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A LONG-RUN EQUILIBRIUM MODEL FOR SWEDEN  
The theory behind the long-run solution  
to the econometric model KOSMOS

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## **ABSTRACT**

A theoretical equilibrium model is derived from assumptions about optimising agents in order to make explicit the theoretical underpinnings of the long-run solution to the econometric model KOSMOS. The theoretical model is further extended to include money, thus creating a framework for the development of KOSMOS. Finally, the theoretical model is employed to compare KOSMOS with the general equilibrium model MECMOD.



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## 1. INTRODUCTION

Computable General Equilibrium (CGE) models are usually considered to be very different tools of empirical economic analysis from the traditional econometric models. CGE models are distinguished by a strict, micro-based theoretical structure and a high degree of disaggregation by type of labour, type of capital, production sectors etc, and also by the lack of a monetary sector<sup>1</sup>. Due to their rigorous equilibrium nature, they are not estimated - since statistical time series data never describe perpetual equilibrium - but, instead, calibrated using information available for a certain point in time.

On the other hand, traditional econometric models are estimated using time series data and therefore trace historical development better. They are designed to show short-run behavior, or adjustment to some (often unknown) equilibrium situation, desirable long-run properties often being sacrificed for the sake of a better short-term fit.

With the introduction of the error-correction formulation<sup>2</sup> the long-run properties of econometric models have started receiving much greater attention than in the past. So too at the National Institute of Economic Research (NIER) in Stockholm, where the ongoing work on the econometric model KOSMOS<sup>3</sup> now has as one of its stated aims the formulation of a long-run solution which would be directly comparable to the institute's CGE model, MECMOD<sup>4</sup>. This should lay the ground for a better understanding of the differences between the simulation results from the two models and enable an analysis of the adjustment paths leading to equilibria.

The purpose of this paper is to present a general equilibrium model of the Swedish economy, which may be identified as the equilibrium version of the macroeconometric model KOSMOS. As it stands now, KOSMOS is a short-run Keynesian model with demand-determined output, not explicitly incorporating any interaction between the real and the financial sides of the economy. The model we present here proposes a long-run equilibrium solution to KOSMOS; it also incorporates some important features of the economy that found no room in the present version of KOSMOS, but which are to be

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<sup>1</sup> Cf. Ginsburgh V. and J. Mercenier (1988).

<sup>2</sup> Cf. Hendry (1993).

<sup>3</sup> Cf LU90, Appendix 1.

<sup>4</sup> Cf LU90, Appendix 2.

included in a forthcoming, full-fledged version. A detailed supply side for output determination, and interaction between the real and financial sides of the economy, are the two most important additions.

It must be pointed out that in some respects, such as sectoral aggregation, the version of the model presented here differs from KOSMOS. Such differences, however, are easily reconciled. Our aim here is to develop an acceptable long-run equilibrium framework.

The model is built on stable microeconomic foundations, and almost all relationships are derived from optimising behaviour by the agents concerned. At first, the model is specified in real terms, much as KOSMOS stands now, but with output determined from the supply side. In the short-run equilibrium that we specify, the output price of the single domestic good in the model is determined by the interaction of supply and demand in the goods market. The long-run equilibrium may be characterized as an "Europa equilibrium", with the domestic rate of inflation equalling the foreign rate.

In the second part of the paper, money is introduced. Then, the real and financial sides of the economy are seen to be linked via the government budget deficit as well as through the equilibrium condition in the money market. The model may be then considered to be a monetary model, somewhat in the spirit of the monetary approach to the balance of payments, but proximate to the structural class of models that determine simultaneously output, prices and the balance of payments. This approach also incorporates many of the ideas included in the econometric model MINIMAC<sup>5</sup>, and in Kanis and Markowski (1990).

In this version of the model, the output price is determined by the equilibrium condition in the money market. A stable equilibrium requires that the domestic rate of inflation coincides with the foreign one; in fact, all nominal variables have to grow at the same rate as the foreign rate of inflation.

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<sup>5</sup> Cf Markowski (1988).

The model is derived from optimising behavior by agents, but is finally specified in terms of standard macroeconomic aggregates. It could actually be estimated as the long-run solution to an econometric model.

It may also be of interest to note how our present work is related to MECMOD, the computable general equilibrium model in use at the NIER. In MECMOD, as in today's KOSMOS, the supply curve is, in effect, horizontal, so that output is demand-determined - prices being determined from the cost side. In contrast, in the present model, prices are either determined from the interaction of supply and demand in the goods market - as in Chapter 2, or in the money market, as in Chapter 3 where money is introduced. Furthermore, MECMOD assumes full employment with wages adjusting accordingly, while our model defines equilibrium as consistent with some level of unemployment. A non-technical review of the results is given in the summary section of the paper.

## 2. A REAL MODEL

In this part we develop a simple computable general equilibrium model with only two goods, one exportable good, which is also consumed at home, and an imported good. Expressions for expenditure, production, imports, exports, factor demands etc., are all derived from optimizing behaviour of the agents involved, and can be assessed numerically for each year, when appropriate parameter values have been chosen.

### 2.1 Consumption demand

The representative consumer is assumed to maximize a CES utility function of the following nested form, involving purchase of the domestic (and exported) good (H), the imported good (I) and leisure (F):

$$(1) \quad U = (a_c C^{-\alpha} + a_F F^{-\alpha})^{-\frac{1}{\alpha}}$$

subject to

$$(2) \quad C = [a_H H^{-\beta} + a_I I^{-\beta}]^{-\frac{1}{\beta}},$$

$$P_c \cdot C = W(1-t) (L_T - F) + \frac{T}{N} + \pi_r (1-t),$$

where the elasticity of substitution between domestic goods and imported goods, and the elasticity of substitution between leisure and consumption, are respectively given as

$$(2a) \quad \epsilon = \frac{1}{1+\beta} ; \rho = \frac{1}{1+\alpha}.$$

The variable  $W(1-t)$  is after-tax wage rate,  $L_T - F$  denotes worktime and  $T/N$  transfers to the consumer.  $\pi_r$  is profits (assumed to be fully distributed) per head. A complete listing of the variable and parameter representation is provided in Table 1 at the end of the paper.



The optimising consumer derives utility from leisure time and from the goods and services purchased for his income which - with the exception of transfers and capital income - is earned during work time. This income should in principle be equal to disposable income; non-wage income is here - for simplicity - predetermined. Since the budget constraint in the optimisation problem (cf. Appendix A) equates total purchases to income, the former should be construed to include savings, or - more exactly - investment goods (which in the present model are the only form of holding savings). Consequently, the variable C is here to be interpreted as disposable income rather than only consumption of goods and services.

Utility maximization gives the following expressions for the demand for goods and leisure (see Appendix A for the derivation of the formulae):

$$(3) C = \left( \frac{P_T}{P_c} \right)^{\rho} \cdot a_c^{\rho} \cdot U$$

$$(4) F = \left[ \frac{P_T}{W(1-t)} \right]^{\rho} \cdot a_F^{\rho} \cdot U$$

$$(5) I = \left[ \frac{P_c}{P_I} \right]^{\epsilon} \cdot a_I^{\epsilon} \cdot C$$

$$(6) H = \left[ \frac{P_c}{P_H} \right]^{\epsilon} \cdot a_H^{\epsilon} \cdot C,$$

where  $P_T$  is the price index for total utility, i.e. the mix of consumption and leisure (see Appendix A) and  $P_c$  is the price index for goods, both those consumed and those invested:

$$(7) P_T = [a_c^{\rho} P_c^{1-\rho} + a_F^{\rho} (W(1-t))^{1-\rho}]^{\frac{1}{1-\rho}}$$

$$(8) P_C = [a_H^{\epsilon} P_H^{1-\epsilon} + a_I^{\epsilon} P_I^{1-\epsilon}]^{\frac{1}{1-\epsilon}}.$$

In the equations above,  $t$  is the tax rate,  $P_H$  the price to the consumer of the home good and  $P_I$  that of the imported good (in domestic currency):

$$P_I = P_w/e,$$

where  $P_w$  is the world price in foreign currency and  $e$  the exchange rate.

Clearly, an increase in the tax rate at unchanged  $U$  results in a smaller labor supply, which is accounted for as

$$(9) L_S = L_T - F,$$

where  $L_T$  is the total number of hours available for allocation between work and leisure and  $L_S$  is the number of hours supplied for work.

The change in the relative price ( $P_R = P_H/P_I$ ) plays a crucial role in the model:

$$(9a) P_R^* = P_H^* - (P_w/e)^*$$

where asterisks denote rates of change.

## 2.2 The supply side

Production in the private sector is assumed to be governed by the CES function<sup>6</sup>

$$(10) Q = \gamma[\delta K^{-\tau} + (1-\delta)L^{-\tau}]^{\frac{-\eta}{\tau}}.$$

The producers maximize

$$(11) Profit = P_H \cdot Q - W \cdot L - K \cdot c,$$

---

<sup>6</sup> Equation (10) is written using the notation of Kanis and Markowski (1990). The forms of the CES functions in equations (10) and (1) are equivalent.

subject to (10), where K and L are capital and labor, respectively, and c is the user cost of capital. The following factor demand functions and implicit supply function are then obtained:

$$(12) \quad L = a_1 \cdot (1-\delta)^\sigma \cdot \left(\frac{P_H}{W}\right)^\sigma \cdot Q^{\frac{1-\sigma(1-\eta)}{\eta}},$$

$$(13) \quad K = a_1 \cdot \delta^\sigma \cdot \left(\frac{P_H}{c}\right)^\sigma \cdot Q^{\frac{1-\sigma(1-\eta)}{\eta}},$$

$$(14) \quad Q = \phi \cdot \left[\left(\frac{W}{P_H}\right)^{-\sigma} \cdot (1-\delta)^\sigma + \left(\frac{c}{P_H}\right)^{-\sigma} \cdot \delta^\sigma\right]^\psi,$$

where

$$(15) \quad \sigma = \frac{1}{1+\tau}; \quad \psi = \frac{\eta}{(1-\sigma)(1-\eta)}; \quad \phi = \left(1 - \frac{1}{\psi}\right)^\psi \cdot \gamma^{-\frac{1}{1-\eta}} \cdot \eta^{\psi \cdot \sigma}; \quad a_1 = \eta^\sigma \cdot \gamma.$$

### 2.3 Factor costs

The user cost of capital c is defined, following Jorgenson (1965), as

$$(16) \quad c = \frac{P_H(r+d-\pi)}{(1-t_k)},$$

where d is the rate of depreciation, assumed constant, and  $t_k$  is the rate of capital taxation. The variable r is the rate of interest, here taken to be fixed in the world credit markets to which the open economy has access. The investment good is assumed to be the home good so that its price is  $P_H$ .  $\pi$  is the rate of inflation, calculated as

$$(17) \quad \pi = \frac{P_H - P_{H(-1)}}{P_{H(-1)}}.$$

where  $P_{H(-1)}$  denotes  $P_H$  lagged by one period.

The nominal wage rate,  $W$ , applicable to all employees, is implicitly determined by the (negative) relationship between the wage share in value added and the rate of unemployment:

$$(18) \quad \frac{WL}{P_F Q} = b_0 + b_1 U_n,$$

where  $b_0$  and  $b_1$  are constants. This formulation, which assumes full compensation for (producer) price increases and productivity gains could be considered to be the outcome of the utility-maximizing behaviour of an all-encompassing trade union.<sup>7</sup>

## 2.4 Unemployment

The rate of unemployment,  $U_n$ , is the difference between total labour supply in hours and total demand - in hours - for labour, expressed as a percentage of total labour supply:

$$(19) \quad U_n = \frac{L_S N - L - L_G}{L_S N},$$

$N$  being the number of members of the active labour force, and  $L_G$  the level of public sector employment.

## 2.5 Trade flows

The import demand function has already been presented in equation (5), derived from utility maximizing behaviour of domestic consumers. Import demand for the economy as a whole is given by

$$(5a) \quad I_T = I.N + g_r G.$$

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<sup>7</sup> See Calmfors and Forslund (1991) and Oswald (1979) for details of this approach.

In (5a),  $g_i$  is the fraction of real government spending on goods,  $G$ , directed towards imports.

The export demand function can be derived similarly, essentially by adopting the two-country model of international trade, where the foreign country represents the rest of the world. Foreign consumers maximize a utility function - say, of the CES type, as in the case of the domestic consumers - where purchase of the domestically produced good, purchase of the imported good and leisure enter as arguments. Then, analogously to the import demand function (5), the following demand function for home country exports is obtained:

$$(20) \quad X = w_f^\zeta \cdot \left( \frac{P_w}{e \cdot P_H} \right)^\zeta \cdot WD ,$$

where  $w_f$  is a parameter in the CES utility function of the foreign consumer,  $\zeta$  is the elasticity of substitution between home goods and foreign goods<sup>8</sup>,  $P_w$  is the price level in the foreign country,  $e$  is the exchange rate and  $WD$  is total real foreign income.

This approach to modelling trade flows invokes the so-called Armington<sup>9</sup> assumption. Home goods and foreign goods are considered to form - in a CES aggregate - a composite good which provides utility upon consumption. The demand for imports will then depend on total domestic demand for the composite good and the price of imports relative to the price of the composite good, which itself is a CES aggregate of the price of the goods produced at home and the price of imports.

The trade balance is, in nominal terms,

$$(21) \quad TB = P_H \cdot X - P_I \cdot I_T .$$

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<sup>8</sup> Strictly speaking, a composite price - rather than the price of the foreign good - should appear in the numerator of (20), but for simplicity the price of the foreign good is used.

<sup>9</sup> Cf Armington (1969).

It is assumed that there is a possibility of international financing of foreign trade imbalances, the latter occurring especially in short-term equilibria.

## 2.6 Disposable income and the level of aggregate consumption

Disposable income, defined in the condition of the representative consumer problem, can be accounted for as

$$(22) Y_D = (P_H Q + W.L_G)(1-t) + T .$$

Corporate profits are assumed to be distributed to the shareholders, meaning that total value added accrues to the public.

Aggregate nominal consumption is, as usually in static CGE models of this type, worked out assuming a stable savings ratio:

$$(22a) C_T = (1-s)Y_D ,$$

where  $s$  is the savings ratio.

An alternative closure rule is to assume fully balanced foreign trade, which gives savings equal to investment.

## 2.7 Government

The government collects taxes, distributes transfers and employs labour<sup>10</sup>. In the absence of a financial sector, these activities obey the following budget constraint:

$$(23) W.L_G + T + (1-g_P).G.P_H + g_I.G.P_I = (P_H Q + W.L_G).t ,$$

where  $G$  represents government real spending on goods and  $g_i$  the fraction of  $G$  spent on imports.

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<sup>10</sup> The output of the government sector does not, however, enter the utility function of the consumer, in the current version of the paper. This extension can readily be made.

## 2.8 Investment in fixed capital and inventory

Gross private fixed investment is obtained as

$$(24) \ i = K - K_{-1} + d.K_{-1}$$

where  $d$  is the depreciation rate.

Investment in inventories,  $v$ , is assumed to follow the simple pattern

$$(25) \ v = \mu.Q ,$$

where  $\mu$  is a constant. Out of  $v$ , a fraction is imported:

$$(25a) \ v_I = m_I v .$$

## 2.9 Price determination in the short run

We obtain what may be characterized as a short-run equilibrium when the goods market clears and the labour market is in a state of "unemployment equilibrium" with an unchanging level of unemployment. Hence, the domestic goods market equilibrium condition (cf (14) and (6))

$$(26) \ \phi.[(\frac{W}{P_H})^{-\sigma} \cdot (1-\delta)^{\sigma} + (\frac{c}{P_H})^{-\sigma} \cdot \delta^{\sigma}]^{\frac{1}{1-\sigma}} =$$
$$[\frac{P_C}{P_H}]^{\epsilon} \cdot a_H^{\epsilon} \cdot C \cdot N + g_H G + X$$

together with equation (19) represent the short-run equilibrium situation. These two equations, together with (8), (12), (14), (16), (17) and (18) can be solved for  $P_H$ ,  $L$ ,  $Q$ ,  $P_C$ ,  $\pi$ ,  $W$ ,  $c$  and  $U_a$ . Hence, the price of the domestic good (in relative

terms<sup>11</sup>) is obtained by matching of demand and supply. For the imported good, the price level is assumed to be determined in the world market, domestic demand being too small to play any part.

The other endogenous variables in the model can be determined recursively, now that the movements of relative prices have been tracked.

The short-run equilibrium described above, can - indeed - be short-lived. In a small open economy, assumed here, any deviation of the domestic price from the world price will necessarily cause an adjustment.

### 2.10 Long-run equilibrium

As the solution procedure indicates, the motion of the economy through time depends on relative prices. Only when the relative prices entering in the eight equation system mentioned above - on which the values of all the endogenous variables depend - are unchanging, will the system reach a stable state. If the ratio  $P_H/P_I$  changes, which will also mean a change in  $P_H/P_T$ , there will be a revamping of the shares of expenditure allocated by consumers to consumption of the two goods, and also a change in export demand, thus disturbing the short-run equilibrium and pushing the economy towards a new equilibrium situation dictated by the new constellation of variables. Hence, in long-run equilibrium, the rates of change of the prices of the goods have to be the same.

Also, in the absence of population growth, capital accumulation - which itself depends on relative prices - must have ceased. So, the long-run static equilibrium conditions will be dictated by the static conditions to the following equations:

- i) The investment equation (24)
- ii) The relative price equation (9a)
- iii) The wage equation (18).

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<sup>11</sup> There is no absolute price level in the real model. The aggregate numeraire price  $P_e$  is arbitrarily fixed at a chosen base level.



In the steady state equilibrium, then,

$$(27a) P_H^* = P_I^* = (P_W/e)^*; \text{ equivalently, } P_H = z_1 \cdot P_I = Z_1 \cdot P_W/e$$

$$(27b) W^* = P_H^*$$

$$(27c) i = d \cdot K_{-1}$$

Equations (27a) and (27b) imply

$$(27d) P_H^* = P_I^* = W^*$$

Thus, equations (27c) and (27d) represent the long-run static equilibrium conditions for the system.

This will mean that the supply curve for the domestic good becomes horizontal in the long-run - at the level indicated by (27), rather than according to a 'mark-up over costs' formula. It thus follows that the long-run production function exhibits constant returns to scale, which implies that

$$(28) \eta = 1$$

in the production function specification (10), as well as in (12)-(14).

Equation (27c), which defines the static equilibrium investment level as mere replacement of capital depreciation, gives implicitly the savings ratio,  $s$ , since without money savings equal investment (both in fixed capital and in inventories) plus the trade balance:

$$(29) s \cdot Y_D = P_H \cdot i + P_H \cdot (1-m_P) \cdot v + P_I \cdot m_I \cdot v + TB.$$

This relation is rarely given explicitly, as it is implicit in the nominal GDP identity, which states that the sum of all incomes (i.e. GDP from the income side) is equal to the sum of all domestic expenditure plus net exports (i.e. GDP from the expenditure side). It is needed here, since in our model the nominal GDP is not introduced.

## 2.11 The system of equations for the long-run specification

To round off the discussion above, the complete system for the long-run is presented below (note that asterisks denote rate of change):

$$(1) U = (a_c C^{-\alpha} + a_F F^{-\alpha})^{-\frac{1}{\alpha}}$$

$$(8) P_C = [a_H^{\epsilon} P_H^{1-\epsilon} + a_I^{\epsilon} P_I^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$$

$$(7) P_T = [a_C^{\rho} P_C^{1-\rho} + a_F^{\rho} (W(1-t))^{1-\rho}]^{\frac{1}{1-\rho}}$$

$$(3) C = \left(\frac{P_T}{P_c}\right)^{\rho} \cdot a_c^{\rho} \cdot U$$

$$(5) I = \left[\frac{P_c}{P_I}\right]^{\epsilon} \cdot a_I^{\epsilon} \cdot C$$

$$(5a) I_T = I.N + g_I G$$

$$(4) F = \left[\frac{P_T}{W(1-t)}\right]^{\rho} \cdot a_F^{\rho} \cdot U$$

$$(12) L = a_1 \cdot (1-\delta)^{\sigma} \cdot \left(\frac{P_H}{W}\right)^{\sigma} \cdot Q$$

$$(13) K = a_1 \cdot \delta^{\sigma} \cdot \left(\frac{P_H}{c}\right)^{\sigma} \cdot Q$$

$$(16) \ c = \frac{P_H(r+d-\pi)}{(1-t_k)} ,$$

$$(17) \ \pi = \frac{P_H - P_{H(-1)}}{P_{H(-1)}}$$

$$(9) \ L_S = L_T - F$$

$$(19) \ U_n = \frac{(L_S N - L - L_G)}{L_S N}$$

$$(20) \ X = w_f^\zeta . (\frac{P_w}{e.P_H})^\zeta . WD ,$$

$$(21) \ TB = P_F X - P_I J_T$$

$$(22) \ Y_D = (P_F Q + W.L_G)(1-t) + T,$$

$$(22a) \ C_T = (1-s)Y_D$$

$$(23) \ W.L_G + T + (1-g_p).G.P_H + g_r G.P_I = (P_H Q + W.L_G).t ,$$

$$(24) \ i = K - K_{-1} + d.K_{-1}$$

$$(29) \ s \cdot Y_D = P_H \cdot i + P_H \cdot (1-m_i) \cdot v + P_I \cdot m_I \cdot v + TB$$

$$(25) \ v = \mu.Q,$$

$$(27a) P_H^* = P_I^*$$

$$(18) \frac{W.L}{P_H Q} = b_0 + b_1 U_n$$

$$(30) Q = \left[ \frac{P_C}{P_H} \right]^{\epsilon} a_H^{\epsilon} C.N + (1-g_p).G + X$$

$$(31) GDP = \frac{C_T}{P_C} + \frac{W.L_G}{P_H} + G + i + v + X - I_T$$

Equations (30) and (31) have not been presented before; (30) gives output in the long-run (using (6)), while (31) aggregates for real GDP. In the latter equation, private consumption,  $C_n$ , is deflated using the price for C, which is a CES aggregate and, furthermore, includes investment. This simplification is introduced in order to avoid further expansion of the size of the model.

These 25 equations solve for  $U, P_T, P_C, C, I, I_T, F, L, K, c, \pi, L_s, U_n, X, TB, Y_D, C_n, t$  (for exogenous  $T, G$  and  $L_G$ ),  $i, s, v, P_H, W, Q$  and GDP.

The above equation system includes four variables which cannot be measured ( $U, C, F$  and  $P_T$ ). The variable  $C$  can, however, be replaced in equations (5) and (30) by real disposable income  $Y/P_c$  using the budget restriction in the consumer's optimising problem. The variable  $F$  in equation (9) can be substituted for from (4). In the resultant formula,  $U$  can be replaced with an income variable and  $P_T$  with  $P_C$ . This approximation gives us labor supply as a function of the real wage and the income level. Equations (7), (3), (4) and (1) - defining  $P_T, C, F$  and  $U$  - can now be eliminated from the system, as they no longer are needed to solve the remaining equations.

Once the non-measurable elements of the utility function are eliminated, the system of nineteen equations - thus obtained - defines the long-run solution to a traditional econometric model of the real sector of the economy, such as today's KOSMOS. Imports and exports are functions of relative prices and income. The consumption ratio is constant. Labour demand and capital stock are proportional to output and depend on relative factor prices. If demand for imports and domestic production is defined at the level of the whole economy, rather than households only, disposable income in equations (5) and (30) should be defined accordingly, giving the standard aggregate demand equations.

### 2.12 Government policies in the real model

The long-run equilibrium conditions, specified above, have strong policy implications. The government budget constraint implies that an increase in government spending is offset by a corresponding increase in taxes. So, when government spending on goods ( $G$ ) increases, private disposable income is reduced by the same amount. The consequent decrease in private consumption is of the same order of magnitude as the income change, the savings ratio being very low. Thus, since the increase in  $G$  is largely offset by the decrease in  $C_p$ , the level of aggregate demand is very little affected. The composition of aggregate demand is, however, changed, with government demand having become larger and private demand smaller. This illustrates the crowding out phenomenon in our simple model.

If the increase in government spending takes the form of greater government employment,  $L_g$ , disposable income remains unchanged. Hence, there is no effect on private consumption, aggregate demand and output. Neither is there any effect on the level of private sector employment. Unemployment is, however, reduced. Thus, assuming that the wage rate does not react to the change in unemployment, this illustrates the case of a pure redistribution policy. Those already employed are taxed to provide jobs for the unemployed, without altering the aggregate level of output and disposable income.

### 3. INTRODUCING MONEY

In this section, a money market is introduced, and it is seen that the interaction between the real and the financial sides of the economy has an important role, this interplay being manifested in the determination of equilibrium in the money market, as well as in the fulfilment of the government budget constraint.

The assumption of constant returns to scale in the production function is retained in this part of the paper.

The consumer is assumed to derive utility also from holding real money balances. Within the framework of the previous model this would mean a double-nested utility function, as an additional nesting level would be needed to determine the distribution of income between money balances (financial savings) and expenditure on goods (both domestic and imported). In order to avoid cumbersome derivations we make the simplifying assumption that leisure time (and thus  $L_s$ ) is given. This eliminates the highest level of nesting and gives us an exogenous labor supply. Our new utility function has, thus, the form:

$$(1') \quad U = [a_c \cdot C^{-\alpha} + a_m \cdot m^{-\alpha}]^{-\frac{1}{\alpha}}$$

where  $m$  denotes real money balances and other symbols are as above.

Utility maximization gives the following money and goods demand functions:

$$(32) \quad m = a_{mm} \cdot U$$

and

$$(3') \quad C = a_{cc} \cdot U,$$

where

$$(32a) \quad a_{mm} = (a_c^p + a_m^p)^{\frac{p}{1-p}} \cdot a_m^p$$

and

$$(3'a) \ a_{cc} = (a_c^\rho + a_m^\rho)^{\frac{\rho}{1-\rho}} \cdot a_c^\rho.$$

It should be noted that  $\rho = 1/(1+\alpha)$  now denotes the elasticity of substitution between goods and real money balances in the utility function. Appendix B provides the derivations of the formulae.

Note that differentiation of (3') gives

$$(3'') \ dC = \frac{a_{cc} \cdot a_m \cdot m^{-\alpha-1}}{1 - a_{cc} \cdot a_c \cdot C^{-\alpha-1}} \cdot U^{1+\alpha} \cdot dm ,$$

so that the change in expenditure is affected by the change in real money balances.

The money market equilibrium condition is

$$(33) \ a_{mm} \cdot U \cdot N = \frac{M}{P_c}.$$

In (33), the left-hand side is the demand for money, defined as equation (32) times the number of consumers. The right-hand side of the expression is real money supply, accounted for as

$$(34) \ \frac{M}{P_c} = \frac{M_t + \Delta D + \Delta R}{P_c},$$

where  $M_1$  is the initial money stock, and the remaining two terms stand for domestic money expansion and the change in foreign reserves, respectively. The change in foreign reserves is, in the current framework, simply equal to the trade balance:

$$(35) \Delta R = TB.$$

Government overspending is in our model the only possible source of domestic money expansion. For given levels of government spending and taxation, the change in the domestic component of the money supply is then determined from the government budget constraint:

$$(36) W.L_G + P_H(I-g_p).G + P_R g_R G + T = \frac{Y_D - T}{1-t} \cdot t + \Delta D.$$

Since the government now can run a deficit, the financial savings identity is complemented with the government budget outcome,  $-\Delta D$ :

$$(29') s.Y_D - \Delta D = P_H.i + P_H.(1-m_f).v + P_f.m_f.v + TB.$$

### 3.1 Price determination and short-run equilibrium

The money market equilibrium condition (33), together with (8) and (16)-(19), represent short-run general equilibrium, solving for  $P_C$ ,  $P_H$ ,  $W$ ,  $\pi$ ,  $c$  and  $U_n$  (using (1'), (3'), (30) and (12)). The money market condition solves for the general price level  $P_C$ ; then from (8),  $P_H$  can be obtained. With the assumption of a constant-returns-to-scale CES production function, the supply curve for the domestic good is horizontal at this price level, and the output of the domestic good is given by (30). Once the relative prices are obtained, the rest of the model can be solved recursively, the entire system being as follows:

$$(3') C = a_{cc}.U$$

$$(1') U = [ a_C C^{-\alpha} + a_m m^{-\alpha} ]^{-\frac{1}{\alpha}}$$



$$(8) \ P_C = [ a_H^\epsilon P_H^{1-\epsilon} + a_I^\epsilon P_I^{1-\epsilon} ]^{\frac{1}{1-\epsilon}}.$$

$$(5) \ I = [\frac{P_c}{P_I}]^\epsilon \cdot a_I^\epsilon \cdot C$$

$$(5a) \ I_T = I.N + g_T G$$

$$(12) \ L = a_1.(1-\delta)^\sigma.(\frac{P_H}{W})^\sigma.Q$$

$$(13) \ K = a_1.\delta^\sigma.(\frac{P_H}{c})^\sigma.Q$$

$$(16) \ c = \frac{P_H(r+d-\pi)}{(1-t_k)} \ ,$$

$$(17) \ \pi = \frac{P_H - P_{H(-1)}}{P_{H(-1)}}$$

$$(19) \ U_{\pi} = \frac{(L_S N - L - L_G)}{L_S N}$$

$$(20) \ X = w_f^{\zeta} . (\frac{P_W}{e.P_H})^{\zeta} . WD \ ,$$

$$(21) \ TB = P_H X - P_T I_T$$

$$(22) Y_D = (P_H Q + W.L_G)(1-t) + T,$$

$$(22a) C_T = (1-s)Y_D$$

$$(24) i = K - K_{-1} + d.K_{-1}$$

$$(29') s \cdot Y_D - \Delta D = P_H \cdot i + P_H \cdot (1-m_I) \cdot v + P_I \cdot m_I \cdot v + TB$$

$$(25) v = \mu \cdot Q,$$

$$(18) \frac{W.L}{P_H Q} = b_0 + b_1 \cdot U_n$$

$$(30) Q = \left[ \frac{P_C}{P_H} \right]^{\epsilon} \cdot a_H^{\epsilon} \cdot C \cdot N + (1-g_p) \cdot G + X$$

$$(31) GDP = \frac{C_T}{P_C} + \frac{W.L_G}{P_H} + G + i + v + X - I_T$$

$$(32) m = a_{mm} U$$

$$(33) a_{mm} \cdot U \cdot N = \frac{M}{P_C}$$

$$(34) \frac{M}{P_c} = \frac{M_t + \Delta D + \Delta R}{P_c}$$

$$(35) \Delta R = TB$$

$$(36) \quad W \cdot L_G + P_H \cdot (1-g_p) \cdot G + P_I \cdot g_I \cdot G + T = \frac{Y_D - T}{1-t} \cdot t + \Delta D.$$

These twenty five equations can be solved for  $P_H$ ,  $C$ ,  $I$ ,  $U$ ,  $L$ ,  $K$ ,  $c$ ,  $W$ ,  $\pi$ ,  $I_T$ ,  $U_n$ ,  $X$ ,  $TB$ ,  $Y_D$ ,  $C_n$ ,  $i$ ,  $v$ ,  $Q$ ,  $GDP$ ,  $s$ ,  $m$ ,  $P_C$ ,  $M$ ,  $\Delta R$  and  $\Delta D$ .

### 3.2 Long-run equilibrium conditions

The long-run equilibrium conditions arrived at for the real model are relevant even after a money market has been added. However, there is one more dynamic relation in the model, equation (34), which describes the change in the stock of the real money supply - which, as is clear from (3''), determines through (32) the change in consumption demand. For no disturbances to the equilibrium to emanate from the money market, the money supply should also be growing at the same rate as the prices. The equilibrium condition to the dynamic equation (34) implies

$$(37a) \quad M^* = \pi.$$

Hence, the long-run equilibrium conditions which should be superimposed on the short-run system are:

$$(37) \quad M^* = \pi = P_H^* = W^* = (P_w/e)^*$$

and

$$(37c) \quad i = d.K_{-1}$$

where an asterisk denotes rate of change.

Again, investments in static equilibrium are equal to the replacement of the capital stock.

As in the case of the real model, the variable  $C$  in equations (5) and (30) can be replaced by an appropriate income variable. Moreover, the variable  $U$  in equations (32) and (33) can also be replaced by the same income variable, since from (3')  $U$  is directly

proportional to  $C$ . After this substitution, equations (3'') and (1') - defining  $C$  and  $U$  - can be eliminated from the system. The remaining equations do not include any non-measurable variables and form a traditional model with money.

The price inflation in this model is not determined by the world price, as was the case in the real model above. Here, the domestic inflation is defined - via the consumer price - by the equilibrium condition for the money market. So a government budget deficit that pumps in money into the economy has a direct link to the rate of price increase. Nevertheless, the equality of the domestic and foreign inflation rates is still a condition for long-run equilibrium. If this condition does not hold, the money stock is affected by the changes in the trade balance (caused by the relative-price changes). The change in the money stock brings about price adjustment.

The model thus illustrates the importance of fiscal (and monetary) policies, since the inflation rate and hence the economic performance is affected (via the money stock) by the government budget deficit. As could be seen, the introduction of money does not change the general equilibrium condition, but introduces into the model the adjustment mechanism through which this condition is enforced. At the same time it makes possible an analysis of the effects of fiscal policy.

### 3.3 Government policies in the monetary model

The introduction of money in the model allows us - unlike the case with the real model - to analyse the effects of an unbalanced government budget. While the analysis is in terms of comparative statics, we will be looking at a series of short-term equilibria that lead to the new long-term equilibrium level. In this way, we can follow the adjustment process without introducing short-term dynamics.

Consider a temporary increase in government non-wage spending ( $G$ ), financed by an expansion of the domestic component of the money supply ( $D$ ). Initially, domestic output goes up, giving rise to an increase in the consumption and import demand. At the same time, the increase in the money stock pushes up the domestic price level ( $P_C$  and  $P_H$ ). Due

to the higher imports and to the rise in relative prices, net exports fall, i.e. the trade balance is negatively affected. This feeds back into the economy through the reduction of the foreign component of the money supply, thus reducing the aggregate money supply.

According to our equilibrium condition, in the long run

$$M^* = P_H^* = P_I^* .$$

Assuming for expositional convenience that  $P_I^* = 0$ , so that we can talk about levels rather than growth rates, the aggregate money supply in the long run should be unaltered. Consequently, the reduction in the foreign component of the money supply, caused by the deterioration of the trade balance, continues until the aggregate money supply ( $M$ ) and the domestic price level ( $P_H$ ) are back to their initial levels. Output, which has risen initially following the increase in  $G$ , falls also back to its original level. The only result of this one-shot policy measure is that the domestic component of the money supply has been increased at the expense of foreign reserves.

Consider now a temporary increase in government employment,  $L_G$ , financed by an increase in the domestic component of the money supply,  $D$ . There is an increase in disposable income, giving rise to an increase in private consumption demand, output and imports. The domestic price level goes up, following an increase in the money stock.

As in the case of the previous policy, there is a downward pressure on aggregate money supply, as foreign reserves fall. In the long run (assuming again zero foreign inflation), the money stock, aggregate output, and the price level are back to their original levels. The only result of the temporary policy measure is once again that the domestic component of the money supply has been increased at the expense of foreign reserves.

As can be seen, one-off government policy measures do not have any long-run effects in our model. However, the medium-term effects of these policies could have been of some importance. On the one hand, net exports have been temporarily crowded out by

increased domestic consumption, on the other hand unemployment has been temporarily reduced.

#### 4. COMPARISON WITH MECMOD

MECMOD, which is a Computable General Equilibrium model, is derived from a similar optimising framework as the real model above. Only few differences can be noted.

In MECMOD, given the horizontal supply curve implied by the constant-returns-to-scale CES function, output is essentially determined by demand, and pricing is by mark-up over factor costs. In the present model, in short-run equilibrium, for the real version, output is determined from the supply side, and the price of domestic goods is the one that brings about equilibrium in the goods market. In long-run equilibrium, the supply curve is horizontal as is the case in MECMOD; however, the price level is determined by the condition that home and foreign rates of inflation should be equal, rather than by a mark-up procedure as in MECMOD.

Note that the long-run equilibrium condition equates home and foreign rates of inflation, and not the absolute price levels at home and abroad.

The treatment of factor markets also differs: in MECMOD there is full employment with complete flexibility of wages (with the labour force being disaggregated into several types), while in our theoretical model a less flexible wage formation mechanism is specified. Labour supply is in MECMOD determined in a simplified manner.

In the model version that includes money, the consumer price is determined in the money market, interaction between the financial and the real sides of the economy being evident in the money market as well as in the importance of the government budget constraint. A change in the trade balance, say, due to a change in foreign prices, brings about a change in the money supply, and hence in the domestic price level. This adjustment mechanism is absent in MECMOD, where the response of the domestic price level to a change in foreign prices is conditioned by the requirement of balanced trade.

## 5. SUMMARY AND CONCLUSION

The econometric model KOSMOS is formulated in the error-correction form. Its long-run solution can thus be interpreted as an equilibrium model. One of the aims of the present paper was to make explicit the theoretical underpinning of this model. This was achieved by deriving a corresponding theoretical model from standard assumptions about optimising producers and consumers. Aggregation over individuals was obtained by recourse to the notion of the representative consumer and the assumption of one (collective) producer, who can be interpreted as the sum of representative producers. The theoretical model, unlike KOSMOS, has only one productive sector.

It was assumed that the government budget always is balanced, as model specification precluded any form of borrowing. The wage-rate equation was based on the utility--maximising behaviour of an all-encompassing trade union. Unlike other prices, the wage rate is not assumed to clear the market; the model allows some level of unemployment in equilibrium.

An analysis of the static equilibrium conditions for the theoretical model gave the well known result that the domestic price inflation should be equal to the inflation rate abroad.

The model, presented in the first part of the paper, was subsequently reduced to eliminate non-measurable variables connected with consumer utility. The resultant equation system summarises well the long-run solution to today's KOSMOS. The main difference pertains to price determination, prices in KOSMOS being determined by costs and in the theoretical model by the goods market equilibrium condition (supply = demand). Another difference is due to the specification of the labour supply function which in KOSMOS is highly simplified. Finally, the government budget constraint is not imposed in KOSMOS.

The second purpose of this paper was to suggest a framework for extending the model to include monetary variables and to show how the properties of the model will be



affected by such an extension. This was done in the second part of the paper where money was introduced into the real model. In order to simplify the analysis, labour supply was there assumed to be given.

In this version of the model, prices are determined by the money market equilibrium condition through an interplay of both real and monetary variables. The money stock is affected by the trade balance and the government budget deficit. This approach gives prominence to the interdependence of the real and the monetary sectors of the economy and makes explicit the effects of government policies.

The introduction of money does not affect the static equilibrium condition for the model. It creates, however, a new adjustment mechanism through which this condition is enforced. This mechanism implies - inter alia - that foreign prices affect domestic prices through the effects of the trade balance on the money stock.

The final aim of this paper was to facilitate a comparison of KOSMOS with the general equilibrium model, MECMOD, employed at the institute. The assumptions behind MECMOD are similar to those of our theoretical real model. MECMOD is, consequently, very close to the long-run solution to today's KOSMOS. Besides the disaggregation into productive sectors, types of capital and types of labour, the main differences pertain to the treatment of the labour market. MECMOD postulates full employment and complete wage adjustment; in KOSMOS wages are less flexible and there is always some degree of unemployment. Labor supply is in both models determined in a simplified way, as the formulae derived from the optimising framework involve non-measurable variables.

The differences between the numerical solutions given by the two models can still be much larger than those arising from unequal wage adjustment and labor supply. A more important source of disparities can be different equation coefficients (or parameters), since MECMOD is calibrated on a single year's data while KOSMOS is estimated on semi-annual time series over twenty years.

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# **APPENDIX A. DERIVATION OF DEMAND FOR GOODS AND LEISURE IN THE REAL MODEL**

Maximize

$$U = (a_c C^{-\alpha} + a_F F^{-\alpha})^{-\frac{1}{\alpha}}$$

subject to

$$P_c C = W(1-t)(L_T - F) + \frac{T}{N} + \pi_r(1-t)$$

Form the Lagrangean

$$L = (a_c C^{-\alpha} + a_F F^{-\alpha})^{-1/\alpha} + \lambda [W(1-t)(L_T - F) + \frac{T}{N} - P_c C].$$

Upon derivation with respect to C and F we obtain the first order conditions:

$$(A.1) \quad W(1-t) = \left(\frac{1}{\lambda}\right) \cdot U^{\alpha+1} \cdot F^{-\alpha-1} \cdot a_F$$

and

$$(A.2) \quad P_c = \frac{1}{\lambda} \cdot U^{\alpha+1} \cdot a_c \cdot C^{-\alpha-1}.$$

From (A.1) and (A.2),

$$\frac{W(1-t)}{P_c} = \frac{a_F}{a_c} \cdot \left(\frac{F}{C}\right)^{-\alpha-1}$$

$$(A.3) \quad \frac{W(1-t) \cdot F}{P_c \cdot C} = \left(\frac{F}{C}\right)^{-\alpha} \cdot \frac{a_F}{a_c}.$$

Adding 1 to both sides of (A.3) we obtain

$$\begin{aligned} W(1-t)F + P_c.C &= \frac{P_c.C}{a_c.C^{-\alpha}}.(a_F.F^{-\alpha} + a_c.C^{-\alpha}) \\ &= P_c.C.\frac{U^{-\alpha}}{a_c.C^{-\alpha}}, \end{aligned}$$

or

$$(A.4) \quad W(1-t)F + P_c.C = \frac{C^{1+\alpha}.P_c.U^{-\alpha}}{a_c}.$$

Now, the left hand side of (A.4) is total expenditure, inclusive of that on leisure:

$$(A.5) \quad W(1-t)F + P_c.C = E,$$

where E is total expenditure.

With the assumption of homothetic utility,

$$(A.6) \quad E = P_T.U,$$

where  $P_T = E/U$  is expenditure per unit welfare, or the unit expenditure function.

Substituting (A.5) and (A.6) into (A.4),

$$P_T.U = C^{1+\alpha}.\frac{P_c}{a_c}.U^{-\alpha},$$

or

$$(A.7) \quad C = \left(\frac{P_T}{P_c}\right)^{\frac{1}{1+\alpha}}.a_c^{\frac{1}{1+\alpha}}.U.$$

Similarly, by inverting (A.3) and following the subsequent steps, we get

$$(A.8) \quad F = \left[ \frac{P_T}{W(1-t)} \right]^{\frac{1}{1+\alpha}} \cdot a_F^{\frac{1}{1+\alpha}} \cdot U.$$

The expressions for I and H can be obtained analogously to (A.7) and (A.8) from the inner utility function

$$C = (a_H H^\beta + a_I I^\beta)^{-1/\beta}.$$

Using (A.7) and (A.8) in (A.5) and (A.6),

$$(A.9) \quad P_T = [a_c^\rho \cdot P_c^{1-\rho} + a_F^\rho \cdot (W(1-t))^{1-\rho}]^{\frac{1}{1-\rho}},$$

$$\rho = \frac{1}{1+\alpha}$$

The partial derivatives of this unit expenditure function with respect to the prices of home good and imported good directly give the quantities demanded H and I, respectively. To obtain this, in (A.9),  $P_c$  has to be written out as a price index in  $P_H$  and  $P_I$ , as in (8) in the main text.

The form (8) can be derived as follows. The consumer minimises the cost  $P_H \cdot H + P_I \cdot I$  of consuming C, subject to (2) in the main text. The Lagrangean is

$$(A.10) \quad L = P_H \cdot H + P_I \cdot I + \lambda [(a_H H^{-\beta} + a_I I^{-\beta}) - C],$$

The first order conditions are

$$(A.11) \quad P_H = \frac{1}{\lambda} \cdot C^{1+\beta} \cdot a_H H^{-(1+\beta)}$$

$$(A.12) \quad P_I = \frac{1}{\lambda} \cdot C^{1+\beta} \cdot a_I I^{-(1+\beta)}.$$

Dividing and manipulating,

$$(A.13) \frac{P_H H + P_I I}{P_I I} = \frac{a_H H^{-\beta} + a_I I^{-\beta}}{a_I I^{-\beta}} = \left(\frac{C}{a_I I}\right)^{-\beta}.$$

Using in (A.13)

$$(A.13a) P_H H + P_I I = P_c C,$$

we obtain

$$\frac{P_c C}{P_I I} = \left(\frac{C}{a_I I}\right)^{-\beta},$$

or

$$(A.14) I = \left(\frac{P_c}{P_I}\right)^{\epsilon} a_I^{\epsilon} C,$$

$$\text{where } \epsilon = \frac{1}{1+\beta}.$$

Similarly,

$$(A.15) H = \left(\frac{P_c}{P_H}\right)^{\epsilon} a_H^{\epsilon} C.$$

Using these expressions for I and H in (A.13a),

$$(A.16) P_c = [a_H^{\epsilon} P_H^{1-\epsilon} + a_I^{\epsilon} P_I^{1-\epsilon}]^{\frac{1}{1-\epsilon}}.$$



## APPENDIX B. DERIVATIONS FOR THE MODEL INCLUDING MONEY

The consumer maximizes the utility function

$$(B.1) \quad U = (a_c \cdot C^{-\alpha} + a_m \cdot m^{-\alpha})^{-\frac{1}{\alpha}}$$

subject to

$$(B.2) \quad P_c \cdot C + P_m \cdot m = [W(1-t)(L_T - F) + \frac{T}{N}] + \pi_r(1-t) + M_0,$$

where  $M_0$  denotes initial money balances.

Forming the Lagrangean and maximising with respect to  $C$  and  $m$  (and dividing the first order conditions into each other),

$$\frac{a_m \cdot m^{-\alpha-1}}{a_c \cdot C^{-\alpha-1}} = 1,$$

or

$$\frac{a_m \cdot m^{-\alpha}}{a_c \cdot C^{-\alpha}} = \frac{P_m \cdot m}{P_c \cdot C}$$

This can be written as

$$\begin{aligned} P_c \cdot C \cdot \frac{U^{-\alpha}}{a_c \cdot C^{-\alpha}} &= P_m \cdot m + P_c \cdot C \\ &= P_T U, \end{aligned}$$

where  $P_T$  is expenditure per unit welfare. Solving for  $C$ ,

$$(B.3) \quad C = a_c^\rho \cdot \left(\frac{P_T}{P_c}\right)^\rho \cdot U$$

$$\rho = \frac{1}{1+\alpha}$$

Similarly,

$$(B.4) \quad m = a_m^\rho \cdot \left(\frac{P_T}{P_c}\right)^\rho \cdot U$$

Using (B.3) and (B.4) in the identity  $P_c C + P_c m = P_T U$ , we get

$$P_T = P_c (a_c^\rho + a_m^\rho)^{\frac{1}{1-\rho}},$$

*giving*

$$(B.5) \quad \left(\frac{P_T}{P_c}\right)^\rho = (a_c^\rho + a_m^\rho)^{\frac{\rho}{1-\rho}}.$$

Substituting (B.5) in (B.3) and (B.4), the expressions (3') and (32') for C and m in the main text are obtained.

**Table 1 List of parameters and variables**

|                    |  |
|--------------------|--|
| $a_{\sigma}$ $a_H$ | = distribution parameters in utility function  |
| $C$                | = total real spending per capita (including real investment)                           |
| $c$                | = user cost of capital   |
| $C_T$              | = aggregate nominal consumption  |
| $D$                | = domestic part of money supply  |
| $d$                | = rate of depreciation of capital stock  |
| $e$                | = exchange rate  |
| $F$                | = leisure time per capita  |
| $G$                | = real government spending on goods  |
| $g_i$              | = fraction of real government spending on goods directed towards imports               |
| $H$                | = real, per capita demand for domestic good  |
| $I$                | = real, per capita demand for imported good  |
| $i$                | = real gross investment  |
| $I_T$              | = total import volume  |
| $K$                | = real fixed capital stock   |
| $L$                | = employment in the private sector   |
| $L_g$              | = government sector employment   |
| $L_s$              | = number of hours supplied for work per person (= $L_T F$ )                            |
| $L_T$              | = total number of hours available per person (which is split between leisure and work) |
| $M$                | = nominal money supply   |
| $m$                | = real money balances of a representative consumer (i.e. per capita)                   |
| $m_i$              | = fraction of inventory investment met by imports                                      |
| $N$                | = number of members in the labour force  |
| $P_C$              | = consumer price index (per unit of total real spending)                               |
| $P_H$              | = price of home good   |
| $P_I$              | = price of imported good   |
| $P_R$              | = $P_H/P_I$  |
| $P_T$              | = price index for utility unit   |
| $P_w$              | = world price of imports in foreign currency   |

$Q$  = output volume (home goods)  
 $R$  = foreign reserves  
 $r$  = rate of interest  
 $s$  = savings-income ratio  
 $T$  = government transfers to the public  
 $t$  = tax rate  
 $t_k$  = rate of capital taxation  
 $TB$  = trade balance  
 $U$  = consumer utility  
 $U_u$  = unemployment rate  
 $v$  = real inventory investment  
 $W$  = wage rate  
 $WD$  = total foreign real spending  
 $X$  = export volume  
 $Y_D$  = nominal disposable income  
 $\alpha$  = substitution parameter in the utility function  
 $\beta$  = substitution parameter in the CES expression for the composite consumption good  
 $\delta$  = distribution parameter in production function  
 $\gamma$  = efficiency parameter in production function  
 $\epsilon$  = elasticity of substitution between home and imported goods  
 $\pi$  = inflation rate (of  $P_c$ )  
 $\pi_r$  = profits per head  
 $\rho$  = elasticity of substitution between goods and leisure in the utility function  
 $\sigma$  = elasticity of substitution between labour and capital  
 $\tau$  = substitution parameter in production function

## SAMMANFATTNING

Ekvationerna i den ekonometriska modellen KOSMOS är formulerade i enlighet med felkorrigeringsansatsen. Modellens långtidslösning kan följaktligen tolkas som en jämviktsmodell. Ett syfte med föreliggande uppsats var att åskådliggöra de teoretiska grunderna för denna modell, genom att från gängse optimeringsantaganden härleda en teoretisk motsvarighet till KOSMOS' jämviktslösning.

Statens budgetsaldo antas i modellen att alltid vara i balans. Timlönens anpassning är inte fullständig; en viss arbetslöshetsnivå tillåts även i jämvikt.

De statiska jämviktsvillkoren för den teoretiska modellen kräver att den inhemska inflationstakten skall vara lika med den utländska.

Den teoretiska modellen - efter eliminering av icke mätbara variabler - sammanfattar väl den långsiktiga lösningen till dagens KOSMOS. Den viktigaste skillnaden avser prisbildningen. Priserna i KOSMOS bestäms av kostnader, medan de i den teoretiska modellen anpassas så att de bringar varumarknaden i jämvikt. En annan skillnad beror på att funktionen för arbetsutbud i KOSMOS är starkt förenklad.

Ett annat syfte med uppsatsen var att undersöka hur modellens egenskaper påverkas av introduktion av finansiella variabler. Den teoretiska modellen utökades följaktligen med pengar. I denna modellversion bestäms priserna av penningmarknadens jämviktsvillkor. Penningmängden påverkas av bytesbalansen samt av statens budgetsaldo. På detta sätt åskådliggörs samspelet mellan ekonomins reala och finansiella sektorer samt finanspolitikens effekter.

Introduktion av pengar förändrar inte de statiska jämviktsvillkoren för modellen. Emellertid skapas på detta sätt den anpassningsmekanism som leder till att jämviktsvillkoren blir uppfyllda.

Det tredje och sista syftet med uppsatsen var att underlätta en jämförelse mellan KOSMOS och allmänna jämviktsmodellen MECMOD, som används vid institutet. Antaganden bakom MECMOD är mycket lika de antaganden som ligger bakom vår teoretiska modell. MECMOD är följaktligen mycket lik långsiktslösningen till dagens KOSMOS. De största skillnaderna avser arbetsmarknaden. I MECMOD är det full sysselsättning och fullständig löneanpassning; i KOSMOS är lönerna mindre flexibla och det finns alltid en viss grad av arbetslöshet. En ytterligare källa till skillnader i numeriska lösningar kan vara olika ekvationskoefficienter, eftersom MECMOD är kalibrerad för ett år medan KOSMOS är skattad på tidsseriedata.

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