LABOUR SUPPLY, HOURS WORKED AND UNEMPLOYMENT IN THE ECONOMETRIC MODEL KOSMOS

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This paper is a progress report on the work with the econometric model KOSMOS. As previous papers on KOSMOS in the Working Paper series, it is intended to constitute part of a future comprehensive report on the whole model, and hence is not completely self-contained.

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LABOUR SUPPLY

The development of the participation rate since 1970 is illustrated in Chart Z.1. The official participation rate series, defined as the ratio of the labour force to population, does not include vocational training (Arbetsmarknadssubbildung - AMU, Arbetslivsutveckling - ALU and Ungdomspraktik) which in this respect is treated as any other form of schooling. However, vocational training - i.e. special training for the unemployed arranged by the Swedish Labour Market Board (AMS) - has been used for some time by the Employment Agencies as a way to secure income for people who no longer were entitled to participate in the relief works. Since, as explained above, persons in training are not reckoned in the labour force, this practice contributed to the sharp drop in the latter variable in the early 1990-ies, illustrated in the chart.

In the model, the labour force is defined to include vocational training. It is included as a separate category, distinct from both those employed and those unemployed. Chart Z.1 shows the participation rate according to both definitions. Participation in the labour force is expressed in the chart in terms of the working-age population, i.e. those aged 16 to 64 years. Semi-annual population data were obtained from the annual numbers by cubic-spline interpolation.

The development of the overall participation rate was significantly affected by a gradual increase in the participation rate for women. This was mainly accentuated in the 1970-ies; in the 1980-ies the increase in women's participation rate decelerated and was, furthermore, partly offset by a slight decrease in the participation rate for men. The participation rates for men and women are depicted in Chart Z.2. As in Chart Z.1, they are based on the working-age population interpolated from annual data.

The increase in women's participation rate was mainly due to changes in social attitudes, a higher education level for women and to measures aiming at facilitating women's participation in the labour force, such as rapid development of day-care centres, provision of sick-child care and legislation extending paid maternity leave and sick-child leave. The abolition of the rule that the incomes of married couples are added together for taxation purposes (resulting in a higher tax rate) in 1971 was also very important in this context.

The legislation giving parents of children below the age of 8 the right to opt for a six-hour work day was most probably another factor having a significant impact on female participation. It was one of the causes of the downward trend in the average work week.
Chart Z.1 Participation rate including vocational training (solid line) 
and excluding vocational training (dashed line)

Chart Z.2 Participation rates for men (solid line) 
and women (dashed line)

The rates do not include vocational training.
in the 1970-ies. This is illustrated in Chart Z.5, where the solid line shows the ratio of hours worked per week to the statutory work time. As can be seen in the chart, the increase in the female participation rate was accompanied by an increase in part-time work in the 1970-ies.

A simple equilibrium relation for labour supply is indirectly derived by Markowski and Nandakumar [1993] from a static optimisation model. Starting from a CES utility function with total income and leisure as arguments and subject to the budget constraint, they obtain upon maximisation the following expression for leisure:

\[ F = a U (w_c)^b, \]

where

- \( F \) - leisure,
- \( U \) - overall utility level,
- \( w_c \) - consumer real wage (i.e. after-tax wage rate deflated by consumer price),
- \( b \) - elasticity of substitution between income from work and "income" from leisure,
- \( a \) - constant parameter.

To be comparable with real income in the utility function, leisure is here to be understood as the "income" derived from the leisure time. Postulating

\[ F = f X, \]

where

- \( f \) - leisure time (hours) and
- \( X \) - "income" derived from one hour of leisure time,

and noting that

\[ f = L_T - L_S, \]

\[ L_T = N_T * H_T, \]

\[ L_S = N_S * H_S, \]

where

- \( L_T \) - total hours available,
\( L_s \) - labour supply in hours,
\( N_T \) - population,
\( H_T \) - statutory working hours per period,
\( N_s \) - total number of employed,
\( H_s \) - hours worked per time period,

we obtain after some transformations:

\[
\frac{f}{L_T} = 1 - \frac{L_s}{L_T} = \frac{(F/X)/L_T}{1 - (N_s/N_T)(H_s/H_T)} = a^*(wc)^b * U/(X \cdot N_T \cdot H_T).
\]

In the third line of the formula, the first parenthesis on the left-hand side is the participation rate and the second one is the relative hours worked, i.e. the ratio of the hours actually worked during a period to the statutory work time.

The model is formulated in terms of work hours since it is basically a micro-model applied to the representative consumer. Since, in practice, one can hardly freely choose the amount of hours of work per day, the ratio \( H_s/H_T \) should not be assumed to be fully controlled by the employee. A more plausible assumption is that the employees vary their supply of labour by entering and exiting the labour market (short-term jobs), i.e. through the participation rate \( N_s/N_T \). Consequently, the long-term (equilibrium) value of the ratio \( H_s/H_T \) is to be treated as a constant.

Since changes in the variable \( X \) (real “income” from one hour of leisure time) can be readily interpreted as productivity growth, we can assume that the ratio \( U/(X \cdot N_T \cdot H_T) \) in the labour supply equation above is constant. This amounts to saying that in equilibrium, with no productivity growth, the overall utility per capita (more exactly: per hour) is constant. Our long-run relation for labour supply thus becomes:

\[
1 - g \cdot (N_s/N_T) = a^* \cdot (wc)^b,
\]

where

\( g \) - constant parameter.

The relation was expressed in log-linear form and complemented by a time trend, to account for non-quantifiable factors that have affected the female participation rate. We can note that the log-linear equation can be approximated as
\[ \frac{N_s}{N_T} = \frac{-a'}{g} + \frac{b}{g} \log(w_c) + \frac{k}{g} T, \]

where \( a', g, b, k \) are constant parameters and \( T \) is a time trend, since

\[ \log(1 - g \frac{N_s}{N_T}) = -g \frac{N_s}{N_T}, \quad 0 < g, \frac{N_s}{N_T} < 1. \]

Thus, the model would require the level of the lagged participation rate rather than its logarithm in the equation below. In fact, both forms of the equation give almost identical results, the main difference being that the coefficient of the level variable is 0.08 instead of 0.06 for the logarithm. The pure log-linear form was retained here as the traditional one. Strictly speaking, this means that our long-run labour supply equation cannot be derived from a CES utility function.

The equation was subsequently employed as the long-run relation in the usual error-correction framework. Attempts to introduce the trend only in the 1970-ies or up to the mid 1980-ies were not successful; it is an open question, whether this trend is going to continue.

The participation rate was postulated to be affected in the short run by the share of employment and vocational training in population, reflecting the fact that e.g. increasing unemployment indicates smaller chances to find a job (the discouraged worker effect). The consumer real wage, defined as in the chapter on wage determination, did not prove to have any short-term effects.

Simultaneous estimation of the long-run and adjustment equations gave better results than the Engle-Granger two step procedure. The equation had the following form:

\[ \begin{align*}
\text{dlog}(\text{labfe/popwa}) &= 0.62311 \times \text{dlog}((\text{emp+vt})/\text{popwa}) \\
&\quad - 0.06169 \times \text{log}(\text{labfe/popwa})_{-1} + 0.02461 \times \text{log}(w_c)_{-1} \\
&\quad + 0.00006 \times T - 0.10883 \quad + 0.00797 \times S2 \\
&\quad (14.3444) \quad (2.78103) \quad (2.67437) \quad (0.99862) \quad (2.95198) \quad (8.11492) \\
\text{Sum Sq} &= 0.0001 \quad \text{Std Err} = 0.0018 \quad \text{LHS Mean} = 0.0026 \quad \text{R Sq} = 0.9769 \quad \text{R Bar Sq} = 0.9740 \quad \text{F} = 5, 39 \quad 330.090 \quad \text{D.W. (1)} = 1.9719 \quad \text{D.W. (2)} = 1.3599 \quad \text{Est. per. 1970:2-92:2}
\end{align*} \]
where

labfe - number of persons in the labour force augmented by the number of persons in vocational training,

popwa - working-age population (16-64 years of age),

emp - number of employed including self-employed,

vt - number of persons in vocational training, i.e. in all the labour market schemes which are not included in the labour force,

wc - consumer real wage (i.e. after-tax wage rate deflated by consumer price),

T - time trend,

S2 - seasonal dummy equal to 1 in the second half-year, otherwise 0.

The fit of the equation as well as a test of its predictive power, are illustrated in Charts Z.3 and Z.4. Due to the strong seasonal pattern, the charts show the levels of the participation rate rather than the changes in its logarithm, which form the dependent variable in the equation.

The low value of the two-lag Durbin-Watson statistic (D.W (2) = 1.36) is mainly due to the errors in 1971 and 1992. The outside-sample forecast errors for 1991-92 - though still relatively small - reflect the unusual labour market situation with massive unemployment and extensive labour market schemes. We can also note, that the adjustment coefficient - indicating the speed of adjustment to equilibrium - is very low, at approximately 0.06.

According to our results, the effect of employment on the labour force (given the working-age population) is in the short run very strong. In particular, at the 1990 levels, when employment increases by 100 persons, the corresponding decrease in unemployment amounts to only 36 persons, additional 64 persons having joined the labour force.

The inclusion of vocational training in the employment variable improved the explanatory power of the equation, indicating that vocational training actually was perceived as a form of employment. Attempts to estimate separately the effect of variation in vocational training were not successful, due to the fact that the latter attains significant numbers only in the last year of the sample. When variation in vocational training was included as a separate variable, its coefficient was much lower than that of employment but the (non-corrected) R² for the equation actually showed a slight decrease.
Chart Z.3 Participation rate: actual values (solid line) and fitted values derived from the equation (dashed line).

Chart Z.4 Participation rate estimated through 1989: actual values (solid line) and fitted values derived from the equation (dashed line).

The last six fitted values are outside-sample forecasts.
The equation implies the following (seasonally adjusted) long-run relation, the intercept being computed under the assumption that the short-run terms are equal to their sample mean values:

$$\log(\text{labfe/popwa}) = 0.399 \log(\text{wc}) + 0.001 T - 1.726.$$  

As can be seen, the long-run elasticity of the participation rate with respect to the consumer real wage is, according to our results, approximately 0.4. The upward trend in the participation rate, attributed to women's participation, is rather limited, at approximately 0.2 of a percentage point per year (note that the long-run equation above is semi-annual).

This long-run relation did not pass the augmented Dickey-Fuller cointegration test. On the other hand, Johansen analysis (cf. (Johansen 1988)) of a two-dimensional VAR including all the variables in the above labour supply function (with endogenous participation rate and real wage rate) clearly indicates the existence of one cointegrating vector with the coefficient 0.7 for the real wage. The value 0.5 for this coefficient is accepted (i.e. cannot be rejected) already at the 10\% level, while the value 0.4 is accepted only at the 1\% level.

When the long run relation was estimated freely in the first step of the Engle-Granger procedure, the trend coefficient was twice as large as above, while the wage rate coefficient was only 0.11. The first-step equation, however, did not pass the Dickey-Fuller cointegration test.

**Hours worked**

The development of the average hours worked per week was commented upon in the introduction to the previous section. The variable is defined in the model as the ratio of total hours worked (including the discrepancy term) to the number of employed (including the self-employed). This definition involves a slight inconsistency, since the number of hours worked is defined in accordance with the National Accounts, while the number of employed follows the Labour Force Survey (AKU). The latter definition is necessary for the computation of the number of unemployed below, as employment in this context should be compatible with the labour force, and the National Accounts do not provide any data for the labour force.
The National Accounts definition of employment is wider than that of the Labour Force Survey and includes, in addition, persons in the military service, youngsters below the age of 16 and those above the retirement age of 65 who still work. Thus, average hours worked per week in the model slightly overstate the true variable. The difference is, however, very small and has been stable in relative terms since the beginning of the 1980-ies. In 1970 the number of employed according to the National Accounts was almost 5% larger than according to the Labour Force Survey. This difference shrunk gradually and has been approximately 2% since 1978.

In accordance with the model derived in the previous section, the ratio of hours worked per week to the statutory work time is postulated to be constant in the long run. A time trend for 1970-78 was added to the equation in order to explain the gradual decrease in the ratio during that period, due to the increased female participation and the increased availability of part-time jobs at the time. This trend variable also accounts for the gradual decrease of the (minor) discrepancy between the National Accounts and Labour Force Survey's definitions of employment, mentioned above.

Hours worked are postulated in the short run to be affected by the cyclical development of the economy, here represented by the change in the real GDP growth rate, and by the growth of employment (in persons). The latter variable is supposed to reflect the trade-off between changes in employment and the use of overtime (or temporary lay-offs).

\[
D(hwa/hws) = 0.00198 \times D(DGDP\%) - 0.00148 \times Dimp\% \\
\quad \quad (8.49611) \\
- 0.09979 \times (hwa/hws)_{-1} - 0.00024 \times T \times D7078 + 0.10659 \\
\quad \quad (1.83954) \\
- 0.06285 \times S2 \\
\quad \quad (15.4226)
\]

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<th>Std Err</th>
<th>R Square</th>
<th>R Bar Sq</th>
<th>D.W. (1)</th>
<th>D.W. (2)</th>
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<td>0.0014</td>
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<td>0.9844</td>
<td>0.9824</td>
<td>2.8884</td>
<td>1.4593</td>
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where

- hwa - number of hours worked per week,
- hws - statutory length of the work week (number of hours),
- DGDP% - rate of growth of the real GDP (percent),
Demp% - rate of growth of employment in persons (percent),
D7078 - dummy variable equal to 1 in 1970-78, otherwise 0,
D(X) = X - X_{t-1}
and other symbols as above.

The fit of the equation as well as a test of its predictive power, are illustrated in Charts Z.5 and Z.6.

As can be seen in the equation, there appears to be a very strong relationship between the weekly hours worked (given the statutory work week) and the cyclical phase, represented by the change in the growth rate of the real GDP. According to our results, when the real GDP growth accelerates by 1 percentage point the average hours worked per week are extended by almost 5 minutes (assuming the statutory work week is 40 hours). The adjustment to the long-run equilibrium level is, on the other hand, very slow, the adjustment coefficient being only 0.1.

The equation implies a long-run ratio of hours worked to the statutory work week of 76%. This long-run relation passed the Dickey-Fuller test.

UNEMPLOYMENT

The number of persons unemployed is computed in the model as the difference between the labour supply and the sum of vocational training and employment (in persons).

\[ \text{un} = \text{labfe} - (\text{emp} + \text{vt}) , \]

where \( \text{un} \) is the number of unemployed.

Labour supply is obtained from the participation rate equation, given the population. The number of persons in vocational training is exogenous, being essentially a policy variable. Employment in persons is obtained from the equations for labour demand in hours (cf. the chapter on the supply side), using the average hours worked per period:

\[ \text{emp} = \frac{\text{total hours worked}}{26 \text{ hwa}} , \]

where the average hours worked per week are multiplied by 26 in order to obtain the corresponding semi-annual number.
Chart Z.5  Hours worked per week as fraction of statutory hours:
actual (solid line) and derived from the equation (dashed)

Chart Z.6  Hours worked per week as fraction of statutory hours: actual
(solid line) and derived from the equation estimated through 1989 (dashed)

The last six fitted values are outside-sample forecasts.
The average hours worked per week are obtained from the equation for relative hours worked, given the exogenous statutory work time.

**Literature**


ARBETSUTBUD OCH ARBETSVECKANS LÄNGD

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