The Labor Market in KIMOD

Johan Lindén

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NIER
Kungsgatan 12-14
Box 3116
SE-103 62 Stockholm
Sweden
Phone: 46-8-453 59 00, Fax: 46-8-453 59 80
E-mail: ki@konj.se, Home page: www.konj.se

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Johan Lindén *

Abstract

This is a description of the labor market sector in the dynamic medium term macroeconomic model KIMOD developed at the National Institute of Economic Research (NIER).

Unemployment is caused by matching inefficiencies of the type described by C. Pissarides in *Equilibrium Unemployment*, 2000. Unemployed workers and firms with vacant jobs are engaged in costly search for a profitable match. Total hirings from unemployment into employment depend on the number of unemployed workers and vacant jobs. Flows into unemployment come from new entrants into the labor force and from exogenous separation of matched job – worker pairs.

Wages are set in individual negotiations between the worker and the firm in a match, according to the Nash bargaining solution. Some inertia in real wages follows from unemployment benefits being indexed to the previous period's market wage.

These features lead to an unemployment rate which adjusts with some inertia towards a long run equilibrium level. Turnover costs provide some incentives for labor hoarding by firms during temporary downturns. The effects on the economy from variations in hours worked due to variations in the labor force are distinct from those due to variations in average working time.

The model is used to estimate the equilibrium unemployment level in Sweden from Swedish labor market data on unemployment and vacancies.

Key words: labor market, matching, modeling, search, unemployment, wage bargaining

JEL classification: C51, C78, E24, E27, J41, J64

* National Institute of Economic Research, e-mail: johan.linden@konj.se, and Mälardalens högskola, e-mail: johan.linden@mdh.se

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Introduction

The National Institute of Economic Research (NIER) develops and maintains the medium term macroeconomic model KIMOD. This paper describes the labor market sector of that model.

KIMOD is a dynamic general equilibrium macroeconomic model of the Swedish economy to be used for aggregate economic-policy analysis and medium-range macroeconomic scenarios. The model was used for the first time in the NIER's ongoing activities during the work on the Wage Formation Report for 2003.

The model is highly aggregated, with a business sector producing one private good, a public sector producing a public good, a foreign sector, and a central bank with an inflation target. All firms in the model are identical, as are all households. In addition, the government sector is consolidated and thus not separated into central, regional and local government. The model is based on microeconomic foundations in the sense that firms and households make optimal decisions on output and consumption, respectively, given rational expectations about the behavior of other agents and about the future development of the model as a whole.

It is dynamic, both in the sense that investments and savings during a period affect future possibilities for output and consumption, respectively, and in that all agents take this into account in their decisions. Time is divided into discrete periods of one year. The projections generated by the model are thus time series with yearly frequency, so the econometric equations of the model are estimated on the basis of annual data. The National Accounts are the preferred data source for initial data and for estimating parameters.

Prices in each period are set so that supply is equal to demand on all markets except on the labor market. In the long run, the model approaches a steady state path that is independent of the initial state of the economy. In this steady state, the economy is on a balanced growth path with a constant relative growth rate.

KIMOD is intended for use in macroeconomic analysis and in medium range scenarios, with a time horizon of two to six years. For other time horizons, the lower limit of usability for the model results from the fact that the length of periods is set at one year. This means that there are no seasonal dynamics, and that data for parts of the current year cannot be used as the initial state for the model. For long range purposes, the usability of the model is limited by insufficient modeling of demography and other structural developments.

This report gives a thorough treatment of the labor market in KIMOD. It is modeled as a matching market, where unemployed workers and firms with vacant jobs are engaged in costly and time consuming search for a profitable match. Wages are set in individual negotiations between matched pairs of employers and employees, with some bargaining power on both sides.

Unemployment arises both from search related friction and from imperfect competition in wage formation. While friction by itself would create unemployment, the wage setting mechanism also contributes to the determination of the unemployment level.

Growth in the labor force and labor productivity is exogenous. Firms and workers maximize their expected profits and utility, discounted by the exogenous interest rate. Workers gain utility from consumption only.

The labor market theory builds largely on Pissarides (2000). The overlapping generations model of the households is based on Blanchard (1985) and on the discrete time treatment in Frenkel and Razin (1992).

Growth in labor force and productivity

Growth in productivity and the labor force are the exogenous sources of long run growth in other variables. Production and capital follow their combined growth rate while real wages grow with productivity.

The number of workers in the labor force, which is the entire population of this model, is an exogenous time series N_t growing at rate n_t between period *t-1* and *t*. In steady state the growth rate is constant.

(1) $$N_t = n_t N_{t-1}$$

In the production function (12) bellow, the labor input is scaled by a productivity factor H_t, which grows at a constant rate *h*.

(2) $$H_{t+1} = hH_t$$

Employment dynamics

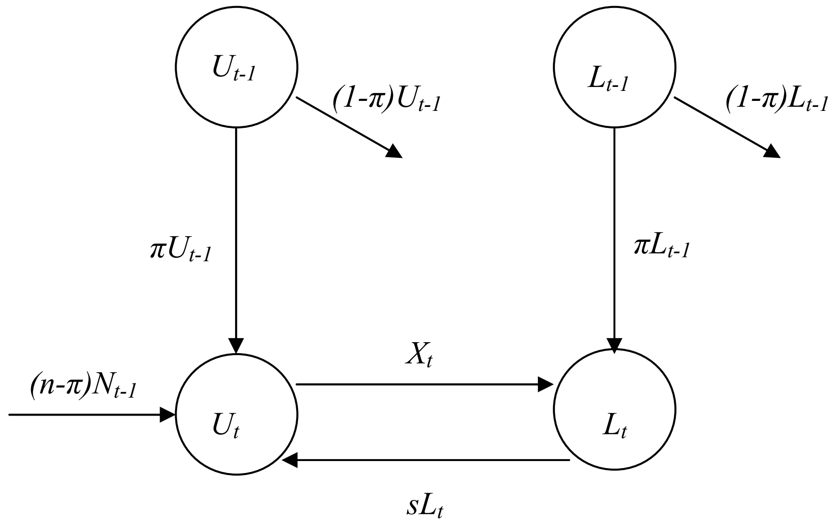
Over a period *t*, the labor force N_t consists of the number of employed workers L_t and unemployed workers U_t.

(3) $$L_t + U_t = N_t$$

Figure 1 illustrates the flows of labor into and out of the labor force, and between employment and unemployment, between period *t-1* and *t*. Between any period *t-1* and *t*, πN_{t-1} survive into the next period, $(1-\pi)N_{t-1}$ die (or retire out of the labor force) proportionally out of employment and unemployment, while $(n_t-\pi)N_{t-1}$ enter the labor force as unemployed. The net growth rate of the labor force is thus a factor n_t.

Within each period, jobs expire, because of lay-offs or quits, at an exogenous separation rate *s*, resulting in a flow sL_t of workers from employment to unemployment. In the other direction, there is a flow X_t of workers hired into employment. Since all jobs are identical, employed workers have no incentive to search for other jobs. Hence labor flows between different kinds of employment without intermediate unemployment spells are not modeled.

Figure 1



Hirings X_t depend on the number of unemployed workers U_t and open vacancies V_t according to a matching function x, increasing in both arguments and linearly homogenous.

(4) $$X_t = x(U_t, V_t)$$

KIMOD has a matching function of the Cobb-Douglas form:

$$X_t = x_0 U_t^{\eta} V_t^{1-\eta}$$

Define labor market tightness ϑ_t as:

(5) $$\vartheta_t = \frac{V_t}{U_t}$$

Define $q(\vartheta_t)$ as the rate at which vacancies are filled. Using (5), q can be expressed in terms of the matching function (4), showing that q is decreasing in ϑ_t.

(6) $$q(\vartheta_t) = \frac{X_t}{V_t} = x(\frac{1}{\vartheta_t}, 1)$$

This also means that the rate at which unemployed workers find jobs is:

(7) $$\frac{X_t}{U_t} = \vartheta_t q(\vartheta_t) = x(1, \vartheta_t)$$

The unemployment and vacancy rates are defined as:

$$u_t = \frac{U_t}{N_t} \qquad \text{and} \qquad v_t = \frac{V_t}{N_t}$$

The flow of workers into employment and unemployment is given by either of the following two equations.

(8) $$L_t = \pi L_{t-1} - sL_t + q(\vartheta_t)V_t$$

(9) $$U_t = \pi U_{t-1} + (n_t - \pi)N_{t-1} + sL_t - q(\vartheta_t)V_t$$

The equations (8) and (9) are equivalent formulations of the employment dynamics, since their sum is the population dynamics (1). Equation (8) is used further on as a restriction (16) on the individual firm's employment decision. The dynamics of the employment rate follow from (9) using the definition (3) and the growth rate (1) of the labor force.

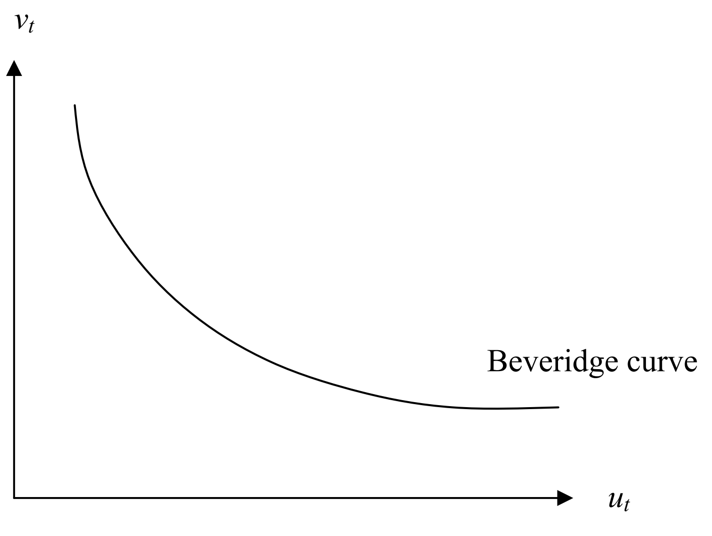
(10) $$u_t = \frac{n_t(1+s) - \pi(1-u_{t-1})}{n_t(1+s+\vartheta_t q(\vartheta_t))}$$

In steady state, unemployment and the labor force growth rate is constant, so the steady state version of (10) is:

(11) $$u_t = \frac{n_t(1+s) - \pi}{n_t(1+s+\vartheta_t q(\vartheta_t)) - \pi}$$

The steady state relation (11) is a static negative relationship between unemployment and vacancies. It is often referred to as the Beveridge curve when plotted in a diagram with unemployment and vacancies on the horizontal and vertical axis respectively, as in *figure 2* bellow.

Figure 2



The position of the economy along the Beveridge curve indicate how demand and supply conditions on the labor market vary with the business cycle, with low unemployment and high vacancy rate, and hence high ϑ_t, in a tight labor market, and vice versa in a recession. The position of the Beveridge curve is considered to depend mainly on structural parameters determining the functioning of the labor market in the long run. However, the separation rate s, though constant in this model, would in a richer model increase in downturns and thus shift the Beveridge curve outward.

The equation (10) for unemployment dynamics, which might be called the dynamic Beveridge curve, describes the adjustment paths of the labor market in response to shocks. Typically these paths describe counter clockwise loops around the static Beveridge curve, converging to the equilibrium point.

Firms

Each firm i produces its output Y_t^i using its capital stock K_t^i and labor force $\varepsilon_t L_t^i$, equal to the number of its employed workers L_t^i times the number of hours ε_t worked per worker and year, which is the same for all firms. It uses a technology given by the production function F, increasing in both arguments and linearly homogenous.

$$Y_t^i = F(K_t^i, H_t \varepsilon_t L_t^i) \tag{12}$$

H_t is the exogenous labor productivity factor, growing at a constant rate according to (2). The workers are homogenous, so ε_t and H_t are the same for all firms. Working hours ε_t are either constant or enter as an exogenous time series. Typical functional forms for the production function are the Cobb-Douglas or the CES forms. Presently, KIMOD has a production function of the Cobb-Douglas form:

$$Y_t^i = (K_t^i)^\alpha (H_t \varepsilon_t L_t^i)^{1-\alpha}$$

Firms are assumed to be large enough for their number of new hirings each period to be equal to its expected value, while still small enough to behave competitively on the product and capital markets. Thus firms take as given the price P_t^p per unit of their produced good and the price P_t^I per unit of the investment good in time period t, and the nominal interest rate R_t from period t to $t+1$.

Wages are set in negotiations, described bellow, between the individual firm and worker in a match. This means that the negotiated wage is specific to the match, but since all firms and workers are alike, the outcome of all negotiations will be the same wage level W_t. This is an hourly nominal wage which is multiplied by the number of hours worked ε_t per worker and year to obtain a worker's yearly nominal wage $\varepsilon_t W_t$. Wage negotiations precede the firms' other decisions each period, so that the negotiated wage for the current period is given in the firms' optimization problem. After firms and workers have agreed on a wage, each firm chooses its investment I_t^i and its number of open vacancies V_t^i to maximize the present value of its profits, taking the wage W_t as given.

Each period t, the firm earns revenue $P_t^p Y_t^i$ from sales of its product. It pays wages $\varepsilon_t W_t L_t^i$ to the workers and an employer's wage tax $\tau_t^e \varepsilon_t W_t L_t^i$ to the government. It also invests in new capital at a cost $P_t^I I_t^i (1 + \psi I_t^i / K_t^i)$ proportional to the price of the investment good and increasing quadratically in the investment level. Finally, it recruits new workers by opening vacancies at a cost $\gamma\ (1+\tau_t^e)\ \varepsilon_t\ W_t\ V_t^i$ proportional to the current wage, reflecting that recruitment is a labor intensive task. Firms do not accumulate financial assets, so the remains are paid out as dividends D_t^i to shareholders according to (13).

(13) $$D_t^i = P_t^y Y_t^i - P_t^I I_t^i (1+\psi \frac{I_t^i}{K_t^i}) - (1+\tau_t^e)\varepsilon_t W_t L_t^i - \gamma(1+\tau_t^e)\varepsilon_t W_t V_t^i$$

Given the series of nominal interest rates R_t, the discount factor ρ_t^s from period s to an earlier period t is compounded as:

$$\rho_t^t = 1 \qquad \text{and} \qquad \rho_t^{s+1} = \frac{1}{1+R_s}\rho_t^s$$

The value of profits maximized by the firm is the present value of its dividends, given by:

(14) $$\Pi_t^i = \sum_{s=t}^{\infty} \rho_t^s D_s^i$$

The firm index i marks the firm specific variables, such as Y_t^i, K_t^i, L_t^i, I_t^i, V_t^i D_t^i and Π_t^i in contrast to aggregate economic variables, such as H_t, ϑ_t, and ε_t, which are beyond the individual firm's control. Summing the firm specific variables over the index i yields the corresponding aggregate variables Y_t, K_t, L_t, I_t, V_t, D_t and Π_t. The distinction is especially important for variables which enter into the firm's decision problem, because if, for example, it could control not only its own vacancies V_t^i but all vacancies V_t in the economy, it would have a monopsony on the labor market and could in effect decide the unemployment rate. However, with identical firms, aggregate variables will equal the firm specific ones times the number of firms.

The firm's budget restriction (13) holds for the corresponding aggregate variables also. The vacancy costs reduce the output given by the production function, so the value added in the private sector VA_t is:

$$VA_t = P_t^y Y_t - \gamma(1+\tau_t^e)\varepsilon_t W_t V_t$$

The firm accumulates capital according to the following dynamics, with a constant depreciation rate δ.

(15) $$K_{t+1}^i = (1-\delta)K_t^i + I_t^i$$

By opening vacancies, a firm controls its labor force according to the following employment dynamics, which is a firm specific version of (8). The vacancy filling rate $q(\vartheta_t)$ depends on aggregate tightness ϑ_t, which is given in the firm's optimization problem.

(16) $$L_t^i = \pi L_{t-1}^i - sL_t^i + q(\vartheta_t)V_t^i$$

The firm's decision problem is to choose investments I_t^i and vacancies V_t^i each period to maximize its profits (14) subject to the technology (12), the budget (13), and the laws of motion (15) and (16) for capital and labor respectively. The following first order conditions determine the individual firm's optimal investments and vacancies.

(17)
$$\frac{\partial \Pi_t^i}{\partial K_{t+1}^i} = \frac{1}{1+R_t}(P_{t+1}^y F_K^{'}(K_{t+1}^i, H_{t+1}\varepsilon_{t+1}L_{t+1}^i) + P_{t+1}^I((1-\delta)(1+2\psi\frac{I_{t+1}^i}{K_{t+1}^i}) + \psi(\frac{I_{t+1}^i}{K_{t+1}^i})^2)) - P_t^I(1+2\psi\frac{I_t^i}{K_t^i}) = 0$$

(18)
$$\frac{\partial \Pi_t^i}{\partial L_t^i} = P_t^y F_L^{'}(K_t^i, H_t\varepsilon_t L_t^i)H_t - (1+\tau_t^e)\varepsilon_t W_t - \frac{(1+s)\gamma(1+\tau_t^e)\varepsilon_t W_t}{q(\vartheta_t)} + \frac{1}{1+R_t}\frac{\pi\gamma(1+\tau_{t+1}^e)\varepsilon_{t+1}W_{t+1}}{q(\vartheta_{t+1})} = 0$$

Since the same first order conditions (17) and (18) holds for all firms, the firm index i in these equations can be dropped to obtain the dynamic equations for aggregate demand for capital and labor. Some simplifying definitions help to make these equations more readable. First introduce the following abbreviations for the partial derivatives of the production function in period t:

(19)
$$F_{K,t}^{'} = F_K^{'}(K_t, H_t\varepsilon_t L_t)$$
$$F_{L,t}^{'} = F_L^{'}(K_t, H_t\varepsilon_t L_t)$$

The real wage cost in period t is denoted:

(20)
$$w_t = \frac{(1+\tau_t^e)\varepsilon_t W_t}{P_t^y}$$

Define the real interest rate r_t in period t as:

(21)
$$(1+r_t) = (1+R_t)\frac{P_t^y}{P_{t+1}^y}$$

Using the definitions (19), (20), and (21), the first order conditions (17) and (18) can now be written as capital and labor demand in real and aggregate form:

(22)
$$F_{K,t+1}^{'} + \frac{P_{t+1}^I}{P_{t+1}^y}((1-\delta)(1+2\psi\frac{I_{t+1}}{K_{t+1}}) + \psi(\frac{I_{t+1}}{K_{t+1}})^2) = (1+r_t)\frac{P_t^I}{P_t^y}(1+2\psi\frac{I_t}{K_t})$$

(23)
$$F_{L,t}^{'}H_t - w_t - \frac{(1+s)\gamma w_t}{q(\vartheta_t)} = -\frac{1}{1+r_t}\frac{\pi\gamma w_{t+1}}{q(\vartheta_{t+1})}$$

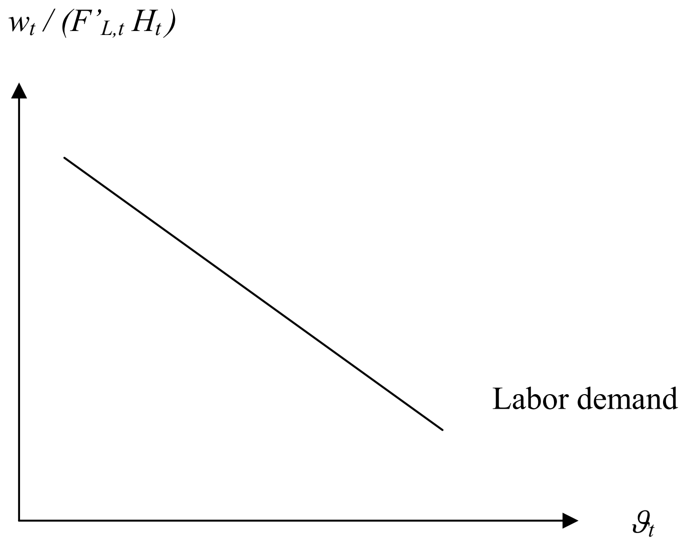
In steady state, the marginal productivity of capital is constant, as is the investments to capital ratio, the relative price of investment goods to output goods, and labor market tightness. The marginal productivity of labor and the real wage both grow at the rate h. Thus, the steady state versions of these two factor demand equations are:

$$(24) \qquad F'_{K,t} = \frac{P_t^I}{P_t^y}((\delta + r_t)(1 + 2\psi \frac{I_t}{K_t}) - \psi (\frac{I_t}{K_t})^2)$$

$$(25) \qquad F'_{L,t} H_t = (1 + (1 + s - \frac{\pi h}{(1+r_t)})\frac{\gamma}{q(\vartheta_t)})w_t$$

The steady state labor demand equation (25) is shown below in *figure 3* as a negative relationship between the labor market tightness ϑ_t and the real marginal wage costs per unit of the produced good $w_t / F'_{L,t} H$.

Figure 3



Wage determination

The matching market described so far may be combined with several alternative wage setting mechanisms. Candidates for wage setting agents are central or local unions, firms, perhaps taking efficiency wage effects into account, or various forms of bargaining between these agents.

The wage setting mechanism assumed here is that individual wages for each period are set in local negotiations between the worker and the firm in a match in the beginning of the period. The outcome of the negotiations is given by the asymmetric Nash bargaining solution, with bargaining power β for the worker and $1-\beta$ for the firm. The rationale behind the Nash bargaining solution is given in Nash (1950). Non-cooperative game theoretic foundations along with possible interpretations of the bargaining power parameter are found in Binmore et. al. (1986).

The outcome of the wage negotiations depends on how firms and workers value all possible outcomes, including agreements on different wage levels as well as breakdown of the negotiations. A worker compares his utility as employed at a certain wage to that of being unemployed, while a firm compares its marginal profits from employing an additional worker at a certain wage to the costs and benefits of a vacancy.

The value functions below give these values of the parties in case of successful or unsuccessful bargaining, and show how, in the former case, they depend on the individually negotiated wage. Here, $\Lambda^V{}_t$ $\Lambda^{J,i}{}_t$ $\Lambda^U{}_t$ and $\Lambda^{E,i}{}_t$ are respectively the value of vacancies and filled jobs to the firms, and of unemployment and employment to the worker. The super index i on $\Lambda^{J,i}{}_t$ and $\Lambda^{E,i}{}_t$ indicate that these values depend on the specific wage W_t^i considered in negotiations within a certain firm-worker pair, which must be distinguished, in the reasoning around the bargaining situation, from the equilibrium outcome W_t of all negotiations.

The Nash bargaining solution requires that the parties' object functions be specified as their von Neumann-Morgenstern expected utility. Firms are taken to be risk neutral profit maximizers of profits as specified by (14). In the present state of the model, workers are assumed to derive utility only from consumption with a constant intertemporal marginal rate of substitution of one. Furthermore, since the model is deterministic, risk preferences may be arbitrarily assigned. Here, workers are assumed to be risk neutral, which is the simplest case. Under these circumstances, the worker's discounted lifetime income measures his money metric expected utility, and the value functions are computed as contributions to this measure.

Specifically, the firm's values $\Lambda^{J,i}{}_t$ and $\Lambda^V{}_t$ are the marginal contributions to its profits from having a worker employed on a job this period at a wage W_t^i or, respectively, to keep an vacancy open into the next period. Similarly, the worker's values $\Lambda^{E,i}{}_t$ and $\Lambda^U{}_t$ are the contributions to its lifetime income from holding a job this period at a wage W_t^i or, respectively, to be unemployed until the next period.

Employed workers work ε_t hours a year at the wage W_t, thus earning a yearly nominal wage of $\varepsilon_t\ W_t$. Unemployed workers receive unemployment transfers T_t. Workers pay a proportional wage tax τ_t^w on wages as well as unemployment benefits. In addition, firms pay an employer's tax τ_t^e on all wage costs.

A firm with a vacant job in period t thus bears the cost $\gamma\ (1 + \tau_t^e)\ \varepsilon_t\ W_t$ of the vacancy this period. Next period it expect to fill it with probability $q(\vartheta_t)$, according to (6), or else remain with the vacancy with probability $1\text{-}q(\vartheta_t)$.

(26) $$\Lambda_t^V = -\gamma \frac{(1+\tau_t^e)\varepsilon_t W_t}{P_t^y} + \frac{1}{1+r_t}((1-q(\vartheta_t))\Lambda_{t+1}^V + q(\vartheta_t)\Lambda_{t+1}^J)$$

A firm with a filled job earns the marginal product of labor this period, while also paying this period's negotiated wage plus taxes $(1 + \tau_t^e)\ \varepsilon_t\ W_t^i$. Next period it runs the risk to loose the employee by death with probability $1\text{-}\pi$, or by separation, with probability $\pi\ s$, or else remains with the filled job with probability $\pi(1\text{-}s)$. In the latter case, the wage will be renegotiated the next period, with that period's market wage as the expected outcome, i.e. an individual worker's wage next period is independent of this period's wage agreement.

$$\Lambda_t^{J,i} = F_L^{'}(K_t^i, H_t \varepsilon_t L_t^i) H_t - \frac{(1+\tau_t^e)\varepsilon_t W_t^i}{P_t^y} + \frac{1}{1+r_t}(\pi(1-s)\Lambda_{t+1}^{J,i} + (1-\pi(1-s))\Lambda_{t+1}^V) \tag{27}$$

Workers pay a proportional wage tax τ_t^w on wages and unemployment benefits. They buy a consumption good priced at P_t^c and pay a value added tax of τ_t^c on all consumption. They therefore discount future income with the consumer price real interest rate r_t^c defined by:

$$(1+r_t^c) = (1+R_t)\frac{(1+\tau_t^c)P_t^c}{(1+\tau_{t+1}^c)P_{t+1}^c} \tag{28}$$

An unemployed worker earns unemployment benefits after tax $(1 - \tau_t^w)\, T_t$ this period. He survives into the next period with probability π, and in that case expect to find a job at the going market wage with probability $\vartheta_t\, q(\vartheta_t)$, as given by (7), or to remain unemployed with probability $1\text{-}\vartheta_t\, q(\vartheta_t)$.

$$\Lambda_t^U = \frac{(1-\tau_t^w)T_t}{(1+\tau_t^c)P_t^c} + \frac{\pi}{1+r_t^c}((1-\vartheta_t q(\vartheta_t))\Lambda_{t+1}^U + \vartheta_t q(\vartheta_t)\Lambda_{t+1}^E) \tag{29}$$

Finally, an employed worker earns his negotiated wage $(1 - \tau_t^w)\, \varepsilon_t\, W_t^i$ this period. He survives into the next period with probability π, and in that case expects to be separated from his present employer into unemployment with probability s, or else to keep his job with probability $1\text{-}s$.

$$\Lambda_t^{E,i} = \frac{(1-\tau_t^w)\varepsilon_t W_t^i}{(1+\tau_t^c)P_t^c} + \frac{\pi}{1+r_t^c}((1-s)\Lambda_{t+1}^{E,i} + s\Lambda_{t+1}^U) \tag{30}$$

From (27) and (30) follow the derivatives of the value functions with respect to the individual wage.

$$\frac{\partial \Lambda_t^{J,i}}{\partial W_t^i} = -\frac{(1+\tau_t^e)\varepsilon_t}{P_t^y} \tag{31}$$

$$\frac{\partial \Lambda_t^{E,i}}{\partial W_t^i} = \frac{(1-\tau_t^w)\varepsilon_t}{(1+\tau_t^c)P_t^c} \tag{32}$$

Subtract (26) from (27) and (29) from (30) and drop the firm index i to obtain the respective value gains from a match for the representative firm $(\Lambda_t^J - \Lambda_t^V)$ and worker $(\Lambda_t^E - \Lambda_t^U)$. The requirement that both of these value gains be non-negative determines the parties' respective reservation wages, between which the wage agreement is bound to fall.

(33) $$(\Lambda_t^J - \Lambda_t^V) - \frac{1}{1+r_t}(\pi(1-s) - q(\vartheta_t))(\Lambda_{t+1}^J - \Lambda_{t+1}^V) = F_L^{'}(K_t, H_t\varepsilon_t L_t)H_t - (1-\gamma)\frac{(1+\tau_t^e)\varepsilon_t W_t}{P_t^y}$$

(34) $$(\Lambda_t^E - \Lambda_t^U) - \frac{\pi}{1+r_t^c}(1-s-\vartheta_t q(\vartheta_t))(\Lambda_{t+1}^E - \Lambda_{t+1}^U) = \frac{(1-\tau_t^w)(\varepsilon_t W_t - T_t)}{(1+\tau_t^c)P_t^c}$$

Profit maximization implies that firms open vacancies until their value is zero.

(35) $$\Lambda^V_t = 0$$

From (26) and (35) follows

(36) $$\Lambda_{t+1}^J = \frac{(1+r_t)\gamma(1+\tau_t^e)\varepsilon_t W_t}{q(\vartheta_t)P_t^y}$$

Now, the Nash bargaining solution requires that the negotiations settle on the wage that maximizes the following Nash product Ω^i_t.

(37) $$\max_{W_t^i} \Omega_t^i = (\Lambda_t^{E,i}(W_t^i) - \Lambda_t^U)^\beta (\Lambda_t^{J,i}(W_t^i) - \Lambda_t^V)^{1-\beta}$$

The first order condition for the Nash bargaining solution requires that the following condition holds for the wage bargaining outcome. The derivatives of the value functions are eliminated using (31) and (32). Since all firm-worker pairs are alike, the wage level will be the same throughout the whole economy, so the index *i* is now dropped.

(38) $$\frac{(1-\tau_t^w)}{(1+\tau_t^c)P_t^c}\beta(\Lambda_t^J - \Lambda_t^V) = \frac{(1+\tau_t^e)}{P_t^y}(1-\beta)(\Lambda_t^E - \Lambda_t^U)$$

The value functions Λ^J_t, Λ^V_t, Λ^E_t, Λ^U_t and the corresponding values for period $t+1$ can now be eliminated from the system (33), (34), (35), (36), and (38), yielding the wage setting equation (39). Some details of this derivation are shown in Appendix A.

(39) $$\frac{(1+\tau_t^e)\varepsilon_t W_t}{P_t^y} = \frac{\beta F_L^{'}(K_t, H_t\varepsilon_t L_t)H_t + (1-\beta)(1+\tau_t^e)T_t / P_t^y}{1 - \beta\gamma\pi((1-\tau_{t+1})\vartheta_t + \tau_{t+1}(1-s)/q(\vartheta_t))}$$

The value τ_{t+1} in the above equation measures the rate of increase from period t to $t+1$ in the wage tax wedge $(1 + \tau_t^e) / (1 - \tau_t^w)$. It is defined by:

(40) $$(1-\tau_t) = \frac{(1+\tau_{t-1}^e)/(1-\tau_{t-1}^w)}{(1+\tau_t^e)/(1-\tau_t^w)}$$

Of course future tax adjustments are relevant in the wage setting equation only to the extent that they are expected at the time of the negotiations.

In the absence of search externalities, the marginal product of labor and the unemployment benefits would be the firm's and worker's respective reservation wages. The wage cost corresponding to the negotiated wage is seen to be an average of these, weighted by the other's bargaining power, times a factor which adjusts for search costs, increasing in labor market tightness, strengthening the worker since the firm pays the vacancy cost. While the wage will never be set bellow the unemployment benefits, it may temporarily rise above the marginal product if the firm expects an increase in its labor demand in the near future, a kind of labor hoarding. In steady state, however, the wage will lie between the marginal product of labor and the unemployment benefits.

If unemployment income T_t stays constant while productivity grows it will become negligible in the long run. More plausibly, it would grow at the same average rate as the other income variables. If interpreted as unemployment benefits, it would reasonably be indexed to the wage level. Here it is assumed to be indexed to the market wage in the previous period.

(41) $$T_t = \lambda \, \varepsilon_{t-1} \, W_{t-1}$$

This implies that the real unemployment benefit and therefore the real wage will be decreasing in the inflation rate p_t, where

(42) $$(1+p_t) = \frac{P_t^y}{P_{t-1}^y}$$

Inserting the assumption (41) into (39) and dividing through by P_t^y yields the real wage equation, which is further simplified using the abbreviated notation introduced in (19), (20), and (42):

(43) $$w_t = \frac{\beta F'_{L,t} H_t + (1-\beta)(1+\tau_t^e)/((1+\tau_{t-1}^e)(1+p_t))\lambda w_{t-1}}{1-\beta\gamma\pi((1-\tau_{t+1})\vartheta_t + \tau_{t+1}(1-s)/q(\vartheta_t))}$$

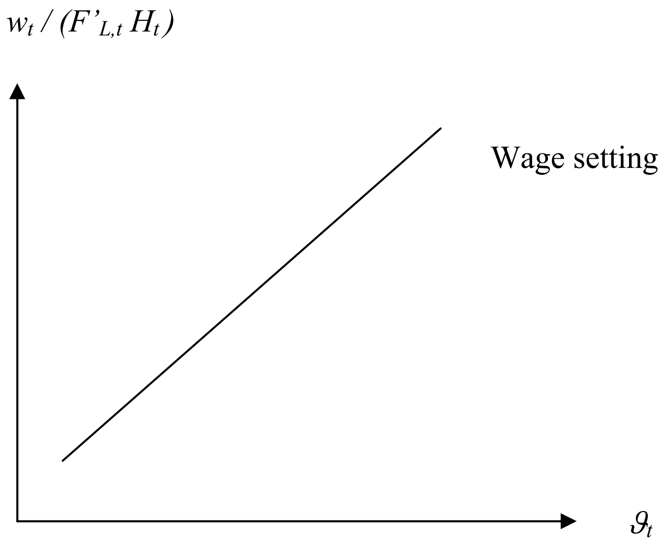
In steady state, $w_t = h\, w_{t-1}$ i.e. the real wage grows at the rate h. Also, the tax system is constant, so $\tau_t = 0$. The steady state wage setting equation then simplifies to:

(44) $$w_t = \frac{\beta F'_{L,t} H_t}{1-\beta\gamma\pi\vartheta_t - (1-\beta)\lambda/(h(1+p_t))}$$

This completes the derivation of the equations which specify the dynamics and steady state of the model.

The steady state labor demand equation (44) is shown in *figure 4* below as a positive relationship between the labor market tightness ϑ_t and the real marginal wage costs per unit of the produced good $w_t / F'_{L,t} H$. Fewer unemployed workers searching and more open vacancies means higher competition for labor and thus higher real wages.

Figure 4



Determination of equilibrium unemployment

In order to obtain the equilibrium unemployment rate, the steady state equation system is solved recursively, beginning with the equilibrium value for labor market tightness ϑ^*. First observe that the steady state versions of the labor demand equation (25) and the wage setting equation (44) both relate labor market tightness ϑ_t to the real marginal wage costs per unit of the produced good *($w_t / F'_{L,t} H$)*. They can be reformulated as:

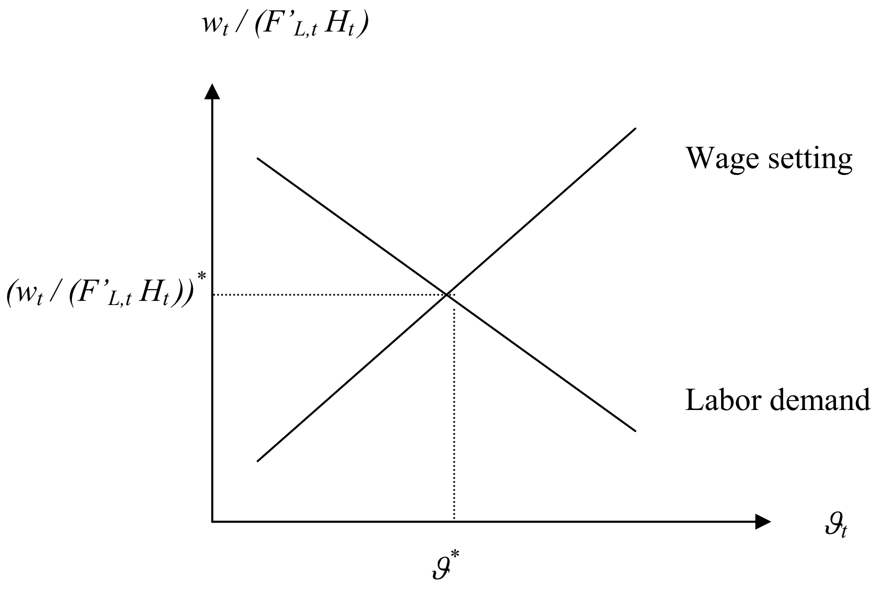
$$\frac{F'_{L,t}H_t}{w_t} = 1 + (1 + s - \frac{\pi h}{(1+r_t)})\frac{\gamma}{q(\vartheta_t)} = \frac{1 - \beta\gamma\pi\vartheta_t - (1-\beta)\lambda/(h(1+p_t))}{\beta}$$

Eliminating wage costs, and after some algebraic simplifications, this reduces to the following equation, with ϑ_t as the only endogenous labor market variable:

$$\text{(45)} \qquad \pi\vartheta_t + (1 + s - \frac{\pi h}{(1+r_t)})\frac{1}{q(\vartheta_t)} = \frac{(1-\beta)}{\beta\gamma}(1 - \frac{\lambda}{h(1+p_t)})$$

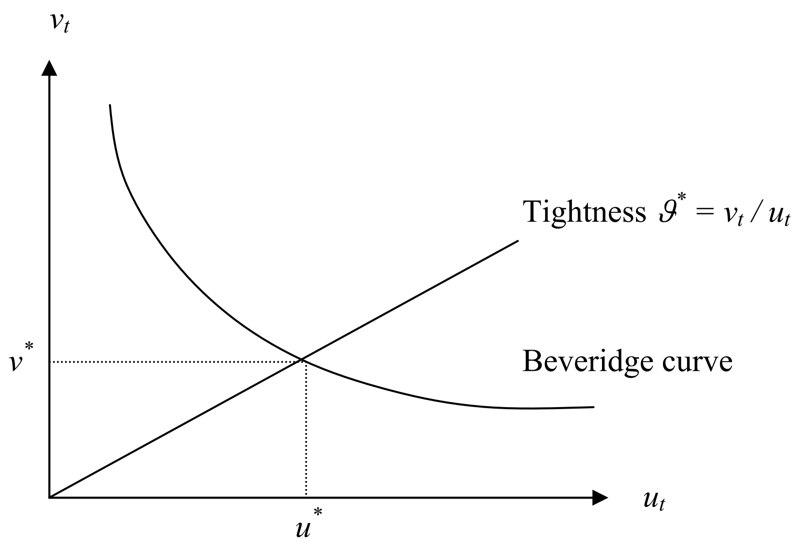
In steady state, p_t and r_t are constant. The value of ϑ_t which satisfies the above equation is the equilibrium tightness ϑ^*. This is illustrated graphically in *figure 5*, which combine the labor demand curve in *figure 3* with the wage setting curve in *figure 4*. The intersection of the two curves determine the steady state values of real marginal wage costs per unit good and labor market tightness.

Figure 5



To determine the equilibrium unemployment rate u^*, substitute this equilibrium tightness ϑ^* for ϑ_t in the steady state version of the Beveridge curve (11). The equilibrium vacancy rate is then simply $v^* = u^* \vartheta^*$. Graphically, in the diagram of the Beveridge curve, with unemployment u_t on the horizontal and vacancies v_t on the vertical axis, the equilibrium labor market tightness $\vartheta^* = v_t / u_t$ describes a straight line through the origin. *Figure 6* shows this line together with the Beveridge curve from in *figure 2*.

Figure 6



The intersection of the tightness line with the Beveridge curve determine the equilibrium unemployment and vacancy rates u^* and v^*.

Other endogenous steady state variables follow recursively from tightness and unemployment. Given the total labor force N_t, equilibrium unemployment u^* determines steady state employment $(1-u^*)\ N_t$ and, for a given capital stock, the marginal product of labor is determined by the production function. The wage level, finally, follows from either the wage equation or from labor demand.

Estimating the Beveridge curve, tightness, and equilibrium unemployment

In order to aid calibration of the model parameters, and to determine a reasonable value for the equilibrium unemployment rate, the Beveridge curve and the labor market tightness line in *figure 6* above are estimated using Swedish data on unemployment and vacancies. Both variables are yearly time series from 1962 to 2002, and expressed as fractions of the labor force. The data is tabled in Appendix C, and plotted in *figures 7* and *8* bellow.

Figure 7: Unemployment in Sweden 1962-2002

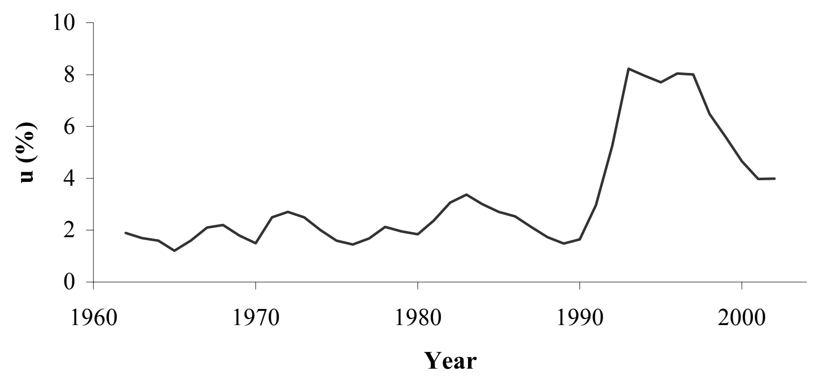
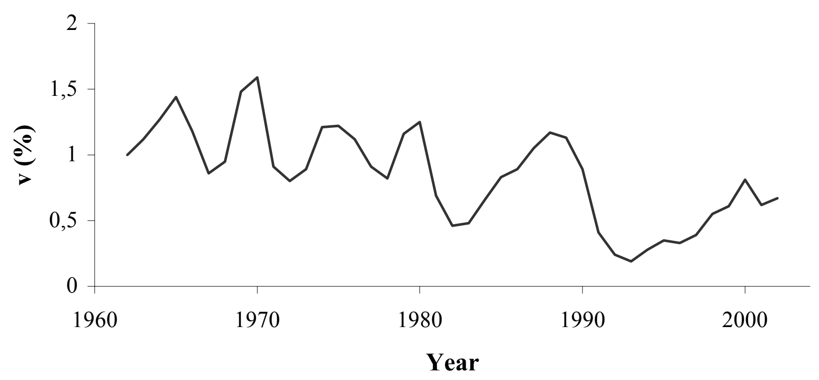
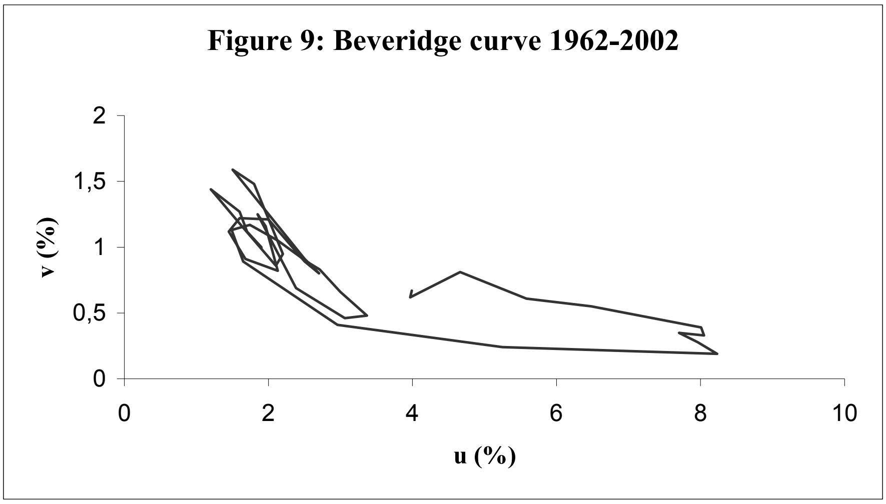


Figure 8: Vacancies in Sweden 1962-2002



As is evident from *figure 7*, unemployment rises sharply in the beginning of the 1990's, to decrease rather slowly towards earlier levels. Yet, while in the period 1962-1991 unemployment never reaches above 3.5 %, it never returns bellow that level in the later period 1992-2002.

Figure 9 shows the unemployment and vacancy data from *figure 7* and *8* combined into an empirical dynamic Beveridge curve comparable to the theoretical unemployment dynamics equation (10). The increase in unemployment in the last decade is seen as a large anti clockwise loop to the lower right in the diagram.



One possible interpretation of the persistence of high unemployment is that it results from slow adjustment down towards low long run equilibrium levels, indicating a high degree of labor market inertia, possibly explainable by search friction of the type assumed in the theoretical matching model presented above. This corresponds to interpreting the large loop in *figure 9* as an adjustment along the same long run Beveridge curve that lies behind the movements in earlier periods. On the other hand, the large difference in unemployment levels between the two periods might be the result of structural changes in the Swedish labor market in the beginning of the 90's, reducing its efficiency in matching unemployed workers with vacant jobs. This corresponds to a shift outward of the Beveridge curve. An attempt is made to test for this possibility, by including a dummy variable for the later period in the estimations.

The vacancy rate in *figure 8* seems to exhibit a decreasing trend. For the latter part of the data period with increasing unemployment, this fits well with the unemployment increase, corresponding to a movement down along the Beveridge curve. However, vacancies seem to fall even early in the data period. This may be due to a transition of the labor force towards sectors with a lower tendency to report vacancies to the Swedish National Labor Market Administration, e.g. from blue collar to white collar jobs, from goods to services, and from the private to the public sector. In that case, it could present a problem for these estimations, which build on a theory implying a constant vacancy rate in steady state.

The estimation method is single equation ordinary least squares regressions. The Beveridge curve is estimated using the following equation, which may be regarded as a version of the dynamic Beveridge curve (10) with v_t / u_t substituted for ϑ_t and solved for v_t, and then linearized in logarithms. D_{9202} is a dummy variable for the period 1992-2002.

$$(46) \qquad \ln(u_t) = \beta_0 + \beta_1 \ln(u_{t-1}) + \beta_2 \ln(v_{t-1}) + \beta_3 D_{9202} + \delta_t$$

The corresponding static Beveridge curve analogous to (11) is

(47) $$(1-\beta_1)\ln(u_t) = (\beta_0 + \beta_3 D_{9202}) + \beta_2 \ln(v_t)$$

Labor market tightness is estimated using the following equation, which may be seen as a dynamic, linear variant of the steady state relationship (45).

(48) $$\ln(\vartheta_t) = \gamma_0 + \gamma_1 \ln(\vartheta_{t-1}) + \gamma_2 D_{9202} + \varepsilon_t$$

The implied steady state value of ϑ^* is given by:

(49) $$\ln(\vartheta^*) = \frac{\gamma_0 + \gamma_2 D_{9202}}{1-\gamma_1}$$

The estimated static equations for the Beveridge curve (47) and the labor market tightness (49) are combined, as illustrated in *figure 6* above, to give the equilibrium unemployment rate u^* expressed in terms of the regression parameters.

(50) $$\ln(u^*) = \frac{\beta_0(1-\gamma_1) + \beta_2\gamma_0 + (\beta_2\gamma_2 + \beta_3(1-\gamma_1))D_{9202}}{(1-\beta_1-\beta_2)(1-\gamma_1)}$$

For each of the two equations (46) and (48), two variants are estimated, one without and one with the dummy included. The four regressions are labeled as follows:

Label	**Regression equation**	**Dummy included?**
A1	Beveridge curve (46)	no
A2	Beveridge curve (46)	yes
B1	Tightness (48)	no
B2	Tightness (48)	yes

Table 1 bellow presents the estimated parameters and the corresponding *t*-values of these four regressions.

Table 1: Parameter estimates

Parameter	**Estimate A1**	**(*t*-value)**	**Estimate A2**	**(*t*-value)**
β_0	0.373	(3.95)	0.556	(5.67)
β_1	0.509	(4.28)	0.205	(1.51)
β_2	-0.494	(3.88)	-0.499	(4.47)
β_3			0.429	(3.48)
	B1		**B2**	
γ_0	-0.169	(1.40)	-0.268	(2.05)
γ_1	0.890	(12.1)	0.692	(5.15)
γ_2			-0.548	(1.74)

Table 2 presents the values of the equilibrium unemployment rate u^* which follow from these parameter estimates according to equation (50). The two variants A1 and A2 of the Beveridge curve (46) may be combined with the two variants B1 and B2 of the tightness equation (48) in

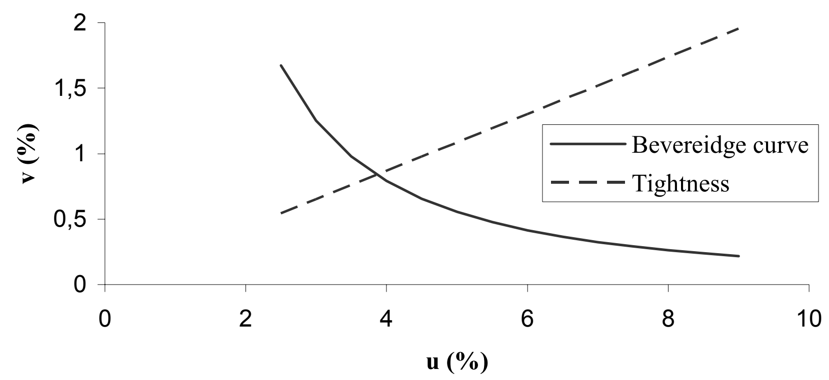
four different ways, each with different values of the equilibrium unemployment rate. Except for the combination of A1 and B1, which contain no dummy variable in either equation, the equilibrium unemployment level differs between the earlier period 1962-1991 and the later period 1992-2002.

Table 2: Equilibrium unemployment (%)

Estimations	1962-1991	1992-2002
A1, B1	3,1	3,1
A2, B2	2,2	6,0
A1, B2	2,3	5,5
A2, B1	2,8	**3,9**

A choice between the different variants of the estimated equations may be based on the statistical significance of the estimated parameters of the dummy variable. The parameter β_3 for the dummy in estimation A2 differs from zero with a significance level of 1%. There thus seems to be some evidence for a shift outward of the Beveridge curve in the early 90's. On the other hand, the parameter γ_2 for the dummy in estimation B2 is not significantly different from zero even on a 5% level. Therefore, on the basis of these regressions there is no need to reject the hypothesis of a constant equilibrium level of labor market tightness during this period. Of the four combinations of Beveridge curve and tightness in *table 2*, the Beveridge curve A2, with a shift, combined with the constant equilibrium tightness B1 seem to fit the data better than the others. These two curves are plotted in *figure 10*. The conclusion is that out of the alternative estimates given, the most plausible estimate of the equilibrium unemployment rate for the period 1992-2002 is 3.9%.

Figure 10: Estimated Beveridge curve and tightness



Appendix A: Derivation of the wage setting equation

Some further details of the algebraic derivation of the wage setting equation (39) are shown here.

The first order condition for maximal Nash product is:

$$\frac{\partial \Lambda_t^{E,i}}{\partial W_t^i}\beta(\Lambda_t^J - \Lambda_t^V) + \frac{\partial \Lambda_t^{J,i}}{\partial W_t^i}(1-\beta)(\Lambda_t^E - \Lambda_t^U) = 0$$

When the partial derivatives (31) and (32) are substituted into the above equation, the Nash bargaining condition (38) results. Rewrite this as the following relation between the firm's and the worker's value gain from a match.

$$(\Lambda_t^J - \Lambda_t^V) = \frac{(1-\beta)}{\beta}\frac{(1+\tau_t^e)(1+\tau_t^c)}{(1-\tau_t^w)}\frac{P_t^c}{P_t^y}(\Lambda_t^E - \Lambda_t^U)$$

Use this equation, and its counterpart for period $t+1$, to substitute the value gains of the firm $(\Lambda^J_t - \Lambda^V_t)$ for those of the worker $(\Lambda^E_{t} - \Lambda^U_t)$ in equation (34) to obtain:

$$\begin{aligned}
&\frac{\beta}{(1-\beta)}\frac{(1-\tau_t^w)}{(1+\tau_t^e)(1+\tau_t^c)}\frac{P_t^y}{P_t^c}(\Lambda_t^J - \Lambda_t^V) - \\
&\frac{\pi}{1+r_t^c}\frac{\beta}{(1-\beta)}\frac{(1-\tau_{t+1}^w)}{(1+\tau_{t+1}^e)(1+\tau_{t+1}^c)}\frac{P_{t+1}^y}{P_{t+1}^c}(1-s-\vartheta_t q(\vartheta_t))(\Lambda_{t+1}^J - \Lambda_{t+1}^V) = \\
&\frac{(1-\tau_t^w)(\varepsilon_t W_t - T_t)}{(1+\tau_t^c)P_t^c}
\end{aligned}$$

Simplify this using the definitions (21) and (28) of the real interest rates for the firm and the consumer respectively, which relates the two real interest rates as follows:

$$\frac{(1+r_t^c)}{(1+r_t)} = \frac{(1+\tau_t^c)P_t^c}{(1+\tau_{t+1}^c)P_{t+1}^c}\frac{P_{t+1}^y}{P_t^y}$$

The result of this substitution is:

$$\begin{aligned}
&(\Lambda_t^J - \Lambda_t^V) - \frac{\pi}{1+r_t}\frac{(1+\tau_t^e)/(1-\tau_t^w)}{(1+\tau_{t+1}^e)/(1-\tau_{t+1}^w)}(1-s-\vartheta_t q(\vartheta_t))(\Lambda_{t+1}^J - \Lambda_{t+1}^V) = \\
&\frac{(1-\beta)}{\beta}\frac{(1+\tau_t^e)(\varepsilon_t W_t - T_t)}{P_t^y}
\end{aligned}$$

Simplify further by introducing the definition (40) of τ_t, the rate of increase in the wage tax wedge:

$$(\Lambda_t^J - \Lambda_t^V) - \frac{\pi}{1+r_t}(1-\tau_{t+1})(1-s-\vartheta_t q(\vartheta_t))(\Lambda_{t+1}^J - \Lambda_{t+1}^V) = \frac{(1-\beta)}{\beta}\frac{(1+\tau_t^e)(\varepsilon_t W_t - T_t)}{P_t^y}$$

Subtract this equation from (33) to eliminate the value gain *(Λ^J_t - Λ^V_t)* in the current period:

$$\frac{1}{1+r_t}(q(\vartheta_t)-(1-\tau_{t+1})\pi\vartheta_t q(\vartheta_t)-\tau_{t+1}\pi(1-s))(\Lambda^J_{t+1}-\Lambda^V_{t+1})=$$
$$F_L^{'}(K_t,H_t\varepsilon_t L_t)H_t-(1-\gamma)\frac{(1+\tau^e_t)\varepsilon_t W_t}{P^y_t}-\frac{(1-\beta)}{\beta}\frac{(1+\tau^e_t)(\varepsilon_t W_t-T_t)}{P^y_t}$$

Eliminate the value gain *(Λ^J_{t+1} - Λ^V_{t+1})* in the next period by substituting the expressions (35) and (36) for these values:

$$(1-\pi((1-\tau_{t+1})\vartheta_t+\tau_{t+1}\frac{(1-s)}{q(\vartheta_t)}))\frac{\gamma(1+\tau^e_t)\varepsilon_t W_t}{P^y_t}=$$
$$F_L^{'}(K_t,H_t\varepsilon_t L_t)H_t-(1-\gamma)\frac{(1+\tau^e_t)\varepsilon_t W_t}{P^y_t}-\frac{(1-\beta)}{\beta}\frac{(1+\tau^e_t)(\varepsilon_t W_t-T_t)}{P^y_t}$$

Finally, solve for the wage W_t to obtain the wage setting equation (39):

$$\frac{(1+\tau^e_t)\varepsilon_t W_t}{P^y_t}=\frac{\beta F_L^{'}(K_t,H_t\varepsilon_t L_t)H_t+(1-\beta)(1+\tau^e_t)T_t/P^y_t}{1-\beta\gamma\pi((1-\tau_{t+1})\vartheta_t+\tau_{t+1}(1-s)/q(\vartheta_t))}$$

Appendix B: The main equations formulated in the Troll modeling language

The equations of KIMOD are solved in the Troll modeling language. The model equations relating to the labor market have been derived above, but will be repeated here in the form they have in the Troll model code. The intention is to provide an easy reference for those working with the model, and possibly for others involved in similar projects elsewhere. Equation numbers in this section refer to the earlier theoretical treatment.

The four main dynamic equations are:

Unemployment dynamics:	(10)	Steady state:	(11)
Capital demand:	(22)	Steady state:	(24)
Labor demand:	(23)	Steady state:	(25)
Wage setting:	(43)	Steady state:	(44)

These equations together determine the development of the variables capital K_t, unemployment u_t, labor market tightness ϑ_t, and the wage rate W_t. The vacancy rate v_t follow from $v_t = \vartheta_t u_t$. A complete list of equations sufficient to solve the model is: (1) - (6), (10), (12), (15), (22), (23), (41), and (43) and their steady state counterparts.

For numerical stability of Troll's model solving algorithms, it is best to avoid mathematically invalid operations during the solution search, such as division by zero, raising negative values to fractional powers or taking logarithms of negative values. Thanks to Troll's backtracking capability, attempts at these operations do not necessarily halt the solution process, but may yet throw it off track. It is often possible to avoid these operations by expressing the equations using logarithms of the original variables.

Therefore, many equations are written in logarithmic form in the Troll model file. The logarithmic variables are defined implicitly using the exponential function rather than explicitly with the logarithmic function. The variables in question are:

```
u = exp(lnu)

v = exp(lnv)

theta = exp(lntheta)

q = exp(lnq)
```

The logarithmic form of the definition of labor market tightness is:

(5) `lntheta = lnv - lnu`

The Cobb-Douglas matching function can simply be expressed as:

(6) `lnq = log(x0) - eta * lntheta`

The unemployment dynamics equation becomes:

(10)
```
u = (n*(1+s)-pi*(1-u(-1))) / (n*(1+s+theta*q))
```

and its steady state counterpart, also known as the Beveridge curve:

(11)
```
u = (n*(1+s)-pi) / (n*(1+s+theta*q)-pi)
```

The first order conditions for the firms' profit maximization are:

Capital demand:

(22)
```
mpk(1) + piy(1) * ((1-delta)*(1+2*psi*i(1)/k(1)) +
psi*(i(1)/k(1))^2) = (1+r) * piy*(1+2*psi*i/k)
```

Capital demand, steady state:

(24)
```
mpk = piy * ((delta+r)*(1+2*psi*i/k) - psi*(i/k)^2)
```

Labor demand:

(23)
```
mpl - wpy - (1+s)/q*gamma * wpy =
- 1/(1+r) * pi/q(1)*gamma * wpy(1)
```

Labor demand, steady state:

(25)
```
mpl = (1 + (1+s-pi*h/(1+r))*gamma/q) * wpy
```

Wage setting:

(43)
```
wpy = (beta*mpl + (1-beta)*(1+taue)/((1+taue(-1)) *
(1+p)) * lambda * wpy(-1)) / ((1 - beta*gamma*pi *
((1-tau(1))*theta + tau(1)*(1-s)/q)))
```

Wage setting, steady state:

(44)
```
wpy = beta*mpl / ((1 - beta*gamma*pi*theta -
(1-beta)*lambda/(h*(1+p))))
```

Appendix C: Unemployment and vacancies in Sweden 1962-2002

Unemployment data is from the Labor Force Survey (LFS) by Statistics Sweden (SCB). Vacancies are administrative data from the Swedish National Labour Market Administration (AMV). Both are yearly averages in percent of the labor force.

Year	Unemployment	Vacancies
1962	1.90	1.00
1963	1.70	1.12
1964	1.60	1.27
1965	1.20	1.44
1966	1.60	1.18
1967	2.10	0.86
1968	2.20	0.95
1969	1.80	1.48
1970	1.50	1.59
1971	2.50	0.91
1972	2.70	0.80
1973	2.50	0.89
1974	2.00	1.21
1975	1.60	1.22
1976	1.45	1.12
1977	1.68	0.91
1978	2.13	0.82
1979	1.96	1.16
1980	1.85	1.25
1981	2.38	0.69
1982	3.06	0.46
1983	3.37	0.48
1984	3.00	0.66
1985	2.71	0.83
1986	2.53	0.89
1987	2.13	1.05
1988	1.74	1.17
1989	1.49	1.13
1990	1.65	0.89
1991	2.96	0.41
1992	5.25	0.24
1993	8.23	0.19
1994	7.96	0.28
1995	7.70	0.35
1996	8.05	0.33
1997	8.01	0.39
1998	6.48	0.55
1999	5.58	0.61
2000	4.66	0.81
2001	3.97	0.62
2002	3.99	0.67

Appendix D: Notation for variables and functions

Symbol	Troll-name	Explanation
Λ^{V}_{t}		value of a vacancy to the representative firm
$\Lambda^{J,i}_{t}$		value of a filled job to firm *i*
Λ^{U}_{t}		value of unemployment to the representative worker
$\Lambda^{E,i}_{t}$		value of employment to worker *i*
Π_t^i		profits earned by firm *i* in period *t*
Π_t		aggregate profits in period *t*
α		elasticity of production w.r.t. capital
β	beta	workers' bargaining power
γ	gamma	vacancy cost relative to wage
δ	delta	capital depreciation rate
ε_t	epsilon	number of hours worked per worker and year
η	eta	elasticity of matchings w.r.t. unemployment
ϑ_t	theta	v_t / u_t, labor market tightness in period *t*
λ	lambda	unemployment benefit replacement ratio
π	pi	labor force survival rate
ρ_t^s		discount factor from period *s* to *t*: $\rho_t^t = 1$; $\rho_t^{s+1} = \rho_t^s / (1 + r_s)$
τ_t	tau	growth rate of wage tax wedge
τ_t^e	taue	employer’s wage tax
τ_t^w	tauw	employee’s wage tax
τ_t^y	tauy	value added tax
ψ	psi	coefficient for quadratic term in investment cost
D_t^i		dividends paid by firm *i* in period *t*
D_t		aggregate dividends in period *t*
F		production function
H_t		labor productivity growth factor in period *t*
I_t^i		investments by firm *i* in period *t*
I_t	i	aggregate investments in period *t*
K_t^i		capital in firm *i* in period *t*
K_t	k	aggregate capital in period *t*
L_t^i		employment in firm *i* in period *t*
L_t		aggregate employment in period *t*
N_t		labor force in period *t*
P_t^c		price of consumption good in period *t*
P_t^I		price of investment good in period *t*
P_t^y		product price in period *t*
R_t		nominal interest rate in period *t*
T_t		nominal unemployment benefits in period *t*
U_t		unemployed workers in period *t*
V_t^i		vacancies in firm *i* in period *t*
V_t		aggregate number of vacancies in period *t*
VA_t		value added in the private sector in period *t*
W_t^i		nominal hourly wage in match *i* in period *t*
W_t		nominal hourly wage in period *t*
X_t		new matches in period *t*
Y_t^i		production in firm *i* in period *t*
Y_t		aggregate production in period *t*

h	h	average productivity growth rate
i		index of firms workers or matches
n	n	labor force growth rate
p_t	p	inflation in the product price: $(1+p_t) = P_t^y / P_{t-1}^y$
$q(\vartheta)$	q	rate of filling vacancies
r_t	r	real interest rate in period t: $(1 + r_t) = (1 + R_t) / (1 + p_{t+1})$
r_t^c		consumer price real interest rate in period t
s	s	separation rate
t, s		time period index
u_t	u	unemployment rate in period t
v_t	v	vacancy rate in period t
w_t	wpy	real wage costs in period t: $(1+\tau_t^e)\ \varepsilon_t\ W_t / P_t^y$
x		matching function
x_0	x0	coefficient in Cobb-Douglas matching function
$F'_{K,t}$	mpk	marginal productivity of capital in period t
$F'_{L,t}$	mpl	marginal productivity of labor in period t
	piy	price of investment goods relative to product price: P_t^I / P_t^y

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