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MIMER - Documentation

CONTENTS

0 NON-TECHNICAL SUMMARY

MIMER[1] is a dynamic general equilibrium model of the Swedish economy featuring overlapping generations of households. The primary purpose of the model is to analyze the long-run evolution of the Swedish economy and the public finances in Sweden. In MIMER, the Swedish economy is modelled as a small open economy where goods are traded in an internationally competitive market, and where the evolution of the Swedish economy has no effect on the prices in international markets.

The economy consists of a production sector, a household sector, a public sector including a public pension system and a private pension system (modelled as the Swedish premium pension system anno 2014). A central feature in the model is that households and firms make rational decisions, by optimizing their respective target functions given the restrictions that they face. This assumption stands in contrast to models where the behaviors of households and firms are completely passive, meaning that their behaviors do not change when the economic environment that they face changes. Furthermore, households and firms are forward-looking, since both their restrictions and target functions contain future variables. Consequently, the households' and firms' expectations about the future of the economy are central for their decisions. It is assumed that these expectations are rational. It is assumed, for simplicity, that households and firms have perfect foresight about the future, meaning that they can correctly project future economic developments and use this information in their decision making. This implies that the model projection neither depends on any present or future business cycle fluctuations, nor on any mistakes in the households' and firms' expectations about the future. In addition, the model contains idiosyncratic uncertainty about how long an individual household will live. This feature, together with the feature that all households have a finite life-span, make the households value the present more than the future.

It is assumed that capital is completely mobile across borders, and that the Sweden is not large enough to affect the international returns to capital. Hence, the interest rate is exogenously determined in the model. Furthermore, MIMER is a real model, meaning that only quantities and relative prices, rather than nominal variables, are affecting the economy. In addition, it is assumed that the labor market is perfectly competitive, implying that wages are perfectly competitive and that the economy is always at full employment. Business cycle movements are not modelled.

Below follows a brief description of the different sectors in the economy.

The Production Sector

There are three types of goods in the economy, one type of intermediate good and two types of final goods. The intermediate good is used as sole input into the two types of final goods. The intermediate good is created using capital and labor as inputs. The first type of final good is the private final good, which is used by the households for consumption and firms for capital investment, as well as for exports. The second type of good is the public good, which is used for consumption by the public sector.

The production technology differs between the two types of final goods in that evolution of productivity with which the goods are created differs. It is assumed that productivity grows faster in the private sector than in the public sector. This implies that the price of public goods relative

[1] Modell för Intergenerationella MakroEkonomiska Räkenskaper

to private goods increases over time. Furthermore, it implies that an increase in public production crowds out private production, which dampens GDP growth.

The two types of final good firms also differ in their assumptions about firm characteristics. The private production good firms operate on a market characterized by perfect competition. In contrast, the output of the public good firms is decided politically, and hence decided by the model user. Given the production decision, the firms minimize their costs of production. An increase in public production leads to a higher demand for intermediate goods, which puts an upwards pressure on the price of such goods which, *ceteris paribus*, leads to dampened private production. The productivity growth in the private production sector is the driver of the wage growth in the economy.

Households

Each year, new cohorts of men and women enter the economy. During their life span, they consume and work. They die with some probability each year until they have reached the age of 105. after which they die with certainty. A household receives utility from consumption, from leisure, and from leaving a bequest. A household's decisions to consume, save, work and to leave a bequest are forward-looking and are made taking into account the household's budget constraint, its preferences regarding when to spend and its life-expectancy. On the aggregate level, there are 106 generations living simultaneously in the model, and as time goes by, some of the generations die and are replaced by new generations. The household start their economic life at the age of 15. Before that, their consumption is assumed to be a part of their parents' consumption.

The income of an individual household consists of wage income, transfers from the public sector, returns on the household's savings and some inheritance from previous generations. The household's wage income, some of its received transfers and its asset returns are taxed. The age of retirement is exogenous in the model, and the household chooses how many hours to work each year until it retires. After retirement, the household receives pension income instead of wage income. The pension income is also taxed. Since the pension income is smaller than the household's wage income, the household builds up a positive net worth during their work life, which they use for consumption after they have retired. In addition, the households also use previous savings to leave a bequest, since they receive utility from leaving a bequest to younger generations.

The labor supply decision of the household is affected by the household's hourly wage, by the amount of time allocated on studies, by the family size and by the household's health. A higher wage has positive effect on hours worked, since the wage reflects the relative price of leisure, which increases as the wage increases.[2] As the household's productivity grows over the course of their life span, their received wage also increases, which leads to an increase in the hours worked in the beginning of the household's work-life. An increased family size will increase the utility of consumption, since the kids' consumption is interpreted in the model as consumption of the parents. This will lead to an increase in hours worked. At the same time, an increase in the number of adults per household will lead to a lower utility of consumption due to increasing returns to scale (for example, the size of the living space per person can be smaller with two grown-ups in a family as compared to one). A deteriorated health leads to lower utility of

[2]The higher wage does also have a negative effect on hours worked since a higher wage increases the household's income which in turn leads to a higher demand for leisure.

consumption relative to the consumption of leisure.

The allocations of the individual households are aggregated into aggregate household consumption, aggregate savings and hours worked in the economy.

The Public Sector

The public sector can be divided into two different autonomous parts. The first is the consolidated central and local government sector (henceforth called the government sector). The second is the public pension system. Together they form the consolidated public sector.

The Government Sector

The government sector delivers welfare services and other goods to the public, for example police protection, a rule of law and medicine. Furthermore, the government also invests and pays out transfers to households and abroad. In addition, it carries a debt, on which interest is paid each year.

The public consumption can be divided into individual consumption and other public consumption. These parts behave differently in the model. The amount of individual consumption differs depending on the individual's age and sex. The behavior of such consumption over time is demographically driven. Since the amount of received welfare services tend to increase with a household's age, an ageing population will lead to an increase in individual public consumption. The projection method of the public consumption may however differ in the model depending on the model user's preferences. The first method is to let the consumption per individual of a particular age and sex be constant in terms of output per capita. The second method is to let the individual public consumption volume per individual of a certain age and sex grow with a constant rate over time. In both cases, an ageing population over time leads to higher public consumption.[3]

The transfers to the households are projected to increase with the wage growth in the economy. As for public consumption, the transfers to the households are higher for old households than for young households, implying that a demographic shift towards an older population leads to higher aggregate transfers. The transfers that goes abroad are assumed to be constant in relation to output.

The lion's share of the government income consists of tax income. The households pay taxes on their labor income, on their transfer and pension income, as well as on their capital income. Furthermore, they pay a tax on their consumption. In addition, the intermediate good firms pay a tax on their profits, and the public production firms pay a tax on their inputs of production.

The Public Pension System

The public pension system is modelled to capture the features of the present Swedish public pension system anno 2014.[4] During a household's work life, it pays fees to the public pension system, where the size of the fee is determined by their wage income. Furthermore, the firms also

[3]Of course, this result does depend on assumed the growth rate of public consumption volumes over time. With a realistically calibrated growth rate however, the absence of any political reforms will lead to higher public consumption over time.

[4]The so-called "balance mechanism" which is present in the Swedish public pension system is however not modelled due to technical reasons.

pay a fee into the public pension system, which size is also determined by the individual's wage income. The fees are transformed into individual pension assets that grow with the wage growth in the economy until the household retires. At that point in time, the individual assets are transformed into a pension payout scheme, in which the individual pension payouts grow with the wage growth in the economy minus 1.6 percent per year.

The Private Pension System

The private pension system is modelled to capture the features of the Swedish premium pension system anno 2014, and is assumed to include both premium pensions and other private pension schemes. The firms pay fees to the system based on the wage income of the households. The fees are transformed into individual pension assets, which grow with the returns to capital in the economy (as opposed to the public pension system where they instead grow with the wage growth). After a household has retired, it receives a pension income from the system each year. The size of the payment is constant over time, and set such that all pension assets are paid out as a generation completely dies out.

The Foreign Sector

Since the economy is assumed to be small and completely open towards the rest of the world, the return to capital is given by the global returns to capital, i.e., the global interest rate. This implies that neither the Swedish demand for goods nor the Swedish demand for capital necessarily needs to equal the supply of capital or goods within Sweden. Any imbalance between the demand and supply of goods is captured by the balance of trade, i.e., by the net exports, while any imbalance between the demand and supply of capital is captured by the current account.

1 INTRODUCTION

In this documentation, the dynamic general equilibrium model MIMER is described. The primary purpose of MIMER is to make economic and public finance projections, and to evaluate long-run effects of changes in fiscal policy. The model can however also be used for other purposes, such as the the analysis of the effect of an increased average life expectancy on households' savings decisions.

MIMER is a quantitative small-open-economy dynamic general equilibrium model featuring overlapping generations of households. Such models have its origin in Auerbach and Kotlikoff (1987), and as been developed further by, among others, De Nardi et al. (1999), Storesletten (2000), Eisensee (2006), Attanasio et al. (2007) and Kotlikoff et al. (2007). For Sweden, such a model has previously been developed by Sundén (2002). Examples of such models used in policy institutions in their analysis of long-term macroeconomic and public finance effects, are the DREAM model in Denmark (see DREAM (2008)) and the GAMMA model in the Netherlands (see Draper and Armstrong (2007)).

The rest of the text is organized as follows: Section 2 describes the model in detail, Section 3 describes the calibration and Section 4 describes how the model is solved. Appendix A contains all equilibrium equations.

2 THE MODEL

MIMER consists of five sectors: households, production, the public sector, the premium pension system and the foreign sector. Households and firms optimize their respective target functions subject to the restrictions that the agents face. The public and private pension systems are modelled to capture the Swedish pension system anno 2014, with the simplification that the rules in the private pension system, assumed to be a consolidated system all private pension systems including the premium pension system, follows the rules of the Swedish premium pension system.[5] The rest of the public sector, i.e., the consolidated central and local government sector, follow a fiscal policy which is decided by the model user depending on his or her preferences. Sweden is modelled as a small open economy that is assumed to not have any effect on the rest of the world, more specifically the global return to capital. Therefore, the interest rate is exogenously determined in the model. Any potential imbalances that might ascend between the demand and supply of private consumption goods and capital are absorbed by the foreign sector.

Below, the five sectors are described in detail. Before that, the demographic assumptions in the model are described.

2.1 Demography

Each time period, t, in MIMER equals a year. In the model, a number of individuals of ages $i \in \{0, 1, ..., 105\}$ and sexes $k \in \{1, 2\}$ die, immigrate and emigrate each year. Furthermore, new individuals of both sexes are born every year. It is assumed that individuals who die within a given year do so in the beginning of that year, before they have time to consume and enjoy leisure. The probability to survive is assumed to be heterogeneous across sex, age and time. Within the same cohort however, i.e., for individuals with the same sex and age in a given year, the survival probabilities are assumed to be identical. The survival probability, conditional on an individual being alive at age $i-1$ is denoted by s_{tik}. The unconditional probability to survive age i, π_{tik}, can be written as

$$\pi_{tik} = \prod_{j=1}^{i} s_{t-i+j,j,k}. \tag{1}$$

The population size of a cohort is denoted by N_{tik}. The immigration rate and the emigration rate are denoted by im_{tik} and em_{tik}. These rates are calculated as the number of immigrated and emigrated relative to the cohort population. Hence, it is possible to write the gross growth rate of the cohort size, n_{tik}, as

$$n_{tik} = s_{tik} + im_{tik} - em_{tik}, \quad i \geq 1. \tag{2}$$

Using the cohort growth rate, the evolution of the cohort size over time can be written as

$$N_{tik} = N_{t-1,i-1,k} n_{tik}, \quad i \geq 1 \tag{3}$$

$$N_{t0k} = \sum_{i=0}^{105} born_{tik} + immi_{t0k} - emmi_{t0k} \tag{4}$$

[5] Despite the choice of modelling year of the pension system, it is possible to partly capture the so called ”pensionsöverenskommelsen”, which is gradually increasing certain age limits in the pension system by letting the retirement gradually increase over time. The model can however not fully capture this reform, or any endogenous retirement response to it.

where $born_{tik}$ denotes the number of individuals born in year t by a mother of age i, $immi_{t0k}$ is the number of immigrated individuals of age zero, and $emmi_{t0k}$ is the number of emigrated individuals of age zero.

It is assumed that individuals die with probability 1 in the beginning of the year they would turn 106. Hence, the total amount of individuals in the economy N_t can be written as

$$N_t = N_{t1} + N_{t2} = \sum_{k=1}^{2} \sum_{i=0}^{105} N_{tik}. \tag{5}$$

2.2 Households

The households are rational and forward looking. There is a heterogeneity between the different cohorts, here defined as individuals of the same sex that are born within the same year. This heterogeneity is however not in terms of preferences, but rather in terms of their productivity and the wage they face, as well as in received non-labor income and survival probabilities. A household of sex k that is born in year j maximize its life-time utility U_k^j at the age of $i = 15$. This is done by choosing consumption c_{ik}^j, hours worked l_{ik}^j, financial asset holdings a_{ik}^j and the size of any left bequest b_{ik}^j for all ages over the life-cycle. U_k^j is given by

$$\begin{aligned} U_k^j &= \sum_{i=15}^{106} \beta^{i-15} \frac{\pi_{j+i-1,i-1,k}}{\pi_{j+14,14,k}} \left[s_{j+i,i,k} u(c_{ik}^j, l_{ik}^j) + (1 - s_{j+i,i,k}) \, \phi \ln \left(b_{ik}^j \right) \right], \\ u(c_{ik}^j, l_{ik}^j) &= h_i \ln \left(c_{ik}^j \right) + \psi \frac{\left(1 - edu_{ik} - l_{ik}^j \right)^{1-\omega}}{1 - \omega} \end{aligned} \tag{6}$$

where β is the household's implicit discount factor. With probability $s_{j+i,i,k} \pi_{j+i-1,i-1,k} / \pi_{j+14,14,k}$ the household survives age i, in which case it receives utility from consumption and leisure. With probability $(1 - s_{j+i,i,k}) \pi_{j+i-1,i-1,k} / \pi_{j+14,14,k}$ it does not survive, in which case it receives utility from leaving a bequest. The utility from consumption changes with age over the life cycle with the index variable $h_i = equiv_i \cdot health_i$. $health_i$ is a health index that captures a deterioration of the household's health over time. This approach of modelling health follows Domeij and Johannesson (2006) who model health and consumption as complementary goods using a similar health index, and show that it improves the fit of their model's life-cycle consumption to Swedish data compared to a model without such an index. $equiv_i$ captures the household size, and the fact that the propensity to consume tend to increase with the number of children in the household (see Kotlikoff et al. (2007)). edu_{ik} captures the fact that the household spends some of its time on education, in which case those hours cannot be allocated on labor.

The household retires at the exogenously given age $Rage_j$. In a given year $t = j + i$, the restrictions that a household faces before it has retired, i < $Rage_j$ are the following:

$$\begin{aligned} (1 + \tau^{cp}) \, c_{ik}^j + a_{ik}^j \leq w_t e_{ik} l_{ik}^j \left(1 - \tau^l - \tau^{ndc_l} \right) + (1 + r_t (1 - \tau^a)) \, a_{i-1,k}^j \\ + tr_{ik}^{j,ntx} + tr_{ik}^{j,tx} \left(1 - \tau^{tr} \right) + beq_{ik}, \quad i < Rage_j \end{aligned} \tag{7}$$

$$b_{ik}^j \leq (1 + r_t (1 - \tau^a)) \, a_{i-1,k}^j \tag{8}$$

$$0 \leq l_{ik}^{j} \leq 1 - edu_{ik} \tag{9}$$

$$a_{ik}^{j} \geq 0 \tag{10}$$

where r_t is the return on the household's asset holdings, e_{ik} is the household's age-dependent productivity, w_t is the productivity-adjusted wage that the household faces, $tr_{ik}^{j,ntx}$ and $tr_{ik}^{j,tx}$ are untaxed and taxed transfers received from the public sector and beq_{ik} is the received inheritance from households who did not survive the period. τ^{cp}, τ^{a}, τ^{l} and τ^{tr} and are tax rates on private consumption, returns on asset holdings, labor income and transfers. τ^{ndc_l} is the fee paid to the public pension system.

For a retired household, i.e., a household of age $i \geq Rage_j$, the budget restriction is instead given by

$$\begin{aligned}(1+\tau^{cp})\, c_{ik}^{j} + a_{ik}^{j} \leq (1-\tau^{p})\, p_{ik}^{j}(l_k^h) + (1 + r_t\,(1-\tau^{a}))\, a_{i-1,k}^{j} \\ + tr_{ik}^{j,ntx} + tr_{ik}^{j,tx}\left(1-\tau^{tr}\right) + beq_{ik}, \quad i \geq Rage_j\end{aligned} \tag{11}$$

where $p_{ik}^{j}(l_k^h)$ denotes received pensions and τ^{p} is the rate of the tax paid on the same pensions. Note that the received pension in a given year is a function of the hours worked during the household's lifetime, $l_k^h = l_{15,k}^{j}, ..., l_{Rage_j-1,k}^{j}$. Furthermore, it is worth noting that it is assumed that households that have reached their age of retirement have exited the labor market.

The pension payments can be divided into public pensions, $p_{ik}^{j,ndc}(l_k^h)$, and private pensions[6], $p_{ik}^{j,dc}(l_k^h)$, such that

$$p_{ik}^{j}(l_k^h) = p_{ik}^{j,ndc}(l_k^h) + p_{ik}^{j,dc}(l_k^h). \tag{12}$$

It is assumed that immigrants are completely identical to Swedish-born in the same cohort. In other words, they have the same productivity, asset holdings and pension payouts as Swedish-born households. Similarly, it is assumed that those who emigrate take all their assets, including individual pension assets, with them when they emigrate.[7]

2.2.1 *First-order conditions*

The household maximizes Equation (6) given the restrictions in Equations (7) to (11) by choosing c_{ik}^{j}, l_{ik}^{j}, a_{ik}^{j} and b_{ik}^{j}. Since it is always optimal to give away all asset holdings as bequest when the household dies, we can write the optimal bequest allocation as

$$b_{ik}^{j} = (1 + r_t\,(1-\tau^{a}))\, a_{i-1,k}^{j}. \tag{13}$$

Denote the Lagrange multiplier of the budget constraint λ_{ik}^{budget}. The utility maximization with respect to c_{ik}^{j} yields the following first-order-condition:

$$\beta^{i} \frac{\pi_{j+i-1,i-1,k}}{\pi_{j+14,14,k}} s_{j+i,i,k} h_i \frac{1}{c_{ik}^{j}(1+\tau^{cp})} = \lambda_{ik}^{budget}. \tag{14}$$

Maximization with respect to asset holdings give the following first-order condition:

$$\lambda_{ik}^{budget} = (1 + r_{t+1}\,(1-\tau^{a}))\, \lambda_{i+1,k}^{budget} + \beta^{i+1} \frac{\pi_{j+i,i,k}}{\pi_{j+14,14,k}} \left(1 - s_{j+i+1,i+1,k}\right) \phi \frac{1}{a_{ik}^{j}} + \lambda_{ik}^{asset} \tag{15}$$

[6] It is assumed that occupational employer pensions also are included in this variable.

[7] This is equivalent to having a state-contingent insurance system within each cohort.

where λ_{ik}^{asset} is the Lagrange multiplier for the financial asset holding restriction. If the restriction does not bind, λ_{ik}^{asset} takes the value zero. Optimizing with respect to l_{ik}^{j} yields the following first-order condition:

$$\begin{aligned}\beta^{i}\frac{\pi_{j+i-1,i-1,k}}{\pi_{j+14,14,k}}s_{j+i,i,k}\psi\left(1-edu_{ik}-l_{ik}^{j}\right)^{-\omega} &= w_{t}e_{ik}\left(1-\tau^{l}-\tau^{ndc_{l}}\right)\lambda_{ik}^{budget} \\ &+\lambda_{ik}^{labor,+}-\lambda_{ik}^{labor,-} \\ &+\sum_{h=Rage_{j}}^{105}\lambda_{h}^{budget}\left(1-\tau^{p}\right)\left(\frac{\partial p_{hk}^{j,ndc}}{\partial l_{ik}^{j}}+\frac{\partial p_{hk}^{j,dc}}{\partial l_{ik}^{j}}\right)\end{aligned} \tag{16}$$

where $\lambda_{ik}^{labor,+}$ and $\lambda_{ik}^{labor,-}$ are the Lagrange multipliers for the time constraints in Equation (9). $\lambda_{ik}^{labor,+}$ captures that $l_{ik}^{j}\geq 0$ while $\lambda_{ik}^{labor,-}$ captures that $l_{ik}^{j}\leq 1-edu_{ik}$. The Lagrange multipliers take non-zero values only when the constraints bind, meaning that $\lambda_{ik}^{labor,+}$ and $\lambda_{ik}^{labor,-}$ cannot be non-zero at the same time. The last term captures the increase in received pensions that an increased allocation of labor at age i yields.

2.2.2 *Aggregation*

The households' allocations are aggregated by taking the sum of the cross-section of individual variables over the population of each cohort in a given year. Aggregate consumption in year t is denoted by C_t, aggregate asset holdings is denoted by A_t, aggregate labor supply is denoted by L_t and aggregate left bequests are denoted by B_t. These variables are given by

$$C_{t}=\sum_{k=1}^{2}\sum_{i=15}^{105}N_{tik}c_{ik}^{t-i} \tag{17}$$

$$A_{t}=\sum_{k=1}^{2}\sum_{i=15}^{105}N_{tik}a_{ik}^{t-i} \tag{18}$$

$$L_{t}=\sum_{k=1}^{2}\sum_{i=15}^{105}N_{tik}e_{ik}l_{ik}^{t-i} \tag{19}$$

$$B_{t}=\sum_{k=1}^{2}\sum_{i=15}^{106}N_{t-1,i-1,k}(1-s_{tik})b_{ik}^{t-i}. \tag{20}$$

Note that in the aggregation of labor supply, individual labor supply is, for convenience, multiplied with the age-dependent productivity variable e_{ik}, so that the total wage sum in the economy is given by w_tL_t.

It is assumed that the bequests left by households that die are aggregated and then redistributed to the surviving population according to an exogenously given age distribution, such that

$$B_{t}=Beq_{t}=\sum_{k=1}^{2}\sum_{i=15}^{105}N_{tik}beq_{i}^{t-i} \tag{21}$$

where Beq_t denotes aggregate received bequests.

2.3 Production

The production sector consists of three types of firms. The first type produces intermediate goods that are in turn produced using labor and capital as inputs. The second type of firm is a private final good firm that uses intermediate goods as the sole input, and produces goods used for consumption, investment and (net) exports. The second type of good is a public production firm, which also uses the intermediate good as the sole input, but delivers its produced goods to the public sector. The two final good producers differ in their level of productivity. First, the intermediate good production sector is described. Then the two final good sectors are described.

2.3.1 Intermediate goods production

The intermediate good production sector consists of a continuum of intermediate good firms that can be represented by a single representative intermediate good firm. The firm's gross output of intermediate goods Y_t^B is produced using the following production technology:

$$Y_t^B = K_{t-1}^{\alpha} L_t^{1-\alpha}. \tag{22}$$

Denote the firm's *taxable* profits in year t by Π_t. These are defined as

$$\Pi_t = P_t Y_t^B (1 + IsFix^g - \delta^g) - w_t(1 + \tau^{dc} + \tau^{ndc_w} + \tau^w) L_t - \delta K_{t-1} - H(K_{t-1}, K_{t-2}) \tag{23}$$

where P_t is the price of the intermediate good, $IsFix^g$ captures public investment, which for simplicity is modelled as a subsidy to the firm, and δ^g captures the capital depreciation of the public sector, which for simplicity is modelled as a tax on the gross output. By introducing public investment as a subsidy, an increase in public investment also increases total output in the economy. The labor cost of the firm is given by $w_t(1 + \tau^{dc} + \tau^{ndc_w} + \tau^w)L_t$, where τ^{ndc_w} and τ^{dc} are fees that go to the two different pension systems, and τ^w is a tax on the labor cost. δ is the depreciation rate of capital and $H(\cdot)$ is a capital adjustment cost, defined, following Domeij and Floden (2006), as

$$H\left(K_{t-1}, K_{t-2}\right) = \frac{\varepsilon}{\eta} \left(\frac{K_{t-1}}{K_{t-2}} - (1 - \delta) \right)^{\eta} K_{t-2} \tag{24}$$

First-order-conditions

The firm takes both the price of their output and the two respective inputs as given in their maximization problem. It maximizes the present value of its future profits according to the following equation:

$$\max_{\{K_{t+i}, L_{t+i}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} \left(\prod_{j=1}^{i} \frac{1}{1 + r_{t+j}} \right) \left((1 - \tau^y) \Pi_{t+i} - r_{t+i} K_{t+i-1} \right). \tag{25}$$

The maximization yields the two following sequences of first-order conditions:

$$\begin{aligned} &\alpha P_{t+1+i} K_{t+i}^{\alpha-1} L_{t+1+i}^{1-\alpha} (1 + IsFix^g - \delta^g) - H_1\left(K_{t+i}, K_{t-1+i}\right) \\ &\quad - \frac{H_2\left(K_{t+1+i}, K_{t+i}\right)}{1 + r_{t+2+i}} = \delta + \frac{r_{t+1+i}}{1 - \tau^y}, \quad i \in \{0, 1, \dots\} \end{aligned} \tag{26}$$

$$\begin{aligned} (1 - \alpha) P_{t+i} K_{t-1+i}^{\alpha} L_{t+i}^{-\alpha} (1 + IsFix^g - \delta^g) &= w_{t+i} \left(1 + \tau^{dc} + \tau^{ndc_w} + \tau^w \right), \\ &\quad i \in \{0, 1, \dots\} \end{aligned} \tag{27}$$

where

$$H_1\left(K_{t+i}, K_{t-1+i}\right) = \varepsilon \left(\frac{K_{t+i}}{K_{t-1+i}} - (1-\delta) \right)^{\eta - 1}$$

and

$$H_2\left(K_{t+1+i}, K_{t+i}\right) = -\varepsilon \left(\frac{K_{t+1+i}}{K_{t+i}} - (1-\delta) \right)^{\eta - 1} \frac{K_{t+1+i}}{K_{t+i}} + \frac{\varepsilon}{\eta} \left(\frac{K_{t+1+i}}{K_{t+i}} - (1-\delta) \right)^{\eta}.$$

The adjustment cost implies that firms do not fully adjust their capital stock immediately after a change in the economic environment. Furthermore, it may capture a time-to-build restriction, which is not explicitly modelled in MIMER.

Some of the produced goods are used to pay the capital adjustment cost. They can be seen as being destroyed in the process of production. The rest, Y_t, is used as inputs in the final goods production sectors. (Net) output Y_t can be expressed as

$$Y_t = Y_t^B - H\left(K_{t-1}, K_{t-2}\right) = X_t^p + X_t^g \tag{28}$$

where X_t^p is the amount of intermediate goods used in the production of private final goods and X_t^g is the amount of intermediate goods used in the production of public goods. Furthermore, for future use, output per capita y_t is defined as

$$y_s = \frac{Y_t}{N_t}. \tag{29}$$

2.3.2 *Private good production*

The private final good sector consists of a continuum of firms that faces perfect competition. The sector can be represented by a signel representative firm. The price of its output, Y_t^p, is normalized to 1. It uses intermediate goods as the sole input in its production. Its production technology is given by

$$Y_t^p = \left(z_t^p\right)^{1-\alpha} X_t^p. \tag{30}$$

where z_t^p is the sectoral productivity. The evolution of productivity can be described by the following process:

$$z_t^p = (1+\gamma^p) z_{t-1}^p. \tag{31}$$

The representative firm's maximization problem is given by

$$\max_{X_t^p} \left\{ Y_t^p - P_t X_t^p \right\}$$

which has the solution

$$P_t = \left(z_t^p\right)^{1-\alpha}. \tag{32}$$

Finally, the goods that are produced by the firm are used for household consumption, investment in capital and (net) exports. The capiatal accumulation function is given by

$$I_t = K_t - (1-\delta) K_{t-1}. \tag{33}$$

2.3.3 *Public good production*

The amount of produced public goods Y_t^g is a political decision, and is therefore not derived through optimization. Instead, the firm optimizes by minimizing their cost of production:

$$(1+\tau^{cg})\, P_t X_t^g,$$

where τ^{cg} is a tax that is paid on the inputs of production.[8] The costs are maximized subject to its production function which is given by

$$Y_t^g = \left(z_t^g\right)^{1-\alpha} X_t^g \tag{34}$$

where z_t^g is the sectoral productivity. The sectoral productivity grows according to the following process:

$$z_t^g = (1+\gamma^g) z_{t-1}^g. \tag{35}$$

Despite the fact that the goods created in this sector are not explicitly priced, an implicit price can be calculated as the cost of producing one public good, relative to the number of private goods that can be created using the same resources. Hence, the implicit price of the public goods P_t^g can be described by the following equation:

$$P_t^g = \left(\frac{z_t^p}{z_t^g}\right)^{1-\alpha} (1+\tau^{cg})\,. \tag{36}$$

2.4 The Public Sector

The public sector consists of two parts: The consolidated central and local government sector, henceforth called the government sector, and the public pension system. These are consolidated into the consolidated public sector, which is described first in this section.

The variables in the government sector are denoted by superscript ps while the variables in the state and regional governmental sector are denoted by superscript g. Finally, the variables in the public pension system are denoted by ndc. For example, PB_t^{ndc} denotes the primary balance in the public pension system.

2.4.1 *The Consolidated Public Sector*

The consolidated public sector's inter-temporal budget constraint is defined as

$$ND_t^{ps} = (1+r_t^{ps})\, ND_{t-1}^{ps} - PB_t^{ps} \tag{37}$$

where ND_t^{ps} and PB_t^{ps} denote the consolidated public sector's net debt and primary balance in year t. r_t^{ps} denotes the interest rate on the public net debt.

The consolidated public sector's net debt and primary revenues are defined as the sum of the net debts and primary revenues in the two different parts of the public sector:

$$ND_t^{ps} = ND_t^g + ND_t^{ndc} \tag{38}$$

$$PB_t^{ps} = PB_t^g + PB_t^{ndc}. \tag{39}$$

[8] The tax on the government production input is included to capture the fact that the government in Sweden pays VAT on purchases. Even if such a tax is budget neutral from the view of the government, it is convenient to include it when calibrating the government's income and expenditures.

The primary balances in the consolidated public sector can be written in terms of primary expenditures $PExp_t^{ps}$ and primary revenues $PRev_t^{ps}$, both of which are functions of the primary revenues and primary expenditures in the two separate parts of the public sector:

$$PB_t^{ps} = PRev_t^{ps} + PExp_t^{ps} \tag{40}$$

$$PRev_t^{ps} = PRev_t^{g} + PRev_t^{ndc} \tag{41}$$

$$PExp_t^{ps} = PExp_t^{g} + PExp_t^{ndc}. \tag{42}$$

The public consolidated gross debt, also known as the Maastricht debt, MD_t^{ps}, is an important policy variable for the European Union, and a part of their stability and convergence criteria. To consolidate the gross debts in the two parts of the public sector, the amount of government bonds that are held by the public pension system needs to be subtracted, since it is treated as within-sector imbalances in the consolidation. The variable for the subtraction is denoted by AGB_t^{ndc}, meaning that the Maastricht debt can be defined as

$$MD_t^{ps} = D_t^{g} + D_t^{ndc} - AGB_t^{ndc} \tag{43}$$

where D_t^g is the gross debt in the state and regional government sector, and D_t^{ndc} is the gross debt in the public pension system. The Maastricht debt does not play any role for the equilibrium of the model, and is included only for informational purposes.

2.4.2 The Government Sector

The government sector has primary expenditures in the form of consumption C_t^g, investment I_t^g and transfers T_t^g. Furthermore, it carries some gross debt D_t^g on which interest is paid. All these expenditures are financed via the primary revenues $PRev_t^g$, which consists of tax income Tx_t^g, by some transfers from the households T_t^g, and by the returns on any assets held by the sector A_t^g.

The net debt ND_t^g is defined as

$$ND_t^g = D_t^g - A_t^g \tag{44}$$

and the primary balance is defined as

$$PB_t^g = PRev_t^g - PExp_t^g. \tag{45}$$

Using the definition of net debt and the primary balance, it is possible to write the budget constraint as

$$ND_t^g = \left(1 + r_t^{ps}\right) ND_{t-1}^g - PB_t^g. \tag{46}$$

Below follows a description of the primary revenues and expenditures.

Primary revenues

The primary revenues are defined as

$$PRev_t^g = Tx_t^g + TRev_t^g + \delta^g Y_t^B \tag{47}$$

where δ^g captures the public capital depreciation, which for simplicity is modelled equivalent to a tax on the intermediate good firms. Taxes are collected both from households and firms. The tax income is defined as

$$\begin{aligned} Tx_t^g = {} & w_t L_t \left(\tau^l + \tau^w\right) + \tau^{cp} C_t + r_t A_{t-1} \tau^a + \left(P_t^{ndc} + P_t^{dc}\right)\tau^p \\ & + \left(TrAgedep_t^{tx} + TrFix_t^{tx}\right)\tau^t + \Pi_t \tau^y + P_t X_t^g \tau^{cg}. \end{aligned} \tag{48}$$

The transfers from the households to the government sector, $TRev_t^g$, is included in order to calibrate the government primary revenues, since there is such a post in data. The variable is assumed to be a constant share of output:

$$TRev_t^g = P_t Y_t TsRev. \tag{49}$$

where $TsRev$ denotes the transfers from households to the government in terms of output.

Primary expenditures

The primary expenditures consist of consumption, investment and transfers:

$$PExp_t^g = C_t^g + I_t^g + T_t^g. \tag{50}$$

How the three different types of expenditures evolve over time is described below.

Public consumption

The public consumption consists of goods created by the public production sector, and is given by

$$C_t^g = P_t^g Y_t^g. \tag{51}$$

Note that the public consumption variable contains both the price and the volume of the public production good. The consumption can divided into individual consumption that is dependent on the household's age and sex, $CAgeDep_t^g$, and consumption that is the same for all households, $CFix_t^g$, such that

$$C_t^g = CAgeDep_t^g + CFix_t^g. \tag{52}$$

In the start-year of the simulation, year s, the public consumption is calculated as a share of output. The age-dependent consumption can be written as

$$CAgeDep_s^g = P_s y_s \sum_{k=1}^{2} \sum_{i=0}^{105} N_{sik} CsAgeDep_{ik}^g \tag{53}$$

where $CsAgeDep_{ik}^g$ is the amount of consumption in terms of output per capita in the start-year of the simulation. The age-dependent consumption can then evolve in two different ways depending on the preferences of the model user. Either, the consumption per cohort is constant in terms of output per capita over time, as in equation (54a), or the consumption volume per cohort evolves with a constant factor θ^g every year, as in equation (54b):

$$CAgeDep_t^g = P_t y_t \sum_{k=1}^{2} \sum_{i=0}^{105} N_{tik} CsAgeDep_{ik}^g \tag{54a}$$

$$CAgeDep_{t|s}^g = P_s y_s \frac{P_t^g}{P_s^g} \left(1 + \theta^g\right)^{t-s} \sum_{k=1}^{2} \sum_{i=0}^{105} N_{tik} CsAgeDep_{ik}^g \tag{54b}$$

where $CAgeDep_{t|s}^g$ denotes consumption in year t, contingent on the level of consumption in the start-year, s, of the simulation. Note that in the case where the consumption volume per cohort grows at a constant pace, total consumption also grows with the price. This means that an increase in the aggregate age-dependent consumption can happen either due to an increase in the price of consumption, or due to an increase in the consumption volume per person, or due to less favourable demographic conditions.

The age-independent part of public consumption in the start-year of the simulation is given by

$$CFix_s^g = P_s Y_s CsFix^g \tag{55}$$

where $CsFix^g$ is the level of age-independent consumption relative to output in the simulation start-year. The age-independent consumption then evolves over time according to the model user preferences, i.e. it either evolves with output, as in equation (A.60a), or with a constant factor, as in equation (A.60b):

$$CFix_t^g = P_t Y_t CsFix^g \tag{56a}$$

$$CFix_{t|s}^g = P_s Y_s \frac{P_t^g}{P_s^g} \left(1+\theta^g\right)^{t-s} CsFix^g. \tag{56b}$$

Public investment

Public investment is for convenience modelled as a subsidy to the intermediate good firms, which is constant over time. This means that we can write public investments as

$$I_t^g = P_t Y_t^B IsFix_t^g \tag{57}$$

Public transfers

The transfers paid out by by the government either go to the households or abroad, and can be written as

$$T_t^g = TFix_t^{g,tx} + TAgeDep_t^{g,tx} + TFix_t^{g,ntx} + TAgeDep_t^{g,ntx} + TAbr_t^g \tag{58}$$

where $TFix_t^{g,x}, x \in \{tx, ntx\}$ are aggregate age-independent transfers to households that are either taxed (superscript tx) or not taxed (superscript ntx), $TAgeDep_t^{g,x}, x \in \{tx, ntx\}$ are age-dependent aggregate transfers to households, and $TAbr_t^g$ are transfers that are sent abroad.

The age-dependent transfers per person $tAgeDep_t^{g,x}, x \in \{tx, ntx\}$ differ between age and sex, and aggregate age-dependent transfers are given by

$$TAgeDep_t^{g,x} = \sum_{k=1}^{2} \sum_{i=0}^{105} N_{tik} tAgeDep_{tik}^{g,x}, \quad x \in \{tx, ntx\}. \tag{59}$$

In the start-year s of the simulation, the individual transfers can be written in terms of output per capita, $TsAgeDep_{ik}^{g,x}$. Over time, the transfers evolve with the wages in the economy. This means that the individual transfers in year t can be written as

$$tAgeDep_{tik|s}^{g,x} = P_s y_s \frac{w_t}{w_s} TsAgeDep_{ik}^{g,x}, \quad x \in \{tx, ntx\}. \tag{60}$$

The aggregate age-independent transfers in year s are calculated as a share of output. They evolve over time with the increase in wages:

$$TFix_{t|s} = P_s y_s N_t \frac{w_t}{w_s} TsFix \tag{61}$$

where $TsFix$ is the age-independent transfers relative to output in year s.

The public transfers that go abroad are defined as being a constant share of output:

$$TAbr_t^g = P_t Y_t TsAbr_t. \tag{62}$$

Note that the previously described evolution of transfers means that we can write the transfers to each individual as

$$tr_{ik}^{j,ntx} = \frac{TFix_t^{ntx} - TRev_t}{N_t} + tAgeDep_{tik}^{ntx} \tag{63}$$

and

$$tr_{ik}^{j,tx} = \frac{TFix_t^{tx}}{N_t} + tAgeDep_{tik}^{tx} \tag{64}$$

where $t = i + j$.

Assets and debt
For the purpose of calculating the Maastricht debt, the net debt must be divided into debt and assets. For simplicity, it is assumed that the assets are constant as a share of output as long as that would not imply a gross debt below zero. In that case the whole stock of net debt is assumed to consist of assets. This can be described by the following equations:

$$A_t^g = \max\left\{P_t Y_t AsFix^g, -ND_t^g\right\} \tag{65}$$

$$D_t^g = ND_t^g + A_t^g \tag{66}$$

where $AsFix^g$ is the stock of assets in year s as a share of output.

2.4.3 The Public Pension System

The public pension system is a system which is separated and autonomous to the rest of the public sector. It is however not a fully funded system, as opposed to the private pension system, but rather a so-called *notional contribution* system. It collects fees from workers, Tx_t^{ndc}, via τ^{ndc_l} and τ^{ndc_w}, and pays out pensions P_t^{ndc} to the households.

The primary revenues, primary expenditures, and primary balance in the public pension system are defined as

$$PRev_t^{ndc} = Tx_t^{ndc} \tag{67}$$

$$PExp_t^{ndc} = P_t^{ndc} \tag{68}$$

$$PB_t^{ndc} = PRev_t^{ndc} - PExp_t^{ndc}. \tag{69}$$

Tx_t^{ndc} is defined as

$$Tx_t^{ndc} = \left(\tau^{ndc_l} + \tau^{ndc_w}\right) w_t L_t \tag{70}$$

and the aggregate pension payouts are calculated as

$$P_t^{ndc} = \sum_{k=1}^{2} \sum_{i=15}^{105} p_{tik}^{ndc} N_{tik} \tag{71}$$

where p_{tik}^{ndc} denotes the pension payout to an individual of age i and sex k in year t. Note that from the view of the household, p_{tik}^{ndc} can be written as

$$p_{tik}^{ndc} = p_{ik}^{j,ndc}(l_k^h), \quad t = i + j. \tag{72}$$

Individual pension assets and pension payouts

During a household's working ages it collects pension assets a_{tik}^{ndc}. These pensions assets are then used to calculate the future pensions as the household retires. The individual asset holdings evolve according to the following function:

$$a_{tik}^{ndc} = a_{t-1,i-1,k}^{ndc} \frac{1+\mu_t}{s_{ti}^{ndc}} + \left(\tau^{ndc_l} + \tau^{ndc_w}\right) w_t e_{tik} l_{tik}, \tag{73}$$

where the pension assets are zero at the start, i.e. when the individual is 14 years of age, and where $\mu_t = w_t/w_{t-1}$ is the wage growth in the economy. s_{ti}^{ndc} captures that individual pension assets from those who die goes to the surviving individuals of the same age, meaning that

$$s_{ti}^{ndc} = \frac{N_{ti1} + N_{ti2}}{N_{t-1,i-1,1} + N_{t-1,i-1,2}} \tag{74}$$

As the household of age h retires in year u, the pension payouts are calculated using expected future pension payouts, and can be written as

$$p_{uhk} = \frac{a_{uhk}^{ndc}}{d_{uh}^{ndc}} \tag{75}$$

after which it evolves according to

$$p_{tik}^{ndc} = p_{uhk}^{ndc} \left(\prod_{j=u+1}^{t} \frac{\mu_j}{1+norm} \right) \quad \forall i > h. \tag{76}$$

The equation implies that the pension payouts grow not with the wage, but is adjusted with the factor $norm = 0.016$. Due to this, the pension payouts just after the retirement are higher than would otherwise be the case, while the pension payouts later in life are lower than they would otherwise be the case.

The variable d_{uh}^{ndc}, which is used to calculate the pension payouts in the first year of retirement, is calculated such that

$$a_{uhk}^{ndc} = p_{uhk}^{ndc} \sum_{i=h}^{105} \left(\frac{\bar{\pi}_{ti}^{ndc}}{\bar{\pi}_{uh}^{ndc}} \frac{1}{1+norm} \right)^{i-h} \tag{77}$$

meaning that

$$d_{uh}^{ndc} = \sum_{i=h}^{105} \left(\frac{\bar{\pi}_{ti}^{ndc}}{\bar{\pi}_{uh}^{ndc}} \frac{1}{1+norm} \right)^{i-h} \tag{78}$$

where $\bar{\pi}_{ti}^{ndc} = \frac{1}{5} \sum_{t-6}^{t-1} N_{ui}/N_{s-i,0}$. In other words, $\bar{\pi}_{ti}^{ndc}$ is the 5-year average of the population size of the cohorts of age i, relative to the same cohorts' population size at the year of their births.

Assets and debt

The pension fees that are collected during year t, but that are not paid out to present retirees, are saved on the market, in the so-called AP-funds, AP_t^{ndc}. These evolve according to

$$AP_t^{ndc} = (1 + r_t^{ps})\, AP_{t-1}^{ndc} - PB_t^{ndc} \tag{79}$$

There are also some debts with duration shorter than a year which has to be paid interest on, that the system carries. These are denoted by $DShort_t^{ndc}$. This debt is assumed to be constant relative to output, and can be written as

$$DShort_t^{ndc} = sDshort^{ndc} P_t Y_t. \tag{80}$$

Since the duration of the debt is not the whole year, the implicit interest payments over the year is also smaller than would otherwise be the case. Therefore, the interest rate payments on the debt is multiplied by $rDShare$, which means that the payments on the debt, $rDShort_t^{ndc}$ are given by

$$rDShort_t^{ndc} = DShort_t^{ndc} rDShare \cdot r_t^{ps} \tag{81}$$

To simplify the model, it is assumed that the assets in the system, A_t^{ndc}, are zero if the funds in the AP-system are emptied. This means that we can write the gross debt, assets and net debt as

$$D_t^{ndc} = \max\left\{0, -AP_t^{ndc}\right\} + DShort_t^{ndc} \tag{82}$$

$$A_t^{ndc} = \max\left\{0, AP_t^{ndc}\right\} \tag{83}$$

$$ND_t^{ndc} = D_t^{ndc} - A_t^{ndc}. \tag{84}$$

Furthermore, to be able to calculate the Maastricht-debt, AGB_t^{ndc} needs to be defined. It is calculated as being a constant share of the total assets in the public pension system. They cannot, however, exceed the amount of supplied government bonds. Therefore, AGB_t^{ndc} is defined as

$$AGB_t^{ndc} = \max\left\{\min\left\{AGBs^{ndc} \cdot A_t^{ndc}, D_t^g\right\}\right\}. \tag{85}$$

It should be noted that the public pension system is not necessarily internally sustainable. In other words, it is not necessarily the case that the present value of all future pension payouts equals the present value of all future fees collected by the system. There are three reasons for this. Firstly, past survival probabilities, rather than future probabilities, are used when calculating the pension payouts. Since the survival probabilities tend to increase over time, this will lead to higher pension payouts than would otherwise be the case. Secondly, the used survival probabilities are the same for both sexes. Since women have higher survival probabilities than men, this leads to higher pension payouts for women, and smaller for men, *ceteris paribus*, than would be the case if the correct survival probabilities were used. Furthermore, since the amount of pension assets differ between men and women, this construction might create an imbalance between how pension payments evolve in the system and how they would evolve if the actual survival probabilities would be used, creating either a surplus or a deficit for the pension system. Thirdly, the assets in the AP-funds have a different return compared to the wage growth. This means that the individual pension assets, which are only implicit, grow in a different pace than the actual aggregate asset savings. These three effects might go in different directions, and which one that dominates depends on the parameterization of the model.

2.5 The Private Pension System

In MIMER, it is assumed that the private pensions, that are fully paid for by the firms, have the same financial structure and the same system for payouts as the premium pension system. The premium pension system is fully funded, and its income and expenditures are modelled according to the 2014 rules of the system.

All variables in the system are denoted by superscript dc, in contrast to the public pension system, where variables are denoted by superscript ndc.

The system does not carry any debt, and its inter-temporal budget constraint is given by

$$A_t^{dc} = A_{t-1}^{dc}(1 + r_t) + PB_t^{dc}. \tag{86}$$

The primary revenues $PRev_t^{dc}$, primary expenditures $PExp_t^{dc}$ and the primary balance PB_t^{dc} are defined as

$$PRev_t^{dc} = Tx_t^{dc} \tag{87}$$

$$PExp_t^{dc} = P_t^{dc} \tag{88}$$

$$PB_t^{dc} = PRev_t^{dc} - PExp_t^{dc} \tag{89}$$

where Tx_t^{dc} denotes the aggregate fees and P_t^{dc} denotes aggregate payouts from the system.

Tx_t^{dc} can be written as

$$Tx_t^{dc} = \tau^{dc} w_t L_t \tag{90}$$

and the aggregate pension payouts can be written as

$$P_t^{dc} = \sum_{k=1}^{2} \sum_{i=15}^{105} p_{tik}^{dc} N_{tik} \tag{91}$$

where p_{tik}^{dc} denotes the pension payout an individual of age i and sex k in year t. Note that from the view of the household, p_{tik}^{dc} can be written as

$$p_{tik}^{dc} = p_{ik}^{j,dc}(l_k^h), \quad t = i + j. \tag{92}$$

Individual pension assets and pension payouts

The individual pension payout in a given year is calculated using the household's individual pension asset holdings, a_{tik}^{dc}, and the future expected return to capital. Before retirement age h, the individual pension asset holdings grow according to

$$a_{tik}^{dc} = a_{t-1,i-1,k}^{dc} \frac{1 + r_t}{s_{ti}^{dc}} + \tau^{dc} w_t e_{tik} l_{tik}, \quad i < h \tag{93}$$

where $s_{ti}^{dc} = \frac{\sum_k N_{tik} a_{t-1,i-1,k}^{dc}}{\sum_k N_{t-1,i-1,k} a_{t-1,i-1,k}^{dc}}$ implies that the surviving household inherits the assets of the households that do not survive. Hence, the individual pension assets grow with the return to capital, with the inheritance from dead households and with the fees that are paid into the system during the year. Note that in contrast to the public pension system, the individual asset holdings grow with the market interest rate rather than the wage growth rate.

When the household retires in year u and at age h, it gets a fixed pension income p^{dc}_{uhk} from the system until death. The level on the pension payout is set such that all funds within the cohort are run out in the year that all individuals in the cohort has died. In other words, each cohort finances its own pension payouts.

To calculate the individual pension payouts note that after retirement, the pension assets grow according to

$$a^{dc}_{tik} = a^{dc}_{t-1,i-1,k}\frac{1+r_t}{s^{dc}_{ti}} + p^{dc}_{uhk}, \quad i \geq h \tag{94}$$

where

$$p^{dc}_{uhk} = \frac{a^{dc}_{uhk}}{d^{dc}_{uh}} \tag{95}$$

Since all individual assets saved by the cohort should are used up as the cohort dies out, it is possible to calculate d^{dc}_{uh} as

$$d^{dc}_{uh} = \sum_{s=t-j}^{106}\left(\frac{s^{dc}_{ti}}{s^{dc}_{uh}}\left(\frac{1}{1+r_s}\right)^{s-(t-j)}\right) \tag{96}$$

Note that in contrast to the public pension system, the yearly return on the individual pension assets are set such that the system is internally sustainable. This means that the net present value of all future payouts equal all future income in the system.

2.6 The Foreign Sector

Sweden is a small open economy, and it is assumed that the Swedish households' and firms' economic decisions have no impact on the prices in the global markets. Furthermore, it is assumed that Swedish and foreign assets are perfect substitutes. Since the model does not include a structural description of the world economy, the return to capital, r_t, is exogenous in the model.

The households' assets in liquid capital or in the private pension system, is domestically used either as production capital in Sweden, or as government bonds. The remainder of the household asset holdings are placed abroad. Therefore, the Swedish net foreign assets A^F_t can be written as

$$A^F_t = A_t + A^{dc}_t - K_{t-1} - ND^{ps}_t \tag{97}$$

The current account CA_t can be written as

$$CA_t = A^F_t - A^F_{t-1}. \tag{98}$$

The balance of trade, BT_t is defined as the private production goods that is not used as investment or consumption, and can therefore be written as

$$BT_t = Y^p_t - C_t - I_t. \tag{99}$$

3 CALIBRATION

For the purpose of calibrating the model, it is assumed that the economy at the year before the start of the simulation, t_0, is in a steady state. In this calibration-steady state, the transfers abroad are adjusted such that the public finances are sustainable for the given fiscal policy, implying that the consolidated public sector debt is constant in relation to output. Furthermore, the survival probabilities in the calibration-steady-state are assumed to be constant over time, and are set to be the same as the average cohort for all ages and years. In addition, migration is adjusted in the calibration-steady-state to keep the aggregate population growth at zero.

3.1 Demography

Up to the year 2110, the demographic projection, i.e., the projection of fertility, mortality, immigration and emigration, is given by the current population forecast by Statistics Sweden (SCB).

For the years after 2110, the fertility rates are calibrated as 10-year averages over the last 10 years in the population projection from SCB, while the mortality rates are assumed to be the same as in the last year of the SCB projection.[9]

The immigration and emigration probabilities after 2110 are, as for the fertility rate, given by the 10-year average of the SCB forecast. The aggregate net immigration is however adjusted such that the population growth in the model is zero at that point going forward.

3.2 Households

In the benchmark calibration, the retirement age is set to $Rage_j = 65\,\forall j$. This can however be varied depending on the experiment.

Age-dependent productivity

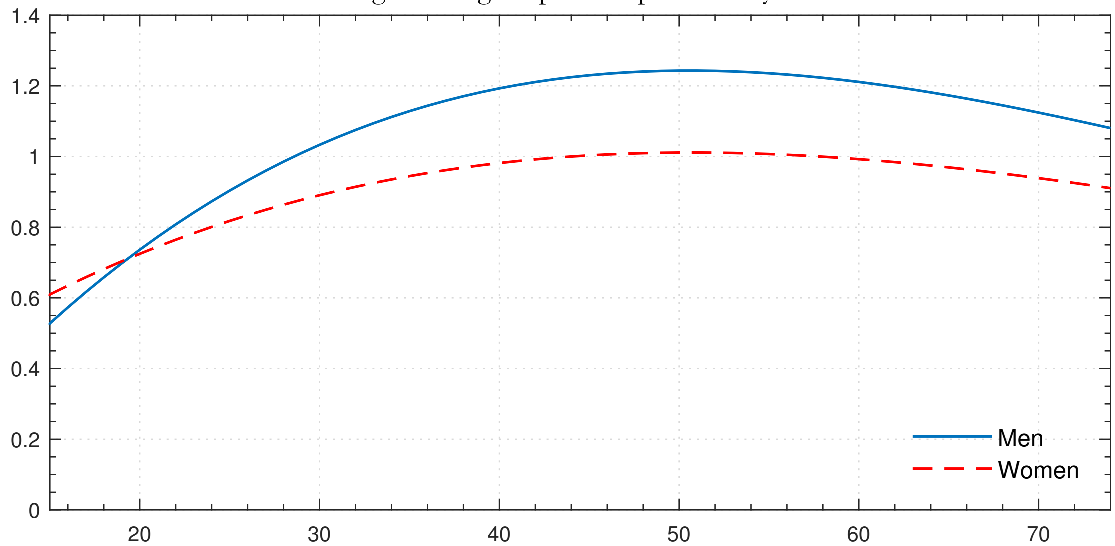
The age-dependent productivity e_{ik} is estimated using a third-degree polynomial on cross-sectional individual wages for the year 2019. For each sex, the following polynomial is estimated: $w_{age} = \beta_0 + \beta_1 age + \beta_2 age^2 + \beta_3 age^3$. The implied productivity series given by the polynomial equations are then normalized such that the average of the two series equals 1. The cross-sectional data used in the estimation is from the structure of wages statistics, which is a survey on Swedish employers conducted by SCB. All public sector employers and their employees are included, while a stratified sample of employers are drawn from the private sector. Large firms with 500 or more employees are always included. Firms with fewer than 500 employees are drawn according to a probability based on firm size. The smaller the firm, the less is the likelihood of being included. Therefore, the estimation is made using probability weights to increase the emphasis on workers employed in smaller private firms. Figure 1 illustrates the age-depenent productivity used in the model.

Health

The construction of the variable $health_i$ follows Domeij and Johannesson (2006), who construct a health index based on data from Lundberg et al. (1999). Lundberg et al. (1999) construct a

[9]The reason for not using an average for the mortality rates is that they are decreasing over time, which would give a higher mortality rate in the model in the years after the last projection-year from SCB than in the last forecast-year.

Figure 1: Age-dependent productivity



Source: Statistics Sweden and National Institute of Economic Research.

health index, using the time-tradeoff method, for individuals in their 20's, 30's and so on, up to age 80. The health index from Lundberg et al. (1999) is then interpolated by Domeij and Johannesson (2006), using cubic hermite interpolation to create an index for each age. In MIMER, linear extrapolation is used to expand the index to households of ages 15-20 and to households of ages 80-105. The index is assumed to be constant over time, and is assumed to not differ between sexes. The index is illustrated in Figure 2.

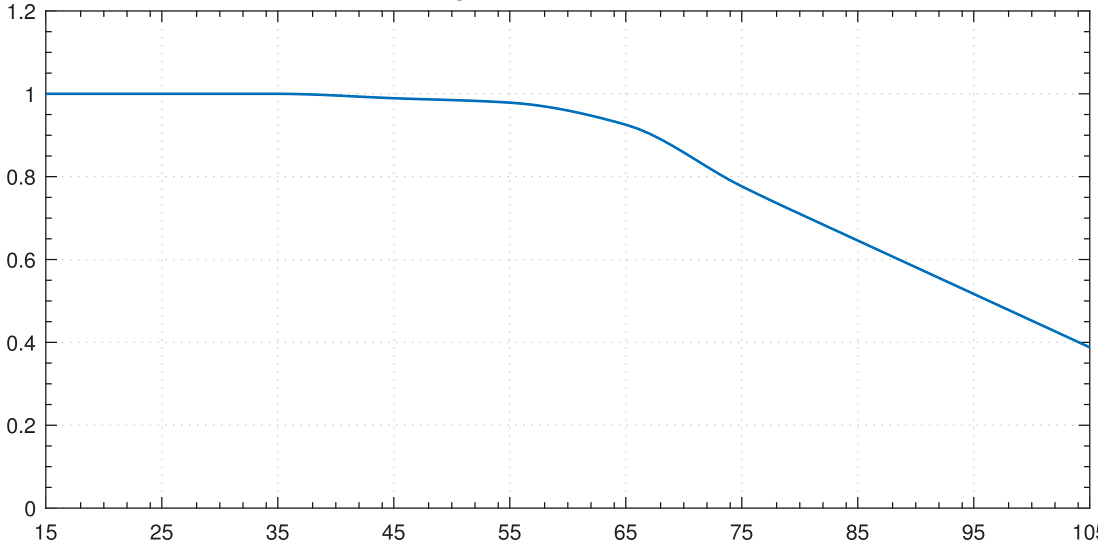
Household size

In addition to health, the variable h_i also contains the variable for household size, $equiv_i$. The utility of consumption is assumed to increase with the number of children in the household, since more individuals are to be supported by the household's income. The utility is however assumed to decrease with the number of adults in the household, due to increasing returns to scale within the household. The household equivalences are calculated by the Swedish Ministry of Finance, using the model SESIM. Figure 3, which illustrates the variable, shows that the utility of consumption is at its largest at the age of 40, since the number of family members that live together takes its maximum value at that age. As the children start to move to their own homes the utility of consumption decreases again, and the index moves down below 1. As households continue to age, 1-person households becomes increasingly frequent, and the variable moves up towards 1 again.

Education

Figure 4 illustrates time spent on education. The series is constructed using data from SCB that contains the number of men and women of different ages that are studying. The variable is constructed into an index by dividing the number of individuals that studies by the total population of the same age and sex. An average over the years 2015-2019 has been used. The data from SCB reports the number of individuals in 1-year-bins for the ages 16-29, a five-year-bin for the ages 30-34 and 10-year bins up to the age of 64. Finally, all individuals 65+ is the last bin. In

Figure 2: Health index



Source: Lundberg et al. (1999), Domeij and Johannesson (2006) and National Institute of Economic Research.

MIMER it is assumed that individuals of age 15 have the same index value as individuals of age 16. To get data in 1-year bins for the ages 30-74, linear interpolation has been used. All individuals of age 69 or above take the value reported in the last bin from SCB. Then a linear interpolation has been used to give values for the ages 30 up to age 68, after which is assumed to be zero.

Household parameters

The parameter values for the households are summarized in Table 1. The parameters in the utility for leisure, $\omega = 1.3000$ and $\psi = 3.3528$, is set to match an average of 23.6 hours worked per week and a so-called Frisch elasticity of 2.5 in the initial steady state.

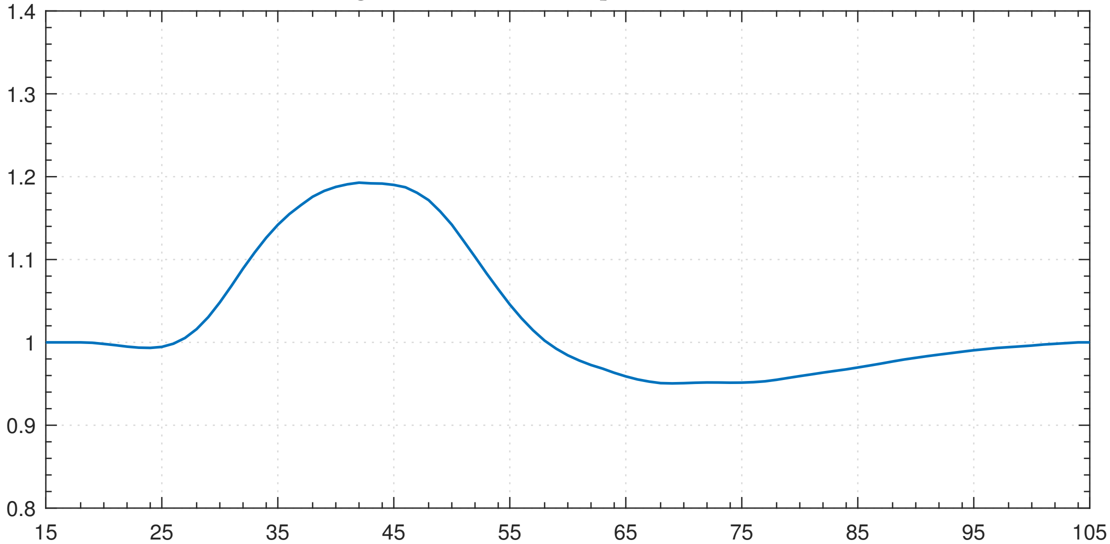
23.6 hours per week is the average between 1995 and 2019 according to Swedish national accounts and population data from SCB. It is assumed, as in Prescott (2004), that each individual has 100 productive hours per week, which gives an average value of $l = 0.236$. A Frisch elasticity of 2.5 is chosen on the basis of Rogerson and Wallenius (2009), who argue that a reasonable value on the Frisch elasticity in an overlapping generations model lies between 2.3 and 3.0. The Frisch elasticity is calculated using the following equation:[10]

$$\left.\frac{dl_{ik}^{j}}{dw_t}\frac{w_t}{l_{ik}^{j}}\right|_{c_{ik}^{j}} = \frac{1}{\omega}\frac{1-l_{ik}^{j}}{l_{ik}^{j}}. \tag{100}$$

The implicit discount factor $\beta = 1.0246$ is set such that average asset holdings A_t/N_t divided by the average wage income (the wage sum divided by population of ages 20-64) in the initial steady state is close to 2.4, which is a value supported by wealth data from SCB over the years 2004-2007. The weight on the bequest $\phi = 0.7000$ is set such that the total inheritence as a share of total

[10]To simplify, in this equation we abstract from time spent on education.

Figure 3: Household Equivalence Index



Source: Swedish Ministry of Finance.

asset holdings lies between 10-20 percent. The inheritence is divided across ages according to the estimated distribution in Klevmarken (2004), and is shown in Table 2.

Table 1: Household parameters

	Parameter value	Matching value	Data period
ψ	3.3528	23.6 average hours worked per week	1995-2019
ω	1.3000	2.5 Frisch elasticity	
β	1.0246	Average net wealth divided by average wage income	2004-2007
ϕ	0.7000	10-20 percent inheritance to net wealth	

3.3 Production

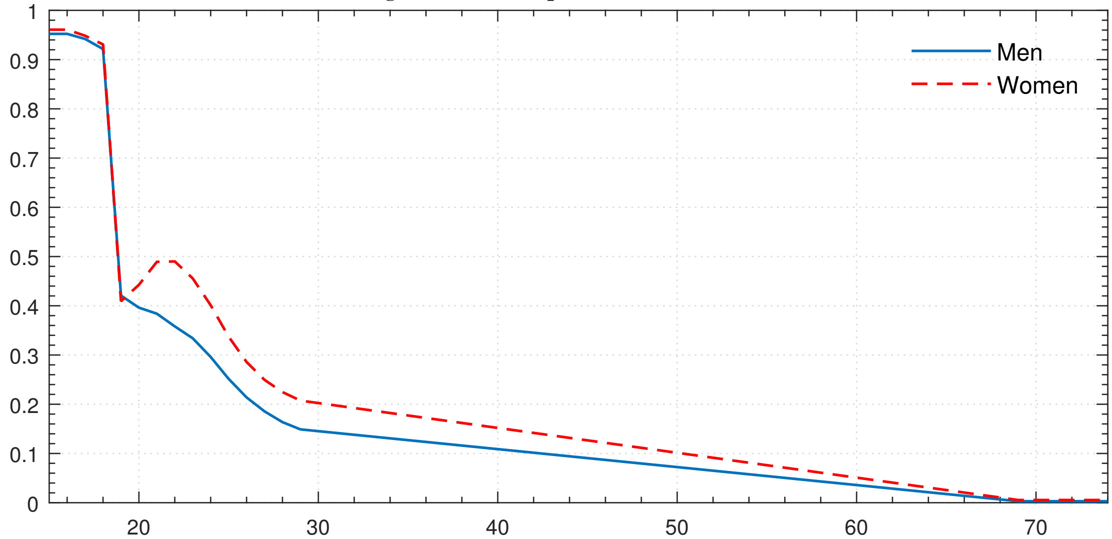
The production parameter values are summarized in Table 3. $\gamma^p = 0.0180$ is set to match the average private sector labor productivity growth, measured as Swedish private sector output per hours worked, over the period 2002-2019, which is a period which is long enough to capture both booms and recessions. The productivity growth in the public production sector is set to match the average growth difference between the public consumption deflator and the private consumption deflator over the period 1997-2019:

$$\frac{P_t^g}{P_{t-1}^g} = \frac{deflator_t^{priv,data}/deflator_t^{pub,data}}{deflator_{t-1}^{priv,data}/deflator_{t-1}^{pub,data}} = \left(\frac{1+\gamma^p}{1+\gamma^g}\right)^{1-\alpha}. \tag{101}$$

After year 2100 however, public productivity growth is assumed, for simplicity, to grow at the same pace as private productivity growth.

The production function is Cobb-Douglas, implying that $1-\alpha$ gives the share of output that goes to workers. The parameter $\alpha = 0.3755$ is therefore calibrated to match the average labor cost share of output over 1980-2019. The interest rate $r_t = 0.0210$ is assumed to be constant over time, and is calibrated to give a growth-interest rate differential in the economy of 0.3 percent.

Figure 4: Time spend on education



Source: Statistics Sweden and National Institute of Economic Research.

Table 2: Inheritence distribution

Age	Percent of year's inheritance
1-20	4.4
21-30	10.4
31-40	20.8
41-50	28.6
51-60	22.6
61-84	13.2

Note: The table shows how the aggregate bequest allocation is distributed as inheritance to households. The numbers are in percent of aggregate bequests in a year per age group.

The parameter governing the convexity of the adjustment cost, $\eta = 2.5385$, was estimated by Eberly (1997), who's estimated value is also used in MIMER. δ and ε are set to match investments to output in the initial steady state to be the average over the period 2010-2019. Furthermore, note that δ is also in line with Olovsson (2009).

3.4 The Public Sector and the Private Pension System

In this section, the calibration of the public sector and the private pension system is described. The public sector interest rate is set to be the same as the private sector interest rate, so that $r_t = r_t^{ps}$.

3.4.1 Income

The calibrated parameter values are summarized in table 4. The taxes are measured as average effective tax rates, or implicit tax rates, following Mendoza et al. (1994), and calibrated using data averages over the period 2010-2019.

The taxes are paid either by the firms or by the households. The payroll taxe rate $\tau^w = 0.2224$ is

Table 3: Production sector parameters

	Parameter value	Matching value	Data period
δ	0.08	Investments as share of output	2010-2019
γ^p	0.0180	Private sector productivity growth	2002-2019
γ^g	-0.0050	Hh cons. defl./Pub. cons. defl.	1997-2017
α	0.3755	Total economy labor cost share	1980-2019
r	0.0210	Interest-growth-differential 0.3	
η	2.5385	From Eberly (1997)	
ε	30	Investments as share of output	2010-2019

calibrated to match the Swedish payroll tax income as share of the Swedish wage sum. $\tau^{ndc_w} = 0.0790$ is celibrated to match the primary income of the public pension system minus the own fees (egenavgifer in Swedish) in data. $\tau^{dc} = 0.1163$ is calibrated to match the ratio between the wage sum and the total labor cost in data, taking as given the payroll taxe rate and the public pension payments from the firms. $\tau^y = 0.4057$ is set to match the capital taxes paid by firms plus other non-good-related taxes as a share of output. The government production tax rate τ^{cg} is calibrated using the model IOR (see Forsfält and Glans (2015)), using data from 2018. τ^{cp} is calibrated to match goods tax income to the public sector adjusted for the tax income from the government given by the tax rate τ^{cg}.

In Sweden, a majority of the labor, pension and transfer tax income goes to local governments. Income tax going to the central government is only paid on income above a certain threshold. To calibrate the taxes rates on labor, pensions and transfers, it is assumed that no central government income tax is paid on transfers and pensions. The labor tax income data is divided into regional government tax and central government tax. Denote the total local labor tax income by $TaxRev^{loc}$. Furthermore, there is data on the total amount of taxed transfers and pensions, and on the wage sum, which is used as the tax base for each respective tax. Hence, we can divide the total labor tax income into three shares, and calculate the implicit tax rates on transfers and pension payouts, $\tau^t = 0.2768$ and $\tau^p = 0.2768$. To get the labor tax rate, $\tau^l = 0.1728$, the remainder of the local tax revenue is added to the central government labor tax revenue, after which the earned income tax credit payouts are removed from that sum. The total tax revenue is then divided by the wage sum to get the implicit tax rate. The pension fee $\tau^{ndc_l} = 0.0702$ is calibrated by dividing the total pension fees paid by households to the public pension system by the wage sum.

The tax on capital returns $\tau^a = 0.6413$ is calibrated to match the sum of the capital tax, ”other tax” and production tax that goes to municipalities relative output in the initial steady-state.

The revenue variable $TsRev^g = 0.0092$ is set to match the average value-to-output over the period 2010-2019.

3.4.2 Expenditures

Table 4 summarizes the parameter values of the public expenditure variables.

For the public consumption, $CsAgeDep^g_{ik}$ is calibrated using data from the STAR-registry for year 2019, which contains public consumption data that can be divided into age sex of the user of the consumption. These profiles are divided into the sub-groups child-related public

Table 4: Government sector parameters

	Parameter value	Matching value	Data period
Production sector taxes			
τ^w	0.2224	Wage tax income-to-wage sum	2010-2019
τ^{ndc_w}	0.0790	(Primary income system - own fees)-to-wage sum	2010-2019
τ^{dc}	0.1163	labor cost-to-wage sum	2010-2019
τ^y	0.4057	Profit taxes/output	2010-2019
τ^{cg}	0.0640	From KI model IOR	
Household and public sector taxes			
τ^{cp}	0.2862	tax income from goods minus tax from gov. purch.	2010-2019
τ^l	0.1728	tax income-to-wage sum	2010-2019
τ^t	0.2768	tax income-to-wage sum as share of output	2010-2019
τ^p	0.2768	tax income-to-wage sum	2010-2019
τ^{ndc_l}	0.0702	tax income-to-wage sum	2010-2019
τ^a	0.6413	tax income to output	2010-2019
Other revenue parameters			
$TsRev^g$	0.0092	Transfer revenues-to-output	2010-2019
Expenditure parameters			
$IsFix^g$	0.0460	Gov. investment-to-output	2010-2019
δ^g	0.0330	Gov. cap. deprecation-to-output	2010-2019
$TsFix^{g,nt}$	0.0375	Aggregate transfers-to-output	2010-2019
$TsAbr^g$	0.0181	Transfers abroad-to-output	2010-2019
$AsTarget^g$	0.2612	Gov. assets-to-output	2010-2019

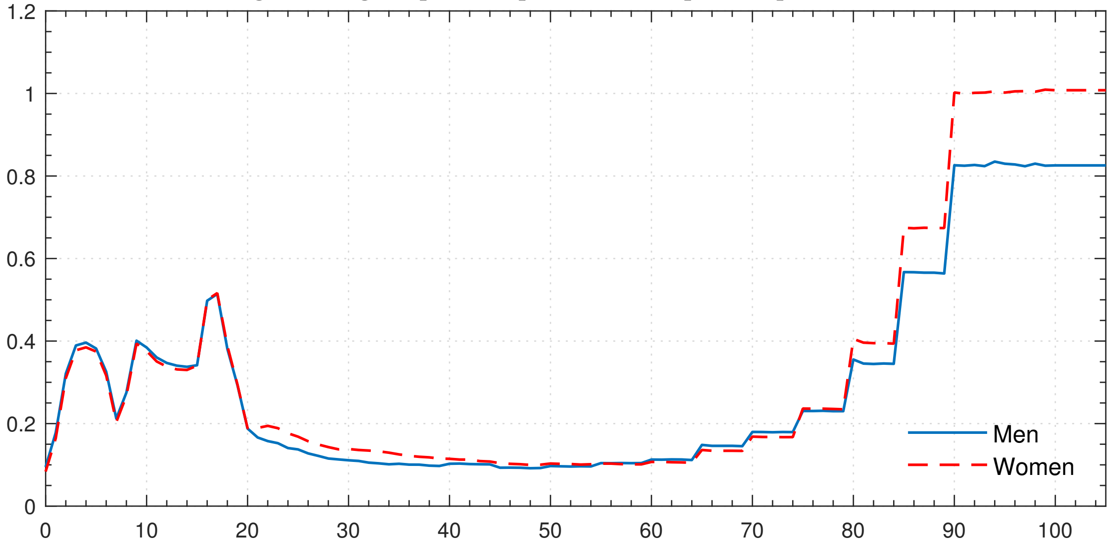
Note: The parameters $IsFix^g$ and δ^g are modelled as shares of gross output, while data is on net output. Therefore, the capital adjustment costs in the model are adjusted for in the calculation of those parameters.

consumption, education, health, labor market consumption and consumption directed towards the elderly. The consumption per person from the registry data is divided with output per capita to get $CsAgeDep^g_{ik}$. The profiles for $CsAgeDep^g_{ik}$ are illustrated in Figure 5. The consumption is higher for the ages 1-19, primarily due to costs for childcare and education. During the prime working age they are lower, and after the age of 60 consumption starts to increase again, as households grow old and begin to require elderly care. It is assumed that all households of a particular sex above the age of 100 have the same public consumption. The age-independent part of public consumption $CsFix^g$ is calibrated residually as the difference between the total public consumption in year in the national accounts $s-1$, where s is the start-year of the simulation and the aggregate age-dependent consumption according to the profiles in $CsAgeDep^g_{ik}$.

The public investment-to-output ratio, $IsFix^g = 0.0460$, which in MIMER is modelled as a subsidy to the intermediate good firms, is calibrated to match the average investment-to-output ratio over the years 2010-2019. The capital depreciation in the public sector, $\delta^g = 0.0330$, is also set to match the average capital depreciation-to-output over the same period. Given that investments are assumed to be a constant share of output, it is also reasonable to let the capital depreciation be so.

As for public consumption, the age-dependent transfers $TsAgeDep^g_{ik}$ are calibrated to match data from the STAR registry for the year 2019, and the data is divided into the five categories

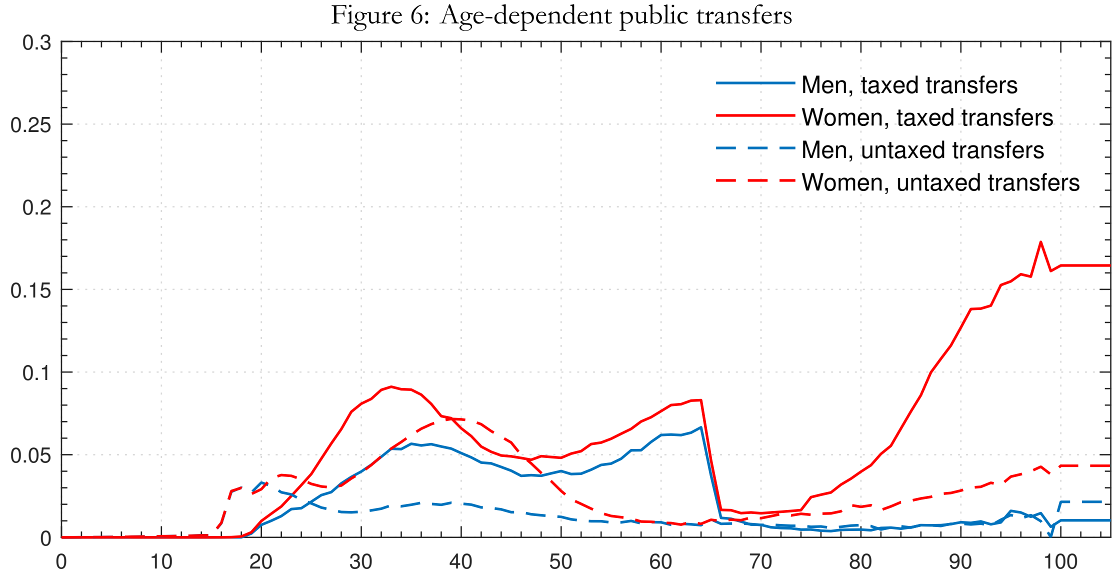
Figure 5: Age-dependent public consumption expenditure



Source: Statistics Sweden and National Institute of Economic Research.

child-related transfers, education, health, labor market transfers and transfers to the elderly. The transfers are illustrated in Figure 6. The majority of transfers for the age-group 20-40 consists of transfers related to children, while for the older working population, the transfers are primarily related to health. The majority of the transfers to individuals above the age of 65 consists of transfers related to pensions that are not part of the public pension system.

Figure 6: Age-dependent public transfers



Source: Statistics Sweden and National Institute of Economic Research.

Is is assumed that all age-independent transfers are untaxed, and part of $TsFix^{g,nt} = 0.0375$ which is set so that the total transfers in the start year of the simulation matches total average transfers over the period 2010-2019.

3.4.3 Assets and Debt

The state and regional government's assets-to-output ratio, $AsTarget^g = 0.2612$, is, as for the other parameter values within the sector, set to match the average over the period 2010-2019.

3.4.4 The Public and Private Pension Systems

The initial individual pension assets and initial individual pension payouts for individuals that are already alive at the start of the simulation are endogenously given by the assets and payouts in the initial steady state. These endogenously given values are then used as input values into MIMER.

The private pension system's initial aggregate assets are set to match the system's initial debt to the households, since the system is autonomous and self-financed.

4 SOLUTION METHOD

This section describes how the model is solved. The model is solved by finding the prices and allocations in the model that are consistent with the general equilibrium. There are however several fiscal policies that are consistent with the general equilibrium, and different fiscal policy experiments imply different model solution methods. All methods do however imply a balanced growth path of all the variables in the model at simulation period going forward, and an aggregate allocation which is consistent with the microeconomic allocations in the model.

A common component for all these solution methods is that the behavior of all individual agents needs to be solved for, for each projected time period. This is done for a given macroeconomic allocations, and hence for given prices. As the different agents' behaviors are solved for, new macroeconomic allocations and new prices can be calculated. This step is called the *inner loop* and is explained first:

Algorithm for the inner loop:
1: Assume that the economy has reached the steady state in year T. Set $t = T$.
2: For a given macroeconomic environment, use the households' first-order-conditions to numerically solve a path for consumption, labor supply and asset holdings for a household born in year $t - 15$, given the terminal condition that all held assets are left as bequest at age 106 (when the household dies with probability 1). Do this for both men and women.
3: Use the (productivity adjusted) asset holdings and labor supply as an initial guess for the household optimization problem in period $t = T - 1$, repeat step 2.
4: Repeat step 3 for all $t \in \{T-2, T-3, ..., s\}$ where s is the start-year of the simulation.
5: Continue to solve for consumption, labor supply and asset holdings for all generations that are still alive, but born before year $s - 15$, i,e., all households that are born between $t = s - 106$ and $t = s - 16$.
6: Given the aggregate labor supply in the years $t = s, ..., T$, use the market clearing condition that labor demand should equal labor supply and solve the firms' first-order-conditions to solve the intermediate good firm's problem for given an initial steady-state level of capital. For the exogenously given public consumption levels, find the levels of public production. Use the first-order condition of the representative private good firm to find the price of intermediate goods and use the market clearing condition for intermediate goods to find the levels of private goods production.
7: Given the production output, solve for the initial individual transfers and the government sector consumption levels, and aggregate these. Solve for the rest of the public sector given the macroeconomic environment solved for in step 6.
8: Calculate the public and private pension variables given the solutions from the household and firm optimization problems.
9: Calculate the Foreign sector variables.

As mentioned before, there are several methods to solve the model, depending on the conducted experiment. The objective is however the same for all methods, namely to change a fiscal tax or allocation such that the general equilibrium is found. The general equilibrium is found when the debt-to-GDP level is constant in the last projected period and going forward in time, or if it is assured that the level will converge to a constant value at some point in future.

In the first algorithm, some tax rate τ^x, $x \in \{cp, l, tr, p, w\}$ is changed such that the general equilibrium is found. This is a way to determine how much the same tax rate needs to be changed to find the general equilibrium, in which the public debt is constant in the steady-state. This is done in the following way:

Algorithm to find the general equilibrium solution by changing the tax rate τ^x, $x \in \{cp, l, tr, p\}$:
1: Define i as the iteration counter. Set the iteration counter $i = 1$. For a given macroeconomic environment, run the inner loop.
2: Change the tax rate with 0.01. Set $i = 2$.
3: Run the inner loop
4: We now have two debt-to-output differences between periods $T - 1$ and T, for $i = 1$ and $i = 2$. When the debt-to-output-ratio is the same in both periods, i.e., when the difference in the debt-to-output-ratios is zero, we have found the general equilibrium. To find a new guess for the tax rate, make a linear interpolation on the tax rate, using the two debt-to-output differences as the explaining variable, and the tax rate as the endogenous variable, and find the the tax rate which sets the debt-to-output difference to 0.
5: The interpolated tax rate is the tax rate for $i = 3$. Run the inner loop using the tax rate for $i = 3$.
6: Iterate steps 4 and 5 until the debt-to-output difference between period $T - 1$ and T is (approximately) zero.

It is important, when the algorithm has converged to a stable path, that the microeconomic behavior gives the same solution for aggregate labor supply and asset holdings as the ones that are used when solving for the inner loop. Otherwise the macroeconomic and macroeconomic behavior is not consistent. Therefore, this also needs to be checked. For the algorithm above, the number of iterations made will be enough to make sure that this is the case. That might however not be the case for all solution methods. Another way of solving the model is the make the counter-factual experiment of setting the public debt level in the start-year of the simulation such that the fiscal policy is sustainable, i.e., such that the general equilibrium is found. The difference between the actual debt level in the economy and the counter-factual initial debt level in the solved model can then be interpreted as the fiscal gap. To to solve the model in this way, the following algorithm is used:

Algorithm to find the general equilibrium where the initial debt level is set so that the public finances are sustainable:
1: Save the initial guess on the aggregate labor supply and asset holdings (the aggregate levels can be exchanged for a matrix of individual values)
2: Run the inner loop.
3: Compare the new aggregate labor supplies and asset holdings with the saved values. If they are not (close to) equal, save new values of aggregate labor supply and asset holdings and go back to step 2. Otherwise, go to step 4.
4: In the general equilibrium, the consolidated budget constraint of the public sector says that the initial net debt level must equal the present value of all future primary balances in the consolidated public sector. Use this fact to calculate the initial debt level.

REFERENCES

Attanasio, O., Kitao, S., and Violante, G. L. (2007). Global demographic trends and social security reform. *Journal of Monetary Economics*, 54(1):144–198.

Auerbach, A. J. and Kotlikoff, L. J. (1987). *Dynamic Fiscal Policy*. Cambridge University Press.

De Nardi, M., Imrohoroglu, S., and Sargent, T. J. (1999). Projected U.S. Demographics and Social Security. *Review of Economic Dynamics*, 2(3):575–615.

Domeij, D. and Floden, M. (2006). The Labor-Supply Elasticity and Borrowing Constraints: Why Estimates are Biased. *Review of Economic Dynamics*, 9(2):242–262.

Domeij, D. and Johannesson, M. (2006). Consumption and Health. *The B.E. Journal of Macroeconomics*, 6(1):1–30.

Draper, N. and Armstrong, A. (2007). Gamma, a simulation model for ageing, pensions and public finances. *CPB Netherlands Bureau for Economic Policy Analysis, CPB Documents*.

DREAM (2008). DREAM dokumentation. Technical report, DREAM.

Eberly, J. C. (1997). International evidence on investment and fundamentals. *European Economic Review*, 41(6):1055–1078.

Eisensee, T. (2006). *Essays on Public Finance: Retirement Behavior and Disaster Relief*, chapter Fiscal Policy and Retirement in the Twentieth century. Dissertation in Economics, Stockholm University.

Forsfält, T. and Glans, E. (2015). IOR – NIER's Input-Output Model of the Swedish Economy. Working Papers 141, National Institute of Economic Research.

Klevmarken, N. A. (2004). On The Wealth Dynamics Of Swedish Families, 1984–98. *Review of Income and Wealth*, 50(4):469–491.

Kotlikoff, L. J., Smetters, K., and Walliser, J. (2007). Mitigating America's demographic dilemma by pre-funding social security. *Journal of Monetary Economics*, 54(2):247–266.

Lundberg, L., Johannesson, M., Isacson, D. G., and Borgquist, L. (1999). Health-state utilities in a general population in relation to age, gender and socioeconomic factors. *European Journal of Public Health*, 9:211–217.

Mendoza, E. G., Razin, A., and Tesar, L. L. (1994). Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. *Journal of Monetary Economics*, 34(3):297–323.

Olovsson, C. (2009). Why Do Europeans Work So Little? *International Economic Review*, 50(1):39–61.

Prescott, E. C. (2004). Why do Americans work so much more than Europeans? *Quarterly Review*, 28(Jul):2–13.

Rogerson, R. and Wallenius, J. (2009). Micro and macro elasticities in a life cycle model with taxes. *Journal of Economic Theory*, 144(6):2277–2292.

Storesletten, K. (2000). Sustaining Fiscal Policy through Immigration. *Journal of Political Economy*, 108(2):300–323.

Sundén, D. (2002). *The Swedish Pension Reform and Projected Demographics*. Dissertation in Economics, Stockholm School of Economics.

A EQUILIBRIUM EQUATIONS

This appendix contains all equations that characterize the economy's equilibrium.

A.1 Demography

Unconditional survival probability:

$$\pi_{tik} = \prod_{j=1}^{i} s_{t-i+j,j,k} \tag{A.1}$$

Cohort population growth rate:

$$n_{tik} = s_{tik} + im_{tik} - em_{tik}, \quad i \geq 1 \tag{A.2}$$

Cohort population evolution:

$$N_{tik} = N_{t-1,i-1,k} n_{tik}, \quad i \geq 1 \tag{A.3}$$

$$N_{t0k} = \sum_{i=0}^{105} born_{tik} + immi_{t0k} - emmi_{t0k} \tag{A.4}$$

Aggregation of population:

$$N_{1t} = \sum_{i=0}^{105} N_{ti1} \tag{A.5}$$

$$N_{2t} = \sum_{i=0}^{105} N_{ti2} \tag{A.6}$$

$$N_t = N_{1t} + N_{2t} \tag{A.7}$$

A.2 Households

Budget restriction for working households:

$$\begin{aligned}(1+\tau^{cp})\, c_{ik}^{j} + a_{ik}^{j} \leq w_t e_{ik} l_{ik}^{j} \left(1 - \tau^{l} - \tau^{ndc_l}\right) + (1 + r_t (1 - \tau^{a}))\, a_{i-1,k}^{j} \\ + tr_{ik}^{j,ntx} + tr_{ik}^{j,tx} \left(1 - \tau^{tr}\right) + beq_{ik}, \quad i < Rage_j\end{aligned} \tag{A.8}$$

Budget restriction for retired households:

$$\begin{aligned}(1+\tau^{cp})\, c_{ik}^{j} + a_{ik}^{j} \leq (1 - \tau^{p})\, p_{ik}^{j}(l_k^h) + (1 + r_t (1 - \tau^{a}))\, a_{i-1,k}^{j} \\ + tr_{ik}^{j,ntx} + tr_{ik}^{j,tx} \left(1 - \tau^{tr}\right) + beq_{ik}, \quad i \geq Rage_j\end{aligned} \tag{A.9}$$

Pensions:

$$p_{ik}^{j}(l_k^h) = p_{ik}^{j,ndc}(l_k^h) + p_{ik}^{j,dc}(l_k^h) \tag{A.10}$$

Given bequests:

$$b_{ik}^{j} = (1 + r_t (1 - \tau^{a}))\, a_{i-1,k}^{j} \tag{A.11}$$

Marginal utility of consumption (adjusted for taxes):

$$u_{c,i,k} = h_i \frac{1}{c^j_{ik}(1+\tau^{cp})} \tag{A.12}$$

Marginal utility of leisure:

$$u_{l,i,k} = \psi \left(1 - edu_{ik} - l^j_{ik}\right)^{-\omega} \tag{A.13}$$

Lagrange multiplier on budget constraint:

$$\lambda^{budget}_{ik} = \beta^i \frac{\pi_{j+i-1,i-1,k}}{\pi_{j+14,14,k}} s_{j+i,i,k} u_{c,i,k} \tag{A.14}$$

Lagrange multiplier on asset holdings:[11]

$$\lambda^{asset}_{ik} = \beta^i \frac{\pi_{j+i-1,i-1,k}}{\pi_{j+14,14,k}} 10^{11} \left(\max\left(0, -a^j_{ik}\right)\right)^2 \tag{A.15}$$

Lagrange multiplier on labor supply constraint 1:[12]

$$\lambda^{labor,+}_{ik} = \beta^i \frac{\pi_{j+i-1,i-1,k}}{\pi_{j+14,14,k}} 10^{11} \left(\max\left(0, -l^j_{ik}\right)\right)^2 \tag{A.16}$$

Lagrange multiplier on labor supply constraint 2:[13]

$$\lambda^{labor,-}_{ik} = \beta^i \frac{\pi_{j+i-1,i-1,k}}{\pi_{j+14,14,k}} 10^{11} \left(\max\left(0, l^j_{ik} - (1 - edu_{ik})\right)\right)^2 \tag{A.17}$$

Euler equation:

$$\lambda^{budget}_{ik} = \left(1 + r_{t+1}\left(1 - \tau^a\right)\right) \lambda^{budget}_{i+1,k} + \beta^{i+1} \frac{\pi_{j+i,i,k}}{\pi_{j+14,14,k}} \left(1 - s_{j+i+1,i+1,k}\right) \phi \frac{1}{a^j_{ik}} + \lambda^{asset}_{ik} \tag{A.18}$$

Labor supply condition:

$$\begin{aligned} \beta^i \frac{\pi_{j+i-1,i-1,k}}{\pi_{j+14,14,k}} s_{j+i,i,k} \psi \left(1 - edu_{ik} - l^j_{ik}\right)^{-\omega} &= w_t e_{ik} \left(1 - \tau^l - \tau^{ndc_l}\right) \lambda^{budget}_{ik} \\ &+ \lambda^{labor,+}_{ik} - \lambda^{labor,-}_{ik} \\ &+ \sum_{h=Rage_j}^{105} \lambda^{budget}_h \left(1 - \tau^p\right) \left(\frac{\partial p^{j,ndc}_{hk}}{\partial l^j_{ik}} + \frac{\partial p^{j,dc}_{hk}}{\partial l^j_{ik}}\right) \end{aligned} \tag{A.19}$$

[11] This multiplier is a numerical approximation to facilitate a continuous equilibrium equation
[12] This multiplier is a numerical approximation to facilitate a continuous equilibrium equation
[13] This multiplier is a numerical approximation to facilitate a continuous equilibrium equation

Effect on public pensions from additional labor supply:

$$\frac{\partial p_{hk}^{j,ndc}}{\partial l_{ik}^{j}} = \frac{\prod_{k=1}^{Rage_j - i} R_{t+k}^{ndc}}{d_{j+Rage_j,Rage_j}^{ndc}} \left(\prod_{k=Rage_j+1}^{h} \frac{\mu_{j+k}}{1+norm} \right) \left(\tau^{ndc_w} + \tau^{ndc_l} \right) w_t e_{ik} \tag{A.20}$$

Effect on private pensions from additional labor supply:

$$\frac{\partial p_{hk}^{j,dc}}{\partial l_{ik}^{j}} = \frac{\prod_{k=1}^{Rage_j - i} R_{t+k}^{dc}}{d_{j+Rage_j,Rage_j}^{dc}} \tau^{dc_w} w_t e_{ik} \tag{A.21}$$

Aggregate consumption:

$$C_t = \sum_{k=1}^{2} \sum_{i=15}^{105} N_{tik} c_{ik}^{t-i} \tag{A.22}$$

Aggregate asset holdings:

$$A_t = \sum_{k=1}^{2} \sum_{i=15}^{105} N_{tik} a_{ik}^{t-i} \tag{A.23}$$

Aggregate labor supply:

$$L_t = \sum_{k=1}^{2} \sum_{i=15}^{105} N_{tik} e_{ik} l_{ik}^{t-i} \tag{A.24}$$

Aggregate left bequests:

$$B_t = \sum_{k=1}^{2} \sum_{i=15}^{106} N_{t-1,i-1,k} (1 - s_{tik}) b_{ik}^{t-i} \tag{A.25}$$

Aggregate inheritance:

$$Beq_t = \sum_{k=1}^{2} \sum_{i=15}^{105} N_{tik} beq_{i}^{t-i} \tag{A.26}$$

Market clearing for bequests:

$$B_t = Beq_t \tag{A.27}$$

A.3 Production

Production function:

$$Y_t^B = K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{A.28}$$

Taxable profits:

$$\Pi_t = P_t Y_t^B (1 + IsFix^g - \delta^g) - w_t (1 + \tau^{dc} + \tau^{ndc_w} + \tau^w) L_t - \delta K_{t-1} - H(K_{t-1}, K_{t-2}) \tag{A.29}$$

Adjustment costs:

$$H\left(K_{t-1}, K_{t-2}\right) = \frac{\varepsilon}{\eta}\left(\frac{K_{t-1}}{K_{t-2}} - (1-\delta)\right)^{\eta} K_{t-2} \tag{A.30}$$

FoC wrt. capital:

$$\begin{aligned} \alpha P_{t+1+i} K_{t+i}^{\alpha-1} L_{t+1+i}^{1-\alpha}(1 + IsFix^{g} - \delta^{g}) - H_1\left(K_{t+i}, K_{t-1+i}\right) \\ -\frac{H_2\left(K_{t+1+i}, K_{t+i}\right)}{1 + r_{t+2+i}} = \delta + \frac{r_{t+1+i}}{1-\tau^{y}}, \quad i \in \{0, 1, \dots\} \end{aligned} \tag{A.31}$$

FoC wrt. labor:

$$\begin{aligned} (1-\alpha) P_{t+i} K_{t-1+i}^{\alpha} L_{t+i}^{-\alpha}(1 + IsFix^{g} - \delta^{g}) = w_{t+i}\left(1 + \tau^{dc} + \tau^{ndc_w} + \tau^{w}\right), \\ i \in \{0, 1, \dots\} \end{aligned} \tag{A.32}$$

Derivative of adjustment cost function, first argument:

$$H_1\left(K_{t+i}, K_{t-1+i}\right) = \varepsilon\left(\frac{K_{t+i}}{K_{t-1+i}} - (1-\delta)\right)^{\eta-1} \tag{A.33}$$

Derivative of adjustment cost function, second argument:

$$H_2\left(K_{t+1+i}, K_{t+i}\right) = -\varepsilon\left(\frac{K_{t+1+i}}{K_{t+i}} - (1-\delta)\right)^{\eta-1} \frac{K_{t+1+i}}{K_{t+i}} + \frac{\varepsilon}{\eta}\left(\frac{K_{t+1+i}}{K_{t+i}} - (1-\delta)\right)^{\eta} \tag{A.34}$$

Output definition:

$$Y_t = Y_t^{B} - H\left(K_{t-1}, K_{t-2}\right) \tag{A.35}$$

Demand for output:

$$Y_t = X_t^{p} + X_t^{g} \tag{A.36}$$

Output per capita:

$$y_s = \frac{Y_t}{N_t} \tag{A.37}$$

Private final goods production:

$$Y_t^{p} = \left(z_t^{p}\right)^{1-\alpha} X_t^{p} \tag{A.38}$$

Final good production technology:

$$z_t^{p} = (1 + \gamma^{p}) z_{t-1}^{p} \tag{A.39}$$

Price of intermediate goods:

$$P_t = \left(z_t^{p}\right)^{1-\alpha} \tag{A.40}$$

Capital accumulation function:

$$I_t = K_t - (1-\delta)K_{t-1} \tag{A.41}$$

Public good production:

$$Y_t^g = (z_t^g)^{1-\alpha} X_t^g \tag{A.42}$$

Public good production technology:

$$z_t^g = (1+\gamma^g) z_{t-1}^g \tag{A.43}$$

Implicit price of public goods:

$$P_t^g = \left(\frac{z_t^p}{z_t^g}\right)^{1-\alpha} (1+\tau^{cg}) \tag{A.44}$$

A.4 The Consolidated Public Sector

Net debt:

$$ND_t^{ps} = ND_t^g + ND_t^{ndc} \tag{A.45}$$

Primary balance:

$$PB_t^{ps} = PB_t^g + PB_t^{ndc} \tag{A.46}$$

Primary revenues:

$$PRev_t^{ps} = PRev_t^g + PRev_t^{ndc} \tag{A.47}$$

Primary expenditures:

$$PExp_t^{ps} = PExp_t^g + PExp_t^{ndc} \tag{A.48}$$

A.5 The Government Sector

Net debt:

$$ND_t^g = (1+r_t^{ps})\, ND_{t-1}^g - PB_t^g \tag{A.49}$$

Primary balance:

$$PB_t^g = PRev_t^g - PExp_t^g \tag{A.50}$$

Primary revenues:

$$PRev_t^g = Tx_t^g + TRev_t^g + \delta^g Y_t^B \tag{A.51}$$

Tax income:

$$\begin{aligned} Tx_t^g = {} & w_t L_t \left(\tau^l + \tau^w\right) + \tau^{cp} C_t + r_t A_{t-1} \tau^a + \left(P_t^{ndc} + P_t^{dc}\right) \tau^p \\ & + \left(TrAgedep_t^{tx} + TrFix_t^{tx}\right) \tau^t + \Pi_t \tau^y + P_t X_t^g \tau^{cg} \end{aligned} \tag{A.52}$$

Transfer income:

$$TRev_t^g = P_t Y_t TsRev \tag{A.53}$$

Primary expenditures:

$$PExp_t^g = C_t^g + I_t^g + T_t^g \tag{A.54}$$

Public consumption, relation to public production:

$$C_t^g = P_t^g Y_t^g \tag{A.55}$$

Public consumption:

$$C_t^g = CAgeDep_t^g + CFix_t^g \tag{A.56}$$

Age-dependent consumption in simulation start-year:

$$CAgeDep_s^g = P_s y_s \sum_{k=1}^{2} \sum_{i=0}^{105} N_{sik} CsAgeDep_{ik}^g \tag{A.57}$$

Age-dependent consumption over time:

$$CAgeDep_t^g = P_t y_t \sum_{k=1}^{2} \sum_{i=0}^{105} N_{tik} CsAgeDep_{ik}^g \tag{A.58a}$$

$$CAgeDep_{t|s}^g = P_s y_s \frac{P_t^g}{P_s^g} (1+\theta^g)^{t-s} \sum_{k=1}^{2} \sum_{i=0}^{105} N_{tik} CsAgeDep_{ik}^g \tag{A.58b}$$

Age-independent consumption in simulation start-year:

$$CFix_s^g = P_s Y_s CsFix^g \tag{A.59}$$

Age-independent consumption over time:

$$CFix_t^g = P_t Y_t CsFix^g \tag{A.60a}$$

$$CFix_{t|s}^g = P_s Y_s \frac{P_t^g}{P_s^g} (1+\theta^g)^{t-s} CsFix^g \tag{A.60b}$$

Government investment:

$$I_t^g = P_t Y_t^B IsFix_t^g \tag{A.61}$$

Aggregate transfers:

$$T_t^g = TFix_t^{g,tx} + TAgeDep_t^{g,tx} + TFix_t^{g,ntx} + TAgeDep_t^{g,ntx} + TAbr_t^g \tag{A.62}$$

Aggregate age-dependent transfers to households:

$$TAgeDep_t^{g,x} = \sum_{k=1}^{2} \sum_{i=0}^{105} N_{tik} tAgeDep_{tik}^{g,x}, \quad x \in \{tx, ntx\} \tag{A.63}$$

Individual age-dependent transfers to households:

$$tAgeDep_{tik|s}^{g,x} = P_s y_s \frac{w_t}{w_s} TsAgeDep_{ik}^{g,x}, \quad x \in \{tx, ntx\} \tag{A.64}$$

Age-independent transfers to households:

$$TFix_{t|s} = P_s y_s N_t \frac{w_t}{w_s} TsFix \tag{A.65}$$

Transfers to abroad:

$$TAbr_t^g = P_t Y_t TsAbr_t \tag{A.66}$$

Relation between public and household untaxed transfer variable:

$$tr_{ik}^{j,ntx} = \frac{TFix_t^{ntx} - TRev_t}{N_t} + tAgeDep_{tik}^{ntx} \tag{A.67}$$

Relation between public and household taxed transfer variable:

$$tr_{ik}^{j,tx} = \frac{TFix_t^{tx}}{N_t} + tAgeDep_{tik}^{tx} \tag{A.68}$$

Assets

$$A_t^g = \max\left\{P_t Y_t AsFix^g, -ND_t^g\right\} \tag{A.69}$$

Debt

$$D_t^g = ND_t^g + A_t^g \tag{A.70}$$

A.6 The Public Pension System

Primary revenues:

$$PRev_t^{ndc} = Tx_t^{ndc} \tag{A.71}$$

Primary expenditures:

$$PExp_t^{ndc} = P_t^{ndc} \tag{A.72}$$

Primary balance:

$$PB_t^{ndc} = PRev_t^{ndc} - PExp_t^{ndc} \tag{A.73}$$

Fees:

$$Tx_t^{ndc} = \left(\tau^{ndc_l} + \tau^{ndc_w}\right) w_t L_t \tag{A.74}$$

Pension payouts, aggregate:

$$P_t^{ndc} = \sum_{k=1}^{2} \sum_{i=15}^{105} p_{tik}^{ndc} N_{tik} \tag{A.75}$$

Relation between individual pension payouts and those received by the household:

$$p_{tik}^{ndc} = p_{ik}^{j,ndc}(l_k^h), \quad t = i + j \tag{A.76}$$

Evolution of individual pension assets:

$$a_{tik}^{ndc} = a_{t-1,i-1,k}^{ndc} \frac{1+\mu_t}{s_{ti}^{ndc}} + \left(\tau^{ndc_l} + \tau^{ndc_w}\right) w_t e_{tik} l_{tik}, \tag{A.77}$$

Inheritance factor:

$$s_{ti}^{ndc} = \frac{N_{ti1} + N_{ti2}}{N_{t-1,i-1,1} + N_{t-1,i-1,2}} \tag{A.78}$$

Individual pensions in first year of retirement:

$$p_{uhk} = \frac{a_{uhk}^{ndc}}{d_{uh}^{ndc}} \tag{A.79}$$

Evolution of individual pensions in years after first year of retirement:

$$p_{tik}^{ndc} = p_{uhk}^{ndc} \left(\prod_{j=u+1}^{t} \frac{\mu_j}{1+norm} \right) \quad \forall i > h \tag{A.80}$$

Division number in the public pension system:

$$d_{uh}^{ndc} = \sum_{i=h}^{105} \left(\frac{\bar{\pi}_{ti}^{ndc}}{\bar{\pi}_{uh}^{ndc}} \frac{1}{1+norm} \right)^{i-h} \tag{A.81}$$

Evolution of AP-fund:

$$AP_t^{ndc} = (1 + r_t^{ps})\, AP_{t-1}^{ndc} - PB_t^{ndc} \tag{A.82}$$

Short-term debt in public pension system:

$$DShort_t^{ndc} = sDshort^{ndc} P_t Y_t \tag{A.83}$$

Interest rate on the short-term debt:

$$rDShort_t^{ndc} = DShort_t^{ndc} rDShare \cdot r_t^{ps} \tag{A.84}$$

Debt:

$$D_t^{ndc} = \max\left\{0, -AP_t^{ndc}\right\} + DShort_t^{ndc} \tag{A.85}$$

Assets:

$$A_t^{ndc} = \max\left\{0, AP_t^{ndc}\right\} \tag{A.86}$$

Net debt:

$$ND_t^{ndc} = D_t^{ndc} - A_t^{ndc} \tag{A.87}$$

Government bond holdings by the public pension system

A.7 The Private Pension System

Assets:

$$A_t^{dc} = A_{t-1}^{dc} \left(1 + r_t\right) + PB_t^{dc} \tag{A.88}$$

Primary revenues:

$$PRev_t^{dc} = Tx_t^{dc} \tag{A.89}$$

Primary expenditures:

$$PExp_t^{dc} = P_t^{dc} \tag{A.90}$$

Primary balance:

$$PB_t^{dc} = PRev_t^{dc} - PExp_t^{dc} \tag{A.91}$$

Fees:

$$Tx_t^{dc} = \tau^{dc} w_t L_t \tag{A.92}$$

Aggregate pension payouts:

$$P_t^{dc} = \sum_{k=1}^{2} \sum_{i=15}^{105} p_{tik}^{dc} N_{tik} \tag{A.93}$$

Relation between individual payouts and household payments:

$$p_{tik}^{dc} = p_{ik}^{j,dc}(l_k^h), \quad t = i + j \tag{A.94}$$

Evolution of individual assets before retirement:

$$a_{tik}^{dc} = a_{t-1,i-1,k}^{dc} \frac{1 + r_t}{s_{ti}^{dc}} + \tau^{dc} w_t e_{tik} l_{tik}, \quad i < h \tag{A.95}$$

Evolution of individual assets after retirement:

$$a_{tik}^{dc} = a_{t-1,i-1,k}^{dc} \frac{1 + r_t}{s_{ti}^{dc}} + p_{uhk}^{dc}, \quad i \geq h \tag{A.96}$$

Individual pension payout in year of retirement:

$$p_{uhk}^{dc} = \frac{a_{uhk}^{dc}}{d_{uh}^{dc}} \tag{A.97}$$

Division number in the pension system:

$$d_{uh}^{dc} = \sum_{s=t-j}^{106} \left(\frac{s_{ti}^{dc}}{s_{uh}^{dc}} \left(\frac{1}{1 + r_s} \right)^{s-(t-j)} \right) \tag{A.98}$$

A.8 The Foreign Sector

Net foreign assets

$$A_t^F = A_t + A_t^{dc} - K_{t-1} - ND_t^{ps} \tag{A.99}$$

Current account

$$CA_t = A_t^F - A_{t-1}^F \tag{A.100}$$

Balance of trade

$$BT_t = Y_t^p - C_t - I_t \tag{A.101}$$

B DERIVATION OF ADDITIONAL PENSIONS RECEIVED FROM ADDITIONAL SUPPLIED LABOR

The household's labor supply condition includes a term that expresses the additional future pension payouts that the household gets from one an additional unit of labor. This expression is derived in this appendix.

We start by noting that derivative of the pension payout of generation j of sex k at age h, $p_{hk}^{j,x}, x \in \{dc, ndc\}$ with respect to the labor allocation l_{ik}^{j} at age i can be written as

$$\frac{\partial p_{hk}^{j,x}}{\partial l_{ik}^{j}} = \frac{\partial p_{hk}^{j,x}}{\partial a_{tik}^{x}} \frac{\partial a_{tik}^{x}}{\partial l_{ik}^{j}} = \frac{\partial p_{hk}^{j,x}}{\partial a_{j+Rage_j,Rage_j,k}^{x}} \frac{\partial a_{j+Rage_j,Rage_j,k}^{x}}{\partial a_{tik}^{x}} \frac{\partial a_{tik}^{x}}{\partial l_{ik}^{j}}, \quad x \in \{ndc, dc\} \tag{B.1}$$

where a_{tik}^{x} is the pension assets in each respective pension system.

Public pensions

The derivative of the public pension assets with respect to labor can be written as

$$\frac{\partial a_{tik}^{ndc}}{\partial l_{ik}^{j}} = \left(\tau^{ndc_w} + \tau^{ndc_l}\right) w_t e_{ik}. \tag{B.2}$$

The derivative of assets at the age of retirement with respect to the pension assets at age i is given by

$$\frac{\partial a_{j+Rage_j,Rage_j,k}^{ndc}}{\partial a_{tik}^{ndc}} = \prod_{k=1}^{Rage_j - i} R_{t+k}^{ndc} \tag{B.3}$$

where $R_t^{ndc} = (1+\nu_t)/s_{ti}^{ndc}$. Finally, the pension payout with respect to assets at retirement is, using Equations (75) and (76), given by

$$\frac{\partial p_{hk}^{j,ndc}}{\partial a_{j+Rage_j,Rage_j,k}^{ndc}} = \frac{1}{d_{j+Rage_j,Rage_j}^{ndc}} \left(\prod_{k=Rage_j+1}^{h} \frac{\mu_{j+k}}{1+norm} \right) \tag{B.4}$$

Putting everything together, we get that

$$\frac{\partial p_{hk}^{j,ndc}}{\partial l_{ik}^{j}} = \frac{\prod_{k=1}^{Rage_j - i} R_{t+k}^{ndc}}{d_{j+Rage_j,Rage_j}^{ndc}} \left(\prod_{k=Rage_j+1}^{h} \frac{\mu_{j+k}}{1+norm} \right) \left(\tau^{ndc_w} + \tau^{ndc_l}\right) w_t e_{ik} \tag{B.5}$$

Private pensions

The derivative of the private pension assets with respect to labor can be written as

$$\frac{\partial a_{tik}^{dc}}{\partial l_{ik}^{j}} = \tau^{dc_w} w_t e_{ik}. \tag{B.6}$$

The derivative of assets at the age of retirement with respect to the pension assets at age i is given by

$$\frac{\partial a_{j+Rage_j,Rage_j,k}^{dc}}{\partial a_{tik}^{dc}} = \prod_{k=1}^{Rage_j - i} R_{t+k}^{dc} \tag{B.7}$$

where $R_t^{ndc} = (1 + r_t)/s_{ti}^{dc}$. Finally, the pension payout with respect to assets at retirement is given by

$$\frac{\partial p_{hk}^{j,dc}}{\partial a_{j+Rage_j,Rage_j,k}^{dc}} = \frac{1}{d_{j+Rage_j,Rage_j}^{dc}} \tag{B.8}$$

Putting everything together, we get that

$$\frac{\partial p_{hk}^{j,dc}}{\partial l_{ik}^{j}} = \frac{\prod_{k=1}^{Rage_j-i} R_{t+k}^{dc}}{d_{j+Rage_j,Rage_j}^{dc}} \tau^{dc_w} w_t e_{ik} \tag{B.9}$$