Preface

This paper is a progress report on the work with the econometric model KOSMOS. As previous papers on KOSMOS in the Working Paper series, it is intended to constitute part of a future comprehensive report on the whole model, and hence is not completely self-contained.

DETERMINATION OF THE EFFECTIVE EXCHANGE RATE IN THE ECONOMETRIC MODEL KOSMOS

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INTRODUCTION

The literature on exchange rate determination is vast as regards both theory and empirical investigations (cf. the references in MacDonald and Taylor [1992]). Few authors, however, claim any success in the econometric modelling of exchange rate variation. Hacche and Townend [1981] wrote in conclusion of their investigation of competing empirical models: "The predominant impression left by our results is one of failure. We have not succeeded in finding any empirical regularities in the data." This generally pessimistic view is shared by a number of applied econometricians and model builders (cf. the references in Currie and Hall [1989]).

A notable exception are Currie and Hall [1989] themselves who note that the equations of the National Institute of Economic and Social Research model "are of some interest because not only are they examples of estimated exchange rate equations which seem to be reasonably data coherent, but they have been tested in practical forecasting exercises for a number of years".

Wallis et al. [1985] distinguish conceptually two approaches to exchange rate determination. According to the first one, the level of the real exchange rate is obtained by inverting the trade balance equation, conditional upon the level of the trade (or current account) balance and other endogenous variables. Alternatively, the capital account equation is solved for the nominal exchange rate, again conditional upon the trade balance (which determines the net capital flow). In a portfolio choice model, a flow version (i.e. the first difference) of the capital account equation is inverted, giving the percentage change of the nominal exchange rate as the dependent variable. Wallis et al. point out that the level of the exchange rate actually is included in the wealth term, which usually is omitted from the right-hand side of the exchange rate equation.

The portfolio choice model belongs to the class of asset market models, employed to determine the nominal exchange rate (cf. Pentecost [1991]). It assumes imperfect asset substitutability and thus involves both money and bonds as exchange rate determinants. The monetary approach to exchange rate determination posits, on the other hand, perfect substitutability between domestic and foreign bonds, thereby reducing the number of assets. The equilibrium exchange rate in this approach depends on excess demands for domestic and foreign money, the bond market being excluded in accordance with Walras law.
Early (flexible price) monetary models assumed continuous purchasing power parity (PPP). As this assumption was subsequently rejected by empirical literature (cf. MacDonald and Taylor [1992]), the sticky-price monetary model (cf. Dornbusch [1976]) allowed for the slow adjustment of goods prices which gave rise to the overshooting of the exchange rate above its long-run (PPP) equilibrium.

Today, the portfolio choice model is the standard tool for deriving the private demand for (net) foreign assets. Its implication is that capital flows are affected by changes in interest rates (or in the interest rate differential) rather than levels. Currie and Hall [1989] postulate a rather unusual model "designed to capture both stock and flow aspects of capital movements" and thus include both interest rate levels and their changes in the capital balance equation. The resultant exchange rate equation, derived using an intertemporal objective function for investors, requires non-standard estimation techniques.

Below, we follow the standard capital-account approach and derive a simple exchange rate equation from the portfolio choice model. No tests of the Currie-Hall model were possible, due to the extreme data shortage to be discussed later on. The data problems called for a relatively simple and "robust" approach that would provide reasonable exchange rate reaction patterns despite the high uncertainty about the coefficient values. Our approach largely follows that of H.M. Treasury (cf. Mellis et al. [1989] and H.M. Treasury [1990]). This applies in particular to the expectations formation, though some tests of the rational expectations hypothesis were also made. Ideally, expectations formation should combine autoregressive and forward-looking elements (cf. Laxton and Tetlow [1992]) rather than being confined to only one of the two approaches. MacDonald and Torrance [1990] demonstrate that the erroneous assumption of (completely) rational expectations is most likely to be one of the reasons for the strong rejection of uncovered interest parity in the empirical literature, cf. also MacDonald and Taylor [1992].

The outline of the paper is as follows. The theoretical model is derived in the next section. Subsequently, data problems and estimation results for the monthly exchange rate function are reported. The last section provides the derivation of the semi-annual exchange rate function.
THE MODEL.

Our exchange rate equation is derived from the (private) foreign-asset market equilibrium condition. The demand for net foreign assets is postulated, within the general framework of the portfolio choice model, as depending on the domestic interest rate, the foreign interest rate adjusted for the expected change in the exchange rate and the total financial wealth:

\[(Y.1) \quad \frac{(F/\epsilon)}{W} = -a_1 r + a_2 [r^f - (\hat{e} - e)],\]

where

- \( F \) - net foreign assets expressed in foreign currency,
- \( \epsilon \) - exchange rate, i.e. foreign price of domestic currency,
- \( W \) - total financial wealth,
- \( r \) - domestic rate of interest,
- \( r^f \) - foreign rate of interest,
- \( \hat{e} = \log(e) \),
- \( \hat{e} \) - expected value of \( e \) one period ahead,
- \( a_1, a_2 \) - coefficients, \( a_1, a_2 > 0 \).

Postulating, further, that the portfolio composition is affected by the relative yields rather than their levels, i.e. \( a_1 = a_2 \), we obtain:

\[(Y.2) \quad \frac{(F/\epsilon)}{W} = -a_1 (r - r^f) - a_1 (\hat{e} - e).\]

The coefficient \( a_1 \) reflects here the degree of capital mobility with perfect mobility resulting in \( a_1 \) equal to infinity.

The supply of net foreign assets to the private sector is derived from the balance of payments identity which states that the sum of the current account balance and the (private and public) capital account balance equals the change in the official foreign reserves:

\[(Y.3) \quad \text{CurrB} + \text{CapB}/\epsilon + \text{CapBPub}/\epsilon = \Delta \text{F/O}/\epsilon,\]

where

- \( \text{CurrB} \) - current account balance,
- \( \text{CapB} \) - private sector capital account balance,
CapBPub - public (i.e. government) capital balance,
DFO - change in the official (central-bank) foreign reserves,
all capital account variables being expressed in foreign currency.

The distinction between the public capital balance and the change in the official foreign reserves is introduced in order to separate the actions of the government and of the central bank in the model.

The supply of net foreign assets to the private sector is obtained from equation (Y.3), keeping in mind that the capital account balance is defined as the change in net foreign *liabilities* rather than assets:

\[(Y.4)\]
\[-\text{CapB}/\epsilon = \text{CurrB} + \text{CapBPub}/\epsilon - \text{DFO}/\epsilon.\]

Since equation (Y.2) defines the demand for market-clearing portfolio investment, i.e. that part of demand for net foreign assets which actually depends on exchange rate expectations, the supply variable is further redefined to exclude non-market-clearing flows, such as direct investment:

\[(Y.5)\]
\[-\text{CapBmc}/\epsilon = \text{CurrB} + \text{CapBPub}/\epsilon + \text{CapBnmc}/\epsilon - \text{DFO}/\epsilon,\]

where
\[
\text{CapBmc} = \text{CapB} - \text{CapBnmc},
\]
\[
\text{CapBnmc} - \text{non-market-clearing private sector capital account transactions (i.e. transactions which do not dependent on exchange rate expectations),}
\]
all expressed in foreign currency.

Equation (Y.2) gives the *stock* demand for net foreign assets, while equation (Y.5) determines the capital *flow*. In order to construct a flow equilibrium condition, the demand equation is rewritten as

\[(Y.6)\]
\[F/\epsilon = [-a_1 (r - rf) - a_1 (\bar{e} - e)] \ W = z \ W,
\]
\[z = -a_1 (r - rf) - a_1 (\bar{e} - e),\]

and then differenced to give:

\[(Y.7)\]
\[D(F)/\epsilon = D(z) \ W + z_{1}D(W) - F_{1}D(1/\epsilon),\]
where

$$D(X) = X_t - X_{t-1},$$

the last term in (Y.7) corresponding to the valuation gain from holding foreign assets.

Below, the last two terms in equation (Y.7) will be neglected as being of secondary importance for the explanation of foreign currency flows. Our flow demand equation thus takes the form:

$$D(F)/\epsilon = -a_1 \left[ D(r - r') - D(\hat{e} - e) \right] W. \tag{Y.8}$$

Substituting equation (Y.8) into the market flow-equilibrium identity, $D(F)/\epsilon = -\text{CapBmc}/\epsilon$, and solving for $D(e)$ we obtain

$$D(e) = D(\hat{e}) + D(r - r') + \frac{1}{a_1} \left( -\text{CapBmc}/\epsilon \right)/W, \tag{Y.9}$$

where $-\text{CapBmc}/\epsilon$ is defined in equation (Y.5).

Since in Sweden exchange rates are traditionally expressed in terms of SEK per foreign currency unit (implying that the thus defined exchange rate increases when the krona depreciates), equation (Y.9) will be rewritten in terms of $e = \log(1/\epsilon)$:

$$D(-e) = D(\hat{e}) - D(r - r') - \frac{1}{a_1} \left( -\text{CapBmc}/\epsilon \right)/W. \tag{Y.10}$$

Thus, according to our equation, the relative change in the exchange rate is determined by the expected rate of change of the exchange rate, the change in the interest rate differential and the market-clearing net capital flow in relation to the financial wealth. Note the unit coefficients in front of the first two variables.

The role of the net capital flow deserves some comment. As can be seen, the change in (market-clearing) net foreign liabilities (i.e. the market-clearing capital account balance rather than its opposite) enters equation (Y.10) with a positive sign. This means that a (private) capital inflow is associated with a depreciating currency, contrary to the case when the currency is pegged. The difference is due to the changed role of the central bank - and its foreign reserves - under the floating exchange rate system.
Assuming that the official foreign reserves are kept constant (as the central bank under the latter regime no longer can be expected to purchase the foreign currency supplied\(^1\)), the net foreign assets of the private sector cannot be changed by exclusively lending or borrowing. In fact, the change in the net foreign assets of the private sector is under the above assumptions equal to the current account balance. Consequently, e.g. a current account surplus implies a corresponding outflow through the capital account. This outflow can take place (given the interest rate differential) only if a currency depreciation is expected. Thus, in our model, a current account surplus gives rise to an immediate (excessive) appreciation, accompanied by a capital outflow caused by the expected depreciation of the currency.

The remainder of this section will be devoted to the treatment of expectations. If rational expectations are postulated, the standard procedure is to replace the expected values with the actual lead values of the variable in question. As an alternative, backward-looking expectations will be considered below.

Following H.M. Treasury [1990], it is assumed that the exchange rate is expected to obey a simple error-correction mechanism of the form:

\[
(Y.11) \quad D(-\delta_e) = r D(-\delta_e) - \delta [(-\delta_e)_t - (-\delta_e)_{t-1}],
\]

where

\[\delta_e - \text{logarithm of the long-run sustainable exchange rate},\]
\[r, \delta - \text{coefficients, } r, \delta > 0.\]

Equation (Y.11) can be obtained by applying the error-correction mechanism to the actual rate and then combining it with static expectations, \(\delta_e = \delta_e\) (cf. Wallis et al. [1985]).

The long-run sustainable exchange rate can in principle be determined from either (or a mixture) of the two distinct assumptions regarding the arbitrage possibility in the goods market. If domestic and foreign goods are assumed to be perfect substitutes, purchasing power parity should prevail and the real exchange rate should in the long run be constant.

If, on the other hand, domestic and foreign goods are not substitutable (the so called Armington assumption), the long-run sustainable exchange rate is the one that brings

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\(^1\) Government foreign borrowing is assumed away as well, for the sake of exposition.
the current account balance to zero (the balance being in this case affected by the price
elasticities of the domestic and foreign demand).

In both cases, non-optimal exchange rates can probably be sustained for very long
periods of time through appropriate adjustment of the interest rate differential, making
the country a long-run importer or exporter of capital.

Since most of the products in the international trade indeed are very close substitutes,
the long-run real exchange rate is here assumed to be constant, subject - as explained
above - to the variation in the interest rate differential. Consequently, the long-run
sustainable nominal rate is determined by the (long-run) relation between domestic and
foreign prices and the interest rate differential:

\[(Y.12)\quad -e^t = p - \beta (r - r^f) + \alpha,\]

where

\[p = \log(p^d/p^f),\]
\[p^d/p^f\ - \text{long-run relation between domestic and foreign prices,}\]
\[\beta, \alpha \ - \text{coefficients, } \beta > 0.\]

Upon substitution of equation (Y.12) into (Y.11) and of the latter into equation (Y.10)
we finally obtain our exchange rate equation:

\[(Y.13)\quad D(-e) = \tau D(p) - (\tau \beta + 1) D(r - r^f) - \delta (e_{-1} - p_{-1})
- \delta \beta (r_{-1} - r^f_{-1}) - (1/a_1) (-\text{CapBmc}/e)/W + \delta \alpha.\]

As can be seen, equation (Y.13) is overidentified, since five structural parameters (\(\tau, \beta,
\delta, a_1 \text{ and } \alpha\)) are determined by six estimated coefficients.

Data problems and data definitions

Estimation of the exchange rate function derived above encountered tremendous
problems due to the recent change of the exchange rate regime in Sweden. The Swedish
krona was floated on the 19-th November, 1992, after a long period of basket peg. This
leaves us with four semi-annual observations which, furthermore, cover a rather
turbulent transition period in the exchange rate market.
In an attempt to circumvent the data problems, we decided to arbitrarily set the values of the structural coefficients, trying to get support from their ability to explain the monthly data for the last two years (1993-94), as well as from the results published in other countries. However, preliminary estimation on monthly data proved to give results that were rather close to our guesses. Therefore, below we simply report the estimated equations. It should be stressed that - due to the limited and shaky data base - we consider the results to be qualified guesses rather than anything else. The standard test statistics are reported, although statistical inference in this case is hardly possible.

The exchange rate is here defined as the effective exchange rate index for fourteen OECD countries. The index was computed as a weighted, geometric mean, with weights based on the sum of exports, imports and capital flows by currency in 1993. The development of the index in 1993-94 is shown in Chart 2. Chart 1 puts the values of those two years in a historical perspective.

The domestic interest rate is defined as the monthly average for the effective rate on ninety-day treasury notes. As for the foreign rate, different weighting schemes in respect of the currencies included were tested. Finally we opted for the monthly average of the three-month Euro-Deutsch Mark rate as the best representative of the foreign interest rate. The interest rate differential was subsequently expressed as a monthly rather than an annual rate and as a fraction rather than a percentage. The development of the interest rate differential on a per cent per annum basis is illustrated in Chart 4.

The market-clearing capital flow is defined as the private sector capital account balance (including the Central Bank lending to banks in foreign currency) minus the (net) direct investment and the (net) portfolio equity investment. (The two latter items represent the structural, i.e. non-market clearing, flows.) This definition of the market-clearing net capital flow exactly corresponds to equation \((Y.5)\) and includes both trade credit and the errors and omissions item in the balance of payments. It should be noted that the variable included in the estimated equation is \(\text{CapBmc}/\epsilon\) rather than \(-\text{CapBmc}/\epsilon\). The net capital flow variable, expressed as a share of SEK M3 lagged by one period, is illustrated in Chart 3.

The scale variable, financial wealth, is represented by SEK M3, defined as the sum of currency in circulation, bank deposits in SEK and the National Savings Scheme (\textit{Allmanskassan}), the latter for the depositor being comparable with a time deposit.

\[\text{Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, The Netherlands, Norway, Switzerland, UK, USA.}\]

\[\text{Consequently the interest rate differential was divided by 1200, i.e. multiplied by 0.00083.}\]
Chart 1. Effective exchange rate index (SEK/foreign currency), 1991 = 1, monthly figures 1980-94

Chart 2. Effective exchange rate index (SEK/foreign currency), 1991 = 1, monthly figures 1993-94
Chart 3. Market-clearing private sector capital balance (solid line) and Government foreign-currency borrowing (dashed line), both expressed as a share of SEK M3. Monthly figures 1993-94

Chart 4. Relative price Sweden-OECD 14 (solid line) and the interest rate differential between Sweden and Germany (dashed line)
The definition of SEK M3 excludes foreign currency deposits and certificates of deposit. The variable was lagged in the estimated equation, in accordance with the practice of discrete differencing (cf. equation (Y.7)), in order to limit the degree of simultaneity in KOSMOS.

The relation between the domestic and foreign prices was expressed in terms of tradeables rather than overall consumer prices, since the exchange rate is here postulated to be primarily affected by foreign-trade arbitrage. The domestic price is defined as the export price index for manufactures, 1991 = 100. The foreign price is a weighted import-price index (1991 = 100) for the 14 OECD countries mentioned above, computed using the Swedish export shares as weights. Due to the lack of data, the monthly index for 1994 was obtained by first extrapolating a corresponding quarterly index and then distributing it by month. The ratio of the two price indices was employed as an approximation of the long-run price relation. This ratio is shown in Chart 4.

THE MONTHLY EXCHANGE RATE EQUATION

Monthly exchange rate equations were estimated for the period 1993:8 - 94:12. The first seven months of 1993 were excluded as the period of transition to the new exchange rate regime. This period was characterised inter alia by extensive foreign borrowing by the government during the periods of weaker currency and (possibly not unrelated) strong currency outflows during the same periods (cf. Chart 3). In fact, estimation over the whole period 1993:1 - 94:12 resulted in the coefficient of the capital flow variable having the (wrong) negative sign.

OLS estimation of the rational-expectations equation (Y.10) gave the following results:

(Y.14)

\[
\text{dlog}(v_x) = \text{dlog}(v_{x+1}) - D(r - r_f) + 0.14369 \times \text{CapF/M3SEK}_{-1} + 0.00109
\]

\[ (0.16135) \quad (0.14804) \]

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<th>Std Err</th>
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<td>R Bar Sq</td>
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<td>D.W. (12)</td>
<td>0.5832</td>
<td>Est. per. 1993:8-94:12</td>
<td></td>
</tr>
</tbody>
</table>
where

- effective exchange rate index expressed in SEK per foreign currency unit,
- interest rate differential between Sweden and Germany, fraction per month,
- change in net foreign liabilities of the private sector, due to market-clearing transactions, SEK mill.,
- SEK M3 (excluding certificates of deposit and including the National Saving Scheme), SEK mill.,
- subscript indicates the time period to which the variable applies.

The coefficients of the first two variables in equation (Y.14) were constrained to unity. The capital flow variable is expected to have a positive sign, since it is defined as a change in net foreign liabilities (corresponding to the capital account balance) rather than in net foreign assets.

OLS estimation of the adaptive-expectations equations (Y.13) gave the following results:

(Y.15)

\[
\begin{align*}
\text{dlog}(vx) &= 0.49863 \times \text{dlog}(RP) - 18.7000 \times D(r - r^f) \\
&\quad - 0.36243 \times \text{log}(vx_{-1}/RP_{-1}) - 12.8915 \times (r_{-1} - r^f_{-1}) \\
&\quad + 0.58920 \times \text{CapF/M3SEK}_{-1} + 0.07587 \\
&\quad \text{(0.73489)} \quad \text{(NC)} \quad \text{(1.24051)} \quad \text{(1.22788)} \quad \text{(0.83879)} \quad \text{(1.36584)}
\end{align*}
\]

Sum Sq 0.0032  Std Err 0.0163  LHS Mean 0.0003
R Sq 0.2197  R Bar Sq -0.0405  F 4, 12 0.8444
D.W.(1) 1.5393  D.W.(12) 0.4214  Est.per. 1993:8–94:12

where

- RP - relation between Swedish prices and foreign prices,
- \(vx/RP\) - real exchange rate.

The coefficient of the second variable in equation (Y.15) was constrained to the value shown, in accordance with the (overidentified) form of equation (Y.13) above. The coefficients and the fit of the estimated equation were relatively little affected by the constraint.
The fit of the estimated equations is illustrated in Charts 5 and 6. As can be seen, equation (Y.15) correctly indicates most of the turning points, but the amplitude of its variation is much more limited than that of the dependent variable. Consequently, it appears that the equation implies a reasonable reaction pattern for the exchange rate, but that it is likely to underestimate its variation.

The fit of equation (Y.14) indicates some major problems. It should be, however, reiterated that our limited data base does not permit any far-reaching conclusions.

Writing equation (Y.15) in the form:

\[
\text{dlog}(vx) = k_1 \text{dlog}(RP) - k_2 D(r - r_f) - k_3 \text{log}(vx_{t-1}/RP_{t-1}) - k_4 (r_{t-1} - r_{f,t-1}) + k_5 \text{CapF}/M3SEK_{t-1} + k_6,
\]

we can - upon comparison with equation (Y.13) - derive the implicit values of the structural coefficients in equations (Y.10), (Y.11) and (Y.12):

\[
\begin{align*}
\tau &= k_{i_1}, \\
\delta &= k_3, \\
\alpha &= k_6/(-k_3) \\
\beta &= k_4/k_3 \quad \text{and} \quad \beta = (-k_2-1)/k_{i_1}, \\
a_1 &= 1/k_5.
\end{align*}
\]

Consequently, we obtain from equation (Y.15):

\[
\begin{align*}
\text{(Y.10')} & \quad D(-\epsilon) = D(-\delta) - D(r - r_f) - (1/1.70) \text{(-CapBmc}/\epsilon)/W. \\
\text{(Y.11')} & \quad D(-\delta) = 0.499 D(-\epsilon) - 0.362 \left[(-\epsilon)_{t-1} - (-\epsilon)^2_{t-1}\right], \\
\text{(Y.12')} & \quad -\epsilon^2 = p - 35.57 (r - r_f) + 0.209.
\end{align*}
\]

When the interest rate differential in equation (Y.12') is expressed in percent per annum, the coefficient \(\beta\) becomes 0.03. This means that - according to our results - an increase of the interest rate differential by one percentage point p.a. in the long run results in an appreciation of the currency by approximately three percentage points. The corresponding short run effect is approximately half as large.

The coefficient \(a_1\) (equal to 1.70), describing the portfolio reaction to changes in the relative yield on foreign assets, implies a relatively high degree of capital mobility. The corresponding number in the Treasury model is 0.9 (cf. Wallis et al. [1985]). The coefficient \(\beta\) amounts there to 0.013 on a per annum basis, it should be however borne in
Chart 5. Actual values (solid line) and fitted values (dashed line) for the adaptive-expectations equation.
Monthly data for 1993:8-94:12

Chart 6. Actual values (solid line) and fitted values (dashed line) for the rational-expectations equation.
Monthly data for 1993:8-94:12
mind that equation (Y.12) is in the Treasury model specified in a different way. The coefficients \( r \) and \( \delta \) (above equal to 0.499 and 0.362, respectively) are, on the other hand, rather close to those of the Treasury model (0.5 and 0.28, respectively).

As already mentioned, the above results are only tentative. The specification of the model itself was restricted by lack of data. In principle, the long-run relation between domestic and foreign prices (\( p \) in equation (Y.12)) should be modelled more thoroughly. Furthermore, the sustainable exchange rate level (\( e^s \)) could be linked to the net foreign debt. However, our attempts to introduce additional variables led to such a high degree of arbitrariness in the results, that we decided to postpone these efforts until significantly longer time series are available.

**The semiannual exchange rate equation**

Since *Kosmos* is a semiannual model, the above monthly equation has to be adapted to allow for a different data definition (monthly figures being replaced by their semiannual aggregates) and for a longer time-period length.

The semiannual exchange rate equation is obtained from the monthly equation (Y.15) upon summation of the latter over six consecutive months. The terms in logarithmic changes become then changes in logarithms computed for six-month intervals, \( \log(X_t) - \log(X_{t-6}) \), and the other terms become the corresponding sums from \( t \) to \( t-6 \). The intercept is, consequently, multiplied by six.

All the variables included in equation (Y.15), with the exception of the capital flow and SEK M3, are on a semiannual basis defined as (arithmetic) averages of monthly values. When the half-yearly variables are employed in the above sum of six monthly equations, the terms in sums of logarithms can be approximated as logarithms of the semiannual averages multiplied by six. This is so, since sums of logarithmic terms can be represented as logarithms of geometric averages multiplied by the number of terms, and geometric averages can be approximated with the arithmetic ones:

\[
(Y.17) \quad 6 \times \log(X^m) = 6 \times \log(\Sigma X_{t-i}/6) \approx 6 \times \Sigma \log(X_{t-i})/6 = \Sigma \log(X_{t-i}), \quad i=0,1,2,...,5,
\]

where

- \( X \) - monthly variable,
- \( X^m \) - semiannual average of \( X \).
In the differences of variables, \([\log(X_i) - \log(X_{i-6})]\), the monthly figures are approximated by six-monthly averages:

\[(Y.18) \quad \log(X^m) = \log(\sum X_{t-i}/6) = \Sigma \log(X_{t-i})/6, \quad i=0,1,2,...5,\]

which can be interpreted as lagged values of \(\log(X)\), namely \(\log(X_{t-2.5})\). The changes in logarithms can, thus, be seen\(^4\) as being lagged by additional 2.5 months:

\[(Y.19) \quad d\log(X^m) = \log(X_{t-2.5}) - \log(X_{t-8.5}).\]

The terms in logarithmic changes are, furthermore, not multiplied by 6, as averages of the monthly figures rather than sums are to be obtained.

Finally, the capital flow term is defined on a semiannual basis as the sum of monthly flows, while the money stock is taken for the last day of the half-year. The sum of six monthly ratios of the two variables can be approximated by the ratio of the semiannual total of the capital flows to the average money stock:

\[(Y.20) \quad Y^s/X^m = \Sigma (Y_{t-i}/X_{t-i}), \quad i=0,1,2,...5,\]

where

\(Y\) - monthly variable,

\(Y^s\) - semiannual sum of \(Y\).

Since the dependent variable and all the terms in logarithmic changes in the semiannual equation are lagged by approximately 0.5 period in comparison with the derived equation, all the level variables on the right-hand side are lagged accordingly. The lag length of all but one lagged variables (which in principle are lagged by one month, i.e. by 1/6) is set at 2/3 (0.67), where \(X_{2/3} = 0.33 X + 0.67 X_{-1}\). The exception is the money stock, defined on ultimo basis, which in accordance with the previous paragraph should not be lagged at all, but is somewhat arbitrarily lagged by 0.5 period in order to reduce the simultaneity of the model.

By analogy to equation (Y.16), our semiannual equation can then be written as:

\(^4\) It should be noted that (logarithmic) changes in semiannual averages can be a poor approximation of the (logarithmic) changes in monthly figures over six months.
(Y.21) \[ \text{dlog}(vx^m) = k_1 \text{dlog}(RP^m) - k_2 D(r^m - r^{fm}) - k_3 * 6^* \text{log}(vx^{m-2/3}/RP^{m-2/3}) \]
\[ - k_4 * 6^* (r^{m-2/3} - r^{fm-2/3}) + k_5 (\text{CapF}/M3SEK^m)_{-1/2} + 6^*k_6. \]

The semiannual equation (Y.21) is only an approximation of the monthly equation (Y.16). In particular, the coefficient \( k_3 \) in the former does not allow for the fact that the lagged monthly exchange-rate values, which in aggregation are added together, depend on each other through the error correction mechanism. The ex-post predictions for the level of the effective exchange rate, derived from equation (Y.21) using the coefficients from equation (Y.15), are illustrated in Chart 7 for the three half-years currently available (the fourth semiannual observation being lost due to the occurrence of lagged variables).
Chart 7. Actual half-yearly values for the level of the effective exchange rate (solid line) and the ex-post predictions from the derived semiannual equation for 1993:2–1994:2 (dashed line)
Literature


Växelkursekvationen utgår från antagandet om jämvikt på marknaden för utländska (netto)tillgångar. Nettoställningen mot utlandet antas utgöra en av tillgångarna i modellens finansiella portfölj. Efterfrågesambandet definieras inom ramen för portföljvalsmodellen som en funktion av differensen mellan den (korta) inhemska räntan och den motsvarande utlandsräntan, korrigerad för den förväntade växel kursförändringen. Utbudet av utländska tillgångar netto definieras utifrån betalningsbalansidentiteten som summan av bytesbalanssaldo och offentligt kapitalbalanssaldo minus förändring av valutareserven.

Efter substitution får man kronans depreciering (d.v.s. den procentuella förändringen i växel kursen) som en funktion av den förväntade deprecieringen, variationer i räntedifferensen mot utlandet samt av (det marknadsklarande) valutaflödet uttryckt som andel av penningmängden. Växel kursförväntningar antogs grundas på antagandet att den faktiska växel kursen närmar sig jämvikts kursen. Den sistnämnda bestämdes utifrån antagandet om långsiktig köpkräftsparitet, korrigerad för variationer i räntedifferensen mellan Sverige och utlandet.