MODEL EVALUATION USING STOCHASTIC SIMULATIONS: THE CASE OF THE ECONOMETRIC MODEL 'KOSMOS'

by

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CONTENTS

1. Introduction 3

2. Analytical analysis of expected forecast errors 5

3. Deterministic and stochastic model solutions 7

4. Purpose of experiments 10

5. Experimental design 11

6. Kosmos: a general description 14

7. Random disturbances 15

8. Random disturbances and random shocks to equation coefficients 23

9. Sampling distributions of forecasts 24

10. Summary and conclusion 30

Literature 34

1. INTRODUCTION[1]

After a period of widespread optimism regarding their usefulness and future role, econometric models came under much criticism in the 1980-ies. It was realised that model forecasts are hardly reliable without extensive (and cumbersome) fine-tuning. Furthermore, due to the interdependence of model variables, even those equations that are estimated with high precision can give poor forecasts within the context of a model.

Despite this general disappointment, econometric models have survived and are still in use as much as ever, perhaps with somewhat less publicity. The reason for that is the fact that some forms of forecasts and assessment of policy effects are necessary for the conduct of economic policy, and models - in one form or another - are a means of formalising (and computerising) the assumptions behind a forecasting procedure or a policy simulation.

In this context, evaluation of econometric models from the point of view of their reliability is of highest importance. As already mentioned, it is not enough to scrutinise each equation separately, the functioning of the whole equation system has also to be investigated.

One way of assessing forecast reliability is to compute the expected range of the forecast error, usually summarised by its variance. The latter can be more or less readily obtained for linear models[2] (dynamic forecasts involving quite formidable formulae), but no formulae are available for non-linear models.

It should be noted that forecast confidence intervals - or any other assessment of expected forecast error dispersion - are hardly ever reported by the forecasters. This is possibly due to the fact that these intervals - when they can be assessed at all - often are very wide and, as such, not very informative. Moreover, any measure of uncertainty connected with

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[1] The authors are indebted to Lasse Koskinen and Lars-Erik Öller for valuable comments and discussions.

[2] See Goldberger *et al.* [1961] for static model results and Schmidt [1973, 1976, 1977] for those for dynamic models. Gajda [1965] gives a review.

estimation of the equation (which is what prediction error variance describes) is probably of limited importance in comparison to such error sources as changing coefficients (due to incorrect linearisation), omitted variables and erroneous assumptions about exogenous variables. The forecasting record of the model (or of the forecaster) in terms of actual past forecast errors is usually considered to be a better description of forecast reliability.

While prediction error variance and past forecast record can be of interest to the users of the model output, there are a number of model evaluation techniques which are mainly of use to the model builder. In the case of non-linear models, where analytical methods usually do not exist, these techniques often involve model simulations.

One aspect of model behaviour that is of interest to the model builder is sensitivity to different forms of errors. This can be investigated using stochastic simulations, as shown by Gajda [1995]. The method involves generating random numbers from a given (usually normal) distribution and introducing them as shocks to the model. In particular, stochastic simulations can be employed to make an empirical investigation into the following aspects of the model:

- the effects on the forecast of random disturbances,
- the effects on the forecast of random variation in equation parameters (sampling errors),
- error propagation and accumulation patterns in the model,
- the effects on the forecast of (random) errors in the exogenous variables.

The results of stochastic simulations can provide information on - *inter alia* - the sampling distribution of the model forecast. In particular, the model builder may be interested in the shape of this distribution. If it is not symmetric, the mean (stochastic) forecast will be different from the median forecast (which under certain conditions is equal to the deterministic forecast). Furthermore, if the distribution is skewed, a typical stochastic forecast (represented by the mode) will systematically underestimate (or overestimate) both the mean and the median forecasts.

The purpose of the present paper is to investigate KOSMOS, the econometric model of the National Institute of Economic Research in Stockholm, from the point of view of the first three aspects mentioned above. The main aim of this exercise is to look for ”weak links” in the model, i.e. to find out which equations introduce most uncertainty and at the same time are crucial for the forecast because of their strong influence on it. Thus, our interest is not only in assessing the forecast error variance as a descriptive statistic, but also - and primarily - in finding those equations that are important for error propagation and those coefficients whose values are crucial for the model.

The outline of the paper is as follows. Section 2 discusses the analysis of expected forecast errors (for linear models) based on analytical formulae. In Section 3, model simulations and stochastic simulations are defined. The two subsequent sections discuss the purpose of our experiments and their design, respectively. Section 6 gives a brief description of the econometric model KOSMOS, whose equations are subject to our investigation. Section 7 describes the results of stochastic simulations with additive equation disturbances. Section 8 presents the results of stochastic simulations with both equation disturbances and disturbances to the estimated coefficients. Sampling distributions of forecasts are discussed in Section 9. Section 10 concludes.

2. ANALYTICAL ANALYSIS OF EXPECTED FORECAST ERRORS

Let us define a simple linear model

(1) $$\mathbf{y} = \mathbf{X}\,\mathbf{b} + \mathbf{u}$$

with $\mathbf{y}$ being a T×1 vector of dependent variable values, $\mathbf{X}$ a T×k matrix of explanatory variables, $\mathbf{u}$ a T×1 vector of the random error term values and $\mathbf{b}$ a k×1 coefficient vector. Estimating $\mathbf{b}$ by OLS

(2) $$\hat{\mathbf{b}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{y}$$

we obtain

(3) $$\mathbf{y} = \mathbf{X}\hat{\mathbf{b}} + \hat{\mathbf{u}}$$

where ^ denotes OLS estimates and the superscript ^{T} a transposed matrix.

For prediction we use the systematic part of the equation:

(4) $$\hat{\mathbf{y}}_{t} = \hat{\mathbf{x}}_{\mathbf{t}}\hat{\mathbf{b}}$$

where x_t is the t-th row of the matrix $\mathbf{X}$, t > T (T being the sample size) and $\hat{\mathbf{x}}_{\mathbf{t}}$ denoting the forecast for the exogenous variables.

The *ex post* forecast error can be written as

(5) $$\hat{\mathbf{u}}_{\mathbf{t}} = \mathbf{y}_{\mathbf{t}} - \hat{\mathbf{y}}_{\mathbf{t}} = \mathbf{x}_{\mathbf{t}}\mathbf{b} + \mathbf{u}_{\mathbf{t}} - \hat{\mathbf{x}}_{\mathbf{t}}\hat{\mathbf{b}} + (\mathbf{x}_{\mathbf{t}}\hat{\mathbf{b}} - \mathbf{x}_{\mathbf{t}}\hat{\mathbf{b}}) = (\mathbf{x}_{\mathbf{t}} - \hat{\mathbf{x}}_{\mathbf{t}})\hat{\mathbf{b}} + \mathbf{x}_{\mathbf{t}}(\mathbf{b} - \hat{\mathbf{b}}) + \mathbf{u}_{\mathbf{t}}$$

each of the terms on the right-hand side describing one error source: exogenous variables values, sampling error (i.e. the error in parameter estimation) and random disturbance.

On an *ex ante* basis, the forecast precision of a static linear regression can be assessed in the form of its mean square error (MSE), which is equal to the prediction error variance when the expected value of the error term is zero:

(6) $$\mathrm{E}(\mathbf{y}_{t} - \hat{\mathbf{y}}_{t})^{2} = \mathrm{E}[(\mathbf{x}_{t}(\mathbf{b} - \hat{\mathbf{b}}) + \mathbf{u}_{t}]^{2} = \mathbf{x}_{t}\,\sigma^{2}\,(\mathbf{X}^{T}\,\mathbf{X})^{-1}\,\mathbf{x}^{T}_{t} + \sigma^{2} = \sigma^{2}\,[\mathbf{x}_{t}\,(\mathbf{X}^{T}\,\mathbf{X})^{-1}\,\mathbf{x}^{T}_{t} + 1]\ ,$$

where σ^2 is the variance of disturbances, usually estimated as the variance of residuals.

In the case of simple regression this formula assumes the form

(7) $$E(\mathrm{y}_{t} - \hat{\mathrm{y}}_{t})^{2} = s^{2}[1 + \frac{1}{T} + \frac{(\mathrm{x}_{t} - \overline{\mathrm{x}})^{2}}{\sum_{i=1}^{T}(\mathrm{x}_{i} - \overline{\mathrm{x}})^{2}}]\ ,$$

showing that the prediction error depends positively on the disturbance variance (σ^2) and the deviation of the forecast values of the exogenous variables from their sample means and that it depends negatively on the sample size (T) and the variability of the exogenous variables in the sample ($\sum_{i=1}^{T}(\mathrm{x}_i - \overline{\mathrm{x}})^2$).

The general formula becomes increasingly more complicated in the cases of a dynamic single equation and of a static linear interdependent system, and hardly tractable in the case of a dynamic linear interdependent system (cf. footnote 2 above, Gajda [1995] and the references therein).

There are no general analytical formulae for the prediction error variance of a non-linear model.

3. DETERMINISTIC AND STOCHASTIC MODEL SOLUTIONS

Linear models can be solved in a non-iterative way using *reduced forms* for static solutions and *final forms* for dynamic solutions. In a reduced form, predetermined variables have been substituted for contemporaneous endogenous variables; in a final form lagged endogenous variables have also been substituted for. In the case of a non-linear model, we are in general not able to derive the reduced and final forms. Instead, iterative procedures are used.[3]

Analogously, impact multipliers (i.e. the contemporaneous effects of exogenous variables on the endogenous ones) in a linear model can be directly obtained from the reduced form. In a non-linear model, multipliers are neither time nor value invariant and can in general

[3] The Gauss-Seidel method is the most commonly employed (cf. Fromm and Klein [1969] and Klein [1983]). The exceptional popularity of this method in economics may be due to (besides its simplicity) the fact that for non-explosive models convergence is always achieved. In numerical methods the approach is known under the name of *simple iterations* (cf. Demidowich and Maron [1966], pp. 148-151, 474-485).

only be computed for a specific data set upon comparison of two solutions. An exogenous input is changed in one of these model runs and the resultant changes in the endogenous variables are observed. Multipliers are obtained as the effects of a unit change in an exogenous variable.

The procedure of obtaining a model solution by means of an iterative method is called *model simulation*. Simulation experiments are the most commonly used tools of investigation of non-linear models. Simulations that abstract from the stochastic character of the model are called *deterministic*. In a deterministic simulation, no account is taken of the uncertainty connected with the estimated model relations. Parameter estimates are treated as the true coefficients[4] and random disturbances are assumed to be equal to zero (i.e. their expected values).

Simulations allowing for the effects of random shocks to model equations are called *stochastic simulations*. They consist of repetitive model solving, each time with a new set of generated random shocks. These shocks are usually introduced as additive disturbances or added to equation parameters. Simulation results are summarised by the mean and the variance of each variable under scrutiny and in each period for which the model has been solved. Other parameters describing the distribution of stochastic forecasts for any variable can also be computed, if needed. Stochastic simulations attempt to imitate the effects of repeated sampling from the real world, a procedure that in reality is not possible in most economic applications.

An important practical difference between deterministic and stochastic forecasts is that the latter require much more computer time, in order to perform all the replications needed, and an extensive programming effort. Another, theoretical difference refers to the statistical properties of the two types of forecasts in non-linear models.

[4] The only (but common) exception to this rule is the use of *constant adjustments*, i.e. the adjustments to equation intercepts.

While both deterministic and stochastic forecasts are unbiased predictors in *linear* models, in general, for a non-linear model a deterministic solution is *not* an unbiased estimator of the mean value of the dependent variables[5]. A deterministic solution is computed (in each period) for the expected values of the explanatory variables and thus is not necessarily equal to the expected value of the (non-linear) function of these variables. This is so, since for the equation system

$$(8) \qquad \mathbf{Y} = g(\mathbf{X}, \mathbf{u})$$

where **Y** is the vector of dependent variables, **X** is the matrix of exogenous variables, **u** is the disturbance vector and g(•) is a non-linear function,

$$(9) \qquad E(\mathbf{Y}) = E(g(\mathbf{X}, \mathbf{u})) \neq g(E(\mathbf{X}), E(\mathbf{u})) .$$

Under the usual assumption that E(**u**) = 0, the right-hand side of equation (9) describes a deterministic solution.

While a large difference between the mean stochastic forecast and the deterministic forecast indicates a high degree of non-linearity of the model, it is not always obvious which of the two forecasts is to be preferred.

The optimal predictor depends on the specific loss function of the forecaster. Hall [1986] shows that for a quadratic loss function, the optimal predictor is the mean of the distribution of the forecasted variable. If the loss function involves the sum of the absolute values of the forecast errors, the optimal predictor is the median. Finally, if the loss function punishes deviations from the most probable realisation, the optimal predictor is the mode.

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[5] The fact that deterministic simulations - contrary to the stochastic ones - produce biased forecasts in non-linear models was already pointed out by Howrey and Keleijan [1971]. The problem is also discussed by Brown and Mariano [1989a, 1989b], Mariano and Brown [1983] and Hall [1985a, 1985b, 1986].

If g(•) is a bijective function (a one-to-one transformation), the deterministic solution is equal to the median value of the dependent variables[6]. In cases where the model produces peculiar distributions of stochastic forecasts, the deterministic forecast may be preferred as a more robust predictor.

4. PURPOSE OF EXPERIMENTS

Stochastic simulation results were here analysed with respect to three questions. The first one refers to the ***difference between the mean stochastic forecast and the deterministic forecast*** (the latter assuming that all disturbances are equal to zero).

As already mentioned, a large difference between the two forecasts indicates a relatively large degree of non-linearity in the model, implying that it may be wise at least to consider the stochastic forecast as an alternative to the deterministic one. In the case when the model does not constitute a bijective transformation, the deterministic forecast is a biased predictor of the mean and has no direct interpretation in terms of the forecast distribution. The decision on which approach to choose depends on the (implicit) loss function, the shape of the forecast distribution and - no doubt - on practical considerations.

The second question addressed referred to the ***standard deviation of the stochastic forecast***. This is a measure of the expected error of a single stochastic forecast. More interestingly, it also shows the dispersion of the stochastic forecasts around their mean. A large dispersion indicates that the model is vulnerable to shocks, which in such a case are - at least temporarily - reinforced by the model.

In principle, it is possible that finite moments do not exist in small-sample distributions of the structural or reduced-form coefficients of the model[7]. The problem was discussed on

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[6] Cf. Hall[1986].

[7] We are indebted to Giorgio Calzolari for a valuable discussion on this point.

theoretical grounds by McCarthy [1972] and Mariano [1982]. Its practical relevance was investigated by Bianchi and Calzolari [1982, 1983]. In Monte Carlo studies, non-existence of moments can result in outliers and non-convergence of the mean squared forecast errors when the number of replications is increased. Bianchi and Calzolari showed that “ the problem of outliers, and the consequent non-convergence of the procedure as the number of replications increase, is not just theoretically possible, but may be encountered in practical applications on real world models (…)”[8].

The last question considered the ***shape of the distribution of the stochastic forecast.*** This distribution provides important information about the statistical characteristics of the forecast error. In a non-linear model, normally distributed disturbances can produce a skewed forecast distribution[9] (in which, in general, the mean differs from the median). In such cases, an inspection of the distribution gives an idea about the location of the deterministic and stochastic forecasts in relation to the mode.

5. Experimental design

In our experiments, random shocks were generated from a standard normal distribution, approximated by the sum of twelve uniformly distributed variables drawn from the interval [0, 1]. The resultant variable has variance equal to 1 and was corrected to have zero mean. The pseudo-random number generator of the computer package AREMOS was employed. The software employed, written in AREMOS, allows complete flexibility regarding the simulated model, the equations shocked and the variables monitored.

Fourteen most important equations of KOSMOS (see below for a brief description of the model) were subjected to random shocks. These were the equations for: total fixed

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[8] Bianchi and Calzolari [1987], p. 219.

[9] Bianchi, Calzolari and Corsi [1979, 1981b] report only marginal non-linearity effects in the models they have studied.

investment, investment in machinery, demand for labour (all three for industry and other business, respectively), wage inflation (for industry, other business, central government and local government, respectively), exports of manufactures, exports of services, imports and private consumption.

In each experiment, 800 replications were made (400 replications when only one equation was subject to shocks). In each replication, the whole model was solved for 13 semi-annual periods starting in 1995:1. Random shocks were drawn for each period of the simulation. Simulation results were computed for twenty variables in the form of the mean and the standard deviation of the stochastic forecast. The variables involved were: GDP, private consumption, total exports, total imports, total fixed investment (all five in current and constant prices respectively), labour supply, employment, real value added (in industry and other business, respectively), capacity utilisation in industry, consumer price level, wage rate in industry, effective exchange rate, short and long interest rates.

In each experiment, the distribution of stochastic forecasts for the variables under study was assessed. This was done in an approximative manner, in order to avoid amassing excessive amounts of information. The averages and standard errors of stochastic forecasts were approximated for each solution period, and each variable, using the first 200 replications. Empirical frequency distributions around those means were subsequently constructed for the remaining replications. Each distribution is divided into 40 intervals altogether covering 4 (estimated) standard deviations.

While our initial experiments indicated that relatively reliable results could be obtained for the mean forecasts and their standard deviations with less than 1000 replications, hundred times larger samples were needed to obtain smooth forecast distributions. The distributions analysed below are based on 190 000 replications.

Two types of experiments were performed. In the first one, stochastic shocks were included additively in the equations, implying non-zero disturbances. In each equation, the disturbance had zero mean and standard deviation equal to the residual standard error of the equation in question.

In the second type of experiment, besides the random disturbance, random shocks were added to all estimated coefficients, with the exception of the intercept and dummy variables. The variance-covariance matrix for coefficient shocks in one equation was equal to the original variance-covariance matrix for the coefficient estimators in this equation. In this way, sample correlation between estimators of structural parameters was preserved. Technically, this was achieved by multiplying the shock series by the ”square root” of the variance-covariance matrix, obtained using the Choleski transformation.

In cases of equations estimated using the Engle-Granger two-stage procedure, the long-run relations were not subjected to random shocks. This approach was chosen since - while the long-run relation constituted part of the short-run one - no common variance-covariance matrix for these two equations was available.

The two types of experiments highlight different aspects of model behaviour. In the first one, the disturbance is the only source of uncertainty. In the second one, uncertainty about the exact parameter values is also considered.

An important question in the experiments of the second type, was what to do with those shocks to the coefficient estimates that led to non-convergent solutions. On quite numerous occasions, random shocks to the equation coefficients resulted in a non-convergent model from which no forecasts could be computed. Those cases can reflect the fact that the standard deviations of the coefficients, routinely computed during the estimation process, in many instances exaggerate the uncertainty connected with the estimates. In reality, the estimation process included a judgmental part whereupon coefficient values within given intervals were prescribed. Consequently, some coefficient values within the standard 95% confidence interval would never be accepted. This aspect of the estimation process is not reflected in the standard errors of the coefficients, which in many cases are unduly large.

Since it was hardly possible to find an acceptable rule for adjustment of the standard errors of coeffients, no adjustment was undertaken and the shocks that led to non-convergent solutions were discarded. The replication count refers only to the convergent solutions. Thus, the distribution of the random shocks to coefficient estimates most probably was effectively trimmed, since we were not able to decide on the correct variance of the distribution.

6. KOSMOS: A GENERAL DESCRIPTION

KOSMOS is a semi-annual econometric model developed at the National Institute of Economic Research in Stockholm. It includes some 500 equations describing six basic sectors: industry, other business, households, central government, local government and the foreign sector. Most of the model equations are identities referring to either technical relations (e.g. aggregate prices as weighted averages of their component price indices) or institutional rules (e.g. computation of different tax revenues and social transfer benefits). Identities describing details of the taxation and transfer systems, and thus building up the government budget balance and household disposable income, constitute nearly half of the model equations.

Private consumption, fixed investment, foreign trade in goods and services, demand for labour, supply of labour, wage rates, export prices and domestic market prices are determined by estimated behavioural equations. There are more than thirty behavioural relations in the model.

Aggregate demand in KOSMOS is distributed between industry and other business *via* an input-output matrix. Industrial output is determined through a supply function, which defines the capacity utilisation rate. Inventory investments constitute the buffer, which makes up for differences between demand and supply. Output in other business is demand determined.

Capacity constraints are introduced into the model through labour force participation and wage formation. An increase in the demand for labour affects labour force participation and unemployment and results in wage inflation. Domestic market prices, determined mainly by costs, follow suit.

The model is non-linear and includes both logarithmic relations and products of the levels of the variables.

7. Random disturbances

In the first experiment, random disturbances were added to the behavioural equations of the model. Each disturbance was normally distributed with zero mean and standard deviation equal to the equation's residual standard error.

The results of the simulation are summarised in Table 1. The first three columns in the table show the deterministic forecast bias, i.e. the difference between the mean stochastic forecast and the deterministic forecast expressed as a percentage of the latter. The last three columns show the coefficient of variation of the stochastic forecast, i.e. the estimated forecast standard deviation expressed as a percentage of the mean forecast. Each statistic is shown for the first, middle and last periods of the overall solution period.

As can be seen from the table, the deterministic forecast bias, i.e. the difference between the deterministic and mean stochastic forecasts is very small. This indicates that in the present version of KOSMOS, the deterministic forecast is a good mean value predictor. We may also draw the conclusion that the special type of non-linearity whereby errors are squared (or multiplied by each other) is not prevalent in the model. Error accumulation, which results in a larger bias at the end of the simulation period than in the beginning of it, is practically inconsequential, possibly with the exception of fixed investment and the long interest rate.

The coefficients of variation, i.e. forecast standard deviations expressed as a percentage of mean stochastic forecast are also rather small. On impact, random disturbances affect most variables by less than one percentage point, notable exceptions being fixed investment, exports, the wage rate and interest rates, where the effect is close to 2%. As already pointed out, these numbers describe the average relative forecast error due to factors not accounted for in the equation. On one hand, one could thus say that a 1.06 percentage points error in the semi-annual forecast for real private consumption expenditure (cf. Table 1) is rather large, on the other hand - as can be seen in the table - this error is almost completely averaged out by the use of deterministic forecast.

Error accumulation is in this case apparent, though quite moderate. The relative error standard deviation triples in many cases, with the largest increases observed for fixed investment and the interest rates.

Upon this brief survey, it is clear that fixed investment, exports and the wage rate deserve further investigation. Some attention could also be given to private consumption, which though not showing excessive errors, is the largest component of GDP. As for the interest rates, errors expressed in relative terms are in this case misleading. In 1998:1 (the mid-period of the simulation) the simulated long and short rate levels were 6.8% and 4.6%, respectively. This means that a 5% relative error (coefficient of variation in Table 1) refers to the inconsequential 0.3 and 0.2 errors in the level of the respective interest rate.

Table 2 shows summary results of a stochastic simulation with random disturbances introduced only into the two equations for fixed investment in industry and in other business, respectively. This time, the number of replication was limited to 400. As can be seen upon comparison of Tables 1 and 2, the investment equations give rise to a relative forecast standard error in (*i.e.* coefficient of variation for) real GDP which amounts to almost one third of the corresponding standard error caused by all the fourteen equations. Investments also strongly affect error variances for employment, real value added and imports. Finally, the disturbances in the investment equations themselves seem to explain

Table 1. Summary results for stochastic simulations with additive random disturbances added to all fourteen equations (800 replications)

Variable Name	Percentage bias			Coeff. of variation, %		
	95:1	98:1	2001:1	95:1	98:1	2001:1
Real GDP	-0.008	-0.06	-0.06	0.60	1.36	1.66
Nominal GDP	-0.003	-0.007	-0.05	0.78	1.79	2.01
Real private consumption	-0.05	0.0004	-0.04	1.06	1.43	1.50
Nom. Private consumption	-0.04	-0.04	-0.05	1.13	1.74	1.85
Consumer prices	0.01	0.03	-0.01	0.42	0.79	0.88
Real fixed investment	0.07	0.05	-0.39	2.17	5.49	6.96
Nominal fixed investment	0.08	0.07	-0.42	2.27	5.65	7.15
Real exports	0.02	-0.05	0.06	1.57	2.44	2.49
Nominal exports	0.02	-0.08	0.06	1.58	2.82	2.86
Real imports	0.02	0.04	-0.02	0.93	2.17	2.50
Nominal imports	0.01	-0.01	-0.03	0.92	2.57	2.93
Employment	0.03	-0.08	-0.09	0.58	1.43	1.78
Labour force	0.02	-0.04	-0.03	0.34	0.73	0.89
Wage rate in industry	0.008	-0.03	-0.05	2.19	2.76	2.91
Long interest rate	0.03	-0.02	0.43	1.68	8.32	9.86
Short interest rate	0.05	0.40	0.10	2.00	7.45	8.60
Effective exchange rate	-0.01	-0.09	-0.05	0.40	1.19	1.31
Real value added in ind.	-0.01	-0.17	0.12	1.07	2.53	2.94
Real value added in other business	0.002	-0.04	0.05	0.76	1.67	2.07
Capacity utilisation in ind.	-0.03	-0.02	0.08	0.94	1.71	1.95

Note: Percentage bias = (mean stochastic forecast)/(deterministic forecast),
Coefficient of variation = (stochastic forecast standard deviation)/(mean stochastic forecast).

the major part of the investment equations' forecast error (cf. the coefficients of variations for real fixed investment in Tables 1 and 2). However, it should be borne in mind that the influence of the remaining twelve equations cannot be computed by subtraction of errors in Table 2 from those in Table 1, since the model is not linear.

While the variables affected reflect the structure of the model (and of the modelled economy), we can conclude that errors in investment forecast appear to be a major source of error in the model. These errors amount to approximately one third of the error in the real GDP forecast (the share of investment in GDP in the simulation being below 20%) and almost half of the error in real imports and real value added in other business. It is important to point out that the above statement does not refer to the actual forecast errors of the model but only to error propagation in the model. It thus implies that uncertainty about investments created by the investment equations is likely to create excessive uncertainty about other aggregates.

As for the consumption equation (cf. Table 3), it affects real GDP, imports and real value added (in particular in other business) . Besides its direct effect on real GDP (of which it is by far the largest component) its effect on forecast errors is rather limited. The two equations for exports of manufactures (to 14 OECD countries) and of services, respectively, exhibit similar effects, though of a larger magnitude (cf. Table 4). One difference is that - in contrast to the case of consumption - value added in industry is affected much more than in other business, another difference is that the effect on employment is more pronounced. Disturbances to the wage rate equations (cf. Table 5) affect slightly - as could be expected - consumer prices but have no pronounced effects otherwise.

Table 2. Summary results for stochastic simulations with additive random disturbance added to the investment equations (400 replications)

Variable Name	Percentage bias			Coeff. of variation, %		
	95:1	98:1	2001:1	95:1	98:1	2001:1
Real GDP	-0,00	-0,02	0,01	0,21	0,39	0,60
Nominal GDP	-0,00	-0,03	-0,02	0,21	0,53	0,72
Real private consumption	-0,00	-0,00	0,01	0,02	0,04	0,12
Nom. Private consumption	-0,00	-0,01	-0,01	0,04	0,20	0,32
Consumer prices	-0,00	-0,01	-0,02	0,03	0,19	0,24
Real fixed investment	-0,02	-0,20	-0,00	2,22	3,78	5,04
Nominal fixed investment	-0,02	-0,21	-0,01	2,28	3,96	5,29
Real exports	0,00	0,00	0,02	0,00	0,11	0,24
Nominal exports	-0,00	-0,01	-0,01	0,02	0,13	0,27
Real imports	-0,00	-0,05	0,01	0,36	0,83	1,20
Nominal imports	-0,00	-0,05	0,00	0,36	0,86	1,26
Employment	-0,00	-0,02	-0,01	0,12	0,41	0,62
Labour force	-0,00	-0,01	-0,00	0,07	0,21	0,35
Wage rate in industry	-0,00	-0,01	-0,04	0,01	0,28	0,39
Long interest rate	-0,00	-0,16	0,04	0,26	2,55	4,11
Short interest rate	-0,00	-0,08	0,11	0,12	1,85	2,56
Effective exchange rate	0,00	-0,01	0,00	0,02	0,13	0,20
Real value added in ind.	-0,00	-0,01	0,01	0,17	0,33	0,53
Real value added in other business	-0,01	-0,04	0,01	0,35	0,65	0,96
Capacity utilisation in ind.	-0,00	-0,01	-0,01	0,09	0,24	0,31

Note: Percentage bias = (mean stochastic forecast)/(deterministic forecast),
Coefficient of variation = (stochastic forecast standard deviation)/(mean stochastic forecast).

Table 3. Summary results for stochastic simulations with additive random disturbance added to the private consumption equation (400 replications)

Variable Name	Percentage bias			Coeff. of variation, %		
	95:1	98:1	2001:1	95:1	98:1	2001:1
Real GDP	0,02	0,04	-0,03	0,36	0,56	0,57
Nominal GDP	0,02	0,05	-0,01	0,36	0,66	0,72
Real private consumption	0,06	0,12	-0,08	1,02	1,39	1,38
Nom. Private consumption	0,06	0,12	-0,06	0,99	1,43	1,45
Consumer prices	-0,00	0,00	0,02	0,02	0,13	0,23
Real fixed investment	0,00	0,03	0,06	0,00	1,18	1,47
Nominal fixed investment	0,00	0,04	0,08	0,01	1,33	1,70
Real exports	0,00	0,00	-0,01	0,00	0,06	0,21
Nominal exports	0,00	0,01	-0,00	0,04	0,18	0,22
Real imports	0,03	0,05	-0,03	0,46	0,79	0,82
Nominal imports	0,03	0,06	-0,03	0,49	0,91	0,93
Employment	0,01	0,02	-0,01	0,13	0,43	0,46
Labour force	0,00	0,01	-0,01	0,08	0,22	0,24
Wage rate in industry	-0,00	0,01	0,03	0,01	0,21	0,40
Long interest rate	0,01	0,12	-0,03	0,17	2,61	2,77
Short interest rate	-0,01	0,02	0,20	0,11	1,40	2,58
Effective exchange rate	0,00	0,01	-0,01	0,08	0,19	0,20
Real value added in ind.	0,01	0,01	-0,03	0,10	0,32	0,33
Real value added in other business	0,03	0,06	-0,04	0,46	0,73	0,74
Capacity utilisation in ind.	0,00	0,00	-0,02	0,07	0,19	0,22

Note: Percentage bias = (mean stochastic forecast)/(deterministic forecast),
Coefficient of variation = (stochastic forecast standard deviation)/(mean stochastic forecast).

Table 4. Summary results for stochastic simulations with additive random disturbance added to the exports equations (400 replications)

Variable Name	Percentage bias			Coeff. of variation, %		
	95:1	98:1	2001:1	95:1	98:1	2001:1
Real GDP	0,06	0,04	0,04	0,45	0,90	0,97
Nominal GDP	0,06¤	0,07	0,04	0,47	1,26	1,35
Real private consumption	0,01	0,01	0,01	0,05	0,17	0,23
Nom. Private consumption	0,00	0,02	-0,00	0,02	0,48	0,60
Consumer prices	-0,00	0,01	-0,01	0,03	0,34	0,40
Real fixed investment	0,00	0,03	-0,05	0,00	1,73	1,93
Nominal fixed investment	-0,01	0,05	-0,06	0,04	2,03	2,21
Real exports	0,22	0,15	0,10	1,67	2,33	2,45
Nominal exports	0,22	0,19	0,12	1,68	2,67	2,72
Real imports	0,06	0,07	0,14	0,49	1,63	1,67
Nominal imports	0,05	0,08	0,14	0,45	1,75	1,69
Employment	0,03	0,03	-0,01	0,22	0,84	0,89
Labour force	0,02	0,02	-0,00	0,13	0,44	0,47
Wage rate in industry	-0,00	0,03	-0,02	0,02	0,79	0,90
Long interest rate	0,06	0,37	0,26	0,44	6,55	7,42
Short interest rate	-0,02	0,13	-0,22	0,16	3,02	3,57
Effective exchange rate	-0,01	0,01	0,01	0,07	0,34	0,35
Real value added in ind.	0,11	0,08	0,13	0,94	2,00	2,01
Real value added in other business	0,07	0,05	0,01	0,58	0,99	1,10
Capacity utilisation in ind.	0,07	0,02	0,11	0,65	1,17	1,30

Note: Percentage bias = (mean stochastic forecast)/(deterministic forecast),
Coefficient of variation = (stochastic forecast standard deviation)/(mean stochastic forecast).

Table 5. Summary results for stochastic simulations with additive random disturbance added to the wage rate equations (400 replications)

Variable Name	Percentage bias			Coeff. of variation, %		
	95:1	98:1	2001:1	95:1	98:1	2001:1
Real GDP	-0,00	-0,03	-0,01	0,13	0,61	0,85
Nominal GDP	0,01	0,05	0,02	0,47	0,72	0,84
Real private consumption	0,00	0,03	0,03	0,11	0,25	0,31
Nom. Private consumption	0,01	0,03	0,05	0,46	0,78	0,79
Consumer prices	0,01	0,01	0,02	0,38	0,61	0,61
Real fixed investment	0,00	-0,27	-0,14	0,00	3,16	4,23
Nominal fixed investment	0,01	-0,27	-0,13	0,35	2,99	4,10
Real exports	-0,00	-0,06	-0,04	0,04	0,72	0,91
Nominal exports	-0,01	0,00	0,01	0,25	0,93	1,05
Real imports	0,01	-0,02	-0,04	0,15	0,74	1,00
Nominal imports	-0,01	-0,01	-0,01	0,25	1,36	1,60
Employment	-0,00	-0,05	-0,02	0,05	0,74	1,04
Labour force	-0,00	-0,01	0,01	0,03	0,31	0,39
Wage rate in industry	-0,05	0,27	0,01	2,14	2,55	2,64
Long interest rate	0,05	-0,11	0,05	1,46	2,98	3,90
Short interest rate	0,06	0,07	0,18	1,84	5,80	6,14
Effective exchange rate	-0,02	0,04	0,02	0,36	1,00	1,06
Real value added in ind.	0,00	-0,06	-0,02	0,36	1,05	1,35
Real value added in other business	-0,00	-0,04	-0,02	0,13	0,80	1,12
Capacity utilisation in ind.	-0,00	0,02	0,02	0,24	0,48	0,47

Note: Percentage bias = (mean stochastic forecast)/(deterministic forecast),
Coefficient of variation = (stochastic forecast standard deviation)/(mean stochastic forecast).

8. Random disturbances and random shocks to equation coefficients

In the second experiment, in addition to the random disturbances, random shocks were added to the coefficients of the behavioural equations of the model. Coefficient disturbances were normally distributed with zero mean and variances and covariances equal to those in the equation's variance-covariance matrix.

This experiment investigates the effects of uncertainty connected with coefficient estimates. In a way, this is the maximum uncertainty connected with the equations, since all the error sources are introduced at the same time. (The equation form and the values of exogenous variables are here taken as given.) Thus, *all* the coefficients, with the exception of intercepts and dummy variables, are subjected to disturbance shocks at the same time.

The results of the simulation are summarised in Table 6. As can be seen in the table, the deviations of the deterministic forecasts from the mean stochastic forecasts are in this case much larger than in the previous experiment. This is obviously so, since most equation coefficients are subject to random shocks. The largest deviations are noted for exports, employment, real value added in other business, and GDP. Since exports of services affect the three latter variables (and exports of manufactures do not affect other business), it appears that the equation for exports of services is the cause of this bias. Moreover, this equation exhibits low accuracy of coefficient estimates and thus the random disturbances to its coefficients are probably very large (as they depend on the coefficients' standard deviations).

Error accumulation is quite apparent here. We may note the peculiar fact that in some cases the percentage bias decreases towards the end of the simulation period.

The coefficients of variation in this case express the (relative) dispersion of forecast errors due to uncertainty about coefficient values and also to random equation disturbances. In the majority of cases, the first-period coefficients are of the order of magnitude of 2%. This is not bad, especially in view of the fact that specification of the equations in KOSMOS was guided more by theoretical considerations than by statistical significance, in some cases resulting in poor t-values.

Error accumulation is substantial, resulting in the coefficient of variation of approximately 7% for real GDP in the last period of the simulation. This can hardly be considered to be excessive in the present context and compares well with our experience of other models. Relatively large coefficients of variation are observed for fixed investment, exports and real value added in industry. As already mentioned, this reflects the poor estimation accuracy in the equations for fixed investment and exports of services.

Finally, we can note the peculiar phenomenon of a decreasing coefficient of variation towards the end of the simulation period for fixed investment and the long interest rate. This results in narrowing confidence bands for these variables.

It should be reiterated that the standard deviations of the coefficient estimators - employed in the sampling process - probably overestimate the uncertainty connected with the estimates, since they do not account for the judgmental part of the estimation process (cf. Section 5 above). Consequently, the sampling results give an exaggerated picture of the uncertainty effects and, as such, are more indicative of our problems with measuring uncertainty than of anything else.

9. SAMPLING DISTRIBUTIONS OF FORECASTS

As already mentioned, computation of relatively smooth sampling distributions required far more replications than our earlier experiments. In fact, in some instances up to 190000 replications were made. The technical details of the computations were briefly outlined in Section 5 above.

Table 6. Summary results for stochastic simulations with additive random disturbance and shocks to equation parameters in fourteen equations (800 replications)

Variable Name	Percentage bias			Coeff. of variation, %		
	95:1	98:1	2001:1	95:1	98:1	2001:1
Real GDP	-2.09	-3.67	-3.50	2.11	6.53	7.28
Nominal GDP	-2.22	-5.14	-4.57	2.27	9.37	12.17
Real private consumption	-0.33	-1.68	-2.69	1.85	5.21	7.64
Nom. Private consumption	-0.26	-2.79	-3.64	1.93	6.30	9.84
Consumer prices	0.08	-1.16	-1.05	0.68	2.68	4.52
Real fixed investment	-0.59	-7.72	-3.90	8.66	41.91	37.20
Nominal fixed investment	-0.45	-7.79	-3.50	8.92	44.82	41.56
Real exports	-5.27	-4.46	-2.52	5.47	9.24	10.70
Nominal exports	-5.13	-5.05	-2.32	5.36	11.10	12.38
Real imports	-0.20	-1.09	1.53	2.27	11.14	12.47
Nominal imports	0.13	0.25	3.45	2.27	11.82	12.72
Employment	-1.46	-5.33	-5.36	2.21	7.26	7.98
Labour force	0.86	-2.52	-1.74	1.29	3.48	3.32
Wage rate in industry	0.05	-1.24	-0.61	2.30	5.68	9.45
Long interest rate	-2.19	-11.91	-3.65	3.30	30.62	26.04
Short interest rate	0.35	-8.69	-6.27	3.25	27.56	46.47
Effective exchange rate	0.47	1.58	2.39	0.77	3.01	3.93
Real value added in ind.	-0.66	-1.18	-0.06	1.87	8.67	10.53
Real value added in other business	-3.92	-6.64	-6.50	3.84	10.55	12.00
Capacity utilisation in ind.	-0.41	-0.58	0.48	1.80	4.36	4.29

Note: Percentage bias = (mean stochastic forecast)/(deterministic forecast),
Coefficient of variation = (stochastic forecast standard deviation)/(mean stochastic forecast).

In general, the sampling distributions of the forecasts for individual variables are not symmetric[10]. However, in the first experiment (where non-zero random disturbances were added) on visual inspection they appear to be quite close to symmetry and normality.

One of the most asymmetric distributions is illustrated in Chart 1, which shows the sampling distribution for the consumer price index (more exactly: the implicit deflator for private consumption) based on 50000 replications. Each curve in the chart represents the sampling distribution for one period in time. The chart is drawn in two rather than three dimensions (simulation periods constituting the original third dimension) in order to facilitate comparison of the shapes of the curves. In order to make a comparison possible, all the distributions are standardised so as to have zero mean and unit variance.

While the picture is obviously too blurred for a thorough analysis of any singular curve, the more or less bell-like shape of the distributions comes out clearly. The three top curves are those for the first three simulation periods, the distributions become somewhat less peaked afterwards.

Once we conclude that some forecast sampling distributions are not symmetric, the question arises as to how the deterministic forecast should be interpreted. In particular, it is interesting to know how the deterministic forecast is related to the standard measures of the central tendency of the distribution, or - in other words - what it tells us about this distribution.

Hall [1986] shows that for a special class of non-linear models the deterministic forecast is equal to the median[11] of the sample distribution of the stochastic forecasts. He also points out that for this class of models - which he calls *bijective* - antithetic shocks drawn from a

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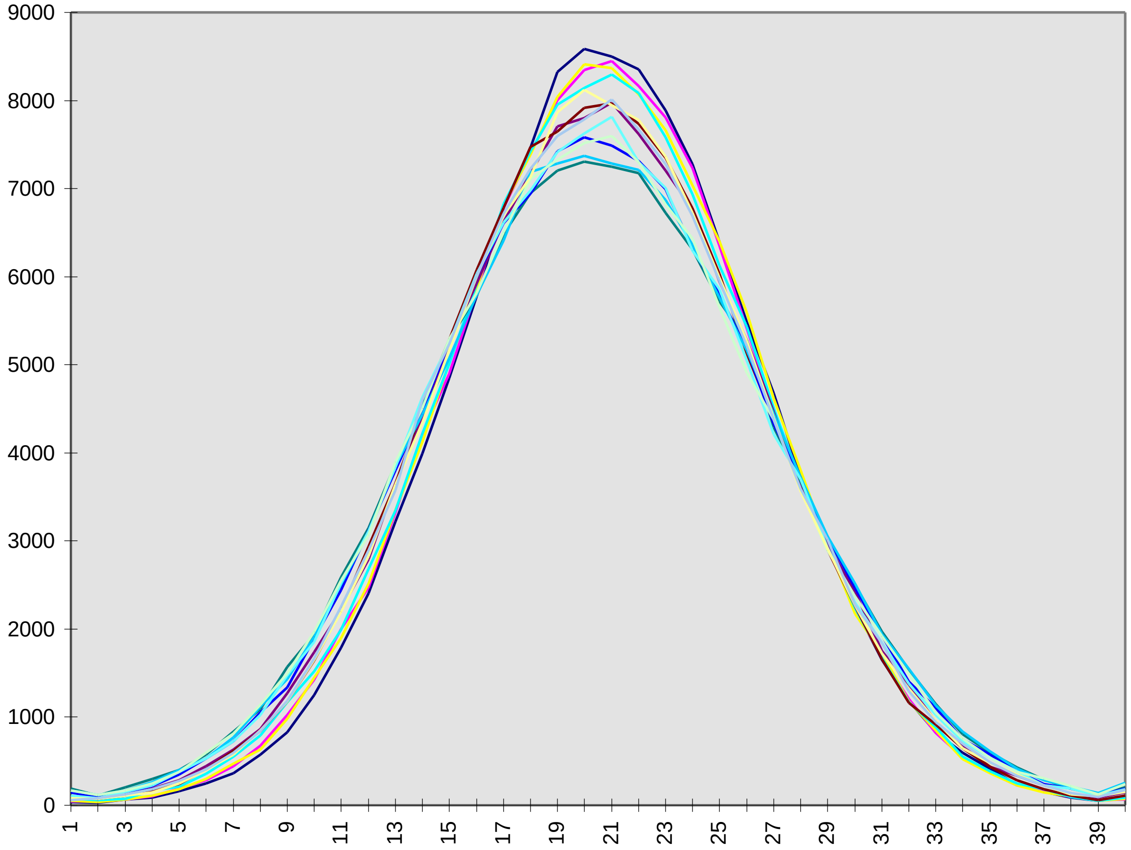
[10] As indicated by the Jarque-Bera test.

[11] Hall[1986] introduces also a definition of the median of a joint distribution.

Chart 1. Sampling distributions for the consumer price index for the thirteen periods of the first experiment.



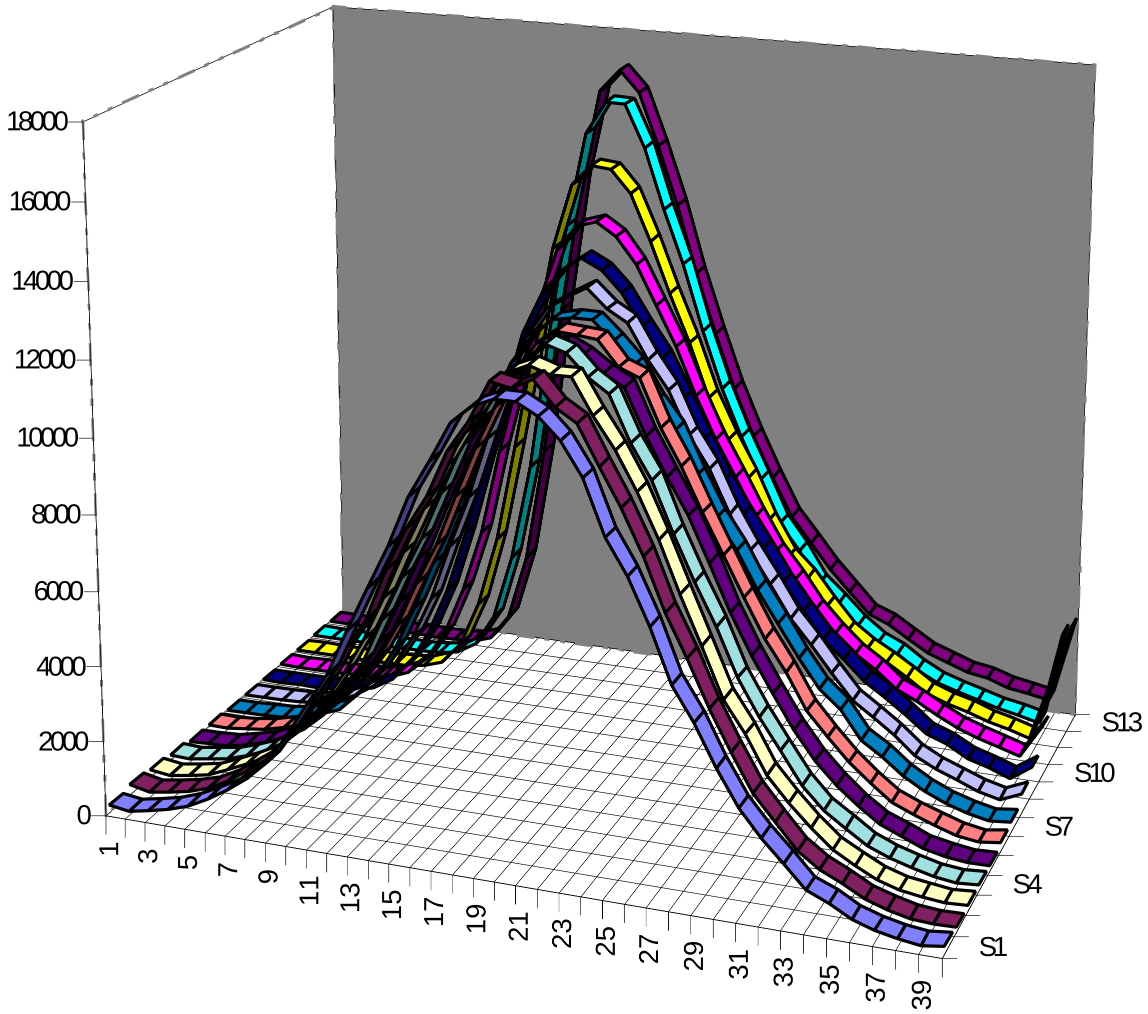
symmetric distribution will give exactly 50% of stochastic forecasts below the median forecast. We investigated this in our experiments by computing, for a number of specific variables, the share of stochastic forecasts that were lower than the deterministic forecast.

According to our results, which are illustrated in Table 7, *in the first experiment* this share varies, but in the majority of cases it differs from 50% by not more than 0.5-1.5 percentage points. Thus, though the deterministic forecast is not equal to the median of the sample distribution, in most cases it is quite close to it. The largest deviation exhibits the above mentioned consumer price index, which has 55.05% of forecasts below the median in 95:1.

Table 7. Percentage share of stochastic forecasts that were lower than the deterministic forecast in the first experiment (15000 replications)

Variable name	95:1	98:1	2001:1
Real GDP	51.80	50.89	50.09
Nominal GDP	51.70	50.40	50.15
Real private consumption	51.31	51.92	51.42
Nom. Private consumption	51.29	51.29	51.31
Consumer prices	55.05	51.66	51.25
Real fixed investment	50.82	50.08	50.09
Real exports	51.20	50.89	50.88
Real imports	51.09	50.85	50.27
Employment	52.50	50.81	49.97
Wage rate in industry	51.09	50.93	50.85
Short interest rate	51.20	50.30	50.29
Effective exchange rate	54.32	52.07	51.38
Real value added in ind.	51.60	50.96	49.67
Real value added in other business	50.55	50.58	49.98
Capacity utilisation in ind.	51.35	51.96	50.07

Chart 2. Sampling distributions for the consumer price index for the thirteen periods of the second experiment. Three-dimensional chart.



In the second experiment, with both non-zero disturbances and shocks to the coefficient estimates, some sampling distributions were, again, relatively symmetric and bell-shaped. However, in other instances the distributions changed over time, showing increasing skewness towards the end of the simulation period. This is illustrated in Chart 2, which shows sampling distributions for the consumer price index in the second experiment. The chart is analogous to Chart 1 but depicted in three dimensions to show the time sequence in the development of the sampling distributions. Chart 3 shows the same data in two dimensions to facilitate comparison of the shape of the distributions. As can be seen, the distributions for the last periods are clearly more skewed than those in the beginning of the simulation period.

In this experiment, the share of the stochastic forecasts that were lower than the deterministic forecast varies from 16.43% for real GDP to 75.72% for the effective exchange rate in 95:1. As can be seen from Table 8, imports and the industrial wage rate exhibit relatively symmetric distributions. In many cases, however, the share in question is persistently lower than 40%, indicating sampling distributions that are skewed to the right. It is interesting to note that real fixed investment, singled out as the most probable main source of error in the first experiment (cf. Section 7), does not show any extreme asymmetry in its sampling distribution. The same is true of the industrial wage rate, but not of real exports, the two other sources of forecast bias.

The skewed sampling distributions may be due to the simultaneous nature of the model itself. In the second experiment, endogenous variables are affected by both error term shocks and shocks to the regression coefficients. When found on the right-hand sides of model equations, endogenous variables are multiplied by regression coefficients that also are subject to random shocks. The resultant sampling distribution of the product of random variables remains to be investigated, but it certainly can be asymmetric. This asymmetry is possibly exacerbated by large shock variances, due to the overestimated standard errors of coefficient estimates, discussed in the previous section.

10. SUMMARY AND CONCLUSION

The stochastic properties of the econometric model KOSMOS were investigated in two experiments, both involving a large number of replications. In the first one, the additive error terms in fourteen behavioural equations were subjected to random (normal) shocks. In the second experiment, random shocks were added to the error terms *and* to all the coefficients (except the intercept and dummy variables) in the same fourteen equations.

Chart 3. Sampling distributions for the consumer price index for the thirteen periods of the second experiment. Two-dimensional chart.

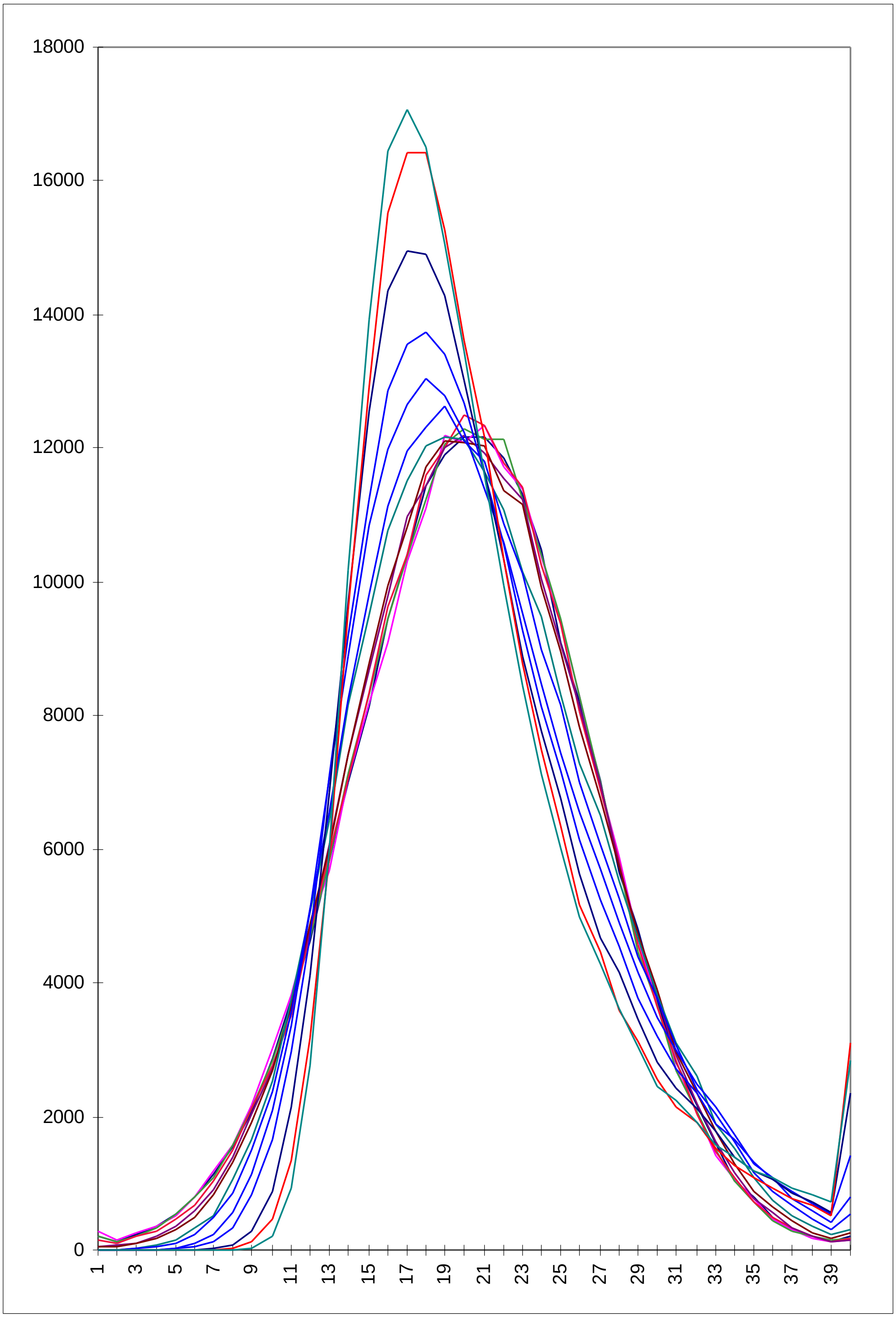


Table 8. Percentage share of stochastic forecasts that were lower than the deterministic forecast in the second experiment (15000 replications)

Variable name	95:1	98:1	2001:1
Real GDP	16.43	28.89	28.25
Nominal GDP	17.72	28.35	31.04
Real private consumption	42.74	40.60	40.92
Nom. private consumption	45.32	35.66	35.23
Consumer prices	56.49	32.00	34.80
Real fixed investment	48.01	36.09	33.83
Real exports	16.54	26.90	35.33
Real imports	44.70	42.64	50.48
Employment	23.84	25.88	29.00
Wage rate in industry	53.27	39.42	41.67
Short interest rate	24.48	32.00	40.70
Effective exchange rate	75.72	70.46	76.04
Real value added in ind.	32.54	45.11	49.62
Real value added in other business	15.14	23.95	23.42
Capacity utilisation in ind.	39.41	50.70	55.53

In the first experiment, the relative bias (i.e. the ratio of the mean stochastic forecast to the deterministic forecast) and the standard deviation of the stochastic forecasts were very small. Error accumulation was practically inconsequential. The investigated sampling distributions of the forecasts were - upon visual inspection - close to symmetry and normality. The equations for fixed investment and exports of services appeared to be the largest sources of forecast uncertainty.

In the second experiment, the relative biases and standard deviations were much larger but (perhaps with the exception of fixed investment) still not excessive as indicators of forecast uncertainty. There was much more error accumulation and the sampling distributions in many cases were clearly asymmetric. This asymmetry had been introduced through the experimental design, since both the endogenous variables (appearing on the right-hand side of model equations) and their coefficients were subject to random shocks. The results were based on random shock distributions that (most probably) were effectively trimmed, since shocks that resulted in non-convergent solutions were replaced by new (random) shocks.

The results of our study indicate that, *as long as model coefficients are taken as given* deterministic forecasts with KOSMOS can readily be used to predict the mean values of the model's dependent variables. There is little uncertainty connected with random disturbances and no excessive error accumulation.

When uncertainty connected with the coefficient estimates is also to be allowed for, a more adequate dispersion measure than the standard error of the coefficient estimate is required. Short samples and poor data quality often lead to coefficient values being set on other grounds than pure statistical estimation. Then, standard errors of estimate do not reflect the actual uncertainty connected with the coefficient values and can easily exaggerate this uncertainty, affecting the stochastic simulation results.

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STOKASTISKA SIMULERINGAR MED KOSMOS

SVENSK SAMMANFATTNING

De stokastiska egenskaperna hos den ekonometriska modellen KOSMOS har undersökts i två experiment. Båda involverade ett stort antal replikationer. I det första experimentet adderades en normal slumpvariabel (som representant för feltermen) till fjorton beteendeekvationer. I det andra experimentet blev även regressionskoefficienterna (med undantag för intercept och dummy-variabler) föremål för additiva chocker.

Den genomsnittliga stokastiska prognosen *i det första experimentet* avvek mycket litet från den deterministiska prognosen. Den stokastiska prognosens standardavvikelse var liten och praktisk taget ingen felkumulering kunde observeras. De stokastiska prognosernas fördelningar såg symmetriska och klockformade ut. Analysen indikerar att ekvationerna för fasta investeringar och tjänsteexport utgör den största källan till prognososäkerhet.

I det andra experimentet var skillnaderna mellan den genomsnittliga stokastiska prognosen och den deterministiska prognosen, liksom prognosstandardavvikelserna, mycket större. De bedömdes dock fortfarande som acceptabla mått på prognososäkerhet. Mycket mer felkumulering kunde dessutom observeras och vissa prognosfördelningar var klart asymmetriska. Denna asymmetri berodde på att både endogena variabler och deras koefficienter (när variabler ifråga förekom i högra ledet) blev föremål för slumpmässiga chocker.

Enligt våra resultat kan deterministiska prognoser med KOSMOS användas som prediktorer för de endogena modellvariablernas medelvärden *så länge modellekvationerna betraktas som givna*. Vill man dessutom ta hänsyn till osäkerheten förknippad med koefficientskattningarna, behövs det ett osäkerhetsmått som tar hänsyn till de subjektiva bedömningar som utgör en del av skattnigsprocessen.