Modelling the Foreign Sector in a Macroeconometric Model of Sweden

Jan Alsterlind, Alek Markowski and Kristian Nilsson

Working Paper No. 88, January 2004

Published by
The National Institute of Economic Research
Stockholm 2004

NIER prepares analyses and forecasts of the Swedish and international economy and conducts related research. NIER is a government agency accountable to the Ministry of Finance and is financed largely by Swedish government funds. Like other government agencies, NIER has an independent status and is responsible for the assessments that it publishes.

The *Working Paper* series consists of publications of research reports and other detailed analyses. The reports may concern macroeconomic issues related to the forecasts of the Institute, research in environmental economics in connection with the work on environmental accounting, or problems of economic and statistical methods. Some of these reports are published in final form in this series, whereas others are previous versions of articles that are subsequently published in international scholarly journals under the heading of *Reprints*. Reports in both of these series can be ordered free of charge. Most publications can also be downloaded directly from the NIER home page.

NIER
Kungsgatan 12-14
Box 3116
SE-103 62 Stockholm
Sweden
Phone: 46-8-453 59 00, Fax: 46-8-453 59 80
E-mail: ki@konj.se, Home page: www.konj.se

ISSN 1100-7818

MODELLING THE FOREIGN SECTOR IN A MACROECONOMETRIC MODEL OF SWEDEN

Jan Alsterlind, Alek Markowski och Kristian Nilsson

Abstract: The purpose of this paper is to estimate a rudimentary model of the “rest of the world”, which may serve as the foreign sector in a model of the Swedish economy. The “rest of the world” is here represented by the US and the euro zone which together cover some two thirds of the Swedish foreign trade. The underlying theoretical model is the so-called Svensson model as of Svensson (1997). This model has the advantage that it is small and simple, but still allows for both supply and demand shocks with realistic responses. The Svensson model is estimated (OLS) for the US and the euro zone separately using quarterly data. Furthermore, following Smeets and Peersman (1999), alternative models with the outputgap treated as an unobserved component are estimated as well. Impulse response analyses indicate that all individual models react reasonably well to both supply and demand shocks. The models for the US and the euro zone are aggregated temporally to annual data (since the model of the Swedish economy uses annual data) and are subsequently aggregated into one model of the foreign sector.

1. INTRODUCTION

National Institute of Economic Research (NIER) is currently working on an aggregate simulation model of the Swedish economy. This paper is a contribution to this ongoing modelling effort. We estimate a rudimentary model of the "rest of the world", which can form the basis for the foreign sector of the model of Sweden, or at least serve as a benchmark for its consistent calibration.

Our model is confined to four variables: the foreign demand (GDP) growth rate, the foreign inflation rate, the foreign interest rate and the exchange rate. As implied by the variables of interest, the foreign sector model should – at least implicitly – include a real demand equation, a price equation (preferably a Philips curve) and monetary policy determination (possibly a reaction function of a "World Central Bank"). Since this is to be a satellite model, it should be small and as simple as possible. At the same time, it should exhibit a fair amount of data coherence in order to give realistic reactions to the shocks.

The world economy is here represented by the US and the euro zone. Together, they cover almost two thirds of the Swedish foreign trade and give a representative picture of that part of the world that has most importance for the Swedish economy. The US and the euro zone are modelled separately and subsequently aggregated into one "world model".

While long time-series data for the US are generally available, the corresponding time series for the euro zone cover only three years, the time that has passed since the third phase of the EMU has started. We have followed many other researchers and used aggregates of national statistics for the period preceding 1999. This means that data with differing definitions and describing different exchange rate and monetary policy regimes were aggregated together. Needless to say, it is uncertain to which extent the results based on these data are relevant as a description of the unified euro zone.

The choice of model variables was determined by the variables routinely forecasted at the NIER. The two sub-models were estimated on quarterly data and then transformed into annual equations. The two reaction functions were not estimated, rather their coefficients were calibrated.

The outline of the paper is as follows. Sections 2 and 3 describe two theoretical models to be estimated. Section 4 gives data definitions. Sections 5 and 6 describe the quarterly estimation results and the quarterly reaction functions, respectively. The impulse responses of the quarterly models are studied in Section 7. The annual models are described in section 8. The last two sections deal with the aggregate annual model.

2. THE BASIC MODEL

The basic model is the so-called Svensson model, a small closed-economy model developed in Svensson (1997) for the purposes of monetary policy analysis and estimated by Rudebusch and Svensson (1999a and 1999b):

$$\left(y_t - y_t^*\right) = a_0 + \sum_{i=1}^{k} a_i \left(y_{t-i} - y_{t-i}^*\right) - a_{k+1}\left(r_{t-1} - \pi_{t-1}\right) + \varepsilon_{1t} \tag{1}$$

$$\pi_t = \pi_t^e + b_{h+1}\left(y_t - y_t^*\right) + \varepsilon_{2t} \tag{2}$$

$$\pi_t^e = b_0 + \sum_{i=1}^{h} b_i \pi_{t-i} \tag{3}$$

$$r_t^* = \left(a_0 + b_0\right) + c_1\left(\pi_t - \pi_t^*\right) + c_2\left(y_t - y_t^*\right) \tag{4}$$

$$r_t = d_1 r_t^* + \left(1 - d_1\right) r_{t-1} \tag{5}$$

where y_t is the logarithm of GDP, y_t^* is the logarithm of the potential GDP (both multiplied by 100), $\left(y_t - y_t^*\right)$ is the output gap i.e. approximately the percentage deviation of the actual GDP from the potential one, π_t is the inflation rate, π_t^e is the expected – or equilibrium – inflation rate, r_t is the central bank intervention rate, r_t^* is the central bank target rate, π_t^* is the central bank inflation target, ε_{1t} and ε_{2t} are demand[1] and supply shocks, respectively, and a_i, b_i, c_i, and d_i are fixed coefficients.

The model consists of an aggregate demand equation, an aggregate supply equation (Philips curve) and a central-bank reaction function. The output gap is affected by its own past values, the (ex-post) short real rate of interest and demand shocks. The ratio a_0/a_{k+1} equals the equilibrium (or steady-state) real interest rate, since the output gap $\left(y_t - y_t^*\right)$ equals zero in equilibrium. (In practice, this requires that the output gap variable employed in estimation should have zero mean.)

The inflation rate is determined by expected inflation, the output gap and supply shocks. The expectations formation mechanism is backward-looking and uses adaptive expectations. If the sum of the lagged inflation coefficients in equation (3) is smaller than 1, inflation expectations are stationary and converge to $b_0/\left(1 - b_1 - b_2 - \ldots - b_h\right)$. The central bank enjoys in this case full credibility provided that $\pi_t^* = b_0/\left(1 - b_1 - b_2 - \ldots - b_h\right)$. If the lagged inflation coefficients in equation (3) sum to 1, the Philips curve has the accelerationist form and inflation expectations are unbounded. The central bank's ability to control inflation is in this case not credible. Rudebusch and Svensson (1999a) cannot reject the hypothesis that the lagged inflation coefficients sum to 1 in their model estimated on US data.

The central bank applies a Taylor rule with interest rate smoothing. This means that the target rate is determined in equation (4) on the basis of the output gap and the inflation rate's deviation from the bank's inflation target. The target rate is imposed gradually, in accordance with equation (5), in order to smooth out interest rate variation.

Despite its highly simplified form, the model has a number of properties that makes it realistic. The central bank controls the short interest rate but not inflation, which it only can affect through

[1] This interpretation assumes that potential output is given.

the output gap. Monetary policy affects the economy with a lag: the output gap reacts first and the inflation rate reacts subsequently with a longer lag.

After substitution of equation (3) into (2) and equation (4) into (5), the model is reduced to three equations. Below, the aggregate supply and aggregate demand equations are estimated, while the reaction function coefficients are set in accordance with the standard results for a Taylor rule. The intercepts in the first two equations can be eliminated by expressing all the variables as deviations from their respective means. This gives the model a true linear form in that all the variables can assume both negative and positive values.

3. THE ALTERNATIVE MODEL

Estimation of the basic model depends on the availability of a time series for the output gap, which is not directly observable. Potential GDP, y_t^*, – which is the unobserved component of the output gap – is exogenous to the model and can be defined in a variety of ways. In the estimation of the basic model we have tested output gap series published by other institutions as well as our own computations, where potential GDP was defined as the actual GDP smoothed using the so called Hodrick-Prescott filter.

An alternative way is to estimate the output gap simultaneously with the model coefficients, using the method of unobserved components, or UC, (cf Harvey 1990 and Kuttner 1994). Following Smeets and Peersman (1999), the following alternative model was estimated:

$$\pi_t = b_0 + \sum_{i=1}^{h} b_i \pi_{t-i} + b_{h+1}\left(y_t - y_t^*\right) + \varepsilon_{2t} \qquad (2')$$

$$\left(y_t - y_t^*\right) = a_0 + \sum_{i=1}^{k} a_i \left(y_{t-i} - y_{t-i}^*\right) - a_{k+1}\left(r_{t-1} - \pi_{t-1}\right) + \varepsilon_{1t} \qquad (1)$$

$$y_t^* = e_0 + y_{t-1}^* + \varepsilon_{3t} \qquad (6)$$

$$y_t = y_t^* + \left(y_t - y_t^*\right) \qquad (7)$$

where ε_{3t} is a random shock.

As in the case of the basic model, equations (4) and (5) were not estimated. Equation (2') was obtained upon substitution of equation (3) into (2). The main difference between the two models is that in the alternative model the log of potential GDP, y_t^*, and the output gap, $(y_t - y_t^*)$, are considered as unobserved and the time series for these variables are obtained from the estimation process. As a result, the assumptions about those variables are imposed on the time series obtained. Thus, y_t^* by definition is a random walk with drift (cf equation (6)) and the output gap is related to the inflation rate and the real rate of interest in accordance with equations (2') and (1).

4. THE DATA

Quarterly data for the US for the period 1956 – 2001 were obtained from the Federal Reserve System's data base, FRED. Price inflation, π_t, is defined as 100 times the quarter-on-quarter change in the log of the seasonally adjusted consumer price index, the interest rate (r_t) – as the federal funds rate *per annum* divided by 4, thus giving the ex-post real interest rate $(r_t - \pi_t)$ on a quarterly basis. Potential GDP is defined as the trend component of the Hodrick-Prescott decomposition applied – using λ = 1600 – to the log of the real seasonally adjusted GDP. The CBO output gap is also used (cf Congressional Budget Office 1995).

Quarterly data for the euro zone for the period 1970 – 2001 were obtained from the data bank of the European Central Bank's Area-Wide Model (cf. Fagan, Henry, Mestre 2001). Price inflation, π_t, is defined as 100 times the quarter-on-quarter change in the log of the seasonally adjusted harmonised consumer price index, the interest rate (r_t) – as the three-month rate *per annum* divided by 4, thus giving the ex-post real interest rate $(r_t - \pi_t)$ on a quarterly basis. Potential GDP is defined as for the US.

5. ESTIMATION RESULTS

The lag lengths and the exact form of the estimated equations were determined on the basis of the residual tests and the significance of the individual terms. The estimation results are compared in Table 1.

The individual coefficients in the autoregressive lag polynomials are sensitive to the number of terms in the polynomial. Insignificant terms within the autoregressive lag polynomials are therefore reported to facilitate comparison. In OLS estimation, the real

Table 1. Estimation results for the basic (OLS) and alternative (UC) model, p-values in parentheses.

Variable	US		Euro zone	
	OLS estimates	UC estimates	OLS estimates	UC estimates
Aggregate demand equation, dependent variable: $(y_t - y_t^*)$				
$(r_{t-1} - \pi_{t-1})$		-0.1732 (0.01)		-0.0863 (0.02)
$(r_{t-2} - \pi_{t-2})$	-0.1862 (0.06)		-0.1486 (0.03)	
$(y_{t-1} - y_{t-1}^*)$	1.0430 (0.00)	1.2671 (0.00)	0.9457 (0.00)	1.2458 (0.00)
$(y_{t-2} - y_{t-2}^*)$	-0.0951 (0.37)	-0.1110 (0.31)	0.0607 (0.62)	-0.0275 (0.44)
$(y_{t-3} - y_{t-3}^*)$	-0.1925 (0.01)	-0.2691 (0.01)	-0.2912 (0.00)	-0.2984 (0.00)
Intercept	0.0929 (0.23)	0.2646[1]	0.0742 (0.26)	0.6213[2]
Dummies = 1 for		1982:1-2001:4[1]	1990:1-1991:4	1990:1-2001:4[2]
R^2	0.76		0.78	
Estimation period	1955:4-2001:4	1965:1-2001:4	1971:1-2001:4	1970:1-2001:4
Philips curve, dependent variable: π_t				
$(y_{t-1} - y_{t-1}^*)$	0.0982 (0.00)	0.1105 (0.00)	0.1490 (0.00)	0.1494 (0.00)
π_{t-1}	0.5255 (0.00)	0.5151 (0.00)	0.3862 (0.00)	0.2820 (0.00)
π_{t-2}	-0.0512 (0.52)	-0.1875 (0.00)	-0.0019 (0.98)	-0.0414 (0.26)
π_{t-3}	0.4124 (0.00)	0.4878 (0.00)	0.1763 (0.07)	0.1274 (0.02)
π_{t-4}		-0.0991 (0.05)	0.2836 (0.00)	0.2341 (0.00)
Intercept	0.1145	1.1689[3]	0.2762 (0.02)	1.7434[4]
Dummies = 1 for		1997:1 – 2001:4	1990:1-2001:4	1990:1-2001:4[2]
R^2	0.79		0.84	
Estimation period	1956:1-2001:4	1965:1-2001:4	1971:2-2001:4	1970:1-2001:4

[1] This is the mean of $(r_t - \pi_t)$ for 1965:1-1981:4. The variables were de-meaned in estimation and the real rate was de-meaned separately for 1965:1-1981:4 and 1982:1-2001:4.
[2] This is the mean of $(r_t - \pi_t)$ for 1970:1-1989:4. The inflation variable was de-meaned for two periods separately, namely 1970:1-1989:4 and 1990:1-2001:4. This has affected the real rate of interest accordingly.
[3] This is the mean of π_t. The variables were de-meaned in estimation.
[4] This is the mean of π_t for 1970:1-1989:4 (cf footnote 2). The variables were de-meaned in estimation.

rate of interest lagged by one quarter was deleted, since its coefficients were close to zero and insignificant.

The OLS estimates are not fully satisfactory. Residual tests indicate non-normality and heteroskedasticity. Residuals in the Philips curve for the euro zone also exhibit some signs of serial correlation, which, however, disappears when seasonal dummies are included. We found this strange, as the data are supposed to be seasonally adjusted, and decided against the inclusion of seasonal dummies.

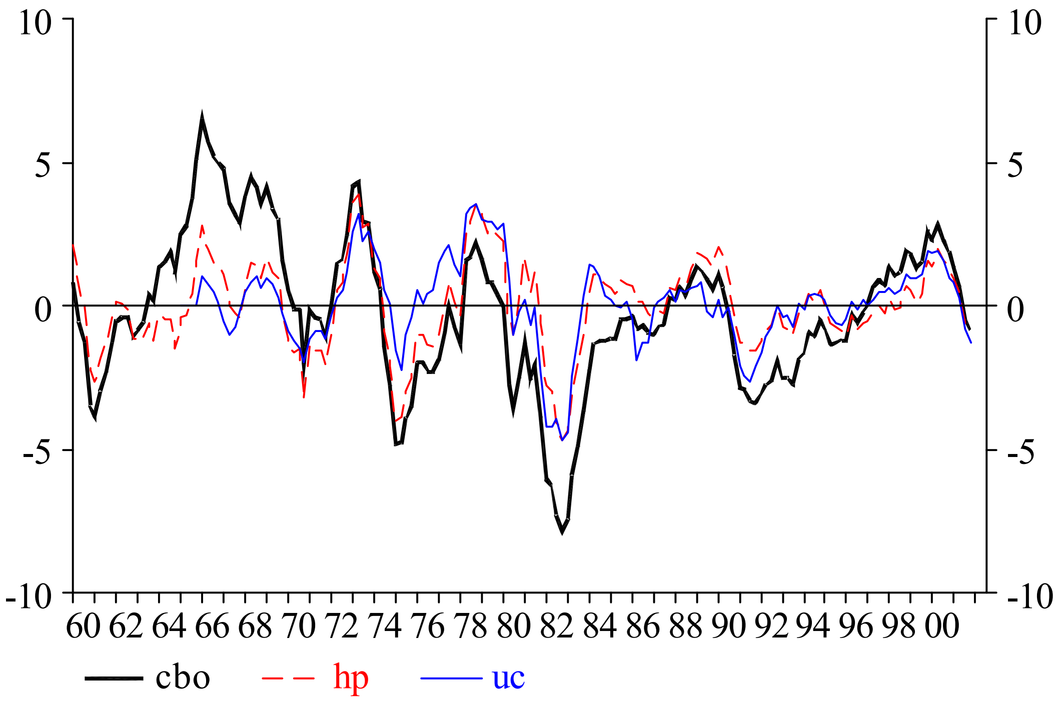
The shift dummies included in the Philips curve represent shifts in inflationary expectations. The hypothesis that the lagged inflation coefficients in the Philips curve sum to unity was rejected for both the US and the euro zone in OLS estimation. Thus, the shifts refer to the expected long-term inflation level. The dummy for 1990:1-2001:4, included in the OLS Philips curve for the euro zone, represents the change in expectations connected with the preparations for and the introduction of the common currency. The shift dummies in the aggregate demand equations affect the equilibrium real rate of interest. The dummy for 1990-91, included in the OLS demand equation for the euro zone, represents the high interest rates (possibly the risk premium) after the unification of Germany.

The output gap estimated for the US using the unobserved components method is – with the exception of the second half of the 1970-ies – comes close to the one obtained from the HP filter (cf. Diagram 1). The gap published by the Congressional Budget Office (CBO) has the same shape but larger amplitude of variation. Apparently, the HP-decomposition gives a potential GDP that co-varies more with the actual GDP than the potential output as defined by the CBO.

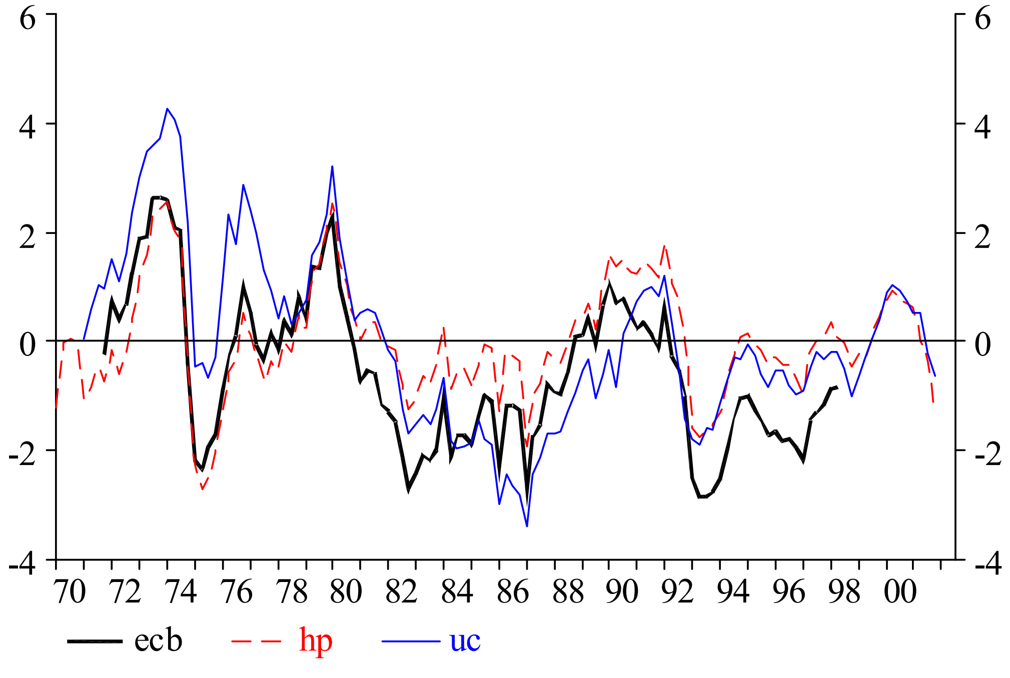
The output gaps estimated for the euro zone with the UC and HP methods are almost identical for the 1990-ies and the first two years of the current century. This period, which includes monetary convergence and introduction of common currency, is of most importance for the analysis. Even in this case, the estimated output gaps appear to have the

Diagram 1. Alternative output gaps for the USA and the euro zone.

USA



Euro zone



same shape as the gap employed in the Area-Wide Model of the ECB (the latter being available only through the first quarter of 1998).

6. THE REACTION FUNCTIONS

The reaction functions of the Fed and ECB have the form of a Taylor rule with interest rate smoothing. Substituting equation (4) into (5) we get:

$$r_t = (1-d_1)r_{t-1} + d_1\left[c_1\left(\pi_t - \pi_t^*\right) + c_2\left(y_t - y_t^*\right) + (a_0 + b_0)\right] \qquad (5')$$

Starting with the coefficient values often employed in the literature[2], the coefficients were subsequently arbitrarily adjusted to secure the resemblance of the interest rate path implied by the rule during the 1990-ies to the actual path observed in that period. The following reaction functions are postulated:

US:

$$r_t = 0.2r_{t-1} + 1.5\left(\pi_t^o - 3.0/4\right) + 0.5/4\left(y_t - y_t^*\right) + (1-0.2)(0.6 + 3.0/4) \qquad (7)$$

Euro zone:

$$r_t = 0.3r_{t-1} + 1.2\left(\pi_t^o - 1.7/4\right) + 0.4/4\left(y_t - y_t^*\right) + (1-0.3)(0.6 + 1.7/4) \qquad (8)$$

where

$$\pi_t^o = \left(\pi_t + \pi_{t-1} + \pi_{t-2} + \pi_{t-3}\right)/4 .$$

The inflation rate is smoothed to avoid reactions to the quasi-seasonal pattern of the inflation rate. The last term in equations (7) and (8) corresponds to the quarterly equilibrium rate of interest. The last parenthesis includes the sum of the equilibrium real rate of interest and the equilibrium rate of inflation. This parenthesis is premultiplied by d_1 in accordance with equation (5'). The equilibrium real rate of interest is in both equations postulated to be 0.6 % per quarter

---

[2] The standard values for c_1 and c_2 are 1.5 and 0.5, respectively. Here, however, c_2 should be divided by four, to allow for the fact that the interest rate and the inflation rate are expressed as quarterly rather than annual rates.

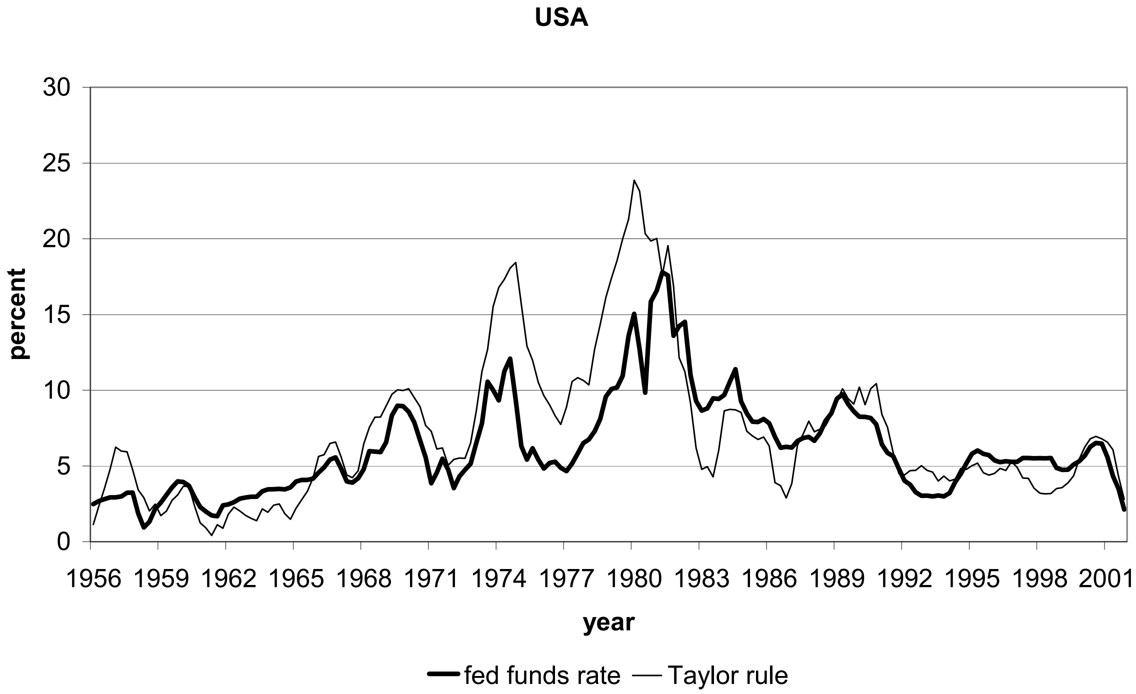
or ca 2.4 % *per annum*. The target inflation rate is postulated to be 3.0 % *per annum* in the US and 1.7 % *per annum* in the euro zone. The interest rate paths implied by the rules are depicted together with the actual interest rates in Diagram 2.

7. IMPULSE RESPONSES

The impulse responses for the two models are shown below. Diagram 3 shows the responses of the basic model, which was estimated with OLS, while Diagram 4 shows the corresponding responses of the alternative model, estimated with the UC method. In each simulation, a one-off shock of 1 percentage point was introduced in one equation in the first quarter of the first year (note that the time axes are scaled in years). The charts show the deviations of the endogenous variables from their equilibrium paths, due to this temporary shock. As can be seen, the reaction patterns are similar for the two models and also for the two geographical zones within each model.

In the case of the supply shock, the central bank reacts with a lag and the real rate of interest increases (in relation to the baseline) in the first quarter, giving rise to an increase in the output gap. Furthermore, the interest rate and the inflation rate move in opposite directions in the first few quarters. This is a result of the choice of the reaction function, where the central bank is assumed to react to the average inflation rate over the previous four quarters rather than to the current quarterly inflation rate. The approach can be interpreted as representing the central bank's forecasting procedure. A reaction function including only the current inflation rate (rather than a moving average) would give an immediate increase of the real rate of interest after the supply shock but also a considerably more variable (less smooth) rate of interest.

Diagram 2 Interest rate paths implied by the Taylor rules and the actual interest rates



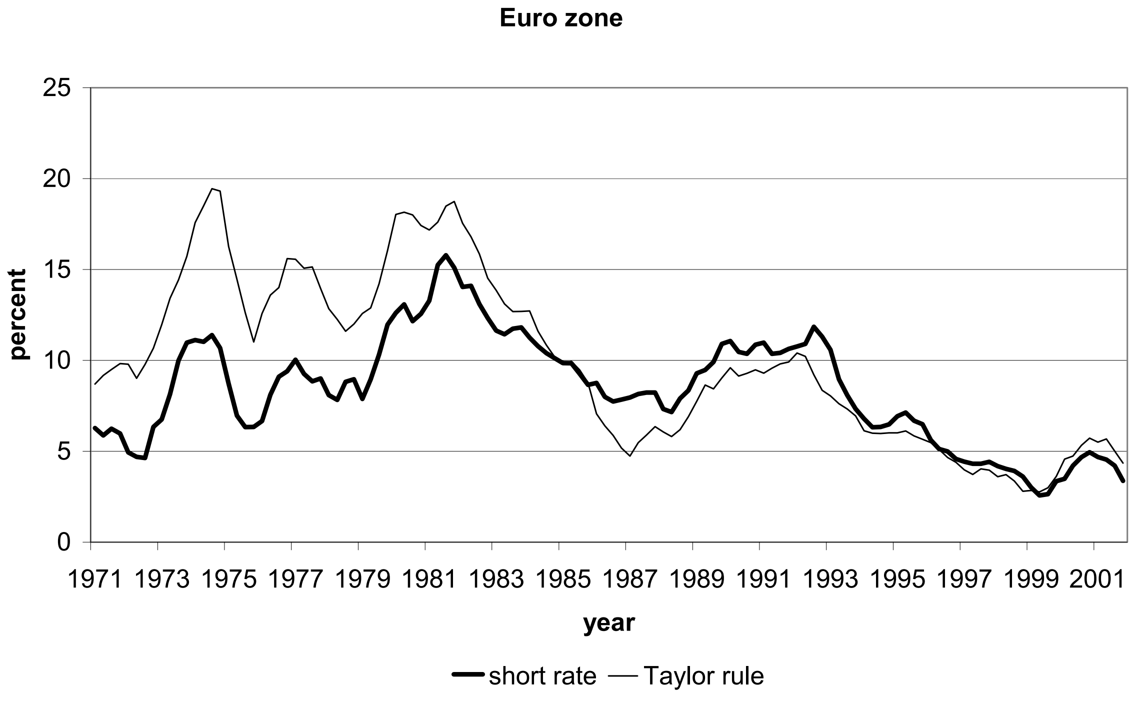
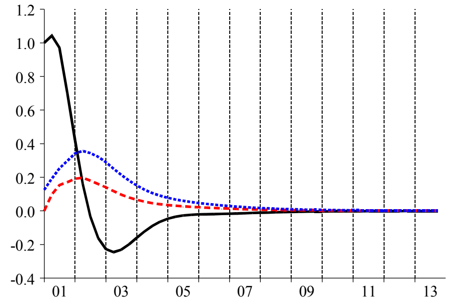
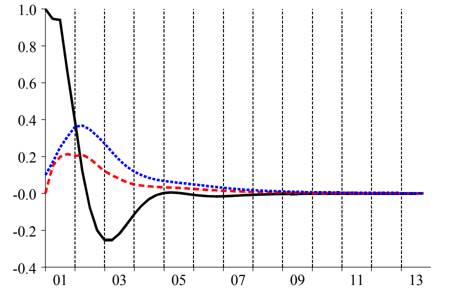
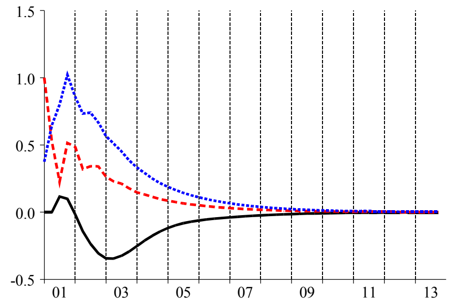
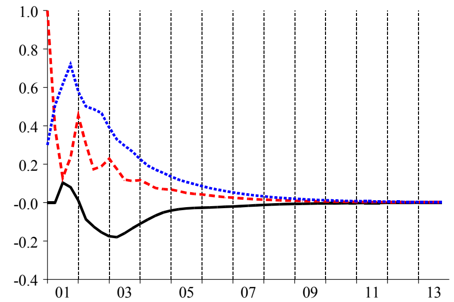
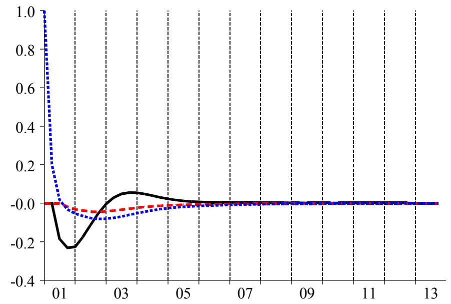
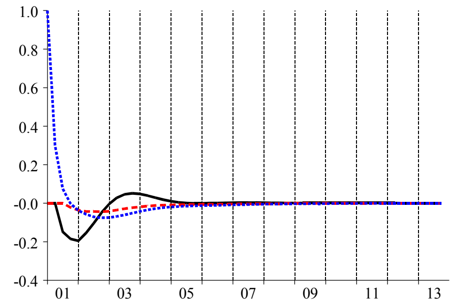
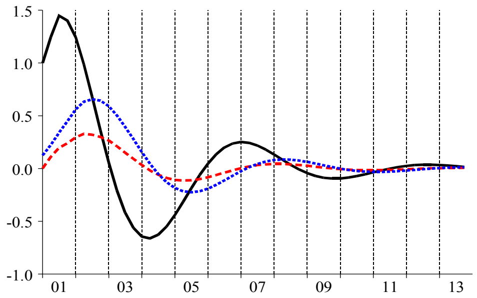
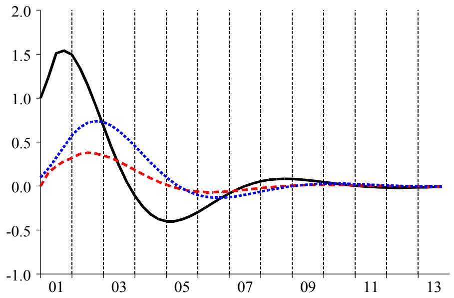
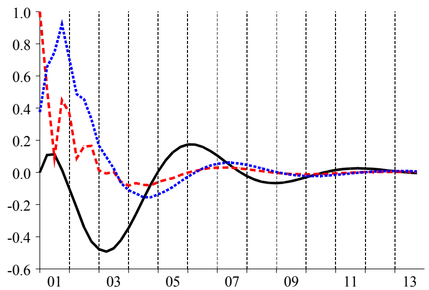
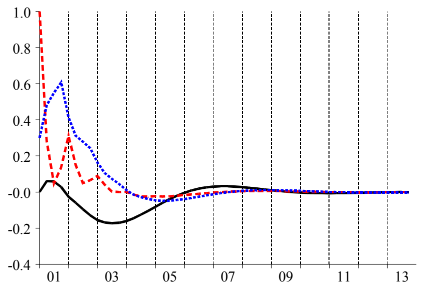
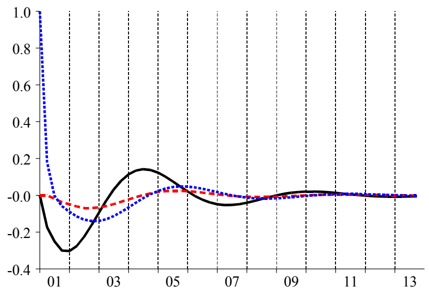
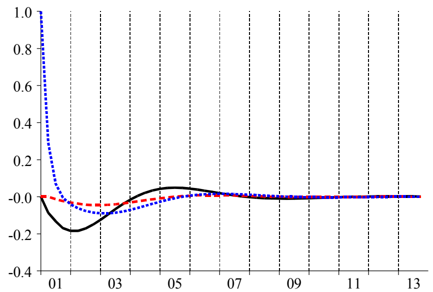


Diagram 4 Simulation results for the basic model

	US model	EMU model
Demand shock		
Supply shock		
Monetary policy shock		

——— Output gap   – – – – inflation rate   .... interest rate

Diagram 4 Simulation results for the alternative mode

	US model	EMU model
Demand shock		
Supply shock		
Monetary policy shock		

—— Output gap   – – – inflation rate   ········ interest rate

8. Aggregation into annual relations

General formulae for temporal aggregation using different operators were derived by Ruist (1996) for aggregation over 2 and 6 subperiods (i.e. quarterly and monthly equations into semi-annual ones). Since we here are interested in aggregation over 4 subperiods (quarterly equations into annual ones) we will use the slightly simplified method employed in Markowski (1995).

In order to obtain an annual expression, the quarterly equation is summed over four consecutive quarters. In other words, the equation is written down as it is and then lagged by one, two and three quarters and the four versions are then summed together. For the Philips curve, equation (2'), we then get (assuming only one autoregressive term):

$$\sum_{i=0}^{3} \pi_{t-i} = 4b_0 + b_1 \sum_{i=1}^{4} \pi_{t-i} + b_2 \left(\sum_{i=1}^{4} y_{t-i} - \sum_{i=1}^{4} y^{*}_{t-i} \right) \tag{9}$$

Since $\pi_t = \ln P_t - \ln P_{t-1}$, where P_t is the price level in quarter *t*, the left-hand side variable in equation (9) equals $\ln P_t - \ln P_{t-1} + \ln P_{t-1} - \ln P_{t-2} + ... + \ln P_{t-3} - \ln P_{t-4} = \ln P_t - \ln P_{t-4}$, which is the year-on-year quarterly inflation rate. This variable can be approximated by the annual inflation rate. By the same token, the second term on the right-hand side of equation (9) can be approximated by the annual inflation rate lagged by ¼ of the year, that is – using linear approximation – a weighted average of the current and lagged annual inflation rates with the weights 0.75 and 0.25, respectively. The last term in parenthesis in the equation is the sum of (logarithmic) output gaps for four consecutive quarters and can be approximated by four times the annual output gap, lagged as above. Using capital letters for annual data (in logs, where appropriate) we get the following annual approximation of equation (9):

$$\Pi_t = 4b_0 + b_1 \Pi_{t-0.25} + 4b_2 \left(Y_{t-0.25} - Y^{*}_{t-0.25} \right) \tag{10}$$

where $X_{t-0.25} = 0.75 X_t + 0.25 X_{t-1}$

The four-quarter sums for equations (1) and (5') are (assuming only one autoregressive term and one quarter's lag on the real rate in (1)):

$$\left(\sum_{i=0}^{3} y_{t-i} - \sum_{i=0}^{3} y_{t-i}^{*}\right) = 4a_0 + a_1\left(\sum_{i=1}^{4} y_{t-i} - \sum_{i=1}^{4} y_{t-i}^{*}\right) - a_2\left(\sum_{i=1}^{4} r_{t-i} - \sum_{i=1}^{4} \pi_{t-i}\right) \tag{11}$$

$$\sum_{i=0}^{3} r_{t-i} = k_1 \sum_{i=1}^{4} r_{t-i} + k_2\left(\sum_{i=0}^{3} \pi_{t-i}^{o} - 4k_3\right) + k_4\left(\sum_{i=0}^{3} y_{t-i} - \sum_{i=0}^{3} y_{t-i}^{*}\right) + 4k_5 \tag{12}$$

where k_3 is the constant inflation target, k_1, k_2, k_4 and k_5 correspond to the fixed coefficients in equation (5'); and in (12)

$$\sum_{i=0}^{3} \pi_{t-1}^{o} = \left(\ln P_t - \ln P_{t-4} + \ln P_{t-1} - \ln P_{t-5} + \ln P_{t-2} - \ln P_{t-6} + \ln P_{t-3} - \ln P_{t-7}\right)/4$$

$$= \left(\sum_{i=0}^{3} \ln P_{t-i} - \sum_{i=0}^{3} \ln P_{t-4-i}\right)\Big/4$$

is the annual inflation rate measured using the average annual price levels.

Following the approach outlined above, we get the following annual equations:

$$\left(Y_t - Y_t^{*}\right) = a_0 + a_1\left(Y_{t-0.25} - Y_{t-0.25}^{*}\right) - \left(a_2/4\right)\left(R_{t-0.25} - \Pi_{t-0.25}\right) \tag{13}$$

$$R_t = k_1 R_{t-0.25} + k_2\left(\Pi_t - 4k_3\right) + 4k_4\left(Y_t - Y_t^{*}\right) + 4k_5 \tag{14}$$

where the last term in (13) is the annual real rate of interest (lagged by 0.25 year).

The annual models for the USA and the euro zone thus comprise equations (10), (13) and (14).

We will use the OLS estimates to construct the annual models. This estimation method is technically less complicated than the UC method and treats the output gap series as given, which makes the future use of the model much easier. At the same time, the output gaps estimated with the UC method appear to be very similar to those estimated using the HP filter (cf. Diagram 1). Coefficient estimates are also rather similar.

The two annual regional models are as follows:

US:

$$\Pi_t = 3.0(1 - 0.5255 + 0.0512 - 0.4124) + 0.5255\Pi_{t-0.25} - 0.0512\Pi_{t-0.5} + 0.4124\Pi_{t-0.75} + 4 \cdot 0.0982(Y_{t-0.25} - Y^*_{t-0.25}) \quad (15)$$

$$(Y_t - Y_t^*) = 1.0430(Y_{t-0.25} - Y^*_{t-0.25}) - 0.0951(Y_{t-0.5} - Y^*_{t-0.5}) - 0.1925(Y_{t-0.75} - Y^*_{t-0.75}) - (0.1862/4)(R_{t-0.5} - \Pi_{t-0.5} - 2.4) \quad (16)$$

$$R_t = 0.2R_{t-0.25} + 1.5(\Pi_t - 3.0) + 0.5(Y_t - Y_t^*) + (1 - 0.2)(2.4 + 3.0) \quad (17)$$

Euro zone:

$$\Pi_t = 1.7(1 - 0.3862 + 0.0019 - 0.1763 - 0.2836) + 0.3862\Pi_{t-0.25} - 0.0019\Pi_{t-0.5} + 0.1763\Pi_{t-0.75} + 0.2836\Pi_{t-1} + 4 \cdot 0.1490(Y_{t-0.25} - Y^*_{t-0.25}) \quad (18)$$

$$(Y_t - Y_t^*) = 0.9457(Y_{t-0.25} - Y^*_{t-0.25}) + 0.0607(Y_{t-0.5} - Y^*_{t-0.5}) - 0.2912(Y_{t-0.75} - Y^*_{t-0.75}) - (0.1486/4)(R_{t-0.5} - \Pi_{t-0.5} - 2.4) \quad (19)$$

$$R_t = 0.3R_{t-0.25} + 1.2(\Pi_t - 1.7) + 0.4(Y_t - Y_t^*) + (1 - 0.3)(2.4 + 1.7) \quad (20)$$

The long-run solutions to the demand and supply relations were adjusted in accordance with the respective reaction functions. The long-run inflation rate is 3.0 % *per annum* in the USA and 1.7 % *per annum* in the euro zone. The intercepts in the Philips curves equal $\pi_t^*(1 - b_1 - b_2 - ... - b_h)$, giving the long-run inflation rate, π_t^*, as the long-run solution (cf. Section 2 on the basic model).

The long-run real interest rate is 2.4 % *per annum* in both regions. The intercepts in the demand equations are $(a_0/a_{k+1})a_{k+1}$, where the term in parenthesis is the equilibrium real rate (cf. Section 2). For clarity of exposition the equations are written in a slightly transformed form, the equilibrium real rate being subtracted from the lagged actual real rate in the last term of the demand relation. The intercept is obtained when the equilibrium real rate is written as a separate term.

9. Aggregation into one foreign sector

If aggregated results for an area comprising both the US and the euro zone are requested, the correct way is to use each regional model separately and then to aggregate the results. This procedure requires exact definitions of aggregate output gap, inflation and interest rate in terms of the corresponding series for both regions.

A simpler approach can be suggested if these aggregate series are taken as given. An approximate model can then be constructed consisting of equations (10), (13) and (14) with coefficients computed as weighted averages of the corresponding coefficients in the two regional models. If the aggregated variables are defined as arithmetic averages with the same weights as those employed in model aggregation, the approximation errors are zero when the two regional models have identical coefficients. Approximation errors depend otherwise on the differences between the corresponding coefficients in the regional models and on the differences between the weights.

10. The foreign sector model

The approximate, aggregate, annual model is constructed using the weights 58% for the US and 42% for the euro zone. These weights are derived from the International Monetary Fund's GDP weights for 2001. Alternatively, Swedish trade weights could also be employed, to allow for the effects of the two regions on Swedish trade.

Aggregate model

$$\Pi_t = 2.4540 \cdot 0.1312 + 0.4670\Pi_{t-0.25} - 0.0305\Pi_{t-0.5} + 0.3132\Pi_{t-0.75} + 0.1191\Pi_{t-1} + 0.4781\left(Y_{t-0.25} - Y^*_{t-0.25}\right) \tag{21}$$

$$\left(Y_t - Y^*_t\right) = 1.0021\left(Y_{t-0.25} - Y^*_{t-0.25}\right) + 0.0055\left(Y_{t-0.5} - Y^*_{t-0.5}\right) - 0.2340\left(Y_{t-0.75} - Y^*_{t-0.75}\right) - 0.0426\left(R_{t-0.5} - \Pi_{t-0.5} - 2.4\right) \tag{22}$$

$$R_t = 0.2420R_{t-0.25} + 1.3740\left(\Pi_t - 2.4540\right) + 0.4580\left(Y_t - Y^*_t\right) + 0.7543\left(2.4 + 2.4540\right) \tag{23}$$

$$\ln P_t = \ln P_{t-1} + \Pi_t / 100 \tag{24}$$

$$Y^*_t = Y^*_{t-1} + g \tag{25}$$

where g is the exogenous potential growth rate, defined as the sum of the labour productivity growth rate and the growth rate of the labour force. The labour productivity growth rate can in turn be distributed into the total factor productivity growth and the effect of productive capital growth on output.

Using the definition of a fractional lag, given after equation (10) above, and solving for the non-lagged left-hand side variables, equations (21) – (23) become:

$$\Pi_t = 2.4540\left(1 - 0.7765\right) + 0.7765\Pi_{t-1} + 0.8149\left[0.75\left(Y_t - Y^*_t\right) - 0.25\left(Y_{t-1} - Y^*_{t-1}\right)\right] \tag{21'}$$

$$\left(Y_t - Y^*_t\right) = 0.2557\left(Y_{t-1} - Y^*_{t-1}\right) - 0.1400\left[0.5\left(R_t + R_{t-1}\right) - 0.5\left(\Pi_t + \Pi_{t-1}\right) - 2.4\right] \tag{22'}$$

$$R_t = 0.0739R_{t-1} + 1.6787\left(\Pi_t - 2.4540\right) + 0.5596\left(Y_t - Y^*_t\right) + \left(1 - 0.0739\right)\left(2.4 + 2.4540\right) \tag{23'}$$

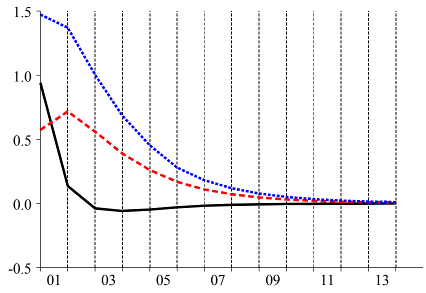
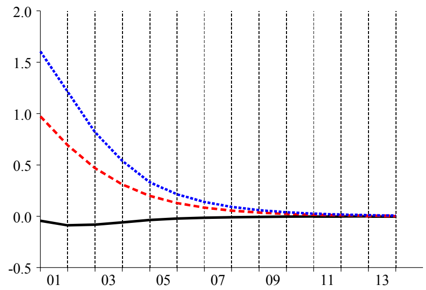
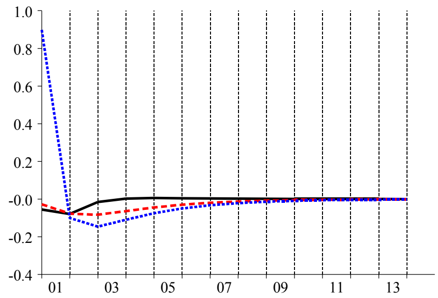
The target inflation rate and the equilibrium real rate of interest, assumed to equal 2.4540 and 2.4 respectively, are written explicitly in the model equations above, to show how the model depends on these equilibrium values. The model is complemented with equations (24) and (25), determining the (log of the) price level and the (log of the) level of the potential output. Thus, a time path for the price level, the GDP and the potential GDP can be determined from model simulations, given initial conditions for those three variables.

Diagram 5 shows the impulse responses obtained from simulations with the aggregate, annual model. As in Section 7, a one-off shock of 1 percentage point was introduced in one equation in year one. The unit of time is here one year, rather than one quarter as in the simulations above (the time axis is scaled in years both in Diagram 5 and in Diagrams 3 and 4).

As can be seen in the diagram, the short-term dynamics of the annual model differ from those of the quarterly models, since temporal aggregation has increased the degree of simultaneity of the annual model as compared to the quarterly ones. This is also the reason why the variables shocked do not increase by a full percentage point the first year. The initial interest rate reaction is much stronger. As a result, the interest rate varies more than in the quarterly simulations, while the output gap varies significantly less. The inflation path is also smoother.

The annual model regains equilibrium approximately nine years after the shock, similarly to the quarterly models. This indicates that temporal aggregation has preserved the models' dynamics in the medium term.

Diagram 5 Simulation results for the aggregate, annual model

<table>
<tr><td>Demand shock</td><td>
</td></tr>
<tr><td>Supply shock</td><td>
</td></tr>
<tr><td>Monetary policy shock</td><td>
</td></tr>
</table>

Output gap — inflation rate — interest rate

LITERATURE

Apel M. and P.Jansson, (1997), *System Estimates of Potential Output and the NAIRU* Sveriges Riksbank Working Paper No. 41.

Congressional Budget Office (1995), *CBO's Method for Estimating Potential Output*, CBO Memorandum, October 1995.

Favero, C.A., *Applied Macroeconometrics* (2001), Oxford: Oxford University Press.

Fagan, J. Henry, Mestre (2001), *An area-wide model for the euro area*, Working Paper No. 42, European Central Bank.

Harvey A. C. (1990), *Forecasting, structural time series models and the Kalman filter*, Cambridge: Cambridge University Press.

Jacobs (1993), *Introduction to Control Theory*, Oxford: Oxford University Press.

Kuttner K. N. (1994), ”Estimating Potential Output as a Latent Variable” *Journal of Business & Economic Statistics*, Vol. 12, No. 3, July 1994.

Markowski, A. (1995), *Determination of the effective exchange rate in the econometric model Kosmos*, Stockholm: National Institute of Economic Research, Working Paper No. 47.

Rudebusch, G.D and L.E.O. Svensson, (1999a) “Policy rules for inflation targeting”, in J.B. Taylor (ed.) *Monetary Policy Rules*, Chicago: The University of Chicago Press.

Rudebusch, G.D and L.E.O. Svensson, (1999b), *Eurosystem Monetary Targeting: Lessons from U.S. Data*, Sveriges Riksbank Working Paper No. 92, September 1999.

Ruist, E, (1996), *Temporal aggregation of an econometric equation*, Stockholm: National Institute of Economic Research, Working Paper No. 52.

Smets F. and G. Peersman (1999), ”The Taylor Rule: A Useful Monetary Policy Benchmark for the Euro Area?” *International Finance*, Vol. 2, No. 1, April 1999.

Svensson, L.E.O., (1999), *Price Stability as a Target for Monetary Policy: Defining and Maintaining Price Stability*, Sveriges Riksbank Working Paper No. 91, September 1999.

Svensson, L.E.O., (1997), “Inflation forecast targeting: Implementing and monitoring inflation targets”, *European economic Review* 41, June 1997.

Söderström, U., (1999), *Should central banks be more aggressive?*, Sveriges Riksbank Working Paper No. 84, May 1999.

Titles in the Working Paper Series

No	*Author*	*Title*	*Year*
1	Warne, Anders and Anders Vredin	Current Account and Business Cycles: Stylized Facts for Sweden	1989
2	Östblom, Göran	Change in Technical Structure of the Swedish Economy	1989
3	Söderling, Paul	Mamtax. A Dynamic CGE Model for Tax Reform Simulations	1989
4	Kanis, Alfred and Aleksander Markowski	The Supply Side of the Econometric Model of the NIER	1990
5	Berg, Lennart	The Financial Sector in the SNEPQ Model	1991
6	Ågren, Anders and Bo Jonsson	Consumer Attitudes, Buying Intentions and Consumption Expenditures. An Analysis of the Swedish Household Survey Data	1991
7	Berg, Lennart and Reinhold Bergström	A Quarterly Consumption Function for Sweden 1979-1989	1991
8	Öller, Lars-Erik	Good Business Cycle Forecasts- A Must for Stabilization Policies	1992
9	Jonsson, Bo and Anders Ågren	Forecasting Car Expenditures Using Household Survey Data	1992
10	Löfgren, Karl-Gustaf, Bo Ranneby and Sara Sjöstedt	Forecasting the Business Cycle Not Using Minimum Autocorrelation Factors	1992
11	Gerlach, Stefan	Current Quarter Forecasts of Swedish GNP Using Monthly Variables	1992
12	Bergström, Reinhold	The Relationship Between Manufacturing Production and Different Business Survey Series in Sweden	1992
13	Edlund, Per-Olov and Sune Karlsson	Forecasting the Swedish Unemployment Rate: VAR vs. Transfer Function Modelling	1992
14	Rahiala, Markku and Timo Teräsvirta	Business Survey Data in Forecasting the Output of Swedish and Finnish Metal and Engineering Industries: A Kalman Filter Approach	1992
15	Christofferson, Anders, Roland Roberts and Ulla Eriksson	The Relationship Between Manufacturing and Various BTS Series in Sweden Illuminated by Frequency and Complex Demodulate Methods	1992
16	Jonsson, Bo	Sample Based Proportions as Values on an Independent Variable in a Regression Model	1992
17	Öller, Lars-Erik	Eliciting Turning Point Warnings from Business Surveys	1992
18	Forster, Margaret M	Volatility, Trading Mechanisms and International Cross-Listing	1992
19	Jonsson, Bo	Prediction with a Linear Regression Model and Errors in a Regressor	1992
20	Gorton, Gary and Richard Rosen	Corporate Control, Portfolio Choice, and the Decline of Banking	1993
21	Gustafsson, Claes-Håkan and Åke Holmén	The Index of Industrial Production – A Formal Description of the Process Behind it	1993
22	Karlsson, Tohmas	A General Equilibrium Analysis of the Swedish Tax Reforms 1989-1991	1993
23	Jonsson, Bo	Forecasting Car Expenditures Using Household Survey Data- A Comparison of Different Predictors	1993

24	Gennotte, Gerard and Hayne Leland	Low Margins, Derivative Securitites and Volatility	1993
25	Boot, Arnoud W.A. and Stuart I. Greenbaum	Discretion in the Regulation of U.S. Banking	1993
26	Spiegel, Matthew and Deane J. Seppi	Does Round-the-Clock Trading Result in Pareto Improvements?	1993
27	Seppi, Deane J.	How Important are Block Trades in the Price Discovery Process?	1993
28	Glosten, Lawrence R.	Equilibrium in an Electronic Open Limit Order Book	1993
29	Boot, Arnoud W.A., Stuart I Greenbaum and Anjan V. Thakor	Reputation and Discretion in Financial Contracting	1993
30a	Bergström, Reinhold	The Full Tricotomous Scale Compared with Net Balances in Qualitative Business Survey Data – Experiences from the Swedish Business Tendency Surveys	1993
30b	Bergström, Reinhold	Quantitative Production Series Compared with Qualiative Business Survey Series for Five Sectors of the Swedish Manufacturing Industry	1993
31	Lin, Chien-Fu Jeff and Timo Teräsvirta	Testing the Constancy of Regression Parameters Against Continous Change	1993
32	Markowski, Aleksander and Parameswar Nandakumar	A Long-Run Equilibrium Model for Sweden. The Theory Behind the Long-Run Solution to the Econometric Model KOSMOS	1993
33	Markowski, Aleksander and Tony Persson	Capital Rental Cost and the Adjustment for the Effects of the Investment Fund System in the Econometric Model Kosmos	1993
34	Kanis, Alfred and Bharat Barot	On Determinants of Private Consumption in Sweden	1993
35	Kääntä, Pekka and Christer Tallbom	Using Business Survey Data for Forecasting Swedish Quantitative Business Cycle Varable. A Kalman Filter Approach	1993
36	Ohlsson, Henry and Anders Vredin	Political Cycles and Cyclical Policies. A New Test Approach Using Fiscal Forecasts	1993
37	Markowski, Aleksander and Lars Ernsäter	The Supply Side in the Econometric Model KOSMOS	1994
38	Gustafsson, Claes-Håkan	On the Consistency of Data on Production, Deliveries, and Inventories in the Swedish Manufacturing Industry	1994
39	Rahiala, Markku and Tapani Kovalainen	Modelling Wages Subject to Both Contracted Increments and Drift by Means of a Simultaneous-Equations Model with Non-Standard Error Structure	1994
40	Öller, Lars-Erik and Christer Tallbom	Hybrid Indicators for the Swedish Economy Based on Noisy Statistical Data and the Business Tendency Survey	1994
41	Östblom, Göran	A Converging Triangularization Algorithm and the Intertemporal Similarity of Production Structures	1994
42a	Markowski, Aleksander	Labour Supply, Hours Worked and Unemployment in the Econometric Model KOSMOS	1994
42b	Markowski, Aleksander	Wage Rate Determination in the Econometric Model KOSMOS	1994
43	Ahlroth, Sofia, Anders Björklund and Anders Forslund	The Output of the Swedish Education Sector	1994
44a	Markowski, Aleksander	Private Consumption Expenditure in the Econometric Model KOSMOS	1994

44b	Markowski, Aleksander	The Input-Output Core: Determination of Inventory Investment and Other Business Output in the Econometric Model KOSMOS	1994
45	Bergström, Reinhold	The Accuracy of the Swedish National Budget Forecasts 1955-92	1995
46	Sjöö, Boo	Dynamic Adjustment and Long-Run Economic Stability	1995
47a	Markowski, Aleksander	Determination of the Effective Exchange Rate in the Econometric Model KOSMOS	1995
47b	Markowski, Aleksander	Interest Rate Determination in the Econometric Model KOSMOS	1995
48	Barot, Bharat	Estimating the Effects of Wealth, Interest Rates and Unemployment on Private Consumption in Sweden	1995
49	Lundvik, Petter	Generational Accounting in a Small Open Economy	1996
50	Eriksson, Kimmo, Johan Karlander and Lars-Erik Öller	Hierarchical Assignments: Stability and Fairness	1996
51	Url, Thomas	Internationalists, Regionalists, or Eurocentrists	1996
52	Ruist, Erik	Temporal Aggregation of an Econometric Equation	1996
53	Markowski, Aleksander	The Financial Block in the Econometric Model KOSMOS	1996
54	Östblom, Göran	Emissions to the Air and the Allocation of GDP: Medium Term Projections for Sweden. In Conflict with the Goals of SO_2, SO_2 and NOX Emissions for Year 2000	1996
55	Koskinen, Lasse, Aleksander Markowski, Parameswar Nandakumar and Lars-Erik Öller	Three Seminar Papers on Output Gap	1997
56	Oke, Timothy and Lars-Erik Öller	Testing for Short Memory in a VARMA Process	1997
57	Johansson, Anders and Karl-Markus Modén	Investment Plan Revisions and Share Price Volatility	1997
58	Lyhagen, Johan	The Effect of Precautionary Saving on Consumption in Sweden	1998
59	Koskinen, Lasse and Lars-Erik Öller	A Hidden Markov Model as a Dynamic Bayesian Classifier, with an Application to Forecasting Business-Cycle Turning Points	1998
60	Kragh, Börje and Aleksander Markowski	Kofi – a Macromodel of the Swedish Financial Markets	1998
61	Gajda, Jan B. and Aleksander Markowski	Model Evaluation Using Stochastic Simulations: The Case of the Econometric Model KOSMOS	1998
62	Johansson, Kerstin	Exports in the Econometric Model KOSMOS	1998
63	Johansson, Kerstin	Permanent Shocks and Spillovers: A Sectoral Approach Using a Structural VAR	1998
64	Öller, Lars-Erik and Bharat Barot	Comparing the Accuracy of European GDP Forecasts	1999
65	Huhtala , Anni and Eva Samakovlis	Does International Harmonization of Environmental Policy Instruments Make Economic Sense? The Case of Paper Recycling in Europe	1999
66	Nilsson, Charlotte	A Unilateral Versus a Multilateral Carbon Dioxide Tax - A Numerical Analysis With The European Model GEM-E3	1999

67	Braconier, Henrik and Steinar Holden	The Public Budget Balance – Fiscal Indicators and Cyclical Sensitivity in the Nordic Countries	1999
68	Nilsson, Kristian	Alternative Measures of the Swedish Real Exchange Rate	1999
69	Östblom, Göran	An Environmental Medium Term Economic Model – EMEC	1999
70	Johnsson, Helena and Peter Kaplan	An Econometric Study of Private Consumption Expenditure in Sweden	1999
71	Arai, Mahmood and Fredrik Heyman	Permanent and Temporary Labour: Job and Worker Flows in Sweden 1989-1998	2000
72	Öller, Lars-Erik and Bharat Barot	The Accuracy of European Growth and Inflation Forecasts	2000
73	Ahlroth, Sofia	Correcting Net Domestic Product for Sulphur Dioxide and Nitrogen Oxide Emissions: Implementation of a Theoretical Model in Practice	2000
74	Andersson, Michael K. And Mikael P. Gredenhoff	Improving Fractional Integration Tests with Bootstrap Distribution	2000
75	Nilsson, Charlotte and Anni Huhtala	Is CO_2 Trading Always Beneficial? A CGE-Model Analysis on Secondary Environmental Benefits	2000
76	Skånberg, Kristian	Constructing a Partially Environmentally Adjusted Net Domestic Product for Sweden 1993 and 1997	2001
77	Huhtala, Anni, Annie Toppinen and Mattias Boman,	An Environmental Accountant's Dilemma: Are Stumpage Prices Reliable Indicators of Resource Scarcity?	2001
78	Nilsson, Kristian	Do Fundamentals Explain the Behavior of the Real Effective Exchange Rate?	2002
79	Bharat, Barot	Growth and Business Cycles for the Swedish Economy	2002
80	Bharat, Barot	House Prices and Housing Investment in Sweden and the United Kingdom. Econometric Analysis for the Period 1970-1998	2002
81	Hjelm, Göran	Simultaneous Determination of NAIRU, Output Gaps and Structural Budget Balances: Swedish Evidence	2003
82	Huhtala, Anni and Eva Samalkovis	Green Accounting, Air Pollution and Health	2003
83	Lindström, Tomas	The Role of High-Tech Capital Formation for Swedish Productivity Growth	2003
84	Hansson, Jesper, Per Jansson and Mårten Löf	Business survey data: do they help in forecasting the macro economy?	2003
85	Boman, Mattias, Anni Huhtala, Charlotte Nilsson, Sofia Ahlroth, Göran Bostedt, Leif Mattson and Peichen Gong	Applying the Contingent Valuation Method in Resource Accounting: A Bold Proposal	
86	Gren, Ing-Marie	Monetary Green Accounting and Ecosystem Services	2003
87	Samakovlis, Eva, Anni Huhtala, Tom Bellander and Magnus Svartengren	Air Quality and Morbidity: Concentration-response Relationships for Sweden	2004
88	Alsterlind, Jan, Alek Markowski and Kristian Nilsson	Modelling the Foreign Sector in a Macroeconometric Model of Sweden	2004