production of non-energy consumption goods

Symbol	Description
$\psi^{C,e} = \vartheta^{C,e} + \frac{1}{1+\omega}(1-\vartheta^{C,e})$	Weight of the domestically produced intermediate goods in the production of energy consumption goods
$\psi^{X} = \vartheta^{X} + \frac{1}{1+\omega}(1-\vartheta^{X})$	Weight of the domestically produced intermediate goods in the production of export goods
$\psi^{I} = \vartheta^{I} + \frac{1}{1+\omega}(1-\vartheta^{I})$	Weight of the domestically produced intermediate goods in the production of investment goods
$\psi_F^{C,xe} = 1 - \frac{1}{1+\omega}(1-\vartheta_F^{C,xe})$	Weight of the domestically produced intermediate goods in the production of non-energy consumption goods, Foreign
$\psi_F^{X} = 1 - \frac{1}{1+\omega}(1-\vartheta_F^{X})$	Weight of the domestically produced intermediate goods in the production of export goods, Foreign
$\phi = (\lambda - 1)\overline{y}$	Fixed cost for intermediate good producers
$\phi^X = (\lambda^X - 1)\overline{x}$	Fixed cost for export good producers
$\phi^{M,C,xe} = (\lambda^{M,C,xe} - 1)\overline{m}^{C,xe}$	Fixed cost for import firms specializing in non-energy consumption goods
$\phi^{M,C,e} = (\lambda^{M,C,xe} - 1)\overline{m}^{C,e}$	Fixed cost for import firms specializing in energy consumption goods
$\phi^{M,I} = (\lambda^{M,I} - 1)\overline{m}^{I}$	Fixed cost for import firms specializing in investment goods
$\phi^{M,X} = (\lambda^{M,X} - 1)\overline{m}^{X}$	Fixed cost for import firms specializing in export goods
$\phi^{Mxe} = \phi^{M,C} + \phi^{M,I} + \phi^{M,X}$	Total fixed cost of the imported good sector
$\phi_F = (\lambda_F - 1)\overline{y}_F$	Fixed cost for Foreign intermediate good producers
$\phi_F^X = (\lambda_F^X - 1)\overline{x}_F$	Fixed cost for Foreign export good producers
$\phi_F^M = (\lambda_F^M - 1)\overline{m}_F$	Fixed cost for Foreign import good producers
$a' = \frac{r^K}{p^I}$	Parameter associated with capital utilization cost
$a'' = a'\sigma_a$	Parameter associated with capital utilization cost
$A_n = \frac{(\overline{\Omega}^C(1-\tau^W)\overline{w})}{(\lambda^W\zeta^n\Theta n^\eta)}$	Parameter associated with labor disutility function
$\rho_{\Pi^{C,trend}}$	
$A_{F,n} = \frac{(\overline{\Omega}_F^C(1-\tau_F^W)\overline{w}_F)}{(\lambda_F^W\zeta_F^n l_F^\eta)}$	Parameter associated with Foreign labor disutility function
$H = \left(1-\tau^K\right)r^K +$ $\left(1-\delta+\iota^K\tau^K\delta\frac{\mu_\gamma}{\Pi}\right)p^K$	Parameter associated with household purchases of installed capital equation
$A_F = \frac{\overline{y}_F}{(1/\lambda_F l_F)}$	Parameter associated with Foreign intermediate good production function

J Appendix: Estimation methodology and the assessment of the posterior

The model is estimated using Bayesian methods, which combine data with prior beliefs. The initial set of beliefs, summarized in the prior distribution, is updated using the observed data and the specified model to yield an updated set of beliefs, known as the posterior distribution. By Bayes' theorem, the relation between these objects is

$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)} \propto p(y|\theta)p(\theta). \tag{J.1}$$

The posterior probability density function (pdf), the object of interest in Bayesian estimation, is denoted by $p(\theta|y)$ while the likelihood and prior are given by $p(y|\theta)$ and $p(\theta)$, respectively. The term in the denominator, $p(y)$, is the marginal likelihood and is used to compare models. Because it is independent of θ it is a constant with respect to it. In the final step of the equation, this constant is simply dropped and the symbol $\propto$ used to indicate that the posterior distribution $p(\theta|y)$ is *proportional to* the likelihood times the prior, $p(y|\theta)p(\theta)$, known as the posterior kernel.

For DSGE models, the posterior distribution lacks a closed, analytical form and simulation methods must be used. These generally rely on the fact that the likelihood and prior can easily be evaluated for a given θ. $p(y|\theta)$ can be evaluated using the Kalman filter, and $p(\theta)$ is typically a product of univariate densities for the parameters such as, beta, normal and inverse-gamma probability density functions and is simple to evaluate for a known θ. DSGE models are predominantly estimated using Markov Chain Monte Carlo (MCMC) methods. The idea behind MCMC methods in general is to create a correlated (explaining the *Markov Chain* part) sequence of random numbers (explaining the *Monte Carlo* part) that represents random but correlated draws from the posterior distribution $p(\theta|y)$. In simplified terms, MCMC methods exploit the fact that Markov chains under certain conditions converge to a long-run, stationary, steady-state distribution. MCMC samplers are therefore constructed in such a way that their stationary distributions are exactly the posterior distributions of interest. Informally, the implication is that a Markov chain (and hence also an MCMC sampling procedure) will eventually reach its stationary distribution, if it runs for a long enough period of time. Once it has reached its stationary distribution, all the draws that it produces can be treated as if they are draws from the posterior distribution of interest. These draws are used to characterize the posterior distribution—for example, by plotting the distribution of the samples (characterizing the posterior itself), computing the mean of the sample (estimating the posterior mean), etc.

Random-Walk Metropolis-Hastings: We use random-walk Metropolis-Hastings (RWMH), a specific MCMC algorithm, for estimating the model. At the current iteration i of the algorithm, a move to a new parameter vector is proposed. The proposal is made using a multivariate normal distribution centered on the current parameter value:

$$\theta^* \sim \mathrm{N}(\theta^{(i)}, c\Sigma). \tag{J.2}$$

The constant c is the tuning (or scale) parameter of the algorithm, and Σ is the covariance matrix for the proposal distribution.

The moves are not always accepted, but instead accepted with a certain probability. This probability is given by

$$r = \min\left\{1, \frac{p(y|\theta^*)p(\theta^*)}{p(y|\theta^{(i)})p(\theta^{(i)})}\right\}. \tag{J.3}$$

That is, with probability r the newly proposed value θ^* is accepted. If the posterior pdf is higher at the newly proposed value $p(y|\theta^*)p(\theta^*) > p(y|\theta^{(i)})p(\theta^{(i)})$ and $r = 1$ and the proposal is always accepted, but the converse is not true. Taking steps to values that yield lower values of the posterior pdf is necessary since MCMC is not an optimization method but a procedure for sampling from a distribution.

Setting Scale and Covariance for the Proposal Distribution: While MCMC procedures under certain conditions are guaranteed to converge to the stationary distribution and eventually provide draws from the posterior distribution, their efficiency and speed is heavily determined by c and Σ. Efficiency in this context means to maximize the amount of information obtained given a fixed number of samples, or given a fixed amount of estimation time. The scaling parameter c determines how large jumps the proposal distribution makes. If it is set to a very small number, then $\theta^* \approx \theta^{(i)}$ and the new draw will very likely be accepted. If it is set to a large value, then the proposed draw will often be useless and not accepted. Whether the value for c is appropriate is monitored through the acceptance ratio, which says how often the proposals are accepted. Acceptance ratios in the range of 25–45% are typically considered appropriate. If it is too high, c is increased, and vice versa. The problem with too high acceptance ratios is that the posterior distribution is explored too slowly—correlation between draws is very high, meaning that the information they carry largely overlaps. [82]

The covariance matrix Σ determines the covariance of the individual proposals. That is, if two parameters in θ are highly correlated, it will be much more efficient to propose moves that are also highly correlated. In principle, Σ can be set to anything, but a Σ that resembles the true correlation in the posterior distribution will increase efficiency. The most popular approach is to maximize the posterior kernel with respect to θ to find the mode, i.e. the peak, of the posterior distribution. The matrix of second-order partial derivatives evaluated at the modal value is the Hessian matrix, and the most popular choice for Σ is the negative of the Hessian. See the posterior mode of the model for each parameter (check plots from DYNARE) below in Figures J - J.

Assessing MCMC Chains and Quality of Estimation There are multiple problems that frequently occur when estimating DSGE models. Some common problems are unidentified parameters, non-convergence of MCMC chains, multimodal posteriors. Evidence of these problems can generally be inferred from trace plots and plots showing prior and posterior distributions, but lack of evidence is typically not evidence of lack of problems.

Trace plots display time series of the sampled parameters, where time here refers to iteration index. Because MCMC relies on Markov chains reaching their stationary distribution, these plots should show time series that appear to be stationary—fluctuations should be around a mean that is constant and the variance, i.e. the spread of the time series, should also not vary but be even across time. Non-convergence usually manifests itself through apparent trends, since the Markov chain was still moving towards the stationary distribution without having reached it yet. Multimodality, i.e., a posterior with multiple peaks, can also often easily be identified by visible regime shifts—the mean might shift from one level to another, which would indicate that the Markov chain has moved to another part of the posterior distribution. Non-identifiability can mostly be seen by comparing prior and posterior distributions, since if these are exactly on top of each other then data did not change our belief of the parameter.

Below, we provide the posterior mode check-plots provided by DYNARE toobox for checking whether the mode-computation found the mode. We use the guidance of Pfeifer(2013) to interpret the graphs and to assess the overall estimation. Each figure shows the log-likelihood and the log-posterior values at the mode and within a certain interval of parameter values around the estimated mode. We would expect that the maximum of the posterior likelihood should be at the estimated mode (vertical line). And the interpretation of the differences in the log-posterior and the log-likelihood would be so that the closer the log-likelihood and the log-posterior to each other, the smaller the effect of the prior on the posterior. The check-plots show that for many variables log-likelihood (data) determines the posterior distribution, largely, rather than the prior distribution, except for a few variables, e.g., elasticity of substitution between imported and domestically produced goods for non-energy consumption goods production and investment goods production, and wage indexation parameter $\nu_{C,xe}$ (nuCxe), ν_I (nuI) and χ_w (chiW) respectively.

We also provide the prior and the posterior distributions below. The comparison of the posterior distributions and the prior distributions doesn't show a particular problem regarding identification of estimated parameters and multimodality. Moreover, from our visual examination of trace plots for each parameters and both univariate and multivariate MCMC convergence diagnostics, we conclude that there is no apparent problem of convergence of the estimation. [83]

[82]The acceptance ratio for all 5 chains in our estimation process is around 32% percent, which falls within the range accepted as reasonable in the literature.

[83]Trace plots and convergence graphs are not shown in this documentation.

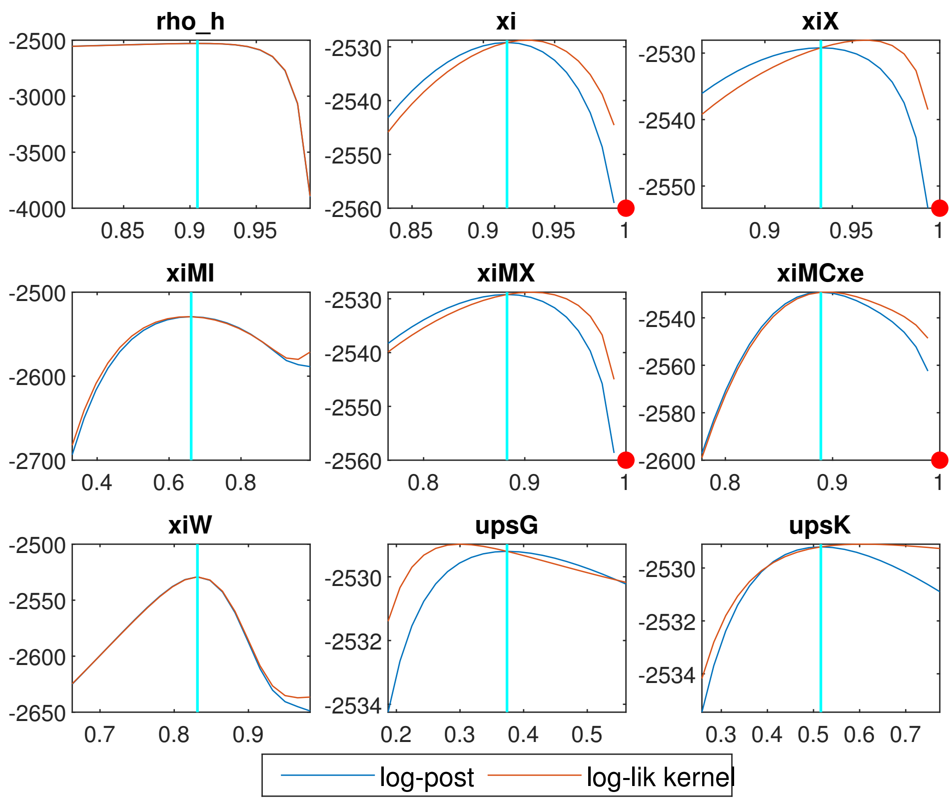

Figure 15: SELMA estimation results-check plots 1

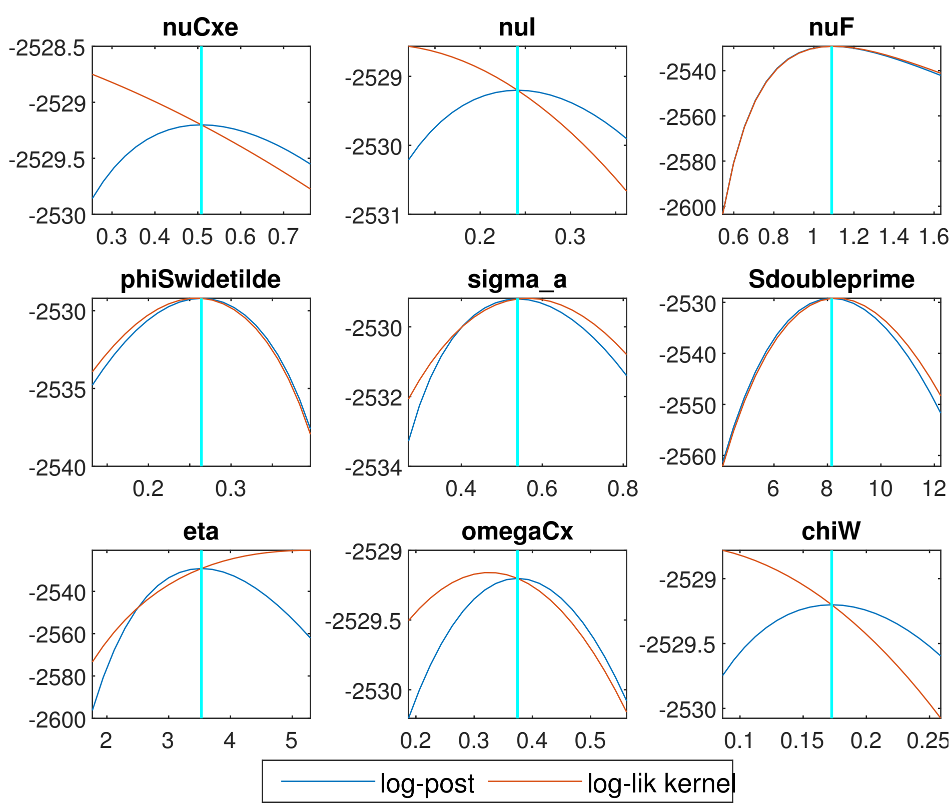

Figure 16: SELMA estimation results-check plots 2

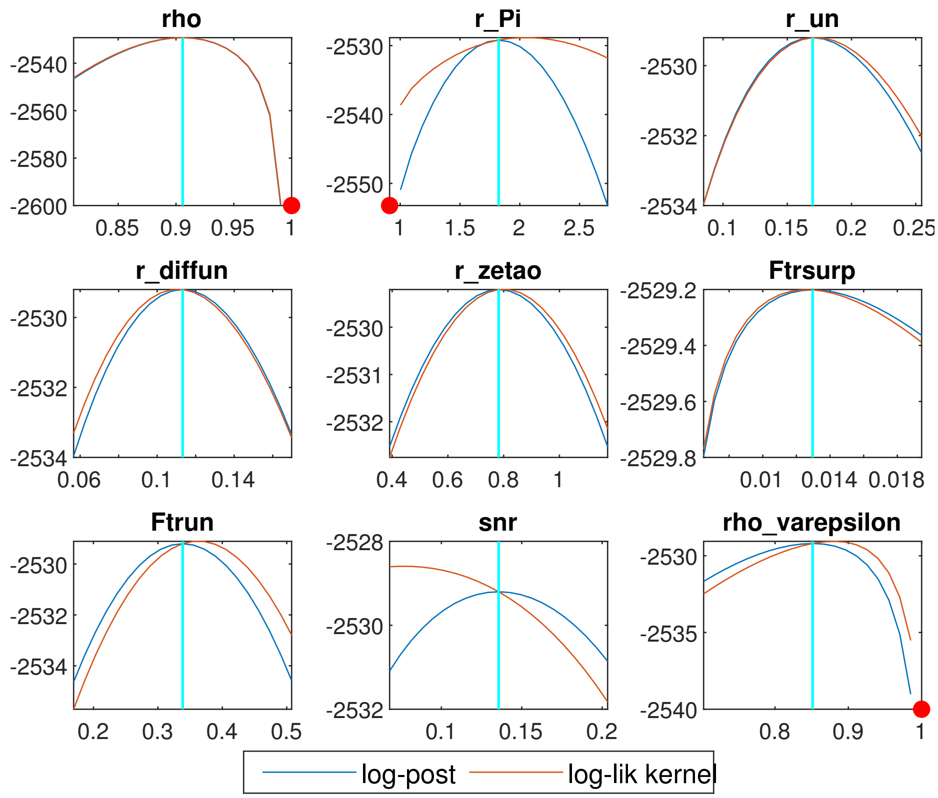

Figure 17: SELMA estimation results-check plots 3

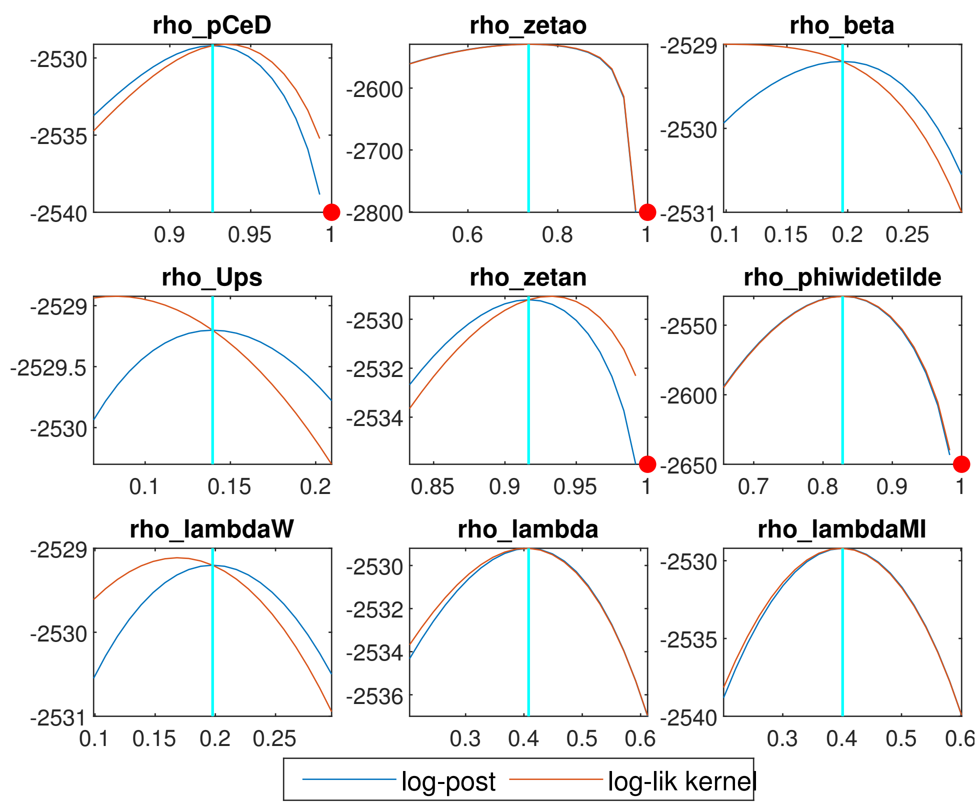

Figure 18: SELMA estimation results-check plots 4

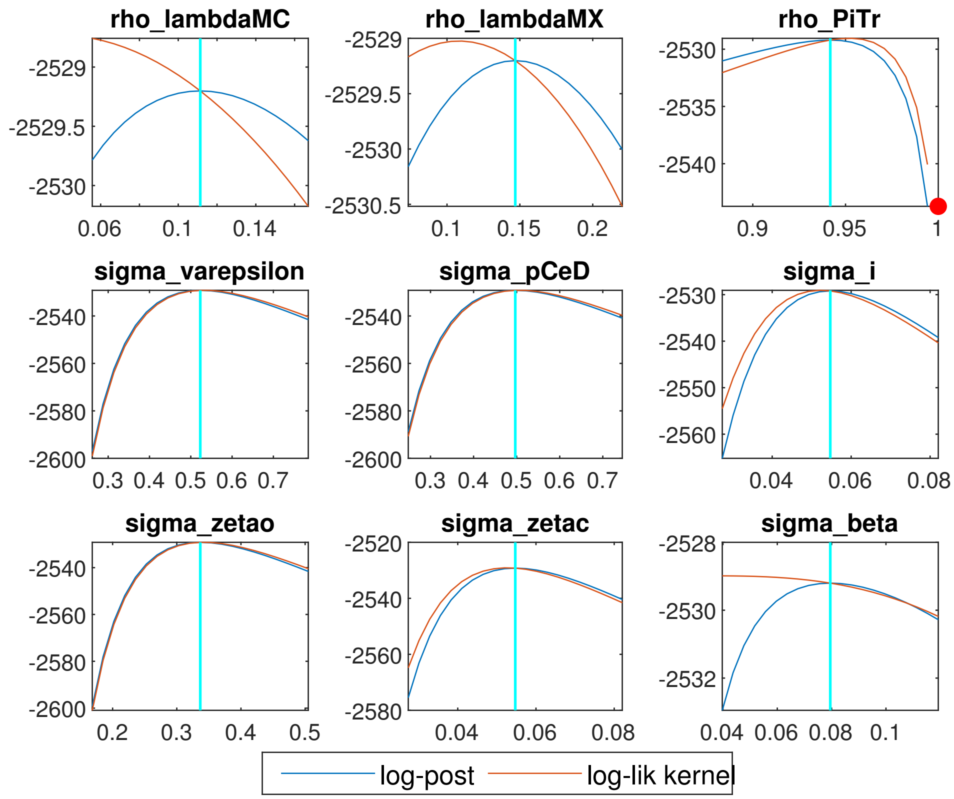

Figure 19: SELMA conditional CheckPlots5

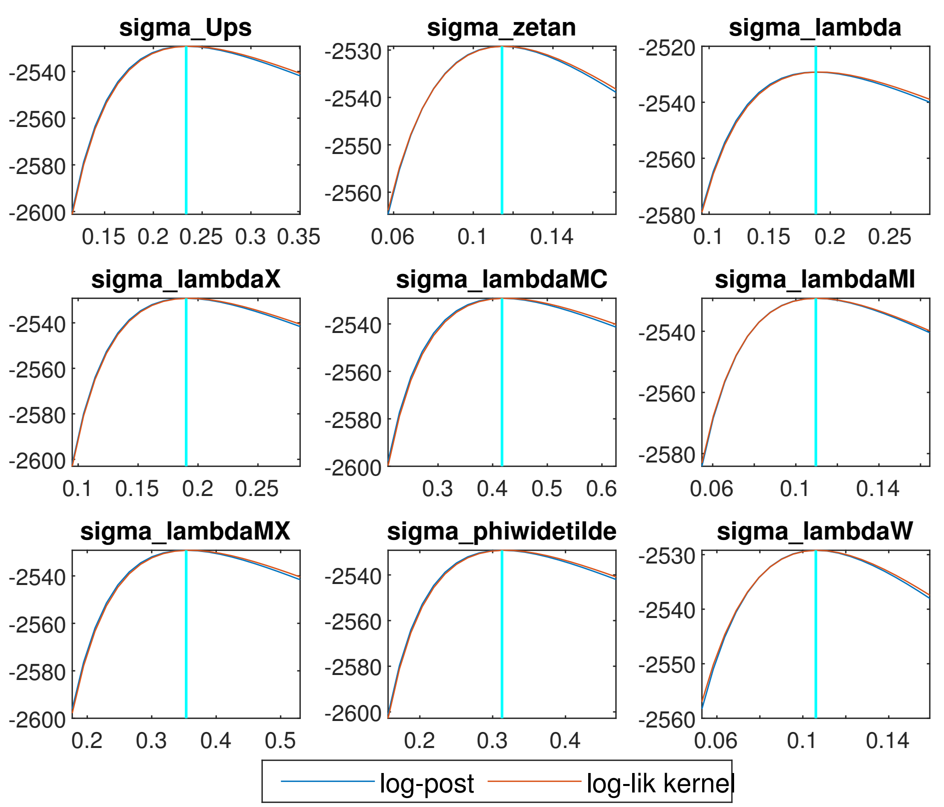

Figure 20: SELMA estimation results-check plots 6

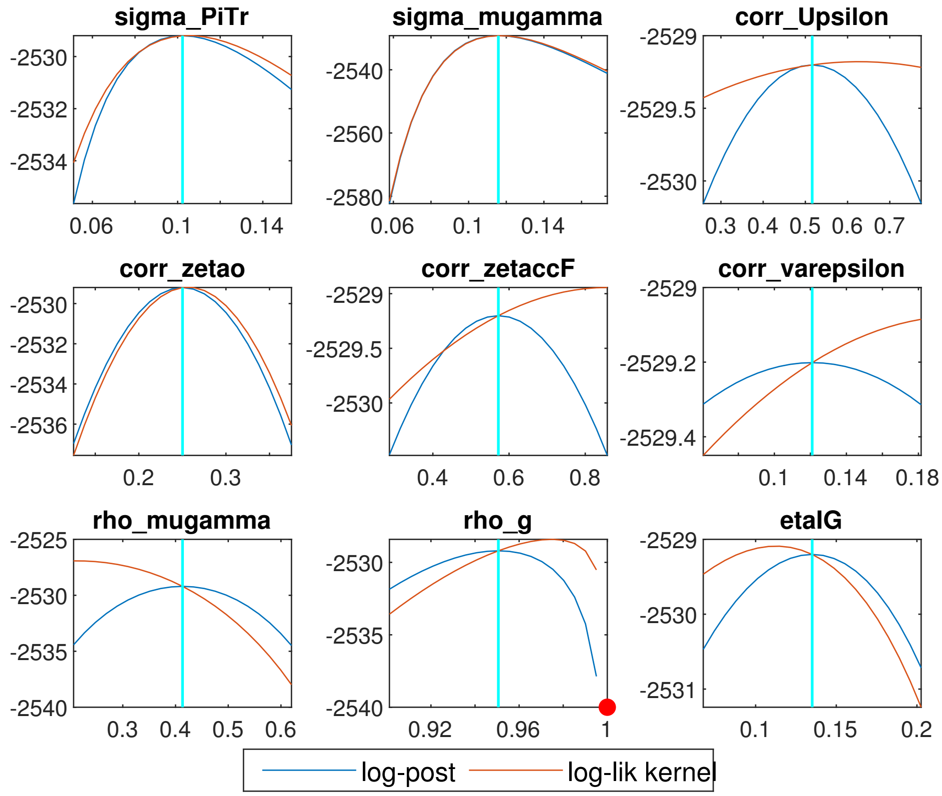

Figure 21: SELMA estimation results-check plots 7

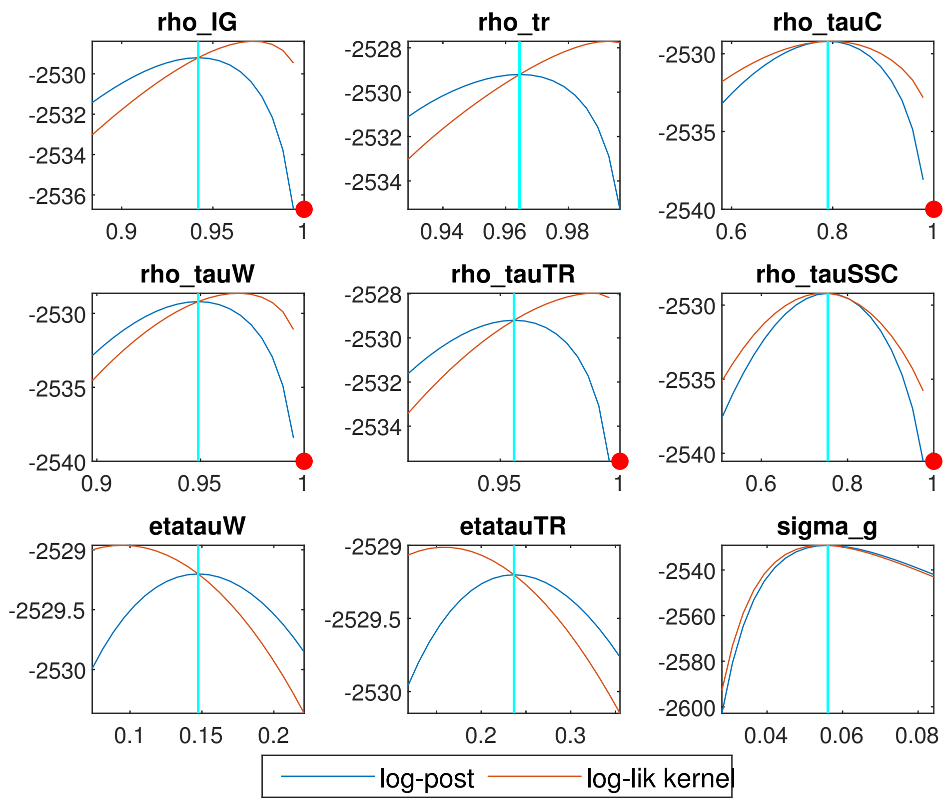

Figure 22: SELMA estimation results-check plots 8

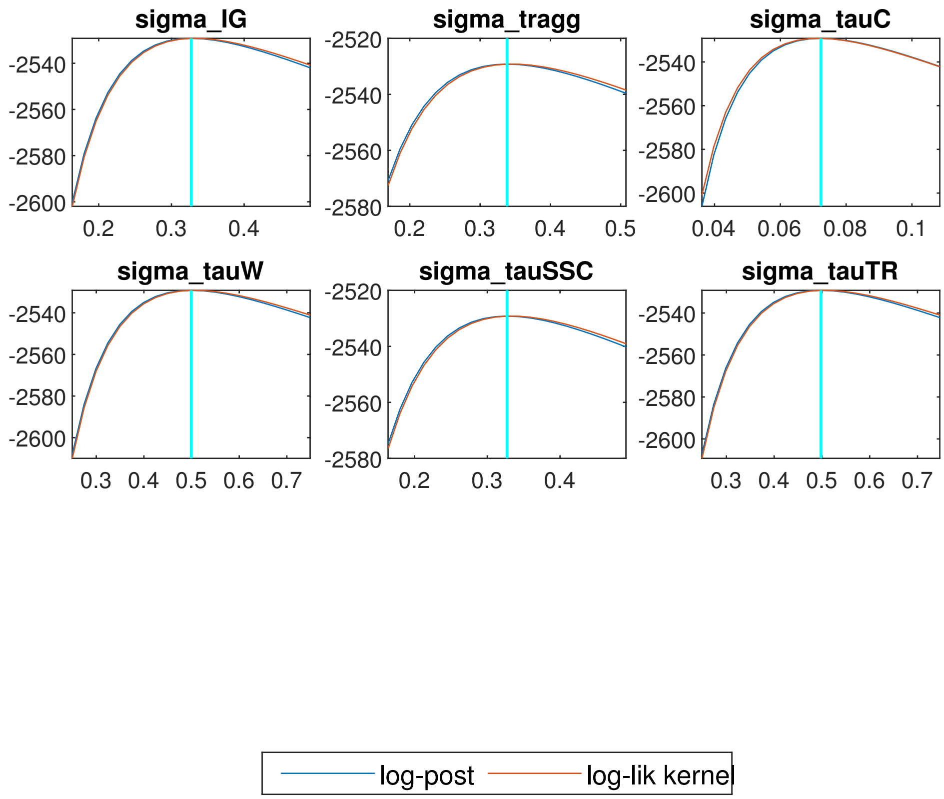

Figure 23: SELMA estimation results-check plots 9

Priors and posteriors Below, we provide prior and posterior distributions provided by the DYNARE toolbox. In each graph, the grey line is the prior distribution, the black line is the posterior distribution and the vertical green line indicates the posterior mode. The x-axis shows a part of the support of the prior distribution and the y-axis shows the densities.

Table 31: Model coding symbols of estimated parameters and model correspondents

Code symbol	**Model symbol**	**Description**
rho_h	ρ_h	Consumption habit
xi	ξ	Calvo domestic prices
xiX	ξ_x	Calvo export prices
xiMI	$\xi_{m,I}$	Calvo import prices, import firms specializing in investment goods
xiMX	$\xi_{m,X}$	Calvo import prices, import firms specializing in export goods
xiMCxe	$\xi_{m,C,xe}$	Calvo import prices, import firms specializing in non-energy consumption goods
xiW	ξ_w	Calvo wages
upsG	υ_G	Elasticity of substitution between private and public consumption
upsK	υ_K	Elasticity of substitution between private and public capital
nuCxe	$\nu_{C,xe}$	Elasticity of substitution between domestic and imported goods used for non-energy consumption goods production
nuI	ν_I	Elasticity of substitution between domestic and imported goods used for investment goods production
nuF	$\nu_{F,C}$	Elasticity of substitution between imported and foreign consumption goods in Foreign
phiSwidetilde	$\widetilde{\phi}_s$	External risk premium parameter associated with exchange rate
sigma_a	σ_a	Capital utilization cost, $\sigma_a = a''/a'$
Sdoubleprime	S''	Investment adjustment cost
eta	η	Inverse of Frisch elasticity

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Table 31 – continued from previous page		
Code symbol	**Model symbol**	**Description**
omegaCx	ω_C^X	Weight on consumption in investment demand
chiW	χ_w	Indexation to previous wage inflation
rho	ρ	Interest rate smoothing, Taylor rule
r_Pi	r_π	Inflation response, Taylor rule
r_un	r_{un}	Unemployment response, Taylor rule
r_diffun	$r_{\triangle un}$	Difference in unemployment response, Taylor rule
r_zetao	r_ζ	Neutral rate response to risk premium
Ftrsurp	$\mathcal{F}_{tr,surp}$	Surplus gap coefficient in aggregate transfer policy rule
Ftrun	$\mathcal{F}_{tr,un}$	Unemployment coefficient in aggregate transfer policy rule
snr	snr	Share of Non-Ricardian households over total population
rho_varepsilon	ρ_ε	Persistence, productivity shock
rho_pCeD	$\rho_{p^{D,C,e}}$	Persistence, domestic energy price
rho_zetao	ρ_ζ	Persistence, private bond risk premium shock
rho_beta	ρ_β	Persistence, discount factor shock
rho_Ups	ρ_Υ	Persistence, stationary investment-specific shock
rho_zetan	ρ_{ζ^n}	Persistence, labor disutility preference shock
rho_phiwidetilde	$\rho_{\widetilde{\phi}}$	Persistence, exchange rate shock (external risk premium shock)
rho_lambdaW	ρ_{λ^W}	Persistence, wage markup shock to intermediate good producers
rho_lambda	ρ_λ	Persistence, markup shock to intermediate good producers
rho_lambdaMI	$\rho_{\lambda^{M,I}}$	Persistence, markup shock to import firms specializing in investment goods
rho_lambdaMC	$\rho_{\lambda^{M,C}}$	Persistence, markup shock to import firms specializing in consumption goods
rho_lambdaMX	$\rho_{\lambda^{M,X}}$	Persistence, markup shock to import firms specializing in export goods
rho_PiTr	$\rho_{\Pi^{trend}}$	Persistence, inflation trend shock
sigma_varepsilon	σ_ε	Standard deviation, productivity shock
sigma_pCeD	$\sigma_{p^{D,C,e}}$	Standard deviation, domestic energy price
sigma_i	σ_i	Standard deviation, monetary policy rate
sigma_zetao	σ_ζ	Standard deviation, private bond risk premium shock
sigma_zetac	σ_{ζ^c}	Standard deviation, consumption preference shock
sigma_beta	σ_β	Standard deviation, discount factor shock
sigma_Ups	σ_Υ	Standard deviation, stationary investment-specific shock
sigma_zetan	σ_{ζ^n}	Standard deviation, labor disutility preference shock
sigma_lambda	σ_λ	Standard deviation, markup shock to intermediate good producers
sigma_lambdaX	σ_{λ^X}	Standard deviation, markup shock to export good producers
sigma_lambdaMC	$\sigma_{\lambda^{M,C}}$	Standard deviation, markup shock to import firms specializing in consumption goods
sigma_lambdaMI	$\sigma_{\lambda^{M,I}}$	Standard deviation, markup shock to import firms specializing in investment goods
sigma_lambdaMX	$\sigma_{\lambda^{M,X}}$	Standard deviation, markup shock to import firms specializing in export goods
sigma_phiwidetilde	$\sigma_{\widetilde{\phi}}$	Standard deviation, exchange rate shock (external risk premium shock)
sigma_lambdaW	σ_{λ^W}	Standard deviation, wage markup shock to intermediate good producers
sigma_PiTr	$\sigma_{\Pi^{trend}}$	Standard deviation, inflation trend shock
sigma_mugamma	σ_{μ_γ}	Standard deviation, non-stationary investment specific shock
corr_Upsilon	$corr_\Upsilon$	Parameter governing correlation, investment efficiency
corr_zetao	$corr_\zeta$	Parameter governing correlation, risk premium
corr_zetaccF	$corr_{\zeta^c}$	Parameter governing correlation, consumption preference
corr_varepsilon	$corr_\varepsilon$	Parameter governing correlation, stationary technology
rho_mugamma	ρ_{μ_γ}	Persistence, non-stationary investment specific shock
rho_g	ρ_g	Persistence, government consumption shock
etaIG	η_{IG}	MA coefficient, government investment shock
rho_IG	ρ_{IG}	Persistence, government investment shock
rho_tr	$\rho_{tr^{agg}}$	Persistence, aggregate transfer shock
rho_tauC	ρ_{τ^C}	Persistence, consumption tax shock
rho_tauW	ρ_{τ^W}	Persistence, labor income tax shock
rho_tauTR	$\rho_{\tau^{TR}}$	Persistence, transfer tax shock
rho_tauSSC	$\rho_{\tau^{SSC}}$	Persistence, social security contribution shock
etatauW	η_{τ^W}	MA coefficient, labor income tax shock
etatauTR	$\eta_{\tau^{TR}}$	MA coefficient, transfer tax shock
sigma_g	σ_g	Standard deviation, government consumption shock

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Table 31 – continued from previous page		
Code symbol	**Model symbol**	**Description**
sigma_IG	σ_{IG}	Standard deviation, government investment shock
sigma_tragg	$\sigma_{tr^{agg}}$	Standard deviation, aggregate transfer shock
sigma_tauC	σ_{τ^C}	Standard deviation, consumption tax shock
sigma_tauW	σ_{τ^W}	Standard deviation, labor income tax shock
sigma_tauSSC	$\sigma_{\tau^{SSC}}$	Standard deviation, social security contribution shock
sigma_tauTR	$\sigma_{\tau^{TR}}$	Standard deviation, transfer tax shock

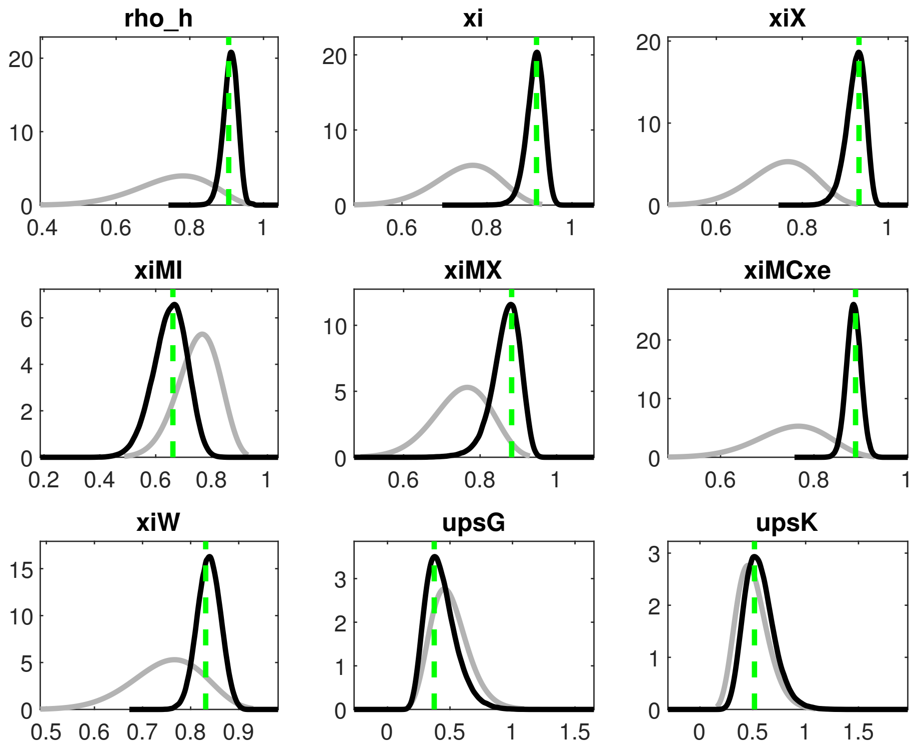


Figure 24: SELMA estimation results-priors and posteriors 1

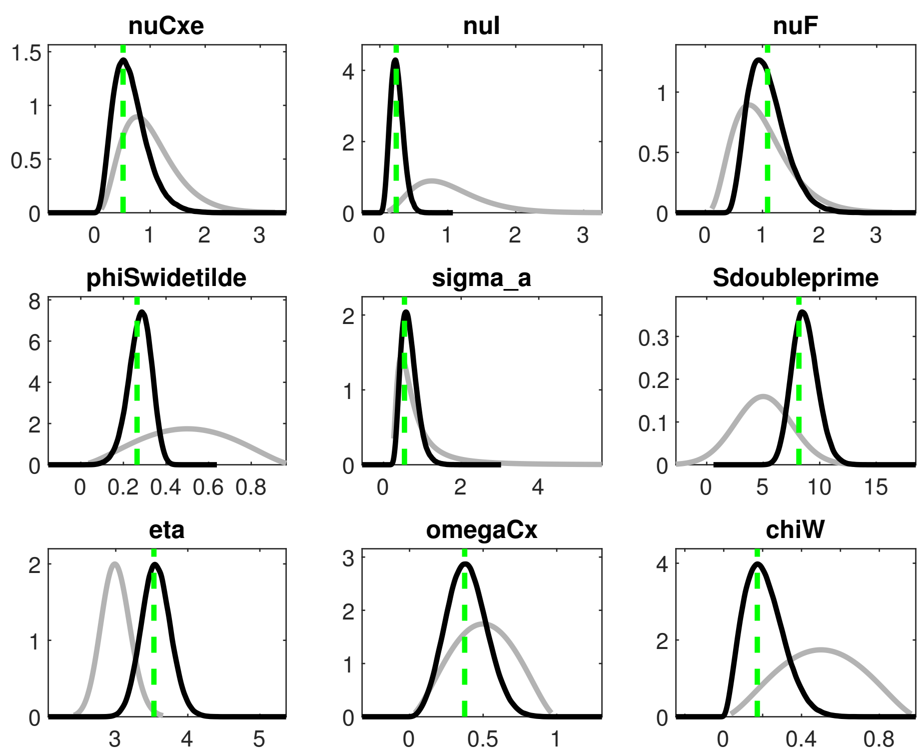


Figure 25: SELMA estimation results-priors and posteriors 2

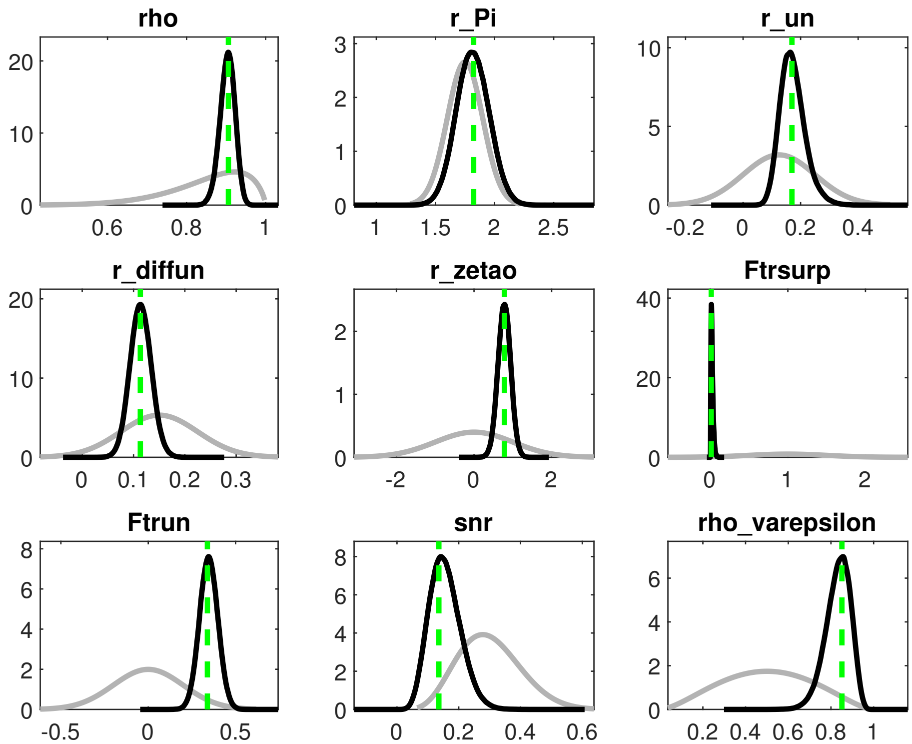


Figure 26: SELMA estimation results-priors and posteriors 3

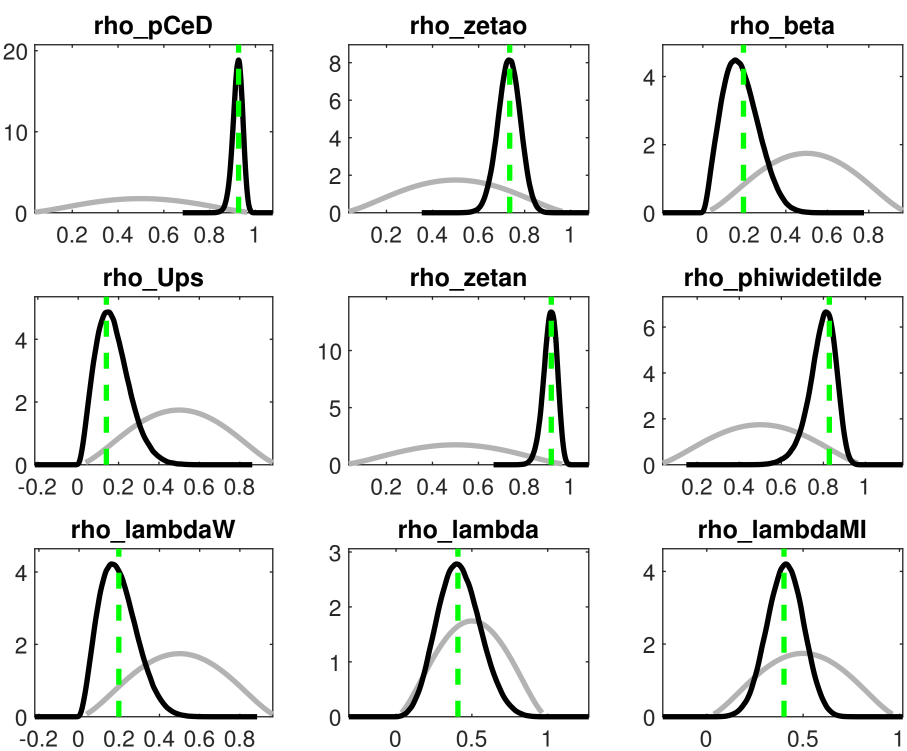


Figure 27: SELMA estimation results-priors and posteriors 4

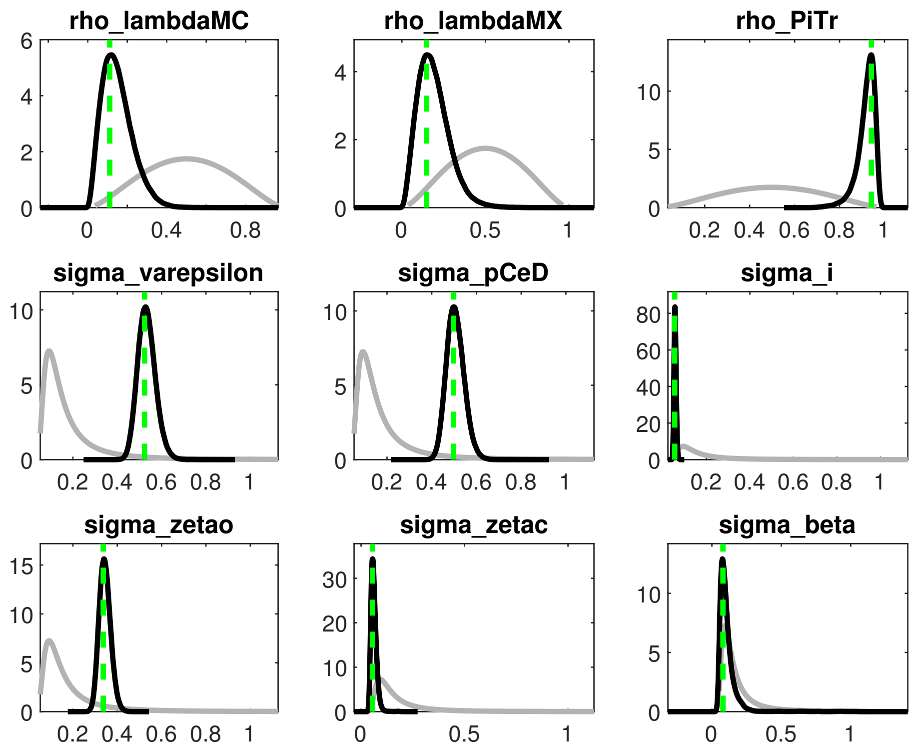


Figure 28: SELMA conditional est PriorsAndPosteriors5

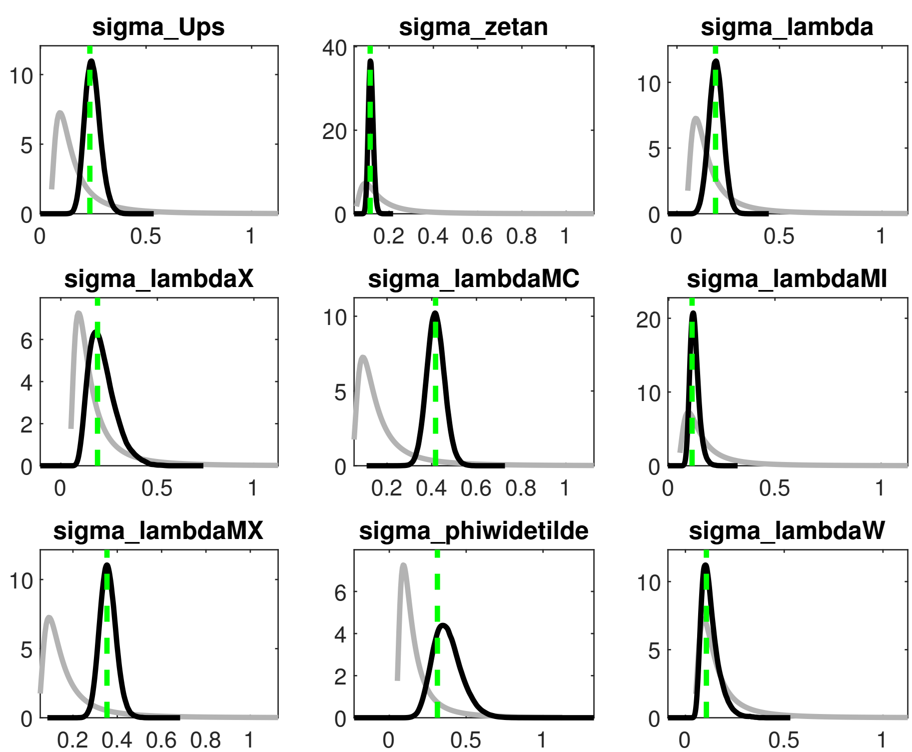


Figure 29: SELMA estimation results-priors and posteriors 6

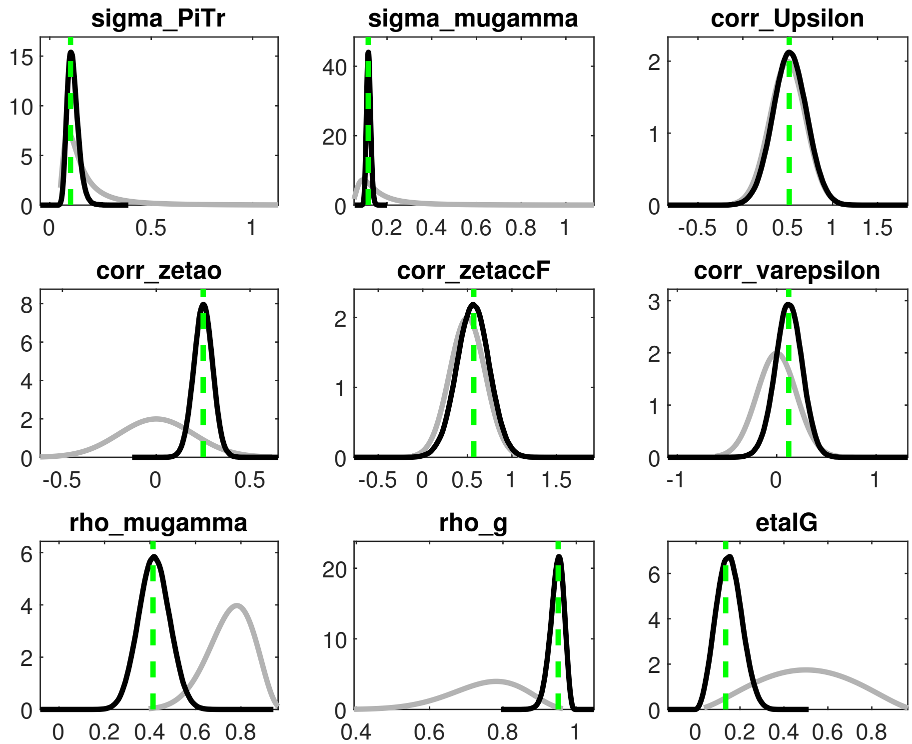


Figure 30: SELMA estimation results-priors and posteriors 7

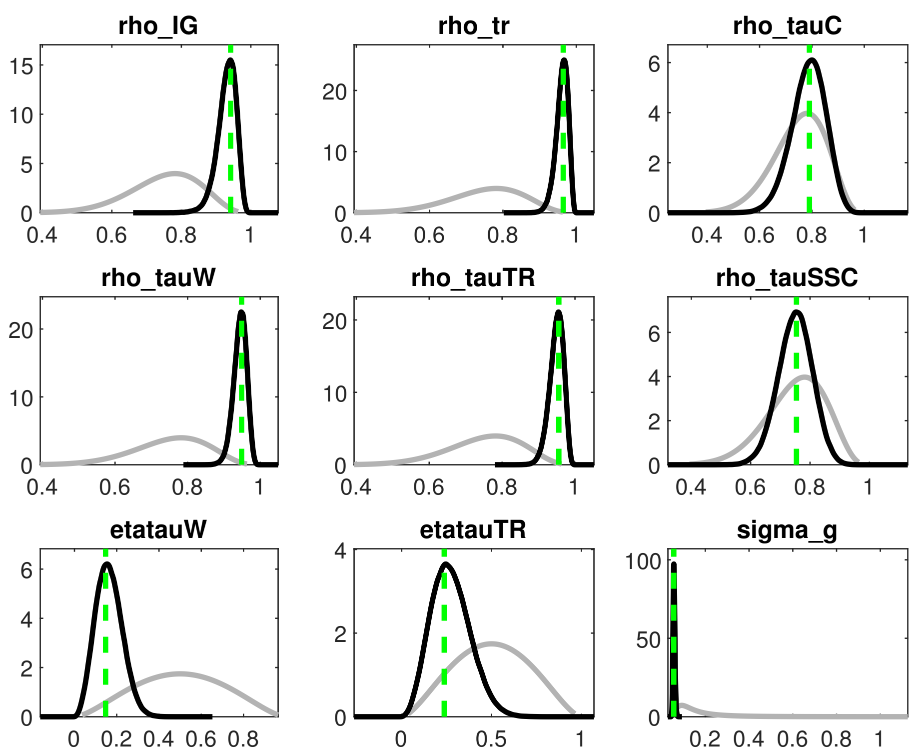


Figure 31: SELMA estimation results-priors and posteriors 8

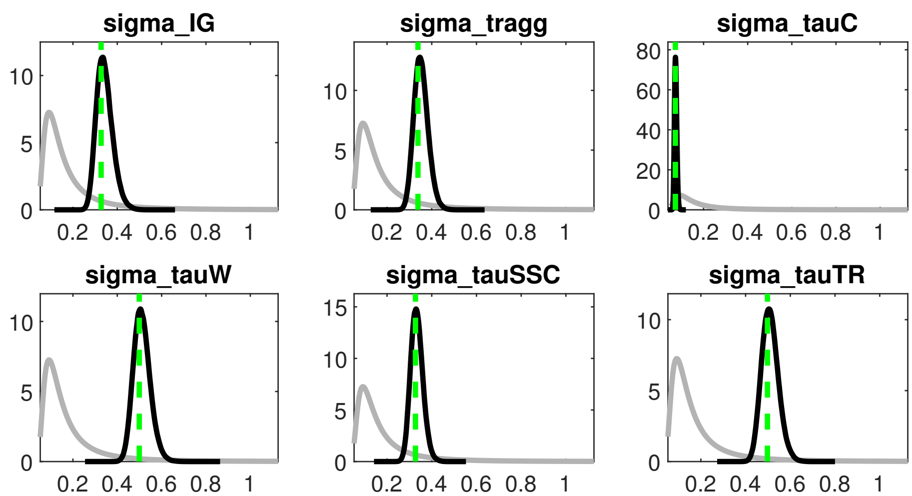


Figure 32: SELMA estimation results-priors and posteriors 9

K Appendix: Data transformations

K.1 Foreign data transformation

KIX-6 series are calculated by taking the weighted average of the series for corresponding countries. Table 32 provides the weights of countries in Sweden's international trade and also the weights in KIX-6 aggregation.While constructing the aggregate KIX-6 series to be used as observable data we must transform some of the country level series. Since we already collect the seasonally adjusted data we don't apply seasonal adjustment to our raw data, except for a few series explained below. We seasonally adjust series with the X13-ARIMA method when necessary.

To be able to get the quarterly growth rates of per capita GDP, private consumption and investment for KIX-6, we first divide quarterly GDP, private final consumption expenditure and gross fixed capital formation series to quarterly working age population series, respectively. However, for the US and Japan the working age population series are available in monthly frequency and we take the quarterly averages to convert them into the quarterly series. The quarterly working age population series for the euro area, Norway and the UK include some missing data points. We fill the missing data points with linear interpolation. Another issue for these series along with the corresponding ones for Denmark is that the quarterly data is not available before 2000. To extend these series back to 1995Q1, we use annual series for these countries. First, we convert annual series into quarterly series by the linear-match last method, which inserts the annual value into the last period of the quarter of that year and performs linear interpolation on the missing quarters (the first three quarters) of the year.[84] Using the quarterly converted series, we calculate the growth rate of the working age population series. Then, we fill the missing values before 2000 by extending the original quarterly series using the growth rates obtained above. Then, it is straightforward to get the quarterly per capita GDP, private consumption and investment growth rates.

In order to compare the series for hours worked across countries better and form a KIX-6 aggregate we have transformed all the hours worked data to weekly hours worked per capita on a quarterly frequency. Moreover, we use the growth rate of the series due to its non-stationarity. From this, one can obtain quarterly percentage change in weekly hours worked per capita. However, we need some transformation of the hours worked data. Norway has two different data series, the first one is for employees and the second one is for self-employed workers, where both series are quarterly. We sum the two series and transform quarterly numbers to weekly numbers by dividing quarterly numbers by 13, which is the number of weeks in a quarter. Then we calculate per capita series by dividing the obtained number by the working age population. For Japan we multiply "monthly hours worked per employee in industries covered" and monthly employment to get the monthly total hours worked since quarterly series of hour worked is not available. The monthly hours worked series is not seasonally adjusted and hence we first seasonally adjust it before we multiply it with monthly employment. To be able to get hours worked per week series at the monthly frequency we multiply the monthly series by 3 (number of months in a quarter) and divide by 13 (the number of weeks in a quarter). Then, we take the 3-month average of each quarter to create the quarterly series of hours worked per week. For the UK, the series is available at the monthly frequency and we also take the 3-month average to convert it to a quarterly frequency. For the euro area, the series are hours worked per quarter in quarterly frequency and we divide it by 13 to have the weekly hours worked numbers in quarterly frequency. For the US, the series includes annual numbers and we divide it by 52, which is the number of weeks in a year, to have the weekly numbers.

The inflation series for all countries has a monthly frequency. Hence, quarterly inflation series are created by averaging monthly series in the respective quarter. Except for Japan and the US, all the inflation series are harmonised index of consumer prices (HICP) indices. The original inflation series are not seasonally adjusted and hence we seasonally adjust them. For Japan and the US we use consumer price index (CPI) indices because we have missing monthly HICP index values for Japan and the US for the period before 2010 M1 and 2001 M12, respectively.

For wages, we use OECD's "Labour Compensation per Unit of Labour Input" quarterly data for all the KIX-6 countries. The KIX-6 credit rates series are constructed by only the euro area and the US data due to non-availability of the corresponding data for other countries. Moreover, the credit data is available for the US only until 2016. For years 2016-2019, only the euro area data is used.

The KIX-6 weighted monetary policy rate series are constructed with five of the KIX-6 countries data and recalculated weights, excluding Japan, for the period 1995Q1 and 1998Q2. The reason is that from April 1998, the Bank of Japan adopted a price stability mandate, and dropped maximum potential output mandate with the new law.

After all the transformations, we annualize the quarterly growth rates of per capita GDP, private consumption, investment and hours worked, quarterly growth of nominal wage, the quarterly inflation rate and the inflation rate excluding energy.

[84] For instance, the annual value in 1999 is inserted into 1999Q4 and the linear interpolation fills in 1999Q1, 1999Q2, 1999Q3.

Table 32: Foreign sector average trading weights between 2000-2019

Countries	Weights in Sweden's trade	Weights in KIX-6
Euro area (19)	0.50	0.61
US	0.10	0.12
UK	0.08	0.10
Denmark	0.05	0.06
Norway	0.05	0.06
Japan	0.04	0.05

K.2 Swedish data transformation

In our data set, there are certain series in annual frequency that need to be transformed into quarterly frequency. We interpolate structural savings, state income tax and state income tax reductions using "the quadratic match sum" to convert them into quarterly frequency. Furthermore, we interpolate gross debt with the quadratic match average method because gross debt is a stock variable.

The observable variables, GDP, consumption, investment, exports, imports, public consumption and public investment, are expressed in per capita annualized quarterly growth rates. Hence, by definition, to construct these variables, we first divide each of these series by the working age population and then calculate the annualized quarterly growth rates.

We compute certain aggregate ratios for the calibration of the model's steady state, including private investment to GDP ratio, exports to GDP ratio, imports to GDP ratio, government consumption to GDP ratio and government investment to GDP ratio, and take the sample average of each ratio to use as the steady state value. To construct these ratios, we use the series in current prices. Macroeconomic stabilization is implemented mainly through public transfers to households, where the variable of interest is public transfers to potential GDP. We calculate this policy variable by dividing transfers excluding pension payments by the potential GDP and take the sample average as the steady state value.[85]

Inflation and inflation excluding energy are constructed using HICP and HICP excluding energy series, respectively. We annualize all the quarterly inflation series.

We construct the quarterly growth rate of the real exchange rate by taking the quarterly growth of the KIX-6 exchange rate series.

We divide government transfers to households, government structural savings and government gross debt by potential GDP to get model's government transfers to GDP, structural savings to GDP and government debt to GDP ratios, respectively.

We calculate the tax rates by dividing the total tax revenue from each tax type by corresponding tax base, we first calculate tax revenue from consumption, labor income, transfers and social security contributions. Tax revenue from labor income is defined by

$$TRL_t = LI_t/(LI_t + TRH_t) * dshk_t + SIT_t - (SITR_t - eaho_t) \tag{K.1}$$

where TRL_t is the total tax revenue from labor income, LI_t is labor income, TRH_t is transfers to households, $dshk_t$ is municipal income tax revenue, SIT_t is state income tax revenue, $SITR_t$ is state income tax reductions and $eaho_t$ is tax deduction for pensioners. Total tax revenue from transfers is defined as

$$TRTr_t = TRH_t/(LI_t + TRH_t) * dshk_t - eaho_t \tag{K.2}$$

where TRT_t is the total tax revenue from transfers, TRH_t is transfers to households, $dshk_t$ is municipal income tax revenue and $eaho_t$ is tax deduction for pensioners.

We take the first difference of all these tax rates to construct the observable variables, except the social security tax rate. For this specific tax rate, we take into account the structural policy change regarding this tax in 2007 by the newly elected government by considering a smooth switch from the pre-2006 equilibrium tax rate to the post-2010 equilibrium tax rate.

K.3 Outliers

We treat some observations of GDP, personal consumption expenditure and investment of the euro area as outliers following Corbo and Strid (2020). However, we deviate from this reference on our choice on outliers in investment data. For example, we treat the data as outlier for the period between 2018Q3-2019Q4. Other observations treated as outliers are the following: the euro area GDP in 2014Q1 and 2015Q1, the euro area

[85] Pension payments are excluded from transfers to calculate the policy variable because this fiscal policy variable is designed to respond to economic fluctuations. In SELMA, government transfers are rule based and respond to deviation of the unemployment rate from its long run equilibrium. Pension payments under government transfers are not linked to economic conditions and thus don't change with business cycles, which approximates the fiscal policy implementation in Sweden.

investment in 2014Q1, 2015Q1, 2017Q2, 2017Q3. We remove these values from the series and apply linear interpolation to fill the missing values.

L Appendix: Observation equations

Observation equations for Foreign data: The observation equation for the Foreign sector GDP is the following:

$$\Delta Y_{F,t}^{obs} = c_{Y_F} + 400(\hat{y}_{F,t} - \hat{y}_{F,t-1} + \hat{\mu}_{z+F,t} + ln(\mu_{z+F})) + \sigma_{\Delta Y_F}^{me} \varepsilon_{\Delta Y_F,t}^{me} \tag{L.1}$$

where $\Delta Y_{F,t}^{obs}$ is the annualized quarterly growth rate of Foreign GDP per capita, $\hat{y}_{F,t}$ and $\hat{\mu}_{z+F,t}$ are log-linearized model variables which are percentage deviations from the steady state GDP and technological growth, respectively. $\sigma_{\Delta Y_F}^{me} \varepsilon_{\Delta Y_F,t}^{me}$ is the measurement error. The steady state annualized quarterly growth rate of GDP in the model is given by

$$\Delta Y_F = SS_Y = 400 ln(\mu_{z+F}) \tag{L.2}$$

where the model quarterly growth rate of Foreign GDP $ln(\mu_{z+F})$ is calibrated to 0.32 percent, which matches the sample average Foreign GDP per capita growth rate. We calibrate the excess parameter c_{Y_F} to zero.[86]

The observation equation for Foreign household consumption is given by

$$\Delta C_{F,t}^{obs} = c_{c_F} + 400(\hat{c}_{F,t} - \hat{c}_{F,t-1} + \hat{\mu}_{z+F,t} + ln(\mu_{z+F})) + \sigma_{\Delta c_F}^{me} \varepsilon_{\Delta c_F,t}^{me}, \tag{L.3}$$

which is very similar to the observation equation for GDP and reflects the model assumption that GDP and household consumption have the same trend growth, $ln(\mu_{z+F})$. As in the previous case, we calibrate c_{c_F} to zero.

The observation equation for Foreign investment is given by

$$\Delta I_{F,t}^{obs} = c_{I_F} + 400(\hat{I}_{F,t} - \hat{I}_{F,t-1} + \hat{\mu}_{z+F,t} + \hat{\mu}_{\gamma F,t} + ln(\mu_{z+F}) + ln(\mu_{\gamma F})) + \sigma_{\Delta I_F}^{me} \varepsilon_{\Delta I_F,t}^{me}, \tag{L.4}$$

which is also very similar to observation equation for GDP with minor differences related to the steady state growth rate of investment, which is given by

$$\Delta I_F = SS_I = 400(ln(\mu_{z+F}) + ln(\mu_{\gamma F})) \tag{L.5}$$

where $ln(\mu_{\gamma F})$ is the Foreign investment-specific growth rate and calibrated to zero, hence Foreign GDP and Foreign investment have the same trend. Again, excess parameter c_{I_F} is set to zero.

The observation equations for Foreign CPI and CPI-excluding energy inflation are given by

$$\Pi_{F,t}^{C,obs} = c_{\Pi_F^C} + 400(\hat{\Pi}_{F,t}^C + ln(\Pi_F^C)) + \sigma_{\Pi_F^C}^{me} \varepsilon_{\Pi_F^C,t}^{me} \tag{L.6}$$

and

$$\Pi_{F,t}^{C,xe,obs} = c_{\Pi_F^{C,xe}} + 400(\hat{\Pi}_{F,t}^{C,xe} + ln(\Pi_F^{C,xe})) + \sigma_{\Pi^{C,xe}}^{me} \varepsilon_{\Pi^{C,xe},t}^{me} \tag{L.7}$$

where $\Pi_{F,t}^{C,obs}$ and $\Pi_{F,t}^{C,xe,obs}$ are the observed annualized quarterly Foreign inflation rates. The steady state inflation rates $400 ln(\Pi_F^C)$ and $400 ln(\Pi_F^{C,xe})$ are calibrated to 2 percent, which is motivated by inflation targets of KIX-6 countries' central banks.

The observation equation for wages in Foreign economy is given by

$$\Pi_{F,t}^{W,obs} = c_{W_F} + 400(\hat{w}_{F,t}^e - \hat{w}_{F,t-1}^e + \hat{\mu}_{z+F,t} + ln(\mu_{z+F}) + ln(\Pi_F^C)) + \sigma_{\Delta w_F^e}^{me} \varepsilon_{\Delta w_F^e,t}^{me} \tag{L.8}$$

where $\Delta W_{F,t}^{obs}$ is the observed nominal wage growth rate, the hatted variables are, as a notation standard, corresponding to model variables in percentage change from their steady state. In the data, the average nominal wage growth rate is lower than the nominal GDP per capita growth rate, which is in line with the decrease in labor share in foreign economy over time. The steady state inflation and GDP growth implies 3.3 percent $(400(ln(\mu_{z+F}) + ln(\Pi_F^C)))$ annualized quarterly nominal wage growth. However, the data sample average for wage growth is 2.4 percent, so that the average nominal wage growth rate in the data sample is 0.7 percentage points lower than the nominal GDP (wage) growth in the steady state. We calibrate the excess parameter accordingly by setting it to -0.9 percent, to get rid of this discrepancy between theoretical model steady state and the data sample average. Recall that an excess parameter represents the component of the data that, by assumption, can not be explained by the model.

The observation equation for the monetary policy rate is given by

$$i_{F,t}^{obs} = c_{i_F} + 400(\breve{i}_{F,t} + (ln R_F)) + \sigma_{i_F}^{me} \varepsilon_{i_F,t}^{me} \tag{L.9}$$

[86] Note that, in the document, we either calibrate the excess parameter to zero or we didn't include it in the observation equation when we don't see any reason to use the excess parameter for reconciling data and the model. Those two cases can be considered as the same.

where $i_{F,t}^{obs}$ is the observed policy rate, $\breve{i}_{F,t}$ is the model policy rate (deviation from its long-run equilibrium), $lnR_F = ln(\mu_{z+F}) + ln(\Pi^C) - ln(\beta)$ is the nominal interest rate on private bonds. The calibration of $ln(\mu_{z+F})$ and $ln(\Pi^C)$ has already been given. We calibrate the discount factor to 0.999, to obtain a steady state monetary policy rate at a reasonable level.[87] Given this calibration, the model-implied steady state policy interest rate is 3.5 percent, which is well above the sample average of 2.2 percent. We calibrate the excess parameter c_{i_F} to -1.25 for both to reconcile the data and the model to some extent, and to make the sum of model steady state and this non-explained component is consistent with the NIER's assessments of the long run monetary policy rates at central banks, which is 2.25 percent currently. As mentioned earlier, the measurement error is calibrated to zero for the monetary policy rate.

The observation equation for the corporate spread in Foreign economy is given by

$$Spr_{F,t}^{obs} = c_{Spr_F} + 400(\hat{\zeta}_{F,t}) + \sigma_{\zeta_F}^{me}\varepsilon_{\zeta_F,t}^{me} \tag{L.10}$$

where $Spr_{F,t}^{obs}$ is the observed corporate spread. The theoretical model's steady state for the corporate spread is calibrated to zero and excess parameter c_{Spr_F} is calibrated to the sample mean, the model's corporate spread gap would capture deviations from the sample mean. Similar to monetary policy rate, the measurement error is calibrated to zero for the corporate spread.

The observation equation for hours worked is given by

$$\Delta N_{F,t}^{obs} = 400(\hat{n}_{F,t} - \hat{n}_{F,t-1}) + \sigma_{n_F}^{me}\varepsilon_{n_F,t}^{me}. \tag{L.11}$$

where $\Delta N_{F,t}^{obs}$ is the observed percentage change in hours worked.

Observation equations for Swedish data:

The observation equation for Swedish GDP is the following:

$$\Delta Y_t^{obs} = c_Y + 400(\hat{y}_t^m - \hat{y}_{t-1}^m + \hat{\mu}_{z+,t} + ln(\mu_{z+})) + \sigma_{\Delta Y}^{me}\varepsilon_{\Delta Y,t}^{me} \tag{L.12}$$

where ΔY_t^{obs} is the annualized quarterly growth rate of Swedish GDP per capita, $\hat{y}_t^m$ and $\hat{\mu}_{z+t}$ are log-linearized model variables which are percentage deviations from the steady state GDP and technological growth, respectively. $\sigma_{\Delta Y}^{me}\varepsilon_{\Delta Y_t}^{me}$ is the measurement error. The steady state annualized quarterly growth rate of GDP in the model is given by

$$\Delta Y = SS_Y = 400ln(\mu_{z+}) \tag{L.13}$$

where the model quarterly growth rate of Swedish GDP $ln(\mu_{z+})$ is calibrated to 0.47 percent, which matches the sample average Swedish GDP per capita growth rate. We calibrate the excess parameter c_Y to zero.

The observation equation for Swedish household consumption is given by

$$\Delta C_t^{obs} = c_C + 400(\hat{c}_t^{agg} - \hat{c}_{t-1}^{agg} + \hat{\mu}_{z+t} + ln(\mu_{z+})) + \sigma_{\Delta C}^{me}\varepsilon_{\Delta C,t}^{me}, \tag{L.14}$$

which is very similar to observation equation for GDP and reflects the model assumption that GDP and household consumption have the same trend growth, $ln(\mu_{z+})$. As in the case for GDP, we calibrate c_C to zero.

The observation equation for Swedish private investment is given by

$$\Delta I_t^{obs} = c_I + 400(\hat{I}_t - \hat{I}_{t-1} + \hat{\mu}_{z+t} + \hat{\mu}_{\gamma,t} + ln(\mu_{z+}) + ln(\mu_\gamma)) + \sigma_{\Delta I}^{me}\varepsilon_{\Delta I,t}^{me}, \tag{L.15}$$

which is also very similar to observation equation for GDP with minor differences due to differences in the trend growth rate of private investment and GDP. We assume that investment growth in Sweden is driven by two technologies, global labor-augmenting technology μ_z and investment-specific technology μ_γ. The trend growth rate of private investment is then given by

$$\Delta I = SS_I = 400(ln(\mu_{z+}) + ln(\mu_\gamma)), \tag{L.16}$$

where $ln(\mu_\gamma)$ is the investment-specific technology growth rate in Sweden and the steady state growth rate is calibrated to the difference between the sample average growth rate of GDP per capita and private investment per capita. Again, excess parameter c_I is set to zero.

The observation equation for Swedish exports is given by

$$\Delta X_t^{obs} = c_{X,t} + 400(\hat{x}_t - \hat{x}_{t-1} + \hat{\mu}_{z+t} + ln(\mu_{z+})) + \sigma_{\Delta X}^{me}\varepsilon_{\Delta X,t}^{me}, \tag{L.17}$$

which is very similar to observation equation for GDP and reflects the model assumption that GDP and exports have the same trend growth, $ln(\mu_{z+})$. However, the excess parameter $c_{X,t}$ is calibrated in a different way than earlier ones due to significant differences in the growth rate of exports per capita and GDP per capita in the sample period, which is not consistent with the model's balanced growth assumption. In the sample period, the annualized average growth rates of exports per capita and GDP per capita are 4.4 and 1.8 percent, respectively.

[87] Values for β which are closer to 1 help to get lnR_F to be closer to $ln(\mu_{z+F}) + ln(\Pi^C)$.

Moreover, the pre-financial crisis episode growth rate of exports per capita is significantly higher than the post-crisis episode.[88] To reconcile the theoretical model assumptions and data sample properties, we calibrate the excess parameter $c_{X,t}$ to 3 percent for the pre-crisis period between $1995Q1 : 2008Q2$, and to 1 percent for the post-crisis period $2008Q3 : 2019Q4$.[89]

The observation equation for Swedish imports is given by

$$\Delta M_t^{obs} = c_{M,t} + 400(\hat{m}_t - \hat{m}_{t-1} + \hat{\mu}_{z+t} + ln(\mu_{z+})) + \sigma_{\Delta M}^{me} \varepsilon_{\Delta M,t}^{me}, \tag{L.18}$$

which is very similar to observation equation for GDP and reflects the model assumption that GDP and imports have the same trend growth, $ln(\mu_{z+})$. However, the excess parameter $c_{M,t}$ is calibrated in the same way as the excess parameter of exports $c_{X,t}$.

The observation equation for government consumption is given by

$$\Delta G_t^{obs} = c_G + 400(\hat{g}_t - \hat{g}_{t-1} + \hat{\mu}_{z+t} + ln(\mu_{z+})) + \sigma_{\Delta G}^{me} \varepsilon_{\Delta G,t}^{me}, \tag{L.19}$$

which is very similar to observation equation for GDP and reflects the model assumption that GDP and government consumption have the same trend growth. Excess parameter c_G is set to zero.

The observation equation for government investment is given by

$$\Delta I_t^{G,obs} = c_{IG} + 400(\hat{I}_t^G - \hat{I}_{t-1}^G + \hat{\mu}_{z+t} + \hat{\mu}_{\gamma,t} + ln(\mu_{z+}) + ln(\mu_\gamma)) + \sigma_{\Delta I^G}^{me} \varepsilon_{\Delta I^G,t}^{me}, \tag{L.20}$$

which is very similar to observation equation for private investment, which reflects the assumption that private investment and government investment have the same trend growth. However, the excess parameter c_{IG} is calibrated to $400(-ln(\mu_\gamma)/2)$. A negative value is motivated by the data property that the average annualized growth rate of government investment per capita is less than private investment per capita.[90]

The observation equation for employment gap is given by

$$n_t^{obs} = 100\hat{n}_t + \sigma_n^{me} \varepsilon_{n,t}^{me} \tag{L.21}$$

where n_t^{obs} is the observable variable for employment gap. Recall that the NIER's employment gap estimates are used as observables in the estimation.

The observation equation for the unemployment rate is given by

$$un_t^{obs} = 100(un + \breve{un}_t) + \sigma_{un}^{me} \varepsilon_{un,t}^{me} \tag{L.22}$$

where un_t^{obs} is the observable variable for the unemployment rate, un is the equilibrium rate of unemployment, which is calibrated to 0.069.

The observation equation for wages is given by

$$w_t^{obs} = 400(ln(\mu_{z+}) + ln(\Pi^C)) + 100(\hat{\Pi}_t^W + \hat{\Pi}_{t-1}^W + \hat{\Pi}_{t-2}^W + \hat{\Pi}_{t-3}^W) + \sigma_w^{me} \varepsilon_{w,t}^{me} \tag{L.23}$$

where w_t^{obs} denotes observable variable for wages, $400(ln(\mu_{z+}) + ln(\Pi^C))$ is the steady state wage growth, which is calibrated to the sum of CPIF inflation and technology growth in Sweden, or in other words, nominal GDP trend growth in Sweden. Since the data is annual wage growth at quarterly frequency, four quarters of model's wage inflation variables are aggregated to map the quarterly model variable to the data.

The observation equation for CPIF inflation is given by

$$\Pi_t^{C,obs} = 400(ln(\Pi^C) + \hat{\Pi}_t^C) + \sigma_{\Pi^C}^{me} \varepsilon_{\Pi^C,t}^{me} \tag{L.24}$$

where $\Pi_t^{C,obs}$ is the observable variable for CPIF inflation, $400(ln(\Pi^C))$ is the annualized steady state CPIF inflation.

[88] In the pre-financial crisis episode, which we consider as 1995Q1-2008Q2, the sample average of annualized exports per capita growth rate is 6.2%, whereas it is 3.4% for the post-financial crisis episode, which we consider as 2010Q1-2019Q4. The crisis episode of 2008Q3-2009Q4, which we exclude while calculating the sample averages, is also considered as post-financial crisis episode while calibrating the excess parameter.

[89] This practical solution to violation of balanced growth assumption for trade variables are borrowed from Corbo and Strid (2020).

[90] The calibrated value of excess parameter for government investment is quite arbitrary but there are other practical reasons behind the chosen value. It is chosen according to the following criterion $-400ln(\mu_\gamma) < c_{IG} < 0$. In the main text, we explain why we choose a negative value for excess parameter, $c_{IG} < 0$. The reason for why we choose a value smaller than the trend growth rate of investment-specific technology in absolute terms for excess parameter, $-ln(\mu_\gamma) < c_{IG}$ is that in the post-crisis period average government investment growth is higher than GDP growth, thus choosing a negative value larger than $-400ln(\mu_\gamma)$ would require model to explain high government investment with "gap" rather than with steady state growth. In this case, model would assign high "government investment gap" for this period and lead to unreasonable gap sizes. Moreover, institutional projections for the government investment growth is higher than GDP growth in the medium term until 2032, which also give us support on our chosen value of excess parameter that model could explain the different growth rates in GDP per capita and government investment per capita partially with different equilibrium growth rates.

The observation equation for CPIF inflation excluding energy is given by

$$\Pi_t^{C,xe,obs} = 400(ln(\Pi^{C,xe}) + \hat{\Pi}_t^{C,xe}) + \sigma_{\Pi^{C,xe}}^{me}\varepsilon_{\Pi^{C,xe},t}^{me} \tag{L.25}$$

where $\Pi_t^{C,xe,obs}$ is the observable variable for CPIF inflation excluding energy, $400(ln(\Pi^{C,xe}))$ is the annualized steady state CPIF inflation excluding energy.

The observation equation for import inflation for non-energy consumption goods is given by

$$\Pi_t^{M,C,xe,obs} = c_{\Pi^{M,C,xe}} + 400(ln(\Pi^{M,C,xe}) + \hat{\Pi}_t^{M,C,xe}) + \sigma_{\Pi^{M,C,xe}}^{me}\varepsilon_{\Pi^{M,C,xe},t}^{me} \tag{L.26}$$

where $\Pi_t^{M,C,xe,obs}$ is the observable variable for import inflation for non-energy consumption goods, $c_{\Pi^{M,C,xe}}$ is excess parameter calibrated to -1.5, $400(ln(\Pi^{M,C,xe}))$ is the annualized steady state import inflation for non-energy consumption goods.

The observation equation for the Riksbank's policy rate is given by

$$i_t^{obs} = c_i + 400(R-1) + 400(\check{i}) + \sigma_i^{me}\varepsilon_{i,t}^{me} \tag{L.27}$$

where i_t^{obs} is the observable variable for the Riksbank policy rate, c_i is the excess parameter calibrated to -2.0 to make the NIER's institutional view on long-run value of the monetary policy rate compatible with the model's theoretical equilibrium (steady state) monetary policy rate R.

The observation equation for the real exchange rate is given by

$$\Delta Q_t^{obs} = c_Q + 100(\hat{Q}_t - \hat{Q}_{t-1}) + \sigma_Q^{me}\varepsilon_{Q,t}^{me} \tag{L.28}$$

where Q_t^{obs} is the observable variable for the real exchange rate, c_Q is the excess parameter calibrated to 0.2 to make the data sample average compatible with the model's steady state value for the change in the real exchange rate, which is zero.

The observation equation for the capital utilization rate is given by

$$u_t^{obs} = 100(u + \hat{u}_t) + \sigma_u^{me}\varepsilon_{u,t}^{me} \tag{L.29}$$

where u_t^{obs} is the observable variable for the capital utilization rate, u is the steady state value for the utilization rate, which is calibrated to the sample average, 0.84.

The observation equation for government structural surplus is given by

$$Stsurp_t^{obs} = c_{surp} + Stsurp_{\bar{y},t}^{Target} + St\check{surp}_t + \sigma_{Stsurp}^{me}\varepsilon_{Stsurp,t}^{me} \tag{L.30}$$

where $Stsurp_t^{obs}$ is the quarterly structural surplus over potential GDP, $Stsurp_{\bar{y},t}^{Target}$ is the Swedish government structural surplus target, which is calibrated 0.33 percent of GDP to match the government's target. We set the excess parameter to the difference between the target level and the data sample average.[91]

The observation equation for government transfers is given by

$$\frac{tr_t^{agg,obs}}{\bar{y}} = tr^{agg}oy + \frac{\check{tr}_t^{agg}}{y} + \sigma_{tragg}^{me}\varepsilon_{tragg,t}^{me} \tag{L.31}$$

where $\frac{tr_t^{agg,obs}}{\bar{y}}$ is the aggregate transfers over potential GDP ratio, $tr^{agg}oy$ is the steady state value of government transfers over potential GDP, which is calibrated to the sample mean, 0.098.[92]

The observation equation for the consumption tax rate is given by

$$\Delta\tau_t^{C,obs} = (\check{\tau}_t^C - \check{\tau}_{t-1}^C) + \sigma_{\tau^C}^{me}\varepsilon_{\tau^C,t}^{me} \tag{L.32}$$

where $\tau_t^{C,obs}$ is the observable variable for the consumption tax rate.

The observation equation for the income tax rate is given by

$$\Delta\tau_t^{W,obs} = c_{\tau^W} + (\check{\tau}_t^W - \check{\tau}_{t-1}^W) + \sigma_{\tau^W}^{me}\varepsilon_{\tau^W,t}^{me} \tag{L.33}$$

where $\tau_t^{W,obs}$ is the observable variable for the labor income tax rate, c_{τ^W} is excess parameter to capture the downward trend growth rate in the sample data.

[91] see the Swedish Fiscal Policy Framework 2017/18:207, where the surplus target is defined as "an average of 0.33 percent of GDP over an economic cycle". Incorporating an excess parameter close to the sample average implies a higher level of model equilibrium of structural surplus than the fiscal framework's target. It reflects the government's precautionary stance during sample period, as the structural surplus is significantly higher than the fiscal framework's target for most of the sample horizon.

[92] See the model definition of aggregate transfers in Section 2. Recall that not all items of public transfers are included in the model definition.

The observation equation for the social security contribution rate is given by

$$\Delta\tau_t^{SSC,obs} = c_{\tau^{SSC}} + \breve{\tau}_t^{SSC} + \sigma_{\tau^{SSC}}^{me}\varepsilon_{\tau^{SSC},t}^{me} \tag{L.34}$$

where $\tau_t^{SSC,obs}$ is the observable variable for the social security contribution rate, $c_{\tau^{SSC}}$ is the excess parameter to capture the structural change in the tax rate after 2006 in Sweden.

The observation equation for the transfers tax rate is given by

$$\Delta\tau_t^{TR,obs} = c_{\tau^{TR}} + (\breve{\tau}_t^{TR} - \breve{\tau}_{t-1}^{TR}) + \sigma_{\tau^{TR}}^{me}\varepsilon_{\tau^{TR},t}^{me} \tag{L.35}$$

where $\tau_t^{TR,obs}$ is the observable variable for the transfers tax rate, $c_{\tau^{TR}}$ is excess parameter to capture the downward trend in the sample data.

Table 33 shows

Table 33: Excess parameters

Symbol	Description	Steady state	Data	Excess parameter Symbol	Value
Π_F^W	Foreign wage inflation	3.3	2.4	c_{I_F}	-0.9
i_F	Foreign monetary policy rate	3.5	2.2	c_{I_F}	-1.3
Spr_F	Foreign corporate spread	0	1.8	c_{Spr_F}	-1.8
ΔX	Growth rate of Swedish exports	1.8	4.4	x_{I_F}	3:1
ΔM	Growth rate of Swedish imports	1.8	3.6	m_{I_F}	3:1
$\Pi^{M,C,xe,obs}$	Inflation rate of imported non-energy goods	2	-0.3	$c_{\Pi^{M,C,xe}}$	-1.5
i	Riksbank's policy rate	4.3	2.3	c_i	-2
ΔQ	Change in real exchange rate	0	0.2	c_Q	-0.2
$Stsurp$	Government structural surplus	0.003	0.005	c_{surp}	0.002

Notes: For exports and imports, excess parameter is time varying. For pre-financial crisis, it is set to 3%, but post-financial crisis, it is set to 1%, given the differences in exports and imports growth rate between these time intervals.

M Appendix: Model properties

M.1 Model-implied theoretical moments

The theoretical moments shown in this section are based on the solution of Lyapunov equation of the model, which is, in turn, obtained by the state space representation of the model. Theoretical moments and mpirical (or sample) moments we show in the text would coincide asymptotically (with infinite draws from the posterior distribution).[93]

Table 34: Model implied standard deviations for Foreign variables (Theoretical)

Variable	Data	Post. dist. percentile		
		5	50	95
GDP	2.03	1.83	2.06	2.37
Investment	4.94	4.87	5.59	6.51
CPI excl. energy	0.50	0.54	0.71	0.95
CPI	1.10	1.09	1.25	1.44
Hours worked	1.51	1.74	1.93	2.16
Monetary policy rate	1.94	1.09	2.20	4.05
Corporate spread	0.44	0.23	0.42	0.65
Wage	0.95	1.28	1.52	1.80
Consumption	1.48	1.40	1.60	1.87

Table 35: Model implied contemporaneous correlations between Foreign variables (Theoretical)

	$\Delta Y_{F,t}$	$\Delta C_{F,t}$	$\Delta I_{F,t}$	$\Pi^{C,xe}_{F,t}$	$\Pi^{C}_{F,t}$	$\Delta N_{F,t}$	$R_{F,t}$	$\zeta_{F,t}$	$\Delta w_{F,t}$
$\Delta Y_{F,t}$	1.00								
$\Delta C_{F,t}$	0.71	1.00							
$\Delta I_{F,t}$	0.75	0.38	1.00						
$\Pi^{C,xe}_{F,t}$	-0.13	-0.11	-0.12	1.00					
$\Pi^{C}_{F,t}$	-0.14	-0.19	-0.09	0.56	1.00				
$\Delta N_{F,t}$	0.73	0.40	0.64	-0.08	-0.13	1.00			
$R_{F,t}$	0.03	0.05	-0.00	0.24	0.13	0.03	1.00		
$\zeta_{F,t}$	-0.07	-0.08	-0.07	-0.29	-0.17	-0.07	-0.33	1.00	
$\Delta w_{F,t}$	0.38	0.30	0.36	-0.01	-0.01	0.22	0.03	-0.04	1.00

Table 36: Model implied contemporaneous correlations between Foreign variables (Theoretical)

Variable 1	Variable 2	Data	Posterior dist. percentile		
			10	50	90
GDP	Investment	0.78	0.71	0.75	0.79
	Consumption	0.80	0.64	0.71	0.77
	Hours worked	0.80	0.68	0.73	0.77
	Monetary policy rate	-0.05	-0.00	0.03	0.07
	CPI	0.25	-0.20	-0.14	-0.09
CPI	CPI excl. energy	0.60	0.48	0.56	0.66
	Monetary policy rate	0.36	0.05	0.11	0.24
	Wage	0.41	-0.10	-0.01	0.08
Hours worked	Wage	0.39	0.17	0.22	0.27

[93] See Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016) for the details of how the Lyapunov equation is obtained.

Table 37: Model implied contemporaneous correlations between Swedish variables (Theoretical)

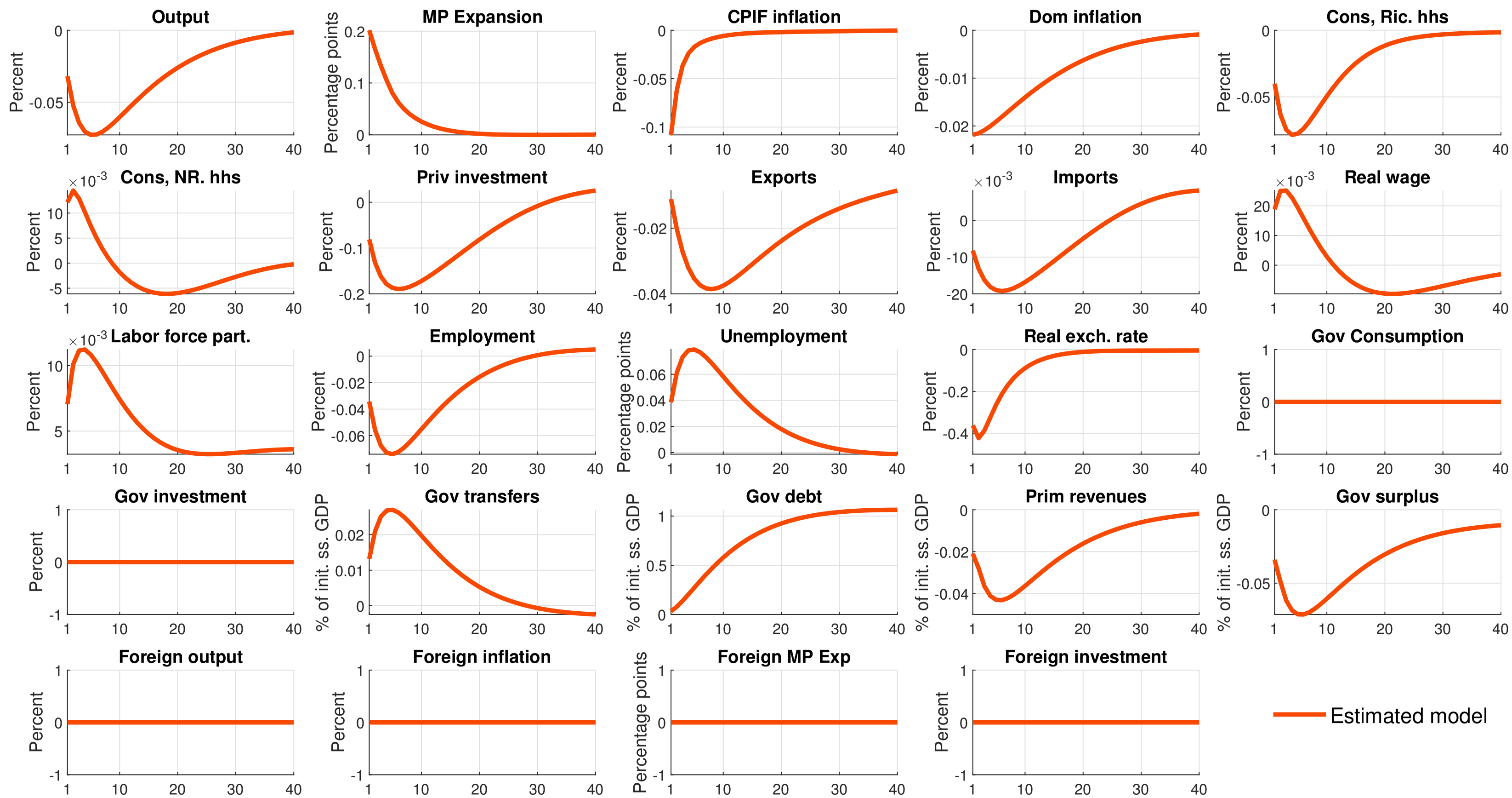
	ΔY_t	ΔC_t	ΔI_t	ΔX_t	ΔIM_t	Π_t^C	$\Pi_t^{C,xe}$	n_t	R_t	ζ_t	ΔW_t	ΔQ_t
ΔY_t	1.00											
ΔC_t	0.30	1.00										
ΔI_t	0.41	0.08	1.00									
ΔX_t	0.56	0.03	-0.00	1.00								
ΔIM_t	0.10	0.24	0.50	0.44	1.00							
Π_t^C	-0.20	-0.15	-0.16	-0.06	-0.10	1.00						
$\Pi_t^{C,xe}$	-0.17	-0.13	-0.16	-0.04	-0.11	0.81	1.00					
n_t	0.17	0.11	0.06	0.08	-0.02	-0.10	-0.10	1.00				
R_t	0.01	-0.01	-0.04	0.02	-0.05	0.04	0.07	0.23	1.00			
ζ_t	-0.01	-0.01	-0.01	-0.00	-0.01	-0.04	-0.05	-0.12	-0.22	1.00		
ΔW_t	0.09	0.12	0.12	0.03	0.11	0.17	0.22	0.26	0.12	-0.43	1.00	
ΔQ_t	-0.17	-0.03	-0.12	-0.09	-0.08	0.21	0.06	-0.07	-0.07	0.07	-0.04	1.00

Table 38: Model implied contemporaneous correlations between Swedish variables (Theoretical)

Variable 1	Variable 2	Data	Posterior dist. percentile		
			10	50	90
GDP	Consumption	0.48	0.26	0.30	0.35
	Investment	0.42	0.37	0.41	0.46
	Exports	0.62	0.53	0.56	0.60
	Imports	0.53	0.00	0.10	0.19
	CPIF	0.10	-0.23	-0.20	-0.17
	R. Exch. rate	-0.29	-0.23	-0.17	-0.12
CPIF	Corporate Spread	-0.15	-0.06	-0.03	-0.01
	R. Exch. rate	-0.13	0.15	0.21	0.28
	Monetary policy rate	0.12	-0.00	0.03	0.08
Exports	Imports	0.74	0.40	0.44	0.47
	R. Exch. rate	-0.14	-0.15	-0.09	-0.03

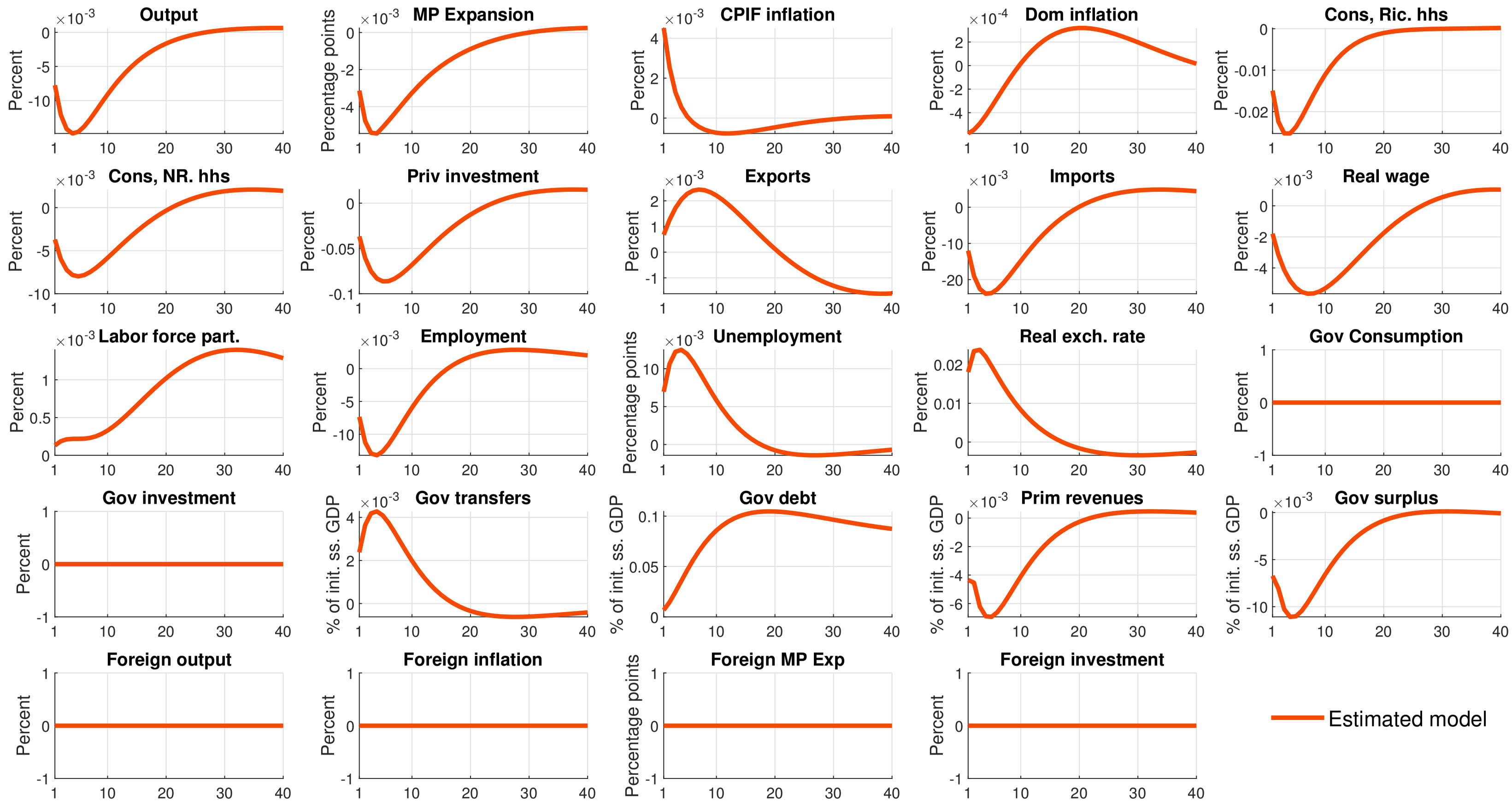
M.2 Impulse response functions

In this section the impulse response functions (IRF) of the main variables to some selected shocks of the model are reported. Since the model has a balanced growth path, the impulse responses are measured as deviations from the balanced growth path steady state.



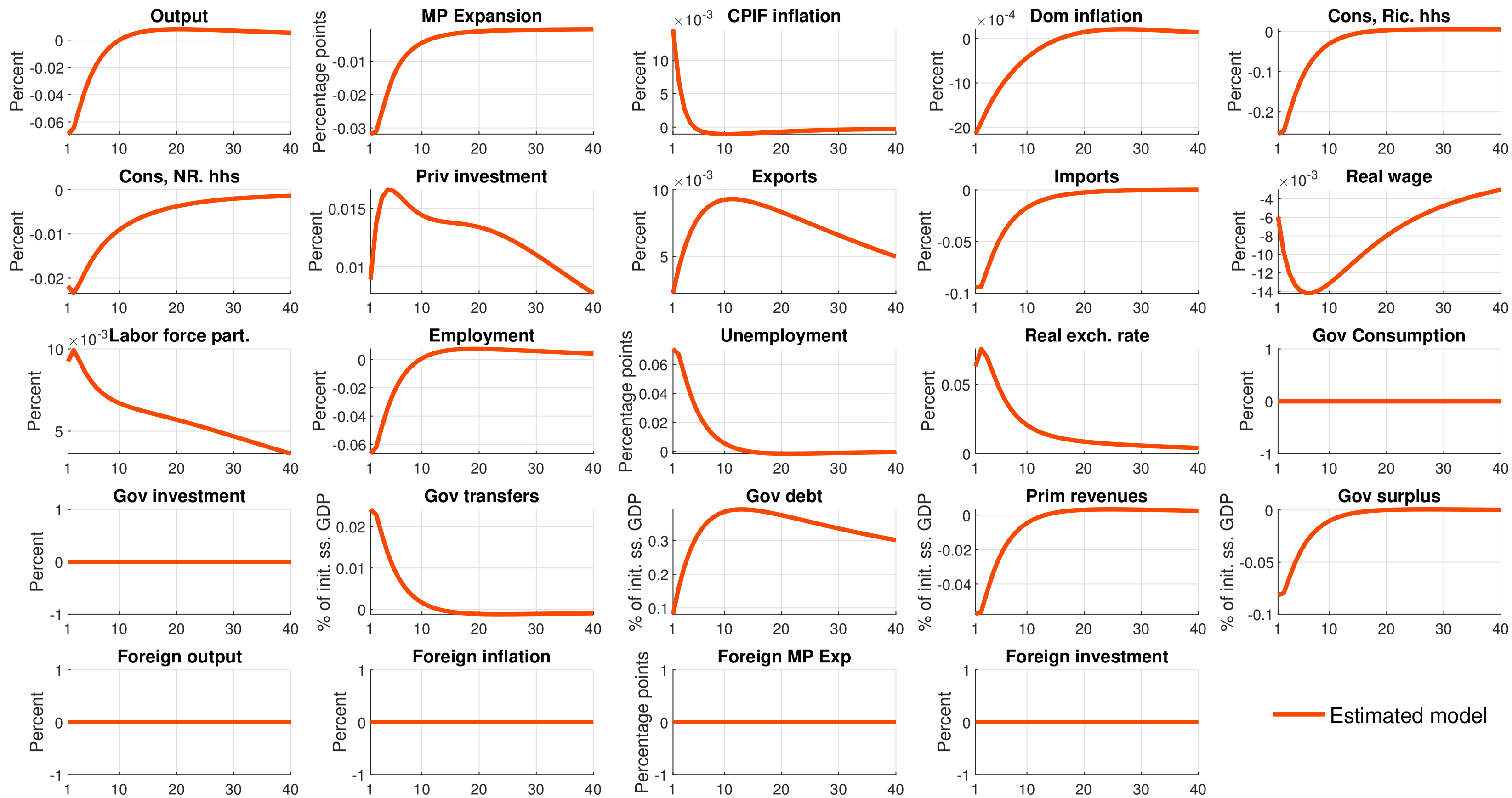
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 33: Monetary Policy Shock



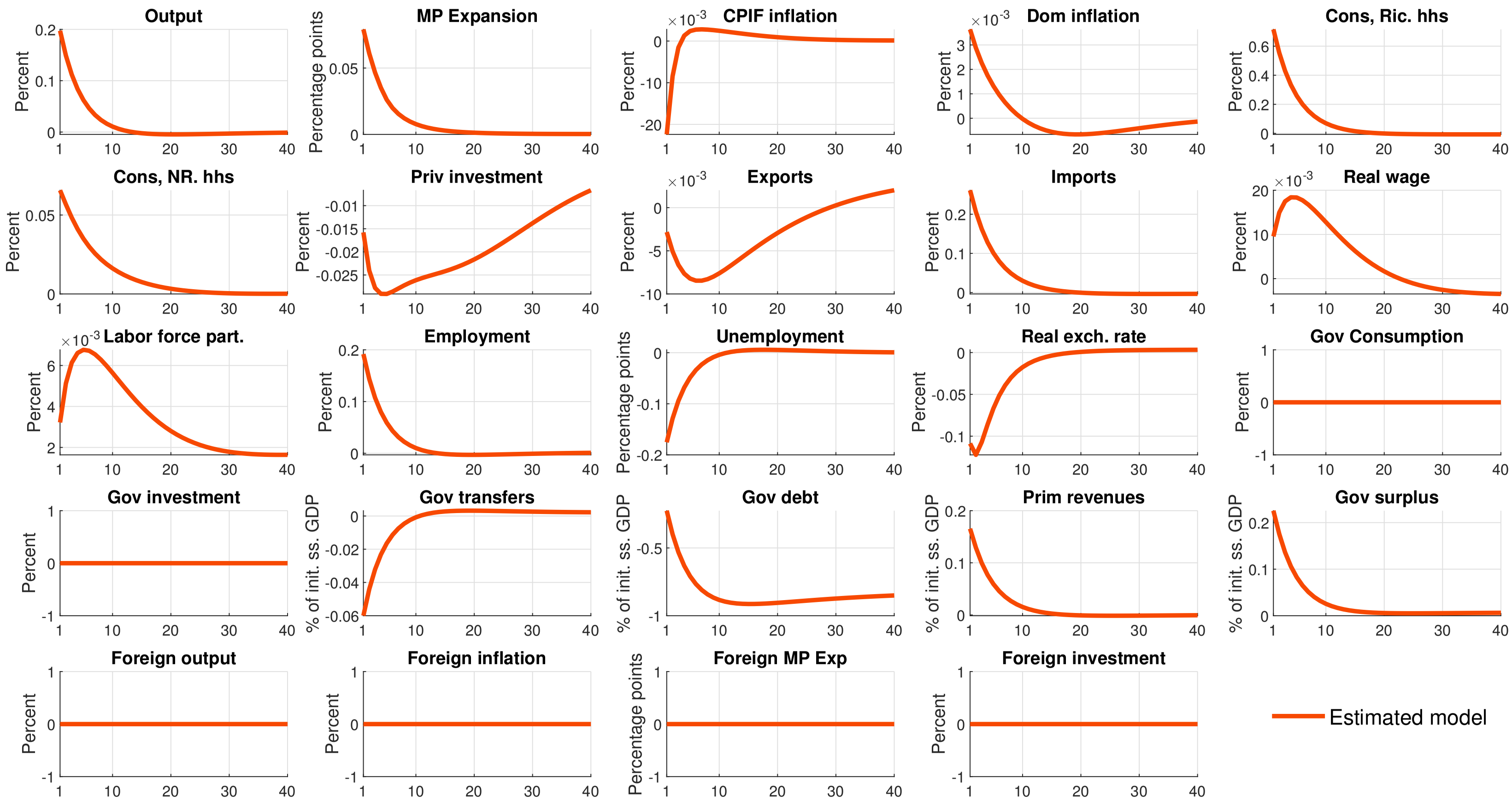
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 34: Risk Premium Shock



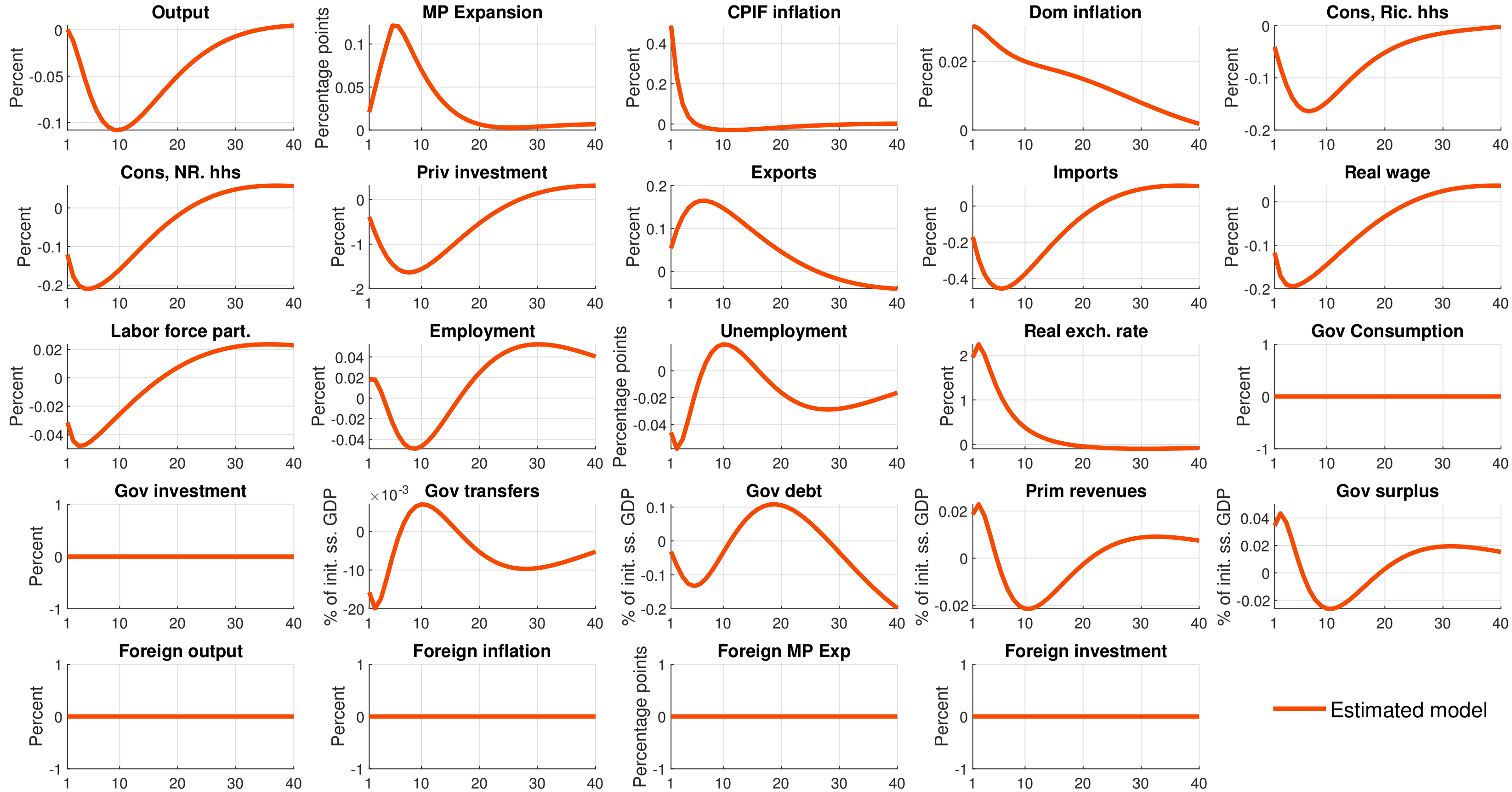
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 35: Discount Factor Shock



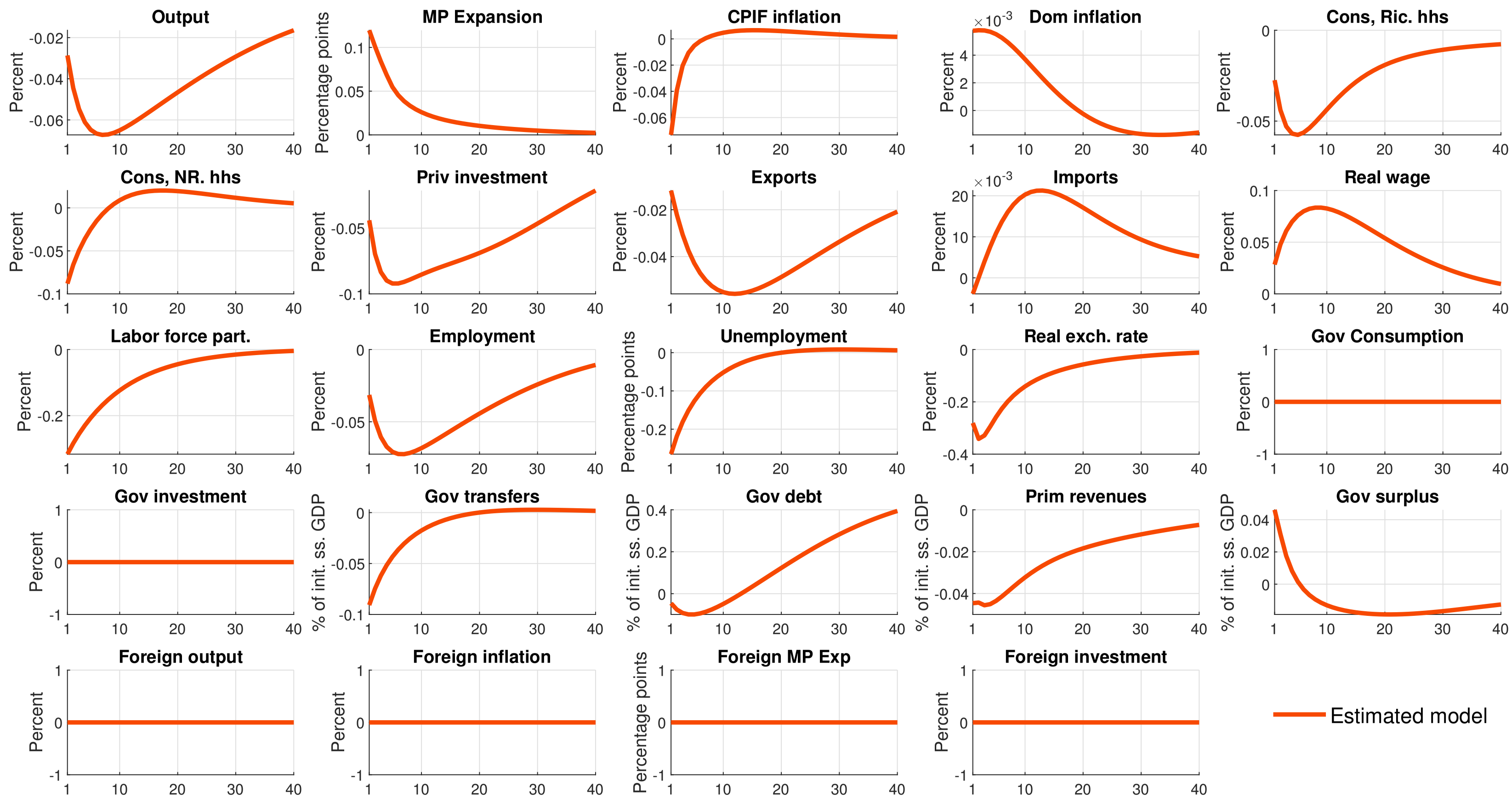
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 36: Consumption Preference Shock



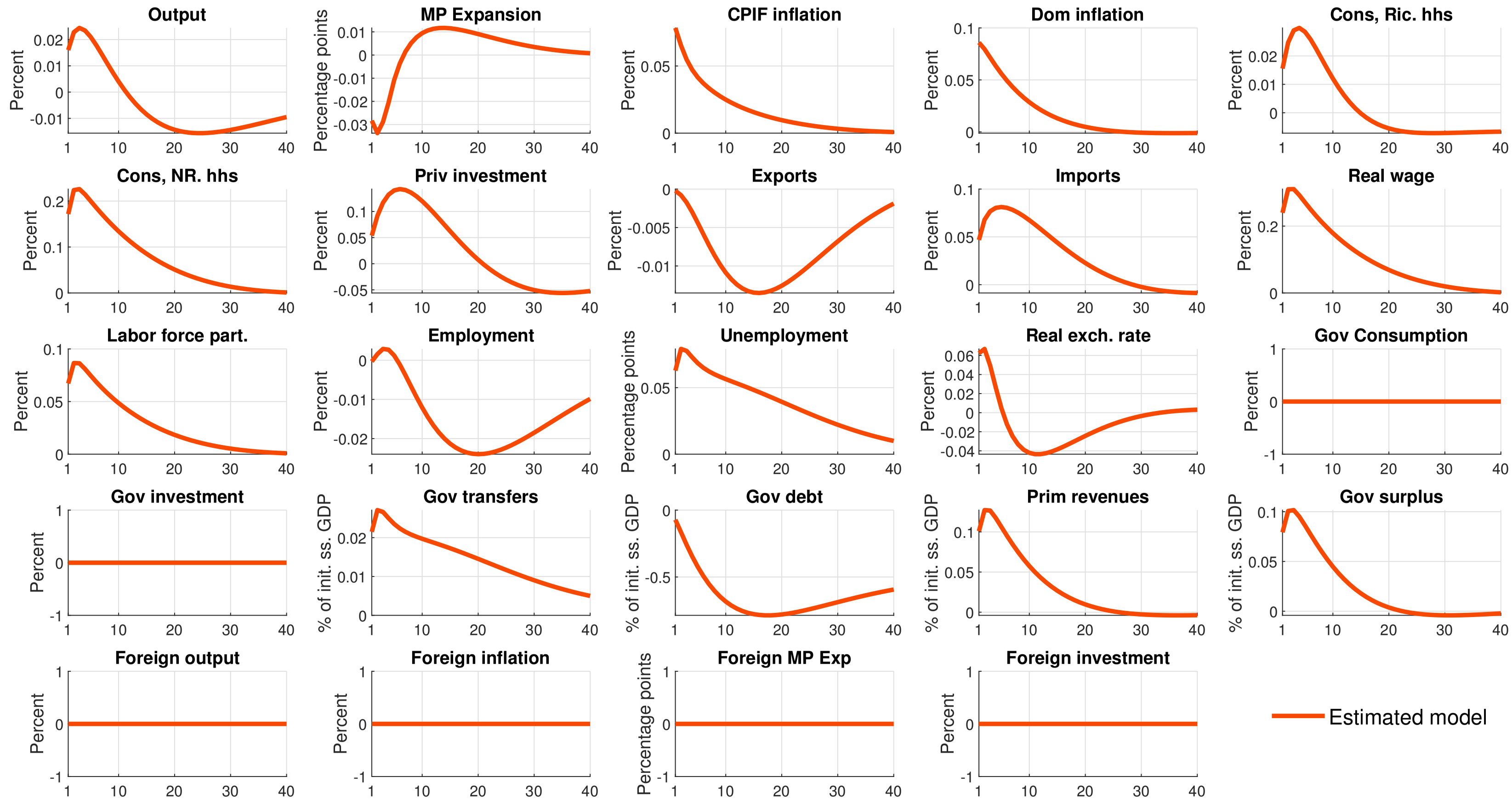
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 37: UIP Risk Premium Shock



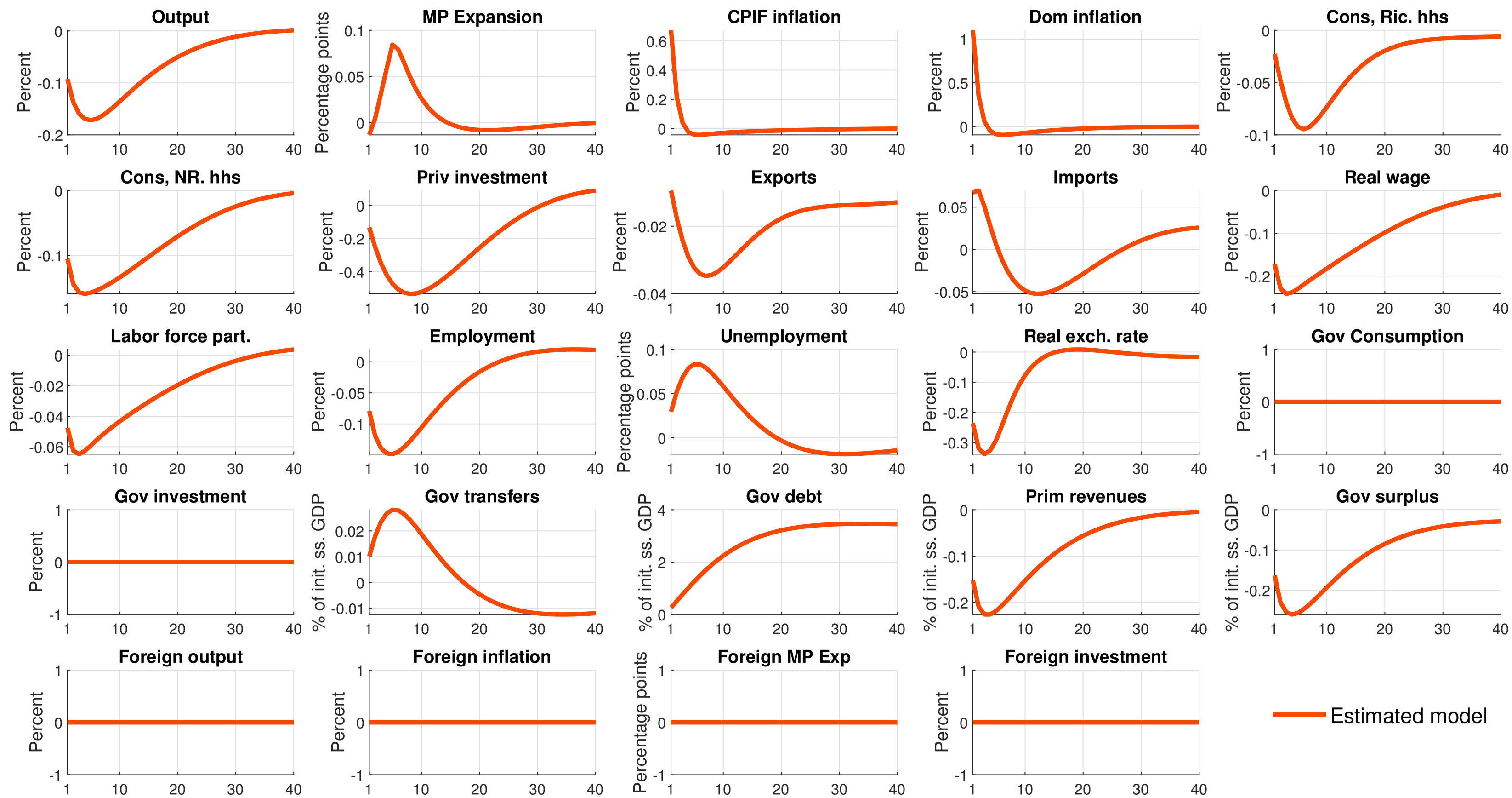
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 38: Labor Preference Shock



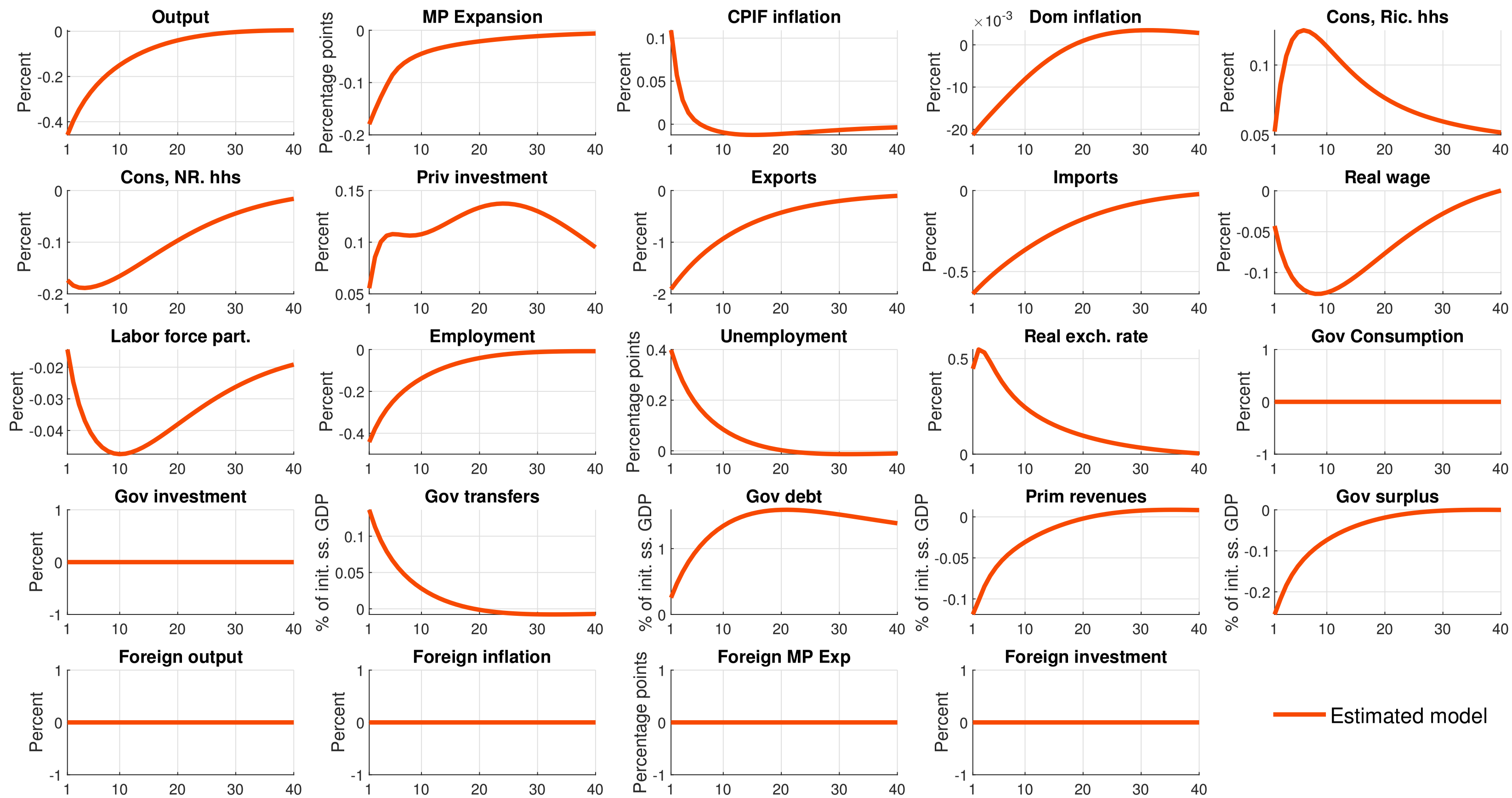
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 39: Wage Markup Shock



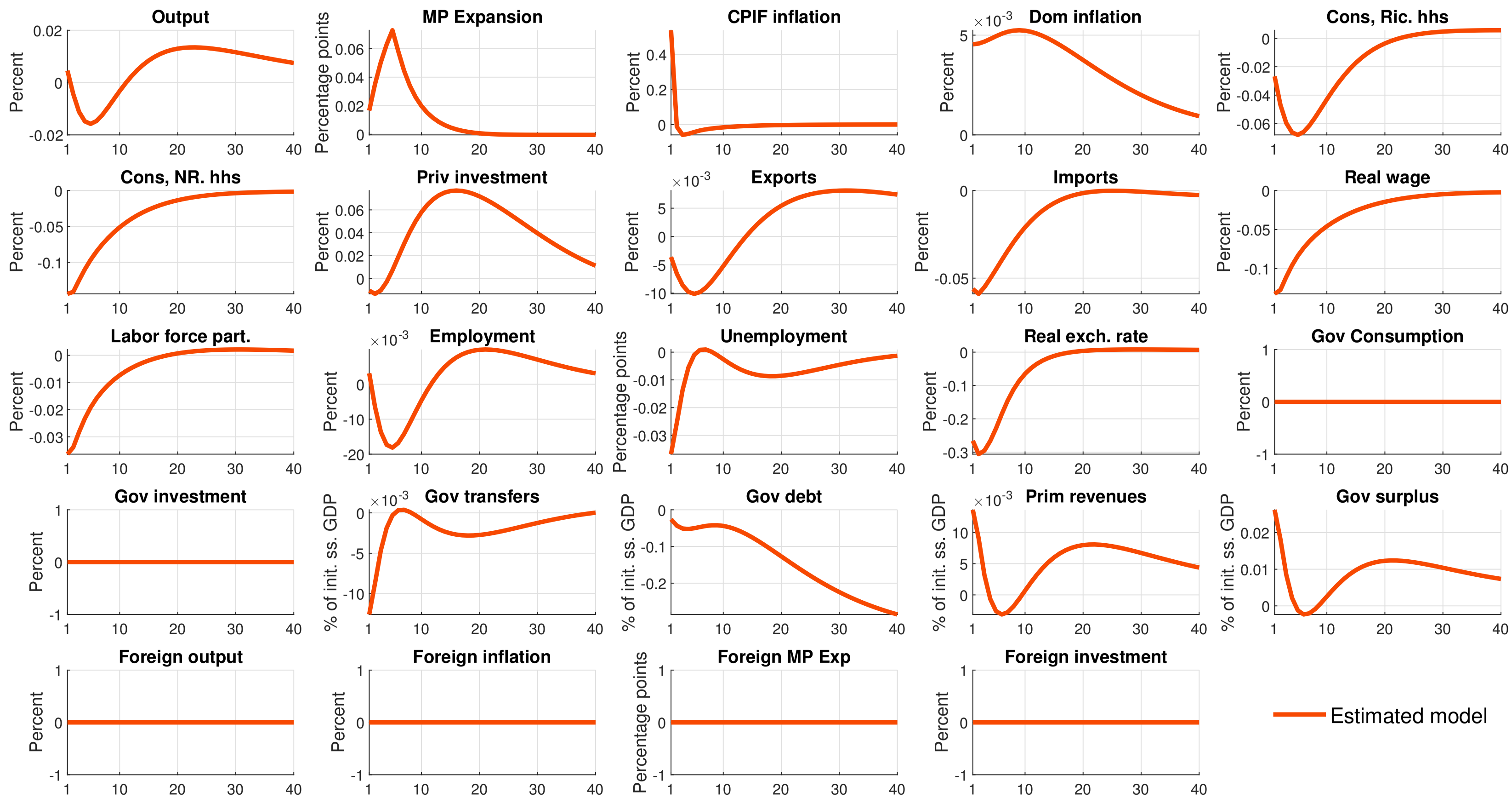
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 40: Intermediate Goods Price Markup Shock



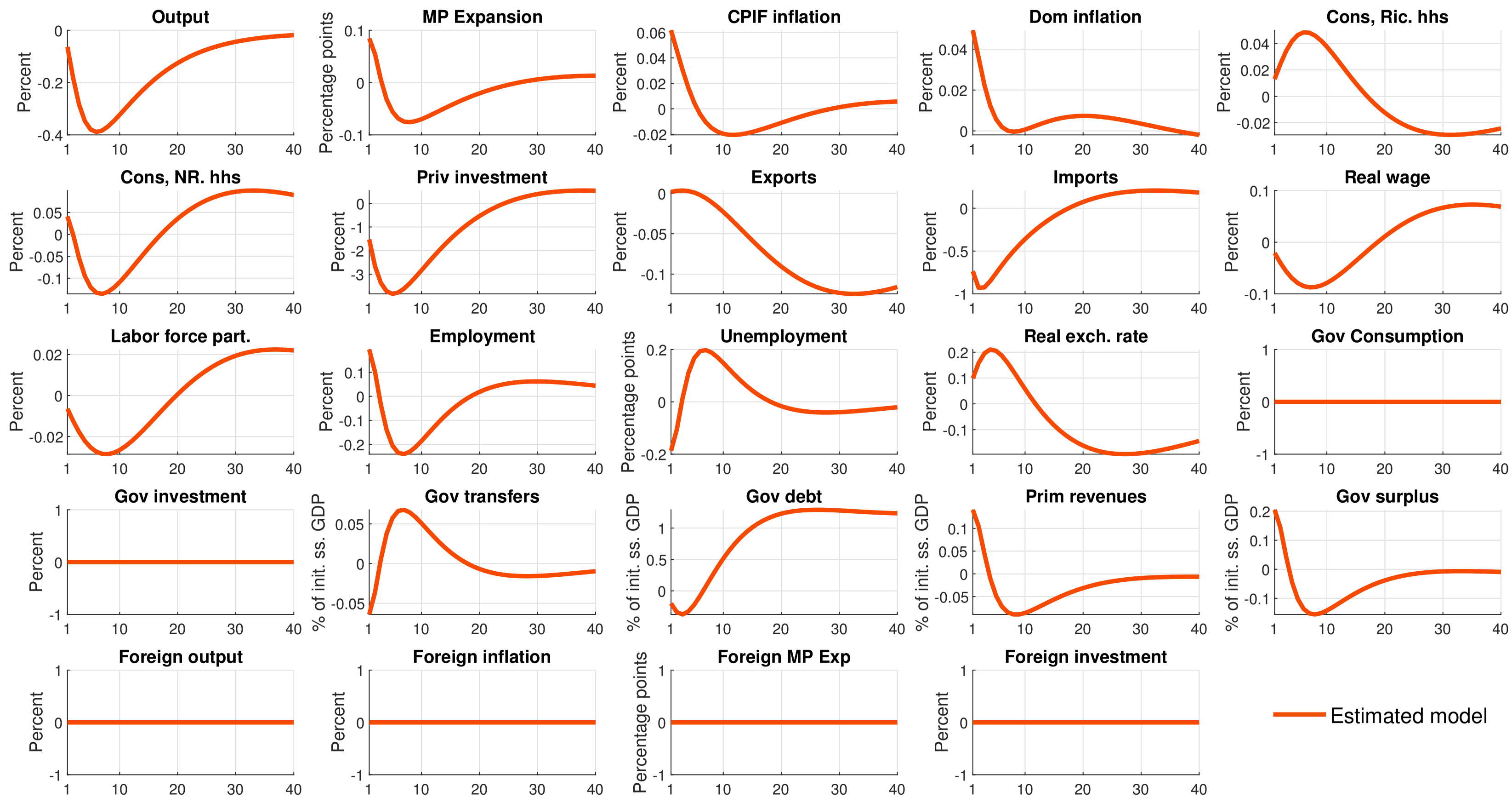
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 41: Export Goods Price Markup Shock



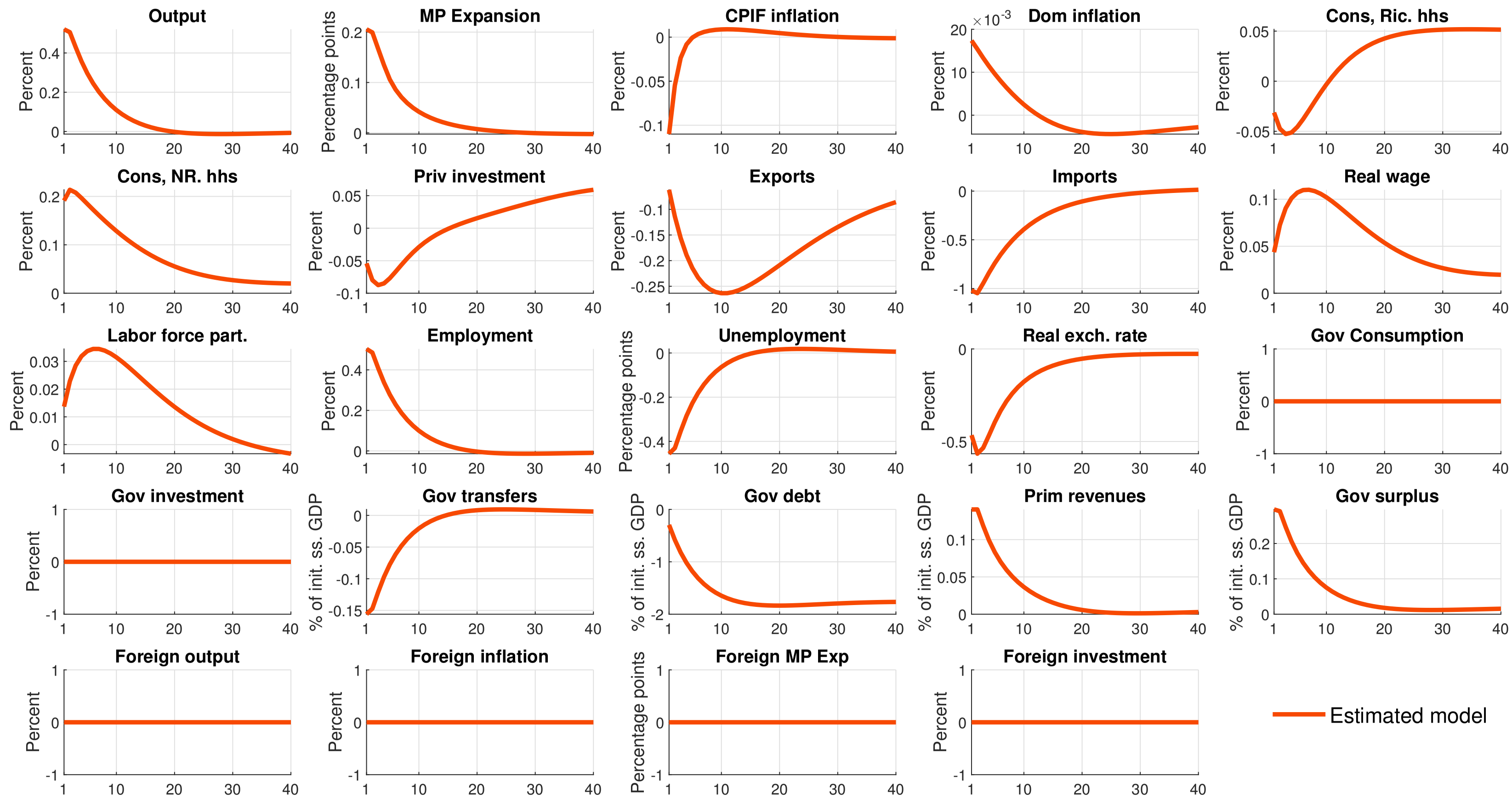
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 42: Import to Non-Energy Consumption Goods Price Markup Shock



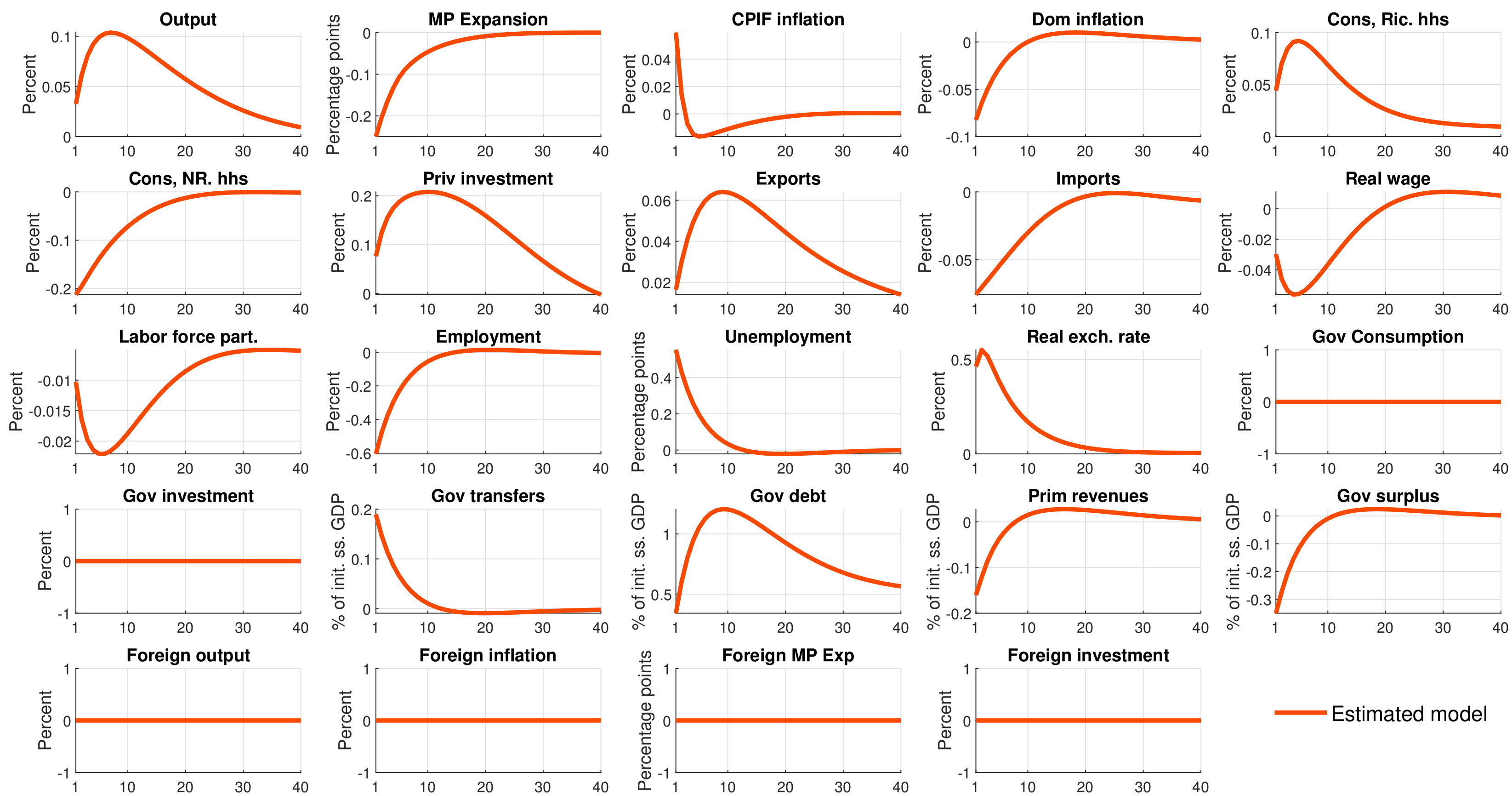
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 43: Import to Investment Goods Price Markup Shock



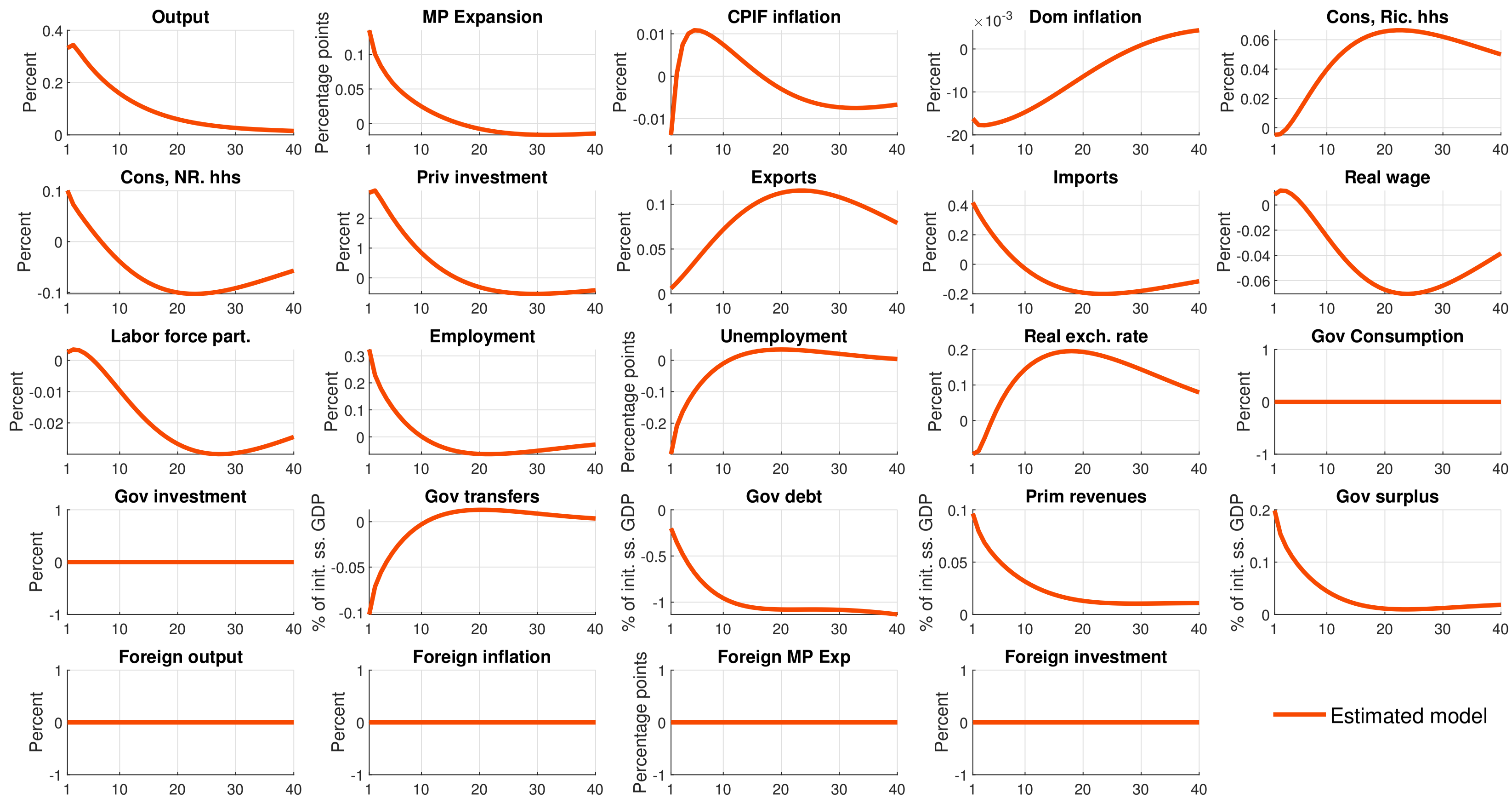
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 44: Import to Export Goods Price Markup Shock



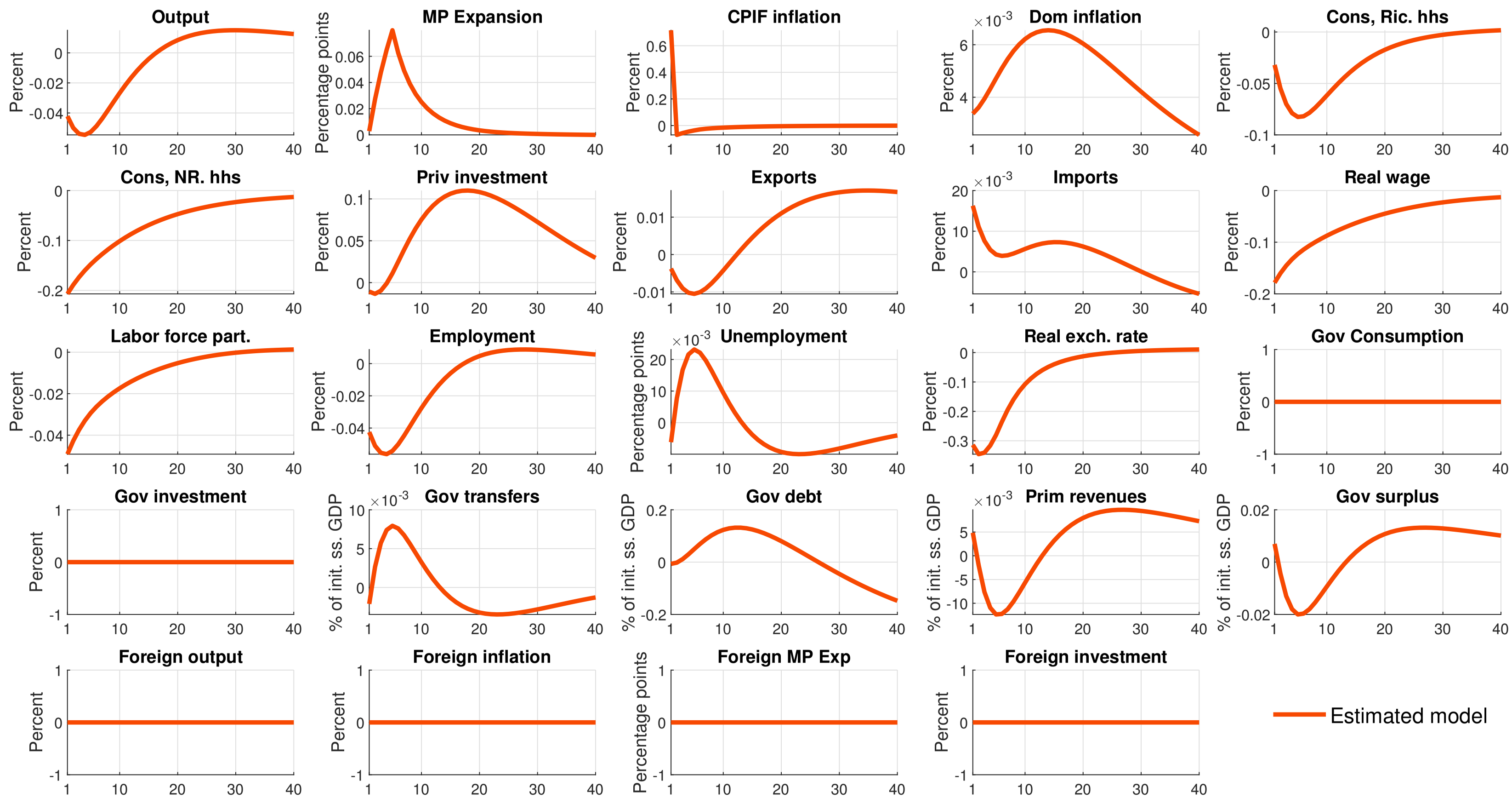
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 45: Stationary Technology Shock



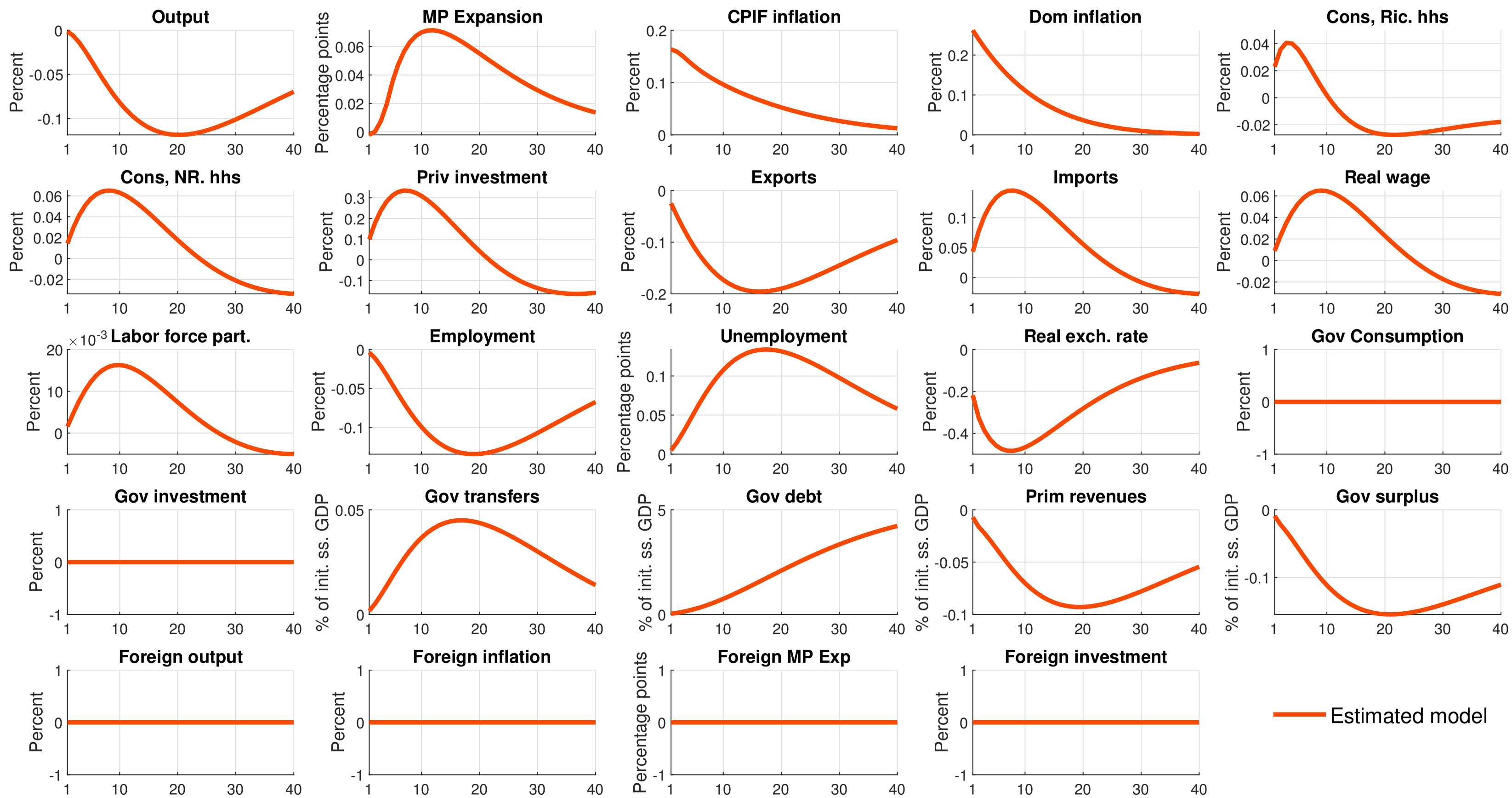
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 46: Temporary investment shock



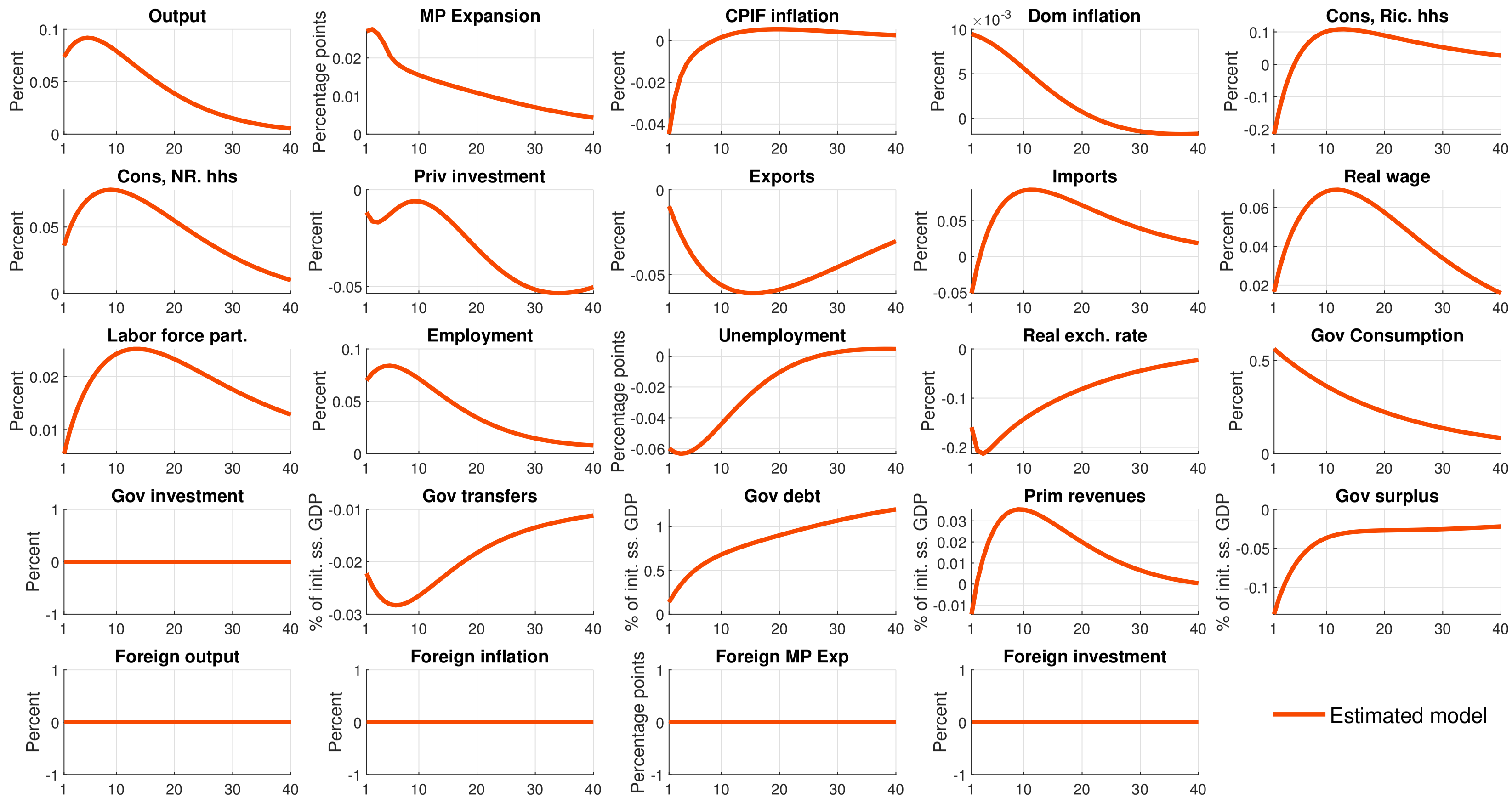
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 47: Domestic Energy Price Shock



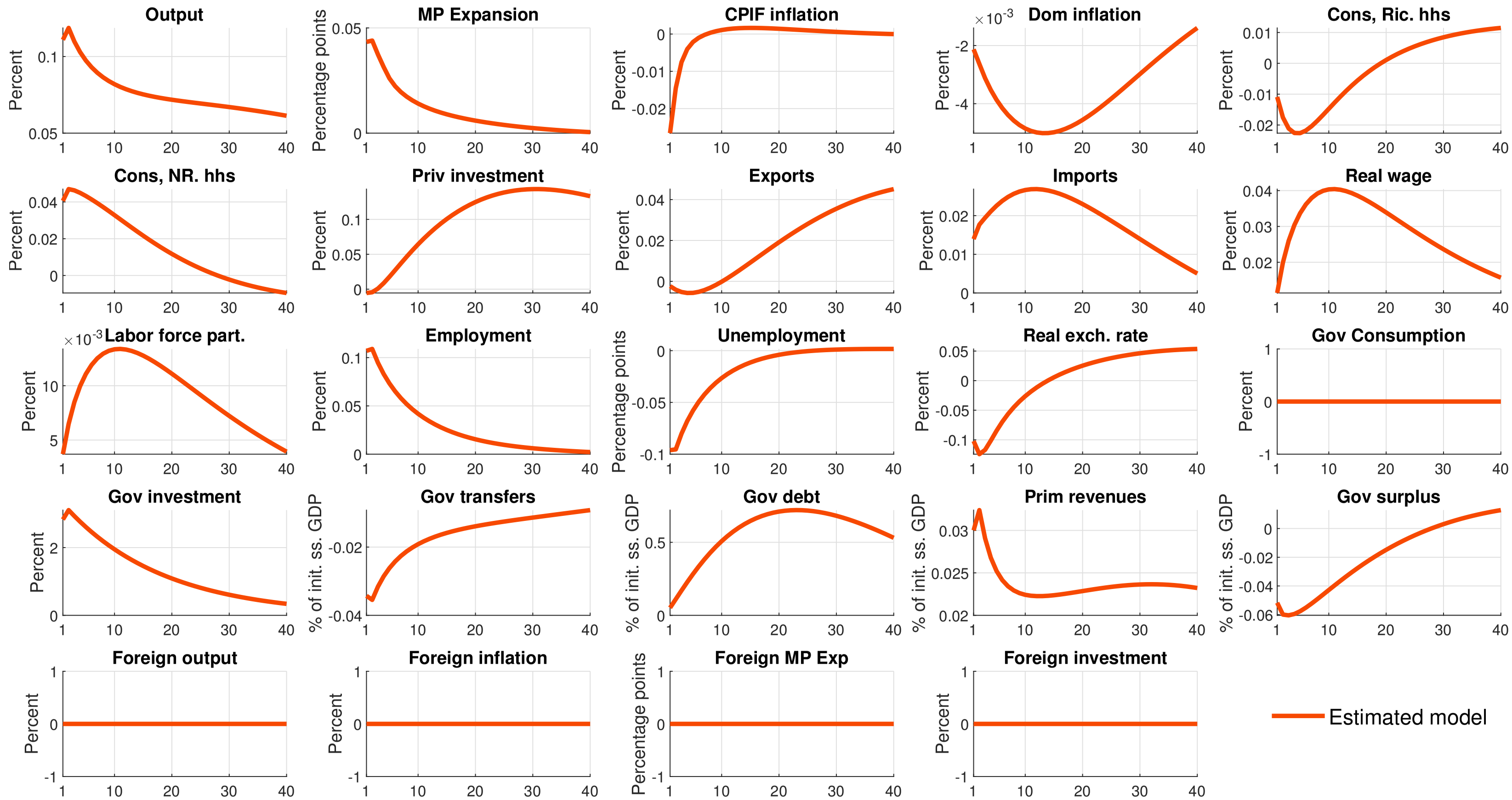
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 48: Inflation Trend Shock



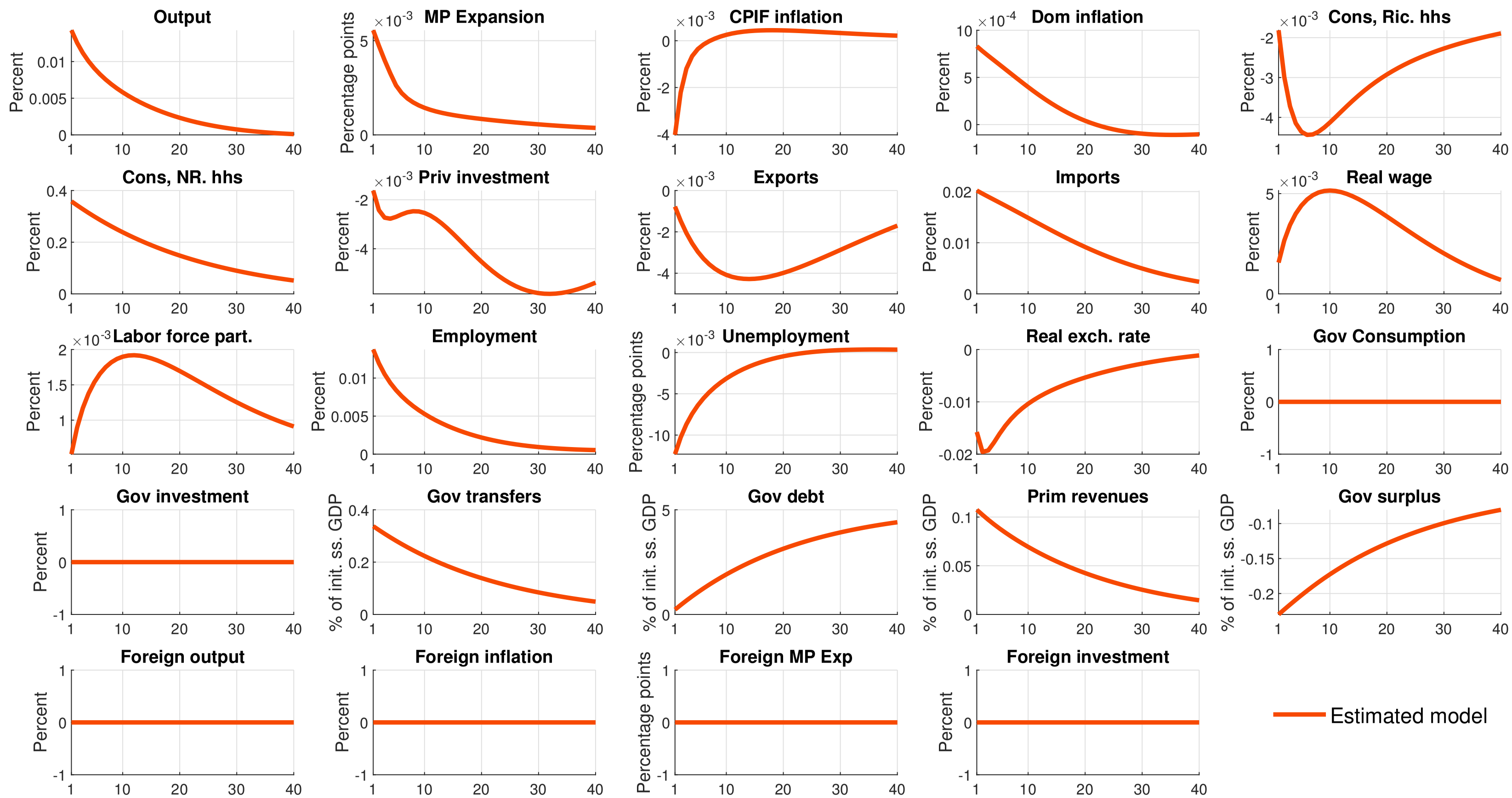
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 49: Government Consumption Shock



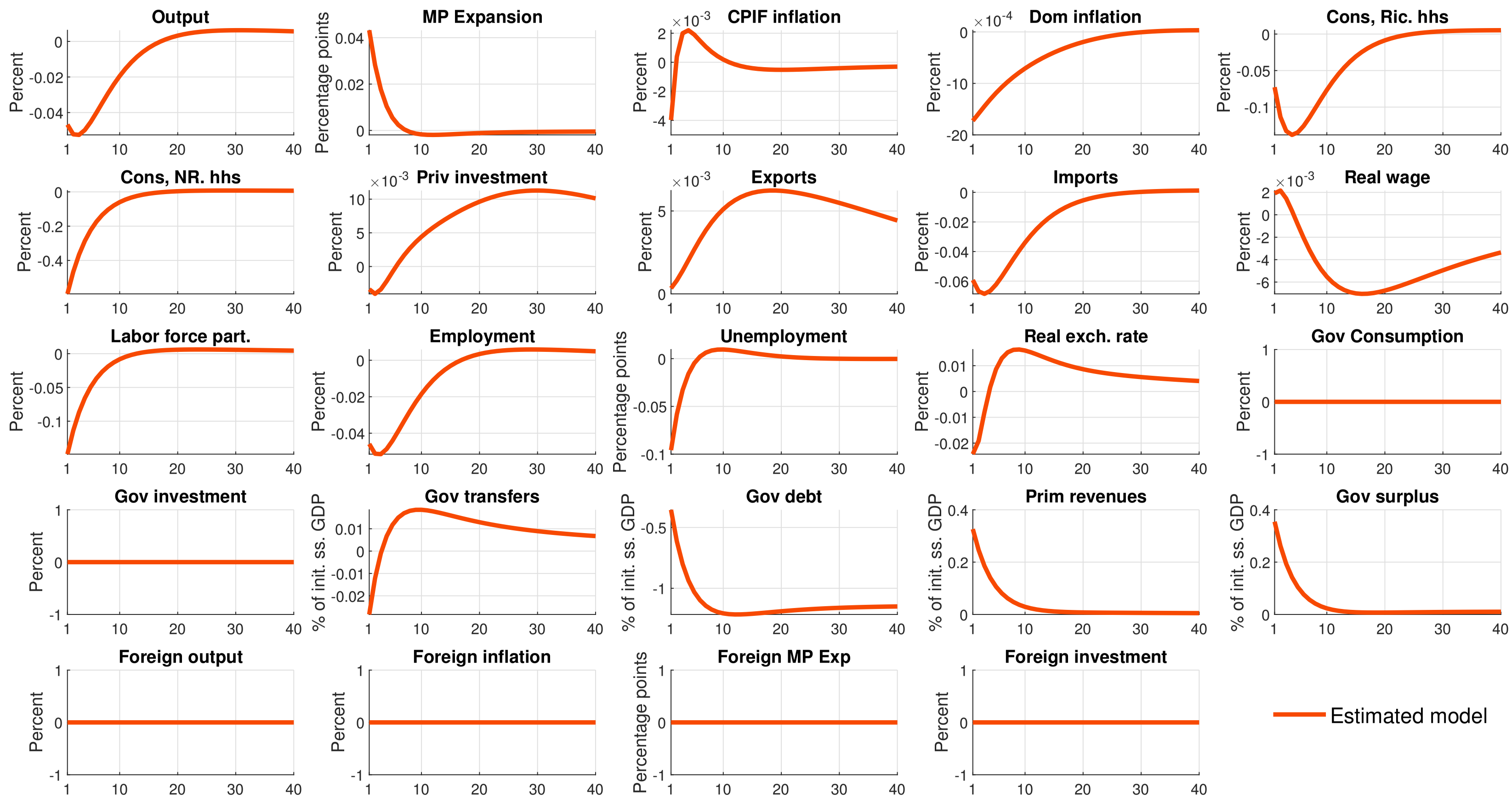
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 50: Government Investment Shock



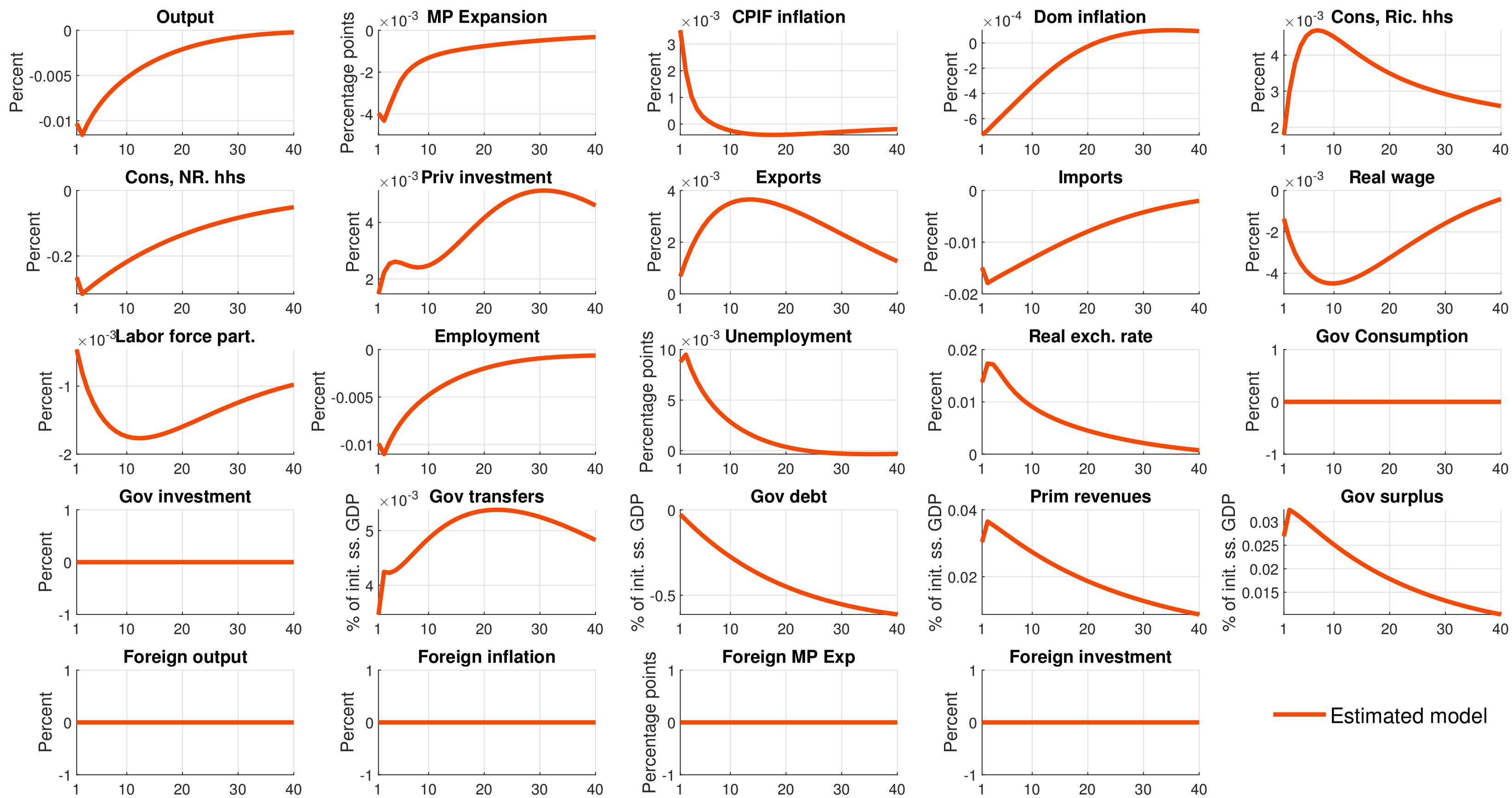
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 51: Aggregate Transfer Shock



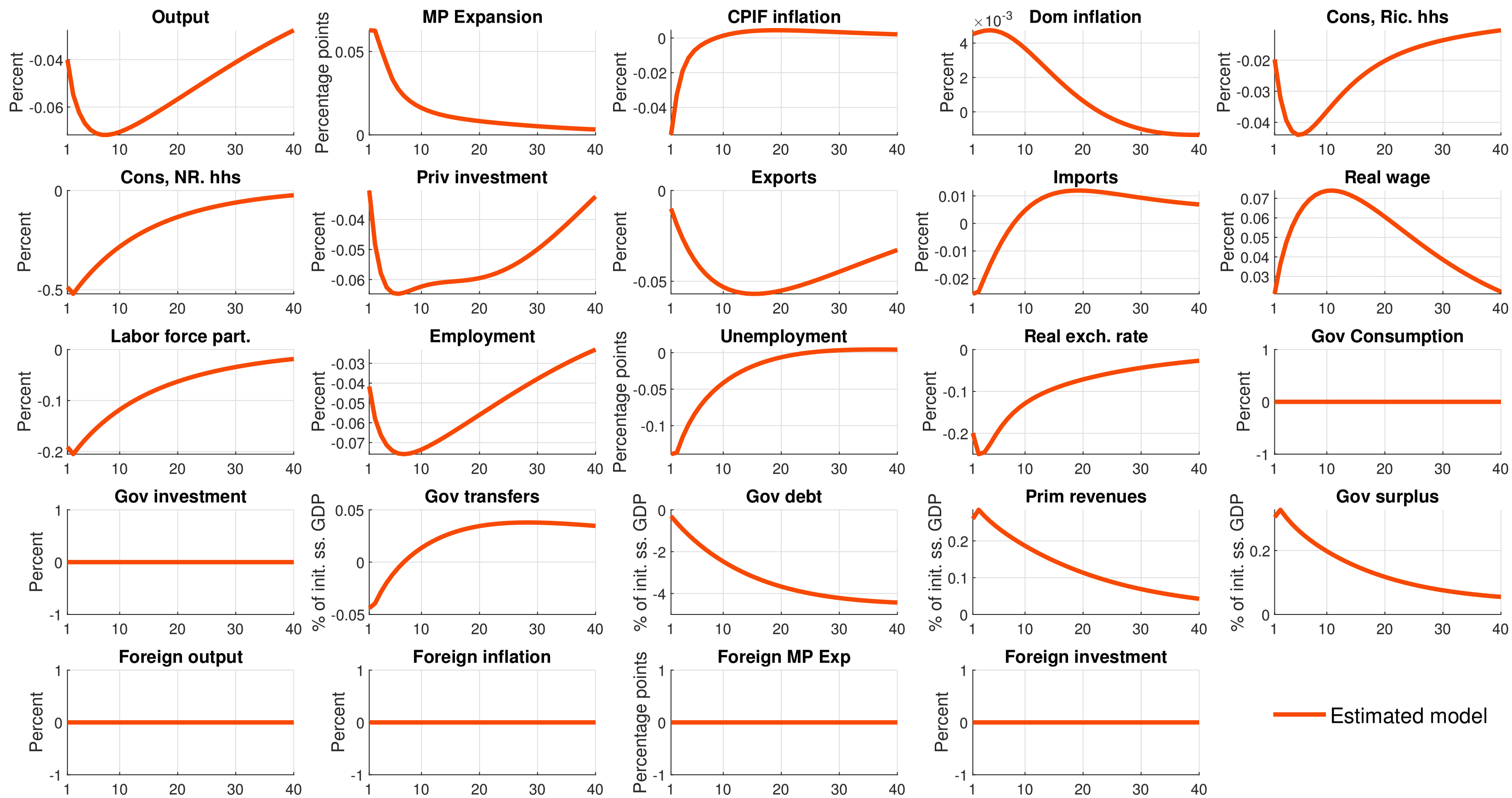
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 52: Consumption Tax Shock



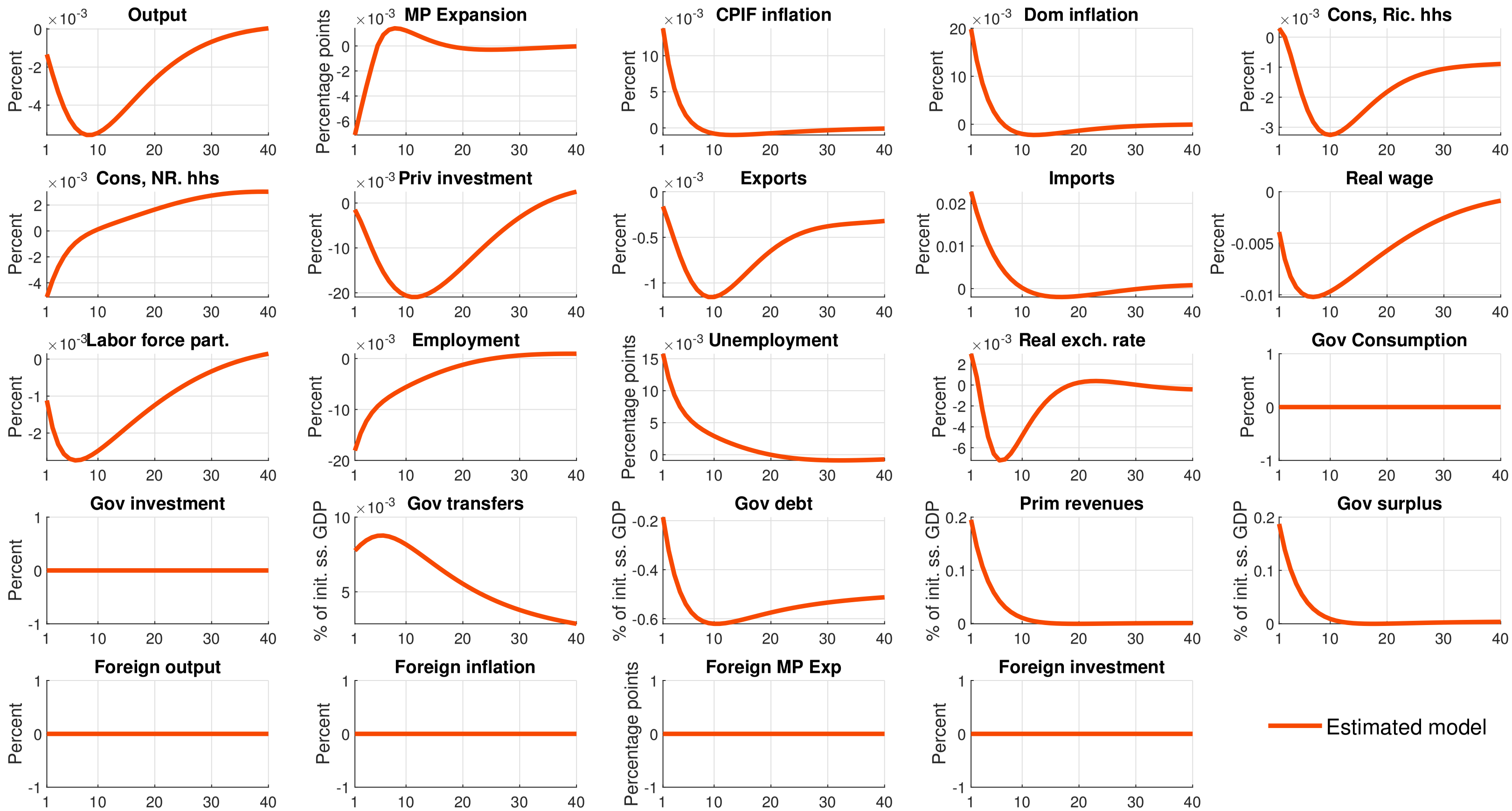
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 53: Transfer Tax Shock



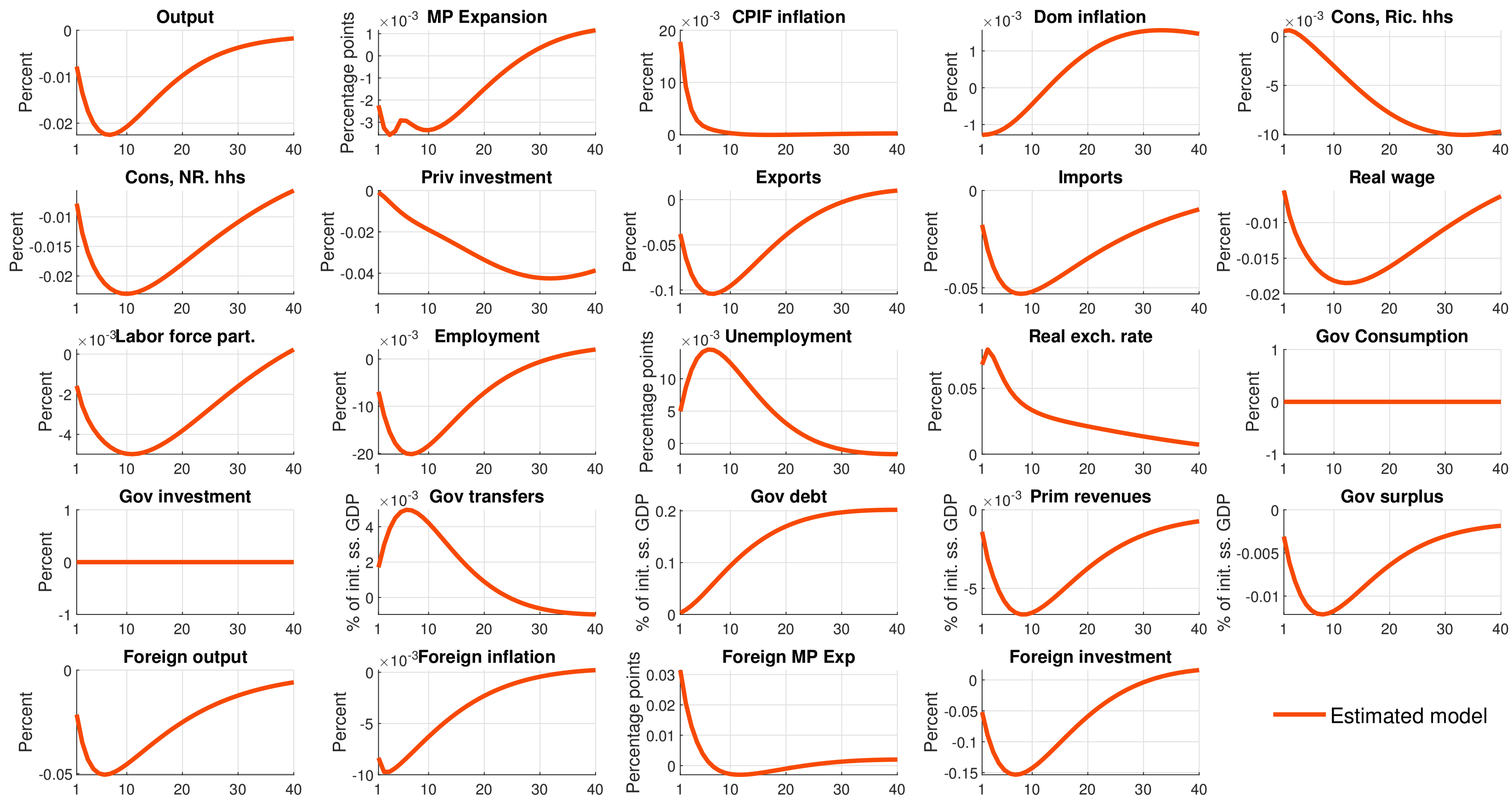
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 54: Labor Income Tax Shock



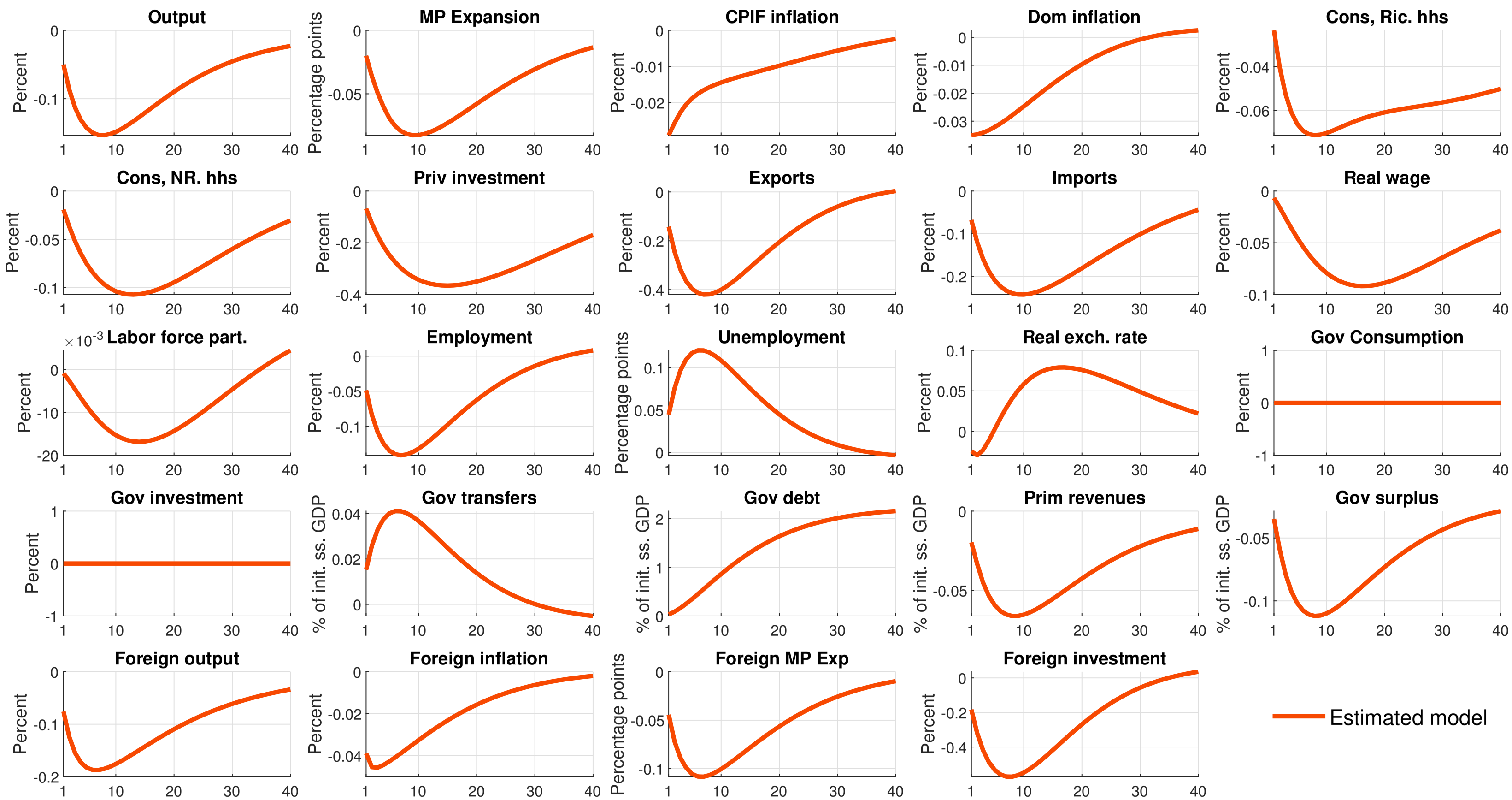
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 55: Social Security Contribution Shock



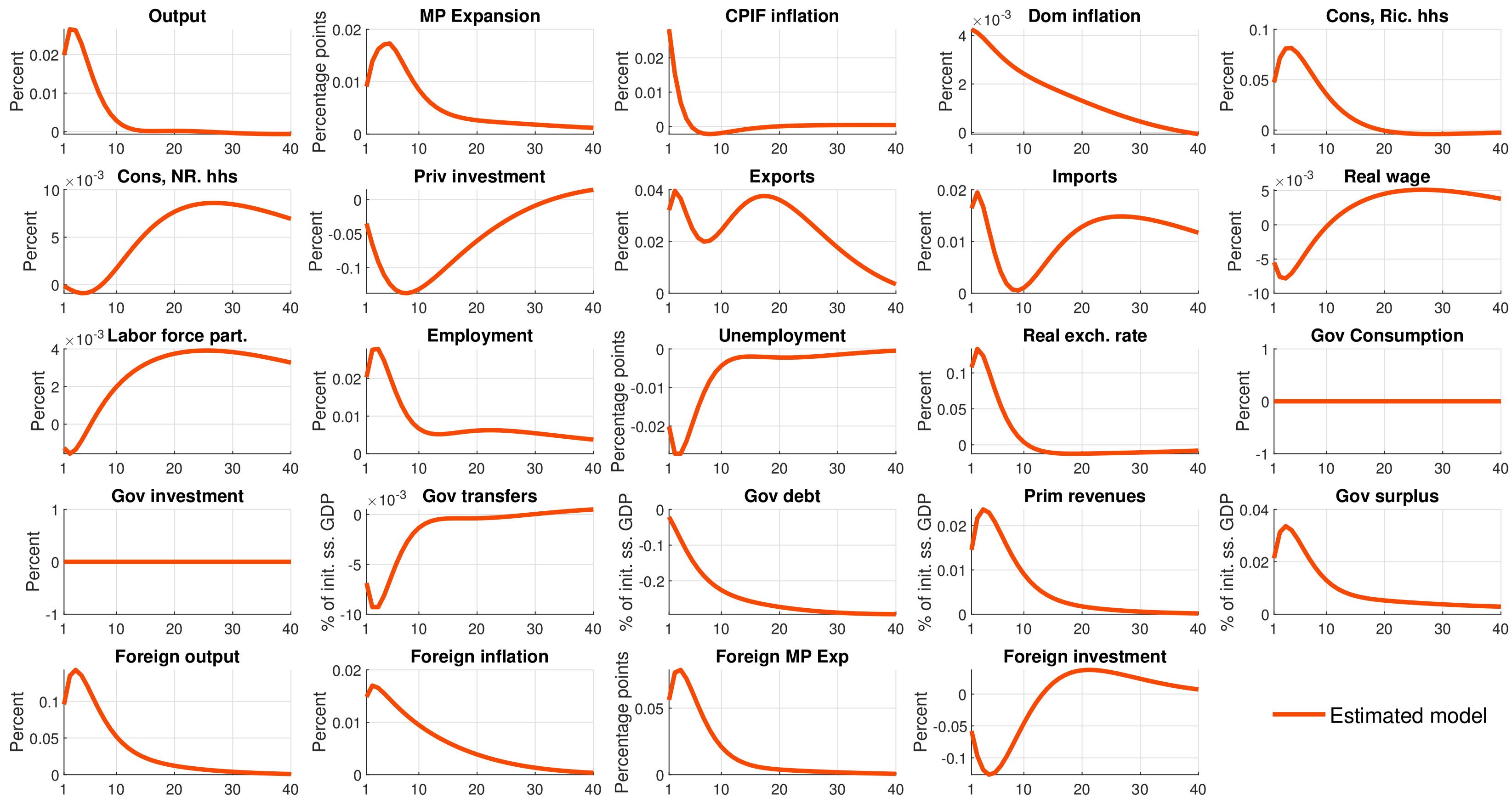
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 56: Foreign Nominal Interest Rate Shock



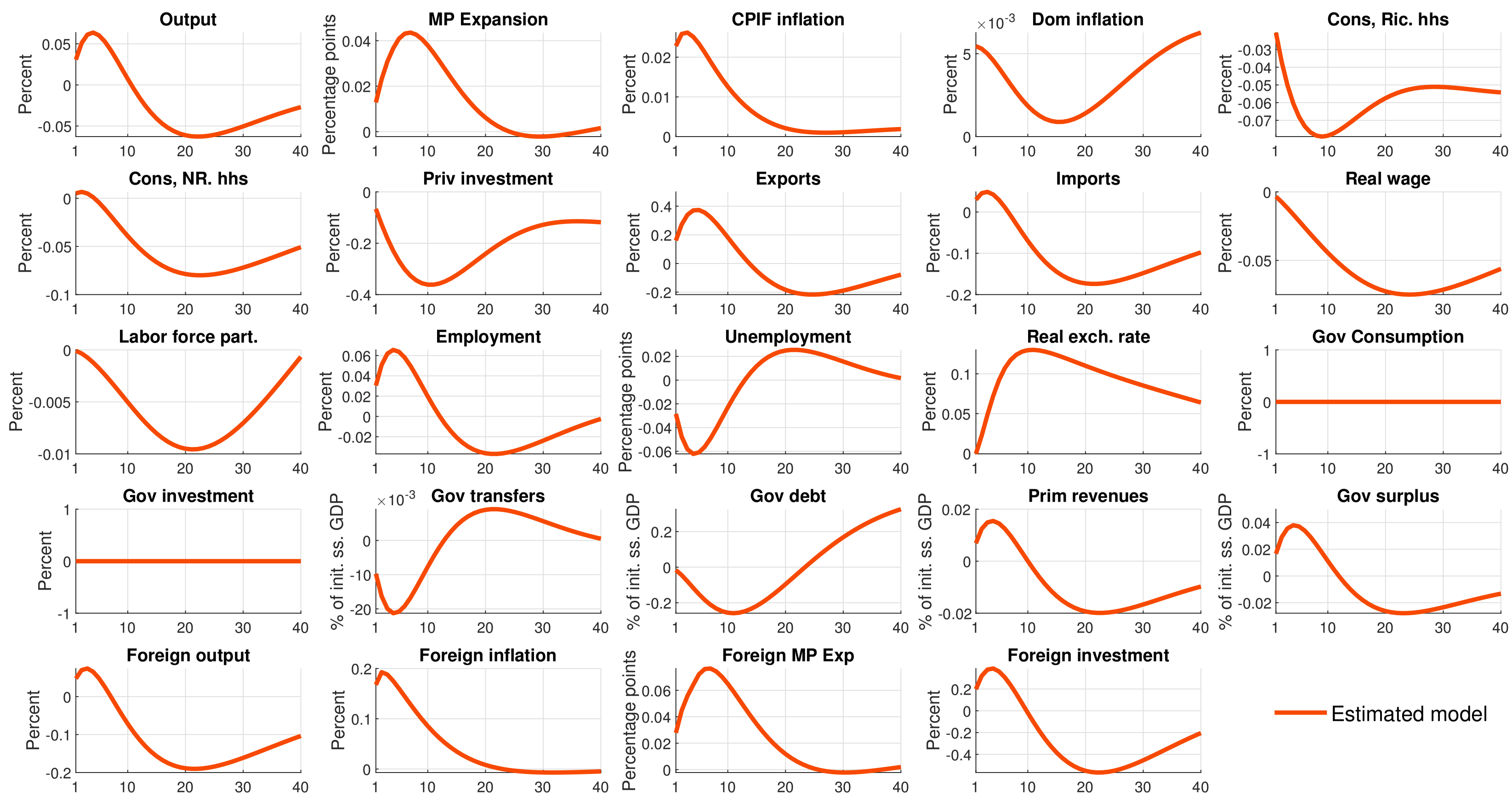
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 57: Foreign Risk Premium Shock



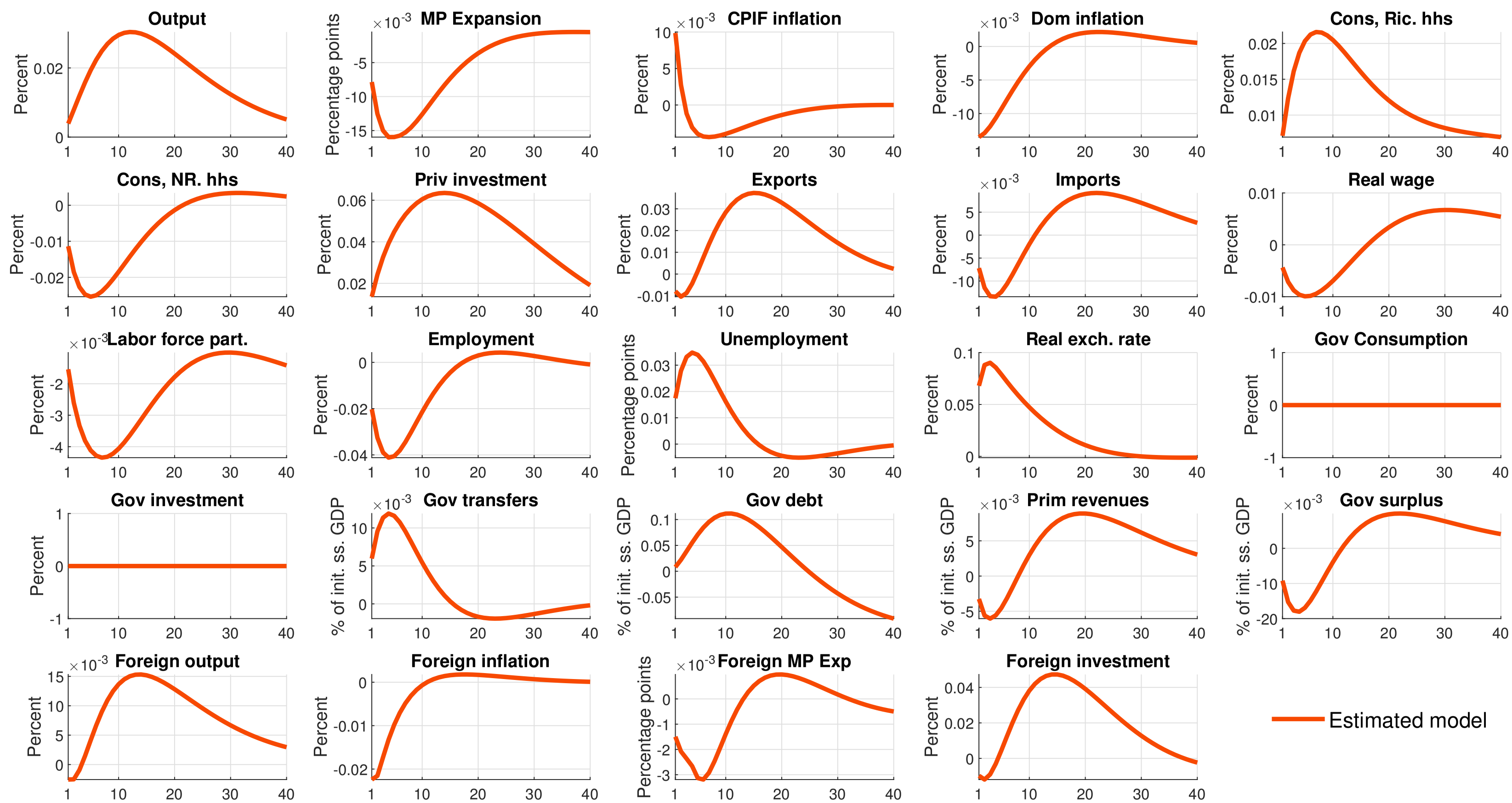
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 58: Foreign Consumption Preference Shock



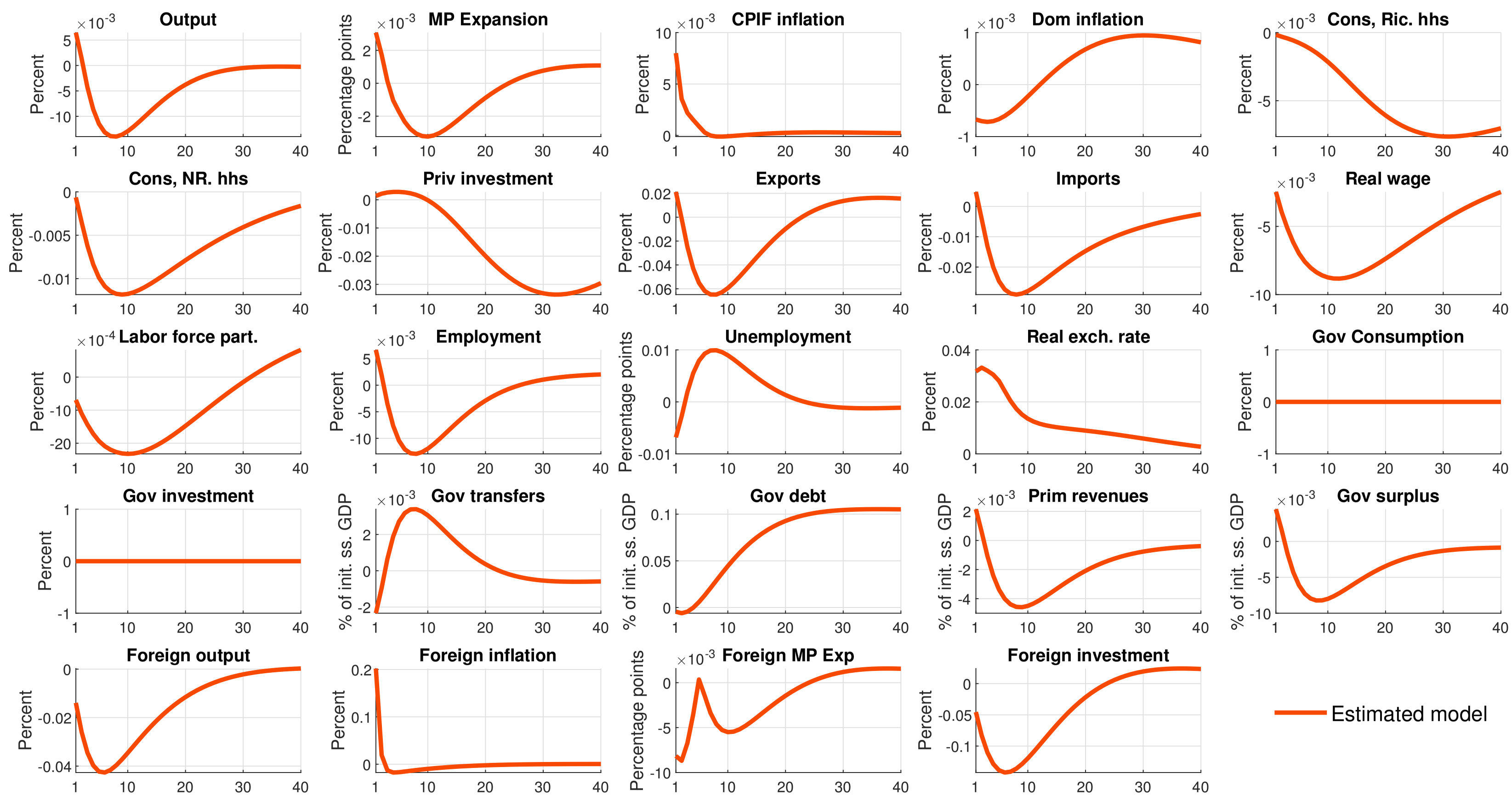
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 59: Foreign Labor Preference Shock



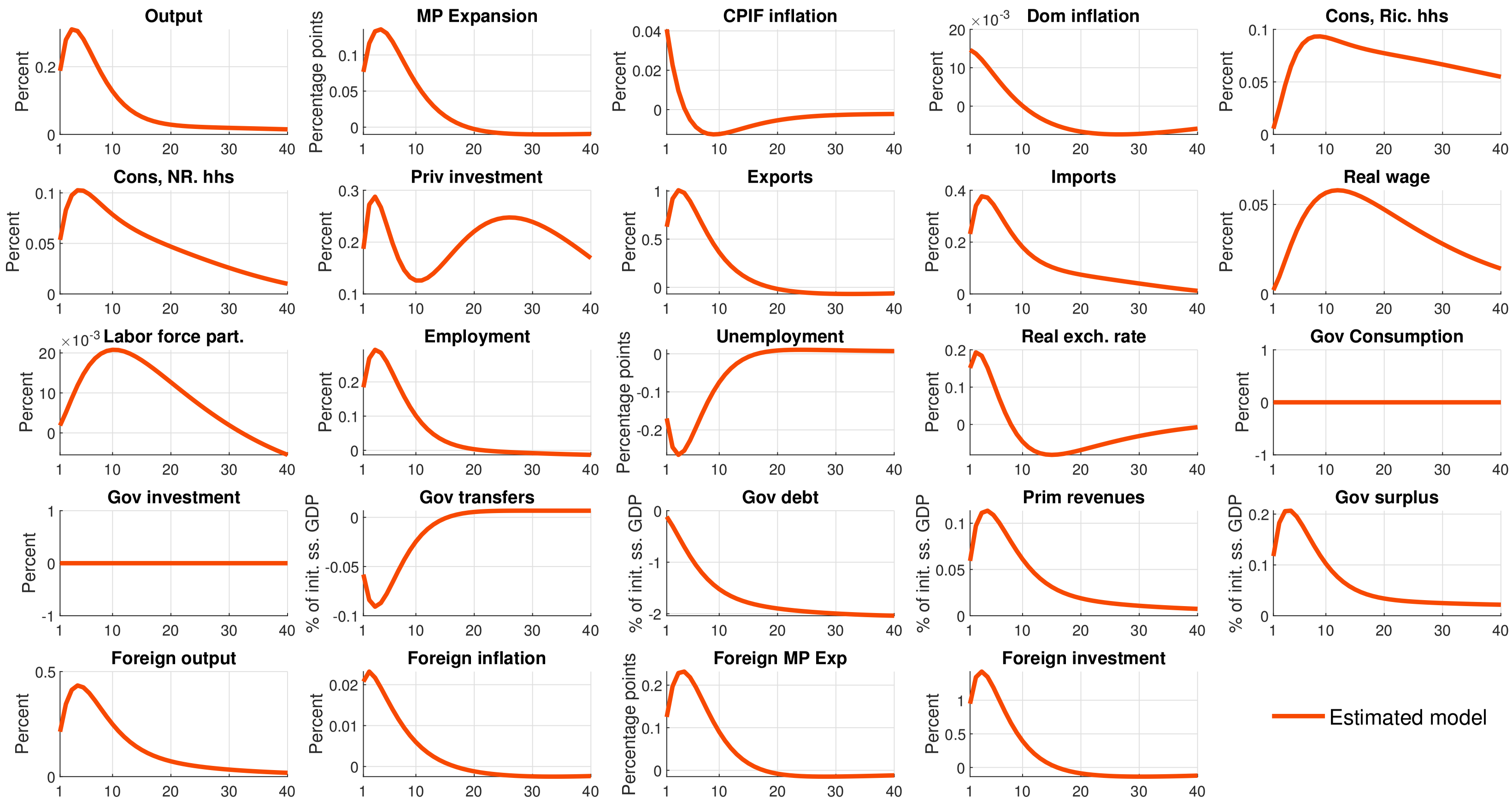
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 60: Foreign Stationary Technology Shock



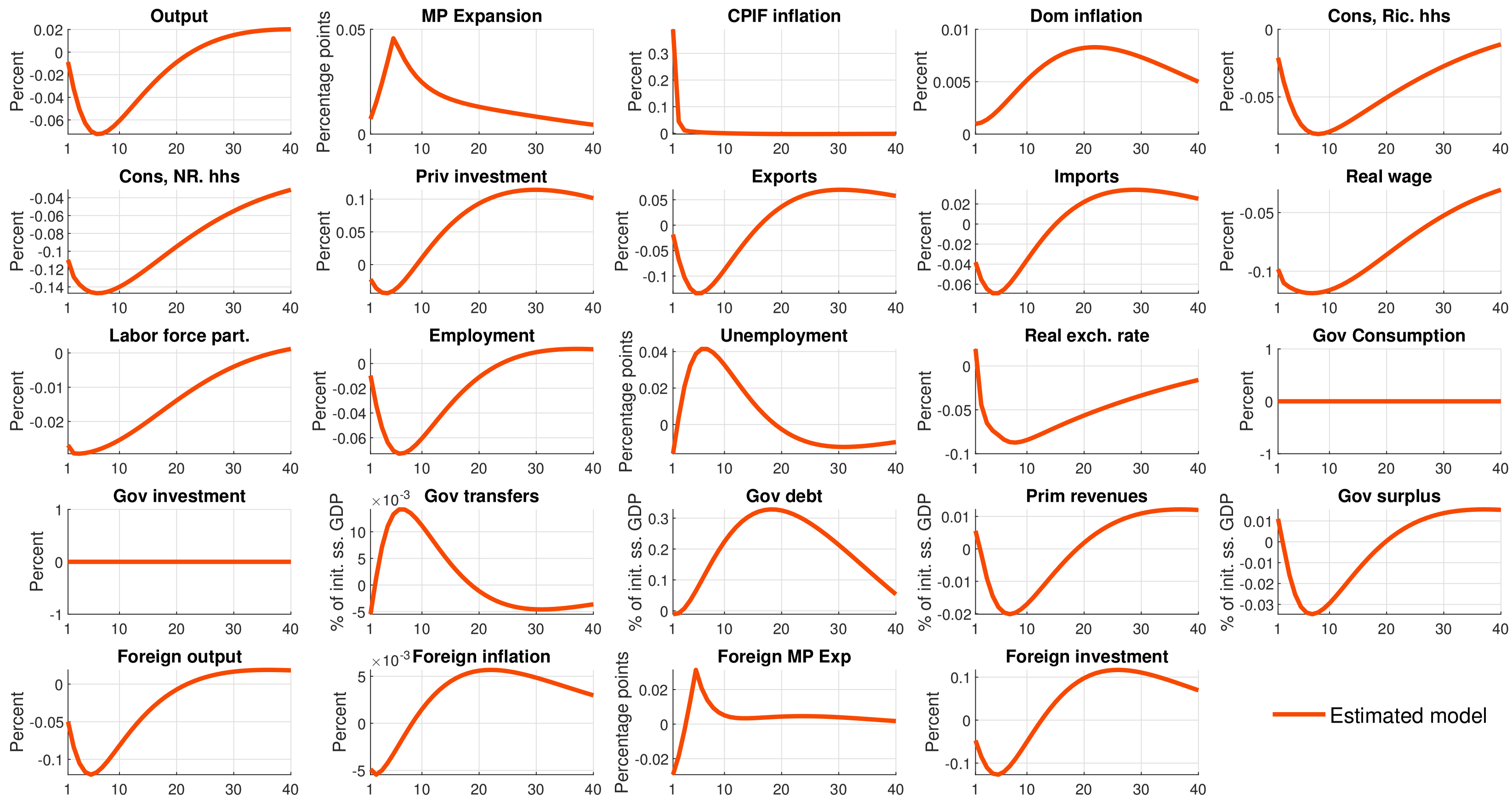
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 61: Foreign Intermediate Goods Price Markup Shock



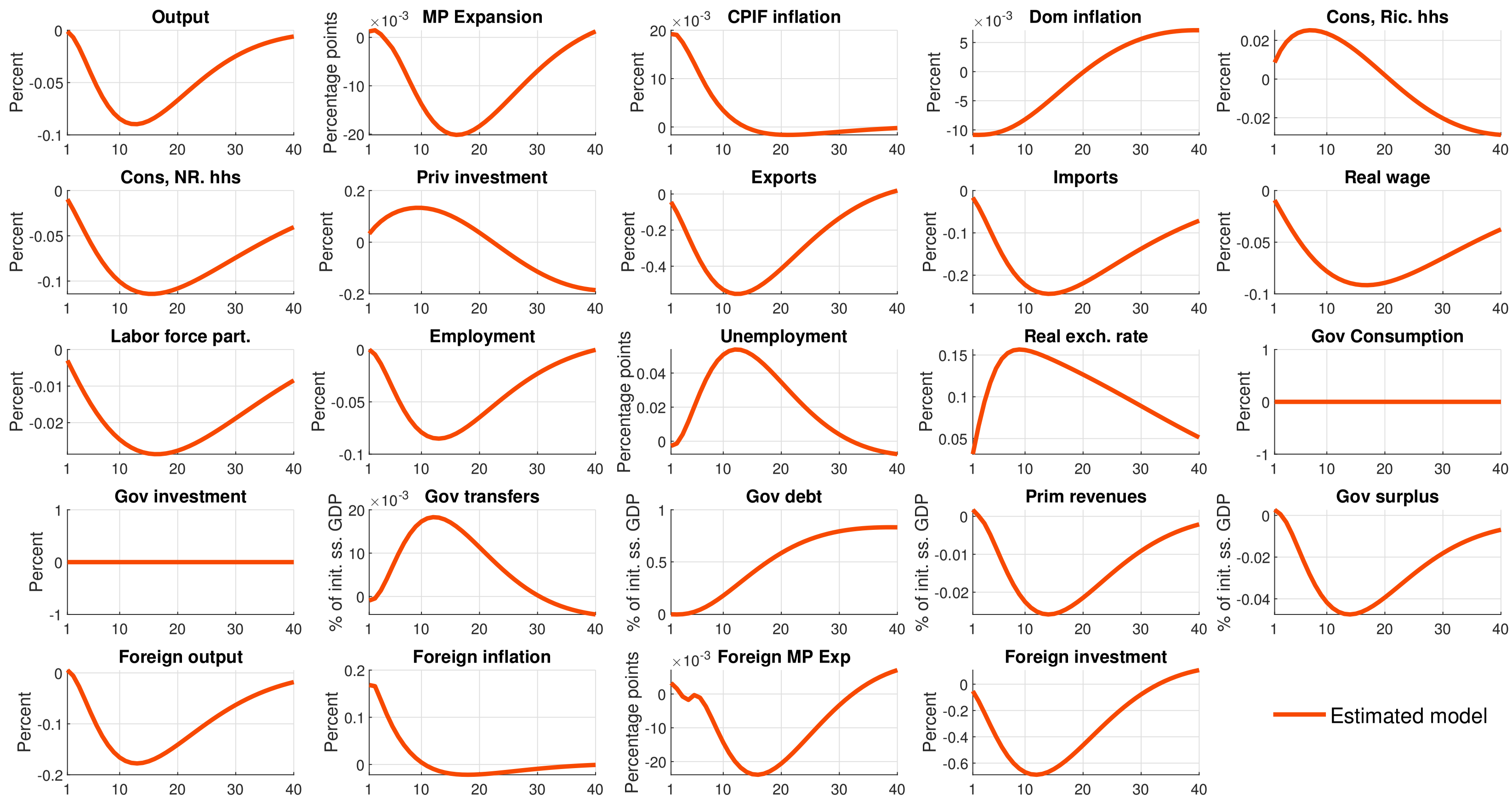
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 62: Foreign Stationary Investment Shock



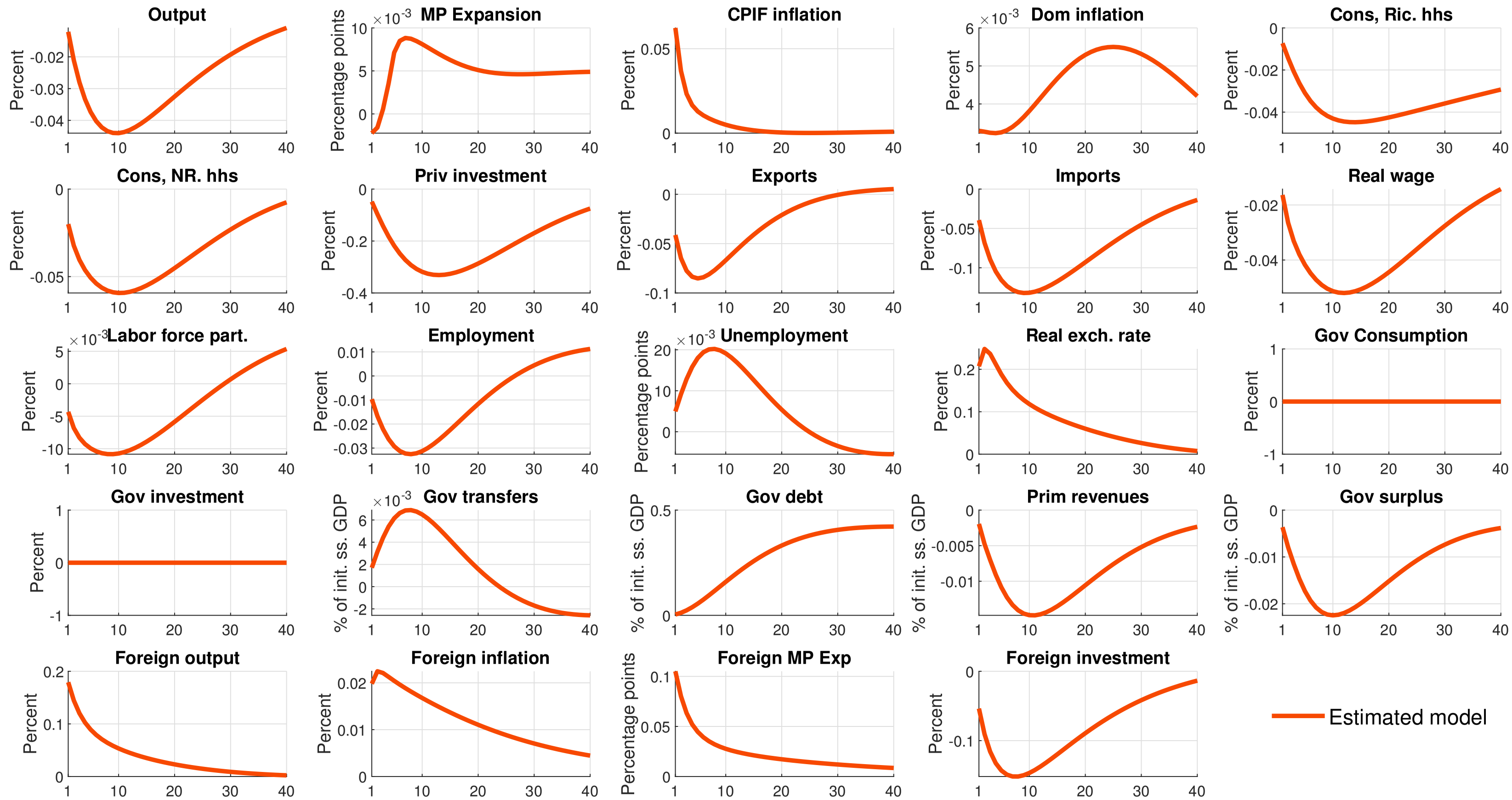
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 63: Foreign Energy Price Shock



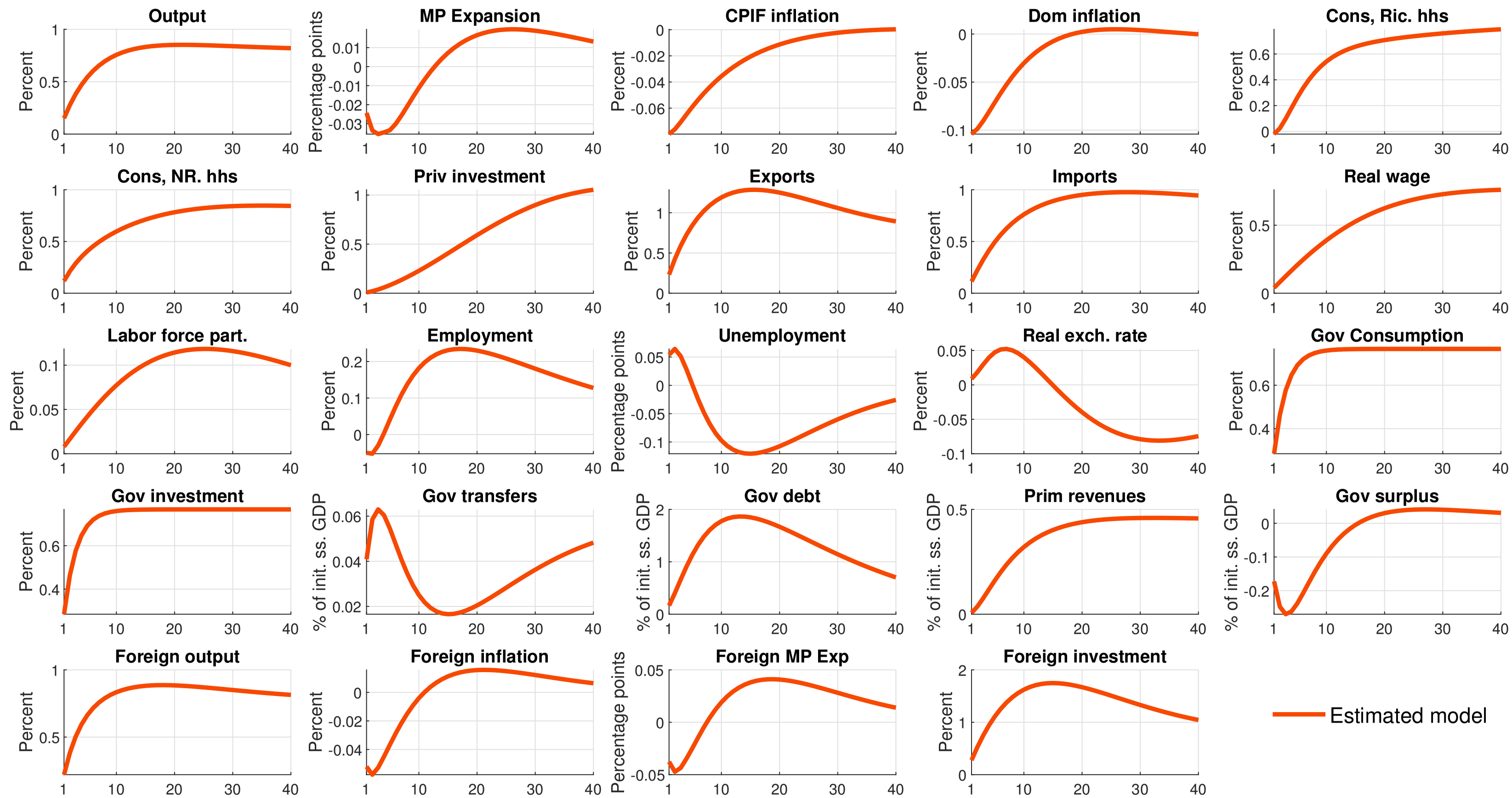
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 64: Foreign Inflation Trend Shock



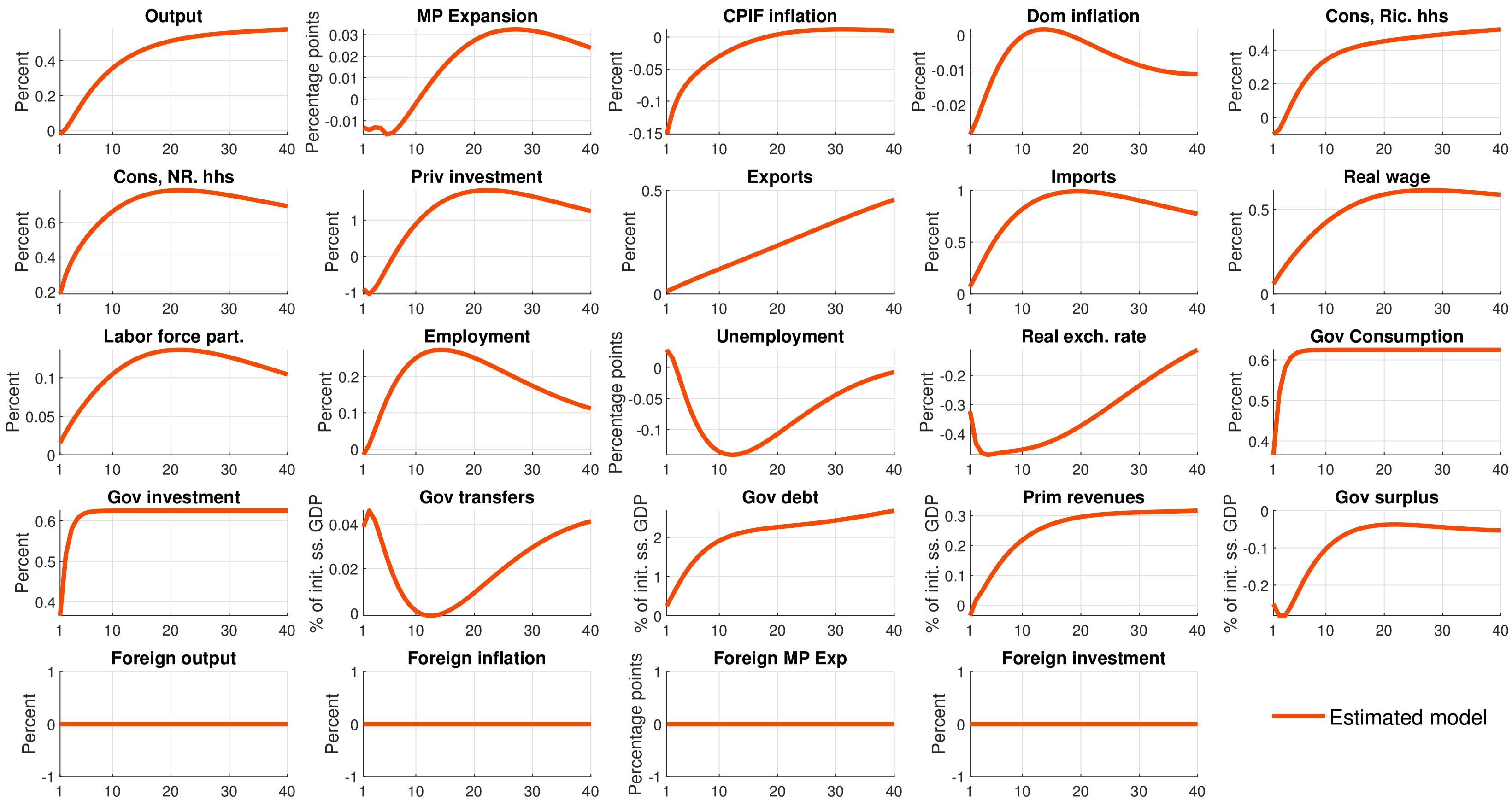
Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 65: Foreign Government Consumption Shock



Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 66: Global Labor Technology Shock



Note: The monetary policy rate in Home and Foreign, all inflation rates and the government bond interest rate are all presented in annualized q/q values

Figure 67: Permanent Investment Technology Shock