

MAMTAX.

A DYNAMIC CGE MODEL FOR TAX REFORM SIMULATIONS

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# 1 INTRODUCTION AND AN OVERVIEW OF THE PAPER

In this paper a small intertemporal perfect foresight general equilibrium tax model is formulated. The model is an extension of the model used by Persson and Svensson [1987] in the Swedish medium-term survey 1987. The main contribution of this paper is the endogenous labour supply and the richer and variable tax structure. Numerical simulations are performed to investigate possible consequences of a tax reform in line with the proposals of the Swedish Income Tax Committee [1989].

Analysis of a large tax reform will probably require a general equilibrium approach. Otherwise a number of important side effects will be disregarded. But most CGE models used for tax analysis are static. There is little doubt that static models are useful for studying eg reallocation of capital and distributional effects of tax reforms - often in a rather disaggregated way. But the static approach is by definition less suitable for discussing saving and investment decisions. The comparative advantage of the model in this paper is a rich dynamic structure, which makes it possible to study the following aspects of a tax reform:

- adjustment paths to a new steady state
- effects on intergenerational distribution
- pre-announcement effects

## 1.1 AN OVERVIEW OF THE MODEL

The model is a highly aggregated dynamic general equilibrium model of a small open economy. The behaviour is based on optimal choices by representative agents with perfect foresight. It is a model in the classical tradition where all markets clear in every period.

The overlapping generations of households (section 2.2) receive transfers from the government, supply labour, consume goods, pay taxes and save. The firms (section 2.3) have adjustment costs for capital, which give rise to smooth transition paths for the capital stock, governed by a "q" investment function. Apart from investing, the firms hire labour and pay payroll taxes. The government (section 2.4) collects taxes on wage payments, wage income, capital income and consumption and it spends on consumption and transfers. Assets (section 2.5) can be freely traded across the border at the given world market interest rate. On the other hand, migration is exogenous. Lists of the model equations and variables are found in appendices 2A and 2B.

The framework for the analysis of the effects of different tax regimes are laid out in section 3, which discusses how different taxes and transfers affect the households.

The model presented in this paper is formulated to converge to a steady state growth path if the parameters (described in section 4) are kept constant long enough. Thus, it is possible to compare steady state solutions for different tax regimes - which is exactly what is done in many static models. It is of interest to compare the steady state effects of a tax reform with the results from a full dynamic simulation in order to see whether the latter has very much more to say. The method for solving the model is described in appendix 4A.

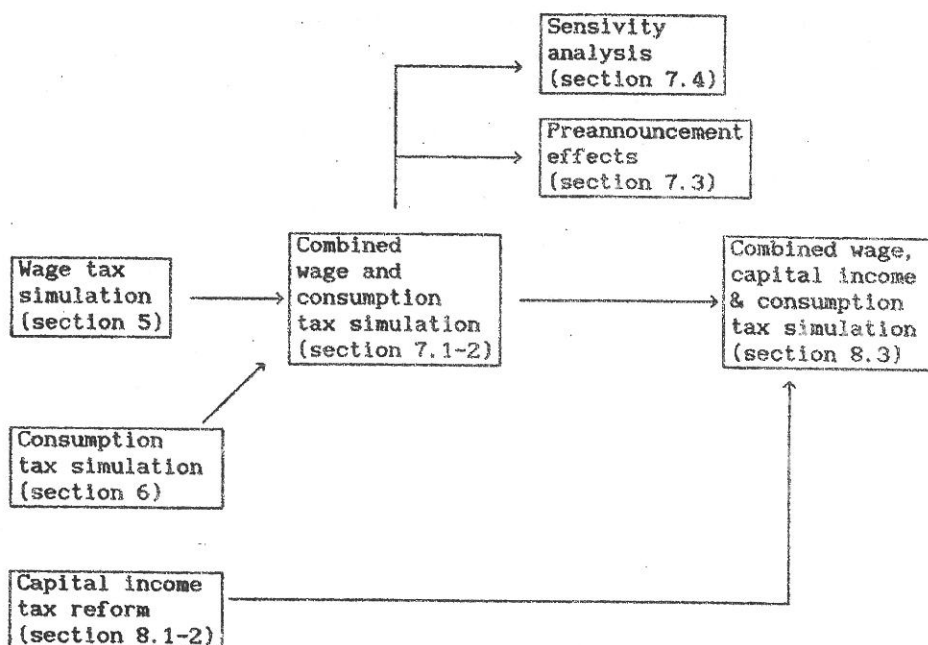
One of the first intertemporal perfect foresight CGE models used in tax reform analysis was developed by Auerbach and Kotlikoff [1983]. It is a model of a closed economy and contains one production sector (without installation costs) and 55 overlapping generations with exogenous labour supply. In later versions of the model<sup>1</sup> the labour supply is endogenous. They simulated the effects of switching from the US tax regime to a consumption tax regime. Bovenberg [1985] developed and simulated a small model of a closed economy with infinitely lived households with exogenous labour supply and adjustment costs for the capital stock. He studied the importance of the adjustment costs for the welfare effects of a switch to a consumption tax regime. Andersson and Norrman [1987] is the only example of a related model that has been calibrated for Sweden. They modelled a semi-open economy with two production sectors and with adjustment costs for capital. One sector is "highly taxed" (normal production), while the other is "lightly taxed" (housing, household activities and leisure). This formulation makes it possible to study the effects on the allocation of capital and labour of a new capital income tax structure. On the other hand, they model the households with infinite length of life, which makes it impossible to draw any conclusions about intergenerational distribution.

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<sup>1</sup>See eg Auerbach, Kotlikoff and Skinner [1983] and Auerbach and Kotlikoff [1987].

## 1.2 AN OVERVIEW OF THE SIMULATIONS

Section 5-8 are devoted to the following question: what can be learnt about a balanced budget (through lump sum taxes) tax reform from a dynamic CGE model? The answer is provided in the form of a number of simulations of stylized tax reforms. The relation between the simulations is illustrated by the following figure



The combined wage and consumption tax simulation in section 7.1-2 is an attempt to study the main part of the tax reform proposal of the Swedish Income Tax Committee [1989]. From a practical point of view, this section is the most important. The two preceding sections (the wage tax simulation in section 5 and the consumption tax simulation in section 6) constitute the runaway for the combined simulation.

Since the wage tax simulation in section 5 turns out to be a simplified approximation of the combined simulation, this section is used for establishing and analyzing the most important results of these simulations. The working of the model is best understood by reading this section.

Section 7.3 studies the effects of pre-announcement of the combined wage and consumption tax reform. This kind of analysis is a privilege of the dynamic

modelling approach and is an area that has received little attention. In section 7.4, the robustness of the results from the simulations in section 7.1-3 are illustrated by varying certain key parameters.

The last section is an attempt to illustrate how the results in section 5-7 are altered when the capital income taxation is changed in stylized way to capture the proposal of the Swedish Income Tax Committee [1989]. Section 8.1-2 presents a stylized capital income tax reform simulation which might capture some of the possible effects of the proposal, namely the overall increase in capital income tax rates. Finally, in section 8.3 the combined wage and consumption tax simulation from section 7.1-2 is brought together with the capital income tax simulation in section 8.1-2 to form a stylized total income tax reform simulation.

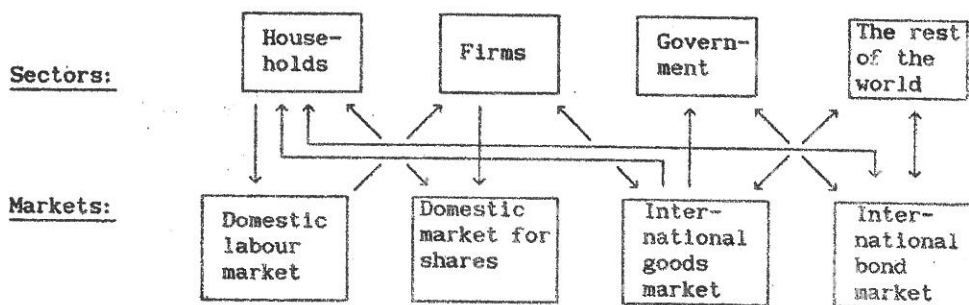
## 2 THE MODEL

This section describes the derivation of the model. The model equations and variables are found in appendices 2A and 2B.

### 2.1 DESCRIPTION OF THE MODEL

#### 2.1.1 The market structure

The following figure illustrates the interaction between the 4 (5) markets and the 4 sectors in the model:



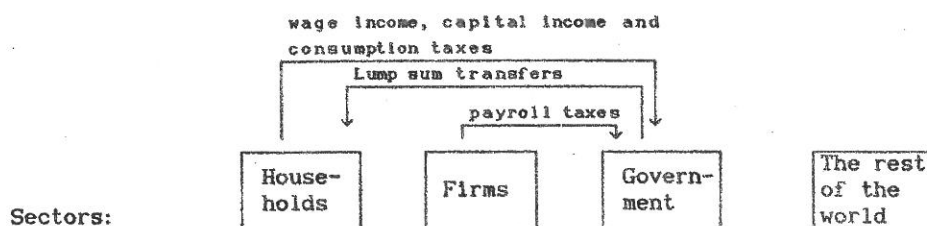
- The households supply and the firms demand labour on the domestic labour market. The government does not demand labour since all domestic production is assumed to take place in the firms. The wage rate equates supply and demand.
- The households supply and demand shares on the domestic market for shares. Hence, the firms are assumed to be owned by the domestic households. Investments are made out of retained earnings and the remaining profits are distributed to the shareholders.
- The households and the government demand goods, while the firms and "the rest of the world" demand/supply goods from/to the goods market at a given world market price. There are no intermediate goods in the model. Hence, the firms do only demand goods for investments. This market could be divided into a market for the good produced abroad (with a given price) and a market for the domestically produced good. In that case, it is convenient to express all values in terms of the foreign good. For

future reference, this is the chosen convention in the derivation of the model, but in the simulations there is only one good whose pretax price is normalized to unity.

- The households, the government and the rest of the world demand and supply bonds on the international bond market at a given world market interest rate. Since the firms finance investments out of retained earnings and distribute the rest of the profits, the firms do not engage themselves on the bond market.

### 2.1.2 The tax and transfer structure

The following figure illustrates the flows of taxes and transfers



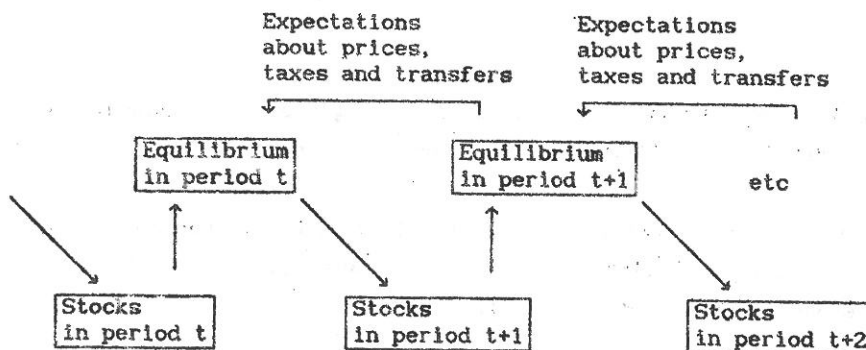
The government receives wage income, capital income and consumption taxes from the households and payroll taxes from the firms. Finally, it pays lump sum transfers to the households. The rest of the world is not affected by domestic taxes or transfers.

### 2.1.3 The dynamic structure

The agents are assumed to have perfect foresight about almost all variables<sup>2</sup>. The only exception is that the individual household has an uncertain life time. The dynamic aspects of the households stem from the fact that households generally live more than one period and are able to lend/borrow on a capital market. Hence, the path of consumption can be separated from the path of earnings. For the firms, the dynamics come from the combination of convex adjustment costs for capital and less than total depreciation of the capital stock. As a result, it is profitable to look ahead and spread out investments

<sup>2</sup>The importance of the degree of foresight is studied in Ballard and Goulder [1985].

during long periods<sup>3</sup>. For both households and firms, the dynamic structure can be described by the following picture



The stock (of assets or capital) in period  $t$  is given by history. Together with expectations about the future, a temporary equilibrium is established. The intertemporal equilibrium is characterized by the fact that the expectations are correct, i.e. identical to the values in the sequence of temporary equilibria.

## 2.2 HOUSEHOLDS

The households are modelled by Yaari-Blanchard's<sup>4</sup> approach to overlapping generations. The model is an extension of a model used by Persson and Svensson [1987]. The main developments in this section are the endogenous labour supply and the richer and variable tax structure.

### 2.2.1 The objective function

The representative household in generation  $j$ <sup>5</sup> maximizes the expected life time utility of consumption of goods and leisure

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<sup>3</sup>In fact, there is a third source of dynamics since the government consumption is assumed to be affected by the size of the government debt.

<sup>4</sup>See eg Blanchard [1985]

<sup>5</sup>The subscript indicating generation  $j$  is dropped in order to simplify notation.

$$E_t \sum_{s=t}^{\infty} \left( \frac{1}{1+\rho} \right)^{s-t} U(C_s, \bar{L}_s - L_s) \quad (1)$$

where  $\rho$  is the time preference rate, and  $C_s$ ,  $\bar{L}_s$  and  $L_s$  are the consumption of goods<sup>8</sup>, labour endowment and labour supply in period  $s$ , respectively.

The household has the probability  $\pi$  to survive into the next period, giving an expected life time of  $1/(1-\pi)$ . With perfect foresight about all other variables the expected life time utility simplifies to

$$\sum_{s=t}^{\infty} \left( \frac{\pi}{1+\rho} \right)^{s-t} U(C_s, \bar{L}_s - L_s) \quad (1')$$

The utility function is assumed to have constant intertemporal elasticity of substitution ( $\sigma$ )

$$U(C_s, \bar{L}_s - L_s) = \begin{cases} \frac{U_s^{1-1/\sigma}}{1-1/\sigma} & \text{if } \sigma \neq 1 \\ \ln(U_s) & \text{otherwise} \end{cases} \quad (2a)$$

where  $U_s$  is an (utility weighted) index of composite consumption in period  $s$ , i.e. of both goods ( $C_s$ ) and leisure ( $\bar{L}_s - L_s$ ). The index  $U_s$  is the value of the CES subutility function

$$U_s = \left[ C_s^{1-1/\eta} + (\theta \lambda^{\eta} (\bar{L}_s - L_s))^{\frac{1}{1-1/\eta}} \right]^{\frac{1}{1-1/\eta}} \quad (2b)$$

where  $\eta$  is the atemporal elasticity of substitution between consumption of goods and leisure and  $\theta$  is the weight of leisure. In order to get a constant

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<sup>8</sup>In the case with several goods,  $C$  is a utility weighted index.



labour supply in an economy with growth, it is necessary to assume either a separable subutility function<sup>7</sup> (eg Cobb-Douglas) or a "productivity" growth ( $\lambda$ ) in leisure which is equal to the growth rate of the wage rate. The second approach is chosen here.

The homothetic specification of the subutility function makes it possible to interpret the normalized expenditure function as a price index of total consumption

$$p_s^u = \left[ (p_s^c)^{1-\eta} + \left( \frac{w_s^n}{\theta \lambda^u} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (2c)$$

where  $p_s^c$  is the price of consumption of goods<sup>8</sup> and  $w_s^n$  is the net of tax wage rate.

By the properties of the expenditure function, we have

$$C_s = \frac{\partial p_s^u}{\partial p_s^c} U_s, \quad \bar{L}_s - L_s = \frac{\partial p_s^u}{\partial w_s} U_s$$

For the subutility function (2b) this gives

$$C_s = \frac{(p_s^c)^{-\eta} p_s^u U_s}{(p_s^c)^{1-\eta} + \left( \frac{w_s^n}{\theta \lambda^u} \right)^{1-\eta}}, \quad \bar{L}_s - L_s = \frac{\left( \frac{w_s^n}{\theta \lambda^u} \right)^{-\eta} p_s^u U_s / \theta \lambda^u}{(p_s^c)^{1-\eta} + \left( \frac{w_s^n}{\theta \lambda^u} \right)^{1-\eta}} \quad (2d-e)$$

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<sup>7</sup>See eg King, Plosser and Rebelo [1988]

<sup>8</sup>In the case with several goods  $p^u$  is the normalized expenditure function for the subutility function over the goods. With only one good, the pretax prices can be normalized to unity and  $p^c=1 + \text{rate of consumption tax}$ .

### 2.2.2 The budget constraint

The *Dynamic* budget constraint for a household is

$$A_{t+1} = \frac{(1+r_{t+1})}{\pi} \left[ A_t + w_t^n \bar{L}_t + S_t - p_t^u U_t \right] \quad (3)$$

where  $A$  are private assets,  $r$  net of tax interest rate and  $S$  transfers from the government. Thus,  $A_{t+1}$  are defined as assets in period  $t+1$  inclusive of interest payments received in the beginning of  $t+1$ .

Note that the household has the return  $(1+r_{t+1})/\pi - 1$  on their assets. This is so because the households sign life insurance contracts. These contracts stipulate that

- A household leaves all its assets to the insurance company in case of death. Thus<sup>9</sup>, the company receives  $(1-\pi)(A_t + w_t^n \bar{L}_t + S_t - p_t^u U_t)$  in the beginning of period  $t+1$ , which immediately gives interest payments of the amount  $r_{t+1}(1-\pi)(A_t + w_t^n \bar{L}_t + S_t - p_t^u U_t)$
- A household receives a "premium" on the assets (added to the interest rate) if it survives. The outlay of the insurance company is thus  $\pi(A_t + w_t^n \bar{L}_t + S_t - p_t^u U_t)$  times the "premium". With zero profit this premium turns out to be  $(1+r_{t+1})(1-\pi)/\pi$ . Adding the "premium" to the gross interest rate  $(1+r_{t+1})$  gives  $(1+r_{t+1})/\pi$ , which is the effective discount rate for a household.

Through iterated recursion on (3) we have

$$A_t = \frac{A_{t+j+1}}{\prod_{s=t+1}^{t+j+1} \alpha_s} - \sum_{s=t}^{t+j} \frac{w_s^n \bar{L}_s + S_s - p_s^u U_s}{\alpha_t \prod_{v=t}^s \alpha_v} \quad (4)$$

where  $\alpha_t = (1+r_t)/\pi$

Let  $j \rightarrow \infty$  and postulate that assets grow at a slower rate than the interest

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<sup>9</sup>For convenience it is assumed that the size of the generation in period  $t$  is equal to one.

rate in the limit<sup>10</sup>. Then the first term on the RHS goes towards zero.

Furthermore, define human capital as the discounted sum of expected "full income" (transfers plus the value of labour endowment)<sup>11</sup>

$$H_t = \sum_{s=t}^{\infty} \frac{w_s \bar{L}_s + S_s}{\alpha_t \prod_{v=t}^s \alpha_v} \quad (5)$$

The intertemporal budget constraint is then

$$\sum_{s=t}^{\infty} \frac{p_s^u u_s}{\alpha_t \prod_{v=t}^s \alpha_v} = A_t + H_t \quad (6)$$

which says that the present value of composite consumption equals total wealth, i.e. the sum of private assets and human capital.

### 2.2.3 The maximization problem

The household maximizes (1') subject to the constraints (2a-c) and (6).

The first order conditions are

$$u_s = \lambda_t \left( \frac{\pi}{1+\rho} \right)^{(s-t)\sigma} \left[ \frac{p_s^u}{\alpha_t \prod_{v=t}^s \alpha_v} \right]^{-\sigma} ; s=t, t+1, t+2, \dots \infty \quad (7)$$

and the budget constraint (6).

$\lambda_t$  is the Lagrange multiplier for the constraint (6). The F.O.C can be shown to give the following function for the value of composite consumption

<sup>10</sup>This is usually called a "No Ponzi game" condition.

<sup>11</sup>It is possible to let the productivity and hence the wage rate of a worker be age dependent, to mimic a freely chosen retirement. See eg Blanchard [1985].

$$p_t^u U_t = \Delta_t (A_t + H_t) \quad (8)$$

where

$$\Delta_t = \frac{1}{\sum_{s=t}^{\infty} \left( \frac{\pi}{1+\rho} \right)^{(s-t)\sigma} \frac{1}{\left[ \frac{1}{\alpha_t} \prod_{v=t}^s \alpha_v \right]^{1-\sigma}} \left( \frac{p_s^u}{p_t^u} \right)^{1-\sigma}} \quad (9)$$

#### 2.2.4 Equivalent variation

The indirect utility function  $v(\cdot)$  corresponding to the maximization problem in section 2.2.3 can be shown to be<sup>12</sup>

$$v(\{r, p^c, w^n\}_t^\infty, A_t, H_t) = \left[ \frac{A_t + H_t}{p_t^u} \right]^{1-1/\sigma} \Delta_t^{-1/\sigma} \quad \text{if } \sigma \neq 1 \quad (10)$$

A monotonic transformation [multiply with  $1-1/\sigma$  and raise to the power of  $\sigma/(\sigma-1)$ ] gives

$$v(\{r, p^c, w^n\}_t^\infty, A_t, H_t) = \frac{A_t + H_t}{p_t^u \Delta_t^{1/(\sigma-1)}} \quad (10')$$

The expenditure function  $e(\cdot)$  is the inverse of (10')

$$e(\{r, p^c, w^n\}_t^\infty, U_t) = p_t^u \Delta_t^{1/(\sigma-1)} U_t \quad (= A_t + H_t) \quad (11)$$

Where  $U_t$  is the value of  $v(\cdot)$  in (10'). Assume that a new tax schedule is implemented, giving wealth  $(A_t + H_t)'$ , prices  $\{r, p^c, w^n\}_t^\infty$  and utility  $U_t'$ . Equivalent variation is then

$$EV_t = e(\{r, p^c, w^n\}_t^\infty, U_t') - e(\{r, p^c, w^n\}_t^\infty, U_t) =$$

<sup>12</sup>The case with  $\sigma=1$  is somewhat messier and will not be shown or used in the paper.

$$\frac{p_t^u \Delta_t^{1/(\sigma-1)}}{(p_t^u \Delta_t^{1/(\sigma-1)})'} * [A_t + H_t]' - [A_t + H_t] \quad (12)$$

Rewriting the definition of equivalent variation yields

$$\frac{{}_jEV_s}{{}_jA_s + H_s} = \frac{p_s^u \Delta_s^{1/(\sigma-1)}}{(p_s^u \Delta_s^{1/(\sigma-1)})'} * \frac{({}_jA_s + H_s)'}{{}_jA_s + H_s} - 1 \quad (12')$$

which is the equivalent variation as a percentage of total wealth for a household in generation  $j$ . In the simulations this measure will be calculated in the first simulation year ( $t=0$ ) or the year of birth, whichever is last. Note that, that the young generations (born  $t \geq 0$ ) do not have any private assets ( $A$ ) in this measure. Assuming that the economy was on a steady state growth path before the reform, it is possible to determine the amount of private assets in  $t=0$  ( ${}_jA_0$ ) for each old generation.

#### 2.2.5 Aggregation of household behaviour

The population consists of an infinite number of cohorts, each of the same initial size  $(1-\pi)^{13}$ . In period  $s$  the size of the generation born in period  $t$  is  $(1-\pi)\pi^{s-t}$ . Hence, the total population is 1. Since composite consumption is linear in total wealth, aggregation is straightforward. A description can be found in Blanchard [1985] (continuous time) and Person and Svensson [1987] (discrete time).

One thing deserves to be mentioned, namely the difference between the rate of return for households and generations. The rate of return for a household is  $(1+r_{t+1})/\pi$  since the surviving members of a generation receive a "premium". As pointed out in section 2.2.2, the total of these "premiums" is equal to the value of the assets of the deceased members of the same generation. Thus, for the generation as a whole "premiums" and bequests to the insurance company balance. Hence, the effective rate of return of assets is  $1+r_{t+1}$  on the aggregated level. In spite of this,  $(1+r_{t+1})/\pi$  is still the correct discount factor in the definition of human capital. This is so because human capital cannot be left as bequest to the insurance company.

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<sup>13</sup>It is possible to let the size of different cohorts and hence the population vary.

### 2.2.6 What are the characteristics of this model of households?

- The finite expected life time  $1/(1-\pi)$  diminishes the scope for intertemporal substitution.
- The constant probability of survival ( $\pi$ ) means that all generations have the same expected remaining life time. Hence, the propensity to consume out of wealth is the same for all generations. This makes aggregation straightforward.
- New generations are born each year and there are no bequests. Thus, generations currently alive can expect that future taxes to some extent will be paid by others. This means that the "Ricardian equivalence" result does not hold in the model except for the special case when  $\pi=1$ .
- The capital market (including the insurance companies) is perfect, which greatly enhances the possibility of intertemporal substitution.

## 2.3 FIRMS

The representative competitive firm maximizes the present value of the cash flow under convex installation costs for capital, taking prices as given on all markets. This gives a "q" -equation for investment.

The model is an extension of Persson and Svensson [1987]. The most important developments are the variable after tax interest rates and the installation cost function that is homogeneous of degree zero in investment and capital stock. The latter draws on Hayashi [1982].

### 2.3.1 The arbitrage condition

For a domestic investor the following arbitrage condition must be fulfilled

$$\left[1 + (1 - \tau_{t+1}^k) r_{t+1}^*\right] (1 - \tau_t^k) D_t + (1 - \tau_{t+1}^k) (V_{t+1} - V_t) = r_{t+1}^* (1 - \tau_{t+1}^k) V_t \quad (13)$$

The LHS is the net of (capital income) tax return of buying shares  $V_t$  in period  $t$ . This is the immediate after tax dividends  $(1 - \tau_t^k) D_t$  which is invested in bonds, giving the return  $(1 - \tau_{t+1}^k) r_{t+1}^*$  in period  $t+1$  plus the after tax capital gain in  $t+1$   $[(1 - \tau_{t+1}^k) (V_{t+1} - V_t)]$ . This must be equal to buying bonds for the same amount ( $V_t$ ) and receiving after tax interest payments equal to  $r_{t+1}^* (1 - \tau_{t+1}^k) V_t$  (the RHS).

Assuming that  $V$  grows at a slower rate than the interest rate in the limit, repeated recursion on (13) gives

$$V_t = \sum_{s=t}^{\infty} \frac{\varphi_s D_s}{\prod_{v=t}^s \psi_v} \quad (14)$$

where  $\psi_t = 1+r_{t+1}^*$  and  $\varphi_t = \frac{[1+(1-\tau_{t+1}^k)r_{t+1}^*](1-\tau_t^k)}{(1-\tau_{t+1}^k)}$

The value of the firm ( $V$ ) is the net of tax present value of current and future dividends (cash flow).

### 2.3.2 The maximization problem

The dividends  $D_t$  are

$$D_t = p_t F_t(K_t, L_t) - p_t^i i_t \left[ 1 + \Gamma(1/K_t) \right] - w_t^c L_t \quad (15)$$

where  $p^{14}$  is the product price,  $F(K, L)$  the production function with capital stock ( $K$ ) and labour (hours) ( $L$ ) as arguments,  $p^i$  the price of investment goods,  $i$  physical investment,  $\Gamma(1/K)$  the convex installation cost function and finally  $w^c$  is labour cost per hour. The formulation of the installation cost function makes overall production technology (i.e. production function and adjustment cost function taken together) linearly homogeneous in  $(L, K, i)^{15}$ . It does also reflect the fact that investments are financed out of retained earnings.

The capital stock evolves according to

$$K_{t+1} = i_t + (1-\delta)K_t \quad (16)$$

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<sup>14</sup>As before, the price indices are used for future reference. In the one good case used in the paper all pretax prices are normalized to unity.

<sup>15</sup>This specification differs from Persson and Svensson [1987] who use  $\Gamma_t(i_t) = \gamma i_t$ . Their formulation does effectively mean that there is decreasing returns to scale in the overall production technology as pointed out by Persson and Svensson [1987].

where  $\delta$  is the depreciation rate.

Maximizing (14) subject to (15)-(16) gives the following first order conditions<sup>18</sup> for  $s=t, t+1, \dots, \infty$

$$p_s F_{sL} - w_s^c = 0 \quad (17a)$$

$$\frac{\varphi_{s+1}}{\psi_{s+1}} \left[ p_{s+1} F_{s+1, K} - p_{s+1}^1 i_{s+1} \Gamma_{s+1, K} + q_{s+1} (1-\delta) \right] - \varphi_s q_s = 0 \quad (17b)$$

$$-p_s^1 \left[ 1 + \Gamma_s + i_s \Gamma_{s, 1} \right] + q_s = 0 \quad (17c)$$

and equation (16)

The shadow price ( $q_s$ ) of installed capital (the multiplier of the restriction (16)) is a parallel to Tobin's marginal (and average) "q".

### 2.3.3 Solution using explicit functional forms

The installation cost function is

$$\Gamma(i_s/K_s) = \frac{\gamma i_s}{2K_s} \quad (18)$$

Together with the F.O.C this gives the investment function

$$i_s = \left( q_s / p_s^1 - 1 \right) \frac{K_s}{\gamma} \quad (19)$$

and  $q$  evolves according to (17b)

A linearly homogeneous CES production function is assumed

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<sup>18</sup>Partial derivatives are denoted by subscripts.



$$F_t(K_t, L_t) = z_t \left\{ \alpha K_t^{1-1/\beta} + (1-\alpha) \left[ \lambda^t L_t \right]^{1-1/\beta} \right\}^{\frac{\beta}{\beta-1}} \quad (20)$$

where  $z$  is total factor productivity,  $\beta$  the elasticity of substitution,  $\alpha$  a distribution parameter and  $\lambda$  the labour augmenting technological growth.

Using (20) and (18) in (17b) gives together with (19) a description of investment behaviour. Employment is found by using (20) in (17a)<sup>17</sup>. The aggregation of the representative competitive firms is straightforward.

#### 2.3.4 A note on the recursivity of the model

The model has a perfectly elastic supply of capital at the given world market interest rate ( $r^*$ ) and a linearly homogeneous overall production technology in the long run<sup>18</sup>. With the world market interest rate equal to an adjusted value of marginal product of capital<sup>19</sup>, the capital-labour ratio and thus the gross wage are defined implicitly in the long run. The long run effect of domestic factors is only to determine the level of production through the labour supply.

#### 2.4 GOVERNMENT AND TAXES

The government spends money on consumption, transfers, interest payments on the debt and it collects taxes.

The value of government consumption ( $p^g G$ ) is an exogenous fraction ( $\xi^g$ ) of production net of installation costs (which corresponds to gross domestic product). In order to make the government obey an intertemporal budget constraint of the same type as for the households, the government consumption is

<sup>17</sup>Using the equilibrium condition on the domestic labour market, equation (17a) can be used to solve labour cost per hour ( $w^c$ ).

<sup>18</sup>The production function is hom(1) in  $(K, L)$  and the installation cost function is hom(0) in  $(K, I)$ .

<sup>19</sup>The exact expression is  $1+r_{s+2}^* = [p_{s+1} F_{s+1, K} - p_{s+1}^I i_{s+1} \Gamma_{s+1, K} + (1-\delta)q]/q$ , which is homogeneous of degree 0 in  $(K, L, I)$ .

further assumed to be affected by the debt<sup>20</sup>. Government consumption does not give any utility to the households.

$$p_t^g G_t = \xi_t^g \left[ p_t Y_t - p_t^1 i_t \Gamma_t (i_t / K_t) \right] - \xi_t^b B_t \quad (21)$$

where  $p^g$  is the price of government consumption.

Government transfers is assumed to be a fraction ( $\xi^s$ ) of net production as well.

$$S_t = \xi_t^s \left[ p_t Y_t - p_t^1 i_t \Gamma_t (i_t / K_t) \right] \quad (22)$$

Taxes are levied on wages with the wage income tax rate  $\tau^w$  and pay roll tax rate  $\tau^a$ , on private consumption of goods and capital income with the tax rates  $\tau^c$  and  $\tau^k$ , respectively. The total tax revenues are

$$TAX_t = (\tau_t^w + \tau_t^a) w_t L_t + \tau_t^k r_t^* \frac{A_t}{1 + r_t} + \tau_t^c p_t^{cc} C_t \quad (23)$$

where  $w$  is wage rate before taxes, making  $w^n = w(1 - \tau^w)$  (cf eq 2c), and  $w^c = w(1 + \tau^a)$  (cf eq 15).  $p_t^{cc}$  is the the price of consumption goods before taxes, making  $p_t^c = (1 + \tau_t^c) p_t^{cc}$  (cf eq 2c). As before  $r^*$  is the world market interest rate, making  $r = (1 - \tau^k) r^*$  (cf eq (3)).

Government debt evolves according to

$$B_{t+1} = (1 + r_{t+1}^*) (B_t + p_t^g G_t + S_t - TAX_t) \quad (24)$$

## 2.5 NATIONAL ACCOUNTS

Foreign assets are defined by

$$FA_t = A_t - V_t - B_t \quad (25)$$

The firms ( $V_t$ ) are assumed to be owned by the domestic households. Thus all capital gains or losses ( $V_t - V_{t-1}$ ) fall on  $A_t$ .  $FA_t$  and  $B_t$  are predetermined variables, ie determined in period  $t-1$ , while  $V_t$  and hence  $A_t$  are a forward

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<sup>20</sup>See Persson and Svensson [1987] for further details.

looking variables that can jump in reaction to new information.

## 2.6 A NOTE ON THE MODEL VERSION USED IN THE SIMULATIONS

The prices  $p$ ,  $p^c$  and  $p^g$  are the pretax price (in terms of the foreign goods) of production, private consumption and government consumption, respectively. In the simulations, there is only one good so the pretax prices can all be normalized to unity. Furthermore, the government's budget is always kept balanced and the government's debt ( $B$ ) is accordingly zero. Instead, transfers ( $S$ ) are calculated endogenously to keep the government's budget balanced.

### 3 HOW DO TAXES AFFECT HOUSEHOLD BEHAVIOUR?

This section describes how changes in the tax rates affect household behaviour.

#### 3.1 INTERTEMPORAL SUBSTITUTION

From the FOC (7) we have the relation between composite consumption ( $U$ ) in subsequent periods

$$\frac{U_{s+1}}{U_s} = \left( \frac{1+r_{s+1}}{1+\rho} \right)^\sigma \left( \frac{p_{s+1}^u}{p_s^u} \right)^{-\sigma} \quad (26)$$

There are a few points to be made about (26):

- if the net of tax interest rate exceeds the time preference rate ( $r_{s+1} > \rho$ ) and prices are constant then the path of composite consumption is rising over time.
- An increase in  $\tau_{t+1}^k$  lowers  $r_{t+1}$  since  $r = (1-\tau^k)r^*$ . The effect is that households substitute current for future composite consumption. Thus, households choose to spend more today since it is less worthwhile to save. Lower saving will of course lead to lower private assets in the future. This effect is larger the higher elasticity of intertemporal substitution ( $\sigma$ ).
- An increase in the ideal price index ( $p^u$ ) over time will give the same type of effect as an increase in  $\tau^k$ . This could be achieved by either raising the consumption tax rate ( $\tau^c$ ) or lowering the wage tax rate ( $\tau^w$ ) over time. The latter can be seen by rewriting the definition of  $p^u$

$$p_s^u = \left[ \left( (1+\tau_s^c) p_s^{cc} \right)^{1-\eta} + \left( \frac{w_s(1-\tau_s^w)}{\theta \lambda^s} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (2c')$$

Though the mechanism is different from the previous example, the ultimate effect is to punish saving. The size of this effect is also governed by  $\sigma$ . If  $0 < \sigma < 1$  ( $> 1$ ) the value of composite consumption will increase (decrease) in response to an increase in  $p^u$ . But as long as  $\sigma > 0$   $U_s$  will be substituted for  $U_{s+1}$ .

### 3.2 ATEMPORAL SUBSTITUTION

From (2d-e) we have that

$$\frac{C_s}{\bar{L}_s - L_s} = \left[ \frac{(1+\tau_s^c)p_s^{cc}}{w_s(1-\tau_s^w)/(\theta\lambda^s)} \right]^{-\eta} \theta\lambda^s \quad (27)$$

In steady state the price of consumption of goods  $[(1+\tau^c)p^{cc}]$  is constant, while  $w(1-\tau^w)$  has a rate of growth equal to  $\lambda$  (the rate of growth in productivity). It can be shown that the value of composite consumption ( $p^u U$ ) also has the rate of growth  $\lambda$  in steady state. According to (2d-e) consumption of goods (C) will then grow at the rate  $\lambda$  too, while leisure ( $\bar{L}-L$ ) is constant.

We note from (27) that

- an increase in consumption tax ( $\tau^c$ ) or in the wage income tax ( $\tau^w$ ) will lower the ratio between consumption of goods and leisure. This effect is stronger the greater the elasticity of substitution  $\eta$  is. The ratio of the money value of consumption of goods and leisure will increase (decrease) if  $0 < \eta < 1$  ( $> 1$ ) as a response to an increase in  $\tau^c$  or  $\tau^w$ . As long as  $\eta > 0$  there will be substitution of leisure for consumption of goods.

### 3.3 EFFECTS ON THE BUDGET CONSTRAINT

Using (5) the budget constraint (6) can be written

$$A_t = \sum_{s=t}^{\infty} \frac{(1-\tau_s^w)w_s\bar{L}_s + S_s - p_s^u U_s}{\frac{\pi}{1+r_t} \prod_{v=t}^s (1+r_v)/\pi} \quad (6')$$

A few points can be made

- Changes in the level or path of transfers (S) (can also be thought of as lump sum taxation) does not trigger off intertemporal substitution as seen from (26). But the higher and the earlier the transfers are, the higher is the general level of  $p^u U$ . This carries over to composite consumption (U).

- The lower and the later the wage income tax ( $\tau^w$ ) is (affecting the value of the labour endowment  $[(1-\tau^w)wL]$  and the ideal price index ( $p^u$ )), the higher is the general level of composite consumption ( $U$ ). The same is true about the consumption tax ( $\tau^c$ ) (which only affects  $P^u$ ).
- If private assets ( $A$ ) are positive an increase in the ideal price index ( $p^u$ ) through raising the consumption tax rate ( $\tau^c$ ) has an element of surprise wealth taxation. Consider an unexpected increase in  $\tau^c$ ; it is obvious that for old generations it would have been preferable to spend more before the increase in  $\tau^c$  and thus to have accumulated less assets. Instead, doing the best they can given a high  $A_t$  means spending to a higher cost (=tax receipts for the government). A similar argument holds for an increase in capital income tax rate ( $\tau^k$ ).
- Lowering the wage tax rate ( $\tau^w$ ) has two effects. First, it creates a lump sum increase in the value of labour endowment, which is equal for all currently alive households. Second, it reduces the subsidy of leisure. The latter effect does as well as the increase in  $\tau^c$  have an element of surprise wealth taxation, since leisure increases with total assets ( $A+H$ ). If the decrease in  $\tau^w$  had been known in advance, intertemporal substitution would have led to a relatively lower accumulation of assets.

## 4 PARAMETERIZATION

This section describes the parameterization of the model and the resulting reference case. The method for solving the model is described in appendix 4A.

A number of parameters must be chosen in order to solve the model. Swedish empirical studies of relevance are extremely scarce. Even though international studies can be used to some extent, this still leaves us with very little evidence on a number of key parameters. In general, values for such parameters have been chosen so that they will be in line with related models while still making the model solution resemble Swedish data during the 1980's.

### 4.1 PRODUCTION PARAMETERS

1. *The elasticity of substitution  $\beta$  (eq 20<sup>21</sup>).* This is perhaps one of the most investigated structural parameters in economics. Unfortunately, there is also a lot of disagreement. In general, cross section studies arrive at values near 1, while time series studies produce estimates around 0.5. Mansur and Whalley [1986] summarize "central tendency values" from the literature. The overall tendency points towards a value around 0.8.  $\beta=0.8$
2. *The distribution parameter  $\alpha$  (eq 20)* is calibrated to give a labour share (inclusive of payroll taxes) of GDP (net production) equal to 68% which is in line with Swedish data during the 1980's.  $\alpha=0.33$
3. *The labour augmenting technological growth  $\lambda$  (eq 20)* is chosen to match the rate of productivity growth in the Swedish private sector during the 1980's which is around 1.6% .  $\lambda=1.015$
4. *The depreciation rate  $\delta$  (eq 16)* is constructed by calculating an adjusted weighted average of Hansson's [1989] depreciation rates for machinery and buildings respectively, with the shares of the total capital stock as weights. The adjustment amounts to disregarding part of the stock of build-

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<sup>21</sup>The equation numbers refer to the most important appearances of the parameter in question.

ings since the imputed rent of private dwellings probably are underestimated in the National Accounts. The idea is to keep the NA definitions and adjust the composition of the capital stock accordingly. This amounts to give machinery a greater weight and the effect is a higher depreciation rate.  $\delta=0.1$

5. The world market real interest rate  $r^*$  (eq 3&13) is notoriously hard to estimate due to the problem of measuring inflation expectations. Similar models do often produce a value around 5-7% (see eg Auerbach and Kotlikoff [1987] and King, Plosser and Rebelo [1988]).  $r^*=0.067$

6. The adjustment cost parameter  $\gamma$  (eq 18). Given the other parameters,  $\gamma$  is calibrated to give an investment share of 18%, which is in line with Swedish experience during the 1980's. This gives a  $\gamma$  around 10, which is lower than the estimates by Abel and Blanchard [1986].  $\gamma=10$

7. Total factor productivity  $z$  (eq 20).  $z=1$

#### 4.2 HOUSEHOLD PREFERENCE PARAMETERS

8. The intertemporal elasticity of substitution  $\sigma$  (eq 2a). Hall [1988] concludes that  $\sigma$  might be zero and probably not above 0.2, while Grossman and Schiller [1981] tend to favour a value around 0.25. Altonij. [1986] investigated the intertemporal elasticity of labour supply for married men and concluded that it is between 0 and 0.35. Blundell [1988] summarizes studies of micro data and states that  $\sigma$  is probably less than 0.5.  $\sigma=0.333$

9. The probability of survival  $\pi$  (eq 1') is chosen to give an expected remaining life time that is a reasonable average for the entire adult population. This expected remaining life time is assumed to be 33 years.  $\pi=0.97$

10. The atemporal elasticity of substitution (eq 2b) between goods and leisure  $\eta$ . Mankiw, Rotemberg and Summers [1985] cannot arrive at reasonable estimates which is interpreted as violating the new classical model of labour supply. Wales and Woodland [1979] on the other hand get an estimate of  $\eta=0.83$ , which is also the result of Ghez and Becker [1975].  $\eta=0.8$

11. The leisure preference rate  $\theta$  (eq 2b) is calibrated to give an average labour supply of about half of the labour endowment.  $\theta=2.5$

12. The time preference rate  $\rho$  (eq 1). There are few direct observations of



the pure time preference rate. It is calibrated to give a reasonable consumption profile and aggregated savings ratio. To achieve this in spite of the low intertemporal elasticity of substitution ( $\sigma$ ) and a growth ( $\lambda$ ) in real wages,  $\rho$  has to be very low in relation to the after tax interest rate ( $r$ ).  $\rho = -0.01$

#### 4.3 GOVERNMENT PARAMETERS

The tax rates have been calculated by Stefan Akerby, Björn Ohlsson, Thomas Olofsson and Paul Söderlind. In short, the effective tax rates are calculated by relating tax revenues to tax bases that are in accordance with the definitions of the model. The tax rates reflect the situation 1989.

13. The tax rate on labour income  $\tau^w$  (eq 23) and transfer share of GDP  $\xi^g$  (eq 22). The income weighted average of the marginal tax rates and the average tax rates have been estimated to be 52.3% and 36.5%, respectively. To capture the difference between marginal and average tax rates a combination of the marginal tax rate ( $\tau^w$ ) and transfer share of GDP ( $\xi^g$ ) is used.  $\tau^w = 0.5$  and  $\xi^g = 0.09$ .

14. The capital income tax  $\tau^k$  (eq 13823). The taxation of capital income is very heterogeneous. In fact the total capital income tax revenues are negative, while certain assets are very heavily taxed. This model is obviously too aggregated to capture the effects of the capital income tax system at present. Nevertheless, it can provide some ideas about how the general level of capital income taxation affects savings. The choice of  $\tau^k$  is more or less arbitrary.  $\tau^k = 0.2$

15. The consumption tax rate  $\tau^c$  (eq 23) is computed by relating VAT and other taxes on goods minus a range of subsidies to total private consumption.  $\tau^c = 0.22$

16. The payroll taxes  $\tau^a$  (eq 23) is that part of the actual payroll taxes that have little or no relation to individual benefits for the employee. This part can be regarded as pure taxes.  $\tau^a = 0.24$

17. Government consumption  $\xi^g$  (eq 21). Given all the other parameters affecting the government revenues and outlays,  $\xi^g$  is calibrated to give a balanced budget.  $\xi^g = 0.43$

18. The reaction parameter  $\xi^b$  (eq 21). This is chosen large enough to guarantee that the government obeys an intertemporal budget constraint.  $\xi^b=0.1$

#### 4.4 THE REFERENCE CASE

The parameters in section 4.1-3 give rise to a steady state growth path solution, which will be the reference case in all tax reform simulations. The results of the simulations will be reported as percentage differences from this reference case.

In the reference case the labour supply (L) is approximately half of the labour endowment ( $\bar{L}$ ). The average tax rate on labour income

$$\bar{\tau}^w = \tau^w - \frac{S}{wL}$$

is about 0.34 which squares with Swedish data for the 1980's.

The households hold private assets (A) that are 3.6 times gross earnings ( $wL+S$ ), the public debt (B) is zero and foreign debt (-FA) is 1/5 of GDP ( $pY-i\Gamma(1/K)$ ). The first magnitude is indeed very hard to observe due to very poor statistics on private wealth but the figure seems to be in line with Södersten's [1989] assessment. The financial assets of the consolidated public sector has been around zero and the foreign debt 20-25% of GDP during the last years of the 80's.

The investment share of GDP ( $1/(pY-i\Gamma(1/K))$ ) is around 18% and the labour share ( $((1+\tau^s)wL/(pY-i\Gamma(1/K)))$ ) is roughly 68%, which also fits Swedish data for the 1980's. The government consumption share of GDP ( $\xi_t^g$ ) is 43% which of course is too high and correspondingly the private consumption of goods is too low. The reason is that an essential part of the transfers to the households has been defined as government consumption. The key point is that all fees and transfers that do not have a direct individual relationship is regarded as taxes and government consumption respectively. This is done in order to avoid blurring the picture with a whole range of transfers that cannot be correctly allocated due to the simplified household sector.

## 5 A BALANCED BUDGET WAGE TAX REFORM

Wage tax  
simulation  
(section 5)

This chapter is devoted to studying the dynamic effects of a wage tax reform.

### 5.1 DESCRIPTION OF THE WAGE TAX REFORM

The tax reform amounts to lowering the tax rate on wage income ( $\tau^w$ ) from 50 to 35 percent. In the simulation, transfers (S) are endogenously calculated in response to an exogenous change in  $\tau^w$  to keep the government budget balanced. This could be thought of as changing the intercept (-S) and the slope ( $\tau^w$ ) in the linear tax schedule:  $\text{taxes} = -S + \tau^w L$

This approach takes into account all indirect (general equilibrium) effects of the change in  $\tau^w$ . This is a difference to the Income Tax Committee [1989], which disregard indirect effects altogether.

Government consumption (G) is kept unchanged between the reference case and the tax reform simulation. This does effectively mean that the government consumption share of GDP ( $\xi_1^g$ ) (see equation 21) is endogenous in the tax reform simulation. This approach has been chosen in order to be able to make clear cut welfare comparisons.

### 5.2 STEADY STATE EFFECTS OF THE WAGE TAX REFORM

The model is constructed to converge to a new steady state growth path after the reform. On this new path the rate of growth is once again equal to the underlying productivity growth rate ( $\lambda$ ). The ultimate effect of the tax reform, after all dynamic adjustment has taken place, is shown by the change in the steady state solution. Such comparisons have many similarities with results from static CGE models.

The most immediate effect of lowering the wage tax rate ( $\tau^w$ ) is to raise the relative price of leisure ( $\bar{L}-L$ ). Thus, consumption of goods (c) is substitut-

ed for leisure. Recalling equation (27) and noting<sup>22</sup> that  $p^{cc}=1$ , we have

$$\frac{C_s}{\bar{L}_s - L_s} = \left[ \frac{1 + \tau_s^c}{(1 - \tau_s^w)w_s / (\theta \lambda^u)} \right]^{-\eta} \theta \lambda^s \quad (27')$$

As noted in section 3.2 the magnitude of this substitution is governed by the size of the atemporal elasticity of substitution between consumption of goods and leisure ( $\eta$ ). It is obvious that  $\eta$  is one of the most crucial parameters in these simulations. Some attempts to estimate  $\eta$  (or something like it) are discussed in section 4.2. Even if the size of  $\eta$  is crucial in determining the overall magnitude of the response to a tax reform, this question is not the key point in this paper. Rather, this study tries to emphasize intertemporal aspects of tax reforms. It will later be shown (see section 7.4) that many of the qualitative results on adjustment paths are not affected by reasonable variations in  $\eta$ .  $\eta=0.8$  is used in the following simulations.

The percentage change from the reference case for 14 variables is found in table 5.1. The substitution of consumption of goods for leisure is obvious, since consumption of goods increases by 16.4% while labour supply increases by 5.7% (decreasing leisure with an equal amount). This could be compared with Blomquist [1989] who estimates the effect of a decrease in marginal tax rates on 10 percentage points (the decrease in the simulation in this chapter is 15 percentage points) to a 3.1-5.2% increase in labour supply and with Aaberge, Ström and Wennemo [1989] who estimate the effects to be a 0.2%-1.0% increase.

Furthermore, the human capital (H) and ideal price index ( $P^u$ ) increase by 18.8 and 10.8 percent, respectively. Thus, the real purchasing power of expected life time resources is increased. As can be seen from equations (2d-e) this income effect has the effect of reinforcing the increase in consumption of goods and reducing the increase in labour supply. The increase in human capital is composed of a 30% increase in the value of labour endowment ( $(1 - \tau^w)w\bar{L}$ ) and 52% decrease in lump sum transfers (S).

Private assets (A) and the value of composite consumption ( $P^u U$ ) increase in

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<sup>22</sup>See section 2.6 for details.

proportion to the human capital. The reason is that this tax reform does not alter the optimal consumption-earnings profile, which is most easily seen from equation (26).

It can also be noted that labour supply, gross production ( $Y$ ), the capital stock ( $K$ ) and the value of the firms ( $V$ ) all increase by the same 5.7 percent, while the wage rate ( $w$ ) is unchanged. This is an illustration of the recursivity of the long run behaviour of the model (see section 2.3.4).

Even though  $\tau^w$  is lowered by 30% (15 percentage points) total tax revenues (TAX) decrease by only 8.8 percent. Disregarding all side effects the decrease would be 16.1 percent. The reform is thus partly self-financing through increased labour supply (wage and payroll tax base) and consumption of goods (consumption tax base).

### 5.3 A FULL SCALE DYNAMIC SIMULATION

In this section a full scale dynamic simulation is presented. It is assumed that the agents in the economy are completely surprised by the implementation of the tax reform in  $t=0$ , which is the first year of the simulation.

The capital stock ( $K$ ) and the foreign assets (FA) in  $t=0$  are inherited from the past, i.e. the old steady state path of the reference case. All other variables can change in  $t=0$  in response to new tax rates in  $t=0$  and to new information about the future. This is eg the case for private assets ( $A_0$ ) which can change in  $t=0$  due to unexpected capital gains (losses) on the stock of shares ( $V$ ). As pointed out in section 2.5 it is assumed that all shares are owned by domestic households.

The model converges to a new steady state growth path since the tax rates (as well as all other exogenous variables) are kept constant after the tax reform. It seems that the major part of the adjustment is completed after 100 years. Therefore, the solution for  $t=101$  (and all years thereafter) is approximated by the the new steady state growth path. The major reason to the long adjustment period is that the old generations are replaced fairly slowly (the average length of life is 33 years) and hence the aggregate private assets adjust slowly.

Table 5.2 provides a summary of the solution: the percentage change from the reference case is reported for 14 important variables (columns) for selected

years (rows). The last row reports the steady state changes and is of course identical to table 5.1. In section 5.2 we noted that the last row have many similarities of the results from a static CGE model. In a way, the comparative advantage of this model is exemplified by the other rows, which give a picture of the adjustment path to the new steady state.

Diagram 5.1 displays the percentage change from the reference case for labour supply (L) and consumption of goods (C) for each year. Labour supply in  $t=0$  is slightly more than 7% higher than in the reference case - a number that increases somewhat during the first 10 years and then starts to fall back to a 5.7% change in the new steady state. Consumption of goods is about 12% higher than in the reference case in  $t=0$  and rises to a 16.4% change in the new steady state.

The main lesson from this exercise is the paths of consumption of goods and labour supply (or equivalently leisure). In diagram 5.2 leisure ( $\bar{L}-L$ ), consumption of goods (C) and the composite consumption (U) are shown. The composite consumption is just the value of the subutility function (2b) which is restated below for convenience

$$U_s = \left[ C_s^{1-1/\eta} + (\theta\lambda^s (\bar{L}_s - L_s))^{1-1/\eta} \right]^{\frac{1}{1-1/\eta}} \quad (2b)$$

The change from the reference case in the composite consumption is around 4% in  $t=0$ , which is split in a 7% decrease in leisure and a 12% increase in consumption of goods (note that leisure has a greater weight in (2b) since  $\theta=2.5$ ). The change in composite consumption grows over time and reaches 7% in the new steady state, with an almost unchanged split in leisure and goods. Note that the substitution of consumption of goods for leisure is almost as important in the short run as in the long run. It is only the development of the wage rate (w) (see diagram 5.4) that dampens the substitution somewhat during the first 10-15 years.

What is the driving force behind this development of composite consumption? The budget constraint (6) which is restated below

$$\sum_{s=t}^{\infty} \frac{P_s U_s}{\alpha_t \prod_{v=t}^s \alpha_v} = A_t + H_t \quad (6)$$

says that the present value of composite consumption should equal the sum of private assets (A) and human capital (H). Diagram 5.3 reveals that human capital in  $t=0$  is 17.0% higher than in the reference case and this increases slowly to reach a 18.8% change in the new steady state.

The private assets do also change somewhat in  $t=0$  due to capital gains on the shares, but much less than the human capital. The important point is that private assets grow slowly over time (after a small stagnation during the first 7 years which is due to intertemporal substitution when wages are low<sup>23</sup>). After 25 years they are still only 7.2% higher than in the reference case, while they are 18.8% higher in the new steady state. It is precisely here the existence of different generation comes into play! The private assets in  $t=0$  are owned by the old generations. They accumulated their assets under a regime with higher wage tax rates. Accordingly, they spent more (than currently young generations) time on leisure and received less after tax for each worked hour. As a consequence they saved less for future consumption of goods. These generations will eventually die off and be replaced by other generations in the role as "old". These new generations will have lived for a longer and longer period of their life in the new tax regime. Hence, they will have accumulated more and more assets. This is the way aggregate assets, consumption of goods and leisure increase over time. This adjustment is completed when the "old" in  $t=0$  is totally replaced by generations born after the tax reform.

It is also worth noting that it takes some 20 years before the increase in private assets is greater than the increase in the value of firms. Since foreign assets (FA) is the difference between private assets and the value of firms (see equation 25)<sup>24</sup>, this means that during this period FA have been lower than in the reference case.

The effects on the production side of the economy are summarized in diagram 5.4. The percentage change from the reference case in labour supply (L), pretax wage rate (w), capital stock (K) and gross production (Y) are shown. The capital stock adjusts slowly due to adjustment costs, governed by the  $\gamma$

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<sup>23</sup>The effects of intertemporal substitution is discussed in some detail in section 7.3.

<sup>24</sup>The government debt is zero and FA is close to zero.

parameter<sup>25</sup>. Hence, the effect of the higher labour supply is to lower pretax wages during the first 15 years. The increase in the capital stock is brought about by expected high labour supply in the future. The "overshooting" in labour supply is long enough to create a substantial "overshooting" of the capital stock too, even if it turns up later and is of a smaller magnitude.

The effects on the tax revenues can be traced from diagram 5.5 where the percentage change from the reference case in total tax revenues (TAX), the sum of wage income and payroll tax revenues  $((\tau^w + \tau^a)wL)$  and the consumption tax revenues  $(\tau^c p^{cc}C)$  are shown. Although  $\tau^w$  is lowered by 30% [ $(\tau^w + \tau^a)$  decrease by 25.4%] labour and payroll tax revenue decrease by merely 16.9% in  $t=0$ . This is explained by the increase in labour supply. Moreover, total tax revenue decreases by only 11.2% in  $t=0$  since the increase in consumption of goods generates tax revenues (without any side effects the decrease would be 16.1%). The decrease in tax revenues is somewhat mitigated during the following 15 years when the wage rate and the labour supply catch up. After  $t=15$  the fall in labour tax revenues are offset by the slowly growing consumption tax revenues.

The second comparative advantage of the dynamic CGE approach is the possibility to study the effect on intergenerational distribution. The effects on the welfare for different generations are illustrated in diagram 5.6. The diagram shows equivalent variation as a percentage of total wealth (private assets+ human capital for old generations in  $t=0$ , i.e. generations  $j < 0$ , but only human capital for young generations, i.e. generations  $j \geq 0$ ). Generations  $j \geq 20$  would be willing to pay around 7.3% of their human capital in order to have the new tax regime, while generation  $j=0$  would be willing to pay 6 percent. Generations  $j \leq 0$  are even less interested in the reform.

The old generations have accumulated private assets (A), which increase somewhat in  $t=0$  due to capital gains on the stock of shares (V). But this change is smaller than the change in human capital. In fact, the real purchasing power of private assets  $(A/p^u)$  decreases. We can confirm from equation (12'), which is restated below

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<sup>25</sup>Some sensitivity analysis for changes in  $\gamma$  is carried out in section 7.4.



$$\frac{{}_jEV_s}{{}_jA_s+H_s} = \frac{p_s^u \Delta_s^{1/(\sigma-1)}}{(p_s^u \Delta_s^{1/(\sigma-1)})'} \cdot \frac{({}_jA_s+H_s)'}{{}_jA_s+H_s} - 1 \quad (12')$$

that young generations get a higher relative increase in the purchasing power than old generations do. Another way of stating the same thing is to note that for old generations part of composite consumption is financed out of assets. The higher price index (i.e. the smaller subsidy of leisure) makes this part shrink, which counteracts the utility gain from the increase in human capital.

The wage tax simulation can be summarized by the following points

- consumption of goods is substituted for leisure
- The increase in human capital make future old generations richer than old generations in  $t=0$ . As a result the increase in labour supply will eventually drop off somewhat.
- Young generations have relatively more to gain from the reform since they are more dependent on labour incomes.

## 6 A BALANCED BUDGET CONSUMPTION TAX REFORM

### Consumption tax simulation (section 6)

Raising the consumption tax rate is one possible way of financing a major tax reform. This section tries to establish some results on higher consumption tax rates per se, before carrying out a combined wage and consumption tax reform simulation in section 7.

#### 6.1 DESCRIPTION OF THE CONSUMPTION TAX REFORM

The consumption tax rate ( $\tau^c$ ) is raised from 22 to 26 percent in the first simulation year ( $t=0$ ). As in the wage tax simulation in section 5, the government budget is balanced by endogenous lump sum transfers ( $S$ ), while government consumption ( $G$ ) is kept unchanged from the reference case. All parameters (and hence the reference case) are the same as in the wage tax simulation.

#### 6.2 A FULL SCALE DYNAMIC SIMULATION

The immediate effect of the increase in the consumption tax rate ( $\tau^c$ ) is to raise the relative price of consumption of goods. Recalling equation (27')

$$\frac{c_s}{\bar{L}_s - L_s} = \left[ \frac{1 + \tau_s^c}{(1 - \tau_s^w)w_s / (\theta \lambda^s)} \right]^{-\eta} \theta \lambda^s \quad (27')$$

we see that leisure will be substituted for consumption of goods. In the wage tax reform simulation presented in section 5, this relative price was lowered. Hence, many of the ultimate effects in this section will be just the opposite to those in the wage tax simulation.

Diagram 6.1 shows the percentage change from the reference case for composite consumption ( $U$ ), consumption of goods ( $C$ ) and leisure ( $\bar{L} - L$ ). In the first simulation year ( $t=0$ ) composite consumption is 0.8% lower than in the reference case. This decrease is somewhat mitigated over time.

The increase in the consumption tax rate sparks off substitution of leisure for consumption of goods. The decrease in composite consumption in  $t=0$  is split in a 0.7% increase in leisure and a 1.7% decrease in consumption of goods. This substitution is reinforced over time due to the downward adjustment of the wage rate ( $w$ ) (see table 6.1). The wage rate increases instantaneously in  $t=0$  due to the combination of the fall in labour supply and the fixed capital stock ( $K$ ). The slow adjustment of the capital stock (and hence the wage rate) is an effect of the adjustment costs for capital.

The path of composite consumption is upward sloping because old generations in  $t=0$  have less private assets ( $A_0$ ) than future old generations (see table 6.1). This is so due to two reasons. First, the fall in labour supply incurs an unexpected capital loss on the shareholders in  $t=0$  ( $V$  falls instantaneously in  $t=0$  in response to new information, which can be seen in table 6.1). Second, the increase in total tax revenues (see diagram 6.3) brought about by the increase in the consumption tax rates is balanced by higher lump sum transfers ( $S$ ), which increase the human capital ( $H$ ). Without any change in the consumption-earning profile, this means that future generations will have accumulated more assets when they become old<sup>28</sup>. The change in both consumption of goods and leisure will thus increase over time.

The combined effect of more leisure and lower wages make the wage and payroll tax revenues  $((\tau^w + \tau^a)wL)$  (diagram 6.3) outweigh the slowly increasing consumption tax revenues  $(\tau^c p^{cc}C)$  (diagram 6.2). Hence, the change from the reference case in total tax revenues ( $TAX$ ) decreases over time (diagram 6.3).

The welfare effects for different generations are illustrated in diagram 6.4, which shows equivalent variation as a percentage of life time resources. It is obvious that all generations loose, but in varying degrees. Generations  $j \geq 20$  would be willing to sacrifice 0.55% of their human capital to avoid the tax reform. Generations  $j \leq 15$  would be willing to pay an even greater share of their total wealth, while the generation born in  $t=0$  is only willing to pay 0.37 percent of the human capital.

The very old generations have much private assets which is supplied inelastically in  $t=0$ . This means that they loose more due to the unexpected fall in

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<sup>28</sup> A similar argument is laid out in more detail in section 5.3.

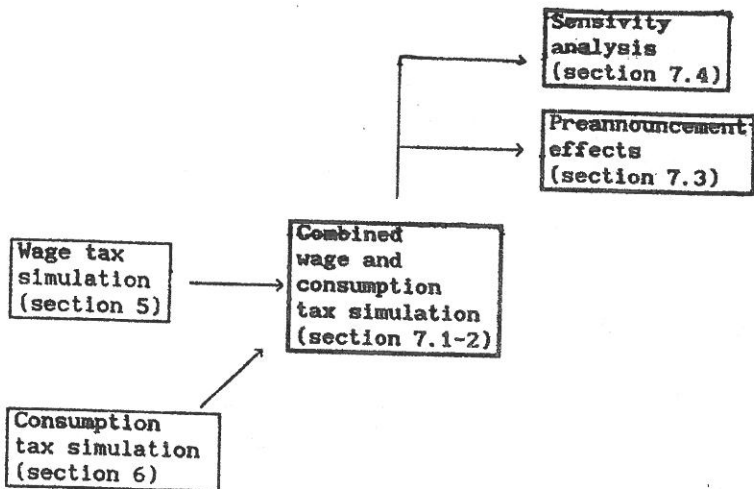
the value of firms and that their total wealth increase relatively less than that of young generation.

The generations born around  $t=0$  fare relatively better off than future young generations, due to the high wage rates and transfers during  $t=0-10$ . The latter is due to both the high wage rates and the relatively high labour supply of (mainly) the old generations.

The consumption tax simulation can be summarized in the following points

- Leisure is substituted for consumption of goods.
- The fall in labour supply is reinforced over time since old generations in the future have accumulated more private assets.
- The welfare is lowered for all generations, most for very old generation and least for generations born around  $t=0$ .

## 7 A BALANCED BUDGET COMBINED WAGE AND CONSUMPTION TAX REFORM



This section combines the two previous simulations in an attempt to illustrate some of the possible effects of the tax reform proposal by the Swedish Income Tax Committee [1989]. The assumptions and the results are reported in section 7.1-2. In section 7.3 pre-announcement effects are studied, while section 7.4 is devoted to testing the robustness of the results in section 7.1-3.

### 7.1 DESCRIPTION OF THE COMBINED WAGE AND CONSUMPTION TAX REFORM

The proposal of the Swedish Income Tax Committee includes among other things a lowering of marginal and average wage income tax rates and a broadening of the consumption tax base. It is estimated that the income weighted average of the marginal and the average labour income tax rates will be lowered from 52.3 and 36.5 percent to 34.8 and 25.9 percent, respectively. Furthermore, the broadening of the consumption tax base is estimated to raise the effective tax rate on private consumption from 22 to 26 percent.

The wage tax part of the reform proposal was studied in the wage tax simulation in section 5 where the wage tax rate ( $\tau^w$ ) was lowered from 50 to 35 percent. If transfers ( $S$ ) are combined with the marginal wage income tax rate ( $\tau^w$ ), the average wage income tax is

$$\bar{\tau}^w = \tau^w - \frac{S}{WL} \quad (28)$$

which was 0.34 in the reference case and 0.28 in the wage tax simulation (in steady state). In the combined simulation it turns out to be 0.26. The consumption tax part was studied in the consumption tax simulation in section 6 where the consumption tax rate ( $\tau^c$ ) was raised from 22 to 26 percent. This section combines the two parts.

As before, the government consumption (G) is unchanged from the reference case and the government budget kept balanced by an endogenous decrease in the lump sum transfers (S).

## 7.2 A FULL SCALE DYNAMIC SIMULATION

Sections 5 and 6 have made it clear that lowering the wage tax rate ( $\tau^w$ ) and raising the consumption tax rate ( $\tau^c$ ) have the opposite effect on many variables. The key point is that while the former lowers the relative price of consumption of goods (i.e. in relation to leisure), the latter raises it. Will the effects of the two parts of the combined wage and consumption tax reform cancel each other? To some extent yes, but the relative magnitudes of the two parts of the reform<sup>27</sup>, suggest that the effects of the lower wage tax rate will tend to dominate.

In table 7.1 the percentage change from the reference case is shown for 14 variables for selected years. In the new steady state, labour supply (L), production (Y), capital stock (K) and the value of firms (V) are all 4.5% higher than in the reference case (in the wage tax simulation the change was 5.7% and in the consumption tax simulation it was -1.1 percent). Consumption of goods (C) increases by 14.8% (16.4% and -1.5%, respectively). Human capital (H), private assets (A) and the value of composite consumption ( $p^u U$ ) increase by 20.7% (18.8% and 1.5%, respectively). In the combined simulation the difference between the change in consumption of goods and leisure (i.e. a measure of the degree of substitution) is some 18 percentage points (22 and -2.5, respectively).

It turns out that the steady state effects of the two parts of the combined simulation are almost additive. As a consequence, the results are not too

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<sup>27</sup> The wage tax rate is lowered by 15 percentage points (30% change) and the consumption tax rate raised by 4 percentage points (18% change).

different from those obtained in the wage tax simulation in section 5. Therefore, most of this section is devoted to discussing some differences between the adjustment paths in the combined simulation and the wage tax simulation.

The relative degree of undershooting (i.e. the difference from the new steady state growth path) of composite consumption ( $U$ ) is greater in the combined simulation. This is illustrated in diagram 7.1, which shows the percentage difference from the new respective steady states (i.e. the amount of undershooting) for composite consumption. In  $t=0$ , the undershooting is 3.3% in the combined simulation and 2.8% in the wage tax simulation. This carries over to consumption of goods and leisure. The degree of overshooting in labour supply is shown in diagram 7.2.

The main reason to this difference is the behaviour of private assets ( $A$ ). The change in private assets in  $t=0$  is lower in the combined simulation than in the wage tax simulation (4.7% and 5.3%, respectively) since the value of the firms ( $V$ ) increases less (4.3% and 4.8%, respectively) (cf tables 7.1 and 5.2). Furthermore, the increase in human capital ( $H$ ) is greater in the combined simulation. Hence, the relative difference between the wealth of the old generation in  $t=0$  and the wealth of the old generation in the new steady state is greater in the combined simulation.

In diagram 7.3 the relative degree of undershooting/overshooting is shown for total tax revenues ( $TAX$ ) as well as for the sum of labour and payroll tax revenues  $[(\tau^w + \tau^a)wL]$ . One can note that in both simulations labour and payroll tax revenues show an initial undershooting (the low wage rate outweighs the high labour supply) followed by an overshooting (normal wages but still high labour supply). Furthermore, the relative degree of undershooting in total tax revenues are almost identical in the two simulations. This is accomplished by greater overshooting (or less undershooting) in labour and payroll tax revenues in the combined simulation (see diagram 7.3), et vice versa for consumption tax revenues. The driving force behind this development is of course the paths for private assets and composite consumption as discussed above.

Finally, the welfare effects for different generations of the combined wage and consumption tax reform are illustrated in diagram 7.4, which shows equivalent variation as a percentage of total wealth. As in the wage tax simulation, current and future young generations gain most and the very old generation least. Compared with the welfare effects on the wage tax simulation, the gain is smaller, especially for the very old generations (cf diagrams 7.1 and



5.6).

The combined wage and consumption tax simulation can be summarized by the following points

- The effects of the lower wage tax rate dominate over the effects of the higher consumption tax rate.
- Consumption of goods is substituted for leisure. Hence, labour supply and production increase.
- The increase in labour supply is initially very high, but falls back over time as the old generations become richer. This effect is even stronger than in the wage tax simulation.
- The young generations gain most from the reform.

### 7.3 PRE-ANNOUNCEMENT EFFECTS

The agents in an economy will surely take into account available information about future tax reforms when planning consumption of goods and leisure. The dynamic CGE approach makes it possible to study such effects within a theoretical consistent framework.

Even though binding pre-announcements of tax reforms are hard to find, it is not unusual that major ingredients in future tax reforms are common knowledge. In the simulations in this section it is assumed that the future tax reform is known for sure. Admittedly, this is a grandiose simplification since it begs the questions about uncertainty and incentive compatibility. Nevertheless, the simulations might be illustrating.

Two new simulations are shortly presented in this section. They differ from the combined wage and consumption tax reform simulation in section 7.1-2 on only one point, namely the year of announcement of the reform. As before, the wage tax rate ( $\tau^w$ ) and the consumption tax rate ( $\tau^c$ ) are changed in  $t=0$ , but this time the reform is assumed to be known in  $t=-3$  and  $-10$ , respectively. Hence, the reform is known 3 or 10 years in advance.<sup>28</sup>

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<sup>28</sup>Technically, this means that the simulation starts in  $t=-3$  (or  $-10$ ) with values for  $K-3$  ( $K-10$ ) and  $FA-3$  ( $FA-10$ ) given from the reference case.



### 7.3.1 Effects on composite consumption

The effect on composite consumption ( $U$ ) of the pre-announcement is shown in diagram 7.5, where the percentage changes from the reference case are displayed for 0, 3 and 10 years pre-announcement. There are two major differences between the curves. First, in the cases with pre-announcement the composite consumption change already in the year of announcement. The reason is that the households are made aware of the future increase in the net of tax income. This is reflected in the change of the human capital ( $H$ ) displayed in diagram 7.6. Remember that human capital is defined as the *expected* discounted value of future net of tax income. As a consequence, the households know that they can afford to raise the general level of composite consumption.

Second, with expected (known) future changes in the tax rates there is scope for intertemporal substitution. The tax reform amounts to a 27% increase in the relative price of leisure. After (atemporal) substitution of consumption of goods for leisure has taken place, this gives an increase in the price index for composite consumption ( $P^u$ )

$$P_s^u = \left[ \left( (1+\tau_s^c) P_s^{cc} \right)^{1-\eta} + \left( \frac{w_s(1-\tau_s^w)}{\theta \lambda^B} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (2c')$$

that is about barely half of that (see table 7.2).

The pre-announcement means that the households realize that the price of composite consumption will increase in the future. Remember that the intertemporal substitution is governed by

$$\frac{U_{s+1}}{U_s} = \left( \frac{1+r_{s+1}}{1+\rho} \right)^{\sigma} \left( \frac{P_{s+1}^u}{P_s^u} \right)^{-\sigma} \quad (26)$$

We see that if  $P^u$  is expected to increase over time, households will substitute current for future composite consumption. As noted in section 3.1 the magnitude of this substitution is governed by the size of the intertemporal elasticity of substitution ( $\sigma$ ). In diagram 7.5 the change from the reference case is about 3.5 percentage points higher in  $t=-1$  than in  $t=0$ , in the cases with pre-announcement. This is about a third of the increase in  $P^u$

$(\sigma=0.333)^{29}$ .

Since  $\sigma < 1$  the value of composite consumption ( $p^u U$ ) will be relatively lower before the tax reform. The change in  $p^u U$  increases by almost 8 percentage points after the implementation of the tax reform (see table 7.2).

It is also worth noting in diagram 7.5 that the change in composite consumption increases slowly until the implementation of the tax reform (i.e. the curves are upward sloping during the pre-announcement phase). The major reason is that the nearer the implementation of the tax reform, the higher is life time resources for new generations and the shorter is the time for intertemporal substitution.

Moreover, the change in composite consumption from the reference case in the year of implementation of the pre-announced tax reform ( $t=0$ ) is small and decreases with the length of preannouncement (2.2 and 0.4 percent in the case with 3 and 10 years preannouncement, respectively). Since the elasticity of intertemporal substitution ( $\sigma$ ) is finite, the amount of composite consumption that is substituted between periods with low and high prices ( $p^u$ ) increases with the time available for substitution (i.e. the length of the preannouncement phase).

### 7.3.2 Effects on the rest of the model

Diagram 7.7 shows the development of private assets ( $A$ ) with 0, 3 and 10 years of preannouncement, respectively. It is readily seen that the decumulation of private assets ( $A$ ) during the preannouncement phase increases with the length of preannouncement. This is of course explained by the intertemporal substitution as discussed above, but also by the fact that net of tax labour income is relatively low before the introduction of the new wage tax rate. In the preannouncement cases the change from the reference case reaches a minimum in the year of implementation of the reform ( $t=0$ ). The small increases of  $A_0$  and  $A_3$  in the cases without and with 3 years preannouncement are explained by capital gains on the stock of share (caused by future increases in the labour supply).

Diagram 7.8 and 7.9 show how the split of composite consumption in consump-

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<sup>29</sup>The sensitivity of the results for changes in  $\sigma$  is studied in section 7.4.

tion of goods (C) and leisure ( $\bar{L}-L$ ) evolves over time, in the case with 3 and 10 years preannouncement, respectively. In both cases, the tendency is clear: consumption of goods and leisure increase almost proportionally during the preannouncement phase. Hence labour supply is very low before the implementation of the reform, which is seen in diagram 7.10. This tends to raise the wage rate and spark off some substitution of consumption of goods for leisure. After the reform, consumption of goods is substituted for leisure in much the same way as in the combined tax reform without preannouncement. An effect of the low composite consumption immediately after the year of implementation of a pre-announced tax reform is to create a considerable overshooting in labour supply (L). The capital stock (K), as shown in diagram 7.11, will therefore also exhibit an overshooting that increases with the length of preannouncement (the maximum change from the reference case case is 8, 7 and 6 percent in the case with 10, 3 and 0 years preannouncement, respectively). It is also worth noting that the adjustment costs for capital almost prevent the capital stock from declining in during the preannouncement phase, in spite of the low labour supply.

The preannouncement/postponement effects of the combined wage and consumption tax reform can be summarized by the following points

- The households substitute current for future composite consumption. As a consequence, labour supply is low and private assets are run down during the preannouncement phase
- In the years following the implementation of the reform, labour supply is very high.

#### 7.4 SENSITIVITY ANALYSIS

The importance of certain parameter values have been pointed out several times in the preceding sections. In this section some of the simulations in section 7.1-3 are repeated with other values on selected parameters. This exercise aims at illuminating certain key points and is in no way exhaustive. There are at least two important limitations. First, only a small subset of the parameters are studied. Second, the parameters are studied one at a time.

It should be noted that the reference cases used in these simulations are not identical to the reference case in section 5-7.3 (and described in section 4.4). Rather, they are unique for each subsection below. Nevertheless, the differences are quite small.

#### 7.4.1 The adjustment cost parameter ( $\gamma$ )

The adjustment cost parameter  $\gamma$  makes rapid adjustments of the capital stock costly. Rewriting equation (18) we have the adjustment costs

$$i_s \Gamma(i_s/K_s) = \frac{\gamma i_s^2}{2K_s} \quad (18')$$

which is convex in investment ( $i$ ). A higher value of  $\gamma$  will increase the adjustment costs and lead to slower and smaller changes in the capital stock.

The combined wage and consumption tax simulation without preannouncement (see section 7.1-2) is repeated with  $\gamma=1$  and  $\gamma=50$  (default  $\gamma=10$ ). Diagram 7.12 shows the percentage change from the respective reference case in the capital stock. It is obvious that a higher adjustment cost parameter deters from rapid movements in the capital stock. For  $\gamma=1$  the maximum value of the change from the reference case is some 7.5% which is reached in  $t=10$ . For  $\gamma=50$  it is 4.8% which is reached much later ( $t=30$ ). There are some differences even in the long run, even if they are of a smaller magnitude (the increase from the reference case is 4.8 and 4.0 percent, respectively).

The flexibility of the production sector has important consequences for the household's ability to effectively substitute between consumption of goods and leisure. Diagram 7.13 shows the change from the reference case in labour supply. Lower adjustment costs reinforce the initial overshooting in the labour supply. The reason is that with lower adjustment costs, the decrease in the wage rate is dampened.

#### 7.4.2 The intertemporal elasticity of substitution ( $\sigma$ )

Recalling equation (26)

$$\frac{U_{s+1}}{U_s} = \left( \frac{1+r_{s+1}}{1+\rho} \right)^{\sigma} \left( \frac{P_{s+1}^u}{P_s^u} \right)^{-\sigma} \quad (26)$$

we see that the value of  $\sigma$  is crucial for the magnitude of the preannouncement effects in section 7.3. In this section the combined wage and consumption tax simulation with 10 years preannouncement (see section 7.3) is repeated using the values  $\sigma=0.8$  and  $\sigma=0.1$  ( $\sigma=0.333$ ). The value of the time preference rate ( $\rho$ ) is adjusted to keep the consumption-earnings profile in the reference case unchanged from section 7.3 ( $\rho=0.027$  and  $\rho=-0.14$ , respectively ( $\rho=-0.01$ )).

Diagram 7.14 shows the change from the reference case in composite consumption ( $U$ ). Remember (from section 7.3) that the implementation of the reform in  $t=0$  amounts to a 12% increase in the ideal price index ( $p^u$ ). With  $\sigma=0.8$ , the change from the reference case in composite consumption is 6.5% in  $t=-1$  and -1.5% in  $t=0$ , i.e. a 8 percentage points difference which is a measure of the intertemporal substitution. In the case with  $\sigma=0.1$ , this intertemporal substitution is only 1 percentage point (2.5% in  $t=-1$  to 1.5% in  $t=0$ ). The intertemporal substitution is financed out of private assets ( $A$ ). With a higher  $\sigma$  private assets are decumulated faster during the preannouncement phase, which is shown in diagram 7.15. In fact, with  $\sigma=0.8$  private assets in  $t=0$  are 30% lower than in the reference case, while they are 15 and 8 percent lower with  $\sigma=0.33$  and 0.1 respectively.

#### 7.4.3 The probability of survival ( $\pi$ )

The finite expected life time will make households (almost) disregard events in the very far future. This is most clearly seen from the utility function

$$\sum_{s=t}^{\infty} \left( \frac{\pi}{1+\rho} \right)^{s-t} U[C_s, \bar{L}_s - L_s] \quad (1')$$

and the intertemporal budget constraint

$$A_t = \sum_{s=t}^{\infty} \frac{w_s^n \bar{L}_s + S_s - p_s^u U_s}{\frac{\pi}{1+r_t} \prod_{v=t}^s (1+r_v) / \pi} \quad (6')$$

Note that future events are discounted by i.a  $\pi$  in both (1') and (6').

In this section the combined wage and consumption tax reform with 10 years preannouncement is repeated with other values on the probability to survive into the next period ( $\pi$ ). The cases with  $\pi=0.99$  and  $\pi=0.96$  ( $\pi=0.97$ ) are investigated. They correspond to an expected remaining life time of 100 and 20 {33} years, respectively. The value of the time preference rate ( $\rho$ ) is adjusted to keep the consumption-earnings profile in the reference case unchanged from section 7.3 ( $\rho=0.0036$  and  $\rho=-0.026$ , respectively ( $\rho=-0.01$ )).

Diagram 7.16 shows the percentage change from the reference case in human capital ( $H$ ). With a higher probability of survival, households have a greater chance to benefit from the high net of tax wage rate after the tax reform.

Hence, the change in human capital in  $t=-10$  increases with the probability of survival (the change from the reference case in  $t=-10$  is 8, 10 and 12 percent in the case with  $\pi=0.95$ ,  $\pi=0.97$  and  $\pi=0.99$ , respectively). After  $t=0$  the difference is negligible. As a result, the change from the reference case in composite consumption ( $U$ ) in  $t=-10$  increases with  $\pi$ , which is shown in diagram 7.17.

With a constant population, a higher  $\pi$  means a lower birth rate ( $1-\pi$ ). Hence, with a higher  $\pi$  the new born generations between  $t=-9$  and  $t=0$  (who have a shorter time available for intertemporal substitution) constitute a smaller share of total population (or wealth). The consequence is to decrease the steepness of the change in composite consumption between  $t=-10$  and  $t=-1$  (see diagram 7.17). Another aspect of the predominance of the old generations is that the upward adjustment of aggregate composite consumption after  $t=0$  is slower with a higher  $\pi$ .

#### 7.4.4 The atemporal elasticity of substitution ( $\eta$ )

In section 5.2, it was noted that the size of the (atemporal) elasticity of substitution between consumption of goods and leisure is crucial for determining the overall magnitude of the effects of the wage tax reform. Remember that this substitution is governed by

$$\frac{C_s}{\bar{L}_s - L_s} = \left[ \frac{1 + \tau_s^c}{(1 - \tau_s^w)w_s / (\theta \lambda^w)} \right]^{-\eta} \theta \lambda^w \quad (27')$$

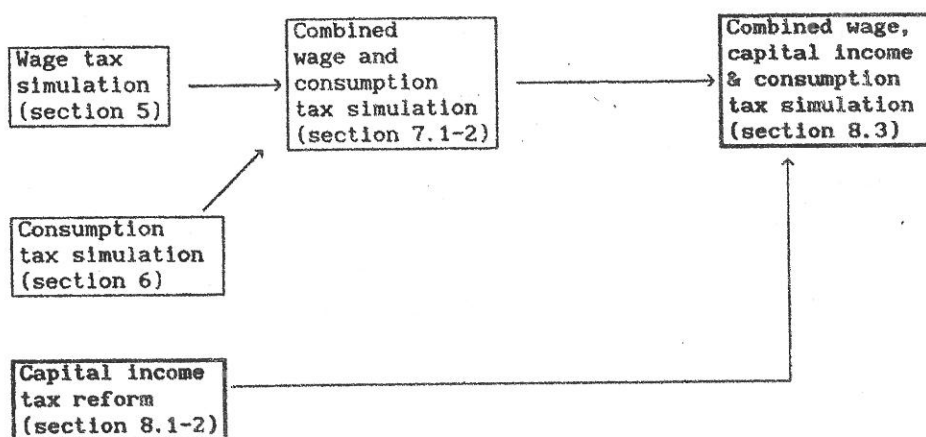
In this section the combined wage and consumption tax simulation without preannouncement (see section 7.1-2) is repeated with  $\eta=0.333$  and  $\eta=1.6$  ( $\eta=0.8$ ). The leisure preference rate  $\theta$  is adjusted to keep the reference case approximately unchanged from section 7.1-2 ( $\theta=0.48$  and  $\theta=0.105$  ( $\theta=2.5$ ), respectively).

Diagram 7.18 shows the percentage change from the reference case in labour supply ( $L$ ). The rate of change increases with the value of  $\eta$ . With  $\eta=0.333$  the long run change is 0.9%, while  $\eta=1.6$  gives a 11.1% change. In all cases an initial overshooting in labour supply is followed by a prolonged downward adjustment towards the new steady state growth path. There are a few differences, though. First, with higher atemporal elasticity of substitution ( $\eta$ ) the overshooting in relation to the new steady state reaches its maximum later (in  $t=15$  for  $\eta=1.6$  but in  $t=0$  for  $\eta=0.333$ ). Second, the relative degree of overshooting decreases with  $\eta$  (2 and 2.4 percent relative overshooting

with  $\eta=1.6$  and  $\eta=0.333$ , respectively). This is explained by the fact that the combination of adjustment costs for capital and a high  $\eta$  will drive down the wage rate to such an extent that the increase in labour supply is dampened.

The effects of changing  $\eta$  on the tax revenues (excluding the lump sum tax) are equally dramatic. With  $\eta=0.8$  the initial change in tax revenues is -8.6% and in the long run -6.4% - far from a totally self financed tax reform. A  $\eta=0.333$  gives of course an even worse effect on the tax revenues. If  $\eta=1.6$  the initial change is -5.0% but in the long run the reform is more than self financed, since tax revenues increase by some 0.5 percent.

## 8 A BALANCED BUDGET CAPITAL INCOME TAX REFORM



The taxation of capital income in Sweden is very heterogeneous. In fact the total capital income tax revenues are negative, at the same time as certain assets are very heavily taxed. Not surprisingly, one of the main proposal of the Swedish Income Tax Committee [1989] is to streamline the taxation of capital income. This model is obviously too aggregated to capture the effects of the capital income tax system at present. Nevertheless, it can provide some ideas about how the general level of capital income taxation affects savings. Section 8.1-2 presents a stylized capital income tax simulation and section 8.3 brings together that simulation with the combined wage and consumption tax simulation in section 7.1-2 in an attempt to illustrate the entire tax reform proposal.

### 8.1 DESCRIPTION OF THE CAPITAL INCOME TAX REFORM

The Swedish Income Tax Committee [1989] estimates that the changes in the capital income taxation will raise additional capital income tax revenues equal to 2% of GDP in the first year after the implementation. In the model this is achieved by increasing the capital income tax rate ( $\tau^k$ ) with 10 percentage points. Consequently, in the simulation in this section the capital income tax rate ( $\tau^k$ ) is raised from 20 to 30 percent<sup>30</sup> in the first simulation year ( $t=0$ ). As in the the earlier simulations, the government budget is

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<sup>30</sup>As noted in section 4.3 the level of  $\tau^k$  is more or less arbitrary.



balanced by endogenous lump sum transfers (S), while government consumption (G) is kept unchanged from the reference case. All parameters (and hence the reference case) are the same as in the simulations in sections 5-7.3 (and described in section 4.4).

## 8.2 A FULL SCALE DYNAMIC SIMULATION

Diagram 8.1 shows the percentage change from the reference case in composite consumption (U) and its two components (consumption of goods (C) and leisure ( $\bar{L}-L$ )). Composite consumption is 1.9% higher than in the reference case in  $t=0$ , but falls back to a 3.6 decrease in the new steady state. The relation between consumption of goods and leisure is almost unchanged (except for the first few years due to the development of the wage rate - see diagram 8.5).

The immediate effect of the increase in the capital income tax rate ( $\tau^k$ ) is to raise the relative price of future composite consumption. Since  $r=(1-\tau^k)r^*$  an increase in  $\tau^k$  lowers  $r$ . Rewriting equation (26)

$$\frac{U_{s+1}}{U_s} = \left( \frac{1+r_{s+1}(1-\tau_{s+1}^k)}{1+p} \right)^{\sigma} \left( \frac{p_{s+1}^u}{p_s^u} \right)^{-\sigma} \quad (26')$$

we see that the effect of raising  $\tau^k$  is that households substitute current for future consumption. The size of this effect is governed by the elasticity of intertemporal substitution<sup>31</sup> ( $\sigma$ ). Another way to put the same thing is to note that households choose to spend more today since it is less worthwhile to save.

Diagram 8.2 shows the planned path of the present value of cumulated net savings (i.e.  $(1-\tau^w)w\bar{L} + S - p^u U$ ) for a household born in  $t=0$  (observe that the diagram shows levels) in the reference case and the capital income tax reform, respectively. Recall the intertemporal budget constraint

$$A_t = \sum_{s=t}^{\infty} \frac{w_s^n \bar{L}_s + S_s - p_s^u U_s}{\frac{\pi}{1+r_t} \prod_{v=t}^s (1+r_v)/\pi} \quad (6')$$

<sup>31</sup> section 7.4 illustrates the importance of  $\sigma$ .

which states that this the present value of (cumulated) net savings should equal  $A_t$  (which is zero for a new born household in  $t$ ) when  $s \rightarrow \infty$ , which seems to be the case in diagram 8.2. It is obvious from the diagram 8.2 that the increase in the capital income tax rate amounts to reducing the life time savings. This is explained by the change in the slope of the consumption path, as expressed in equation (26'). The latter is also illustrated in diagram 8.3, which shows the change from the reference case in planned composite consumption for two generations (generation  $j=-30$ , i.e. born in  $t=-30$  and generation  $j=0$ ). The two curves exhibit higher composite consumption than the reference case in the early stages of the life cycle, et vice versa for the late stages. Hence, the increase in  $\tau^k$  amounts to tilting the path of composite consumption towards the early part of the life cycle.

As a result of the capital income tax reform the total tax revenues (TAX) are some 2% higher than in the reference case (see table 8.1) and the transfers (S) is accordingly higher. Together with the lower discounting of future full income this generates a 12% change from the reference case in human capital (H), as seen in diagram 8.4. It should be noted that the increase in capital income tax revenues is totally levied on the old generations, while the increased transfers are spread out on all. Since the private assets (A) in  $t=0$  is supplied inelastically, the increase in the capital income tax rate has an element of a surprise wealth taxation (see section 3.3 for a discussion). This explains why generation  $j=-30$  (born in  $t=-30$ ) fare worse off than generation  $j=0$  in diagram 8.3. Diagram 8.4 verify that the private assets (A) are decumulated over time.

Diagram 8.5 shows the percentage change from the reference case for the production sector. The driving force behind the development is of course the labour supply (L). The low initial labour supply (mainly by the old generation) leads to a small decumulation of capital during the first 7 years. When the old generations are replaced by new born generations the labour supply increases. This sparks off an upward adjustment of the capital stock (K), which in the new steady state it is 3.7% higher than in the reference case.

The welfare effects for different generations are illustrated in diagram 8.6, which shows equivalent variation as a percentage of total wealth. The generations older than 17 years ( $j < -17$ ) are the losers, eg generation  $j=-30$  (cf diagram 8.3) would need a 1% increase in total wealth in order to be as well off as in the reference case. This is of course a reflection of the lump sum element of the increase in the capital income tax rate as well as the unexpected capital loss on shares (V) (see table 8.1). The generation born in  $t=0$

(generation  $j=0$ ) fare much better off, since they gain from the high transfers (which are financed by the increase in the capital income tax ( $r^*kA/(1+r)$ )) and the relatively high wage rate ( $w$ ). Generation  $j=1-30$  are somewhat worse off (but they still gain) than generation  $j=0$ . There are two reasons to that. First, they receive lower transfers since the capital income tax revenues drop as the old have decumulated their assets. Second, the wage rate falls as the labour supply increases rather rapidly. Finally, generations  $j>70$  would be willing to pay around 1% of their human capital to get the reform.

All in all, the very old generations bear the entire burden of the capital income tax reform, since they supply capital inelastically. Young generations are also affected by the lower return on savings, but since their supply of capital is rather elastic, the positive effect of higher transfers ( $S$ ) dominates.

It might be hard to conceive that the welfare for all future generations can increase at the same time as the aggregate composite consumption ( $U$ ) is lower in every period (see diagram 8.1). From the function for life time utility

$$\sum_{s=t}^{\infty} \left( \frac{\pi}{1+\rho} \right)^{s-t} U(C_s, \bar{L}_s - L_s) \quad (1')$$

we see that this is quite possible if the path of composite consumption is tilted towards the earlier periods in life (i.e. closer to  $t$ ).

The positive welfare effects in the new steady state of raising a distorting capital income tax is of course non-standard. But, one should bear in mind that the model depicts a small open economy with free access to the world capital market. As a consequence, there is no (long run) adverse effect on the real wage. Moreover, there exists a range of other distorting taxes, i.e. we are in a second best situation.

The capital income tax simulation can be summarized by the following points

- An increase in the capital income tax rate will make households substitute current for future composite consumption. As a consequence, life cycle savings decrease.
- Labour supply of the old generations is very low after the tax reform. As these generations are replaced, labour supply increase over time.
- The tax burden is shifted towards the old generations. Hence, they will

loose substantially on the reform, while all young generations gain.

#### 8.4 A STYLIZED TOTAL TAX REFORM SIMULATION

This section brings together the capital income tax simulation in section 8.1-2 and the combined wage and consumption tax simulation in section 7.1-2 in an attempt to illustrate the entire tax reform proposal. Hence, the wage income tax rate ( $\tau^w$ ) is lowered from 50% to 35%, the consumption tax rate raised from 22% to 26% and the capital income tax rate raised from 20 to 30 percent. As before, the government consumption ( $G$ ) is unchanged from the reference case and the government budget kept balanced by an endogenous decrease in the lump sum transfers ( $S$ ).

In table 8.2 the percentage change from the reference case is shown for 14 variables for selected years. In the new steady state, labour supply ( $L$ ), production ( $Y$ ), capital stock ( $K$ ) are all 8.7% higher than in the reference case (in the combined wage and consumption tax simulation the change was 4.5% and in the capital income tax simulation it was 3.7%). Consumption of goods ( $C$ ) increases by 9.8% (14.8% and -3.6%, respectively). This means that the long run effects are not far from being additive.

In the combined wage and consumption tax simulation in section 7.1-2, the composite consumption showed an initial undershooting (see table 7.1). This was explained by the fact that the old generations in  $t=0$  was relatively poorer than the old generations in the future, which in turn was due to the unfavourable tax regime in the past. This is reversed when the capital income tax rate is increased, as seen from diagram 8.7. The diagram shows the change from the reference case in composite consumption ( $U$ ) and its two components consumption of goods ( $C$ ) and leisure ( $\bar{L}-L$ ). The composite consumption is 5.8% and 2.2% higher than in the reference case in  $t=0$  and in the new steady state, respectively - to compare with 3.6% and 6.8% in the combined wage and consumption tax simulation. Hence, the tilting of the consumption path (current composite consumption is substituted for future composite consumption) caused by the lower net of tax interest rate (see section 8.2) makes the composite consumption show an initial overshooting.

In diagram 8.8 the labour supply ( $L$ ) shows a relatively small immediate increase (4.7% in  $t=0$  to compare with 6.4% in the combined wage and consumption tax simulation) and a gradual upward adjustment. Consequently, both the change in production ( $Y$ ) and in the capital stock ( $K$ ) adjust monotonically to

the new steady state.

The welfare effects are illustrated in diagram 8.9. The effects for the young and future generations are very similar to those in the combined wage and consumption tax simulation (cf diagram 7.4) . These are willing to pay some 7% of their human capital in order to get the reform implemented. In contrast, the welfare gains of the old generations are effectively reduced by the higher capital income taxation. In fact, generations born in  $t=-60$  (or earlier) are worse off than without any reform.

In comparison with the combined wage and consumption tax simulation in section 7.1-2, the addition of a higher capital income tax rate means that

- Labour supply increases less in the short run, but more in the long run. There is no overshooting in labour supply or capital stock.
- The welfare gains of the old generations are reduced substantially.

## APPENDIX 2A: LIST OF MODEL EQUATIONS

The relevant and somewhat rewritten model equations are listed below.

### 2A.1 HOUSEHOLDS

Human capital

$$H_t = \sum_{s=t}^{\infty} \frac{w_s^n \bar{L}_s + S_s}{\frac{\pi}{1+r_t} \prod_{v=t}^s (1+r_v)/\pi} \quad (5')$$

Value of composite consumption

$$p_t^u u_t = \Delta_t (A_t + H_t) \quad (8)$$

Propensity to consume

$$\Delta_t = \frac{1}{\sum_{s=t}^{\infty} \left( \frac{\pi}{1+\rho} \right)^{(s-t)\sigma} \frac{1}{\left[ \frac{\pi}{1+r_t} \prod_{v=t}^s (1+r_v)/\pi \right]^{1-\sigma}} \left( \frac{p_s^u}{p_t^u} \right)^{1-\sigma}} \quad (9')$$

Consumption of goods

$$C_t = \frac{(p_t^c)^{-\eta} p_t^u u_t}{(p_t^c)^{1-\eta} + \left( \frac{w_t^n}{\theta \lambda^s} \right)^{1-\eta}} \quad (2d)$$

Labour supply

$$L_t = \bar{L}_t - \frac{\left( \frac{w_t^n}{\theta \lambda^t} \right)^{-\eta} p_t^u u_t / \theta \lambda^s}{(p_t^c)^{1-\eta} + \left( \frac{w_t^n}{\theta \lambda^t} \right)^{1-\eta}} \quad (2e')$$

Ideal price index of composite consumption

$$p_t^u = \left[ (p_t^c)^{1-\eta} + \left( \frac{w_t^n}{\theta \lambda^t} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} \quad (2c)$$

Difference equation private assets

$$A_{t+1} = (1+r_{t+1}) \left[ A_t + w_t^n \bar{L}_t + S_t - p_t^u U_t \right] \quad (3')$$

where  $w_t^n = w(1-\tau^w)$ ,  $p_t^c = (1+\tau_t^c)p_t^{cc}$ ,  $r_t = (1-\tau_t^k)r_t^*$  and equation (3) is re-written according to the discussion in section 2.2.5.

## 2A.2 FIRMS

Value of firms

$$V_t = \sum_{s=t}^{\infty} \frac{\varphi_s \left[ p_s F_s(K_s, L_s) - p_s^1 i_s (1 + \Gamma_s(i_s/K_s)) - w_s^c L_s \right]}{\prod_{v=t}^s (1+r_{v+1}^*)} \quad (14')$$

Difference equation capital stock

$$K_{t+1} = i_t + (1-\delta)K_t \quad (16)$$

Wage rate

$$w_t^c = p_t F_{t,L} \quad (17a)$$

Installation cost

$$\Gamma(i_t/K_t) = \frac{\gamma i_t}{2K_t} \quad (18)$$

Physical investment

$$i_s = \frac{(q_s/p_s^1 - 1)K_s}{\gamma} \quad (19)$$

Difference equation Tobin's marginal  $q$

$$q_t = \frac{\varphi_{t+1}}{\varphi_t (1+r_{t+2}^*)} \left[ p_{t+1} F_{t+1, \kappa} + p_{t+1}^1 \frac{\gamma}{2} \left( \frac{1_{t+1}}{K_{t+1}} \right)^2 + q_{t+1} (1-\delta) \right] \quad (17b')$$

### Production

$$Y_t = z_t \left\{ \alpha K_t^{1-1/\beta} + (1-\alpha) \left[ \lambda^t L_t \right]^{1-1/\beta} \right\}^{\frac{\beta}{\beta-1}} \quad (20)$$

where  $w_t^c = w_t (1+\tau_t^*)$  and  $\varphi_t = \frac{[1+(1-\tau_{t+1}^k) r_{t+1}^*] (1-\tau_t^k)}{(1-\tau_{t+1}^k)}$

## 2A.3 GOVERNMENT AND NATIONAL ACCOUNTS

### Value of government consumption

$$p_t^g G_t = \xi_t^g \left[ p_t Y_t - p_t^1 l_t \Gamma_t(i_t) \right] - \xi_t^b B_t \quad (21)$$

### Government transfers

$$S_t = \xi_t^s \left[ p_t Y_t - p_t^1 l_t \Gamma_t(i_t) \right] \quad (22)$$

### Tax revenues

$$TAX_t = (\tau_t^w + \tau_t^*) w_t L_t + \tau_t^k r_t^* \frac{A_t}{1+r_t} + \tau_t^c p_t^{cc} C_t \quad (23)$$

### Difference equation government debt

$$B_{t+1} = (1+r_{t+1}^*) (B_t + p_t^g G_t + S_t - TAX_t) \quad (24)$$

### Foreign assets

$$FA_t = A_t - V_t - B_t \quad (25)$$



## APPENDIX 2B: LIST OF MODEL VARIABLES

### 2B.1 ENDOGENOUS MODEL VARIABLES

Variable	Explanation
H	Human capital
U	Composite consumption
$\Delta$	Propensity to consume
C	Consumption of goods
$L^u$	Labour supply
$P^u$	Ideal price index of composite consumption
A	Private assets
V	Value of firms
K	Capital stock
$w, w^n, w^c$	Wage rate, wage rate net of wage income tax and wage rate inclusive of payroll tax
i	Physical investment
q	Tobin's marginal q
Y	Production
G	Government consumption
S	Government transfers
TAX	Tax revenues
B	Government debt
FA	Foreign assets

### 2B.2 EXOGENOUS MODEL VARIABLES

Variable	Explanation
$p^{cc}$	Relative price (net of tax) private consumption
$p^c$	" private consumption: $p^{cc}(1+\tau^c)$
$p^g$	" government consumption
$p$	" domestic production
$r$	Interest rate (net of tax): $r=r^*(1-\tau^k)$
$\varphi$	Discount factor cash flow: $\varphi_t = \frac{[1+(1-\tau_{t+1}^k)r_{t+1}^*](1-\tau_t^k)}{(1-\tau_{t+1}^k)}$

### 2B.3 PRODUCTION PARAMETERS

Variable	Explanation
$\beta$	Elasticity of substitution
$\alpha$	Distribution parameter
$\lambda$	Labour augmenting technological growth
$\delta$	Depreciation rate
$r$	World market real interest rate
$\gamma$	Adjustment cost parameter
$z$	Total factor productivity

## 2B.4 HOUSEHOLD PREFERENCE PARAMETERS

Variable	Explanation
$\sigma$	Intertemporal elasticity of substitution
$\pi$	Probability of survival
$\eta$	Atemporal elasticity of substitution
$\theta$	Leisure preference rate
$\rho$	Time preference rate

## 2B.5 GOVERNMENT PARAMETERS

Variable	Explanation
$\tau^w$	Tax rate on labour income
$\tau^g$	Transfer share of GDP
$\tau^k$	Capital income tax rate
$\tau^c$	Consumption tax rate
$\tau^a$	Payroll tax rate
$\tau^g$	Government consumption share of GDP
$\alpha$	Reaction parameter

tions in this paper is written in the PC language GAUSS, using a GAUSS-supplied Newton algorithm for solving the system of non-linear equations. The program draws on a program developed by McKibben&McKibben [1987].

Define            i)  $K_t$     a vector of predetermined variables in t  
                   ii)  $Q_t$      "                    jump                    "  
                   iii)  $Q_{t+1}^e$     "                    expected  $Q$ 's in t+1

Moreover, let  $\Phi_t(K_{t+1}, Q_t; K_t, Q_{t+1}^e) = 0$  be the system of non-linear difference equations (ie the model) in period t.

The algorithm contains the following steps where  $t=0$  is assumed to be the first simulation period:

- 0) Calculate the steady state growth path solution.
- 1) Guess  $Q_s^e$   $s=\{1,2,\dots,T\}$  and assume that the value in  $t=T$  can be approximated by the steady state value (thus T must be large).
- 2) Given  $K_0$ , i.e inherited values of the predetermined variables, solve

$$\begin{array}{ll} \Phi_0(K_1, Q_0; K_0, Q_1^e) = 0 & \text{solve for } K_1, Q_0 \text{ given } K_0, Q_1^e \\ \downarrow & \\ \Phi_1(K_2, Q_1; K_1, Q_2^e) = 0 & \text{" } K_2, Q_1 \text{ " } K_1, Q_2^e \\ \vdots & \\ \downarrow & \\ \Phi_{T-1}(K_T, Q_{T-1}; K_{T-1}, Q_T^e) = 0 & \text{" } K_T, Q_{T-1} \text{ " } K_{T-1}, Q_T^e \end{array}$$

- 3) Update the guess of  $Q_s^e$   $s=\{1,2,\dots,T\}$ , using a weighted average of the former guess and the solution for  $Q_s$   $s=\{1,2,\dots,T\}$ . Iterate on 2) until the difference between  $Q_s^e$  and  $Q_s$  is negligible.
- 4) Choose a new terminal period  $T' > T$  and repeat 1)-3). Compare this and the former solution for  $t=0-T$ . If the difference is small, accept the solution, otherwise iterate on 1)-4).

There is no available proof of the uniqueness of this solution, but experience suggests that this is a problem of minor importance. Although much effort has been made in order to speed up the execution, it is still time consuming. Solving the system of non-linear equations repeated times is the most burdensome part of the algorithm.

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Diagram 5.1

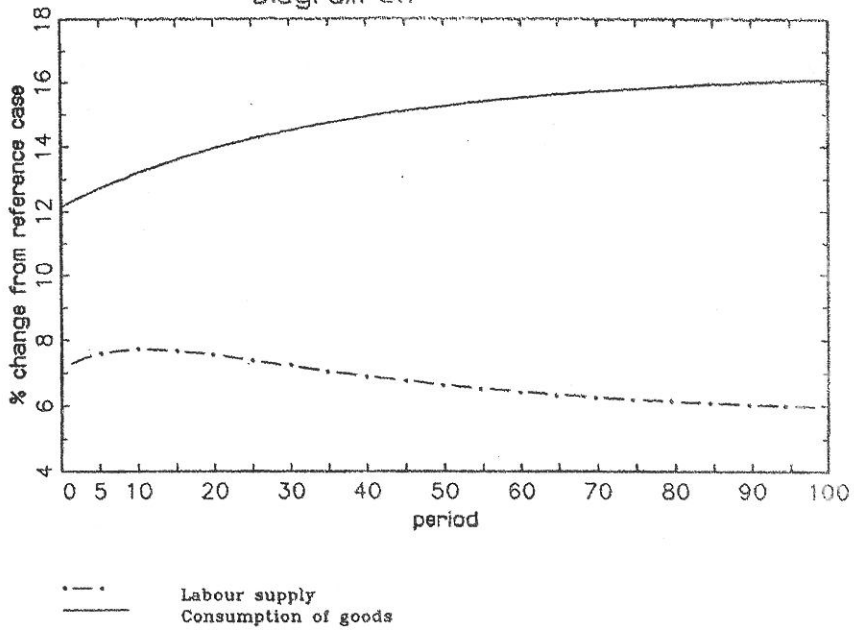
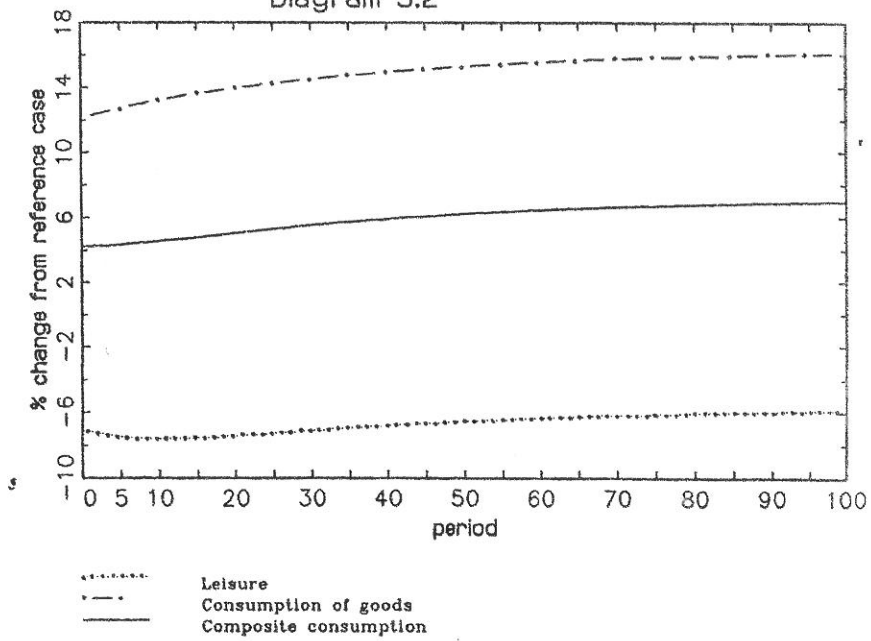


Diagram 5.2



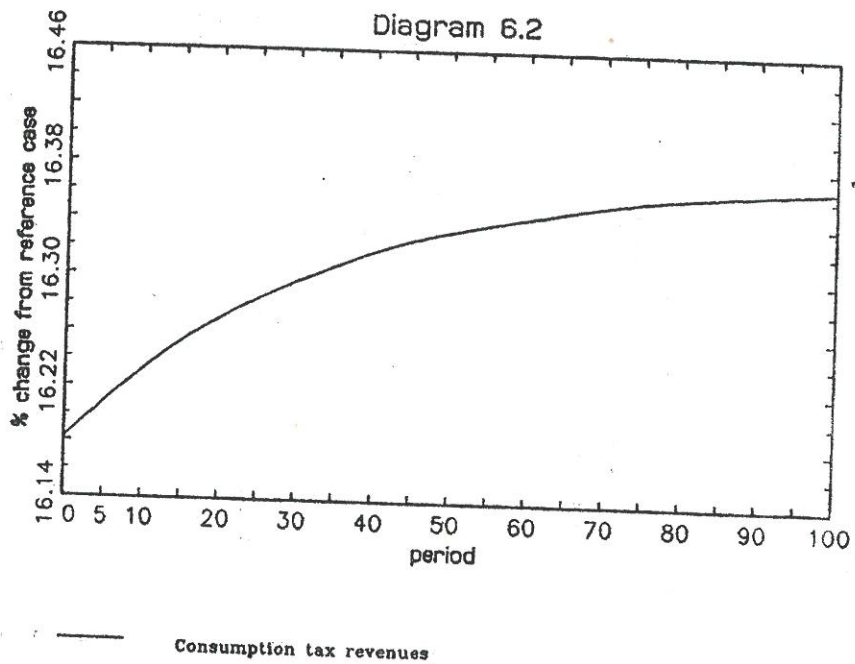
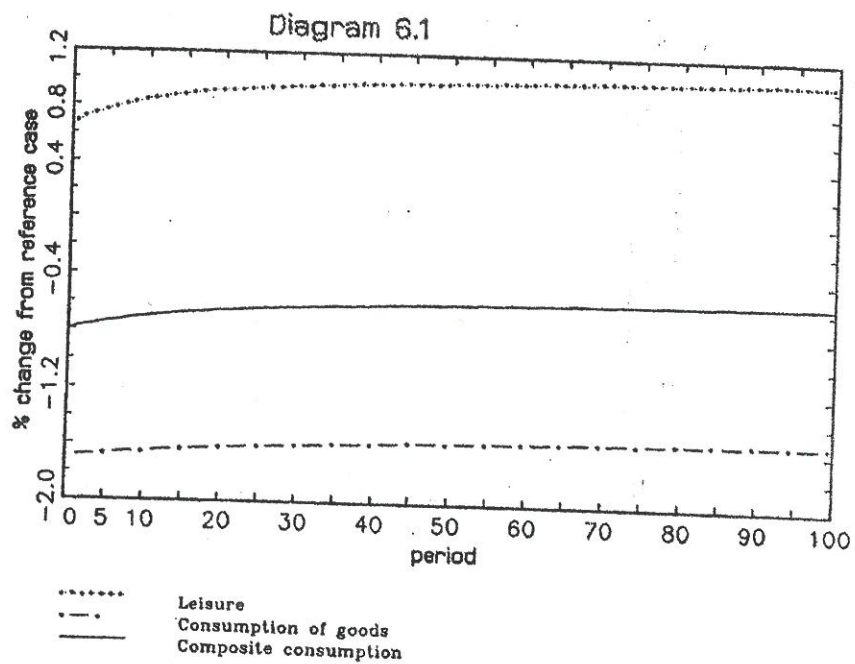


Diagram 6.3

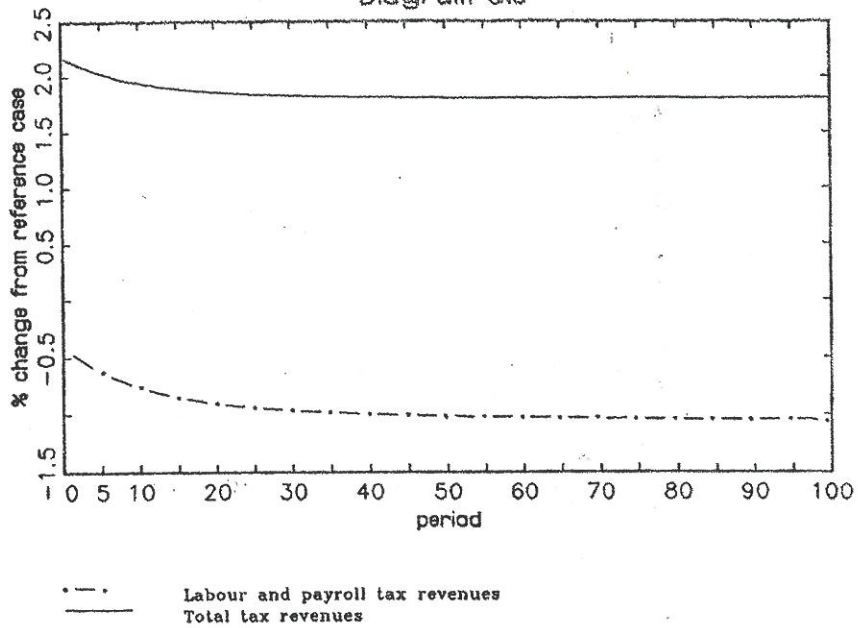


Diagram 6.4

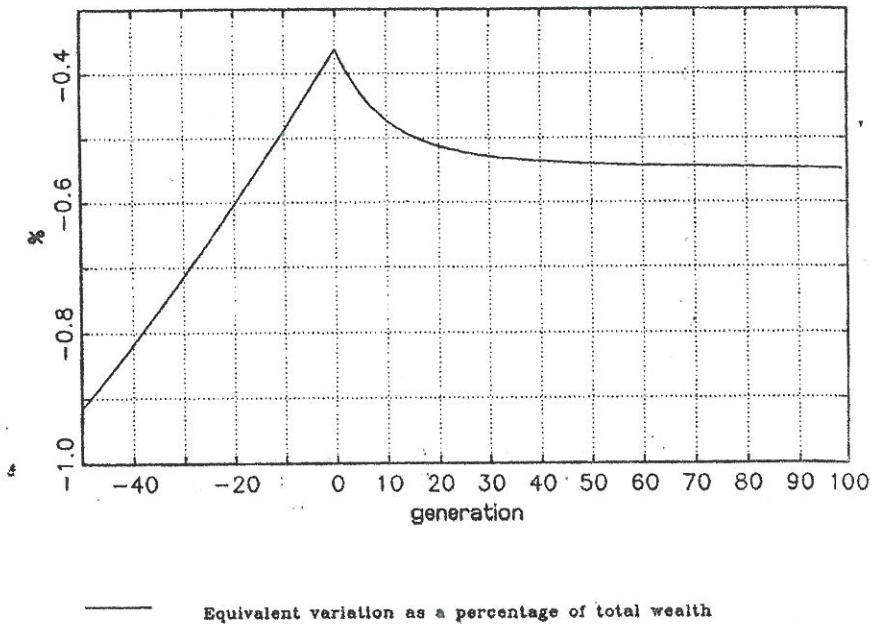




Diagram 7.1

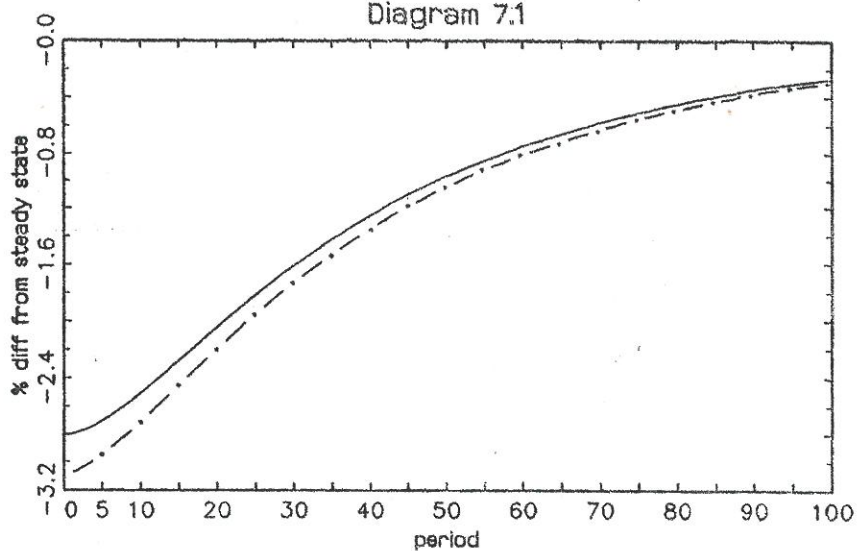


Diagram 7.2

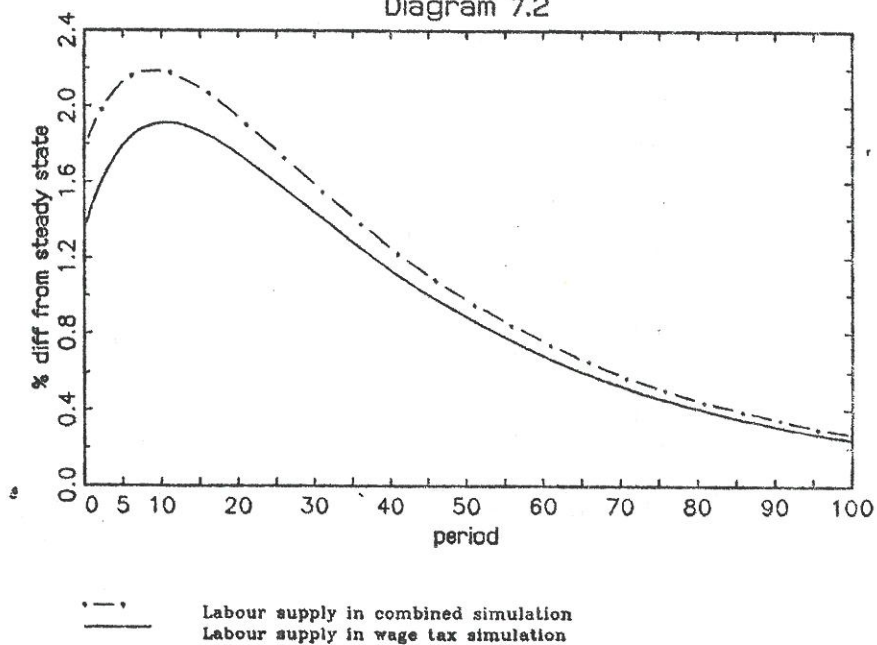


Diagram 7.3

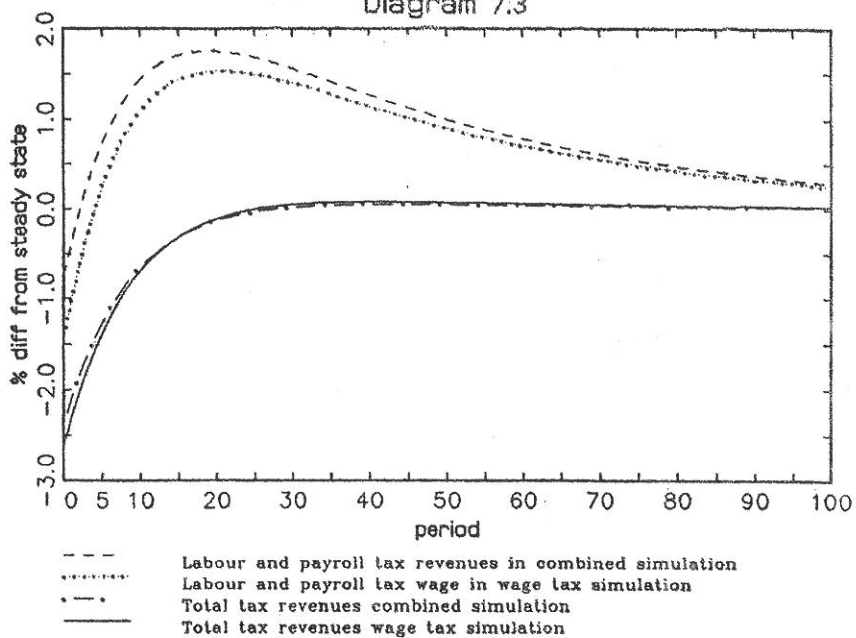
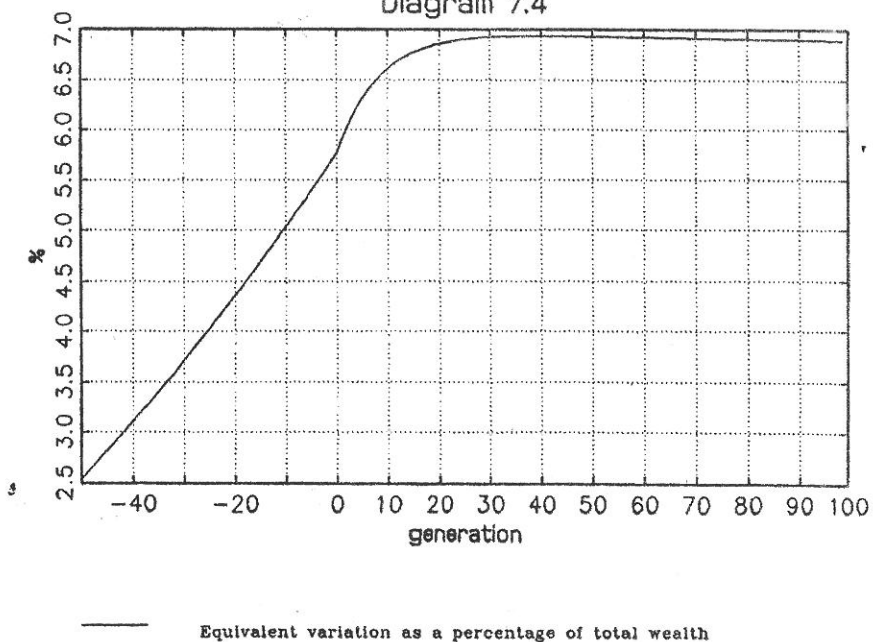


Diagram 7.4



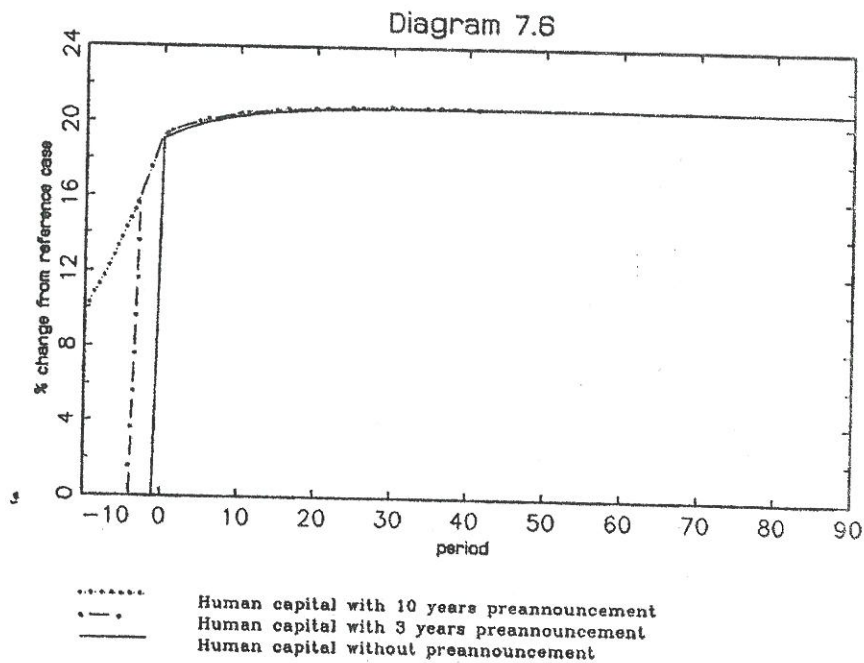
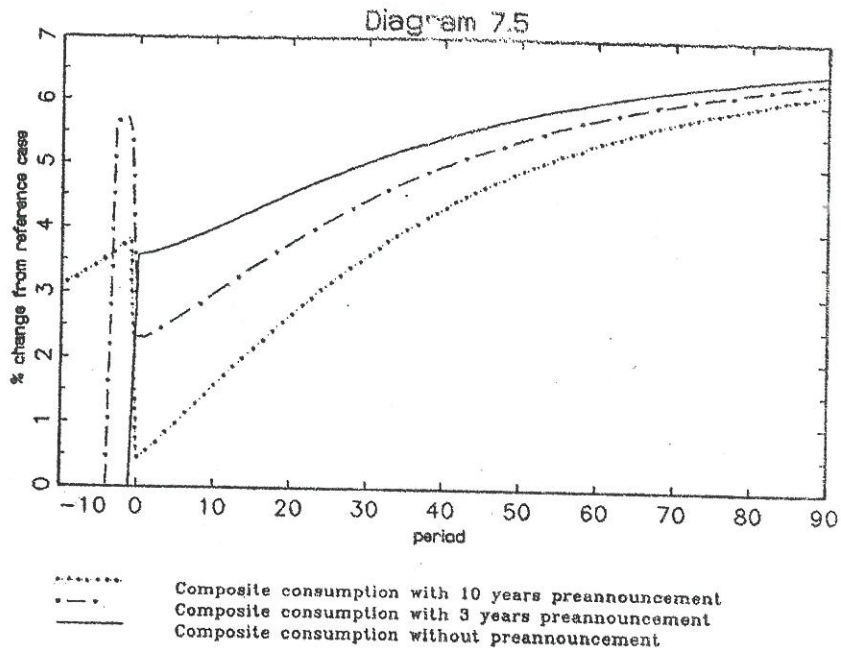


Diagram 7.7

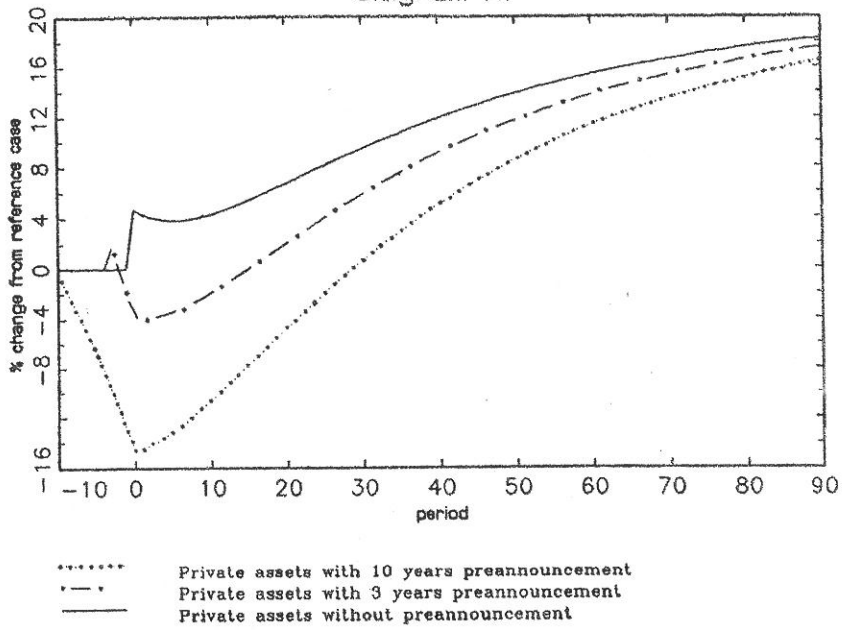


Diagram 7.8

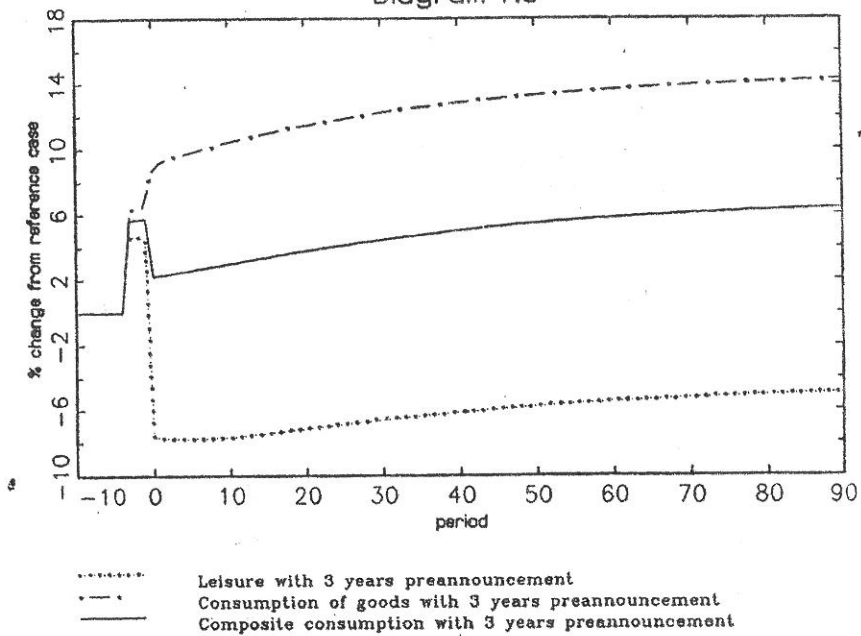


Diagram 7.9

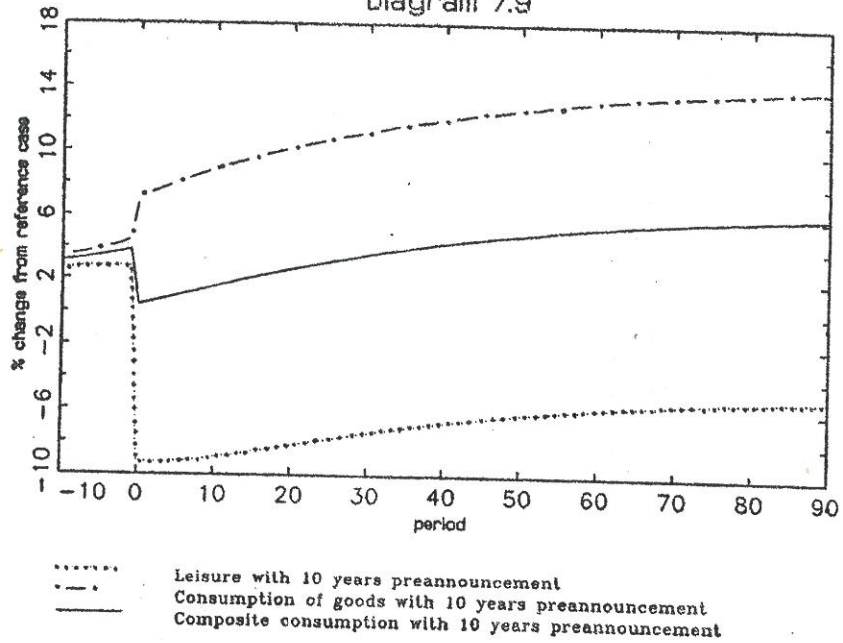


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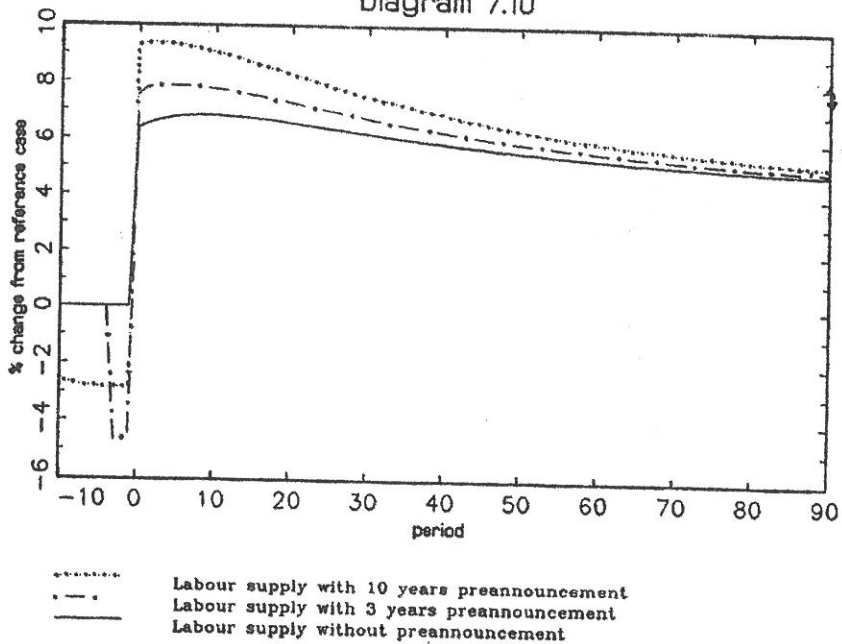


Diagram 7.11

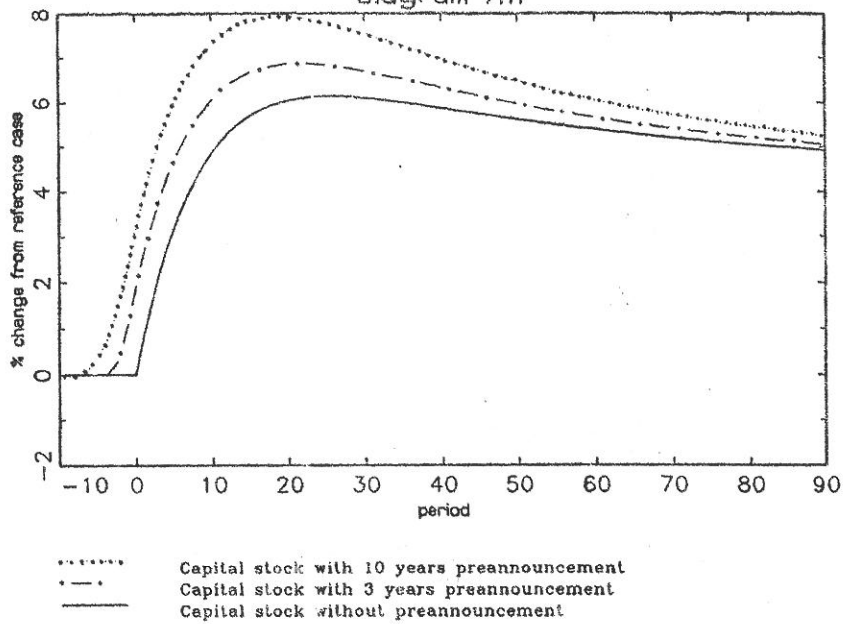


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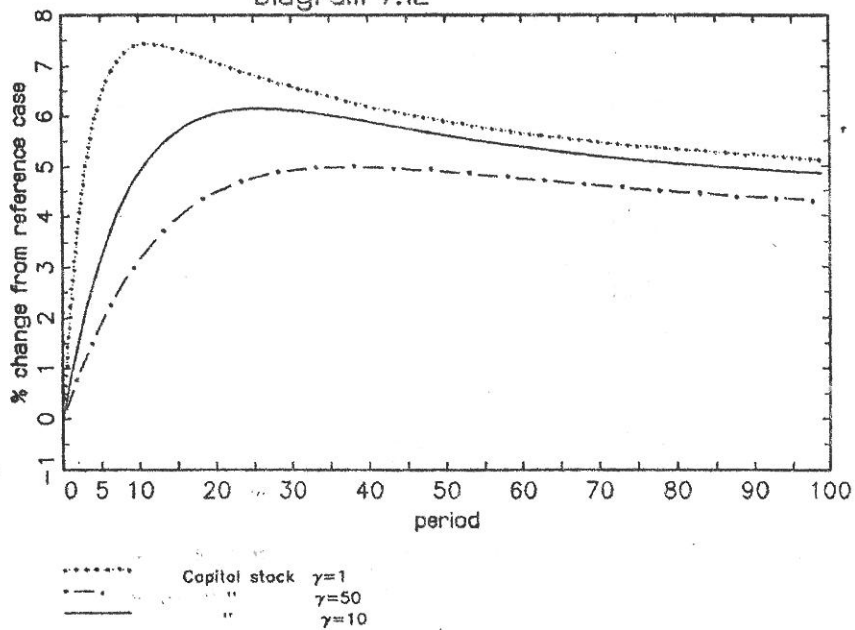


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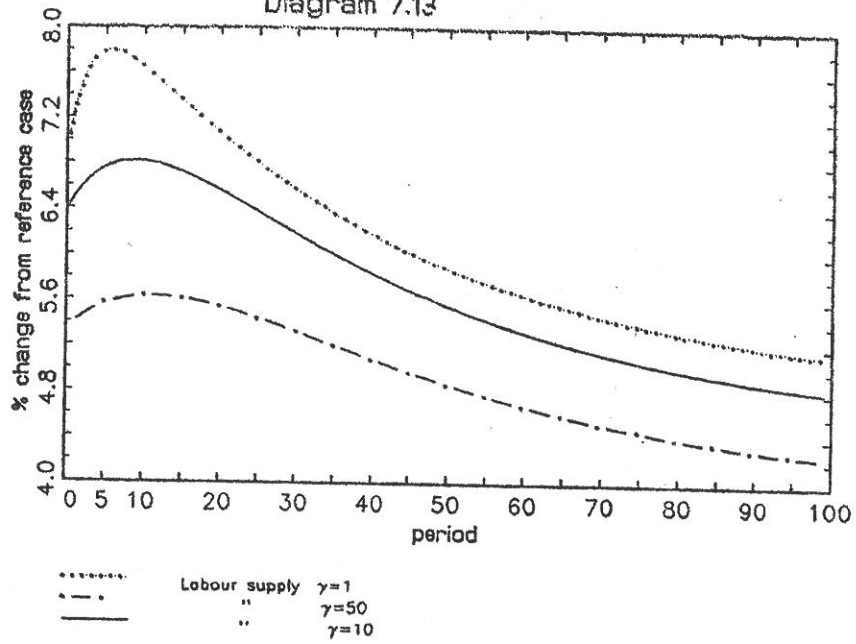


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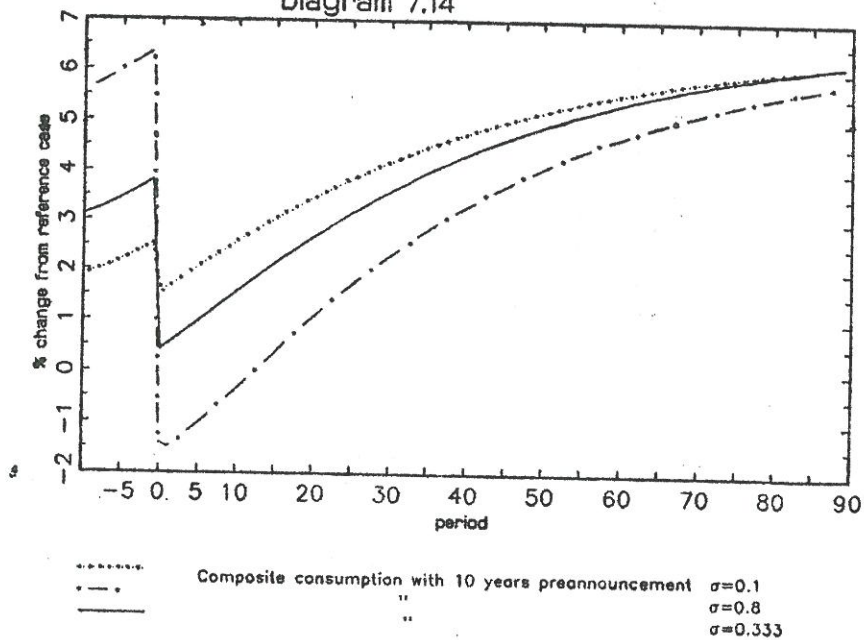


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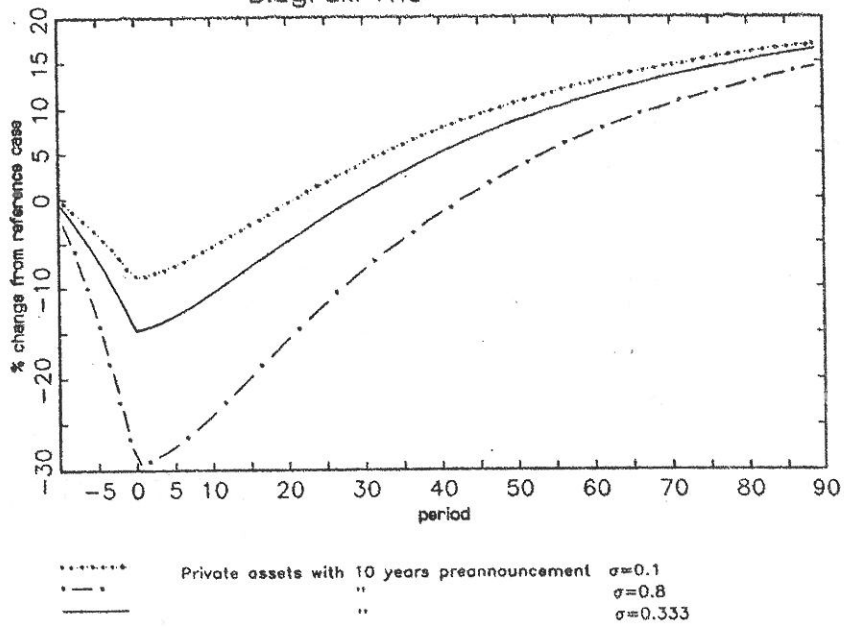
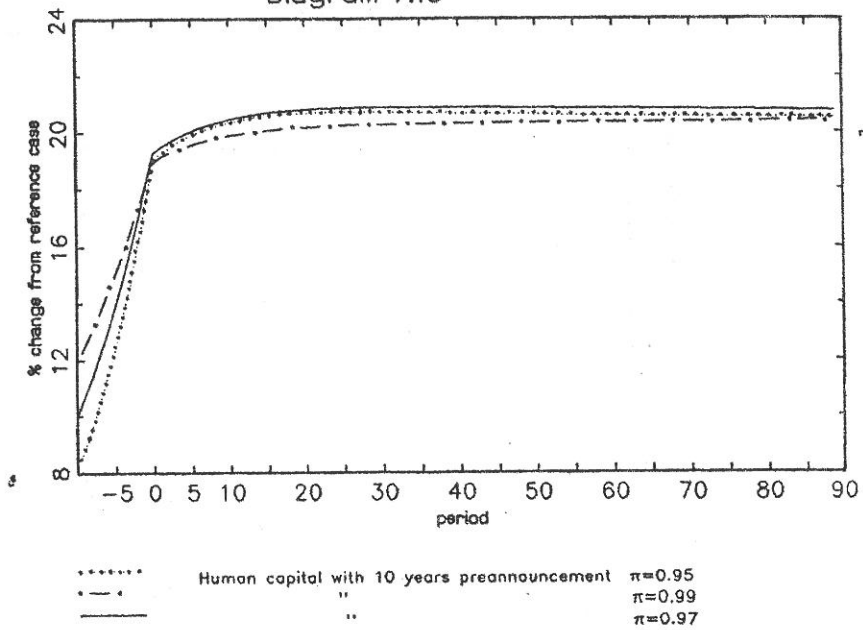


Diagram 7.16





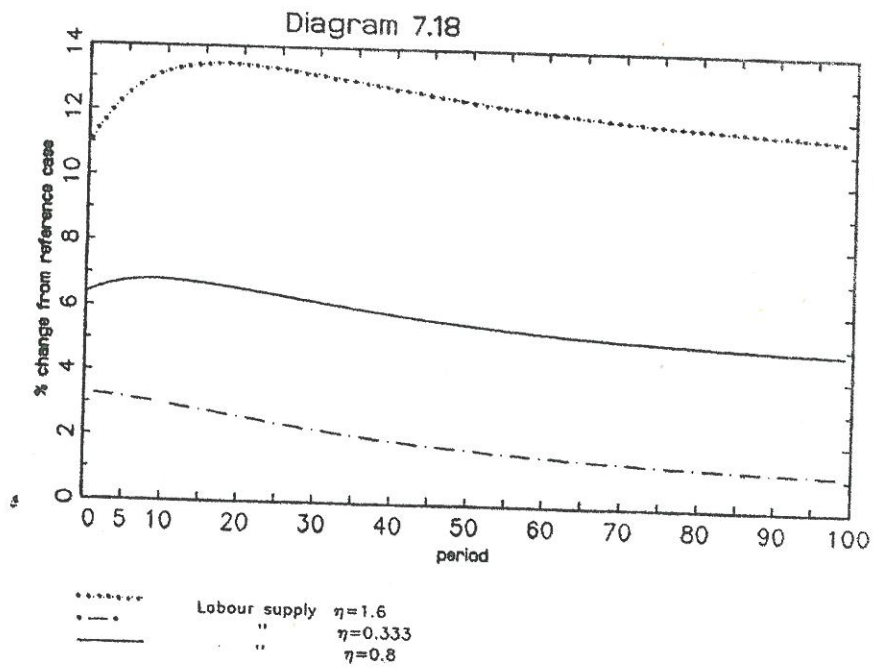
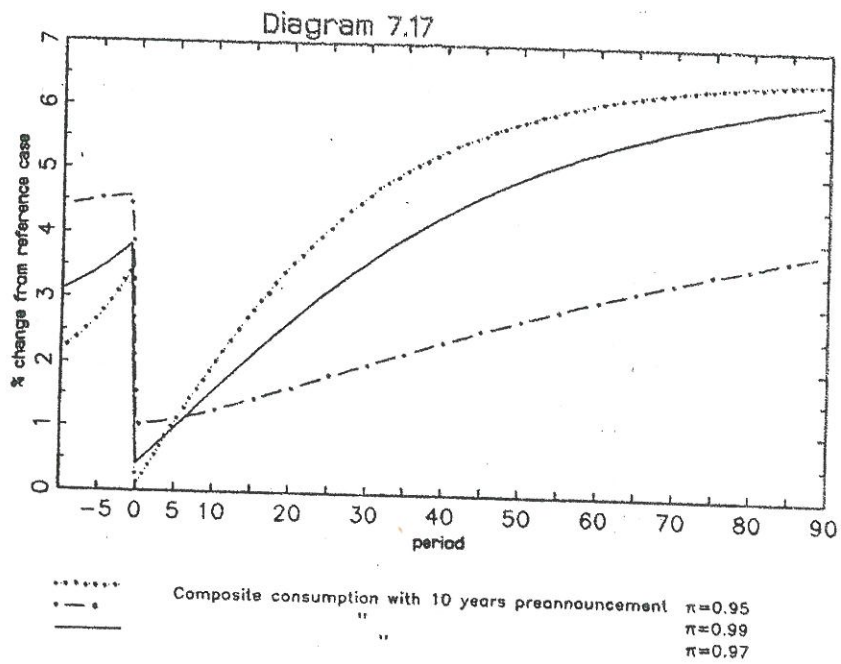


Diagram 7.19

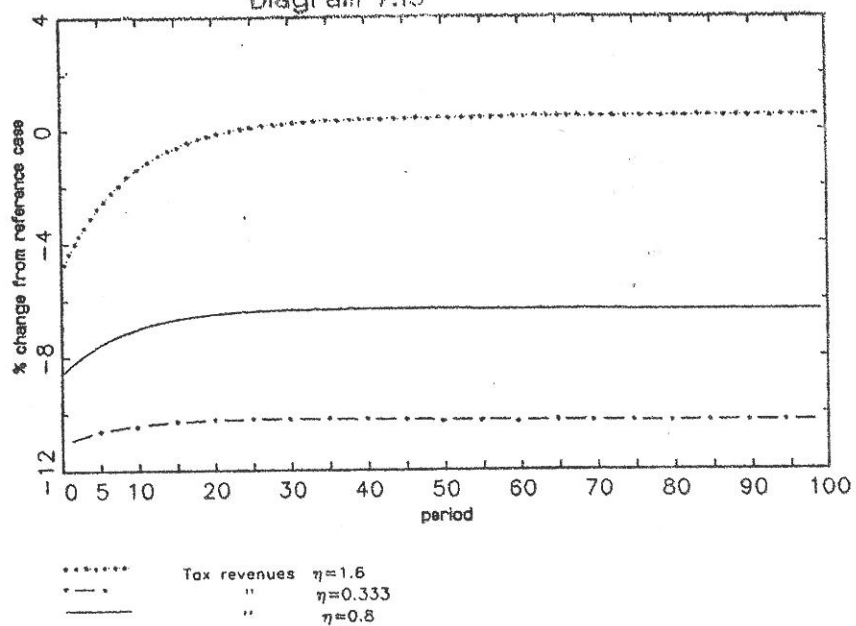


Diagram 8.1

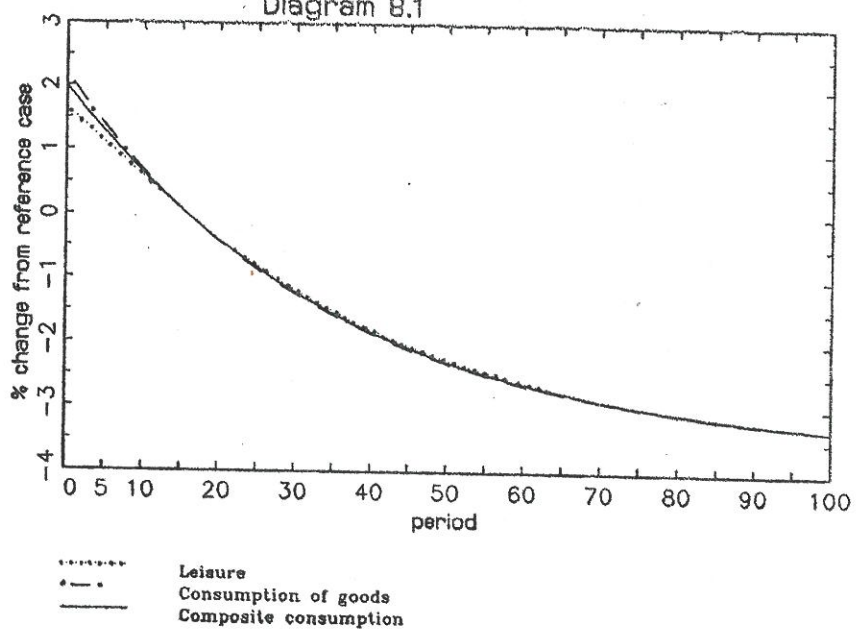


Diagram 8.2

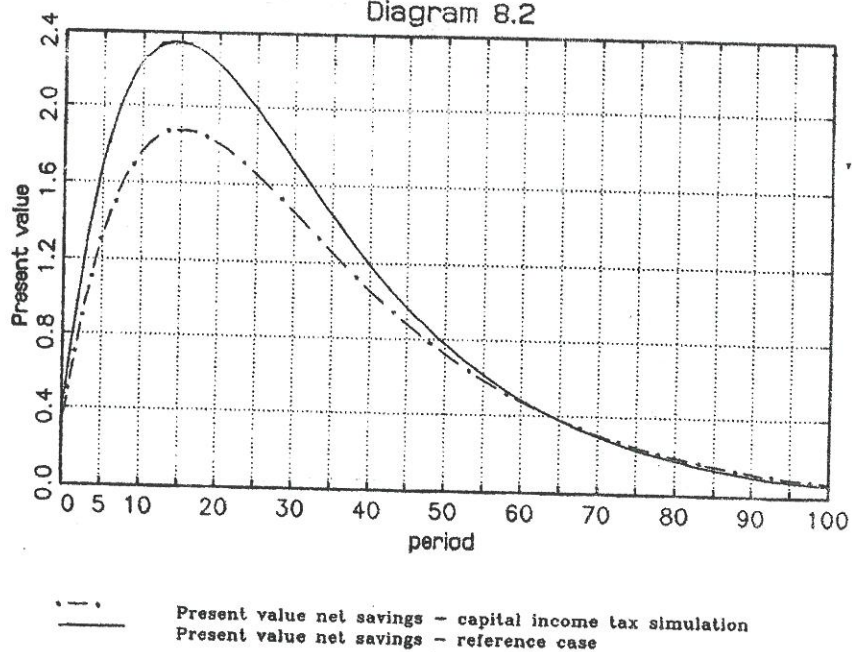
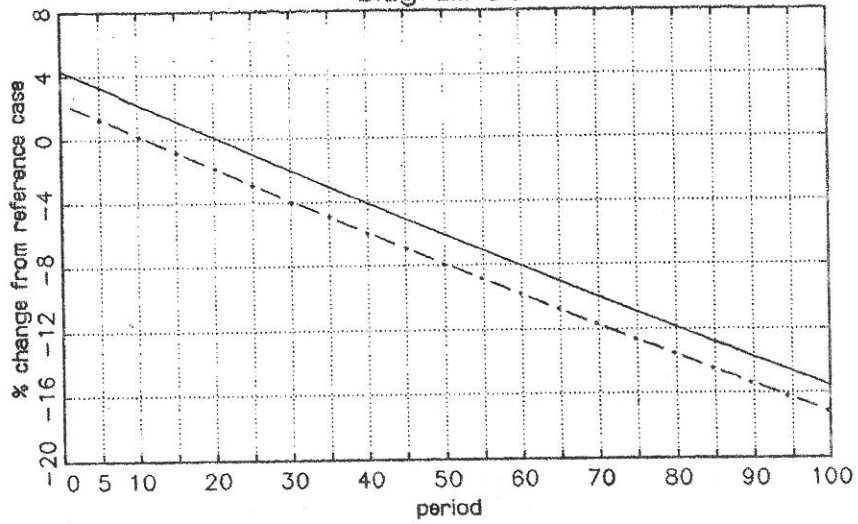
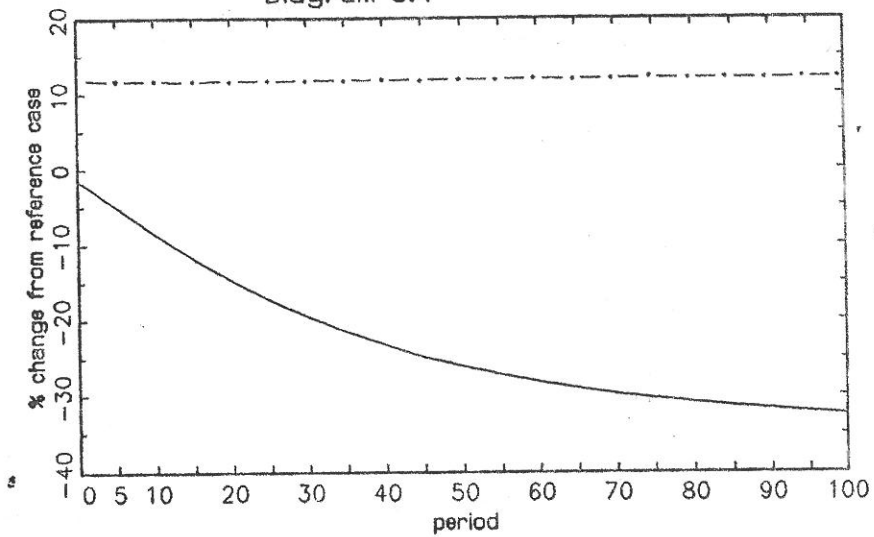


Diagram 8.3



- - - Planned composite consumption generation  $j=-30$   
 — Planned composite consumption generation  $j=0$

Diagram 8.4



- - - Human capital  
 — Private assets

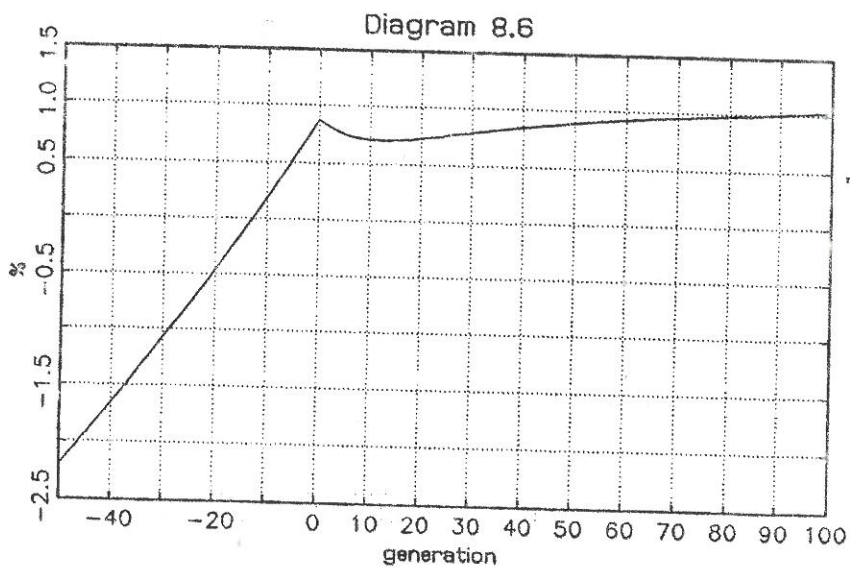
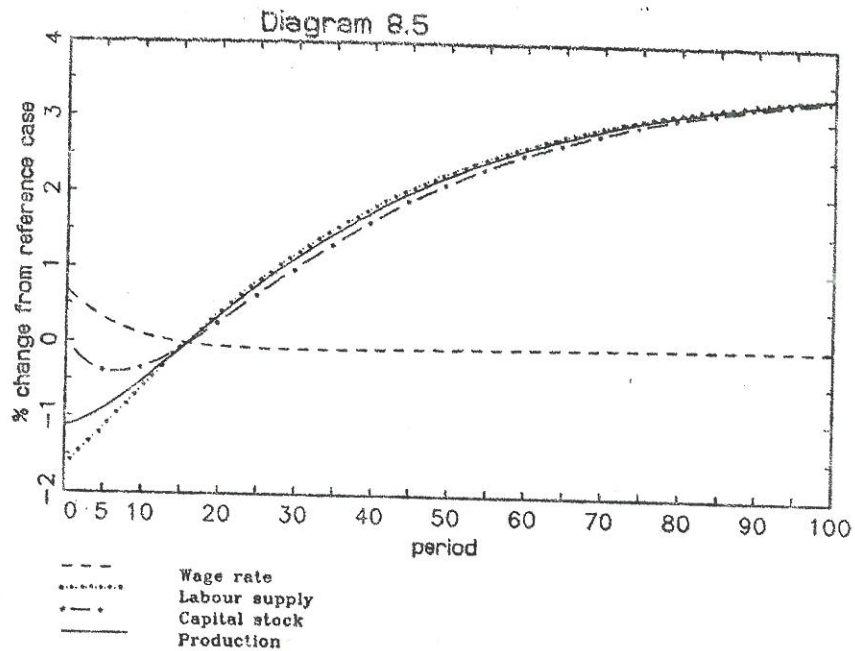


Diagram 8.7

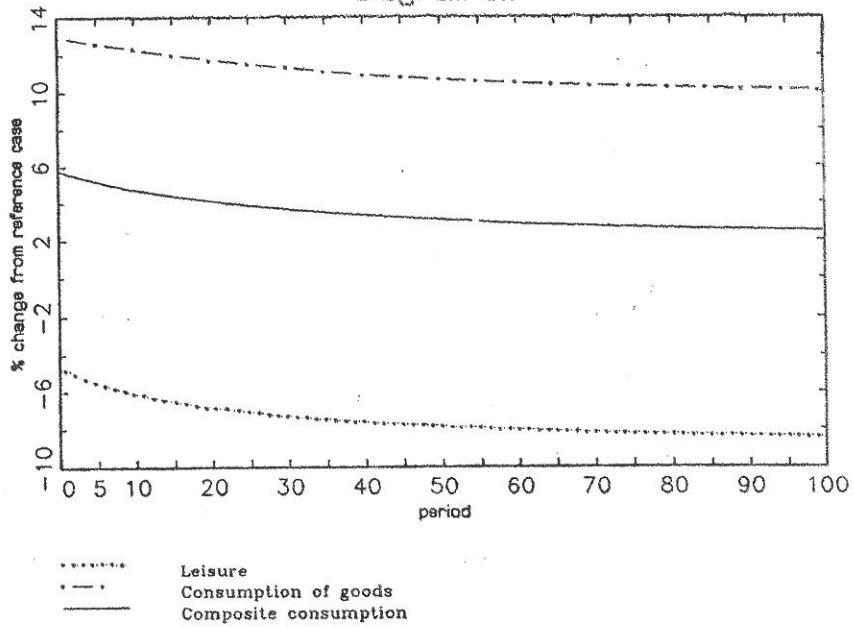


Diagram 8.8

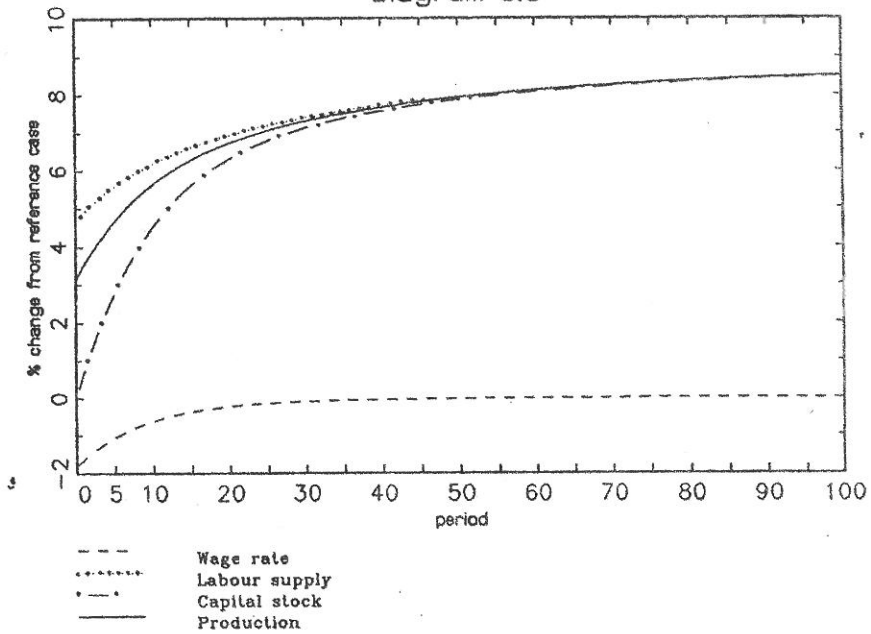
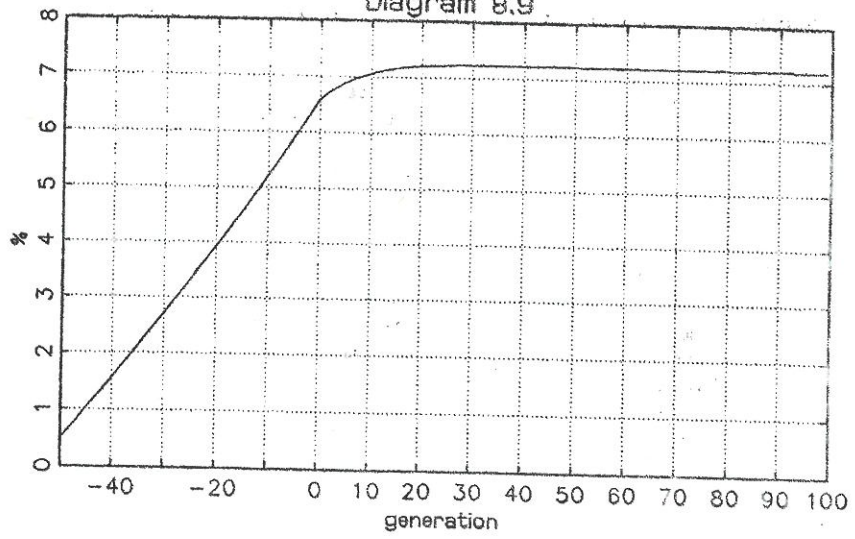


Diagram 8.9



Equivalent variation as a percentage of total wealth

Table 5.1: % Changes from the reference case in steady state values

a)	Consumption of goods (C)	Labour supply (L)	Private assets (A)	Production (Y)	Capital stock (K)	Tax revenues (TAX)
	16.4	5.7	18.8	5.7	5.7	-8.8

b)	Value of composite consumption ( $P^U$ )	Human capital (H)	Propensity to consume ( $\Delta$ )	Ideal price index ( $P^u$ )	Wage rate (W)	Value of firms (V)
	18.8	18.8	0.0	10.8	0.0	5.7

TABLE 5.2: % Changes from the reference case for selected years (t)

a)	Consumption of goods (C)	Labour supply (L)	Private assets (A)	Production (Y)	Capital stock (K)	Tax revenues (TAX)
t						
0	12.1	7.1	5.3	4.8	0.0	-11.2
1	12.3	7.3	4.8	5.2	0.9	-10.9
3	12.5	7.5	4.2	5.8	2.4	-10.4
5	12.7	7.6	3.9	6.3	3.6	-10.0
10	13.2	7.7	4.0	7.0	5.5	-9.4
25	14.3	7.4	7.2	7.3	7.1	-8.8
50	15.3	6.6	12.5	6.7	6.7	-8.7
ss	16.4	5.7	18.8	5.7	5.7	-8.8

b)	Value of composite consumption ( $P^U$ )	Human capital (H)	Propensity to consume ( $\Delta$ )	Ideal price index ( $P^u$ )	Wage rate (W)	Value of firms (V)
t						
0	14.2	17.0	-0.5	9.6	-2.7	4.8
1	14.4	17.2	-0.4	9.7	-2.4	5.2
3	14.7	17.6	-0.3	9.9	-1.9	5.8
5	14.9	17.9	-0.3	10.1	-1.5	6.2
10	15.5	18.4	-0.1	10.4	-0.8	6.9
25	16.6	18.8	0.0	10.7	-0.1	7.2
50	17.7	18.9	0.0	10.8	0.0	6.6
ss	18.8	18.8	0.0	10.8	0.0	5.7



TABLE 6.1: % Changes from the reference case for selected years (t)

a)	Consumption of goods (C)	Labour supply (L)	Private assets (A)	Production (Y)	Capital stock (K)	Tax revenues (TAX)
t						
0	-1.7	-0.7	-0.6	-0.5	0.0	2.2
1	-1.7	-0.7	-0.4	-0.5	-0.1	2.1
3	-1.7	-0.7	-0.2	-0.6	-0.3	2.1
5	-1.7	-0.8	0.0	-0.7	-0.4	2.0
10	-1.7	-0.9	0.3	-0.8	-0.6	1.9
25	-1.6	-1.0	0.9	-0.9	-0.9	1.8
50	-1.6	-1.0	1.2	-1.0	-1.0	1.8
ss	-1.5	-1.1	1.5	-1.1	-1.1	1.8

b)	Value of composite consumption( $P^U$ )	Human capital (H)	Propensity to consume ( $\Delta$ )	Ideal price index ( $P^u$ )	Wage rate (W)	Value of firms (V)
t						
0	1.3	1.7	0.0	2.1	0.3	-0.5
1	1.3	1.7	0.0	2.1	0.2	-0.5
3	1.3	1.6	0.0	2.1	0.2	-0.6
5	1.3	1.6	0.0	2.1	0.2	-0.7
10	1.3	1.5	0.0	2.0	0.1	-0.8
25	1.4	1.5	0.0	2.0	0.0	-1.0
50	1.4	1.5	0.0	2.0	0.0	-1.0
ss	1.5	1.5	0.0	2.0	0.0	-1.1

TABLE 7.1: % Changes from the reference case for selected years (t)

a)	Consumption of goods (C)	Labour supply (L)	Private assets (A)	Production (Y)	Capital stock (K)	Tax revenues (TAX)
t						
0	10.4	6.4	4.7	4.3	0.0	-8.6
1	10.5	6.5	4.4	4.6	0.8	-8.3
3	10.7	6.7	4.0	5.2	2.1	-7.9
5	11.0	6.8	3.9	5.6	3.2	-7.6
10	11.5	6.8	4.3	6.2	4.9	-7.0
25	12.6	6.4	8.2	6.3	6.1	-6.4
50	13.6	5.6	13.9	5.6	5.6	-6.3
ss	14.8	4.5	20.7	4.5	4.5	-6.4

b)	Value of composite consumption( $P^U$ )	Human capital (H)	Propensity to consume ( $\Delta$ )	Ideal price index ( $P^u$ )	Wage rate (W)	Value of firms (V)
t						
0	15.8	19.1	-0.4	11.8	-2.5	4.3
1	16.0	19.3	-0.4	12.0	-2.2	4.6
3	16.3	19.6	-0.3	12.2	-1.7	5.1
5	16.5	19.9	-0.2	12.3	-1.4	5.5
10	17.1	20.3	-0.1	12.6	-0.7	6.1
25	18.3	20.8	0.0	12.9	-0.1	6.2
50	19.5	20.8	0.0	13.0	0.0	5.5
ss	20.7	20.7	0.0	13.0	0.0	4.5

TABLE 7.2: % Changes from the reference case for selected years (t)

t	Ideal price index ( $P^u$ )			Composite Consumption (U)		
	Length of preannouncement (in years)			Length of preannouncement (in years)		
	0	3	10	0	3	10
-10	0.0	0.0	0.4	0.0	0.0	3.1
-3	0.0	0.7	0.6	0.0	5.6	3.6
0	11.8	12.0	11.9	3.6	2.2	0.4
5	12.3	12.4	12.4	3.7	2.6	1.0
10	12.6	12.7	12.7	4.0	3.0	1.5
25	12.9	12.9	12.9	4.8	4.1	3.2
50	13.0	13.0	13.0	5.8	5.4	4.9
ss	13.0	13.0	13.0	6.9	6.9	6.9

TABLE 8.1: % Changes from reference for selected years (t)

a)	Consump- tion of goods (C)	Labour supply (L)	Private assets (A)	Produc- tion (Y)	Capital stock (K)	Tax revenues (TAX)
t						
0	2.2	-1.7	-1.5	-1.1	0.0	2.3
1	2.0	-1.6	-2.2	-1.1	-0.1	2.2
3	1.7	-1.4	-3.6	-1.0	-0.3	2.1
5	1.4	-1.1	-5.1	-0.9	-0.4	2.0
10	0.7	-0.6	-8.6	-0.5	-0.3	1.9
25	-0.9	0.8	-17.3	0.8	0.6	1.9
50	-2.3	2.3	-26.1	2.2	2.1	2.0
ss	-3.6	3.7	-34.3	3.7	3.7	2.3

b)	Value of composite consump- tion( $P^u$ )	Human capital (H)	Propen- sity to consume ( $\Delta$ )	Ideal price index ( $P^u$ )	Wage rate (W)	Value of firms (V)
t						
0	2.2	11.9	-6.6	0.3	0.7	-1.4
1	2.1	11.9	-6.6	0.2	0.6	-1.3
3	1.7	11.8	-6.6	0.2	0.4	-1.3
5	1.4	11.8	-6.6	0.1	0.3	-1.2
10	0.7	11.7	-6.6	0.0	0.1	-0.8
25	-0.9	11.7	-6.7	0.0	-0.1	0.4
50	-2.3	11.9	-6.7	0.0	-0.1	1.7
ss	-3.6	12.1	-6.7	0.0	0.0	3.0

TABLE 8.2: % Changes from reference for selected years (t)

a) t	Consump- tion of goods (C)	Labour supply (L)	Private assets (A)	Produc- tion (Y)	Capital stock (K)	Tax revenues (TAX)
0	13.0	4.7	3.2	3.1	0.0	-5.9
1	12.9	4.9	2.1	3.5	0.7	-5.7
3	12.8	5.3	0.1	4.1	1.8	-5.5
5	12.6	5.6	-1.6	4.7	2.8	-5.3
10	12.3	6.2	-5.0	5.6	4.5	-5.0
25	11.5	7.2	-11.2	7.1	6.8	-4.7
50	10.7	7.9	-16.1	7.9	7.9	-4.7
ss	9.8	8.7	-21.3	8.7	8.7	-4.8

b) t	Value of composite consump- tion( $P^u$ )	Human capital (H)	Propen- sity to consume ( $\Delta$ )	Ideal price index ( $P^u$ )	Wage rate (W)	Value of firms (V)
0	18.6	33.2	-7.0	12.1	-1.8	2.9
1	18.6	33.3	-6.9	12.2	-1.6	3.2
3	18.4	33.5	-6.9	12.4	-1.3	3.8
5	18.3	33.7	-6.8	12.5	-1.1	4.3
10	18.0	34.0	-6.8	12.7	-0.6	5.2
25	17.3	34.3	-6.7	12.9	-0.2	6.5
50	16.4	34.3	-6.7	12.9	0.0	7.3
ss	15.5	34.3	-6.7	13.0	0.0	8.0

