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Documentation of NIER´s Dynamic Macroeconomic General Equilibrium Model of the Swedish Economy

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1 Overview

KIMOD 1.0 is an annual large-scale macroeconomic model[2] of the Swedish economy and is the result of a project that started in 2002 at the National Institute of Economic Research (NIER) in Sweden. In 2003, the model was used for the first time in policy analysis (see NIER, 2003) and from 2004 onwards it has also been applied for forecasting purposes. In November 2005, the time had come to document the first official version of the model, *KIMOD 1.0*. This document is a resulting part of the documentation project.

In this first chapter, the reader is introduced to the main features of the model (sections 1.1 to 1.4). This description includes the purpose of the model, the basic model structure, some key design features as well as the relationship with other large-scale macroeconomic models. Finally, in section 1.5, the structure of this documentation is outlined.

1.1 The purpose of KIMOD

The NIER publishes projections of the Swedish and international economy four times a year. In addition, the NIER continuously identifies and analysis policy issues that are important to the Swedish economy (see, e.g., NIER, 2005).

KIMOD is designed for both policy analysis and projection purposes. The model's comparative advantage is on the policy side. The reason is that it incorporates major parts of the Swedish economy, it is based on solid theoretical foundations, it keeps track of stocks and flows and it respects national account identities. As such, the model allows us to ask economically relevant questions (e.g. "what will happen with the real exchange rate if the Swedes start to consume more?"), let the model has it say, and trace out the theoretical underpinnings of the answer. Indeed, the model ensures that there are no "free lunches" and that all stocks and flows add up in a consistent manner. It thus relaxes the pressure on economists to keep a model of the

[2] The term "large-scale macroeconomic model" is used for describing the class of models KIMOD belongs to (see section 1.4 for examples of similar models in use around the world). There is, however, no formal definition of this term. In the context in which KIMOD operates, the expression could be said to stand for theory-based general equilibrium models which incorporate the major markets (goods, labour, and financial markets) and agents (firms, households, government, central bank, and a foreign sector) of an economy.

entire economy in their minds! Instead, economists have the model explicitly in front of them which often is a requirement for a fruitful discussion and sensible decisions.

Although large-scale macroeconomic models can be helpful for projection purposes, they are usually not well adapted to incorporate the latest real time data such as confidence indicators, investment and employment plans etc. For example, a model with a generally specified consumption function is often too restrictive for a consumption expert who also has access to the latest information on consumer confidence indicators and household's investment plans. A good model can and should, however, be an important contribution to the overall projection process. KIMOD is designed to serve as such a model.

KIMOD has, however, one comparative advantage in the projection process; the possibility of carrying out so-called *alternative scenarios*. In other words, the model can take as its starting point the projection suggested by the sector experts, and then let a variable (such as e.g. oil prices) take a different route, and trace out the effects on the other variables. For example, the model can incorporate the full projection of the Swedish economy for the period 2006–2008, assign alternative values for the oil price 2006–2008, and generate a fully fledged alternative scenario for all model variables. This operation can be both quickly and consistently performed by the model group as compared to the necessary interaction and recalculation that otherwise would have to be carried out by the sector experts.

In summary, we believe that KIMOD is well adapted to both the policy and the projection environments of the NIER.

1.2 The use of models at the NIER

KIMOD is the only large-scale macroeconomic model at the NIER. Until now, different small-scale macroeconomic models have been used to analyze various issues. For example, the Öller and Tallbom (1996) model is frequently used for analyzing business cycle turning points. KLEM is a small open-economy model based on Svensson (2000) and is mainly used for analyzing macroeconomic effects of monetary policy. The unobservable components model of Apel and Jansson (1999) and the structural vector autoregression model of Fabiani et al (2000) have been used to estimate the NAIRU for Sweden. For short-run GDP forecasting, the quarterly model by Jansson, Hansson and Löf (2005) is applied.

The purpose of KIMOD is not to be a substitute for these small-scale models, for several reasons. Although small-scale models only deal with a partial analysis of the economy, they are often very good at doing their specific job. For example, the business cycle turning point model by Öller and Tallbom (1996) includes much more specific business cycle information, such as various types of indicators, than what would be feasible in a large-scale model. Another example is the purely data-driven forecasting model by Jansson, Hansson and Löf (2005) or, for developments in public finances, the accounting model FIMO.

In short, different small-scale macroeconomic models will still be used at the NIER to complement KIMOD.

1.3 KIMOD – a brief overview

KIMOD can be situated within the synthesis of neoclassical and neokeynesian models. The neoclassical features dominate in the longer run (say 7–15 years) and constitute the core theory of the model. In the long run, prices are fully flexible and agents are fully rational. This implies that the economic variables based on the core theory generally responds very quickly to shocks.[3] Such responses are not intended to represent the actual behaviour of the Swedish economy in the short run. Rather, the core theory serves to generate a realistic and consistent long run behaviour of the economy.

In the short run (say 1–6 years), neokeynesian nominal rigidities stemming from imperfect information, bounded rationality and adjustment costs become operative. The nominal rigidities imply that volume variables such as production and employment respond sluggishly to shocks so that the model can realistically mimic the business cycle dynamics of the Swedish economy.

In section 1.3.3, the economics of KIMOD is discussed in more detail. First, however, section 1.3.1 describes in general terms our strategy of merging neoclassical core theory and neokeynesian business cycle theory within one model structure.

[3] There are two real rigidities – a search cost in the labour market and an installation cost of investment – included in the core theory. These rigidities imply that, despite fully flexible prices and fully rational agents, it takes some time for the economy to reach a new long run equilibrium after a shock.

1.3.1 Three models in one

KIMOD consists of three separate, although highly interrelated, models. Simplifying somewhat, we could say that the three models are valid at different time horizons. In the (very) long run, when the effects of nominal and real rigidities have faded out, the values of the variables are the same across the three models.

The *Steady state model* is a static model based on the neoclassical theory of consumption, investment, production, and the labour market. In steady state, all real variables, such as GDP, consumption, and investments grow at the same rate, which implies that, for example, the consumption-GDP ratio is constant.[4] Moreover, the steady state model is a pure *real* model, in that there are no nominal prices, nominal interest rates or nominal exchange rates, only relative prices, real interest rates, and a real exchange rate. These are all constant in steady state, which implicitly implies that all nominal prices grow at the same rate.

The second part of KIMOD is the *Flex-price model*. Prices and wages are fully flexible and all agents are fully rational and have perfect foresight. Obviously, the Steady state model is the static version of the Flex-price model. The dynamics are mostly due to the introduction of two real rigidities, one in the capital market and one in the labour market. In the capital market, there are adjustment costs to investments. This implies that the capital stock adjusts only gradually to a changed economic environment, i.e. a change in the steady state. The labour market is modelled as in Pissarides (2000). Vacancy costs imply that employers hoard labour if a fall in demand is not permanent. Equilibrium unemployment is determined by replacement ratios, separation rates, vacancy costs, matching efficiency as well as other variables incorporated insearch theory.

In practical applications, KIMOD shares the same equilibrium values of for example output, unemployment and the real exchange rate used elsewhere at the NIER. It is not likely, though, that all the variables in the economy initially take their flex-price equilibrium values. The time it takes for them to reach the flex-price counterparts is, however, less than it takes to reach the steady state. In absence of shocks, the econ-

[4] The steady state condition that all real variables grow at the same rate derives from the one-good-assumption, which obviously is not fulfilled in reality. The most obvious example is the heterogenous growth of the private and public sectors, respectively, where former grows faster than the latter. The problem of imposing the same growth rate is, however, not severe in the use of the model. For projection purposes, the problem is reduced by letting the steady state restrictions only be restrictive far away (say 2030) from the projection period.

omy should normally reach the flex-price equilibrium within 5–10 years. We emphasize that the flex-price equilibrium reaches the steady state only after the effects of the real rigidities in the markets for capital and labour have faded out.

The third and final part of KIMOD is the *Sticky-price model*. Here, nominal rigidities in prices and wages are introduced which lead to sluggish volume adjustment. The nominal rigidities stem from the presence of imperfect information and bounded rationality. This implies, for example, that consumers, investors, and wage setters are partially backward looking; expectations are partly adaptive. As time goes by after a shock, however, the agents learn and become more informed. As a result, variables adjust gradually towards their flex-price equilibrium values. The nominal rigidities give room for stabilization policies, such as monetary and fiscal policies, wich aim to move the economy in the direction of the flex-price equilibrium. It is important to stress, though, that stabilization policies can not alter any equilibrium values. For projection purposes, the Sticky-price model variables start in actual data, i.e. the data used elsewhere at the NIER.

Fig 1: Different frequencies in KIMOD

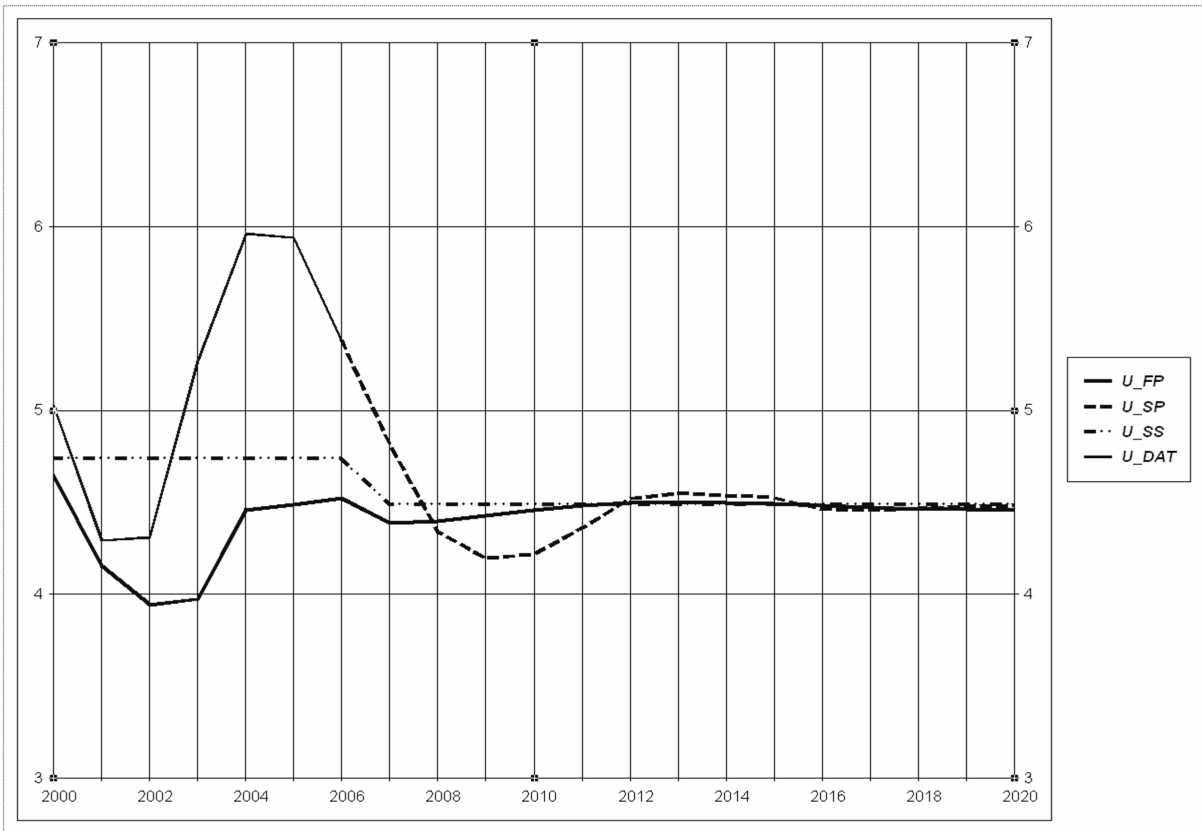


Figure 1 gives an example in order to illustrate the differences between these concepts. For any stationary variable, U_t, there is an anticipated permanent shift in

2007 in its steady state value, displayed by the *U_SS* path. The time variant flex-price analogue, *U_FP*, converges eventually on the steady state path while the business cycle counterpart, *U_SP*, starting in observed data, *U_DAT*, oscillates somewhat more before converging upon the long run equilibrium.

1.3.2 Permanent and temporary shocks

The three models in KIMOD – Steady state, Flex-price, and Sticky-price – can also be understood in the light of permanent and temporary shocks. By definition, a temporary shock only affects the Sticky Price model and does not alter any flex-price or steady state values. For example, a rise in world oil prices that is assumed to be of temporary nature, will affect prices and volumes in the short run (i.e. in the Sticky-price model) while equilibrium values in the Steady state and Flex-price models are unaltered.

On the other hand a permanent rise in the world oil price might affect equilibrium values of output, relative prices, terms of trade and the real exchange rate. A permanent shock must therefore be introduced both in the Steady state and Flex-price model, as well as in the Sticky-price model. In practice, a permanent shock is first introduced in the Steady state model to describe the new static long run equilibrium. This will give rise to a set of terminal conditions which, in turn, are introduced to the Flex-price model generating a dynamic (due to real rigidities) flex-price equilibrium path to the new steady state. Finally, the shock is introduced in the Sticky-price model. As the new equlibrium paths mentioned above are exogenous to the Sticky-price model this will give rise to a business cycle response and show how, through the help of stabilization policies, the economy reaches the new flex-price equilibrium.

Effects of permanent and temporary shocks will be examined in chapter 5.

1.3.3 The economic structure of KIMOD

KIMOD contains the main economic variables in the Swedish economy, such as nominal and real GDP as well as its components, employment, vacancies, wages, interest rates, and exchange rates. The set of agents include individuals, firms, a central bank, a government, a foreign sector and the different markets include goods, labour, capital, and financial markets.

Consumption: In the Steady state and Flex-price models, consumers maximize their life time utility and smooth their consumption due to a decreasing marginal utility of consumption. In each period they consume a fraction of the discounted life time wealth and save by buying international bonds or domestic stocks. In the short run (i.e. in the Sticky-price model), a fraction of the consumers are liquidity constrained (see Campbell and Mankiw, 1991) which, together with uncertainty, affects their consumption decision.

Production: In the Steady state and Flex-price models, firms maximize profits by hiring homogenous labour and capital services. In the long run, growth of production equals the sum of the rates of growth in labour supply and labour-augmenting technical change in productivity. This (intermediate) output is used, together with imported intermediate goods to generate consumption, investment, and export goods. In the short run, firms are uncertain about demand and form expectations about future values which affect their employment and pricing decisions. Moreover, in the short run, firms' investment decisions depend on lagged profits.

Labour market: In the labour market, unemployed workers bargain with firms over wages according to a Nash bargaining game (Pissarides, 2000). In this process, factors such as the replacement ratio, vacancy costs, the separation rate and relative bargaining strength determine the outcome. The number of successful matches is determined by a matching function. In the short run, (i.e. in the Sticky-price model) the outcome of the bargain also depends on lagged wages as well as expectations about future wages as well as companies pricing behaviour and expected monetary policy.

Government: The government produces output by buying labour services at a wage that is not necessarily equal to the wage in the business sector. The government also purchases goods produced by the business sector. In addition to the expenditure on wages and goods, the government pays transfers to households and unemployment benefits. On the revenue side, the government collects indirect taxes, labour related taxes and taxes on dividends. A fiscal policy rule assures that a long run debt-to-GDP target is reached.

Monetary policy: An independent central bank is responsible for the monetary policy. According to the central bank's reaction function the nominal interest rate is set to steer inflation towards the inflation target. As nominal prices are sticky in the short run, the central bank can influence the real interest rate. The real interest rate, in turn, affects consumption and investment decisions as well as the real exchange rate (and thereby net exports). The time it takes to reach the target inflation depends on the initial state and on the specific shocks that have hit the economy.

Financial markets: The bond market is assumed to be international whereas the stock market is national. The real interest rate of the Steady state and Flex-prices models is determined by the rest of the world and thus exogenous. The real exchange rate model is based on Nilsson (2004), in which the real exchange rate in equilibrium depends on relative productivity, terms of trade as well as net foreign assets. In the short run, real exchange rate is determined by the uncovered real interest parity condition and, therefore, by the monetary policy stance.

Expectations: In the Steady state and Flex-price models, expectations are rational and agents have perfect foresight. In the Sticky-price model, expectations are partially rational and partial backward looking. The agents learn, however, as time goes by after a shock, and they gradually become better informed. For example, if there is a permanent productivity shock, firms and employees are at first uncertain as to whether the shock is temporary or permanent. However, the longer the time gone by after the shock, the more have agents learnt about its nature. They can thereby gradually formulate more accurate expectations which guide their decisions.

The core theory of the Steady state and Flex-price models is described in chapter 2 and the business cycle theory of the Sticky-price model is described in chapter 3.

1.3.4 Key design features of KIMOD

As described in section 1.1 above, KIMOD's two major purposes are to provide a tool for policy analysis and projection. As mentioned, policy analysis is the most important of the two because KIMOD has a clear comparative advantage in this area. A closely related issue is the maxim "system goes first", which has been a leading principle in the development of KIMOD. By this we mean that it is more important that the whole system of equations and variables behaves well than it is to maximize the projection accuracy of single variables, such as consumption growth. Consider again the example of the permanent productivity shock. It is of greatest importance that KIMOD can tell a reliable story of how the effects are spread throughout the economy and how they affect steady state values, short run exchange rates, wage bargaining, employment, central bank actions, the government budget balance and other relevant variables. The reason is that such complicated scenarios can neither be included in small-scale macroeconomic models, nor in any one's mind.

Adhering to the "system goes first" principle, the purpose of economic projection needs to be fulfilled in a reasonable way. This is partly achieved by estimating consumption, export and investment equations and by including them in the Sticky-price model. Moreover, the expectation formation has to be calibrated so that the model's response to short run shocks be plausible. A detailed discussion of the models responses to permanent and temporary shocks is described in chapter 5.

KIMOD is also designed to respect the major definitions and identities of the National Accounts. Whenever possible, KIMOD uses the same definitions for output, consumption, prices, wages, exchange rates etc. as the rest of the NIER. KIMOD also keeps track of stock-flow dynamics. For example, investment flows builds the capital stock, the current account balance builds the Swedish net foreign asset position, government budget balance builds the government debt etc. In short, KIMOD is built so that there are neither "leaks" nor "free lunches" in the economy and economic and national accounts identities are satisfied.

Finally, KIMOD is solved using the *Troll* modelling language. Although there are several other possible software products on the market, *Troll* is well suited to solve large, non-linear, forward looking models.

1.4 Relationship with other large-scale macroeconomic models

KIMOD is built in the spirit of a model framework that was initiated and implemented in the mid 1990s. The QPM model developed by the Bank of Canada has been a role model that has inspired many others (Black et al, 1994). Examples include the FPS model of Reserve Bank of New Zealand (Black et al, 1997), RIXMOD of Bank of Sweden (Karlsson, 1998), the FRB model of the Federal Reserve in US (Brayton and Tinsley, 1996), MULTIMOD Mark III of the IMF (Laxton et al, 1998), and BEQM of Bank of England (Harrison et al, 2005).

The models (including KIMOD) within this tradition all share the objective of theoretical consistency, a well-defined steady state, a rational expectations equilibrium in the long run and a variety of neokeynesian rigidities in the short run. Although the overall structure of the models mentioned above is similar, there are of course theoretical differences both in the core theory part and in the business cycle theory part. Examples of the former are different production functions, utility functions, and the modelling of the labour market. The differences in the short run behaviour of the model might be even greater, since short run dynamics often depend on estimated equations and calibration based on historical information which differs substantially between countries.

However, the similarities between the models become striking when one examine so-called standard shocks such as productivity, taste, and wage shocks. Since the theoretical structures are similar, the models often imply the same direction and the same sign of the responses to these shocks, especially in the medium and long run. As a result, they all tell the same qualitative story, i.e. the *economics* of the model responses are the same. Of course, they differ somewhat in the short run, partly due to country-specific short run calibration.

As mentioned, these models are based on work that began in the mid 1990s. In recent years, a number of models have been developed that aim to take these models a bit further, see TOTEM (Murchison and Rennison, Bank of Canada, 2006), AINO (Kilponen et al, Bank of Finland, 2004), Smets and Wouters (2003) or Adolfsson et al (2005) The aim in this line of research is to introduce more rigidities (and thereby dynamics) within the theory-based core model and, as a result, rely less on ad hoc short run dynamics.

1.5 The structure of this documentation

The rest of this document is organized as follows:

Core theory (chapter 2) In this chapter, the main steady state and flex-price equations are discussed. The subsections include consumption (including the external sector), production, labour market, and government.

Business cycle theory (chapter 3) In this chapter, the most important Sticky-price model equations are discussed. Estimated equations are presented in some detail. The subsections include volumes, prices, monetary policy, fiscal policy, and the formation of expectations.

Calibration (chapter 4) In this chapter, the principles behind the calibration are discussed.

Dynamic model properties (chapter 5) In this chapter, a number of permanent and temporary shocks (see section 1.3.2 for a definition) are reported. The responses to these shocks display the economics of the model, and we relate the effects to the core and business cycle theories outlined in chapters 2 and 3.

Appendix: Graphs and variable list (chapter 5) In this section graphical illustrations of the shocks discussed in the preceding chapter are displayed and a complete list of variables is appended

In addition to this document, a similar but more detailed version has been written, aimed for internal use. It includes more details of the derivations of the model's equations, useful for the future development of KIMOD. This *internal* documentation is available on request.

Furthermore, the documentation project also includes a *User guide,* available to those who want to run the model in practice. It aims to give a "hands on" description of how to use KIMOD in the *Troll* modelling language. The subsections include Troll commands, policy analysis, data management, and presentation of results. The User guide is written in Swedish and is available upon request.

2 Core theory

One of the key features of KIMOD is that it should be based on robust theory. This is especially important for the long-run and equilibrium properties of the model. In practice, this means that the behavioural equations of the Steady-state and the Flex-price models are derived from mainstream macroeconomic theory. This chapter presents the theories involved. The assumptions underlying individual consumer behaviour are presented in section 2.1, which yield equations for aggregate consumption demand and wealth dynamics. The theoretical labour market model is set up in section 2.2, which, together with the private production properties discussed in section 2.3, determines employment and wage setting behaviour as well as investment decisions. The modelling of public sector production and consumption are presented in section 2.4, where we also discuss the aims of fiscal policy. Finally, the equations describing the interaction with the external sector are derived in section 2.5.

2.1 Consumer behaviour

The model of consumer behaviour presented below is based on the overlapping generations (OLG) model by Blanchard (1985).[5] Finite lives are introduced in a very flexible way, making the model well suited for economic policy analysis. For example, the model allows for deviations from Ricardian equivalence which implies that fiscal policy can have real effects. Moreover, the OLG framework allows countries to have different relationships between subjective discount factors and real interest rates. As a result, a country can be a net debtor or a net creditor which, for example, determines trade flows in steady state.

In Blanchard's model, one new generation (consisting of a certain number of individuals) is "born" each period, while some individuals within each generation die. When a new generation is "born", it directly enters the workforce. There are no explicit formulations for pensioners, students, and people on leave.[6]

[5] Blanchard (1985) based his continuous time overlapping generation framework on Yaari (1965). Our model follows the discrete time framework by Frenkel and Razin (1996). See also Buiter (1988) and Weil (1989) for similar models with some extensions.

[6] In the literature, this kind of OLG model is sometimes called "perpetual youth" model.

All individuals within a generation are assumed to be identical. Thus, we can solve the individual's optimization problem, derive the consumption function and then aggregate it over the generation as well as over the entire economy.

Individuals maximize their expected lifetime utility. They face one uncertainty; they do not know when they will die. Following Blanchard (1985), a constant probability of death is assumed to affects many aspects of the solution. Furthermore, labour supply is assumed to be exogenous. Hence, an individual does not choose between work and leisure. The individual's real choice each period instead consists only of how much to consume and how much to save out of his/hers expected life time wealth.

The population dynamics of the present version of KIMOD, are presented in section 2.1.1. A brief presentation of the different steps involved in the derivation of the aggregated consumption function is given. Individual preferences and the associated utility function are described in section 2.1.2. The intertemporal budget constraint is described in section 2.1.3, and various wealth measures are discussed in section 2.1.4. The dynamic solution of the optimization problem is presented in section 2.1.5, and the individual's optimal consumption, as a function of total wealth, is derived in section 2.1.6. Finally aggregation is described in sections 2.1.7 and 2.1.8.

2.1.1 Population dynamics

An individual is either employed or unemployed. The total population is then simply equal to the sum of the individuals in the working population, aged between 16 and 64, and equal to the labour supply (LS). The flows into and out from the population are exogenously given. A new generation of individuals enters the working population each year, and a fraction of the old population exit.

Formally, a share $1-\pi$ of each generation exit each year. If $\pi = 1$, then individuals would expect to survive each period and the planning horizon becomes infinite. In other cases, the expected remaining life time for an individual is finite and given by $1/(1-\pi)$, i.e. 25 years if $\pi = 0.96$ (a reasonable number for Sweden).

The number of "births" of individuals in a new generation is assumed to be proportional to the old population. Let a denote the exogenous birth rate, in terms of those surviving from last period (i.e. πLS_{t-1}).

Summing up, the population in period t consists of *(i)* those individuals in the old generations who survive between period $t-1$ and t, and *(ii)* the new generation entering in period t, so that

$$LS_t = \pi LS_{t-1} + a\pi LS_{t-1}$$
$$= (1+\nu) LS_{t-1}, \tag{2.1}$$

where ν is defined as the constant growth rate of labour supply, $\nu = (1+a)\pi - 1$, that is,

$$\frac{LS_t}{LS_{t-1}} = 1+\nu. \tag{2.2}$$

2.1.2 Preferences and the utility function

Since labour supply is exogenous, leisure is not an argument of the utility function. An individual receives utility solely from consumption. The instantaneous utility function is logarithmic, which implies that income and substitution effects of a change in the interest rate cancel out. The interest rate, however, will change the present value of labour income and the human capital wealth which, in turn, affects the optimal path for consumption (i.e., the wealth effect remains).

Since there is a probability of exit, the *expected* utility of individual i evaluated in period t is given by the expected present value of the flow of utility, given a specific consumption path,

$$U_t^i = E\left[\sum_{s=t}^{\infty} (\beta)^{s-t} u\left(C_s^i\right)\right]$$
$$= \sum_{s=t}^{\infty} (\pi\beta)^{s-t} \ln C_s^i. \tag{2.3}$$

The effective (and subjective) discount factor ($\pi\beta$) consists of two parts. Parameter β is the pure subjective discount factor. The individual prefers consumption this year instead of next year, so that $0 < \beta < 1$ holds. The lower the probability of surviving until the next period (π), the more the individual prefers consumption this year compared to next year. In other words, we can incorporate the idea of an impatient individual. As we will see, the finitely lived individual consumes more out of his/her wealth compared to a model with an infinite horizon (in which $\pi = 1$), since the effective planning horizon is shorter.

2.1.3 The individual's intertemporal budget constraint

Each individual in KIMOD is an amalgam that represents private employees, public employees, the unemployed, as well as bond and share holders. The budget constraint, in real terms, of a representative individual in generation i in period t is thus given by expenditures and revenues from various sources:

$$P_t^{C\_P}C_t^i+\phi_t^iV_t+P_t^{M\_P}B_t^{P,i}=\left(1-\tau_t^W\right)W_t^{i,A}LH_t^i+TR_t^i+\left(\frac{1+R_{t-1}}{\pi}\right)P_{t-1}^{M\_P}B_{t-1}^{P,i}$$
$$+\left(\frac{\phi_{t-1}^i}{\pi}\right)\left(1-\tau_t^D\right)D_t+\left(\frac{\phi_{t-1}^i}{\pi}\right)V_t, \tag{2.4}$$

where

$$W_t^{i,A}\equiv W_t^P PB_t^{YPG\_P}\left[\left(\frac{LH_t^{i,P}}{LH_t^i}\right)+\alpha\left(\frac{LH_t^{i,G}}{LH_t^i}\right)+\zeta\left(\frac{UH_t^i}{LH_t^i}\right)\right]. \tag{2.5}$$

In these equations, $P_t^{C\_P}$, $P_t^{PM\_P}$, and $PB_t^{YPG\_P}$ are the relative prices of consumption, imports and private production vis-à-vis the GDP-deflator (P). The left hand side of (2.4) consists of consumption expenditure (C) during the period and the value of shares (of aggregated equities) (ϕ^iV) and bonds (B) at the end of the period. The individual's average wage rate, denoted by $W_t^{i,A}$, depends on the real wage in the private sector[7] (W_t^P), the real wage in the government sector and unemployment benefits. It is a weighted average, where the weights are equal to shares of total hours supplied by the individual (LH_t^i), see equation (2.5). The government wage rate is a fraction (α) of the private sector wage rate and the same is true for unemployment benefits, the level of which is effectively defined by the replacement ratio (ζ).The right hand side of (2.4) thus consists of working income from hours spent in private employment ($LH_t^{i,P}$), government employment ($LH_t^{i,G}$), and income generated by hours in unemployment (UH_t^i). The individual also receives government transfer payments (TR), interest on bonds ($R\cdot B$) and dividends (D). The individuals pay income taxes on labour and capital, represented by τ_t^W and τ_t^D. Furthermore, on the right hand side of (2.4) we include the value of the bonds and stocks at the beginning of the period, denoted with subscript "$t-1$".

The stock market is domestic while the bond market is international. The value of bonds is translated into Swedish crowns by using the import deflator (P_t^M). Note that the returns on bonds, dividends, bond holdings and the share of stocks are divided by π. This stems from the introduction of finite lives (or, equally, the constant probability of death) in the model. The survivors inherit the financial assets, which

[7] The real wage is measured in units of private (gross) production, $W^PP^{YPG\_P}$ is the real wage in terms of GDP.

increases their income and wealth. For example, the total rate of return on bonds between period t–1 and t is R_{t-1} while the effective rate for the individual is $(1+R_{t-1})/\pi-1$, where $(1+R_{t-1})/\pi-1>R_{t-1}$ if $\pi<1$.

2.1.4 The stock measures of total, human, and financial wealth

There are several wealth measures in KIMOD; human (capital) wealth, financial wealth and their sum, total wealth. *Total wealth* (TW) is simply measured as the expected present value of consumption expenditures, given any consumption path. More precisely, total wealth is given by

$$\begin{aligned} TW_t^i &= E_t\left[\sum_{s=t}^{\infty}\left(\frac{1}{1+\bar{R}_s}\right)^{s-t} P_s^{C-P}C_s^i\right] \\ &= \sum_{s=t}^{\infty}\left(\frac{\pi}{1+\bar{R}_s}\right)^{s-t} P_s^{C-P}C_s^i, \end{aligned} \tag{2.6}$$

or, in other words, the discounted real value of future consumption possibilities.[8] A lower probability of surviving or a higher real interest rate implies a lower weight on future consumption. By substituting consumption as given by the budget constraint (2.4) into (2.6), we can separate this expression into two parts. Human wealth (HW) is defined as the part associated with labour income, while the rest is defined as financial wealth. Financial wealth, in turn, is the sum of bond wealth (BW) and stock wealth (SW), so that

$$TW_t^i = HW_t^i + BW_t^i + SW_t^i. \tag{2.7}$$

By definition, *human wealth* is given by the following expression:

$$\begin{aligned} HW_{\mathrm{t}}^{\mathrm{i}} &= \sum_{s=t}^{\infty}\left(\frac{\pi}{1+\bar{R}_s}\right)^{s-t}\left[(1-\tau_s^W)W_s^{i,A}LH_s^i + TR_s^i\right] \\ &= \left(1-\tau_t^W\right)W_t^{i,A}LH_t^i + TR_t^i + \left(\frac{\pi}{1+\bar{R}_{t+1}}\right)HW_{t+1}^i. \end{aligned} \tag{2.8}$$

[8] The effective discount rate is equal to the individual's average effective rate on return on bonds, so that the uncertainty of one's lifetime is taken into account. The probability of "being alive" in period s for the individuals in period t is $\pi_{t+1}\cdot\pi_{t+2}\cdot\ldots\cdot\pi_s=\pi^{s-t}$, and the average annual interest rate is equal to $1/\left(1+\bar{R}_s\right)$, where $\bar{R}_s=\left(\prod_{i=t}^{s}(1+R_i)\right)^{1/(s-t)}-1$.

Current human wealth is thus equal to this period's labour income and transfer payments, plus the expected discounted value of next period's human wealth. The human wealth expression is thus forward looking, which implies that total wealth is also forward looking.

Furthermore, assuming a constant real interest rate, *bond wealth* is given by

$$\begin{aligned} BW_t^i &= \sum_{s=t}^{\infty}\left(\frac{\pi}{1+R}\right)^{s-t}\left[\frac{(1+R)}{\pi}P_{s-1}^{M\_P}B_{s-1}^{P,i} - P_s^{M\_P}B_s^{P,i}\right] \\ &= \left(\frac{1+R}{\pi}\right)P_{t-1}^{M\_P}B_{t-1}^{P,i}, \end{aligned} \tag{2.9}$$

where a solvency requirement is assumed to hold, so that $\left(\pi/(1+R)\right)^t P_t^{M\_P}B_t^{P,i} = 0$ as $t \to \infty$. Individuals are not allowed to expect a nonzero debt in the long run. The interpretation of equation (2.9) is that the expected value of bond wealth is equal to the value of today's bond holdings.

Since there is no financial risk in KIMOD, the return on shares must equal the return on bonds, i.e. R_t. This non-arbitrage condition gives the following dynamic path of the value of the total number of shares, that is, the value of the firm (see section 2.3 below):

$$V_{t+1} + (1-\tau_{t+1}^D)D_{t+1} = (1+R_t)V_t. \tag{2.10}$$

Equation (2.10) is derived using the fact that the return on shares stems from two sources; dividends and changes in the market value of the firm (V). Assuming a constant real interest rate, an expression for the individual's *stock wealth* can be derived as

$$\begin{aligned} SW_t^i &= \sum_{s=t}^{\infty}\left(\frac{\pi}{1+R}\right)^{s-t}\left[\frac{\phi_{s-1}^i}{\pi}V_s + \frac{\phi_{s-1}^i}{\pi}\left(1-\tau_s^d\right)D_s - \phi_s^i V_s\right] \\ &= \frac{\phi_{t-1}^i}{\pi}\left[V_t + \left(1-\tau_t^d\right)D_t\right], \end{aligned} \tag{2.11}$$

where $\left(\pi/(1+R)\right)^t \phi_t^i V_t = 0$ as $t \to \infty$ is assumed. Similarly to bond wealth, the expected stock wealth is defined as the value of the shares in the beginning of the period plus dividends that accrue during the period. Note that there is no forward locking part in neither the bond wealth nor the stock wealth. This implies that there is no forward looking part in the financial wealth, only in the human capital wealth.

2.1.5 Optimization and the expected path for the individual's consumption

Choosing optimal consumption is equivalent to choosing optimal real savings. The individuals thus act as if they were maximizing the utility in equation (2.3) with respect to bond holdings. The latter is obtained by first solving the budget constraint in equation (2.4) for consumption and substituting this into the utility function. The first order condition for optimal bond holdings in period *t* and *t+1* yields

$$\frac{\partial U_t^i}{\partial B_{t+1}^{P,i}} = 0 \qquad \Rightarrow \qquad C_{t+1}^i = \beta\left(1+R_t\right)\frac{P_t^{C\_P}}{P_{t+1}^{C\_P}} C_t^i, \tag{2.12}$$

which is the standard Euler condition. The optimal time path of consumption depends on the subjective discount rate, the interest rate and changes in the relative price of consumption. If $\beta \cong 1/\left(1+R\right)$, and if there is no change in relative prices, consumption is constant over time. Note that the intertemporal allocation of consumption is not distorted by the probability of death.[9] Yet the finiteness of lifespan affects the initial consumption level, as will be shown next. The Euler condition will be used below in the derivation of the consumption as a function of wealth.

2.1.6 The individual's consumption function

Total wealth was defined above as the expected present value of any consumption path. Using the expression of total wealth in equation (2.6) and the Euler condition for the optimal consumption path in equation (2.12), the optimal level of consumption as a function of total wealth is derived. The individual's consumption function is derived as

$$P_t^{C\_P} C_t^i = \left(1-\pi\beta\right) TW_t^i, \tag{2.13}$$

Hence, the optimal consumption level in the current period is a fixed proportion of total wealth in the beginning of the period (*the marginal propensity to consume out of wealth, MPC,* $=1-\pi\beta$). More impatient individuals (i.e. lower β) and/or higher probability of death (i.e. lower π), yields higher consumption as of today. The spending propensity does not depend on age, which is a reflection of the assumption that the probability of death does not depend on age. As we shall see in the next section, this implies that the consumption function can be aggregated in a simple way. Finally, it is important to note that the interest rate is not an argument in the con-

[9] According to Frenkel and Razin (1996), this property ensures the existence of a Pareto optimal equilibrium (in the absence of other distortions of course).

sumption function. This is due to the use of logarithmic utility. A property does not hold for more general utility functions.

2.1.7 Aggregation

So far, we have considered an individual in generation *i*. That is, we have only shown expressions valid at this highly disaggregated level. There exist an infinite number of generations at each point of time. More specifically, the number of individuals in generation i in period t is denoted LS_t^i and the total number of individuals in the economy in period t is denoted LS_t, where

$$LS_t = \sum_{i=-\infty}^{t} LS_i^i. \tag{2.14}$$

This definition and others shown below will be used when aggregating, in turn, bond wealth, stock wealth, human wealth, total wealth, the consumption function and, finally, the budget constraint.

Aggregating *bond wealth* over all individuals yields

$$BW_t = (1+R_{t-1}) P_{t-1}^{M-P} B_{t-1}^P, \tag{2.15}$$

which is obtained by multiplying both sides of equation (2.9) with the number of individuals in generation i (LS_t^i) and summarize over all generations, using the facts that

$$\begin{aligned} &LS_t^i = \pi LS_{t-1}^t \qquad \text{and} \\ &X_t = \sum_{i=-\infty}^{t} LS_t^i X_t^i, \end{aligned} \tag{2.16}$$

for any stock variable X_t^i. As mentioned earlier, note the difference between the rate of return on bonds at the individual and the aggregated levels. In the former case, the effective rate of return was increased with the factor $1/\pi$. At the aggregated level, the payments represent transfers *within* society and do not alter the social rate of return.

Aggregating *stock wealth* given by equation (2.11) over all individuals yields

$$SW_t = V_t + (1-\tau_t^D) D_t, \tag{2.17}$$

obtained by following the same steps as above and using the fact that the weights on the total stock (ϕ_t^i) sum to one over the population.

Human wealth at the individual level is displayed in equation (2.8). Aggregating this expression yields the following expression:

$$HW_{t+1} = \frac{1+R_{t+1}}{\pi}\left[HW_t - (1-\tau_t^W)W_t^P PB_t^{YPG\_P} LH_t \left[\left(\frac{LH_t^P}{LH_t}\right) + \alpha\left(\frac{LH_t^G}{LH_t}\right) + \zeta\left(\frac{UH_t}{LH_t}\right)\right] - TR_t \right], \quad (2.18)$$

where the total number of hours (including unemployed hours), human wealth, and transfer payment $(LH, HW, \text{and } TR)$ satisfies the last equation in (2.16).

In equations (2.18), the probability of death (π) is included, in contrast to the expression for bonds and stocks in equations (2.15) and (2.17), respectively, above. The reason is that there is no way to transfer the human wealth of individuals who die. Hence, each generation (and therefore the society as a whole) takes the probability of death into account when considering the development of human wealth.

Aggregate *total wealth* is the sum of the above aggregate wealth components, that is,

$$TW_t = FW_t + HW_t. \quad (2.19)$$

The aggregate *consumption function* is derived by aggregating the individuals' consumption given in equation (2.13) in a similar way as wealth, which gives

$$P_t^{C\_P} C_t = (1-\pi\beta) TW_t, \quad (2.20)$$

where consumption and total wealth $(C, \text{and } TW)$ satisfies the last equation in (2.16).

Aggregation of the *budget constraint* finally yields

$$\begin{aligned} P_t^{C\_P} C_t + P_t^{M\_P} B_t^P = {} & (1-\tau_t^W)W_t^P PB_t^{YPG\_P} LH_t \left[\left(\frac{LH_t^P}{LH_t}\right) + \alpha\left(\frac{LH_t^G}{LH_t}\right) + \zeta\left(\frac{UH_t}{LH_t}\right)\right] \\ & + TR_t + (1+R_{t-1})P_{t-1}^{M\_P} B_{t-1}^P + \left(1-\tau_t^D\right) D_t. \end{aligned} \quad (2.21)$$

2.1.8 The aggregate expressions of total wealth and private consumption

The Euler equation (i.e., the dynamics of consumption at the individual level) was derived in equation (2.12) above. There is no counterpart to that equation for the economy as a whole. Due to the introduction of finite live spans, one can not simply aggregate the individual's utility maximization problem. The dynamics of aggregate consumption must instead be derived implicitly, through the aggregate consumption function, using the dynamics of aggregate total wealth, that is,

$$TW_t = (1+R_{t-1})\pi\beta TW_{t-1} + (1-\pi) HW_t. \quad (2.22)$$

By substituting C from the budget constraint for TW in equation (2.22), the dynamic behaviour of aggregate consumption is finally given by

$$P_t^{C-P} C_t = (1+R_{t-1})\pi\beta P_{t-1}^{C-P} C_{t-1} + (1-\pi\beta)(1-\pi)HW_t. \tag{2.23}$$

2.2 The labour market

The purpose of this section is to derive the labour market equations used in KIMOD.[10] The labour market in KIMOD's Steady state and Flex-price models builds heavily on Pissarides (2000). Wage setting is characterized by the "right-to-manage" assumption which implies that wages are negotiated before firms decide on investment and production. Firms with vacancies and unemployed workers are engaged in a costly search for a profitable match and unemployment is caused by matching inefficiencies. As prices and wages are fully flexible, the outcome can be characterised as *equilibrium* unemployment. As we shall see in the derivation below, it depends on relative bargaining strengths, the job separation rate each period, the reservation wage, the cost of vacancies, the probability of survival, and the growth rate of the labour force.

2.2.1 Employment dynamics

The employment dynamics concern the stocks and flows out of and into employment and unemployment. The equations derived in this section are included in all three sub models of KIMOD.

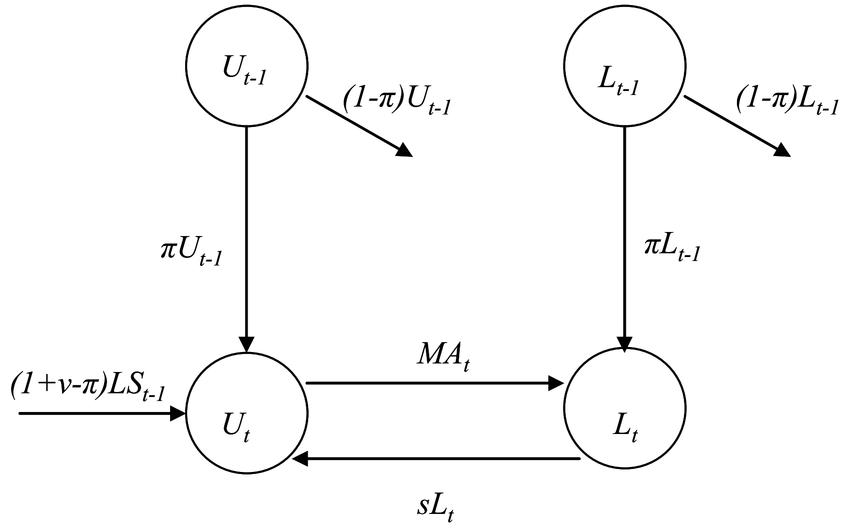
Figure 2.1 illustrates the flows on the labour market between two periods. As was discussed in section 2.1.1 above, the working population is equal to the labour supply and grows at an exogenous rate, denoted ν.[11] The dynamics of the labour supply are thus simply given by

$$LS_t = (1+\nu)LS_{t-1}. \tag{2.24}$$

[10] Part of the derivation below follows Lindén (2004) closely (although the notation differs).

[11] It was shown that $\nu = \pi(1+a)-1$, where π is the probability of non-exit, and a is the "birth-rate", both of which are exogenous.

Figure 2.1 Flows in the labour market in KIMOD



Note. *LS*: labour supply, *L*: no. of employed, *U*: no. of unemployed, π: survival rate, ν: growth rate of labour supply, *MA*: no. of matches, *s*: separation rate.
Source: Lindén (2004).

Furthermore, an individual is either employed or unemployed. It is sufficient to determine unemployment in order to determine employment, due to the exogenous growth rate in equation (2.24). Formally, the following identity holds in all periods:

$$LS_t = L_t + U_t, \tag{2.25}$$

where L_t is the number of employed individuals, and U_t is the number of unemployed.

The number of unemployed people in period *t* is equal to the unemployed last period who did not exit (πU_{t-1}), plus the flow of newcomers into the working population in the beginning of the period ($a\pi LS_{t-1} = (1+\nu-\pi)LS_{t-1}$), plus the number of separations (firings) during the period (sL_t, $0 \le s \le 1$), minus the number of successful matches (hirings) during the period (MA_t), that is,

$$U_t = \pi U_{t-1} + (1+\nu-\pi)LS_{t-1} + sL_t - MA_t, \tag{2.26}$$

where we have used the fact that $a\pi = 1+\nu-\pi$ is the *gross* growth factor of the labour supply.

The process of matching vacancies and unemployed is viewed as an ordinary production process. Matching is easier with more individuals unemployed or more vacancies to fill. Following the search-theoretical literature, we assume that the matching function is linear homogenous. Thus, if both unemployment and vacancies increase with one percent, then the number of successful matches also increases with one percent. In KIMOD, the number of successful matches between firms vacancies (VA_t) and unemployment is determined by the following matching function which is increasing in its both arguments and linearly homogenous:

$$MA_t = Z^{MA} U_t^{\eta^{MA}} VA_t^{1-\eta^{MA}}, \tag{2.27}$$

where Z^{MA} is a positive constant (often denoted "matching efficiency") and η^{MA} is the elasticity of matching with respect to unemployment. Using equations (2.24), (2.25) and (2.27) above, equation (2.26) implies that the dynamics of the unemployment rate (U_t / LS_t) is given by

$$\frac{U_t}{LS_t} = \frac{(1+\nu)(1+s) - \pi\left(1 - \frac{U_{t-1}}{LS_{t-1}}\right)}{(1+\nu)\left(1+s+\vartheta_t J(\vartheta_t)\right)}, \tag{2.28}$$

where labour market tightness is defined as

$$\vartheta_t = VA_t / U_t, \tag{2.29}$$

and the rate at which vacancies are filled is

$$J(\vartheta_t) = MA_t / VA_t. \tag{2.30}$$

Unemployment and vacancies are the only endogenous variables in equation (2.28) and we can calculate the vacancy rates (VA_t / LS_t) corresponding to different unemployment rates (U_t / LS_t). Combinations of unemployment and vacancy rates can be combined in the so-called Beveridge curve, (we return to this below). Note that the unemployment rate is constant in steady state.

Solving (2.30) for the number of vacancies (VA) and using the expressions in (2.24), (2.25) and (2.26), we get the following dynamic expression for employment:

$$\begin{aligned} VA_t &= \frac{MA_t}{J(\vartheta_t)} \\ &= \frac{\pi U_{t-1} + (1+\nu-\pi) LS_{t-1} + sL_t - U_t}{J(\vartheta_t)} \\ &= \frac{(1+s)L_t - \pi L_{t-1}}{J(\vartheta_t)}, \end{aligned} \tag{2.31}$$

or

$$L_t = \frac{1}{1+s} J(\vartheta_t) \cdot VA_t + \frac{\pi}{1+s} L_{t-1}. \tag{2.32}$$

Expression (2.32) is used below in section 2.3.3 were we discuss the behaviour of a representative firm. By opening vacancies, the firm can increase employment. The number actually in work at the end of the period depends also on the rate at which employees leave the firm (s), the rate at which employees formerly employed do survive (π), and the efficiency of the matching process ($J(\vartheta_t)$).

2.2.2 Labour demand

Labour demand in the Steady state and Flex-price models is determined by the representative firm's behaviour, discussed in section 2.3 below. This yields a first order condition involving the marginal productivity of labour and the labour cost, where both wages and hiring costs are taken into account. In the Flex-price model the marginal productivity (MPL_t) expression is

$$MPL_t = W_t^P \left(1 + \frac{\gamma(1+s)}{J(\vartheta_t)}\right) + W_{t+1}^P \left(\frac{1}{1+R_t}\right) \left(\frac{\gamma\pi \dfrac{LH_{t+1}}{LH_t}}{J(\vartheta_{t+1})(1+\nu)}\right), \tag{2.33}$$

where W_t^P is the real hourly wage rate, γW_t^P is the vacancy cost (cf. equation (2.44) below), R_t is the real interest rate, and LH_t is total working hours per year. In steady state, real wages grow at the rate A^{GRF} while the growth rate in hours and labour supply is zero. Hence, in the Steady state model the following expression is used:

$$MPL_{ss} = W_{ss}^P \left[1 + \left(\frac{\gamma}{J(\vartheta_{ss})}\right)\left(1 + s - \frac{\pi A_{ss}^{GRF}}{1+R_{ss}}\right)\right]. \tag{2.34}$$

The expression within the square brackets is greater than one, since $\pi A_{SS}^{GRF} / (1+R_{SS}) < 1$. This implies that the marginal product of labour exceeds the real wage, as the firms have to bear the search cost due to unfilled vacancies. Equations (2.33) and (2.34) implicitly give labour demand, measured as vacancies, as a function of the real wage.

2.2.3 Wage setting

At the beginning of each period, local negotiations between a firm and a worker over the real wage are assumed to take place. Firms aim at low wages and they would like to fill vacancies. Workers like high wages but dislike unemployment. The firm's value function describes how firms value vacancies and filled jobs, and the worker's value function describes how workers value unemployment and employment. The negotiation ends up in a so-called Nash bargaining solution. [12] This equilibrium wage rate is derived below. A number of factors together determine the bargain outcome:

- The relative bargaining strength of the worker, (μ, $0 < \mu < 1$), and of the firm ($1-\mu$). The higher is μ, the higher the negotiated wage rate.
- Unemployment benefits (i.e. the worker's reservation wage) are modelled as a share ($0 \leq \zeta \leq 1$) of the current real wage rate, ζW_t^P. The higher the reservation wage of the worker, the higher the negotiated wage rate.
- The workers' marginal product, MPL_t, is the reservation wage of the firm.[13] The higher the marginal product of labour, the higher the negotiated wage rate.
- The cost of a vacancy, relative to the wage rate, γ. The higher the cost of vacancies, the lower the negotiated wage rate.
- The greater the labour market tightness, VA_t / U_t, the higher the negotiated wage rate.
- The greater the share of exogenous job separations in each period (due to e.g. a continuous structural change) the more costly is the search for successful matches that must be carried out, and, hence, the lower the negotiated wage rate.

Before negotiation, a firm and a worker each have to value two different states (or outcomes). The firm has to calculate (i) the value of an unfilled vacancy (Λ_t^V), the case in which the negotiations break down, and (ii) the value of a filled vacancy (Λ_t^J). The worker has to calculate the value of (i) being unemployed (Λ_t^U), the case in which the negotiations break down, and (ii) the value of being employed (Λ_t^E).

Now, the difference between the value of a filled job and a vacancy ($\Lambda_t^J - \Lambda_t^V$) must be greater than zero. This is also true for the difference between the value of being employed and unemployed ($\Lambda_t^E - \Lambda_t^U$). We assume "free entry" of vacancies, or, in

[12] See Nash (1950) and Binmore et al (1986).

[13] It is possible however that the wage rate temporarily exceeds the marginal product if the firm expects an increase in its labour demand in the near future. This is a type of labour hoarding behavior.

other words, to open a vacancy costs nothing. This implies that $\Lambda_t^V = 0$, that is, firms open vacancies until their value is zero.

The Nash product that is to be maximized can now be formulated as follows:

$$\max_{W_t^i} \Omega_t^i = \left[\Lambda_t^{E,i}(W_t^i) - \Lambda_t^U\right]^{\mu} \left[\Lambda_t^{J,i}(W_t^i) - \Lambda_t^V\right]^{1-\mu}, \tag{2.35}$$

and the first order condition implies that

$$(\Lambda_t^J - \Lambda_t^V) = \frac{(1-\mu)(1+\tau_t^E)(\Lambda_t^E - \Lambda_t^U)}{\mu(1-\tau_t^W)}, \tag{2.36}$$

where μ is the relative bargaining strength of the worker; τ_t^E and τ_t^W are tax wedges associated with profits respectively wages. The above equation for period t and t+1 can be used to substitute the value functions of the firm $(\Lambda_t^J - \Lambda_t^V)$ for those of the worker $(\Lambda_t^E - \Lambda_t^U)$.

Eliminating the value functions in (2.36) yields an equation describing the equilibrium wage rate, i.e. a so called wage setting curve,

$$W_t = \frac{\mu MPL_t}{\left[1 - \mu\pi\vartheta_t\gamma\left(\frac{1-\tau_{t+1}^W}{1-\tau_t^W}\right) - (1-\mu)\zeta - \mu\pi\gamma\left(1 - \frac{1-\tau_{t+1}^W}{1-\tau_t^W}\right)\left(\frac{1-s}{J(\vartheta_t)}\right)\right]}. \tag{2.37}$$

Equation (2.37) is included in the flex-price part of KIMOD. The steady state version is given by

$$W_{ss}^P = \frac{\mu MPL_{ss}}{1 - \mu\pi\vartheta_{ss}\gamma - (1-\mu)\zeta}, \tag{2.38}$$

from which it is clear that the assumed bargaining strength of the workers (μ) is important for the determination of the equilibrium wage rate. A higher bargaining strength implies a higher wage. In steady-state, however, the wage cannot exceed the *firm's* reservation wage (MPL_{ss}) This implies that the parameters of KIMOD have to meet the following condition:

$$W_{ss}^P \leq MPL_{ss} \quad \Leftrightarrow \quad \frac{\mu}{1 - \mu\pi\vartheta_{ss}\gamma - (1-\mu)\zeta} \leq 1. \tag{2.39}$$

In the normal case this inequality is strict, so that the negotiation puts a wedge between the real wage and the marginal productivity, compared to the case of perfect competition.

2.2.4 Determination of labour market tightness, unemployment and vacancies in equilibrium

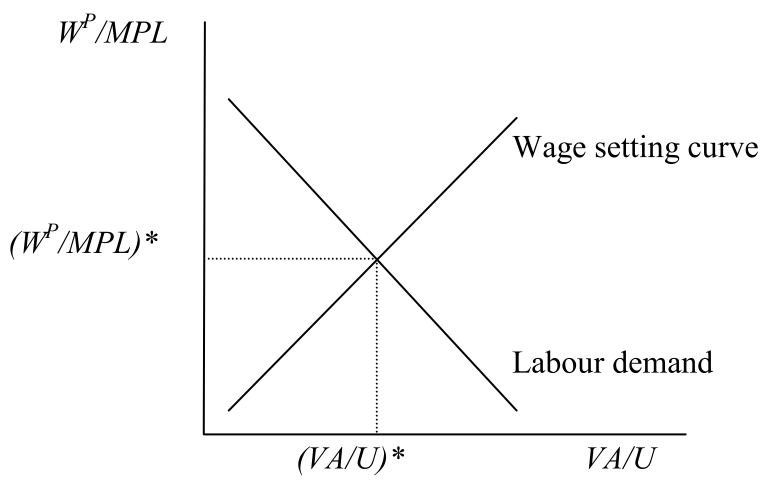
To sum up, *the labour demand curve* in equation (2.34) is only determined by the firms' optimization behaviour, taking the real wage as given. *The wage setting curve* in equation (2.38) is determined by negotiations between both workers and firms; the workers thus indirectly also control the labour demand. When the two curves intersect, the labour market is in equilibrium. Figure 2.2 illustrates the result.

Formally, eliminating W^P/MPL from equations (2.34) and (2.38) yields the following expression:

$$\pi\vartheta_t + \left(1 + s - \frac{\pi A_t^{GRF}}{1+R_t}\right)\frac{1}{J(\vartheta_t)} = \frac{1}{\gamma\mu}\left[1 - \mu - (1-\mu)\zeta\right]. \quad (2.40)$$

Labour market tightness (ϑ_t) is the only endogenous variable in the above equation and can therefore be determined. This equilibrium value can then be inserted in either the labour demand or the wage setting equation to calculate the equilibrium value of the real wage to marginal productivity ratio, W^P/MPL.

Figure 2.2 Labour market equilibrium

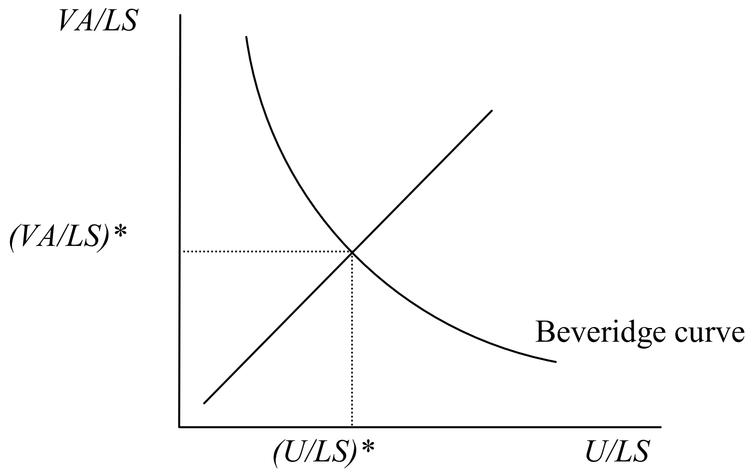


Note. W^P: real wage in private sector, *MPL*: marginal productivity of labour, *VA*: no. of vacancies, *U*: no. of unemployed.

Source: Lindén (2004).

Equilibrium unemployment and vacancies is then given by the dynamics of the labour market. By definition, $\vartheta_t = VA_t / U_t$, which is a straight line from the origin in the vacancies-unemployment space, see figure 2.3, where the slope is equal to the labour market tightness. The equilibrium unemployment rate is given by the intersection of this line and the Beveridge curve (see equation (2.28), under equilibrium labour market tightness. Finally, the equilibrium number of vacancies can be calculated using the definition of labour market tightness.

Figure 2.3 The Beveridge curve and equilibrium labour market tightness



Note. *VA*: no. of vacancies, *U*: no. of unemployed, *LS*: labour supply.
Source: Lindén (2004).

2.3 Private production

In KIMOD, the production of private goods for final consumption takes place in a two step procedure. The first step uses resources and generates added value in the form of intermediaries while the second step simply combines domestic and imported intermediate goods into final goods. The first three sections below present the theory behind the intermediate production. In section 2.3.4, the production of final goods is discussed.

2.3.1 Preliminaries

An infinite number of identical competitive firms use a constant return to scale technology (Cobb-Douglas) to produce an intermediate good. In this process, resources are used in the form of labour and capital services. Resources are also used for installation of capital and for hiring labour. The basics of a representative firm are given by the following equations.

Technology:

$$\begin{aligned} Y_t^{PG} &= f\left(K_{t-1}^P, A_t^H LH_t^P\right) \\ &= \left(K_{t-1}^P\right)^{\eta^{YPG}} \left(A_t^H LH_t^P\right)^{1-\eta^{YPG}} \end{aligned} \tag{2.41}$$

Capital accumulation:

$$P_t^I K_t^P = P_t^I I_t^P + (1-\delta) P_t^I K_{t-1}^P \tag{2.42}$$

Total cost of investment:

$$IC_t^P = I_t^P + \lambda \frac{\left(I_t^P\right)^2}{K_{t-1}^P} \tag{2.43}$$

The cost of hiring labour:

$$VC_t^P = \gamma \frac{NW_t}{P_t} \left(\frac{L_t^P}{L_t}\right) VA_t \left(\frac{LH_t^P}{L_t^P}\right) \tag{2.44}$$

The production function in equation (2.41) describes the private gross production (Y^{PG}), where η^{YPG} is the output elasticity of capital (equal to the capital share in equilibrium), AH_t denotes labour-augmenting (i.e., Harrod-neutral) productivity, and LH_t^P is hours worked in the private sector; thus $A_t^H \cdot LH_t^P$ is the amount of efficient labour. Harrod-neutrality implies the property that the factor proportions only change if relative factor prices change. The capital accumulation equation (2.42) implicitly assumes that the depreciation rate (δ) captures physical depreciation only, not the effect of a change in asset prices (P_t^I). Furthermore, equation (2.43) shows that a standard assumption of a quadratic installation cost is made, with parameter λ, $\lambda > 0$. According to equation (2.44), the unit cost for vacancies during a period is assumed to be proportional to the annual wage of an employed person ($NW_t\left(LH_t^P / L_t^P\right)$, where LH_t^P / L_t^P is the average working hours in the private sector) with the (constant) factor γ, $\gamma > 0$, indicating the proportion. The number of vacancies in the private sector is a fraction (L_t^P / L_t) of the total number of vacancies (VA_t).

The value added of the firm (denoted Y_t^P) is equal to the value of the gross production, less the installation cost of capital and the cost of hiring labour, that is, [14]

$$PB_t^{YP} Y_t^P = PB_t^{YPG} Y_t^{PG} - P_t^I \left(IC_t^P - I_t^P \right) - P_t VC_t^P . \qquad (2.45)$$

The net cash flow from the firm to the household is defined as the amount available for dividends in the current period. Corporate incomes are not taxed at the corporate level, but the dividends are taxed at the household level. Net operating surplus (OS) is defined as the value added less the cost of wages and consumption of fixed capital, that is,

$$P_t OS_t = PB_t^{YP} Y_t^P - NW_t LH_t^P - P_t^I \delta K_{t-1}^P , \qquad (2.46)$$

where NW is the nominal wage in the private sector, including employers' social security contributions.[15]

Dividends ($P_t D_t$) are finally given by the value added less the cost of wages and investments, that is, they are equal to the operating surplus less net investment cost, that is,

$$\begin{aligned} P_t D_t &= PB_t^{YP} Y_t^P - NW_t LH_t^P - P_t^I I_t^P \\ &= P_t OS_t - P_t^I \left(I_t^P - \delta K_t^P \right). \end{aligned} \qquad (2.47)$$

2.3.2 Investors' perspective

It is assumed that firms act on behalf of their owners and maximize their profits. The household can choose between consumption and investing in bonds or shares. The planning horizon differs between households and firms. For an individual household there exist two financial assets, one nominal bond with prices determined on a world market and one entitlement to future dividends from the firms.

Solving the household's optimization problem yields an expression for the real value of the firm's equity, that is, the real value of the total number of shares:

$$V_t = \frac{1}{1+R_t} \left(V_{t+1} + \left(1 - \tau_{t+1}^d \right) D_{t+1} \right), \qquad (2.48)$$

[14] PB refers to "basic prices" according to the National accounts, that is, factor prices plus other taxes (net of subsidies) on production not included "taxes on products" (e.g. VAT), or, equivalently, market prices less taxes on products (VAT).

[15] The real rate of return on capital can be measured as $P_t OS_t / P_t^I K_{t-1}^P$.

which is a dynamic equation. It can be solved to yield the following familiar expression:

$$V_t = \sum_{j=1}^{\infty} \gamma_{t+j} (1-\tau_{t+j}^d) D_{t+j}, \tag{2.49}$$

where the discount factor is $\gamma_{t+j} = 1/\left(1+R_{t+j}\right)\gamma_{t+j-1}$, $\gamma_t = 1$, and where we have assumed that V and D are bounded from above and that the "No-Ponzi-game condition" is fulfilled. Note that the value of the firm is independent of the generation index. One interpretation of equation (2.49) is that the equilibrium (arbitrage free) value of the outstanding shares depends on the expected present value of pre-tax dividends, discounted with the opportunity cost given by the return on bonds. At the same time, this is the value which the household wants the firms to maximize.

2.3.3 Behaviour of the representative firm

The firm will choose production, hiring and investment as if it would maximize the value of the firm for its owners. Using the expression for V_t in equation (2.49), this implies that the firm solves

$$\max\left[\sum_{j=1}^{\infty} \gamma_{t+j} (1-\tau_{t+j}^d) D_{t+j}\right], \tag{2.50}$$

subject to the "budget constraint" in equation (2.47), the dynamics of the vacancies in equation (2.31), and appropriate boundary and non-negativity conditions.

Furthermore, wage negotiations as described in section 2.2 are assumed to take place and determine real wages in advance of the firm's other decisions, including the number of vacancies to open. Therefore the firm will take real wages as given.

The optimal investment path is given by the first order condition, cf. Tobin's q,

$$q_t = \left(1 + 2\lambda \frac{I_t^P}{K_{t-1}^P}\right), \tag{2.51}$$

i.e. the value of a marginal unit of capital is equal to the marginal cost for investment and the increase in installation costs. If q is greater than the total marginal cost, it will be profitable to invest further. Due to the homogenous specification of installation costs in equation (2.43), marginal q and average q coincide.

The first order condition for choosing an optimal path for capital is

$$\left(1-\tau_t^d\right)q_t P_t^I = \frac{\left(1-\tau_{t+1}^d\right)}{1+NR_t}\left[PB_{t+1}^{YPG} f_{K,t+1} + \lambda \frac{P_{t+1}^I \left(I_{t+1}^P\right)^2}{\left(K_t^P\right)^2} + q_{t+1} P_t^I \left(1-\delta\right)\right], \tag{2.52}$$

where the marginal product of capital is given by: $f_{K,t+1} = \frac{\eta^{YPG} Y_{t+1}^{PG}}{K_t^P}$.

The interpretation of this is that the shadow price today is equal to the present value of the next period's increase in marginal productivity and the decrease in costs for installation, as well as the next period's value for q.

An interpretation of q can be suggested by forward substitution of (2.52), slightly simplified by assuming constant rates of taxation, giving

$$q_t = \sum_{j=0}^{\infty} \gamma_{t+1+j} (1-\delta)^j \left[\frac{PB_{t+j+1}^{YPG} f_{K,t+j+1}}{P_{t+j}^I} + \lambda \frac{P_{t+j+1}^I}{P_{t+j}^I} \left(\frac{I_{t+j+1}^P}{K_{t+j}^P}\right)^2\right], \tag{2.53}$$

that is, the shadow price of installed capital is equal to the discounted future incomes from the marginal product of that investment and the reduction of installation costs due to the expansion of the capital stock.

The first order condition for hiring labour is

$$PB_t^{YPG} f_{L,t} = NW_t\left(1+\frac{\gamma(1+s)}{J(\vartheta_t)}\right) - NW_{t+1}\left(\frac{1}{(1+NR_t)}\frac{\gamma\pi LH_{t+1}^{GRF}}{J(\vartheta_{t+1}) LS_{t+1}^{GRF}}\right). \tag{2.54}$$

The second term on the right hand side shows that profits are increasing in future increases in π, due to savings in costs for vacancies when more of the employed today survive until tomorrow, and because of the resulting reduction in future efficiency of matching $J\left(\vartheta_{t+1}\right)$.

2.3.4 Final good production and relative prices

There are three types of final privately produced goods in the economy: a consumption good, an investment good, and an export good. Competitive firms produce these goods using a constant return to scale technology. They use both the domestically produced intermediate good (discussed above), and an imported good as factors of production. Thus, these firms essentially combine the intermediate good and the imported good, there is no value added created in this production and no profits either, as the firms just cover their costs. The producers of the intermediate good are not able to price discriminate between the final goods producers. The price of the input to each final good is thus the same, and is therefore equal to the price of total private production. It follows that the value added of private production (Y_t^P) must satisfy the following identity:

$$Y_t^P = C_t^H + G_t^H + I_t^H + X_t^H, \tag{2.55}$$

where superscript H denotes *home* produced goods, used as input to the private consumption good, the government consumption good (discussed in section 2.4), the investment good, and the export good, respectively. Total imports must also satisfy a similar adding up constraint,

$$M_t = C_t^F + I_t^F + X_t^F, \tag{2.56}$$

where superscript F denotes imported *foreign* input. It is assumed that the production of the government consumption good does not use any imported input and that the import price is the same regardless final use.

Consider the production of the consumption good. The market value (at basic prices) of private consumption is denoted $PB_t^C C_t$ and the cost of producing it consists of the cost for the home produced input and the imported foreign produced good. Hence, the competitive firms seek to maximize profits, defined as

$$\Pi_c = PB_t^C C_t - PB_t^{YP} C_t^H - P_t^M C_t^F, \tag{2.57}$$

subject to the technology constraint

$$C_t = A_c \left(C_t^H\right)^{\eta^C} \left(C_t^F\right)^{1-\eta^C}. \tag{2.58}$$

The first order conditions leads to the conditional demand functions:

$$C_t^H = \left(\frac{\eta^C}{1-\eta^C}\frac{P_t^M}{PB_t^{YP}}\right)^{1-\eta^C}\frac{C_t}{A_c} \tag{2.59}$$

$$C_t^F = \left(\frac{\eta^C}{1-\eta^C}\frac{P_t^M}{PB_t^{YP}}\right)^{-\eta^C}\frac{C_t}{A_c}, \tag{2.60}$$

which together with similar expressions for the other good and the identities in equation (2.55) and (2.56), are used to determine total private production and imports. The first order conditions also determine the relative price of consumption, and in a similar way, the relative price of investment and exports,

$$PB_t^C = \left[\left(\frac{\eta^C}{1-\eta^C}\right)^{(1-\eta^C)} + \left(\frac{\eta^C}{1-\eta^C}\right)^{-\eta^C}\right]\frac{\left(PB_t^{YP}\right)^{\eta^C}\left(P_t^M\right)^{1-\eta^C}}{A_c}. \tag{2.61}$$

Due to competition, firms make zero profits in equilibrium, which gives marginal cost pricing according to equation (2.61). Marginal costs, that is the right hand side of (2.61), depend on domestic and import prices, all measured in national currency, productivity as well as the demand elasticity.

2.4 The government

The formulation of government behaviour in KIMOD is much simpler than that of the household and firm outlined in the previous sections. There are several reasons for this. First, there is only one agent, the general government. That is, we do not have to aggregate expressions over individuals or generations as we did for the households. Second, whereas individuals and firms solve explicit maximization problems, the majority of the choices of the government are simply *imposed* in the steady state and flex-price parts of KIMOD. Debt and spending targets are such examples. Other variables (e.g., the labour income tax rate and the budget balance), although endogenous, are restricted in such a way that appropriate long run targets are met.

The government affects the behaviour of individuals and firms in several ways. For example, due to the assumption of finite lives, Ricardian equivalence does not hold in the model. Hence, the timing of taxes affects the perceived wealth of the individuals, which in turn affects their consumption and saving decisions. Note, however, that the labour income tax does not affect labour supply since the latter is exogenous in the current version of the model.[16] The government influences firms through the aggregate demand channel and thereby expected returns. Moreover, profitability is affected by the tax rate on dividends.

In sections 2.4.1 and 2.4.2, we describe government production and consumption, respectively. In section 2.4.3 we present the public sector's budget constraint. In order to exclude explosive paths of government debt, the model is written in a way that ensures that imposed debt targets are met in steady state. These dynamic processes are discussed in section 2.4.4.

2.4.1 Government production

The government consumes all of the goods and services it produces. The production function is simple and corresponds to the National Accounts measure of production in

[16] The individuals behave as if the labour income tax were a lump sum tax.

the public sector. Both labour and capital are used in production and the value added from government production is defined as

$$PB_t^{YG} Y_t^G = \alpha NW_t^P LH_t^G + P_t^I \delta K_{t-1}^G, \tag{2.62}$$

where Y_t^G is the government real value added, or real production, PB_t^{YG} is the government production deflator at basic prices, αNW_t^P is the nominal hourly wage rate in the public sector, assumed to be proportional to the wage in the private sector, LH_t^G is total hours worked in the government sector, δ is the depreciation rate of capital, and K_t^G is the government capital stock.

2.4.2 Government consumption

Government consumption refers to the traditional part included in the GDP identity. The major component of government consumption is government production. The second, and final, component is the amount of the home produced consumption good purchased by the government, G_t^H.[17] The nominal consumption identity for the government is

$$PB_t^G G_t = PB_t^{YP} G_t^H + PB_t^{YG} Y_t^G, \tag{2.63}$$

where PB_t^G is the government consumption deflator, PB_t^{YP} is the deflator of production of consumption goods, all in basic prices.

2.4.3 Budget constraint

Government consumption is an important part of the expenditure side of the public sector's budget constraint. The public budget balance is given by:

$$\begin{aligned} P_t BB_t = {} & \tau_t^W W_t^P PB_t^{YPG} LH_t \left[\left(\frac{LH_t^P}{LH_t} \right) + \alpha \left(\frac{LH_t^G}{LH_t} \right) + \zeta \left(\frac{UH_t}{LH_t} \right) \right] \\ & + \tau_t^C PB_t^C C_t + \tau_t^G PB_t^G G_t + \tau_t^D P_t D_t + \delta P_t K_t^G + NR_{t-1} P_{t-1}^M B_{t-1}^G \\ & - P_t^G G_t - P_t^I I_t^G - UH_t \xi W_t^P PB_t^{YPG} - P_t TR_t . \end{aligned} \tag{2.64}$$

The first four terms in the equation are tax revenues from, respectively, labour income, private consumption, government consumption, and dividends. The next terms are gross value added of government production[18] and net interest payments. On the

[17] It is straightforward to allow government consumption to include imported consumption goods as well.

[18] Gross operating surplus equals the value of production minus wage costs.

expenditure side we have government consumption, government investments, unemployment benefits, and other net transfer payments. Note that the tax base for taxes on labour income includes income of the unemployed.

2.4.4 Dynamic expressions for the public sector

Although most of the government variables are just imposed exogenously, there are some exceptions. The evolution of debt is naturally a variable that is endogenous and depends on the dynamics of the budget balance (which includes the interest rate on net bond holdings) and changes in the relative price of imports ($P^{M\_P} \equiv P^M / P^P$). More specifically, the dynamics of real government bond holdings are defined as:

$$P_t^{M\_P} B_t^G = BB_t + P_{t-1}^{M\_P} B_{t-1}^G + (P_t^{M\_P} - P_{t-1}^{M\_P}) B_t^G. \tag{2.65}$$

Note that there is no interest term in this expression as it was accounted for in the expression for the budget balance above.

In order not to allow the government to choose explosive paths of its net bond holdings (or, equally, net debt), at least one policy variable must be determined endogenously so that a preset target for net bond holdings is fulfilled in steady state. That is, although net bond holdings can temporarily be off target in the flex-price part of KIMOD, they are forced to go in the direction determined by the exogenously given steady state target. In the present version of KIMOD, we use the labour income tax rate to be the policy variable that ensures that the net bond holdings target is fulfilled in the long run. More specifically, we let the labour income tax rate in the flex-price part of KIMOD be a function of both the steady state labour income tax rate (τ_{ss}^W) and the steady state target for net bond holdings ($P_{ss}^{M\_P} B_{ss}^G / Y_{ss}$), that is,

$$\tau_t^W = \chi \tau_{t-1}^W + (1-\chi)\left[\tau_{ss}^W - \chi\left(\frac{P_t^{M\_P} B_t^G}{Y_t} - \frac{P_{ss}^{M\_P} B_{ss}^G}{Y_{ss}}\right)\right]. \tag{2.66}$$

Equation (2.66) is of course only one of many possible formulations. Note, finally, that any other policy variable can be used in order achieve the steady state net bond holding target.

2.5 The external sector

KIMOD is designed for analyses of the Swedish economy. The model thus satisfies the standard properties of a domestic small open economy. The interaction with the rest of the world, i.e. the external sector, is discussed in this section.

2.5.1 Net foreign assets, net exports, and the current account

Some of the parameters in the model are crucial for the determination of the dynamic paths and the steady state values of net foreign assets, net exports and the current account. Important examples of these so-called 'deep parameters' are the real interest rate (R) and the subjective discount rate (β).

The dynamics of the net foreign asset position are determined by the development of net exports. If net exports are positive, domestic claims on the rest of the world increase or, equivalently, net foreign assets increase. Since government bonds are traded on an international market, net foreign assets are equal to the annual change in the sum of private and government bonds ($B^P + B^G$). The formal relationship between net exports (X_t^{NET}) and net foreign assets is the following:

$$X_t^{NET} = (B_t^P + B_t^G) - (1 + R_{t-1}) \frac{P_{t-1}^{M\_P}}{P_t^{M\_P}} (B_{t-1}^P + B_{t-1}^G). \tag{2.67}$$

This equation is included in the flex-price part of KIMOD. The steady state version is

$$X_{ss}^{NET} = \left[1 - \frac{1 + R_{ss}}{A_{ss}^{GRF}(1+\nu)} \right] (B_{ss}^P + B_{ss}^G), \tag{2.68}$$

were A_{ss}^{GRF} is the growth factor of productivity, and ν is the growth rate of labour supply. Note that if consumers become, say, more patient and savings increase, then private bond holdings (B^P) rise and so do net foreign assets. This gathering of assets will allow for higher net exports in steady state according to (2.68).

Finally, the current account balance (CA) is simply defined as the sum of net exports and interest rate earnings on the stock of net foreign assets, that is,

$$CA_t = P_t^{M\_P} X_t^{NET} + R_{t-1} P_{t-1}^{M\_P} (B_{ss}^P + B_{ss}^G). \tag{2.69}$$

2.5.2 Real exchange rate

What regards the determination of the real exchange rate, the law of one price are *not* assumed to hold in KIMOD, not even in the long run. That is, the model is not required to respect the *Absolute* Purchasing Power Parity (PPP). However, we do require (some) respect for the relative version of PPP. This hypothesis says that the domestic inflation, for a specific consumption basket, should in the long run be equal to the expected foreign inflation, measured in a common currency. This would imply that the *expected* real exchange rate should be constant over time in equilibrium.

Furthermore, the assumption in KIMOD of a common international bond market implies that the real interest rates are the same for all agents in the model world. This, in turn, implies that the expected rate of future depreciation or appreciation is zero, to satisfy the uncovered interest rate parity (UIP). This is another reason why long-lasting trends of the real exchange rate are not allowed in the Flex-price model. Only unanticipated shocks could change the real exchange. Strictly speaking, in between two steady states, real exchange rate jump instantaneously to a new equilibrium, such that UIP holds.

In practice, the model must also be able to explain the persistence behaviour of the real exchange rate in data, besides the theoretical parities. Therefore, we relax the theoretical results in the Sticky-price model by allowing smooth adjustments over a few years to a new steady state, instead of a discrete jump (see chapter 3). The model used in the equilibrium part of KIMOD for determining the real exchange rate is based on, and described in, Nilsson (2002). Two equations included in the Steady-state model are as follows:

$$RER_t = Z^{RER} \left(\frac{A_t^H}{A_t^F} \right)^{\eta^{RER2}} \left(\frac{P_t^X}{P_t^M} \right)^{\eta^{RER1}} \exp\left\{ \frac{P_t^M \left(B_t^P + B_t^G \right)}{P_t Y_t} \right\}^{\eta^{RER3}} \tag{2.70}$$

$$\frac{P_t^M}{P_t} = Z^{PM\_P} RER_t \frac{P_t^{MF}}{P_t^F}. \tag{2.71}$$

The real exchange rate (RER_t) is explained by relative TFP-levels (A_t^H / A_t^F), terms of trade (P_t^X / P_t^M) and net foreign asset to GDP ratio ($(B_t^P + B_t^G)/Y_t$). Furthermore, the relative price of imports (P_t^M / P_t) are (almost by definition) equal to the relative import price denominated in foreign currency, multiplied with the real exchange rate.

3 Business cycle theory: the Sticky-price model

The third and final part of KIMOD is the *Sticky-price model*. Here, imperfect information and bounded rationality are present, yielding nominal as well as real rigidities. Imperfect information and bounded rationality make consumers, investors as well as price and wage setters partially backward looking, which in turn implies that adjustments to a new equilibrium are time consuming. These rigidities imply a role for stabilization policies, such as monetary and fiscal policies, with the aim of moving the economy in the direction of the flex-price equilibrium. In short, the Sticky-price model aims at describing the properties of the Swedish economy at business cycle frequencies, say 1–6 years.

By construction, the Sticky-price model takes equilibrium developments as exogenous and all deviations from equilibrium are of transitory nature. In particular, nominal shocks have no lasting real effects and cannot change relative prices permanently. This means that stabilization policy, in the form of monetary and fiscal interventions, has no impact on equilibrium levels of real variables such as GDP and unemployment.

Since economic theory is more eclectic in its explanations of business cycle movements, we need to sacrifice some theoretical stringency in order to be able to describe the Swedish economy at business cycle frequency. Hence, in the Sticky-price model, we introduce structure and variables in a somewhat ad hoc manner, as compared to the theoretically based Steady state and Flex-price models.

This chapter describes how volumes, prices, expectations, monetary policy and fiscal policy are modelled in the Sticky-price model.

3.1 General structures for volumes and prices

The Sticky-price model is designed to deliver both policy advice and support to the forecasting process at the NIER. A macroeconomic model like KIMOD needs to be tractable in order to be able to tell a reliable economic “story” around policy issues including forecasts. Telling reliable stories is an important part of the presentation of results from KIMOD. Compared to more traditional forecasting processes, a model like KIMOD provides a consistent structure to the discussion. The model ensures that all the main economic restrictions, consistency requirements and relationships are included in the analysis. As a consequence, the Sticky-price model *is not* built to minimize forecast errors. Saying this does not imply that statistical fit is not an issue but, rather, that statistical fit is not the only (or the most important) issue.

To formulate a tractable short run model, we need to model volume and price responses separately. The motion equations for most real variables therefore lack an explicit dependence on associated prices.[19] This is not that far fetched in a staggered price setting where volumes in the short run are demand driven anyway. The following sections will discuss the general principles behind volume and price behaviour.

3.2 Volume equations

In this section, we first discuss the general structure of the volume equations (section 3.2.1). Then we turn to the principles of estimating volume relations (section 3.2.2). Finally, we discuss some important volume equations in the Sticky-price model (sections 3.2.3-3.2.11).

3.2.1 The basics for volume equations

As discussed in chapter 1, the three models of KIMOD – Steady state, Flex-price and Sticky-price – are valid at different frequencies (long run, medium run and short run). Still, a variable included in all of the three models could be determined by similar processes across the models. Private consumption, for example, is driven by total wealth in both the Steady state and Flex-price models. Total wealth, however, is somewhat sticky in the Flex-price model due to real rigidities. As a result, the private consumption paths of the Steady state and Flex-price models generally coincide only in the long run. In the Sticky-price model, private consumption is still driven by total wealth through the latter's dependence on the flex-price value of private consumption. But, as we add imperfect information and imperfectly rational behaviour to the Sticky-price model, private consumption now becomes also driven by other processes such as uncertainty and habits.

The generic form of the dynamic equations for volumes in the Sticky-price model is derived here. The derivation illustrates the close connection to the Flex-price model

[19] If prices and volumes were to be determined simultaneously, the model would become untractable. Prices do, however, affect volumes indirectly through their effects on monetary policy and thus interest rates and exchange rates.

discussed above. If we let variable X be driven by the same process in both the Sticky-price (superscript "*SP*") and Flex-price (superscript "*FP*") models, we have:

$$\begin{aligned} X_t^{SP} &= a_0 + a_1 X_{t-1}^{SP} + a_2 Y_t^{SP} + a_3 Z_t^{SP} \\ X_t^{FP} &= a_0 + a_1 X_{t-1}^{FP} + a_2 Y_t^{FP} + a_3 Z_t^{FP}, \end{aligned} \tag{3.1}$$

where the variables X_t and Y_t are endogenous, Z_t is exogenous[20], the coefficients (assumed here to be the same in the two models) are constants and $-1 < a_1 < 1$. In order to derive the generic form of the volume equations in the Sticky-price model, we first perform a purely algebraic operation and set the two equations in (3.1) equal to each other:[21]

$$X_t^{SP} - a_1 X_{t-1}^{SP} - a_2 Y_t^{SP} - a_3 Z_t^{SP} = X_t^{FP} - a_1 X_{t-1}^{FP} - a_2 Y_t^{FP} - a_3 Z_t^{FP}. \tag{3.2}$$

If we then add $\left(X_{t-1}^{FP} - X_{t-1}^{SP}\right)$ to both sides and rearrange, we get:

$$X_t^{SP} = X_{t-1}^{SP} + \left(X_t^{FP} - X_{t-1}^{FP}\right) - \left(1 - a_1\right)\left(X_{t-1}^{SP} - X_{t-1}^{FP}\right) + a_2\left(Y_t^{SP} - Y_t^{FP}\right) + a_3\left(Z_t^{SP} - Z_t^{FP}\right), \tag{3.3}$$

which is the generic form for the dynamics of volumes in the Sticky-price-model. In short, we make use of the fact that variable X is driven by the same processes in the two models and model the development of X^{SP} as *deviations* from the Flex-price model.[22] More specifically, the interpretation of the first two expressions on the right-hand-side of equation (3.3) is that the annual change in the variable is basically equal to the annual change in the equilibrium counterpart. The third expression is an error-correction term, increasing the annual change if the variable was below equilibrium last period, i.e. if the "gap" were negative.[23] The fourth expression implies that it is the gap, not the level, of other endogenous variables that influences the result. Finally, the fifth expression concerns the effect on X^{SP} stemming from the deviation of the exogenous variable Z from its flex-price value.

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[20] As we will discuss later on (see section 3.2.2), we try to avoid the inclusion of exogenous variables. Of course, there are cases in which exogenous variables have to play an important role. For example, foreign productivity growth is exogenous and included in the equation determining Swedish productivity growth.

[21] This is valid as both the left and right hand sides of equation (3.2) are equal to a_0 due to equation (3.1).

[22] Note that equation (3.3) is just a generic example. In practice, due to the inclusion of imperfect information and non-rationality, a variable like X^{SP} may be driven by variables not included in the Flex-price model.

[23] Assuming $0 < a_1 < 1$.

Equation (3.3) also implies that the Sticky-price variable converges to the flex-price variable if Y^{SP} and Z^{SP} converge to the flex-price counterparts. Hence, one has to include a structure (e.g. using error-correction terms) that is compatible with this requirement.

3.2.2 Choice of volume equations

In order to be able to project the model into the future, we have to take into account historical economic relationships. These historical relationships may have more or less of a theoretical foundation. As stressed above, however, we do not aim at minimizing forecasting errors at the cost of not being able to tell economically reliable stories.

We try to only include *endogenous* variables when possible in the various volume specifications, that is, variables which are determined elsewhere in the Sticky-price model. Again the reason is the importance of being able to support the result with a reliable story. By definition, an exogenous variable is determined (and predicted) outside the model and can therefore not contribute to an explanation of the overall model response.[24]

In short, the following criteria have been guiding the choice of volume equations:

- The specification must ensure that the sticky-price variable in question converges to the flex-price counterpart within reasonable time. This is generally ensured by the inclusion of error-correction terms.
- The variables included must be determined elsewhere in the model, i.e. they must be *endogenous*.
- We use variables suggested by economic theory whenever possible.
- We aim at good fit.
- We eliminate serial correlation and endogeneity problems by using lags and instrumental variable in the estimation phase.

In the following sections, we present the general structure of some important volume equations. The exact specifications are not reported as they are in the process of revision.

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[24] Of course, there are cases in which exogenous variables should be included. One such case is the labour supply, which currently is exogenous in KIMOD. This variable is important for determining the unemployment rate (and thereby short run nominal wage growth etc.) in the Sticky-price model.

3.2.3 Private consumption expenditure

The consumption function is in the spirit of Muellbauer and Lattimore (1994) who incorporate stochastic expectations, precautionary behaviour, credit constraints, habit information, alternatives to rational expectations as well as assets in their consumption function. Although all of their theoretically justified variables have not been included in KIMOD 1.0, some of them (such as assets) may be included in the future. The following variables are included:

$$\Delta \ln C_t = f\left(\Delta \ln C_t^{FP}, \Delta \ln C_{t-1}, \Delta \ln YD_{t-1}, \Delta U\_LS_t, \Delta R_{t-1}, \ln C_{t-1} - \ln C_{t-1}^{FP}\right), \tag{3.4}$$

where superscript "FP" refers to equilibrium values from the Flex-price model, YD is disposable income, U_LS is the unemployment rate, R is the real interest rate and $\ln C_{t-1} - \ln C_{t-1}^{FP}$ is an error-correction term. In general, unemployment represents (otherwise unmodelled) uncertainty whereas lagged consumption growth and lagged disposable income account for habits and credit constrained individuals, respectively.

3.2.4 Gross capital formation in the business sector (investments)

In the Steady state and Flex-price models, investments are determined by Tobin's q. This theory, although probably valid in the long run, has difficulties explaining short run movements in investment flows.

According to the research literature, current and historical cash flows[25] can be important variables in determining short run investment flows (see Gilchrist and Himmelberg, 1998, and Hubbard, 1998). That is, firms seem to prefer to use their own cash instead of new loans or new issues of stocks. In the estimated investment function, change in cash flow is approximated by production growth (ΔY^P). The following variables are included in the investment equation of the private sector (I^P):

$$\Delta \ln I_t^P = f\left(\Delta \ln I_t^{P,FP}, \Delta \ln Y_{t-1}^{P,FP}, \Delta \ln Y_{t-1}^P, R_{t-1}, R_{t-1}^{FP}, \ln I_{t-1}^P - \ln I_{t-1}^{P,FP}, \ln K_{t-1}^P - \ln K_{t-1}^{P,FP}\right), \tag{3.5}$$

where K^P is the capital stock in the private sector. One can note that two error-correction terms are needed here, with respect to both investments and the capital stock, in order to ensure that both investment and capital stock levels converge to their equilibrium counterparts.

[25] The cash flow is generally defined as the value of sales minus cost of production. In KIMOD the latter corresponds to the cost of investments and labour (including installation costs of capital and vacancy costs).

Total investment (I) in equation (3.5) is the sum of private investment (I^P) and government investment (I^G). The latter is exogenous in KIMOD 1.0 and determined by the government sector experts at the NIER.

3.2.5 Exports

As can be seen from the consumption function, equation (3.4), and the investment function, equation (3.5), the real interest rate (R) enters only with a lag. Exports, on the other hand, depend on the real exchange rate (which, in turn, is determined by the real interest rate, see section 3.5) in the current period, according to equation (3.6) below:

$$\Delta \ln X_t = f\left(\Delta \ln X_t^{FP}, \Delta \ln X_{t-1}^{FP}, \Delta \ln X_{t-1}, RER_t, RER_t^{FP}, \ln X_{t-1} - \ln X_{t-1}^{FP}\right). \qquad (3.6)$$

Monetary policy, which affects the real exchange rate without any (yearly) lag through the nominal interest rate, thus exerts a direct channel into aggregate demand.

3.2.6 Imports

Imported goods (superscript "*F*") are simply a function of consumption, investment, and export demand. The weights are identical to those of the Flex-price model and consist of the imported share of the variable in question. Hence, we have:

$$M_t = \left(\frac{C_t^{F,FP}}{C_t^{FP}} C_t\right) + \left(\frac{I_t^{F,FP}}{I_t^{FP}} I_t\right) + \left(\frac{X_t^{F,FP}}{X_t^{FP}} X_t\right). \qquad (3.7)$$

3.2.7 Gross domestic product (GDP)

GDP at fixed market prices is then simply the sum of private consumption (3.4), government consumption (G , exogenous, see section 3.8), private investments (3.5), government investments (I^G), exports (3.6) minus imports (3.7).

3.2.8 Average working hours

To describe the modelling of the labour market in the Sticky-price model, we first consider the empirical specification of business cycle movements in average working hours. There is a fairly clear business cycle pattern for average working hours in the Swedish economy. Average working hours increase in (especially in the beginning of) upturns and falls in downturns, according to a clear pro-cyclical pattern. The specification for average working hours (LH / L) is:

$$\Delta \ln\left(\frac{LH_t}{L_t}\right) = f\left(D_t^{UPTURN}, \ln\left(\frac{LH_{t-1}}{L_{t-1}}\right) - \ln\left(\frac{LH_{t-1}^{FP}}{L_{t-1}^{FP}}\right)\right), \tag{3.8}$$

where $D_t^{UPTURN} = 1$ in upturns, i.e. when actual GDP growth is higher than potential growth. It turns out that if average working hours are initially in equilibrium (i.e., the error-correction term is zero), an upturn implies that average working hours grow by about 0,3 percentage points per year. In downturns, average working hours fall by about 0,4 percentage points per year.

3.2.9 Employment and unemployment

As described in chapter 2, the full labour market model by Pissarides (2000) is applied in the Steady state and Flex-price models. Since the Pissarides model deals with *equilibrium* unemployment only, it needs to be adjusted in order to deal with business cycle variations in unemployment.

Instead of letting (the entirely forward looking) equilibrium labour demand determine labour demand in the Sticky-price model, we model labour demand as a weighted average of backward (superscript "*BACK*") and forward looking (superscript "*FOR*") labour demand (in hours), LH_t^P:

$$\ln LH_t^P = \eta^{LH1} \ln LH_t^{P,BACK} + (1-\eta^{LH1}) \ln LH_t^{P,FOR}, \tag{3.9}$$

where the backward looking part simply follows the (trend) development of the Flex-price model:

$$\ln LH_t^{P,BACK} = \ln LH_{t-1}^{P,BACK} + (\ln LH_t^{P,FP} - \ln LH_{t-1}^{P,FP}). \tag{3.10}$$

For the forward looking part, we first invert the production function so that working hours becomes the left hand side variable:

$$\begin{aligned} Y_t^{PG} &= \left(K_{t-1}^P\right)^{\eta^{YPG}} \left(A_t^H LH_t^P\right)^{1-\eta^{YPG}} \\ &\Leftrightarrow \\ \ln LH_t^P &= \left(\frac{1}{1-\eta^{YPG}}\right)\left(\ln Y_t^{PG} - \left(1-\eta^{YPG}\right)\ln A_t^H - \eta^{YPG} \ln K_{t-1}^P\right). \end{aligned} \tag{3.11}$$

Since it is costly to hire and fire labour, the firms determine their labour demand by forming expectations about future production (i.e. demand), taking future productivity and capital stock into account. Currently in KIMOD, firms form expectations (superscript "*E*") for period t, $t+1$, and $t+2$ when determining labour demand in period t. Hence, the forward looking part of labour demand is:

$$\ln LH_t^{P,FOR} = \eta^{LH2}\left[\left(\frac{1}{1-\eta^{YPG}}\right)\left(\ln Y_t^{PG^E} - \left(1-\eta^{YPG}\right)\ln A_t^{H^E} - \eta^{YPG}\ln K_{t-1}^{P}\right)\right] +$$
$$\eta^{LH3}\left[\left(\frac{1}{1-\eta^{YPG}}\right)\left(\ln Y_{t+1}^{PG^E} - \left(1-\eta^{YPG}\right)\ln A_{t+1}^{H^E} - \eta^{YPG}\ln K_t^{P^E}\right)\right] + \quad (3.12)$$
$$\left(1-\eta^{LH2}-\eta^{LH3}\right)\left[\left(\frac{1}{1-\eta^{YPG}}\right)\left(\ln Y_{t+2}^{PG^E} - \left(1-\eta^{YPG}\right)\ln A_{t+2}^{H^E} - \eta^{YPG}\ln K_{t+1}^{P^E}\right)\right],$$

where $\eta^{LH2}, \eta^{LH3}, \left(1-\eta^{LH2}-\eta^{LH3}\right)$ are the weights on the different horizons. Expectation formation is discussed in section 3.4.

The hours demanded in the government sector (LH^G) are proportional to government production. At present, this variable is exogenous in KIMOD 1.0 and is determined by the government sector experts at the NIER.

As we now have labour demand in hours and average working hours from equations (3.11) and (3.8), respectively, we can easily calculate employment in the economy (L):

$$L_t = \frac{LH_t^P + LH_t^G}{\left(LH_t\right)/L}, \quad (3.13)$$

where the denominator is average working hours determined in equation (3.8). The unemployment rate (U_t / LS_t) can then easily be calculated as follows:

$$\frac{U_t}{LS_t} = \frac{LS_t - L_t}{LS_t}. \quad (3.14)$$

3.2.10 Number of matches and vacancies

In order to calculate the number of matches and vacancies, we assume that both the labour market flow equation and the matching function are valid in the Sticky-price model as well. The former was:

$$U_t = \pi U_{t-1} + \left(1+\nu_t - \pi\right)LS_{t-1} + sL_t - MA_t, \quad (3.15)$$

where ν_t is labour supply growth rate, $\left(1+\nu_t - \pi\right)LS_{t-1}$ is the number of newcomers to the labour force who become initially unemployed, s is the exogenous share of separations of the current labour force due to ongoing structural change and MA_t is the number of successful matches. See Figure 2.1 in chapter 2 for a visual description of the labour market flows in equation (3.15).

As we already have information on the unemployment rate U_t, thanks to equation (3.14), and on employment L_t, due to equation (3.13), we can calculate the number of successful matches MA_t in equation (3.15). Finally, we get the number of vacancies, VA_t by applying the matching function:

$$MA_t = Z^{MA} U_t^{\eta^{MA}} VA_t^{1-\eta^{MA}}. \tag{3.16}$$

In summary, the Pissarides (2000) labour market model is used to calculate the number of matches and vacancies in the Sticky-price model.

3.2.11 Production, dividends, and disposable income

The production function of the business sector is assumed to hold in the Sticky-price model as well, see equation (3.11). The inputs, capital stock and hours, of course come from the Sticky-price model. The investment equation (3.5) builds the capital stock. Labour demand for hours is given by equation (3.11).

Government production is currently captured by a simple error-correction equation so that the sticky-price value converges to the flex-price value in the case in which they are not the same initially. As mentioned above, government production is exogenous in the Sticky-price model and borrowed from the government sector experts at the NIER.

Dividends are calculated in the same way as in the Flex-price model. The arguments, production, investment costs, and labour costs, are taken from the Sticky-price model, with two exceptions. The installation cost of investment and the vacancy costs are currently equal to the values of the Flex-price model. The reason is that dividends get very volatile if they include the Sticky-price counterparts. This calls for further analysis.

Finally, disposable income is defined as in the Flex-price model while of course the values of the variables included come from the Sticky-price model.

3.3 Nominal prices and wages

This section describes the price and wage setting behaviour in the Sticky-price model. In the New Keynesian literature, there are many explanations and formulations of *why* prices and wages are sticky. All of these can of course not be included in the Sticky-price model. However, as will be outlined below, the spirit of some of them is taken into account when specifying the price and wage equations. We begin with price setting.

3.3.1 The generic pricing problem

On this occasion, we need to be a bit agnostic. We introduce a hypothesis about price setting firms in the short run without supporting this by assuming monopolistic competition in the equilibrium part of the model. This is not fully satisfying and should be an entry for future development.

In the short run, firms are assumed to be price setters. As mentioned above, there are many explanations of why prices are sticky in the short run. The pricing equations in KIMOD are in the spirit of Calvo (1983) and the menu cost literature (see Mankiw, 1985). In Calvo (1983), it is assumed that only a fraction of all firms get the opportunity to reset prices every period. Knowing that it will take a while before it gets the opportunity again, the firm will behave in a forward looking manner. It will plan for the foreseeable future and set prices in accordance to perceived future costs and mark-ups.

It is important to note, though, that Calvo (1983) does not offer any explanation of *why* firms can not change their prices every period. The menu-cost literature focuses on the actual costs of changing prices as an explanation. These and other related theories of the root cause of price stickiness provide possible explanations of why inflation is often found to be both autocorrelated and forward looking in data. Krieger and Tinsley (2001) model this observed behaviour by assuming that firm i minimizes the following quadratic loss function when setting its' price $PB^{i,YPG}$:

$$\underset{\ln PB_t^{i,YPG}}{\arg\min}\, E_{t-1}\left\{\sum_{s=0}^{\infty}(b)^s\left[\frac{1}{2}\left(\ln PB_{t+s}^{i,YPG}-\ln PB_{t+s}^{*}\right)^2+\frac{1}{2}\left(c(L)\ln PB_{t+s}^{i,YPG}\right)^2\right]\right\}, \qquad (3.17)$$

where b is a discount factor, PB^* is the equilibrium price level and $c(L)$ is a lag polynomial that represents price frictions. Equation (3.17) implies that it is costly for firms both to deviate from the equilibrium price level PB^* (as this price level, by definition, maximizes profits) and to change prices (see the second term in the square brackets of (3.17)). In KIMOD, the last term in the square brackets of (3.17) is modelled as the deviation of the price inflation of the firm ($\ln PB_{t+s}^{i,YPG}-\ln PB_{t+s-1}^{i,YPG}$) from the trend rate of inflation in the economy, $\overline{\pi}^{YPG}$ (assumed to be constant in the following[26]):[27]

$$\underset{\ln PB_t^{i,YPG}}{\arg\min} E_{t-1}\left\{\sum_{s=0}^{\infty}(b)^s\left[(1-d)\left(\ln PB_{t+s}^{i,YPG}-\ln PB_{t+s}^{YPG,*}\right)^2+d\left(\ln PB_{t+s}^{i,YPG}-\ln PB_{t+s-1}^{i,YPG}-\overline{\pi}^{YPG}\right)^2\right]\right\}, \quad (3.18)$$

Equation (3.18) is a very flexible tool for modelling pricing behaviour. As $d \to 1$, the behaviour becomes more and more myopic (non-forward looking), and, in the limit, $\ln PB_t^{i,YPG} = \overline{\pi}^{YPG} + \ln PB_{t-1}^{i,YPG}$, i.e. the firm follows the expected general price development of the economy. On the other hand, as $d \to 0$, the firm becomes more and more forward looking and in the limit set its price equal to the equilibrium price, $PB_t^{i,YPG} = PB_t^*$.

Equation (3.18) is a forward looking quadratic problem and the solution takes the form:

$$\ln PB_t^{i,YPG} = \frac{e(1-b)}{1-eb}\overline{\pi}^{YPG} + e\ln PB_{t-1}^{i,YPG} + \frac{1-d}{bd}\sum_{j=0}^{\infty}(be)^j E_{t-1}\left\{\ln PB_{t+j}^{YPG,*}\right\}, \quad (3.19)$$

where e is a function of b and d.[28]

For an adjustment cost between zero and one, $0<d<1$, the firm is forward looking and has an infinite horizon. In practice, we need to truncate the horizon, which will be a subjective choice, i.e. choosing the number of leads and lags in equation (3.19). The price equation has to respect homogeneity in prices. This means that the weights in the lead and lag polynomials must add up to one.

The general structure of the pricing equations in KIMOD can be described as follows, where the expected price level PB^* is substituted for marginal cost (MC):

$$PB_t^{i,YPG} = f_0 + f_1 A(L) PB_t^{i,YPG} + (1-f_2)B(F)MC_t, \quad (3.20)$$

where f_i are parameters, $A(L)$ represents the finite lag polynomial in any price level and $B(F)=B(L^{-1})$ is the lead operator. In order to give a specific example of the general structure of equation (3.20), we assume that no future values are taken into

[26] This is not a restriction. One could for example assume that $\overline{\pi}_t^{YPG} = \ln PB_t^{YPG,E} - \ln PB_{t-1}^{YPG}$, where $PB_t^{YPG,E}$ is the expected general price level in period *t*.

[27] Although Calvo (1983) does not include any explicit costs of changing prices, his model generates a similar minimization problem.

[28] More specifically, e is the positive root from: $e = (1/2b)\left[\left((1+bd)/d\right) \pm \sqrt{\left((1+bd)/d\right)^2 - 4b}\right]$.

account, i.e. $s=0$ in equation (3.18), and solve for the price $PB^{i,YPG}$ that minimizes the loss:

$$\begin{aligned} \ln PB_t^{i,YPG} &= d\left(\ln PB_{t-1}^{i,YPG} + \overline{\pi}^{YPG}\right) + (1-d)\ln PB_t^{YPG,*} \\ &= d\left(\ln PB_{t-1}^{i,YPG} + \overline{\pi}^{YPG}\right) + (1-d)\ln MC_t^E, \end{aligned} \tag{3.21}$$

as $PB = MC$ in equilibrium. The information structure in KIMOD is such that variables in the current period are usually not known by the agents. As a result, agents must form expectations (superscript "E") about aggregate price and marginal cost levels in the current period and these terms will thus be treated as lead variables. The modelling of expectations is discussed in section 3.4.

Finally, in order to satisfy the relative price structure of the Flex-price model, here represented by $\left(PB^{YPG}/P\right)^{FP}$, we extend the expression of $\ln MC_t^E$ in equation (3.21):

$$\ln PB_t = d\left(\ln PB_{t-1}^{i,YPG} + \overline{\pi}^{YPG}\right) + (1-d)\left[d_2 \ln P_t^{E0} + (1-d_2)\ln MC_t^E + d_2 \ln\left(\frac{PB^{YPG}}{P}\right)_t^{FP}\right], \tag{3.22}$$

where the term in the square brackets reduces to $\ln MC_t^E$ in equilibrium. In short, we have introduced a pricing structure that allows for both backward and forward looking behaviour where the former can be explained by different types of restrictions of flexible price setting.

3.3.2 Nominal prices

In this section, we apply the generic price equation in (3.21) on some important prices in the Sticky-price model.

Price of gross production

Production is carried out by two types of producers, as discussed in chapter 2. The first type uses capital and labour to produce an intermediary domestic good and the second type simply puts intermediate domestic and imported goods together. The firms of the first type have marginal costs in form of rental prices for capital and nominal labour costs. The generic price equation in (3.21) is applied on the price of gross production (PB^{YPG}) in equation (3.23) below. We have added a term for the output gap ($\ln Y_t - \ln Y_t^{FP}$) to mimic the observed positive correlation between the output gap and prices:[29]

$$\ln PB_t^{YPG} = \eta^{PBYPG2}\left(\ln PB_{t-1}^{YPG} + \overline{\pi}^{YPG}\right) + \left(1-\eta^{PBYPG2}\right)\ln PB_t^{LEAD} + \eta^{PBYPG3}\left(\ln Y_t - \ln Y_t^{FP}\right). \quad (3.23)$$

The leading variable PB^{LEAD} can be compared with MC^E in equation (3.21). From equation (3.22) we then know that:

$$\ln PB_t^{LEAD} = f\left(\ln P_t^E, \ln MC_t^E, \ln\left(\frac{PB^{YPG}}{P}\right)_t^{FP}\right). \quad (3.24)$$

Currently, we let $\ln PB_t^{LEAD}$ be a function of expected prices and marginal costs in the current period and the two periods after that:

$$\begin{aligned}\ln PB_t^{LEAD} = {} & \eta^{PBL1}\left[\eta^{PBYPG1}\ln P_t^E + \left(1-\eta^{PBYPG1}\right)\ln MC_t^E\right] + \\ & \eta^{PBL2}\left[\eta^{PBYPG1}\left(\ln P_{t+1}^E - \overline{\pi}^{YPG}\right) + \left(1-\eta^{PBYPG1}\right)\left(\ln MC_{t+1}^E - \overline{\pi}^{MC}\right)\right] + \\ & \left(1-\eta^{PBL1} - \eta^{PBL2}\right)\left[\eta^{PBYPG1}\left(\ln P_{t+2}^E - 2*\overline{\pi}^{YPG}\right) + \left(1-\eta^{PBYPG1}\right)\left(\ln MC_{t+2}^E - 2*\overline{\pi}^{MC}\right)\right] + \\ & \eta^{PBYPG1}\ln\left(\frac{PB}{P}\right)_t^{FP},\end{aligned} \quad (3.25)$$

where $\overline{\pi}^{MC}$ is the trend change in nominal marginal costs (assumed here to be constant). Note that $\ln P_{t+1}^E$ reflects expectations in period *t* about the GDP deflator in period *t*+1 and so on. Note, finally, that we need to adjust future price expectations (for example, $\ln P_{t+1}^E$) for the future trend inflation rate ($\overline{\pi}^{YPG}$) in order to "discount" future prices so that they be expressed as period *t* values.

Marginal costs and mark-ups

In equilibrium, perfect competition applies so that marginal costs equal prices and no mark-ups are present. Although we do not model imperfect competition explicitly, the Sticky-price model allows for time-varying mark-ups. More specifically, we define marginal costs as the relationship between the nominal wage (NW) and the equilibrium real wage ($W^{P,FP}$). The latter is close to the equilibrium marginal product of labour (MPL^{FP}). Hence, the marginal cost could thus be said to represent unit labour costs:

[29] Note that prices and the output gap are determined simultaneously. Hence, we can not say that the output gap generates a price response.

$$
\begin{aligned}
MC_t &= \frac{NW_t}{W_t^{P,FP}} \\
&\approx \frac{NW_t}{MPL_t^{FP}}.
\end{aligned}
\tag{3.26}
$$

The mark-up is, in turn, defined as prices over marginal costs:

$$\frac{PB_t^{YPG}}{MC_t}. \tag{3.27}$$

We calibrate the nominal wage dynamics to be stickier than producer prices in the short run. This implies that producer prices increase faster than wages, given a positive temporary demand shock. The positive spread between prices and marginal costs in the first phase of the shock is interpreted as a pro-cyclical, although implicitly defined, mark-up. Eventually wages picks up speed and real wages increase so that, in the longer run, all nominal prices and wages have increased by the same amount. The latter is a general feature of macroeconomic models with inflation targeting and nominal neutrality.

<u>Price of public sector production</u>

The price equation of public sector production (Y^G) is mainly cost driven, as in the National Accounts. The public production deflator (PB^{YG}) adjusts to increases in nominal wages and to depreciation (δ) of the public capital (K^G), so that:

$$PB_t^{YG} = \alpha \frac{NW_t LH_t^G}{Y_t^G} + \frac{\delta P_t K_t^G}{Y_t^G}. \tag{3.28}$$

By introducing the parameter α, we can allow nominal wage costs per hour to differ from those of the private sector (NW_t). Note that equation (3.28) implicitly assumes that there is no stickiness in the government's pricing behaviour. The assumption is natural for two reasons. First, the deflator seldom represents actual prices, at least not market prices. Second, we have no specific knowledge of the dynamic nature of public charging.

<u>Exports</u>

In the current version of the model, we do not assume any dynamic specification at all for the price behaviour of exporting firms. Instead, export prices follow the equilibrium relative price movements $\left(P^X / P\right)^{FP}$ of the Flex-price model. Formally, we have the following simple export price (P^X) equation:

$$P_t^X = \left(\frac{P^X}{P}\right)_t^{FP} P_t. \tag{3.29}$$

Imports

The import price P^M is measured in domestic currency. It will therefore depend on the nominal exchange rate NER. For importing firms, we assume that a share of the importing costs measured in the foreign currency can be passed on to the consumers, although not fully in the short run. There is also a positive response to increases in domestic demand, modelled as the output gap $\left(\ln Y_t - \ln Y_t^{FP}\right)$. The import price equation is:

$$\begin{aligned}\log\left(P_t^M\right) = \eta^{PM1}\left(\ln P_{t-1}^M + \overline{\pi}^M\right) + \left(1-\eta^{PM1}\right)\left(\ln NER_t + \ln P_t^{M,F} + \ln Z^{PM\_P}\right) + \\ \eta^{PM2}\left(\ln Y_t - \ln Y_t^{FP}\right),\end{aligned} \tag{3.30}$$

where $\overline{\pi}^M$ is the trend inflation rate of import prices (assumed to be constant here), $P^{M,F}$ is the price of import goods denominated in foreign currency and Z^{PM_P} is a constant. Equation (3.30) is derived from the real exchange rate equation 2.69 in the Flex-price model (see chapter 2) and implies that import prices are sticky and do not respond one-to-one to changes in the nominal exchange rate in the short run.

Private consumption

The price of private consumption (P^C) including taxes (τ^C) is given by:

$$P_t^C = \left(1+\tau^C\right) PB_t^C, \tag{3.31}$$

where PB^C is the consumption deflator at basic prices. This variable, in turn, is assumed to follow the general trend set by the core inflation measure ($CORE$, see section 3.6 below), and to respect the relative price development from the flex price model, $\left(P^C/P\right)^{FP}$:

$$\ln PB_t^C = \eta^{PBC}\left(\ln PB_{t-1}^C + \ln CORE_t - \ln CORE_{t-1}\right) + \left(1-\eta^{PBC}\right)\left(\ln\left(\frac{P^C}{P}\right)_t^{FP} + \ln P_t\right). \tag{3.32}$$

The core inflation is determined by unit labour costs and import prices (see section 3.6 below).

Public consumption

The price of public consumption is, like the price of public production, not a market price. The price, including taxes (τ^G), is given by:

$$P_t^G = \left(1+\tau^G\right) PB_t^G, \tag{3.33}$$

where PB^G is the public consumption deflator at basic prices. The latter is driven by the cost of buying private sector services (G^H) at basic prices (PB^{YPG}) and the cost of public production ($PB^{YG}Y^G$). The equation below mimics the one in the National Accounts.

$$PB_t^G = \frac{PB_t^{YPG} G_t^H}{G_t} + \frac{PB_t^{YG} Y_t^G}{G_t}. \tag{3.34}$$

<u>Investment</u>

Finally, investment prices are not explicitly modelled. They are determined residually, using the nominal GDP identity for final demand, that is:

$$P_t Y_t = P_t^C C_t + P_t^G G_t + P_t^I I_t + P_t^X X_t - P_t^M M_t. \tag{3.35}$$

3.3.3 Nominal wage setting

In the Flex-price model, real wages are determined by the Nash bargaining solution of an individual firm and an unemployed person (see chapter 2). As all individual bargaining outcomes turn out to be identical, higher wage claims imply higher unemployment and lower search costs for the firms. In the negotiations between a single firm and a single unemployed person, however, both parts act without taking into account the repercussions on the general equilibrium outcome.

Although the above wage bargaining structure might be a reasonable approximation in equilibrium, it is not likely to be a good explanation of how nominal wages are set in disequilibrium with imperfect information and bounded rationality. In order to capture these realistic features, nominal wage setting is modelled in analogy with the firms' generic pricing equation, see (3.21) and (3.23). Equation (3.36) below is a general formulation of nominal wages NW:

$$\ln NW_t = h_1\left(\ln NW_{t-1} + \overline{\pi}^{NW}\right) + \left(1-h_1\right)\ln NW_t^{LEAD} + h_2\left(\frac{U_{t-1}}{LS_{t-1}}_t - \frac{U_{t-1}^{FP}}{LS_{t-1}^{FP}}\right), \tag{3.36}$$

where $\overline{\pi}^{NW}$ is the trend change in nominal wages (assumed to be constant here).[30] The forward looking part (NW^{LEAD}) of equation (3.36) depends on expected nominal wages (NW^{E}), expected prices (P^{E}), the equilibrium real wage ($W^{P,FP}$) and the trend change, $\overline{\pi}^{NW}$:

$$\ln NW_t^{LEAD} = f\left(\ln NW_t^{E}, \ln P_t^{E} \ln W_t^{P,FP}, \overline{\pi}^{NW}\right). \tag{3.37}$$

Currently, two lags and two leads are applied in the nominal wage equation. In short, nominal wages are set in a fashion similar to that of the model of overlapping contracts in Taylor (1980). That is, in each period only a part of the labour market contracts are renegotiated. In these negotiations, firms and workers look backward on the contracts that were negotiated previous periods and they look forward and form expectations about the likely contracts that will be settled in future periods. As can be seen from equation (3.37) above, these expectations concern both future nominal wages and prices while the future equilibrium real wage is assumed to be known.

3.4 Expectation formation

In the price and wage equations above, variables representing expectations (those with superscript "*E*") have been included. In this section, we show how expectations are modelled in KIMOD. First, however, we need to recapitulate a bit.

The short-run dynamics of KIMOD can be understood as stemming from three sources. The first (intrinsic) part originates from fixed adjustment costs (e.g. long-term contracts and menu costs) and was discussed above in the price and wage equations. The second part, which we will discuss in this section, originates from expectation formation, i.e. the assumption of imperfectly rational expectations at business cycle frequency. The third part is the dynamics generated by the policy rules. This is discussed in section 3.6.

There is a clear distinction between the first two parts of stickiness – fixed adjustment costs and expectation formation. We are interested in separating adjustment costs into one part that is immune to policy changes (i.e. to the Lucas critique) and one that is not. As will be clear in this section, fixed adjustment costs are immune to

[30] This nominal trend change is equal to the sum of the change in the real wage rate of the Flex-price model (i.e. the result of the Nash bargaining solution) and the inflation target.

policy changes (for example, a new inflation target) while the formation of expectation can be affected by policy.

In the modelling of expectations in highly aggregated models, weights are assigned to backward and forward looking variables. This structure can be interpreted in at least two ways. One interpretation is that every agent does a little bit of this and a little bit of that, i.e. every agent is partially backward-looking and partially forward-looking. Another possible interpretation is that different groups in the society behave differently. For instance, some consumers are "rules-of-thumb" consumers and act in a backward-looking way while the rest of the consumers are truly forward-looking. We do not restrict ourselves to either of these interpretations.

Expectations are explicitly modelled as separate equations and are formed as convex combinations of backward and forward looking parts. We start with the backward looking part. We want a fairly general structure that allows us to readily change the length of the backward looking part (that is, how many periods backwards in history agents are assumed to consider when forming expectations) and extend it linearly into the future. Consider:

$$\ln P_{t+i}^{E} \quad = \quad \ln P_{t-1} + k\left(\ln P_{t-1} - \ln P_{t-1-j}\right), \tag{3.38}$$

where P_{t+i}^{E} is the price level in period $t+i$ as expected at time t and j is the length of the historic sample that the agents consider. We now need a way to calculate the parameter k in such a way that it (i) captures the unweighted average growth during the backward looking period (here t-1-j) and (ii) project's i periods ahead using this mean growth rate starting from the last known data, P_{t-1}. Such a formula is given by:

$$k = \left(\frac{1+i}{j}\right). \tag{3.39}$$

Let's try it. If we put $j=3$ and $i=2$ we have $k=1$. Insert this and let:

$$\begin{aligned} P_{t+2}^{E} &= P_{t-1} + k\left(P_{t-1} - P_{t-4}\right) \\ &= P_{t-1} + (1+i) * \frac{\left(P_{t-1} - P_{t-4}\right)}{j} \\ &= P_{t-1} + 3 * \overline{\pi}, \end{aligned} \tag{3.40}$$

where $\overline{\pi}$ is the observed mean growth between t–4 and t–1. Hence, equation (3.40) shows that our (linear) forecast of t+2 dated in period t is 3 times the average historic growth computed between t–4 and t–1.

Now we add a forward looking part (with the weight $1-\eta^{P^E}$) by using the model consistent (that is, rational) value (P_{t+i}) for our forward prediction:

$$\ln P_{t+i}^{E} \;=\; \eta^{P^E}\left[(1+k)\ln P_{t-1} - k\ln P_{t-1-j}\right] + \left(1-\eta^{P^E}\right)\ln P_{t+i}. \tag{3.41}$$

Due to the random walk behaviour of price levels in a model with an inflation target for monetary policy, there is no economically plausible interpretation of nominal "equilibrium" levels in the long run. For volumes and *relative* prices, on the other hand, we do have well defined equilibrium values. For these variables (in equation (3.42) exemplified below by "*X*"), we thus expand the convex combination to include the equilibrium value (X^{FP}) as an attractor:

$$X_{t+i}^{E} \;=\; l_1\left[(1+k)X_{t-1} - kX_{t-1-j}\right] + l_2 X_{t+i} + (1-l_1-l_2)X_{t+i}^{FP}. \tag{3.42}$$

Finally, for variables demanding very long forward horizons, the number *i* being large, we need to be able to downplay the backward looking information against the forward looking one, based on the assumption that people eventually will learn, although there is no explicit model for how agents learn in KIMOD. Consider:

$$X_{t+i}^{E} \;=\; \left(l_1 m^i\right)\left[(1+k)X_{t-1} - kX_{t-1-j}\right] + l_2 X_{t+i} + \left(1-\left(l_1 m^i\right)-l_2\right)X_{t+i}^{FP}, \tag{3.43}$$

where the parameter m, $0 < m < 1$, is used to downplay the backward looking part of (3.43) when *i* increases. Equation (3.43) implies that expected values further into the future (i.e. when *i* increases) will have increasing weight on the equilibrium value and decreasing weight on the historic information. This is helpful to speed up convergence towards the new equilibrium if the economy is hit by a permanent shock. By using large values for m, we are in effect assuming that people are slow "learners". As mentioned earlier, we do not have an explicit theory of learning in KIMOD and if we truncate the forward horizon heavily, this specification will only have a small impact.

Finally, as the interested reader might have noticed, we use lagged values in the price and wage equations because of different types of adjustment costs (see section 3.3). These costs are fixed and do not change if policy changes. On the other hand, we also use, as we have seen in this section, lagged values in the formation of price of wage expectations. Hence, in the expectation formation, we assume that historic information is *beneficial* (i.e. not a cost) when predicting future values. This benefit is, however, highly sensitive to policy changes (cf. the Lucas critique). If politicians try to exploit the way individuals form their expectations, the expectation formation will change. In summary, the inclusion of lags in the price/wage equations and the expectation formation should be kept apart.

3.4.1 Trend growth rates in expectation formation

Earlier on we showed that trend growth rates for different variables showed up in the non-stationary equations for prices, wages and volumes. Generically we have:

$$P_t = n\left(P_{t-1} + \pi_t^{P,E}\right) + (1-n)\left(P_{t+1} - \pi_{t+1}^{P,E}\right), \tag{3.44}$$

where $\pi^{P,E}$ is the expected growth of price P. Although we assumed for simplicity that $\pi^{P,E}$ is constant and equal to the trend growth, this needs not to be the case. As mentioned in footnote 26, $\pi^{P,E} = \ln P_t^E - \ln P_{t-1}$ is one alternative where $\ln P_t^E$ could be determined by equation (3.41).

However, in the expectation formulations we let the individuals know the trend growth ($\overline{\pi}^P$) for certain, i.e, we exclude superscript "*E*":[31]

$$P_{t+i}^E = \eta^{P^E}\left[(1+k)\left(P_{t-1} + \overline{\pi}^P\right) - k\left(P_{t-1-j} + j * \overline{\pi}^P\right)\right] + \left(1-\eta^{P^E}\right)\left(P_{t+i} - i * \overline{\pi}^P\right). \tag{3.45}$$

3.5 Exchange rates and UIP

In modern macro modelling, forward looking behaviour is the natural benchmark. This is especially true in a model designed to be used in policy analysis. KIMOD is developed exactly to support such studies and incorporates an explicit modelling of expectation formation, as we saw in the above section.

A model just needs one forward looking variable to become messy to analyse numerically. The reason is that this variable will in a particular year depend on the rest of the model and one needs to compute *the full time profile for all variables* before the forward looking one can be solved.

Uncovered Interest Parity (UIP) is exactly such an economic relation. It incorporates one forward looking variable, i.e. the expectation of the next period value for the nominal exchange rate, but the solution requires the time path for both domestic and foreign nominal interest rates as well. The domestic interest rate, in turn, is determined by the monetary policy authority and their decision presupposes the full model calculation of the variables included in the model.

[31] The alternative to include an *expectation* based $\pi^{P,E}$ in the expectation formation equations is not satisfying for obvious reasons.

In KIMOD, the determination of the nominal variables connected to monetary policy “starts” with the determination of the expected real exchange rate. We do not describe this process in terms of cause and effects, since everything is solved simultaneously. Yet there is something to be learned from how the software *Troll* solves such a forward looking problem. Given a tentative solution of the expected next period value for the real exchange rate (RER_{t+1}^{E}) and the expected domestic price level (P_{t+1}^{E}), we solve for the expected *nominal* exchange rate (NER_{t+1}^{E}):

$$\ln NER_{t+1}^{E} = \ln RER_{t+1}^{E} - \ln P_{t+1}^{F} + \ln P_{t+1}^{E}, \tag{3.46}$$

where P_{t+1}^{F} is the foreign GDP deflator. Now, equipped with the short term nominal interest rate paths, we use UIP to solve for the current value for the nominal exchange rate:

$$\ln NER_{t} = \ln NER_{t+1}^{E} - \ln\left(1 + NR_{t}\right) + \ln\left(1 + NR_{t}^{F}\right), \tag{3.47}$$

where NR^{F} is the foreign nominal interest rate. This means that the current value of the nominal exchange rate *adjusts* (jumps) to accommodate differences between interest rate paths in order to prohibit arbitrage opportunities. It is well known that UIP is often rejected in empirical work. Still, it is a necessary ingredient in a consistent forecast. We cannot have known “free lunches” in a consistent description of future developments.

The interest rate path, in turn, is the central bank response to the inflationary/deflationary pressure due to the recent shocks. As such, the central bank response involves, and is based on, the impact of interest rates on nominal and real exchange rates and the latter variables’ repercussions on net exports. Finally, because of the Fisher relation, nominal interest rates translate into expected real interest rates (R_t) which, in turn, will have an impact on domestic private consumption and business investment:

$$\left(1 + R_{t}\right) = \frac{\left(1 + NR_{t}\right) P_{t+1}^{E}}{P_{t}^{E}}. \tag{3.48}$$

Finally, one can combine equations (3.47) and (3.48) with the definition of the real exchange rate:

$$RER_{t} = \frac{NER_{t} P_{t}^{F}}{P_{t}}, \tag{3.49}$$

to see that KIMOD respects uncovered *real* interest parity as well:

$$RER_t = \frac{RER^E_{t+1}\left(1+R^F_t\right)}{\left(1+R_t\right)}. \tag{3.50}$$

This overall specification, together with the neutrality assumption on monetary policy, means that the lasting effect of monetary policy actions (as well as temporary shocks) on price levels is the same for all nominal variables and that domestic prices respect relative PPP. Although the model is specified to respect UIP both in real and nominal terms, it is easy to show that temporary shocks to the policy rule or the nominal exchange rate will momentarily produce deviations from pure theory prediction due to transitory expectation errors.

3.6 Core inflation

According to the law governing the conduct of monetary policy, the central bank in Sweden is responsible for maintaining price stability. The central bank has made its objective operational as a 2 percent annual increase in the consumer price index (CPI) with a margin of ±1 percentage point. Due to specific features of the CPI index, the central bank instead targets a measure of core inflation, also called UND1X.

3.6.1 Modelling the target rate

The highly aggregated nature of KIMOD prohibits replicating the CPI. However, the core inflation measure should in some way be a function of endogenous variables, and monetary policy should affect these variables. To fulfill these requirements, we have settled on a specification of consumer prices from the producer's point of view. The core measure ($CORE_t$) is modelled as a weighted average of unit labour costs (ULC) and import prices (P^M):

$$\Delta \ln CORE_t = \frac{PB^{YP}_t C^H_t}{PB^C_t C_t} \Delta \ln ULC_t + \frac{P^M_t C^F_t}{PB^C_t C_t} \Delta \ln P^M_t, \tag{3.51}$$

where the weights in front of ULC and P^M sum to one and equal, respectively, the share of consumption produced at home (C^H) and abroad (C^F).

In KIMOD, we try to simultaneously mimic different features of the data. The time series for the import deflator P^M_t and ULC_t are fairly volatile and, at the same time, the measures for domestic inflation (UNDH1X) and imported inflation (UNDIMP1X) are peculiarly sticky and smooth, as is the core measure (UND1X) from Statistics Sweden itself. In order to succeed with these diverse endeavours, we need to smooth the series included in the core equation as well as KIMODs core measure itself. In the calibration, we have to balance this sticky feature of prices against a fairly "aggressive" central bank.

3.7 Monetary policy

The central bank is responsible for creating and sustaining conditions in which public expectations are consistent with the goals for monetary policy. Since 1993, Sweden has chosen an explicit inflation targeting regime, with an explicit target for consumer price inflation. The central bank does have a monopoly in supplying the economy with currencies and overnight loans, which gives it the possibility to peg the short term nominal interest rate (i.e. the "repo rate"), according to its perception of the current liquidity position of the banking system.

The central bank needs to be forward looking, due to the time lag between action and effect, and it also needs to have an explicit view of the transmission mechanism that will transmit the interest rate movements into a successful implementation of the policy goal. The central bank carries out forecasts on economic variables, conditioning upon its own actions at least two years ahead. It trades off the importance in reaching the target in the shortest possible time with the costs that this policy will impose on the economy, i.e. the output loss and unemployment associated with managing the monetary policy in an environment with less than perfect credibility. With perfect credibility there are, of course, no costs at all in changing policy.

KIMOD does not have an interest rate rule explicitly dependent on output or unemployment gaps in the sense of Svensson (2000), and the modelled central bank does not conduct optimal monetary policy. The central bank is assumed to set interest rates within a two year horizon, conditioned on an inflation forecast that is supplied by the model. This forecast is equal to the model consistent solution of future core inflation, which amounts to a perfect foresight assumption. Uncertainty is addressed by the inclusion of a weight on last period monetary stance, suggesting a dislike to move quickly upon the latest information (i.e. a smoothing behaviour). The policy rule is:

$$1+NR_t=\eta^{NR2}\left(1+NR_{t-1}\right)+\left(1-\eta^{NR2}\right)\left(1+NR_t^{FP}\right)+\eta^{NR1}\left(\Delta\ln CORE_{t+2}-\pi^*\right), \qquad (3.52)$$

where π^* is the inflation target and NR^{FP} is the equilibrium nominal interest rate. The central bank law dictates the central bank's responsibility for consumer price stability and allows other considerations only if they can be implemented without jeopardising the main objective. It is possible to pay explicit attention to employment, unemployment or the output gap in the rule but it is not necessary from a model perspective. The channel behind price stabilisation goes through the output gap which means that the rule implicitly takes that into consideration. By an adequate choice of the parameters in the rule including the length of the horizon, one can always mimic a rule with an explicit role for the output gap.

For demand shocks, there is no trade off in the first place, but supply shocks generate a conflict between the two goals. This will show up as a more cautionary and gradual behaviour in that the central bank wants to raise and lower interest rates at the same time. This gradual behaviour induces longer and larger deviations for inflation relative to the target rate and could well be mimicked by a greater weight on historic rates and a lesser on the inflation gap.

In the class of models where KIMOD belongs, it is in general impossible to derive an optimal monetary policy rule analytically. That leaves one with the question of whether to try to model the perceived behaviour of the central bank for forecasting or to develop a conditional optimization rule. The latter will be optimal conditional on the chosen structure of the model and the set of shocks applied. In the current work we have chosen another strategy. As described in Chapter 4, parameters (including those in the reaction function of the central bank) are chosen taking the full system properties into account. In some way one can say that we close the model by calibrating the monetary policy rule.

Monetary policy is thus assumed to be both neutral and super neutral, i.e. it has no permanent effect on real variables or on the rate of growth. Nominal shocks will give rise to transitory inflationary impulses which in the end produce drift in the levels of prices but nothing more.

3.8 Fiscal policy

Fiscal policy is modelled in the same way as in the Steady state and Flex-price models. This partly reflects the fact that modelling fiscal policy at business cycle frequency has so far not been a priority in developing KIMOD 1.0. The only difference with the Flex-price model is that we include error-correction terms in the equations for government production and government consumption, to account for the case in which the starting values do not correspond to the values of the Flex-price model.

The labour income tax (τ^W) is endogenous in the Sticky-price model and ensures that the equilibrium level of government net position as a percentage of GDP is reached:

$$\tau_t^W = \chi\tau_{t-1}^W + (1-\chi)\left[\tau_{ss}^W - \chi\left(\frac{P_t^{M\_P}B_t^G}{Y_t} - \frac{P_{ss}^{M\_P}B_{ss}^G}{Y_{ss}}\right)\right], \tag{3.53}$$

where P^{M_P} is the relative price between imports and the GDP deflator and $P^{M\_P}B^G/Y$ is the government nominal net asset position in relation to nominal GDP.

3.9 Production, tax wedges and productivity

Production is determined by demand in the Sticky-price model. The indirect taxes impose a wedge between GDP to market prices (demand) and GDP to basic prices (production). Formally, the following nominal identity defines private and public production:

$$P_tY_t = \left(1+\tau^{YL}\right)\left(PB_t^{YP}Y_t^P + PB_t^{YG}Y_t^G\right), \tag{3.54}$$

where τ^{YL} is used as the correct, exogenous measure of the wedge. Both sides of the identity represent added value although measured at different prices. It is tempting to think of the different Y as volumes and use them to formulate a volume identity, similar to equation (3.54). With the chain index system of the National Account, this is not correct, however, except for the base year and the preceding year. For other years, the volume parts do not generally add up. The endogenous variable τ^Y is introduced to capture the "chain index drift":

$$Y_t = \left(1+\tau^Y\right)\left(Y_t^P + Y_t^G\right), \tag{3.55}$$

where τ^Y converges to the flex-price value according to:

$$\tau_t^Y = \tau_{t-1}^Y + 0.5\left(\tau_{t-1}^Y - \tau_{t-1}^{Y,FP}\right). \tag{3.56}$$

Public production, Y^G, is determined by the government sector experts at the NIER. Equation (3.55) determines private production (Y_t^P). With production given by equation (3.55), and with employment explained in section 3.2.9, private sector's productivity (A^H) can be solved and interpreted as a business cycle residual by inverting the production function:

$$Y_t^{PG} = \left(K_{t-1}^P\right)^{\eta^{YPG}} \left(A_t^H LH_t^P\right)^{1-\eta^{YPG}}$$
$$\Leftrightarrow \tag{3.57}$$
$$\ln A_t^H = \frac{1}{1-\eta^{YPG}} \left(\ln Y_t^{PG} - \eta^{YPG} \ln K_{t-1}^P - \left(1-\eta^{YPG}\right) LH_t^P\right),$$

where added value production (Y^P in equation (3.55)) is transformed into gross production (Y^{PG}), in order to use the production function. This is obtained by the identity:

$$PB_t^{YP} Y_t^P = PB_t^{YPG} Y_t^{PG} - COST_t^{FP}, \tag{3.58}$$

where the cost variable consists of installation cost of capital and vacancy costs. This is exogenous in the Sticky-price model and equal to the flex-price equilibrium value.

4 Calibration

In the class of models where KIMOD belongs, specifications run from highly abstract and theoretic equations to very hands-on National Accounting identities. To accommodate this, the values for the different parameters in the three models which constitute KIMOD are chosen in a fairly eclectic way. The ideal method would be a full system estimation of the entire model which would respect all the relevant cross-restrictions. A full system estimation is unfortunately not a realistic option, however, for several reasons. The amount of parameters is quite large and the model frequency is annual, so that the degrees of freedom would be in short supply. Furthermore, major structural shocks hit the Swedish economy in 1990–1992. On top of this, Statistics Sweden changed radically its way of measuring GDP components in the mid-1990s.

On the other hand, we cannot rely entirely on an eclectic calibration approach that borrows from the research literature. Firstly, the research literature is seldom representative for Sweden. Parameter estimations mostly exploit data from other countries whose economic institutions may be quite different from those of Sweden. Moreover, KIMOD needs a spectrum of parameters of very different types. Some are true representations of technology and preferences, classical "deep parameters", some are exogenous rates of growth, and some are weights in fairly general dynamic specifications.

In practice, we end up with a mixed strategy which we can compactly state as:

"Estimate when possible, then calibrate
to generate reasonable behaviour as a system"

The sources/methods used to calibrate KIMOD can be summarized in four points:

- Results from mainstream economic literature
- Own estimates on Swedish data
- Focus on the system, not single variables
- Impulse-response analysis

The last point is discussed further in chapter 5.

4.1 Treating different frequencies differently

Basically, KIMOD's three parts can be envisioned as a three parts decomposition of a generic time series into different frequencies. The parts of the model supposed to trace out medium term to long run behaviour, i.e. the Flex-price model and the Steady-state model, are moistly driven by unobserved equilibrium variables. These can only be assessed through their impact on the end product, that is, the sticky price variables.

In the Steady-state model, most parameters are chosen to yield a static replication of the NIER's common view on steady state levels, with some additional calibrated dynamic parameters. By choosing parameters in this way one has to face the trade off between striving for the correct levels and trend rates and influencing the comparative static performance. In order to illustrate the last argument, consider the following small dynamic model:

$$\begin{aligned} & X_t = a + bX_{t-1} + cY_t \\ \Rightarrow \qquad & X^{SS} = \frac{a}{1-b} + \frac{c}{1-b} Y^{SS}. \end{aligned} \tag{4.1}$$

In this trivial example, one has to choose the parameters $\{a,b,c\}$ to induce correct steady state levels and ratios, as well as correct comparative static's (i.e. elasticities). On top of that, the parameters should induce an appropriate dynamic behaviour. If only the parameter b is used in order to respect certain steady state conditions, the decision will have a direct impact on the dynamics. If one instead uses the intercepts (i.e. a), one could end up with undesired comparative static properties.

There are more dimensions to the problem. Parameters differ not only as to their functional aspects, as described above, but to their interpretation as well. Many parameters are hybrids in the sense that, in a small model, the parameters have to carry too much of the explanation. Consider, as an example, consumer theory and the choice of a utility function to represent household preferences. In a realistic description, one might want to model saving decisions, intertemporal substitution, risk aversion and precautionary decisions. In a complex specification all this can be parameterised but if one has settled for a log utility specification, as we have, then there are actually no parameters at all to accommodate the model!

Another example is the parameter π, which represents the probability for survival in the equilibrium part of KIMOD. It will determine the life expectancy of the households, the discrepancy between social and private discount factor and the degree (or lack) of Ricardian equivalence. Now, if one has some sort of information, or opinion, on the magnitude of all of these features, which one should govern the choice for π?

Furthermore, some of the theoretically based mechanisms, such as installation costs for capital goods, much needed to yield plausible adjustment rates, cannot be extracted from data directly. Such parameters are in actual practice used in a more "residual" way, so that they are used to "close" the model. Technically speaking, they are treated as endogenous variables and given an imputed value.

For the less stringent theoretical parts of the model, that is, for the Sticky-price model, we have chosen another strategical mix. Volumes and real rigidities are estimated, whereas prices, expectations and policy rules are calibrated. For instance, we have to balance the "excess" rigidity and perfect foresight in the core-measure of inflation with a fairly aggressive monetary policy reaction.

To sum up, all the parameters in the full system will actually resemble coefficients in a VAR-model. The absolute value of a specific coefficient or parameter cannot tell much. One needs to study the dynamic behaviour of the full system, mostly with the help of impulse-response experiments. For a thorough discussion of this, see chapter 5.

4.2 The gradual approach

Models never provide the final answers to questions posed at a forecasting institute, and we take a similar stance on the calibration issues. Theoretical insights, equation specifications and parameterization are always going to change and develop.

We start out with estimated coefficients or parameters borrowed from the literature and adjust them to give the system the desired properties. Frequent use of the model in the forecasting process will eventually give the modellers more information that will enable an updating of one's initial specifications.

5 Dynamic model properties

To illustrate the dynamic properties of KIMOD 1.0, this chapter reports the model's responses to various shocks. Several relevant shock simulations help to illustrate the dynamic behaviour of KIMOD. Section 5.1 describes five permanent shocks: A productivity shock, a labour supply shock, a permanent shock to the relative bargain strength of the workers, a reduction in the ratio of government debt to GDP and a shock to the inflation target. In section 5.2 we focus on five transitory shocks: a shock to nominal wages, a shock to the nominal exchange rate, an interest rate shock, a temporary labour supply shock and a shock to household spending.

We have chosen these shocks to illustrate the interaction of households, firms and policymakers in goods, labour and financial markets. By definition, a transitory shock does not alter any flex-price or steady state values. For example, a shock to household spending that is of temporary nature will affect relative prices, nominal prices and volumes in the short run while equilibrium values in the Steady state and Flex-price models are unaltered. A permanent shock, on the other hand, affects equilibrium values of output and relative prices. A permanent shock must therefore be introduced both in the Steady state, Flex-price, and Sticky-price models. In practice, a permanent shock is first introduced in the Steady state model which describes the static long run effect. Secondly, the permanent shock is introduced in the Flex-price model which describes the dynamic (due to real rigidities) flex-prices equilibrium path to the new steady state. Finally, the shock is incorporated into the Sticky-Price model, which generates the business cycle response to such a shock, illustrating the process by which the economy reaches its new flex-price equilibrium, eventually with the help of appropriate stabilization policies.

For each shock, we show a standard set of charts. For the permanent shocks we also show some additional charts which illustrate the sluggish adjustment in the Flex-price model. For these shocks, the dynamic path of the Flex-price variables is shown as a dashed line and labelled with a suffix FP. In these figures, the development in the Sticky-price model is labelled with a suffix SP.

The shocks responses are intended to highlight specific mechanisms of the model, and we focus on a few stylized experiments. Some points are worth noting about the nature of these results.

First, we assume that the economy is initially at its long-run equilibrium when the shock hits the economy. This is a different type of exercise compared to forecasting, which generally describes how an economy that is away from a sustainable long-run equilibrium could move toward such a position.

Second, each shock represents an isolated change of a single exogenous or endogenous variable. This helps to clarify the properties of the model but is a simplification compared with most historical episodes and forecasts, which often involve simultaneous shocks to a number of variables. Moreover, in the simulation presented in this chapter, we assume that the rest of the world does not respond to the shocks in Sweden.

5.1 Permanent shocks

This section first discusses the behaviour of the full model – Steady state, Flex-price and Sticky price – in face of permanent shocks to productivity, labor supply, the relative bargaining strength of workers and the ratio of government debt to GDP. These permanent shocks affect the equilibrium values of output and relative prices.

Finally, we analyze a permanent nominal shock – a shock to the inflation target. In this case the equilibrium values of output and relative prices are unaltered, but the shock will have permanent effects on nominal variables. Figures 5.1 to 5.5 in Appendix A show the responses of KIMOD to these shocks and table 5.1 in Appendix A summaries the impact on production, inflation and the output gap.

5.1.1 A productivity shock

Figure 5.1 shows the effect of a 1 per cent permanent increase in the level of TFP (total factor productivity). The productivity shock is specific to Sweden, since foreign output is exogenous in the simulation. This simulation is of interest because it highlights the importance of stock-flow dynamics and the wealth dimension of KIMOD, and it also illustrates how demand and policy react to supply shifts.

In the long run, the implied increase in labour productivity is accompanied by a rise in real wages paid by firms. Household spending is positively affected by the higher wage, while investment flows rise as the desired capital stock increase. Government expenditure rises by the same amount as output in the long run, to restore the steady-state share of government. Imports rise permanently, in line with the permanent increase in consumption and capital investment. The real exchange rate appreciates due to the Balassa – Samulesson effect (i.e. the real exchange rate of countries with high productivity growth in the tradable sectors appreciates). This reduces net exports, but exports still rise in absolute terms (see figure 5.1.k).

The appreciation also has a “second-round” effect on the real economy, increasing the overall impact on long-run supply. This is due to the fact that the price of domestically produced goods rises, with respect to the price of imported goods, so that Sweden’s terms of trade improve. This raises the ratios of imported to domestically produced consumption and capital goods. This relative price effect reduces the overall cost of capital, which adds to the increase in capital and output. Hence, output increases more than 1 % (see figure 5.1.a).

The exchange rate movements are associated with permanent shifts in relative prices, but inflation, real interest rate and nominal interest rates return to the initial equilibrium implied by the Riksbank’s inflation target and the world equilibrium real interest rate. If this shock had been shared with the rest of the world, it would have had a neutral effect on the real exchange rate in the long run, and output would have risen by exactly 1 %.

Turning to the short run, firms are immediately able to produce more output, for given inputs of capital and labour. Although demand reacts quickly, adjustment costs imply that the shock initially creates an excess supply. This encourages firms both to reduce prices to stimulate demand and to reduce their demand for labour in the short run (see figures 5.1.c and 5.1.f). The labour market gap, however, closes rapidly as demand adjusts to its new equilibrium. Monetary policy responds to low inflation, with lower interest rates, which stimulates demand. This prevents the inflation from falling further, and the inflation rate gradually rises back towards the inflation target.

5.1.2 A labour supply shock

In this scenario, a supply shock stems from the assumption that labour supply permanently increases by 1 percentage point relative to the base case scenario (see figure 5.2). In the simulation, hours worked in the public sector are assumed to be unaltered. The extra supply of labour is therefore completely absorbed by the private sector in the long run. Hours worked in the private sector will therefore increase more than 1 % (see figure 5.2.c).

In a small open economy model, in which there is no imported capital, a given increase in employment would in the long run be matched by a proportional increase in capital and output. In KIMOD, however some capital goods are imported. Therefore, the interaction between the real exchange rate and the long-run supply discussed above for the case of a productivity shock means that the increase in production in the private sector is slightly lower than the increase in hours worked in the private sector (see figures 5.2.c – 5.2.e). This is so because the real wage falls slightly and the cost of capital increases as the real exchange rate depreciates.

In the long run, aggregate labour income increases because the fall in the real wage is less than the increase in hours worked. Household spending therefore increases, while investment flows rise as the desired capital stock increases. Imports rise permanently, in line with the permanent increase in consumption and capital investment. Exports increase more than imports because the real exchange rate depreciates (see figure 5.2.k).

Turning to the short run, the direct effect of a higher labour supply is higher unemployment. Higher unemployment, and hence a negative output gap, constrains both nominal and real wage growth. The lower rate of wage increase is reflected in the pricing behaviour of the firms and the rate of inflation shrinks. The Riksbank, with its target of maintaining the rate of inflation at an average of 2 percent, cuts the repo rate in order to bring back inflation to the target.

Lower real wages also imply that firms demand more labour. Employment increases sharply over the first three years, which means that the unemployment rate gradually falls back to the equilibrium level (see figure 5.2.a). Consumption is initially curbed by lower real wages and higher unemployment.

5.1.3 A shock to the relative bargaining strength of workers

This scenario analyzes the economic outcome when the labour market parties suddenly starts to show more concern for the welfare of the unemployed and will accept a somewhat lower real wage (see figure 5.3). Technically, this is achieved by lowering the relative bargaining strength of workers (see chapter 2 for a description of the labour market model in KIMOD 1.0). Firms then demand more labour, which implies higher GDP and employment in the long run.

The labour market parties, who now place more emphasis on the level of unemployment, settle on a lower real wage. At this new, lower real wage, firms demand more hours worked, partly because labour is relatively cheaper, and partly because the stock of capital increases as investment in Sweden becomes more profitable for a transitional period. Unemployment therefore drops and the number of hours worked rises correspondingly in the long run.

With more hours worked, higher GDP and lower unemployment general government finances are permanently strengthened and the general government consumption can increase. This means, therefore, that lower unemployment and higher output create a margin for the government for increasing general government expenditure and/or reducing taxes.

More hours worked implies higher GDP. The improvement in GDP, however, is proportionally lower than the increase in hours worked, since the stock of capital does not increase as much as the number of hours worked. The explanation for this is that the real exchange rate depreciates, making imported investment goods more expensive. In addition, lower real wages imply that firms to some extent substitute capital by employing more personnel. Investment, exports and imports increase in the long run. Investment goes up because the stock of capital must be adjusted to the higher level of employment. Exports increase because prices of exports in foreign currency decrease. On the other hand, higher imports do not entail any change in prices of imports in foreign currency. Therefore, the combined impact of the increases in the volume of imports and exports is deterioration in the terms of trade.

Thus, with lower equilibrium unemployment and a somewhat lower real wage, the number of hours worked increases, GDP rises and general government finances improve.

5.1.4 A permanent reduction in the ratio of government debt to GDP

In this scenario, we lower the steady-state target ratio of government debt to GDP by 1 % (see figure 5.4.a). A permanent reduction in the ratio of government debt to GDP is accompanied by a rise in the ratio of net foreign assets to GDP. The real exchange rate will therefore appreciate. This reduces net exports, but exports still rise in absolute terms. The appreciation also has an effect on the desired capital stock. This is due to the fact that the price of domestically produced goods rises relative to imported goods, so that terms of trade improve. This rises the ratios of imported to domestically produced consumption and capital goods. This relative price effect reduces the overall cost of capital and hence increases the desired capital stock (see figure 5.4.e). Higher capital stock leads to a permanent increase in potential output. In the long run, the increase in production is accompanied by a rise in real wages paid by firms. Household spending is therefore positively affected by higher wage. Imports rise permanently, in line with the permanent increase in consumption and capital investment.

5.1.5 A shock to the inflation target

In this scenario, the Riksbank decides to permanently reduce the inflation target by 1 % (see figure 5.5). To convince agents that the reduction in inflation is permanent, the Riksbank needs to raise the real interest rate through altering the nominal repo rate, due to lack of full credibility. This dampens consumption and investment in the short run. As a result of the initial increase in the real interest rates, both nominal and real exchange rates appreciate. Net exports therefore decrease in the short run. The nominal exchange rate will display an appreciating trend in the long run as compared to the base scenario, because the domestic inflation rate is permanently reduced (see figure 5.5.g).

Overall, an excess supply lasting six years and peaking at around 0.4 per cent is created. The sacrifice ratio (i.e. the cumulative loss of output needed to reduce the inflation target by 1 per cent) is 1.2 in the current calibration of KIMOD (see figure 5.5).

5.2 Transitory shocks

This section discusses the behaviour of KIMOD in face of transitory shocks to nominal wages, the exchange rate, the interest rate, labour supply and household consumption. For these shocks, the equilibrium values of output and relative prices are unaltered, but they will have permanent effects on nominal variables. Figures 5.6 to 5.10 in Appendix A show the responses of KIMOD to these shocks and table 5.2 in Appendix A summarize the impact on inflation and the output gap.

5.2.1 A shock to the nominal wage

This scenario illustrates the effects of a settlement providing for wage increases that are 1 % higher for one year (see figure 5.6), as compared to the base scenario. Firms react in the short run to the increase in wage costs by reducing both their output and the number of their employees.

The higher rate of wage increases is reflected in the pricing behaviour of firms, and the rate of inflation increases (see figure 5.6.b). The Riksbank, with its target of maintaining the rate of inflation at an average of 2 %, initially raises the repo rate and then gradually lowers it again to its original level.

As a result of the initial increase in interest rates, both real and nominal exchange rates appreciate. Net exports decrease, as the real exchange rate appreciates in the short run. Investment is negatively affected by higher real interest rates and lower output. Higher real wages imply a higher disposable income, which stimulates consumption. However, consumption is curbed by higher real interest rates and higher unemployment. The net effect on consumption is therefore close to zero in both the short and the medium term. The aggregate effect on GDP is shown in figure 5.6.b. At most we have a negative output gap of 0.1 %. The number of hours worked is lower than in the main scenario (the labour market gap in hours worked is the same as the output gap in this scenario, because labour productivity is not effected). Unemployment is highest after about 3 years.

Thus, this simulation illustrates how a higher wage settlement can affect the economy negatively (i.e. lower output and hours worked compared to the base case scenario).

5.2.2 A shock to the nominal exchange rate

In this scenario, there is a 1 % temporary depreciation of the nominal exchange rate for one year (see figure 5.7). Hence, in this simulation the agents in the economy know (expect) that the exchange rate will be weaker for exactly one year and act according to that. The direct channel through which this variable affects the economy is relative import prices, exports and the relative investment prices. The weaker exchange rate leads to a 0.2 % increase in net exports around year one of the simulation. The positive effect in net exports is offset by a deterioration in investment, peaking at about 0.2 % after 3 years. This negative response in investment is mainly attributed to the higher real interest rates. The shock has virtually no impact in the other components of domestic demand. The output gap is increased by around 0.25 % at its peak.

Inflation is higher than in the base case scenario in line with the output gap response and the increase in import prices. It peaks at about after three years. Reacting to the expected inflation gap, the Riksbank raises the repo rate by slightly less than 0.5 percentage points.

5.2.3 A shock to the interest rate ("Policy shock")

Figure 5.8 illustrates the responses of the model to a shock of 50 basis points in the nominal interest rate for one year. It is designed to show the impact of interest changes on the economy. In the other simulations presented in this chapter, interest rates react endogenously to movements in inflation and expected inflation that come about because of some other shocks. Here, we start instead with an interest rate movement and follow the reaction of the remaining variables of the model. In particular, we fix the nominal interest rate for one year at a level 50 basis point higher than the equilibrium interest rate (see figure 5.8). After that period the Riksbank will again react according to the normal reaction function.

The increase in the nominal interest rate leads immediately to an increase in the real interest rate, and hence the cost of borrowing to finance consumption and investment. Also, the nominal and the real exchange rate immediately appreciate with the increase in the interest rate.

Higher interest rates reduce both consumption and investment in the early years of the simulation. At its peak, consumption is about 0.3 % lower than in the base case, and investment is about 1.0 % lower than the base case scenario. This reflects the greater sensitivity of investment to interest rates. Exports fall in response to the initial appreciation in the exchange rate. Imports fall as well, as the short-run effect that results from the reduction in private income dominates. The effect on net exports is therefore quite small.

Faced with unchanged potential supply, firms immediately reduce factor utilization and start reducing employment. Hours worked decrease and the unemployment rate increases (see figure 5.8.c).

Inflation falls below the starting point as firms react to the fall in demand. Also, nominal and real wages fall back quickly as a result of lower demand for labour.

The shock lasts for one year. After one year, the Riksbank reverts to its standard reaction function. Faced with a negative output gap and inflation below target, the Riksbank responds rapidly and cuts the interest rate aggressively to bring inflation back to target (see figure 5.8.a).

5.2.4 A temporary labour supply shock

In this scenario, a supply shock stems from a temporary increase in labour supply by 1 percentage point, relative to the base case scenario (see figure 5.9). The direct effect of a higher labour supply is higher unemployment; the unemployment rate initially increases by almost 1 percentage point (from 4.7 to 5.7 %).

Higher unemployment constrains both the nominal and real wage growth. After two years the nominal wage increase is around 1 % lower than in the base case scenario (see figure 5.9.h). The lower rate of wage increase is reflected in the pricing behaviour of the firms and the rate of inflation is lower. The Riksbank, with its target of maintaining the rate of inflation at an average of 2 %, initially cuts the repo rate and then gradually increases it again to its original level.

Lower real wages also imply that firms demand more labour. Both average hours worked and the number of workers increase and unemployment falls (see figure 5.9.c).

As a result of the initial cut in the nominal interest rate, both the real and the nominal exchange rates depreciate. Net exports therefore increase, as the real exchange rate depreciates in the short run. Investment is positively affected by higher production and lower real interest rates. Consumption is initially curbed by lower real wages and higher unemployment.

5.2.5 A demand shock

This scenario analyzes the effects of a positive demand shock stemming from household spending which increases by 1 % (see figure 5.10 and table 5.2). The shock lasts for one year. A positive demand shock to household spending translates into a positive output gap and somewhat higher inflation peaking after about two to three years. Reacting to the expected inflation gap, the Riksbank, initially raises the repo rate (see figure 5.10). As a result of the initial increase in the nominal interest rate, both real and nominal exchange rate appreciates and the real interest increases.

Net export decrease, as the increases in domestic demand has a positive effect on imports while exports fall in response to the initial appreciation of the exchange rate. Higher demand and production imply higher investment. However, investment is curbed somewhat by higher real interest rates.

Higher domestic demand also implies that firms demand more labour. Hours worked increase initially and unemployment falls. The tighter labour market also leads to higher nominal and real wages. Higher real wages, lower unemployment and higher average hours worked imply a higher disposable income, which stimulates future consumption.

Two years after the initial demand shock the output gap is close to zero. This follows from the fact that consumption, investment and net export are curbed by the higher real rate of interest and by the stronger real exchange rate.

6 Conclusion

Based on the assumption of forward-looking rationality of agents, economic theory imposes a number of powerful constraints on the behaviour of economic variables. KIMOD incorporates many prominent ideas of the modern macroeconomics of the neoclassical synthesis: forward-looking rationality of agents, adherence to neoclassical assumptions of production and investment technology, an explicitly modelled search framework for the labour market, money neutrality and superneutrality in the long run and the Blanchard – Yaari way of modelling finite life spans in an otherwise infinite horizon model. In a model like KIMOD, the dynamic solution is computed backwards from the future and the interpretation of current and changing economic information is only possible against the "background" of a fully specified economic future.

In theoretical general equilibrium models, it is a natural benchmark to assume fully informed economic agents and fully flexible price setting, and the steady state of KIMOD is of course also based on these assumptions. As this documentation makes apparent, it is difficult or impossible to use such a long term model to mimic the short run behaviour of actual macroeconomic data. To generate model behaviour that resembles the real economy, a number of other restrictions have to be imposed. In this respect, the applied macro modeller has no generally accepted code of conduct to appeal to. Instead, ideas are adopted from the research literature on a somewhat ad hoc basis, with the aim of achieving the most progress in reconciling the model and economic reality with the least complicated and most plausible extra assumptions. In KIMOD, due to assumptions of imperfect information and bounded rationality, there are nominal rigidities in both price and wage setting. For example, there is sluggish price adjustment because only a part of all firms in each period undertake price revisions. Furthermore, households' expectations are partially adaptive.

Moreover, KIMOD incorporates a search model of the labour market that by its very nature and without compromising any rationality assumption introduces a time-consuming adjustment process, due to the realistic assumption that vacant jobs and unemployed workers cannot all meet instantaneously. The inclusion of a search model of the labour market in an otherwise neoclassical growth model is probably one comparative advantage of KIMOD. It generates a plausible model connection between the labour market and monetary policy and can be richly contrasted with labour market data, but it also introduces a small but welcome degree of freedom in an otherwise neoclassical growth model: with vacancy costs, total output need not be exactly exhausted by labour and capital shares and there is some – though quantitatively limited – latitude in the model for short run fluctuations that would otherwise be difficult to reconcile with rigorous neoclassical assumptions. By these means, the search process of the labour market becomes an intermediate production process, using vacancies and unemployed workers as inputs, that in turn produces the labour input used in the production of goods.

A sophisticated macroeconomic model only thrives in an institutional environment that is geared to macroeconomic analyses. A model like KIMOD has a natural field of applications in policy simulations and shock simulations. Once there is a long term solution in place, it is "relatively" straightforward to simulate the effects of permanent or transitory economic shocks on the main macroeconomic variables (cf. chapter 5 on the dynamic properties of KIMOD). In addition to providing some interesting answers, the KIMOD exercise often compels the analyst to pose the economic question in a sharp way. For example, from the point of view of a policymaker or economic journalist, it seems like an innocent question to ask about the "effect of an interest rate increase of 50 points" or "the effect of a SEK 10 billions more expansive fiscal policy". When such questions are be incorporated as alternative policies in KIMOD, the questions become anything but simple. In the case of an interest hike: what were the public's expectations of the hike? Is the hike part of a revised monetary policy rule? If so, what is the underlying reason for the revision? Was it anticipated? Or is the hike a temporary shock due to, say, misleading current economic information or a temporary majority in the Central Bank Board? Different answers to such questions generate altogether different outcomes, and the merciless obligation to write down, in KIMOD, an exact specification for all of such cases can prompt a much more insightful view on the issue that has to be analysed.

Besides simulation purposes, the institutional environments that demand sophisticated macro modelling are most often also preoccupied with forecasting. This is also the case of KIMOD, since it has been designed and developed at the NIER. Yet the interplay with forecasters is by its nature challenging, since a model like KIMOD is by itself not particularly "clever" when it comes to interpreting the latest revisions of economic variables. As long as the KIMOD analyst is not provided with an interpretation of the long term repercussions – if any – of a revision of a macroeconomic variable, it is difficult to use that revision to generate interesting new information in KIMOD.

This "interface" between the short term NIER forecast and the long term constraints imposed by KIMOD is one challenge of future work with KIMOD in a forecasting environment. It is of course not the only one. To take one example, KIMOD with its about 80 endogenous variables is a fairly large economic model but still quite crude with respect to the hundreds and thousands economic variables that are routinely updated in a forecasting round. This is not a trivial problem of aggregation. For example, economic reality consists of many production sectors (industries), some of which are on trends that may be expansive or contractive. If linearly projected in the future, some industries would dominate the economy and others would soon disappear (and many do disappear in the course of time). In KIMOD, by contrast, any long term steady state must exhibit balanced growth with labour-augmenting technical change. To analyse real economies with real structural change by using a model that imposes balanced long term growth can lead to unforeseen technical problems.

Of course, such problems are also an indispensable sign of vitality. A macro model that would become “complete” in the sense that it would not be in need of any further developments would also die instantly, since it is only the process of improvements and new challenges that sustains the curiosity and ability of the modelling team. Thus, the only worse thing than to have to struggle with modelling challenges is not to have to struggle with them at all. This truth should be appreciated by model builders and appliers, as well as any large organisation that embeds such activity: that sustaining a large scale macroeconomic model is not about designing a perfect tool that can easily be used by anyone, but rather about sustaining a research and analysis infrastructure that contributes to the way an organisation thinks about the economy.

7 References

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8 Appendix A Figures and tables illustrating the dynamic model properties

Figure 5.1: A 1 % permanent productivity shock
Responses in %

Figure 5.1.a: Labour productivity

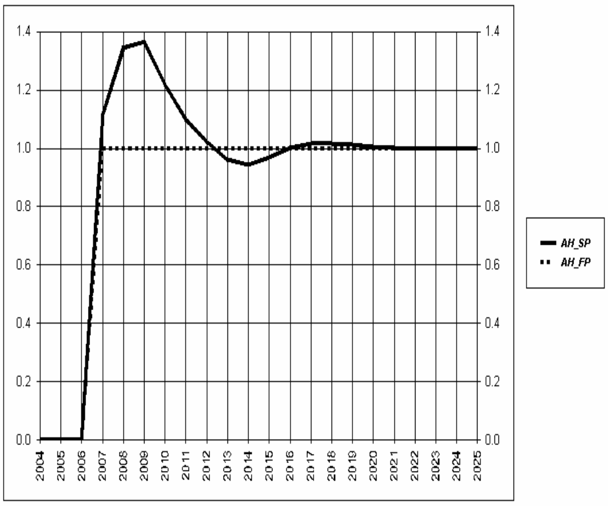


Figure 5.1.b: Capital stock

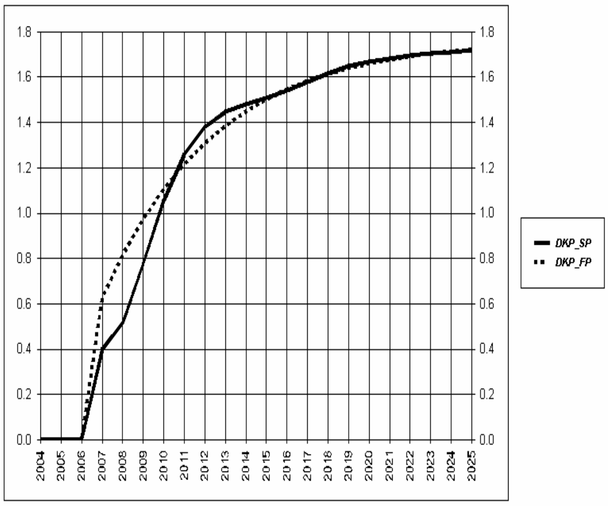


Figure 5.1.c: Hours worked, private sector

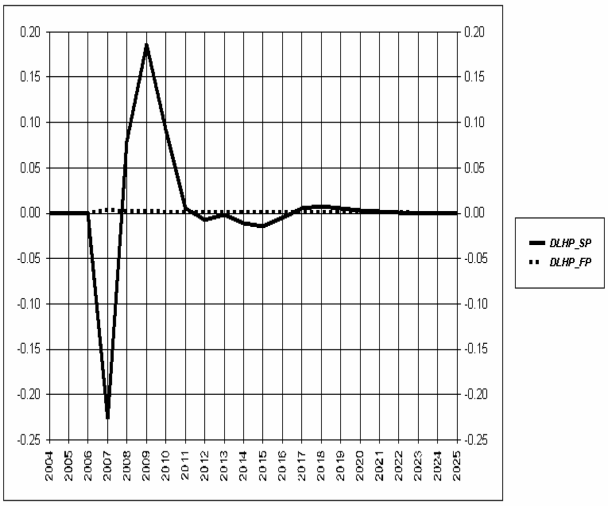


Figure 5.1.d: Production, private sector

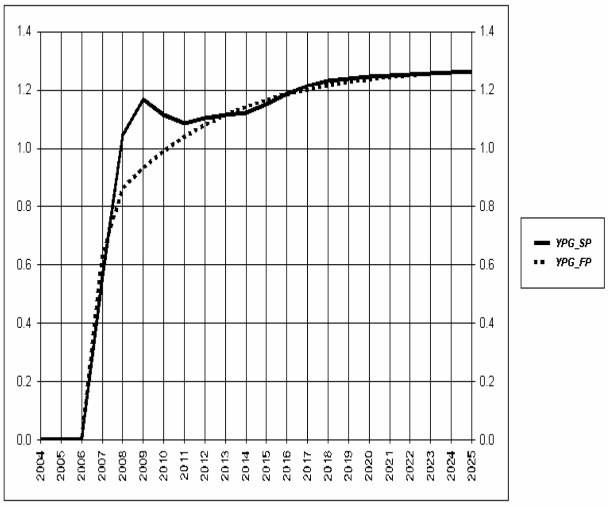


Figure 5.1.e: Total production

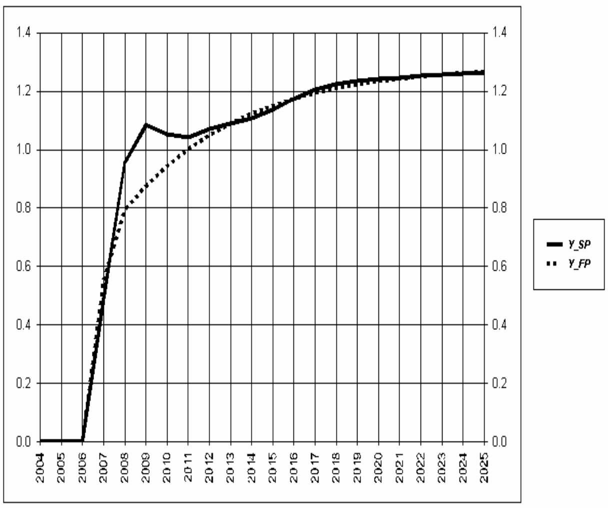


Figure 5.1.f : Output gap and inflation

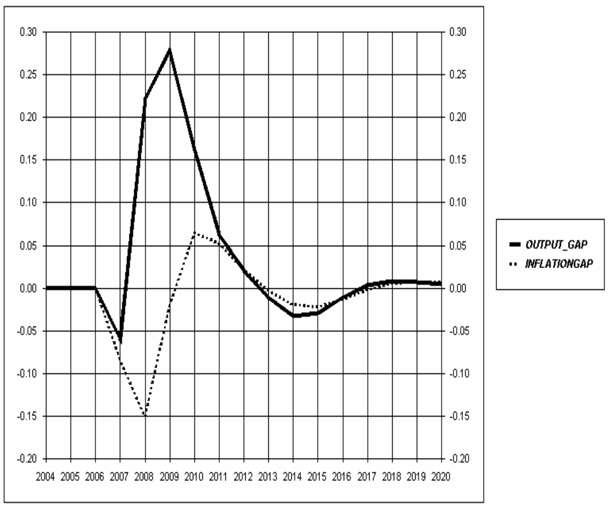


Figure 5.1: (continued) A 1 % permanent productivity shock
Responses in %

Figure 5.1.g: Real wage (level)

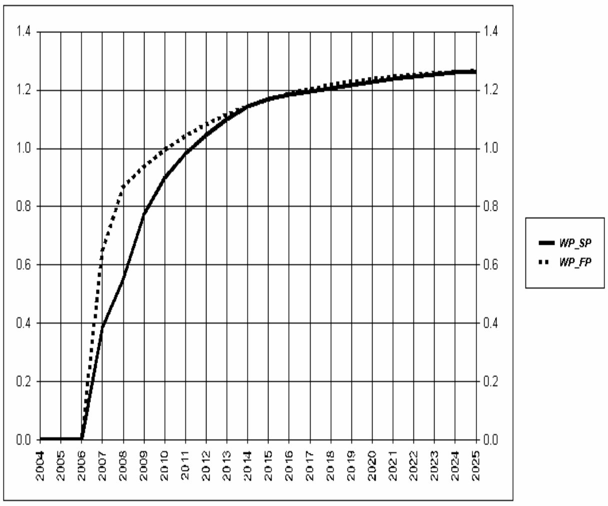


Figure 5.1.h: Nominal and real interest rate

Figure 5.1.i: Real and nominal exchange rates, terms of trade

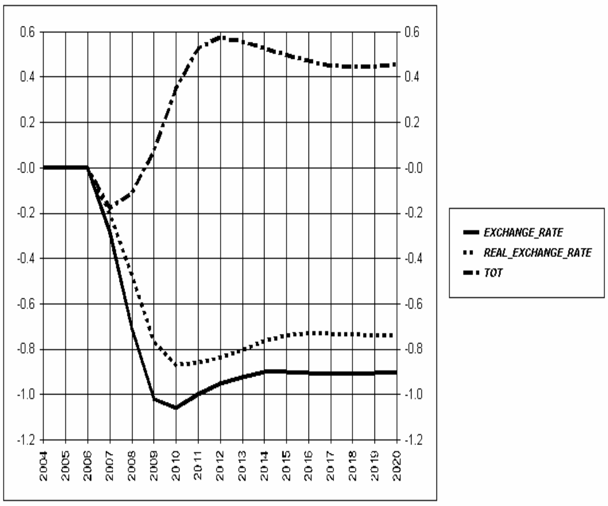


Figure 5.1.j: Nominal and real wages

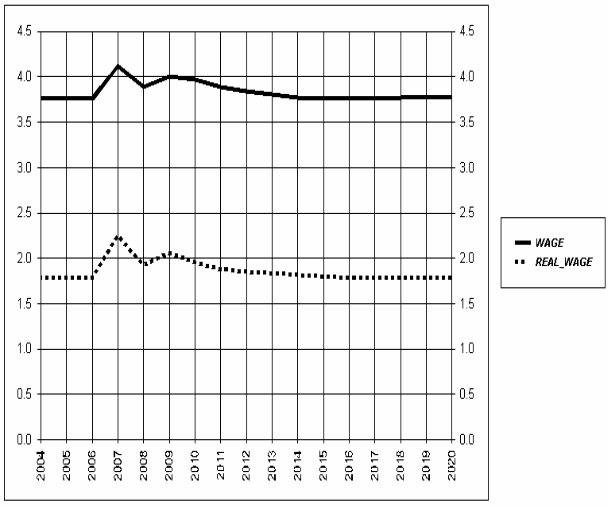


Figure 5.1.k: Export and import

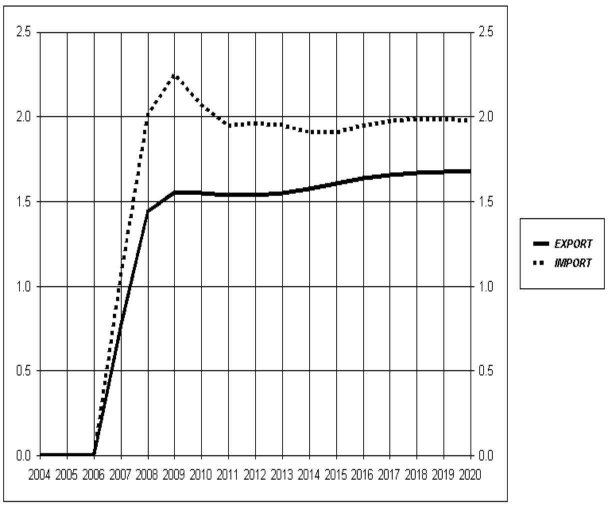


Figure 5.1.l: Private consumption and investment

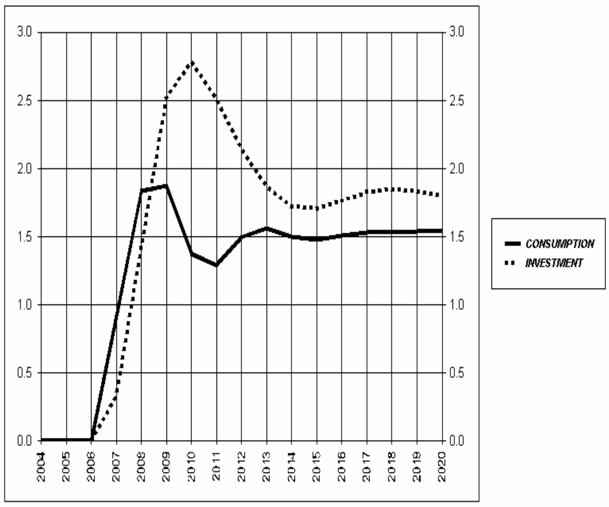


Figure 5.2: A permanent labour supply shock
Responses in %

Figure 5.2.a: Labour supply, hours worked and unemployment rate

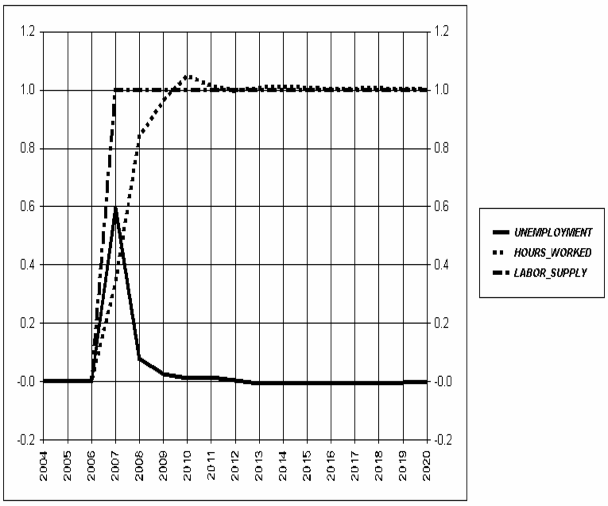


Figure 5.2.b: Total production

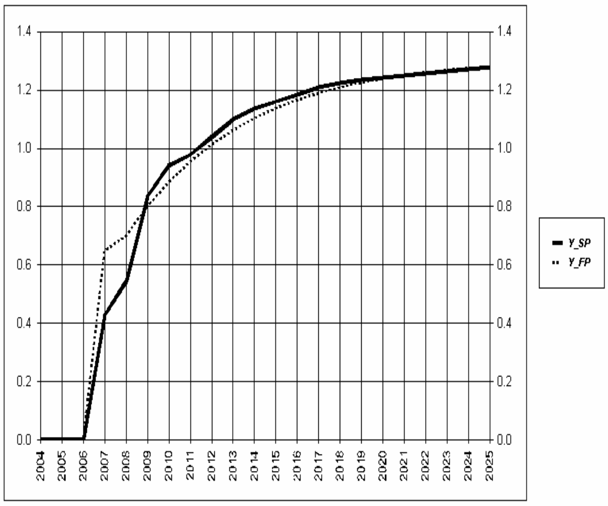


Figure 5.2.c: Hours worked, private sector

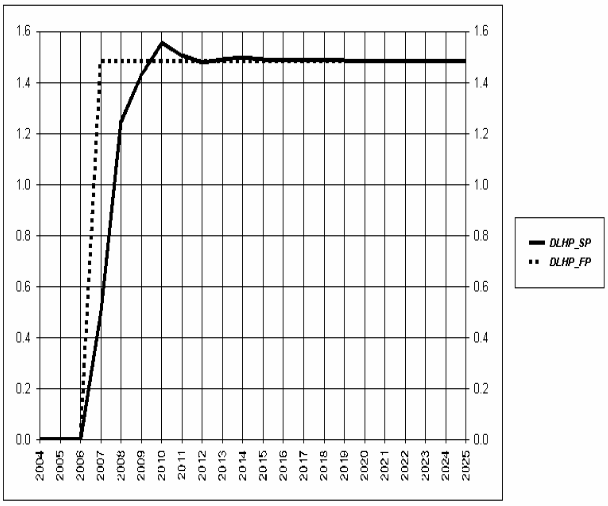


Figure 5.2.d: Production, private sector

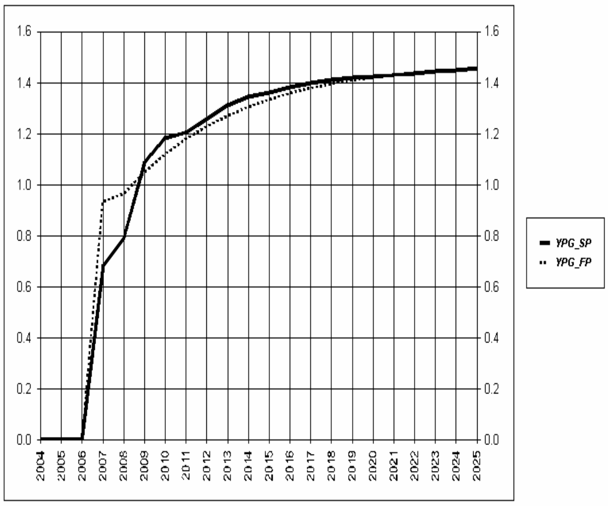


Figure 5.2.e: Capital stock, private sector

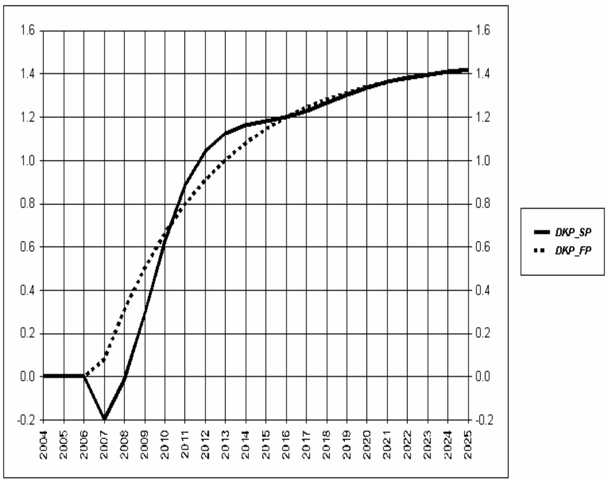


Figure 5.2.f : Labour productivity, private sector

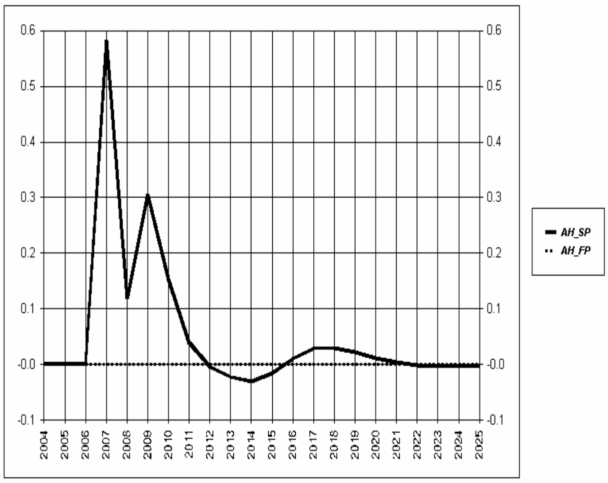


Figure 5.2: (continued) A permanent labour supply shock
Responses in %

Figure 5.2.g: Output gap and inflation

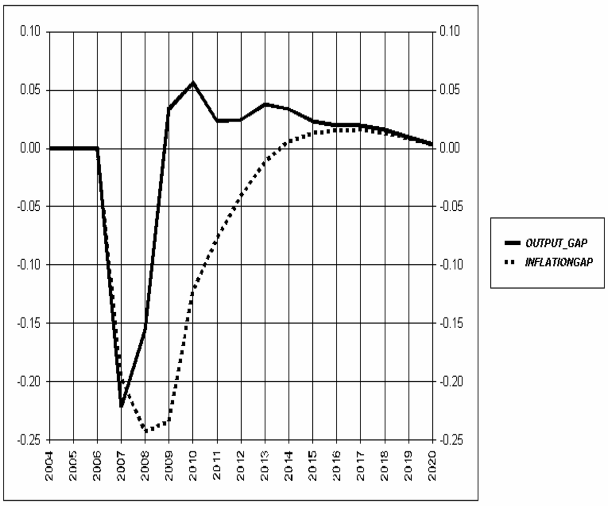


Figure 5.2.h: Nominal and real interest rate

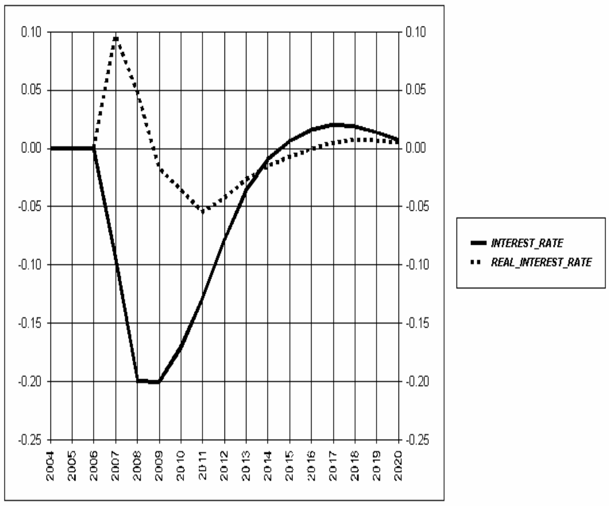


Figure 5.2.i: Real and nominal exchange rates, terms of trade

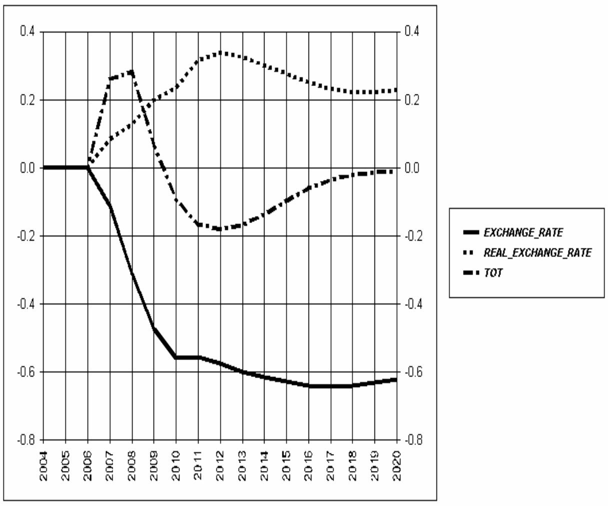


Figure 5.2.j: Real wage (level)

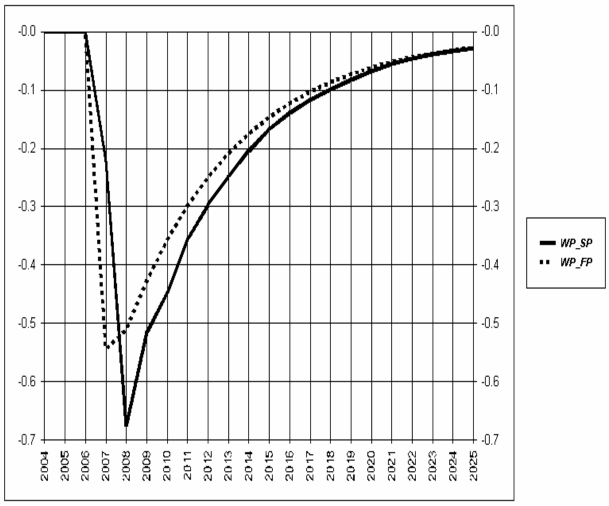


Figure 5.2.k: Export and import

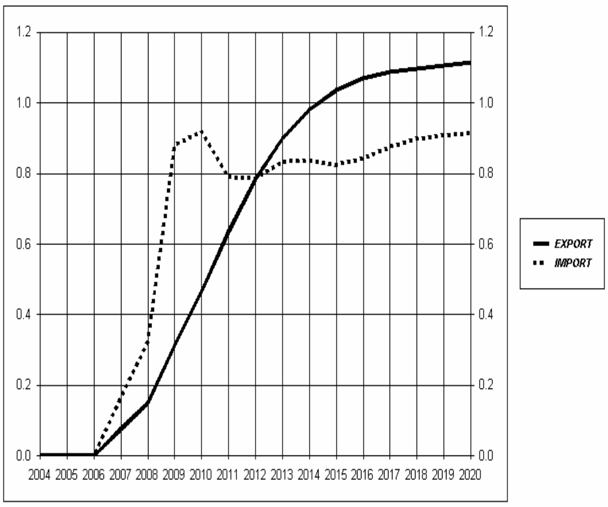


Figure 5.2.l: Private consumption and investment

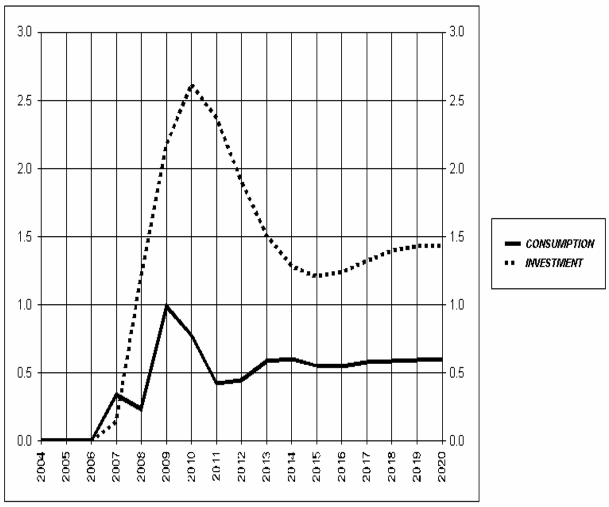


Figure 5.3: A shock to the relative bargain strength of workers

Responses in %

Figure 5.3.a: Labour supply, hours worked and unemployment rate

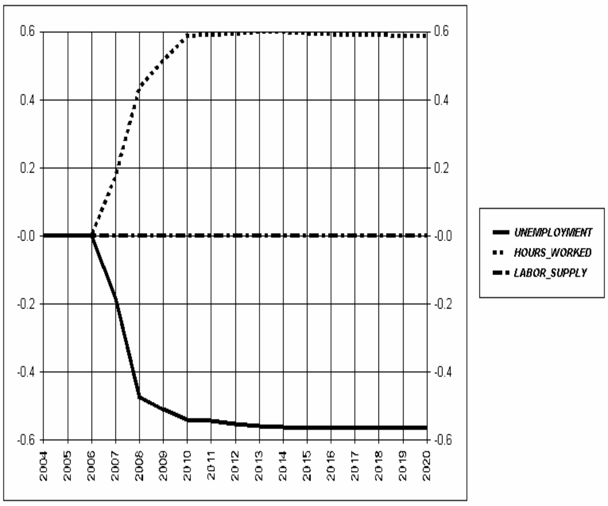


Figure 5.3.b: Total production

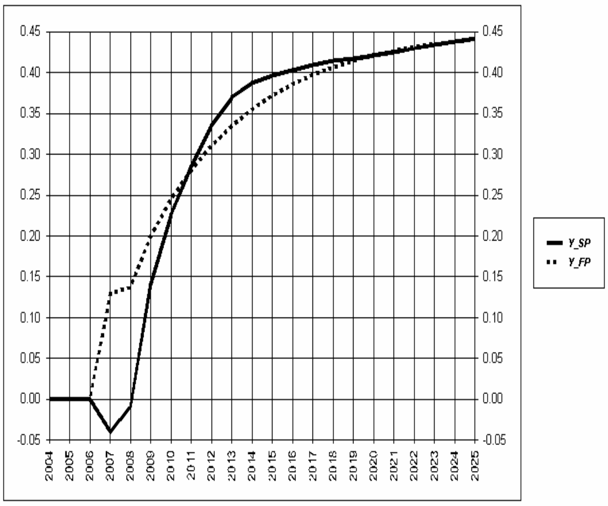


Figure 5.3.c: Hours worked, private sector

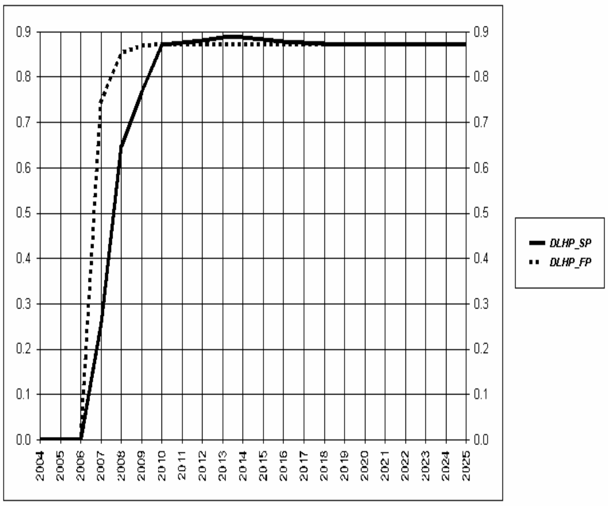


Figure 5.3.d: Production, private sector

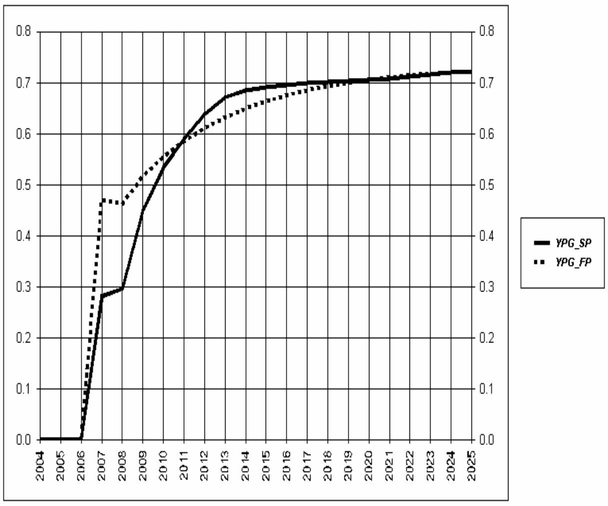


Figure 5.3.e: Capital stock, private sector

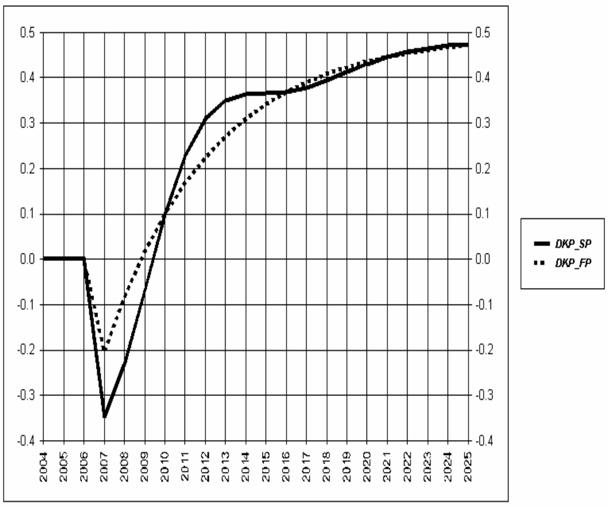


Figure 5.3.f : Labour productivity, private sector

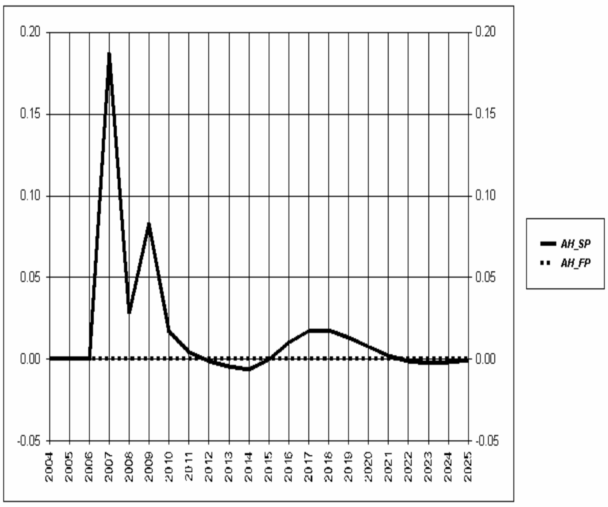


Figure 5.3: (continued) A shock to the relative bargain strength of workers
Responses in %

Figure 5.3.g: Output gap and inflation

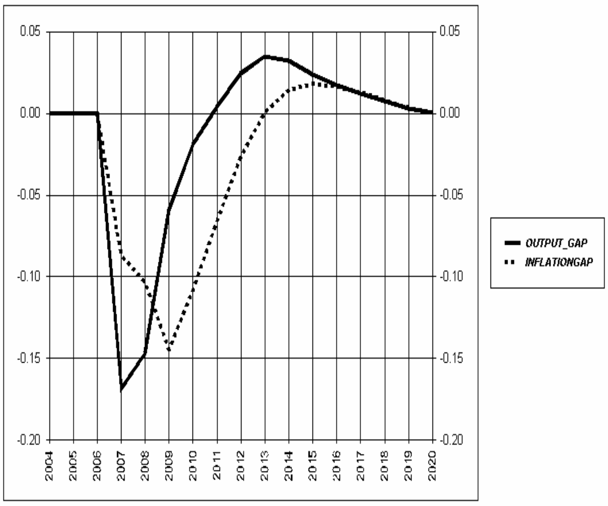


Figure 5.3.h: Nominal and real interest rate

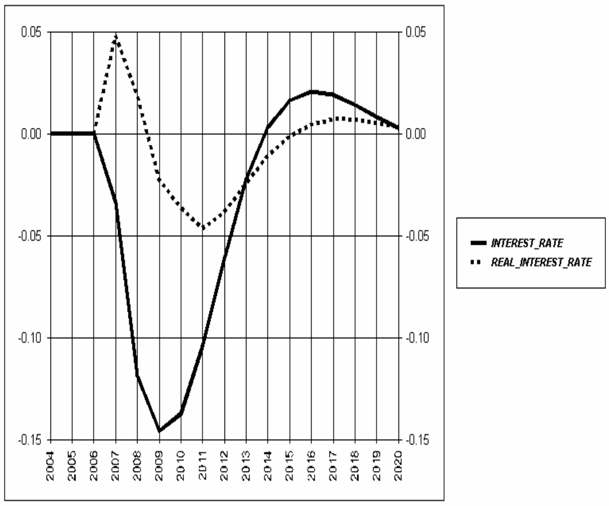


Figure 5.3.i: Real and nominal exchange rates, terms of trade

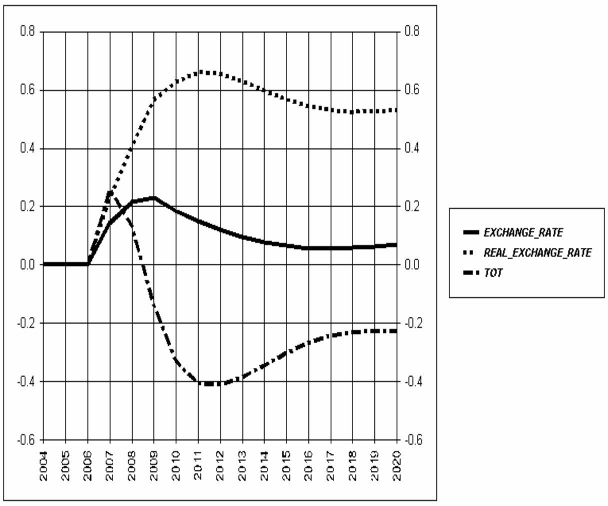


Figure 5.3.j: Nominal and real wages

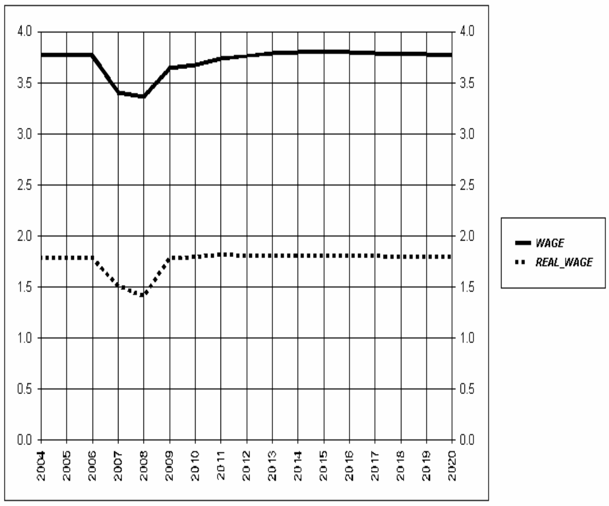


Figure 5.3.k: Net export and government consumption

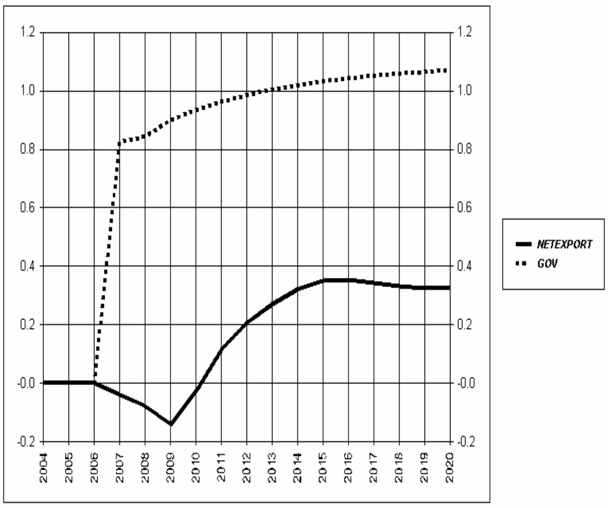


Figure 5.3.l: Private consumption and investment

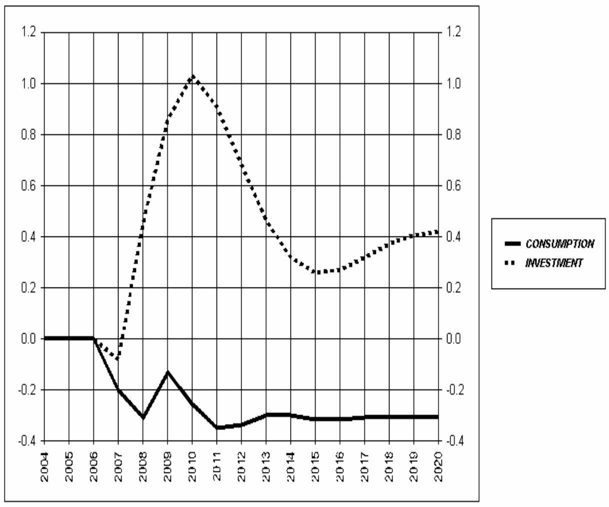


Figure 5.4: A permanent reduction in the ratio of government debt to GDP
Responses in %

Figure 5.4.a: Government debt to GDP

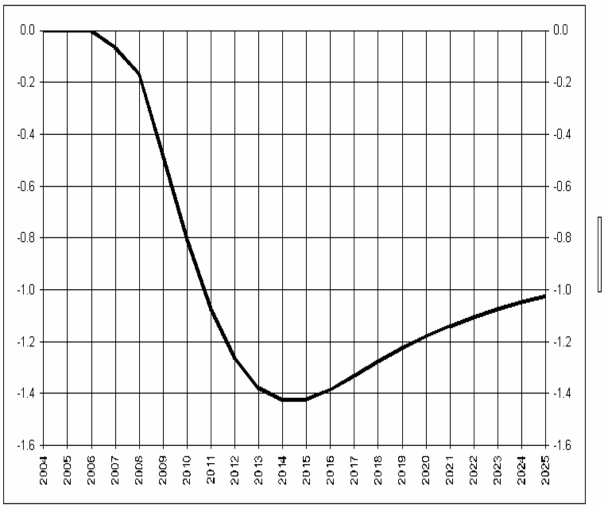


Figure 5.4.b: Total production

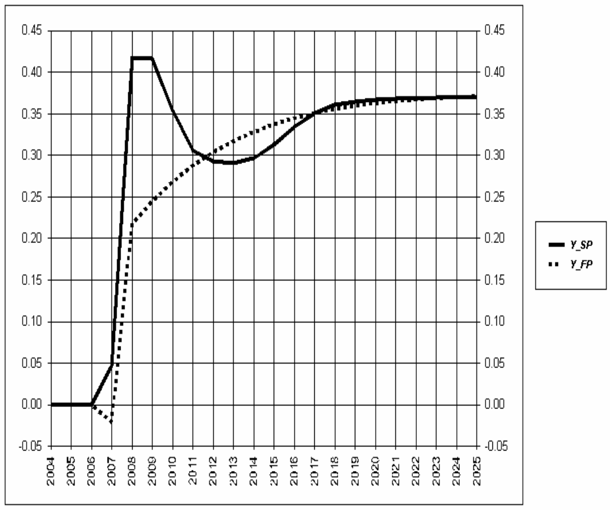


Figure 5.4.c: Hours worked, private sector

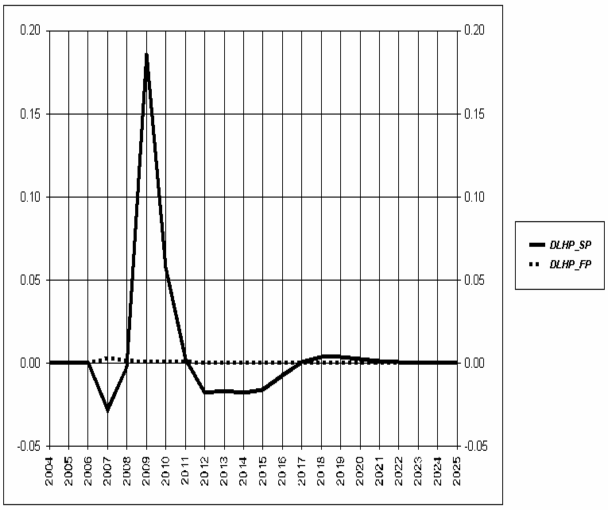


Figure 5.4.d: Production, private sector

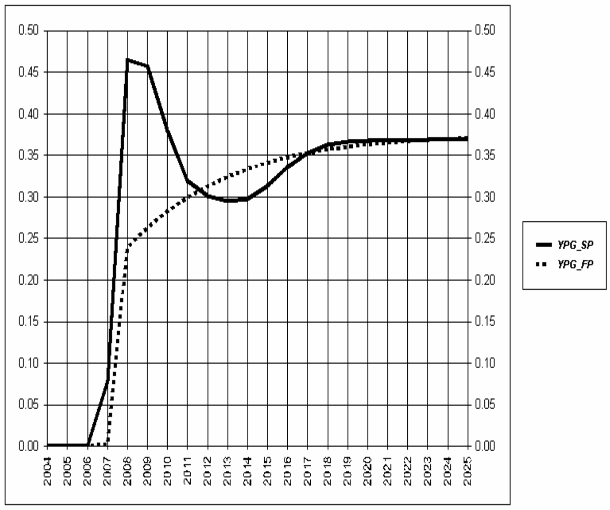


Figure 5.4.e: Capital stock, private sector

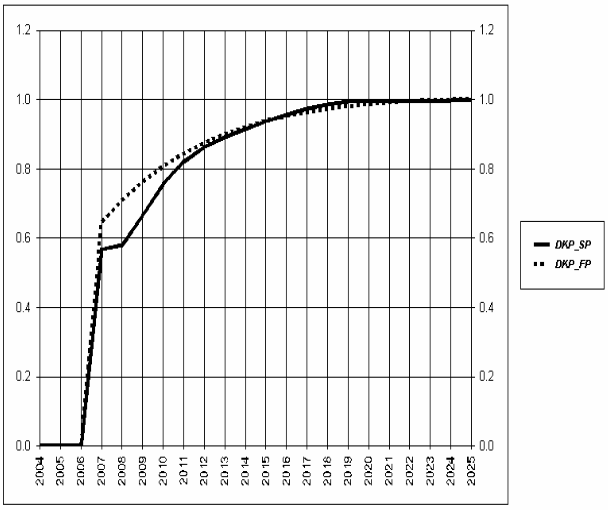


Figure 5.4.f : Labour productivity, private sector

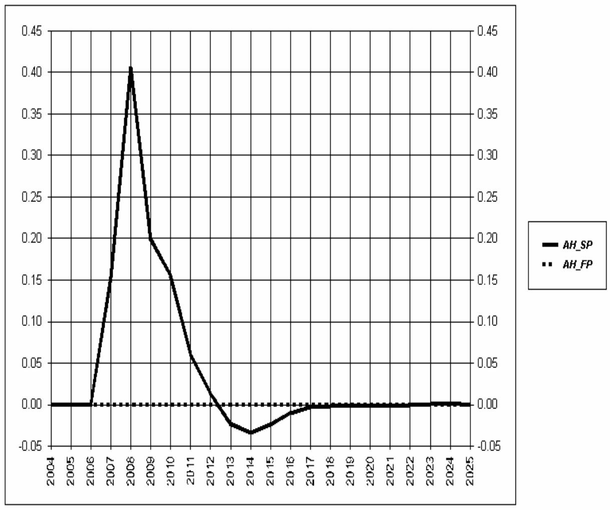


Figure 5.4: (continued) a permanent reduction in the ratio of government debt to GDP
Responses in %

Figure 5.4.g: Real wage

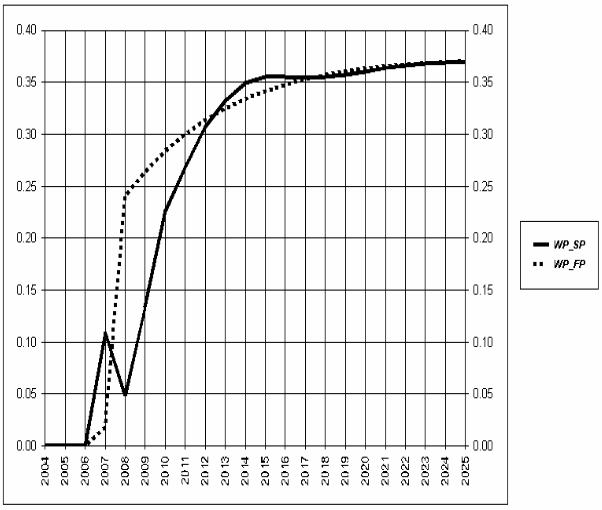


Figure 5.4.h: Nominal and real interest rate

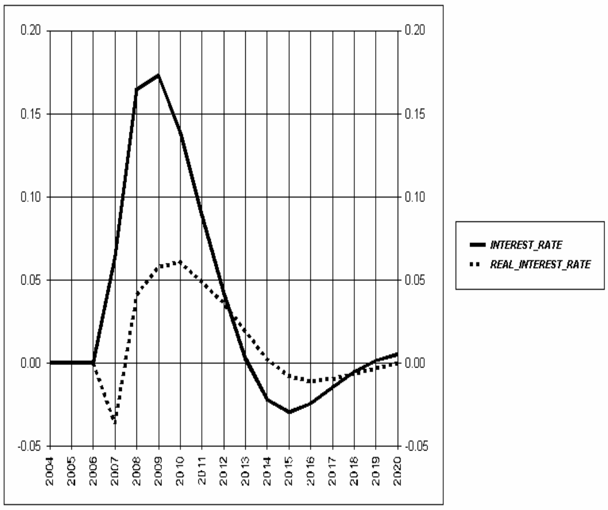


Figure 5.4.i: Real and nominal exchange rates, terms of trade

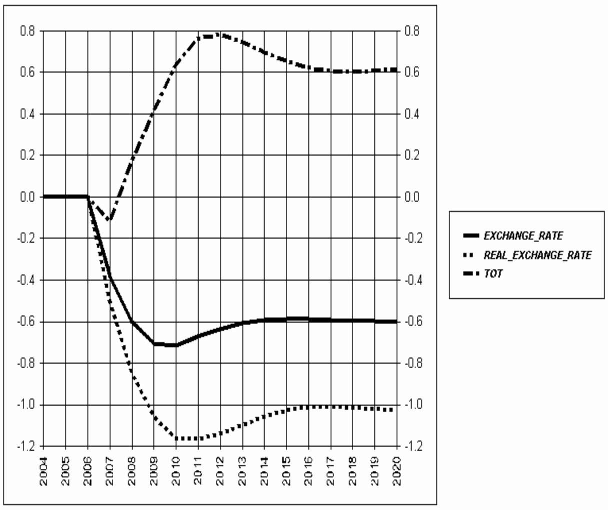


Figure 5.4.j: Output gap and inflation

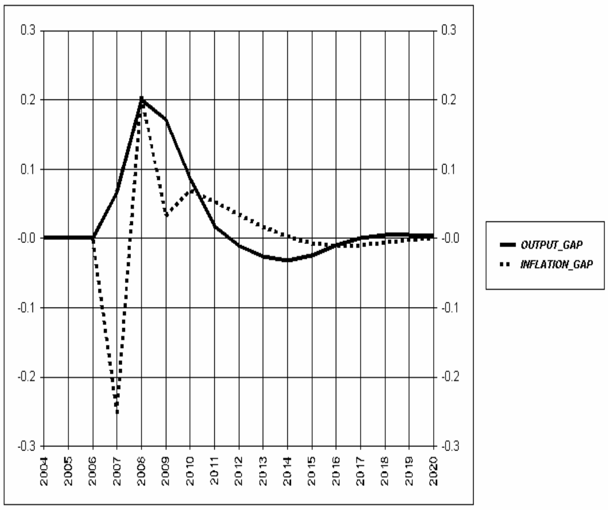


Figure 5.4.k: Net export and government consumption

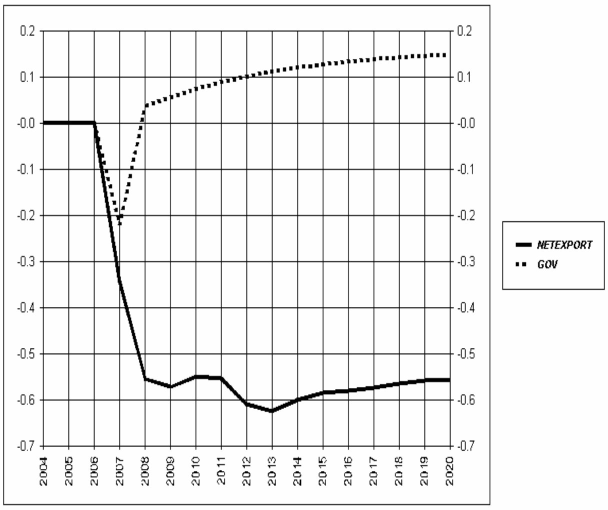


Figure 5.4.l: Private consumption and investment

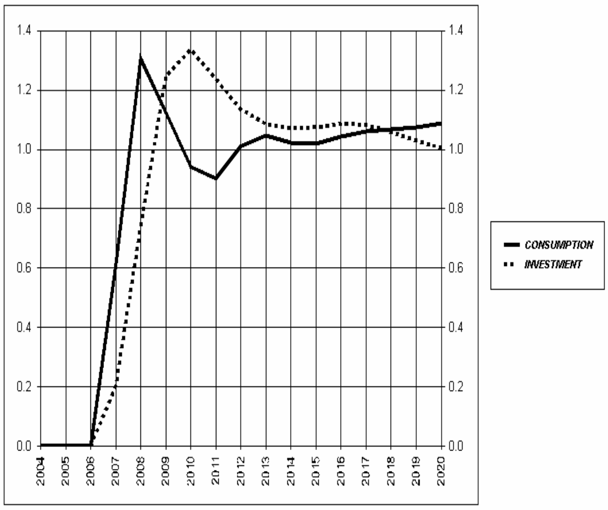


Figure 5.5: A 1 % permanent reduction in the Inflation target
Responses in %

Figure 5.5.a: Output gap and inflation (deviation from old target target)

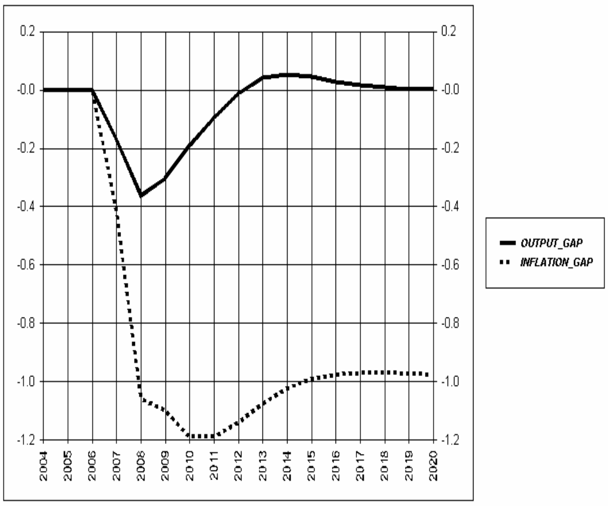


Figure 5.5.b: Nominal and real interest rate

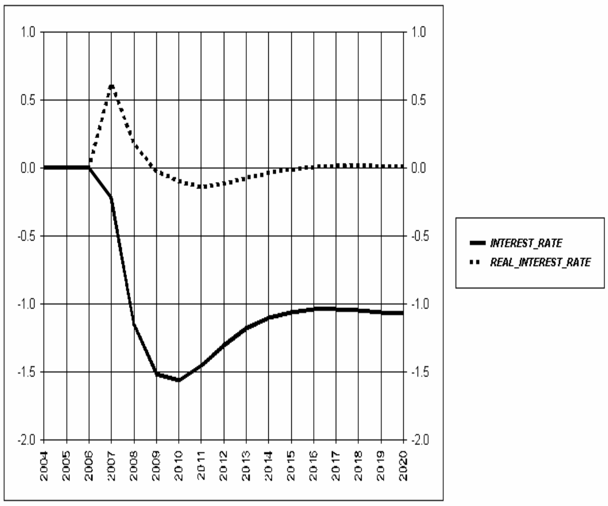


Figure 5.5.c: Labour market

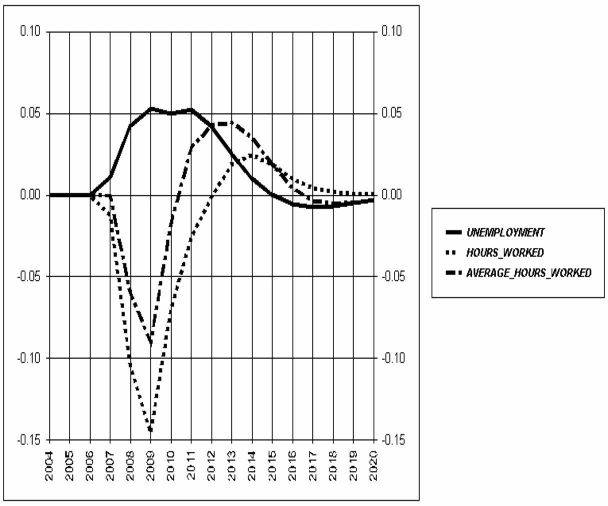


Figure 5.5.d: Nominal and real wage

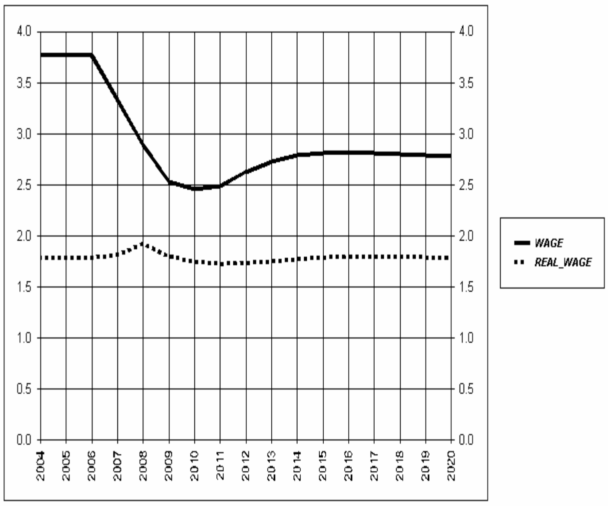


Figure 5.5.e: Net export and government consumption

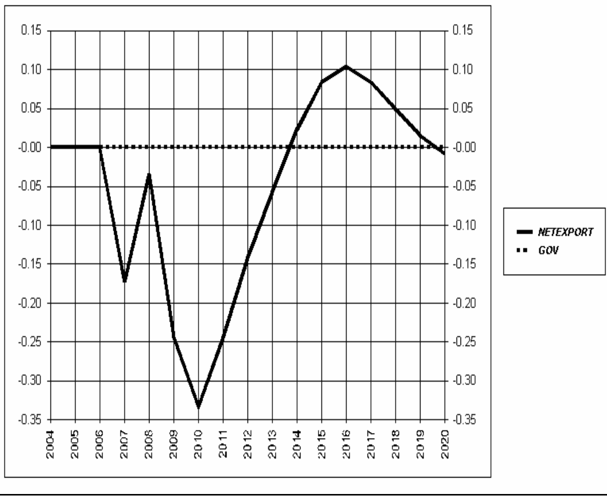


Figure 5.5.f: Private consumption and investment

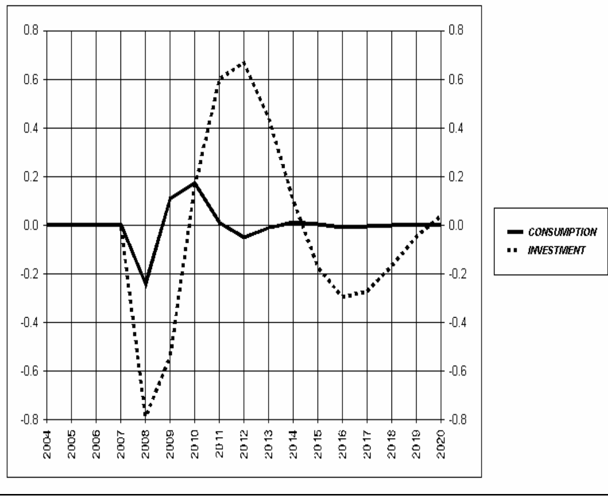


Figure 5.5: (continued) A 1 % permanent reduction in the Inflation target
Responses in %

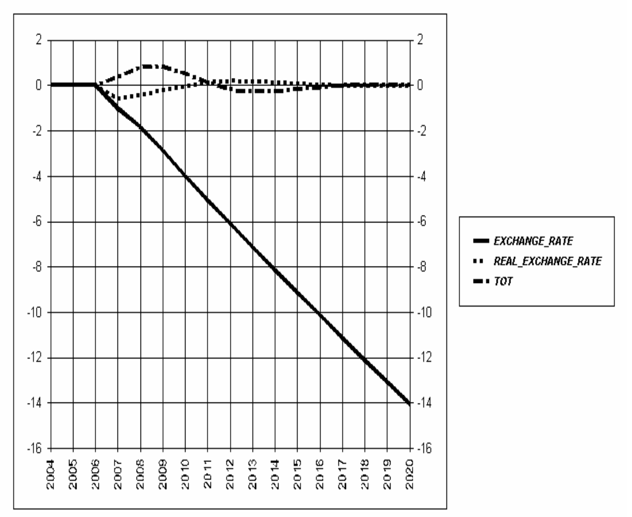
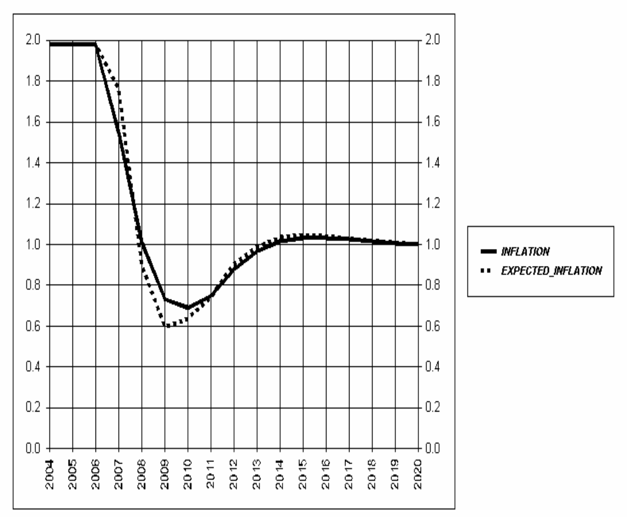
Figure 5.5.g: Real and nominal exchange rates, terms of trade	Figure 5.5.h: Inflation and expected inflation
	

Figure 5.6: A 1% shock to nominal wages

Responses in %

Figure 5.6.a: Nominal wage

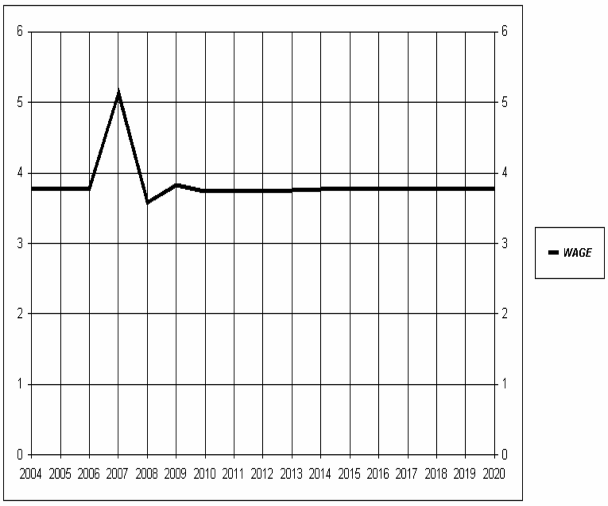


Figure 5.6.b: Output gap and inflation (deviation target)

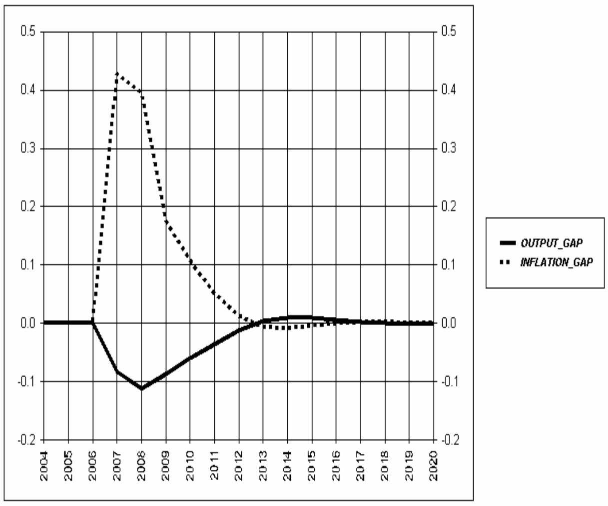


Figure 5.6.c: Labour market

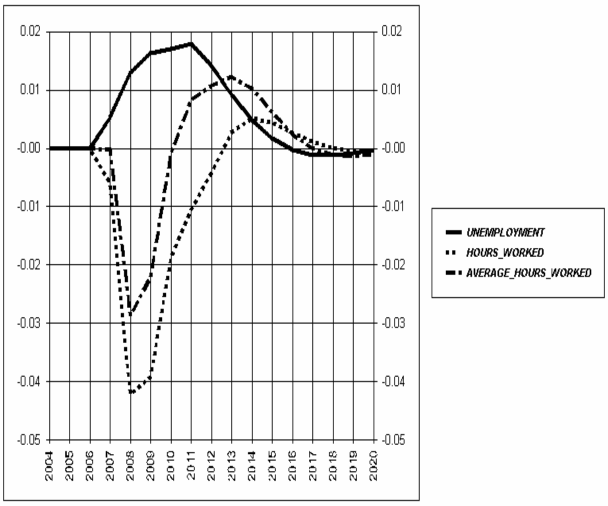


Figure 5.6.d: Nominal and real interest rate

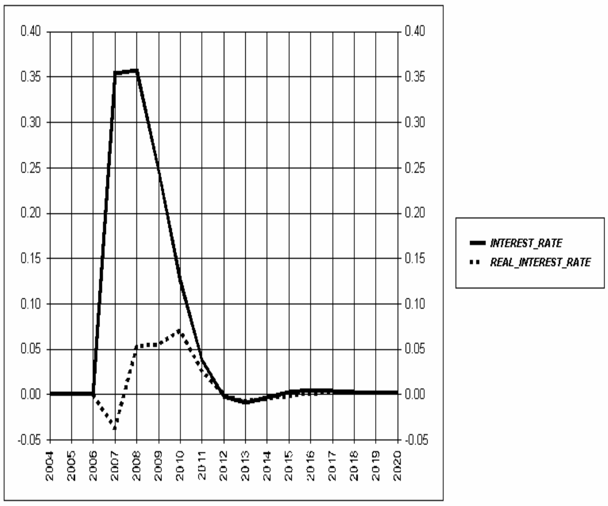


Figure 5.6.e: Net export and government consumption

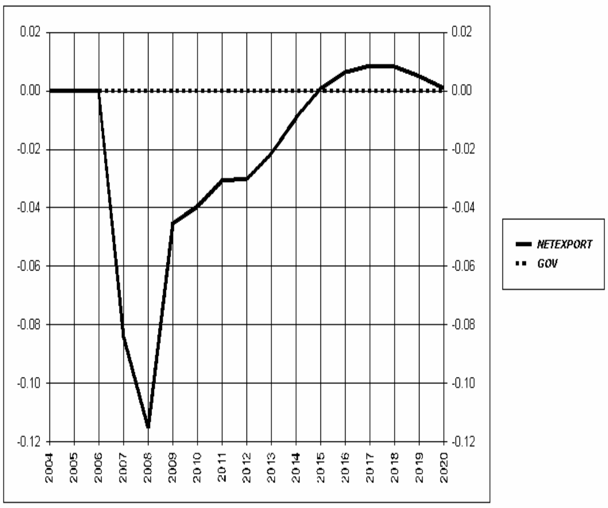


Figure 5.6.f: Private consumption and investment

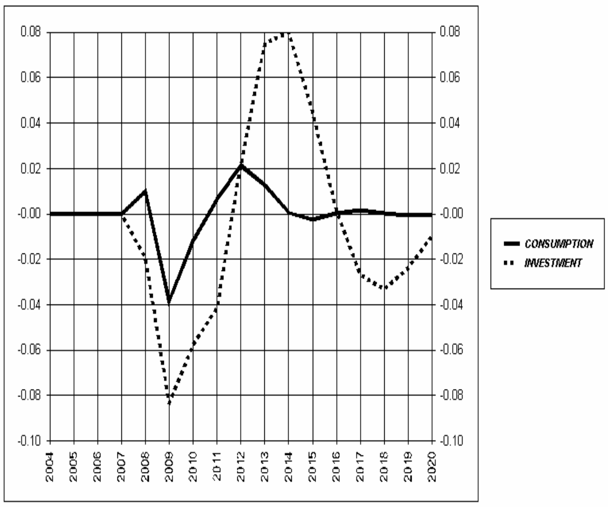


Figure 5.6: (continued) A 1% shock to nominal wages

Responses in %

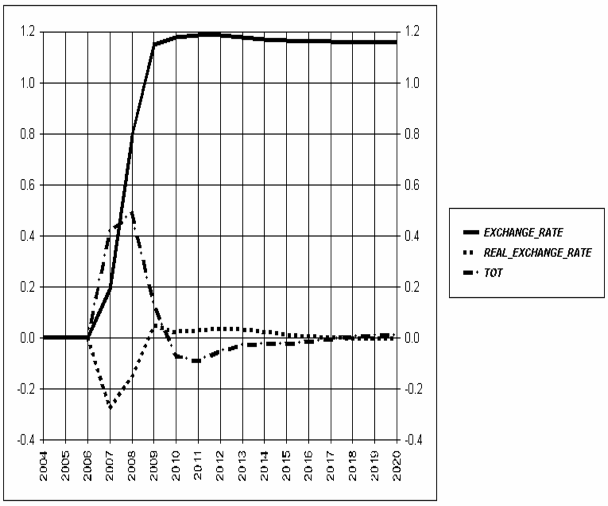
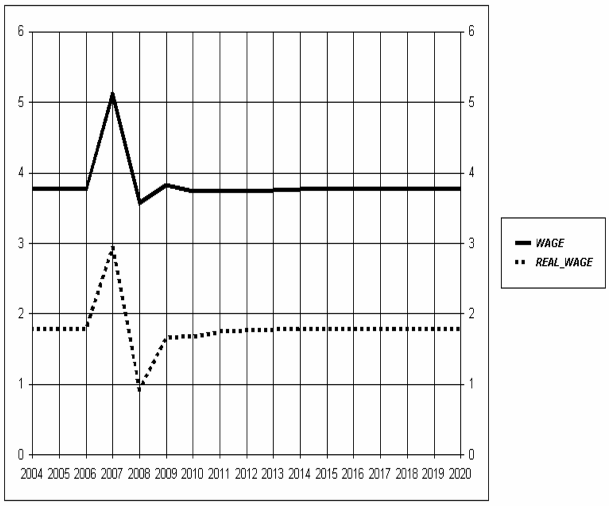
Figure 5.6.g: Real and nominal exchange rates, terms of trade	Figure 5.6.h: Nominal and real wage
	

Figure 5.7: A 1% shock to the nominal exchange rate
Responses in %

Figure 5.7.a: Nominal exchange rate

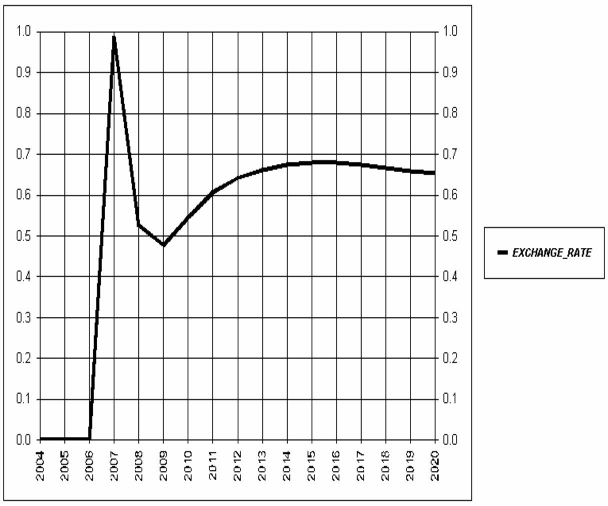


Figure 5.7.b: Output gap and inflation (deviation target)

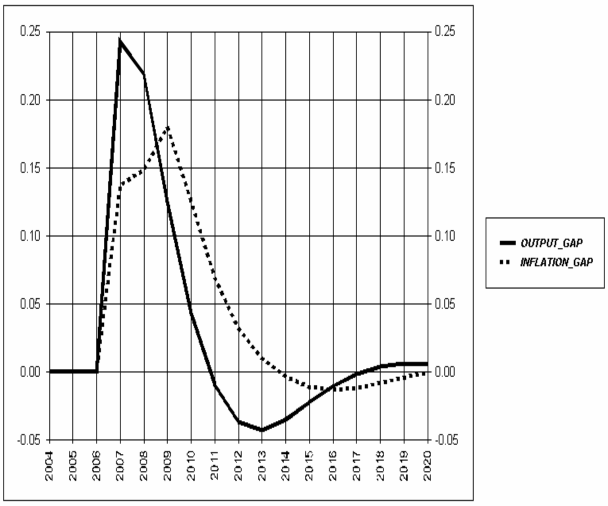


Figure 5.7.c: Labour market

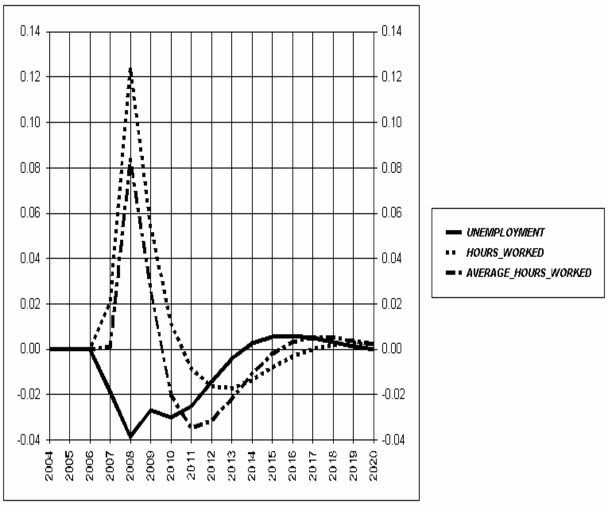


Figure 5.7.d: Nominal and real interest rate

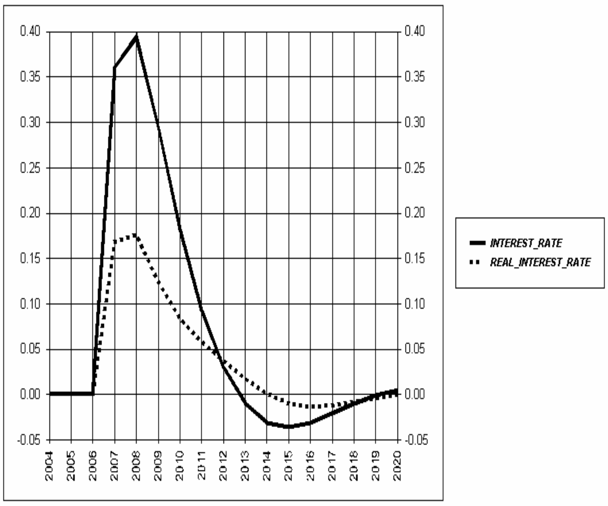


Figure 5.7.e: Net export and government consumption

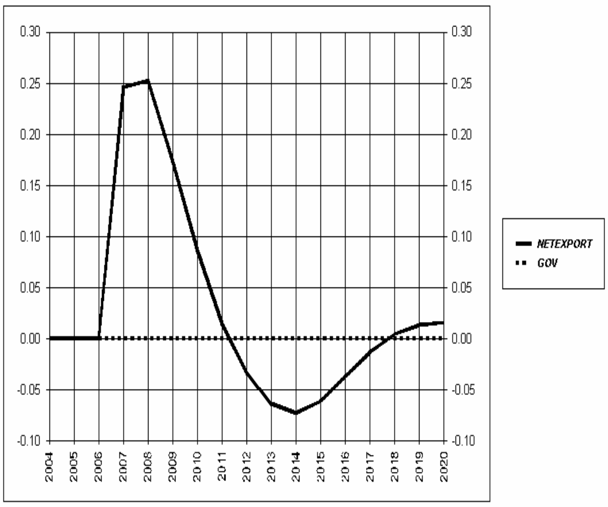


Figure 5.7.f: Private consumption and investment

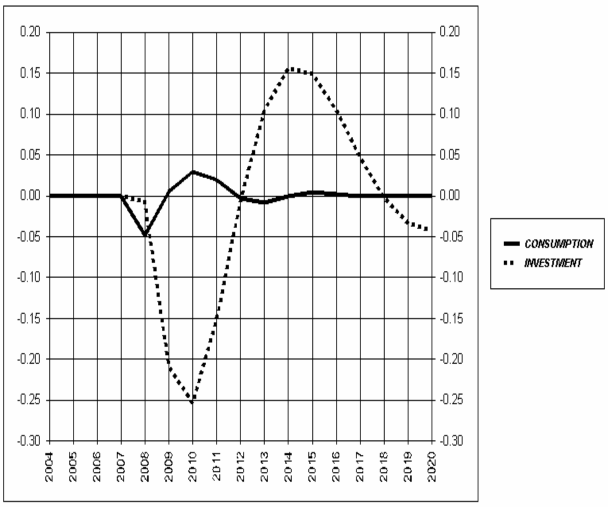


Figure 5.7: (continued) A 1% shock to the nominal exchange rate

Responses in %

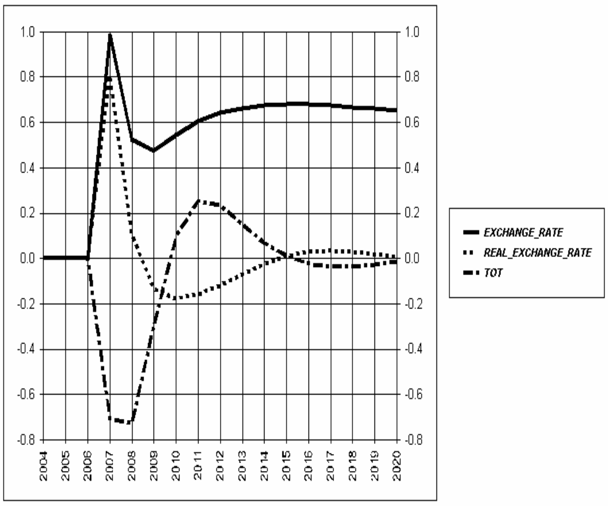
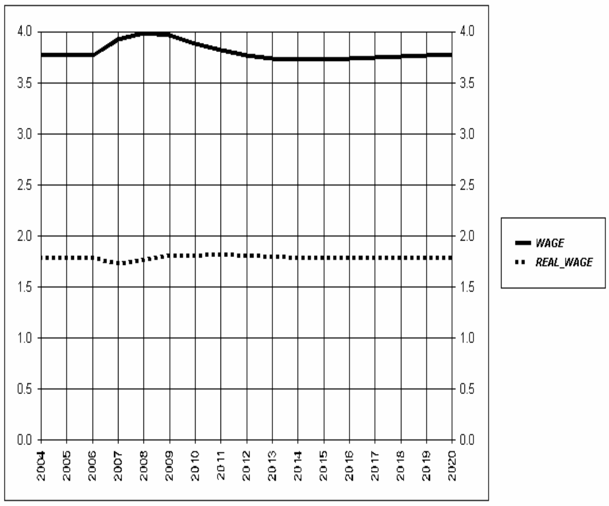
Figure 5.7.g: Real and nominal exchange rates, terms of trade	Figure 5.7.h: Nominal and real wage
	

Figure 5.8: A shock to the interest rate

Responses in %

Figure 5.8.a: Nominal interest rate

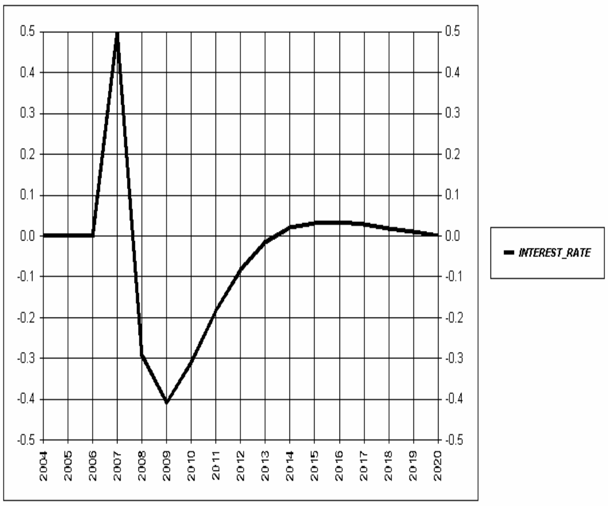


Figure 5.8.b: Output gap and inflation (deviation target)

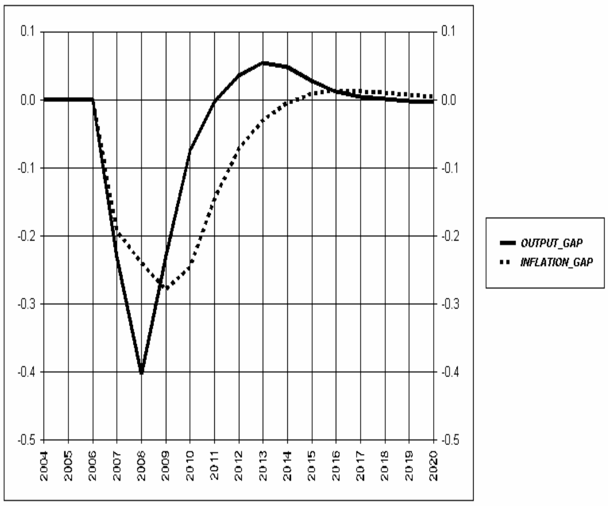


Figure 5.8.c: Labour market

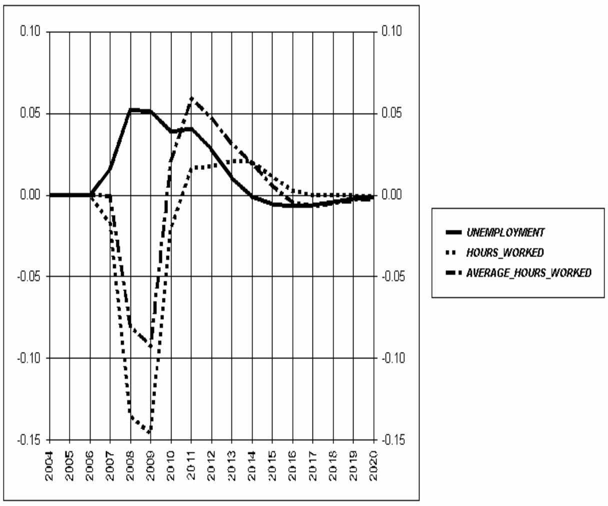


Figure 5.8.d: Nominal and real interest rate

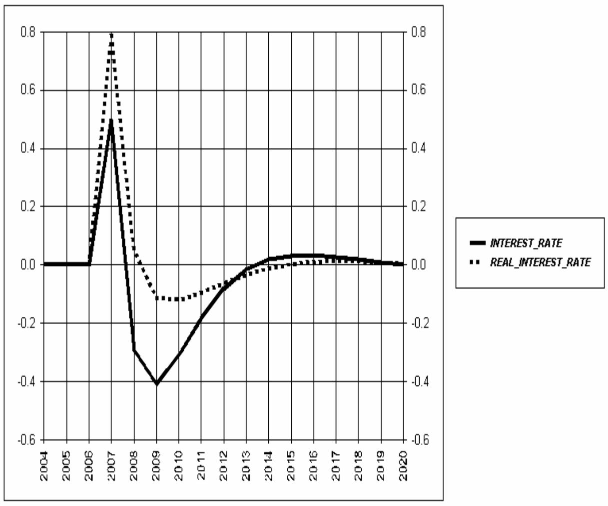


Figure 5.8.e: Net export and government consumption

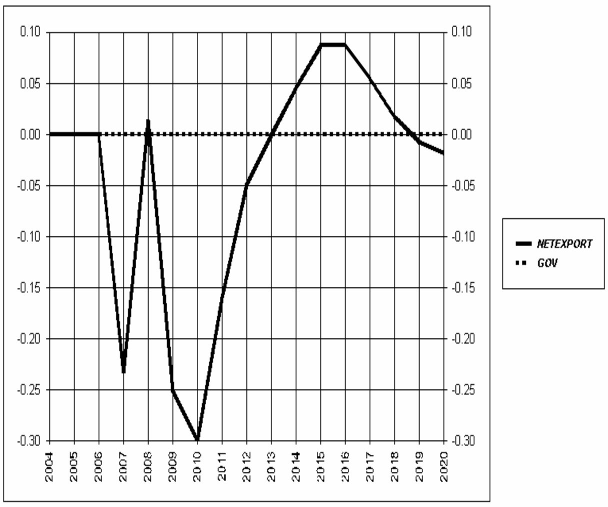


Figure 5.8.f: Private consumption and investment

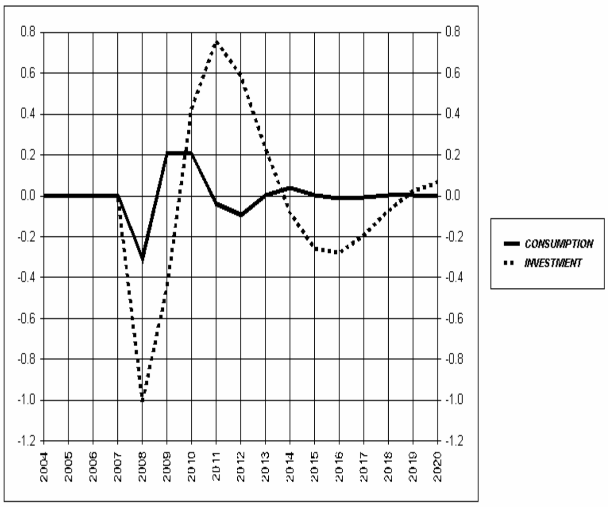


Figure 5.8: (continued) A shock to the interest rate
Responses in %

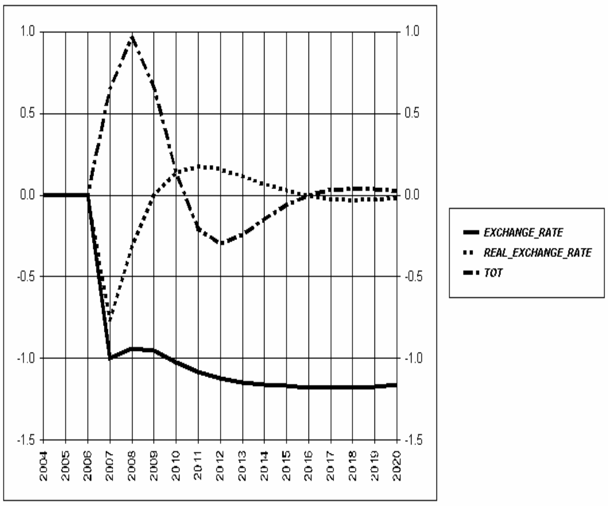
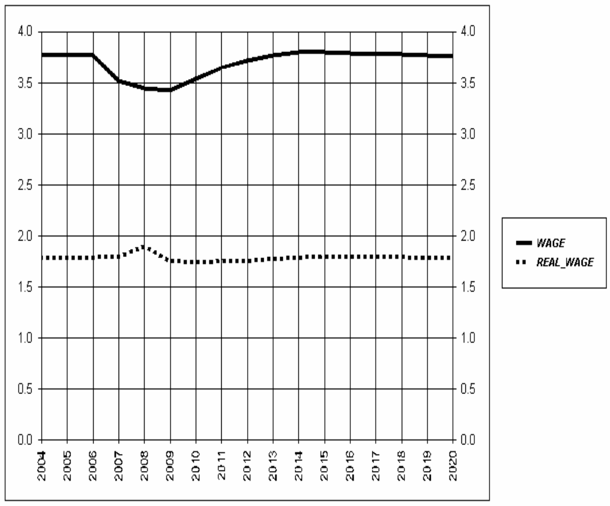
Figure 5.8.g: Real and nominal exchange rates, terms of trade	Figure 5.8.h: Nominal and real wage
	

Figure 5.9: A temporary labour supply shock
Responses in %

Figure 5.9.a: Labour supply

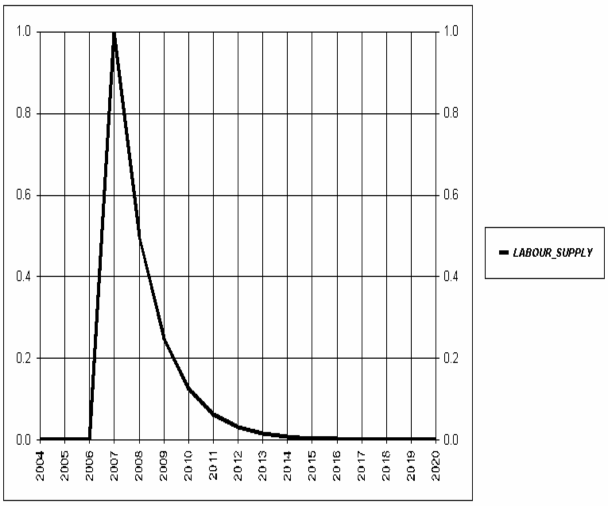


Figure 5.9.b: Output gap and inflation (deviation target)

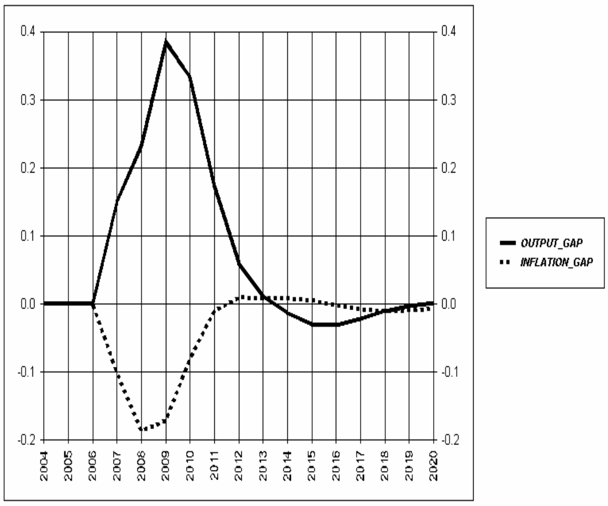


Figure 5.9.c: Labour market

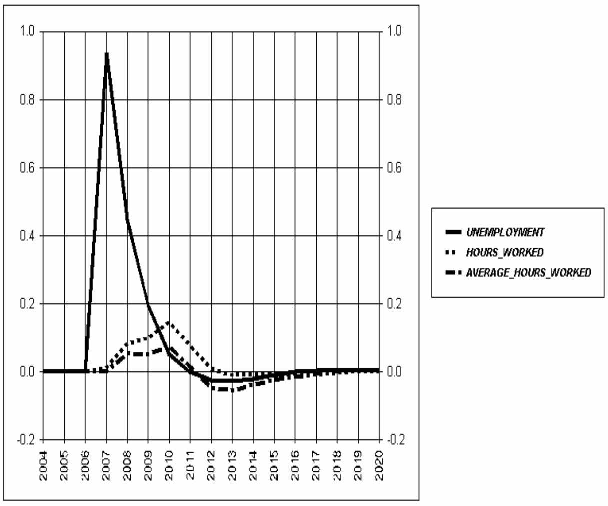


Figure 5.9.d: Nominal and real interest rate

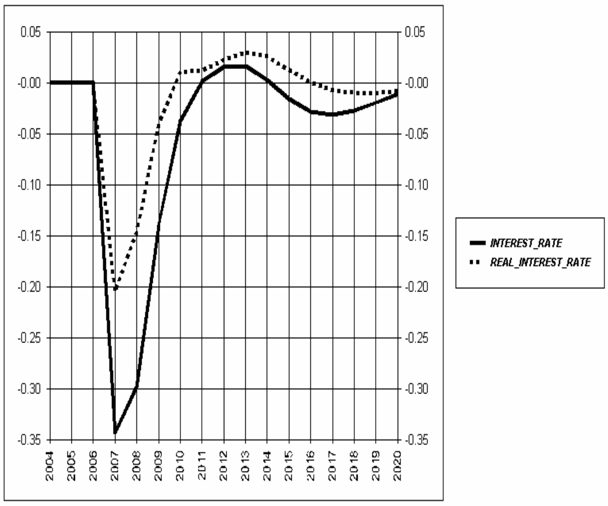


Figure 5.9.e: Net export and government consumption

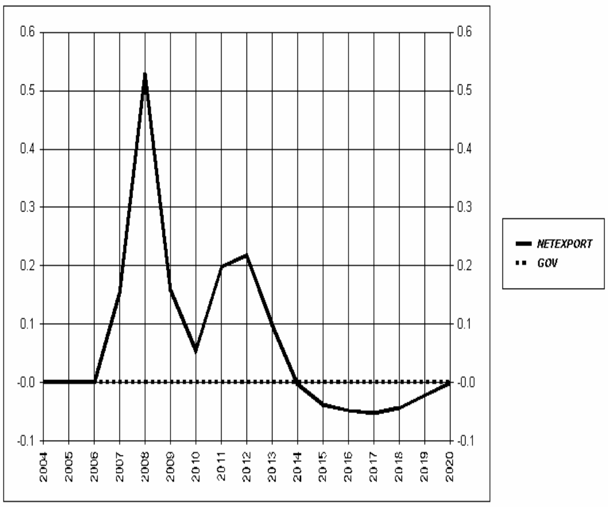


Figure 5.9.f: Private consumption and investment

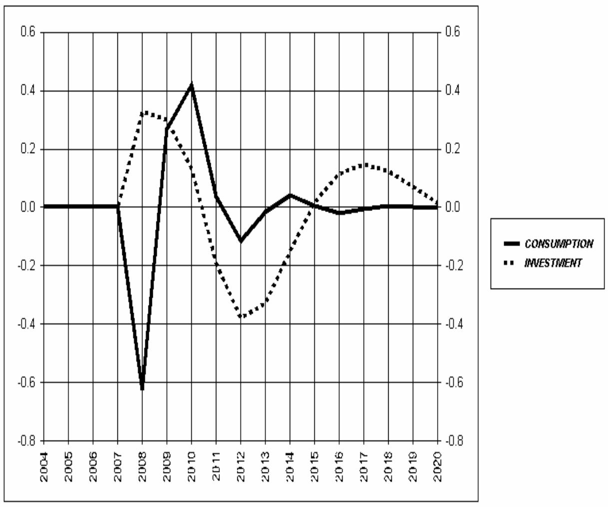


Figure 5.9: (continued) A temporary labour supply shock
Responses in %

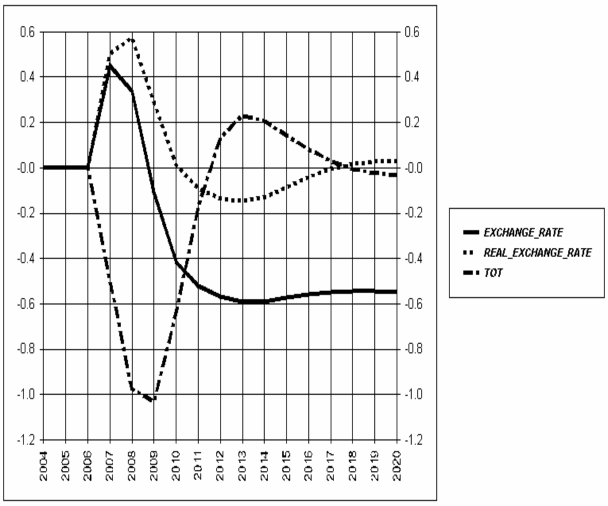
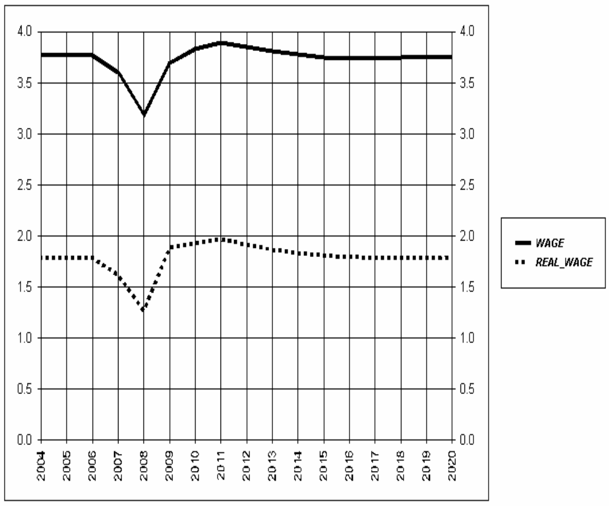
Figure 5.9.g: Real and nominal exchange rates, terms of trade	Figure 5.9.h: Nominal and real wage
	

Figure 5.10: A 1% shock to private consumption
Responses in %

Figure 5.10.a: Private consumption

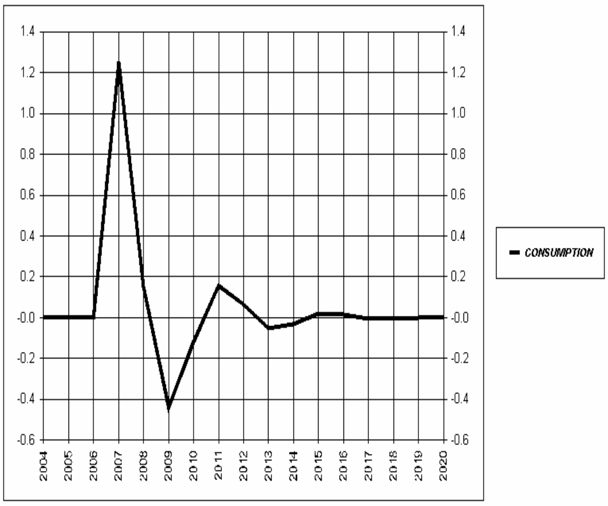


Figure 5.10.b: Output gap and inflation (deviation target)

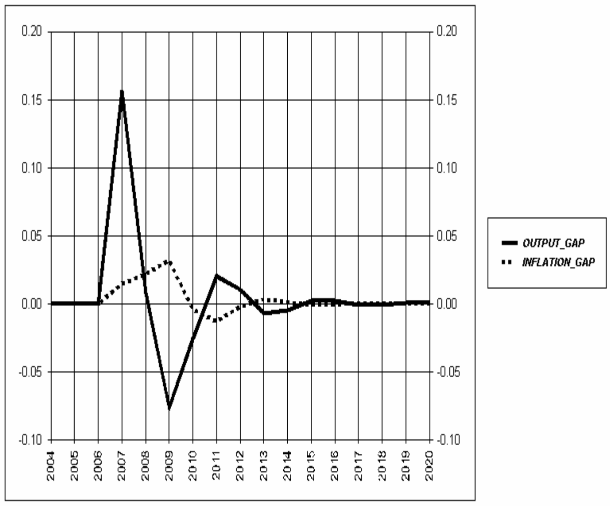


Figure 5.10.c: Labour market

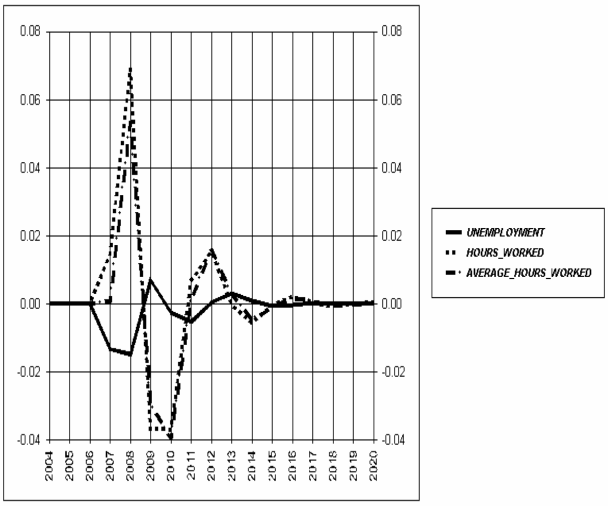


Figure 5.10.d: Nominal and real interest rate

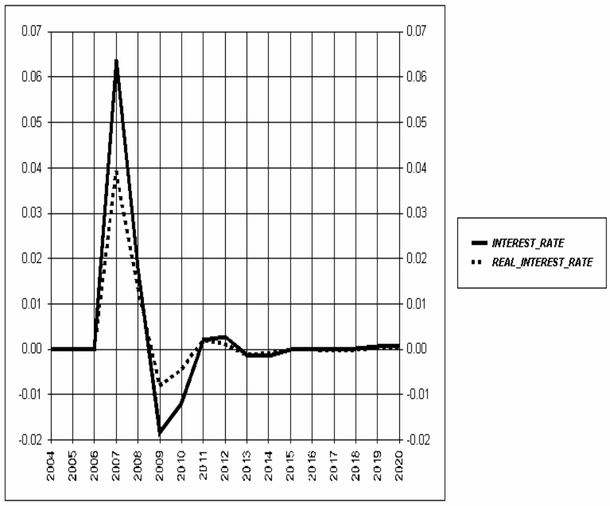


Figure 5.10.e: Net export and government consumption

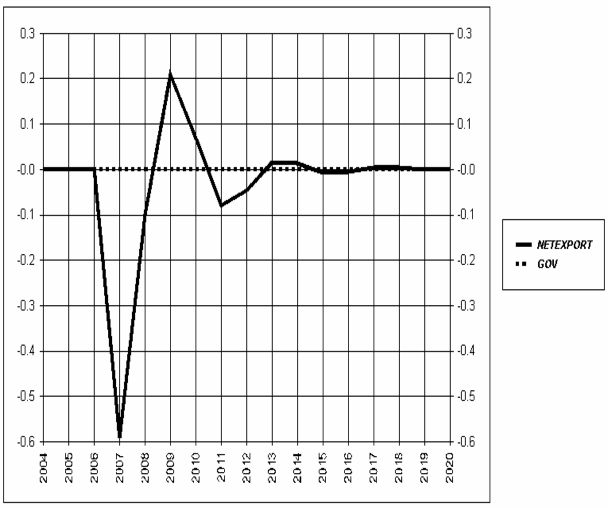


Figure 5.10.f: Private consumption and investment

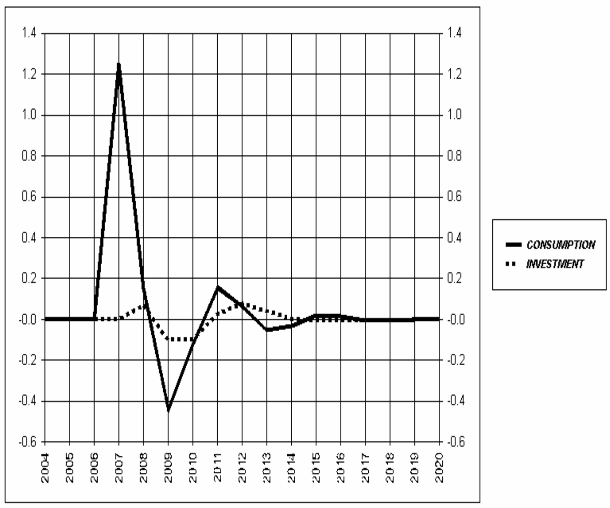


Figure 5.10: (continued) A 1% shock to private consumption
Responses in %

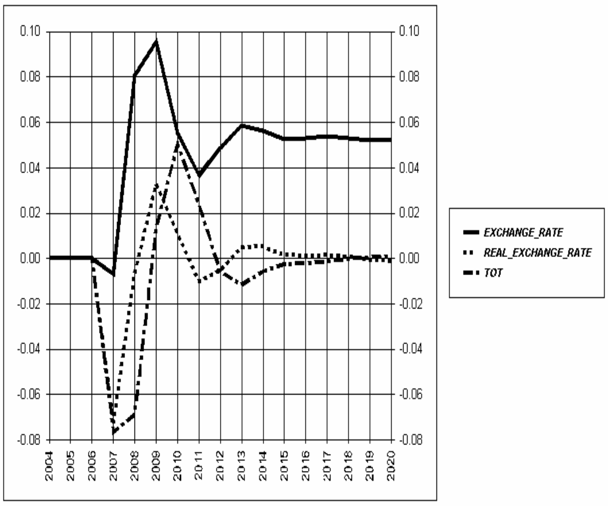
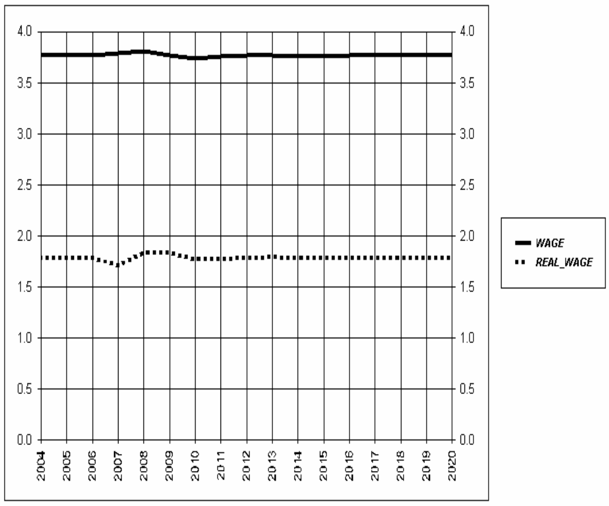
Figure 5.10.g: Real and nominal exchange rates, terms of trade	Figure 5.10.h: Nominal and real wage
	

Table 5.1: Impact on production, output gap and inflation after permanent shocks
Percent

Shock	Production*	Output gap after 1 year	Inflation gap after 1 year	Output gap Max.effect**	Inflation Max.effect**
Productivity	1.25	–0.06	–0.09	0.27(3)	–0.15(2)
Labour supply	1.30	–0.22	–0.20	–0.22(1)	–0.25(2)
Bargain strength of workers	0.44	–0.17	–0.08	–0.17(1)	–0.15(3)
Government debt to GDP	0.37	0.08	–0.07	0.20(2)	–0.24(2)
Inflation target**	0.00	–0.18	–0.40	–0.38(2)	–1.20(4)

* The long-run effect on the production level
**The figure in parentheses indicate after how many years the maximum effect occur
** The inflation gap is compared to the old inflation target

Table 5.2: Impact on output gap and inflation after transitory shocks
Percent

Shock	Output gap after 1 year	Inflation gap after 1 year	Output gap Max. effect**	Inflation Max. effect**
Nominal wage	–0.08	0.43	–0.12(3)	0.43(1)
Exchange rate	0.24	0.14	0.24(1)	0.18(3)
Interest rate	–0.23	0.19	–0.41(2)	–0.27(3)
Labour supply	0.15	–0.10	0.38(3)	–0.19(2)
Private consumption	0.15	0.02	0.15(1)	0.08(3)

** The figure in parentheses indicate after how many years the maximum effect occurs

9 Appendix B: Definition of variables and parameters

In this appendix, variables and parameters are listed in alphabetical order. All volumes are in real terms. Superscript "*E*", "*SS*" and "*FP*" in the text concerns expectations, steady state and flex-price values, respectively, and are not repeated here.

9.1 Variables

A^F	Foreign labour-augmenting productivity.
A^{GRF}	Growth factor, total factor productivity, A_t^H / A_{t-1}^H.
A^H	Labour-augmenting productivity.
B^P	Bonds hold by the private sector.
B^G	Bonds hold by the government sector.
BB	Budget surplus, government.
BW	Bond wealth.
C	Private consumption.
C^F	Foreign produced intermediate private consumption goods.
C^H	Home produced intermediate private consumption goods.
CA	Current account.
$CORE$	Core inflation.
$COST$	Installation cost of capital plus vacancy costs.
D	Dividends.
f_K	Marginal product of capital.
f_L	Marginal product of labour.
G	Government consumption.
G^H	Government consumption of home goods.
HW	Human wealth.

I Total investments.

I^F Foreign produced intermediate investment goods.

I^G Government investments.

I^H Home produced intermediate investment goods.

I^P Private sector investments.

IC Investment cost in the private sector (I^P plus installation cost of capital).

K Capital stock, whole economy.

K^G Capital stock, government sector.

K^P Capital stock, private sector.

L Employment, whole economy.

L^G Employment, government sector.

L^P Employment, private sector.

LH Hours worked, whole economy.

LH^G Hours worked, government sector.

LH^P Hours worked, private sector.

LS Labour supply in persons.

M Imports.

MA Number of successful matches at the labour market.

MC Marginal cost.

MPC Marginal propensity to consume.

MPL Marginal product of labour.

NER Nominal exchange rate (depreciation implies an increase in NER).

NR Nominal interest rate (repo rate).

NR^F Foreign nominal interest rate.

NW	Nominal hourly wage in the private sector.
OS	Operating surplus.
P	Deflator, GDP.
P^{C}	Deflator, private consumption.
PB^{C}	Deflator, private consumption (basic prices).
P^{F}	GDP deflator, rest of the world.
P^{G}	Deflator, government consumption.
PB^{G}	Deflator, government consumption (basic prices).
P^{I}	Deflator, investments.
P^{M}	Deflator, imports.
$P^{M,F}$	Deflator, foreign goods in foreign currency, exported to Sweden.
P^{X}	Deflator, exports.
P^{C_P}	Relative price, P^{C} / P.
P^{M_P}	Relative price, P^{M} / P.
PB^{YG}	Deflator, government production (basic prices).
PB^{YP}	Deflator, private production (basic prices).
PB^{YPG}	Deflator, private gross production (basic prices).
PB^{YPG_P}	Relative price, P^{YPG} / P.
π^{*}	Inflation target of the central bank.
$\overline{\pi}^{MC}$	Trend change in marginal costs, MC.
$\overline{\pi}^{NW}$	Trend change in nominal wages, NW.
$\overline{\pi}^{YPG}$	Trend inflation rate, PB^{YPG}.
q	Tobin's q.
R	Real interest rate.
R^{F}	Foreign real interest rate.

RER	Real exchange rate (depreciation implies an increase in RER).
SW	Stock wealth.
TR	Transfers.
TW	Total wealth.
U_LS	Unemployment rate
U	Number of unemployed persons.
UH	Number of unemployed hours.
U^i	Utility of generation i.
ULC	Unit labour cost.
V	Value of shares.
VA	Number of vacancies.
VC	Cost of hiring labour (vacancy costs).
W^P	Real hourly wage, private sector.
X	Exports.
X^F	Foreign produced intermediate export goods.
X^H	Home produced intermediate export goods.
X^{NET}	Net exports.
Y	Gross domestic product (GDP).
Y^G	Production, government sector.
Y^P	Production, private sector.
Y^{PG}	Gross production, private sector (production plus installation and vacancy costs).
YD	Disposible income.

9.2 Parameters

α	Share (in percentage) of government hourly wage to the hourly wage in the private sector.
β	Subjective discount factor.
γ	Vacancy cost parameter.
δ	Depreciation rate of capital.
ζ	Replacement ratio in the unemployment insurance.
η_C	Productivity parameter in the production function of consumption goods.
η^{LH1}	Elasticity w.r.t. backward looking expectations of demand for hours.
η^{LH2}	Elasticity w.r.t. expected production in t when determining demand for hours in the private sector.
η^{LH3}	Elasticity w.r.t. expected production in t+1 when determining demand for hours in the private sector.
η^{MA}	Elasticity w.r.t. unemployment in the matching function.
η^{NR1}	Elasticity w.r.t. the inflation target in the policy reaction function of the central bank.
η^{NR2}	Elasticity w.r.t. lagged interest rate in the policy reaction function of the central bank.
η^{PBC}	Elasticity w.r.t. core inflation of the private consumption deflator (basic prices).
η^{PBL1}	Elasticity w.r.t. the forward looking part (dated in t) of private gross production deflator.
η^{PBL2}	Elasticity w.r.t. the forward looking part (dated in t+1) of private gross production deflator.
η^{PBYPG1}	Elasticity w.r.t. the equilibrium relative price in the forward looking part of private gross production deflator.
η^{PBYPG2}	Elasticity w.r.t. backward looking expectations of private gross production deflator.

η^{PBYPG3}	Elasticity w.r.t. the output gap of private gross production deflator.
η^{P^E}	Elasticity w.r.t. backward looking part of the GDP-deflator expectation.
η^{PM1}	Elasticity w.r.t. the backward looking part of the import deflator.
η^{PM2}	Elasticity w.r.t. the output gap of the import deflator.
η^{YPG}	Elasticity w.r.t. capital in the production function of the private sector.
η^{RER1}	Elasticity w.r.t. terms of trade in the real exchange rate equation.
η^{RER2}	Elasticity w.r.t. productivity level (relative the rest of the world) in the real exchange rate equation.
η^{RER3}	Elasticity w.r.t. net foreign assets in the real exchange rate equation.
ϑ	Labour market tightness, VA/U.
$J(\vartheta)$	Rate at which vacancies are filled, MA/VA.
λ	Installation cost (of capital) parameter.
Λ^V	The value of a vacancy for a firm.
Λ^J	The value of a filled vacancy for a firm.
Λ^U	The value of unemployment for an unemployed.
Λ^E	The value of a job for an unemployed.
μ	Relative bargaining strength of workers.
ν	Growth rate of labour supply.
π	Probability of survival.
Π	Profits.
s	Separation rate.
τ^C	Indirect tax, private consumption.
τ^E	Social security contributions by the firm (zero presently in KIMOD, incorporated in the labour income tax, τ^W, instead).
τ^G	Indirect tax, government consumption.
τ^W	Tax on labour income.

τ^{Y}	Tax wedge between real GDP at market and basic prices, respectively.
τ^{PY}	Tax wedge between nominal GDP at market and basic prices, respectively.
ϕ^{i}	Share of total stocks hold by generation *i*.
χ	Elasticity in the labour income tax equation.
Ω	Nash product.
Z^{MA}	Constant i in the matching function ("matchning efficiency").
Z^{PM_P}	Constant in the real exchange rate equation.

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