DOES ROUND-THE-CLOCK TRADING RESULT IN PARETO IMPROVEMENTS?

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Does Round-the-Clock Trading Result in Pareto Improvements?

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Abstract

Exchanges have recently expressed considerable interest in moving towards a round-the-clock trading mechanism. This paper explores the effects of mandated market closure in an intertemporal economy in which liquidity is provided by risk averse agents (termed 'market makers'), who are an imperfect mechanism for transferring risk over time. These market makers trade with risk averse outside investors who find it costly to monitor prices continually, and therefore trade only once in the time-period covered by the model. Within this framework, it is found that market closure lowers market risk premia and increases the net welfare of the outside investors under a large parameter set. Some empirical and policy implications of our analysis are discussed.
Introduction

In recent months, exchanges have taken a tremendous interest in extending trading hours and have generally shown a strong inclination to move towards a 24-hour trading technology. The NYSE has indicated on several occasions that an ultimate objective of the exchange is to facilitate trade round-the-clock. For example, the extended one-hour trading session (between 4 and 5 pm) recently introduced by the NYSE was quoted by an NYSE official as representing a "small step towards continuous trading.\textsuperscript{1}" Another Wall Street Journal article recently quoted the NYSE President as stating that "we are on our way to a 24-hour market.\textsuperscript{2}" In addition, the Securities and Exchange Commission approved a recent proposal by the National Association of Securities Dealers (NASD) to allow extended trading hours for over-the-counter stocks. It was claimed that the goal was to "allow investors and institutions to eventually participate in [the] trading of U.S. stocks around the clock.\textsuperscript{3}" This paper analyzes the effects of mandated market closure on market risk premia and investor welfare.

The central theme of our analysis can be illustrated by examining two informal arguments in support of the notion that increasing an exchange's hours of operation will improve the welfare of all investors. First, suppose that all investors can continuously monitor the market. Then, closing the market can only hinder pareto improvements. While this statement may be valid, it seems unrealistic to assume that all agents can costlessly monitor financial markets throughout the day. This brings up the second informal argument in favor of twenty-four hour trading. Suppose some investors find it costly to remain in the market at all times, but market makers facilitate intertemporal trades. In this situation, it can again

\textsuperscript{1}"Big Board Mulls Early Start, Begins Late Hours," \textit{Wall Street Journal}, 6/14/91, p. C1.

\textsuperscript{2}"Big Board Plans to Start Trading 1/2 Hour Earlier," \textit{The Wall Street Journal}, 7/5/91, p. C1. As the article heading indicates, the context in this case was a proposal by the NYSE to start trading thirty minutes earlier: at 9 am instead of 9:30 am.

\textsuperscript{3}"SEC Approves Early Trading for OTC Stocks," \textit{The Wall Street Journal}, 11/10/91, p. C1. The extra session was to run from 3:30 am to 9 am.
be argued that closing the market reduces welfare, since the market makers effectively allow agents to trade with each other across periods. This paper shows that the second argument is not necessarily true. Our analysis demonstrates that if one makes the plausible assumption that agents cannot afford to stay continuously in the market, then closing the exchange for a period can lower market risk premia and improve the welfare of particular groups of traders.

For modeling the impact of market closure on a financial market, a multiperiod setting seems most appropriate. More specifically, to fully capture the impact of market closure, a model must distinctly allow trading to begin, halt, and resume; otherwise, it is evident that the welfare consequences on an investor from an inability to trade will not enter the welfare calculations. Further, to address welfare issues, it is desirable that all traders possess explicit utility functions, which they maximize.⁴ Also, when examining markets over a short period of time, such as a day or less, microstructure distinctions between types of traders become significant. The vast majority of traders find it costly to remain constantly in the market,⁵ and instead obtain intertemporal liquidity from financial institutions and floor brokers, who buy and sell against their own accounts in response to order flow fluctuations.

Keeping the above observations in mind, market participants in our model are divided into two groups, termed "market makers" and "outside investors." The market makers are competitive risk averse agents who continually monitor stock prices and trade whenever a suitable opportunity arises. These agents represent various professional groups such as floor brokers, "scalpers," and large financial

⁴Though microstructure models in which "liquidity" traders buy and sell unmodeled quantities (e.g., Kyle (1985), Glosten and Milgrom (1985), Ho and Stoll (1983)) have provided numerous insights, they seem less than appropriate for our purposes since in our case one must balance the gain from the elimination of the trading costs incurred by the liquidity traders (by precluding trade) with the reduction in welfare from losing the opportunity to trade. Without further modeling the liquidity traders, it does not seem possible to provide definitive answers to welfare questions.

⁵The costs may arise due to outside employment activities and/or cognitive limitations. See Hirshleifer (1988) and Allen and Gale (1992) for models in which agents have restricted access to financial markets.
institutions who are always present on the trading floor. The outside investors represent the vast majority of agents (e.g., individual investors) who find the cost of continuously monitoring and reacting to market fluctuations to be prohibitively high. This class is intended to capture the behavior of a typical individual whose primary income does not derive from stock-market related activities. For example, consider the case of a corporate executive. Most of the time he ignores market-related news, concentrating on activities related to his employment. However, at some point in time he receives income, say a bonus, that induces him to invest in the financial markets. At this time he determines, on the basis of current market prices, an allocation between "stocks" and "bonds." He then calls his broker with instructions on the quantity and time of trade. After doing so, he stops monitoring the financial markets, returning to his employment activities. The paper captures this type of investment activity by assuming that the outside investors can only trade once and arrive at an exogenously specified date. However, once they arrive at the market, they may postpone their trades, although in equilibrium they choose not to do so.

In our framework, market closure is modeled as a trading halt in round two. This stoppage in trade forces the outside investors who were potential round 2 traders to postpone their trades until round 3. Now consider the effect of the halt on the risk premia in rounds 1 and 3. The batching of the round 2 traders with the the round 3 traders increases the risk-bearing capacity of the market when it reopens; this effect tends to lower the risk premium in round 3. The offsetting effect is that since the total hedgeable endowment in round 3 is larger, the risk premium in this round tends to increase. Turning now to the risk premium in round 1, though market makers are forced to hold a unhedged position for a longer period upon market closure in round 2, the endowment of the round 2 traders does not need to be hedged till round 3, when the market makers receive the risk sharing benefit discussed earlier. The latter effect tends to lower the round 1 risk premium as well. We find that when all information shocks are identically distributed and the endowments brought to the market in each period are homogeneous and positive, halting trade in round 2 unambiguously reduces the risk premium in rounds 1 and 3. This
improves the welfare of those outside investors who arrive in the market during rounds 1 or 3. However, the round 2 outside investors are worse off, because the cost of maintaining an unhedged position between rounds 2 and 3 dominates the benefit of increased risk sharing opportunities by having their trades batched with the round 3 investors. Our analysis also indicates that the tendency for market closure to increase the aggregate welfare of the outside investors is strongest when the market closure occurs during periods with low levels of information arrival. Since the intensity of information arrival is small overnight, our analysis suggests that the net welfare of the outside investors is likely to be higher when markets are closed overnight than when they are open round-the-clock.

Note that in the present paper, the term "market maker" refers to any financial concern which monitors the market in search of profitable trading opportunities. Thus, the role of the "specialist" in our model is not to take positions on his account, but rather to act as a Walrasian auctioneer, clearing the market in each time-period through price adjustments. This viewpoint accords with that taken by the NYSE. For example, the NYSE Fact Book (1990) states on p. 4 that specialists act as "...catalysts, bringing buyers and sellers together, insuring that all offers to buy are matched with all offers to sell." The Fact Book goes on to state that buying and selling for specialists' own accounts represents "a very small portion of trading" and that "the vast majority of NYSE volume is a result of public order meeting public order - individuals, institutions and member firms interacting directly with each other."

Our analysis exemplifies the point that in an economy with imperfections (a "second best" economy), restricting trade may lead to social benefits. This general observation has been made by other papers in more abstract frameworks. For example, Hart (1975) and Newbery and Stiglitz (1984) show that under incomplete markets, welfare increases can be guaranteed only by completing markets; eliminating only some impediments to insuring against states is not sufficient. We provide an analogous result in a framework which differs from those above and adheres more closely to the institutional features of trading on actual exchanges. In our model, different agents have access to the market at different
points in time. We show that closing the market for a period of time can lead to overall welfare increases by forcing some agents to trade in time-periods other than those in which they originally would have traded. Our structure allows us to shed light on specific applied issues such as the effect of financial market closure on trading costs and the relative desirability of closing the market during periods of high versus low information arrival.

Madhavan (1992) examines the consequences of moving from a continuously open specialist system to one in which orders are batched and cleared simultaneously at regular intervals. He finds that under some conditions the specialist system may fail, in which case a batched order system offers welfare improvements. Unlike the present paper, the analysis in Madhavan, though insightful, is of two completely different market mechanisms, in the sense that the order batching process he examines does not derive from a specialist model in which markets are closed for certain periods of time.

Admati and Pfleiderer (1991) examine the welfare implications of "sunshine trading" (preannouncement of trades) in a one-period model which contains "speculators:" agents who act to absorb demand shocks from other investors. The role of these agents is similar to that of our market makers. As our goal is to address the intertemporal consequences of market closure, unlike Admati and Pfleiderer, we use a multi-period model in the present paper.

In other related work, Grossman and Miller (1988) consider a two-date model somewhat similar to ours. While the interpretation of the two classes of traders is the same in both models, there are three primary differences. First, Grossman and Miller require the endowment shocks received by the period 1 and period 2 outside investors to exactly offset each other; we do not impose this requirement. Second, in Grossman and Miller all traders arriving in period 1, including the outside investors, can trade in period 2. In contrast, in our model there is an explicit distinction between outside investors and market makers in the sense that each outside investor trades only once, while market makers always remain in the market. Finally, the focus in Grossman and Miller is on the role of market makers in providing
intertemporal liquidity, while we focus on the effects of market closure on trading costs and investor welfare.

This paper is organized as follows. Section I describes the basic model. Sections II and III deal with the homogeneous and heterogeneous cases respectively. Section IV concludes.

I. The Model

We consider an economy which contains a risky security and a riskless bond that can be exchanged in three rounds of trading. The bond yields a return of zero. All security payoffs are realized at the end of trade in the third round.

There are two types of potential traders in the market. As motivated in the introduction, the first type, termed "market makers" trade in all three rounds, while the second type, "outside investors," trade only once. The outside investors enter in a specified round but can postpone their trades if they wish. Since the model is designed to cover a short time span of about a day, the fact that the outside investors cannot trade in every period does not seem restrictive.\(^6\) If one simply assumes that an individual's day-to-day schedule makes it prohibitively costly to trade, except at one particular point in time, similar results are achieved. For example, an executive may be unable to monitor the market, except for a period between business meetings. Alternatively, an individual investor may not discover his endowment shock until the late evening hours (after he completes his employment-related activities), and then may find it costly to monitor the market and trade more than once.

All traders possess negative exponential utility. The risk aversion of the market makers is denoted by \(\theta\) and that of the outside investors is denoted by \(\phi\). There is a continuum of each type of trader in the economy, and the masses of the market makers and the outside investors arriving at the market in a particular period are each normalized to unity. Thus, in a competitive environment, one can treat an

\(^6\)Consequences of restricted market participation by agents have been analyzed by several authors in the market microstructure literature; for examples, see Grossman and Miller (1988), Admati and Pfleiderer (1988), and Foster and Viswanathan (1991).
agent's individual demand as the aggregate demand of all agents belonging to that agent's class. The goal of both classes of agents is to maximize their future wealth at the time when the security is liquidated.

The liquidation value of the security subsequent to the 3rd round of trading is denoted by

\[ V = \mu + e_1 + e_2 + e_3. \]  

Each \( e_i \) is revealed following the \( t \)th round of trading and prior to the next round of trading. We assume that \( e_i \sim N(0, \sigma_i^2) \) for \( i = 1, 2, 3 \) and that \( e_1, e_2, \) and \( e_3 \) are mutually independent. Prices are set by a Walrasian auctioneer to equate supply and demand in each period. Since we focus on intraday price movements over short time-periods, we ignore discounting, as this is likely not a significant issue in our context.

The fact that information arrives after each round of trading allows for a commonly advanced argument that market closure is undesirable because it forces individuals to hold unhedged positions. However, as will be seen, other consequences of market closure can result in a net benefit for some if not all outside investors.

We now conjecture that the equilibrium prices in each trading round take the form

\[
\begin{bmatrix}
    p_1 \\
    p_2 \\
    p_3
\end{bmatrix} =
\begin{bmatrix}
    \mu - r_t \\
    \mu + e_1 - r_t \\
    \mu + e_2 - r_t
\end{bmatrix}
\]  

where \( r_t, t=1,2,3 \) is the risk premium at time \( t \). We will later confirm that these functional forms do indeed hold in equilibrium. Subsections A and B below consider the optimization problems of the market makers and the outside investors respectively.
A. The Market Makers’ Problem

The market makers are endowed with $B_0$ units of the riskless bond and possess an initial endowment $S_0$ of the risky security. Denote the quantities of the risky security and the bond held by the market makers after trading at time $t$ by $S_t$ and $B_t$, respectively. The market makers then face the following budget constraints:

$$(S_{t} - S_{t-1})p_t + B_{t} - B_{t-1} = 0, \quad t = 1, 2, 3.$$  

After the third trading round the liquidation value of their position is $S_3(\mu + \sum_{j=1}^{3} e_j) + B_3$. Using the budget constraint above and successively eliminating $B_1$, $B_2$, and $B_3$ from the final payoff, the market maker’s problem reduces to

$$\max_{S_1, S_2, S_3} \quad S_3 r_3 + S_2 (r_2 - r_3) + S_1 (r_1 - r_2) - S_0 r_1 + B_0 - \frac{\theta}{2} (S_3^2 \sigma_3^2 + S_2^2 \sigma_2^2 + S_1^2 \sigma_1^2)$$

(3)

Solving the above problem, one obtains

$$\begin{bmatrix}
    S_1 \\
    S_2 \\
    S_3
\end{bmatrix} = \frac{(r_1 - r_2)/\theta \sigma_1^2}{(r_2 - r_3)/\theta \sigma_2^2}$$

(4)

B. The Outside Investors’ Problem

Consider now the outside investors. For expositional convenience, we refer to an outside investor who trades in period $t$ as a "period $t$ trader." Let $a_t$, $t = 1, 2, 3$ respectively denote the total (positive) endowments of the three groups of outside investors. The numbers $a_t$ are common knowledge to all market makers. This assumption can be justified on the grounds that unpredictable shocks to endowments are idiosyncratic across individuals and therefore by the law of large numbers, the aggregate endowments equal the mean of the endowment process in a large market. The analysis remains unchanged if each outside investor receives an endowment shock $a_t + \eta_{tk}$, where the $\eta_{tk}$’s are random shocks which integrate
to zero across all outside investors arriving in a particular period. The period t outside investors are endowed with $B_\alpha$ units of the riskless bond. Let $S_t$ and $B_t$ denote the risky security and bond holdings of the period t outside investor after their trading is complete. One can write the budget constraint faced by a period t outside investor as

$$ (S_t - a_t)P_t + B_t = B_\alpha = 0. $$

After he trades to a position of $S_t$ and $B_t$, the period t trader's terminal payoff can be calculated as

$$ S_t \left( \mu + \sum_{i=1}^{3} e_i \right) + B_t. $$

Eliminating $B_t$ in the above expression and using the budget constraint, the problem of the period t trader reduces to

$$ \max \quad S_t r_t + a_t P_t + B_\alpha - \frac{\phi}{2} \sum_{i=1}^{3} \sigma_i^2. $$

$$ S_t $$

Thus, the vector of the outside investors' demands is given by

$$ \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} r_1/[\phi(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)] \\ r_2/[\phi(\sigma_1^2 + \sigma_2^2)] \\ r_3/[\phi \sigma_3^2] \end{bmatrix}. $$

The market clearing conditions require that in each period, the total supply of the risky security equal the aggregate demand of all the traders. In period 1, this requires $S_0 + a_1 = S_1 + S_2$. In period 2,
the outside investors enter with \(a_2\) units of the security. Therefore, market clearing requires that
\[ S_0 + a_1 + a_2 = \hat{S}_1 + \hat{S}_2 + S_2. \]
Applying a similar reasoning for period 3, the market clearing conditions can be written compactly as
\[ S_0 + \sum_{i=1}^{t} a_i = \sum_{i=1}^{t} \hat{S}_i + S_t, \quad t = 1, 2, 3. \tag{7} \]
Substituting for \(\hat{S}_i\) and \(S_t\) from (6) and (4), (7) becomes
\[ S_0 + \sum_{i=1}^{t} a_i = \sum_{i=1}^{t} \frac{r_i}{\phi \sum_{j=1}^{3} \sigma_j^2} + \frac{r_i - r_{i+1}}{\theta \sigma_i^2}, \quad t = 1, 2 \tag{8} \]
\[ S_0 + \sum_{i=1}^{3} a_i = \sum_{i=1}^{3} \frac{r_i}{\phi \sum_{j=1}^{3} \sigma_j^2} + \frac{r_3}{\theta \sigma_3^2}. \]
The next section presents the analysis for the case of homogeneous endowments and identically distributed information shocks, while Section III addresses the general case.

II. The Homogeneous Case

In this section, we assume that the total endowments brought to the market in each period are equal and positive (i.e., \(S_0 + a_1 = a_2 = a_3 > 0\)). The case of positive endowments clearly is of primary interest since stocks are, by definition, in positive net supply. The common value for the endowments is denoted by \(a\). Further, let \(\sigma_t^2 = \sigma^2\) for all \(t\). Denote \(\phi a \sigma^2 = k\) and \(\theta / \phi = p\). From (8) the equilibrium risk premia in our economy then solve the system
\[
\begin{bmatrix}
1/3 + 1/p & -1/p & 0 \\
1/3 & 1/2 + 1/p & 1/p \\
1/3 & 1/2 & 1 + 1/p
\end{bmatrix}
\begin{bmatrix}
r_1 \\
r_2 \\
r_3
\end{bmatrix} =
\begin{bmatrix}
k \\
2k \\
3k
\end{bmatrix}.
\tag{9}
\]
The solutions for \(r_1\), \(r_2\), and \(r_3\) are stated in the following lemma.
Lemma 1: Under the above assumptions, the equilibrium risk premia are given by

\[
\begin{bmatrix}
  r_1 \\
  r_2 \\
  r_3 \\
\end{bmatrix} = \begin{bmatrix}
  3kp(p^2+8p+12)/D \\
  2kp(p^2+9p+15)/D \\
  kp(p^2+9p+18)/D \\
\end{bmatrix}
\] (10)

where \( D = p^3 + 9p^2 + 18p + 6 \).

Note that the sign of the risk premium corresponds to the sign of the endowments. It can also be shown that all the risk premia increase in \( p \) (i.e., increase as the market makers' risk aversion increases relative to that of the outside investors).

The analysis next examines the issue of whether the outside investors may wish to postpone their trades. If they wish to delay their exchanges, then it can be argued that closing the market becomes irrelevant, since no trade will take place even if it is open. However, the next Lemma, which follows from straightforward calculations, shows that the outside investors prefer to rebalance their portfolios as soon as they enter the market, i.e., that they do not wish to postpone their trades, as to do so would entail their holding an unhedged position for a length of time.

Lemma 2: It is an equilibrium for period one traders to trade in period one and period two traders to trade in period two (i.e., period 1 traders maximize their utility by trading in period one and period two traders maximize their utility by trading in period two).

A. Comparison of the Equilibrium Risk Premia With and Without Market Closure

If the market is closed in period 2, the orders of the period 2 traders are batched and executed along with the orders of the period 3 traders and the market makers in period 3. This causes the period 2 traders to hold an unhedged position between dates 2 and 3 and to then hedge the shock \( e_3 \) in the final period. The trades of the period 1 and period 3 traders, as a function of the respective risk premia,
remain unchanged by the closing of the market. This simply results from the fact that market closure does not affect their trading opportunities. The demands of the market makers in period 1 (as a function of the risk premia), however, are affected if the market is closed in period 2.

Denote as $S_{2c}$ the after-trade holdings of the period 2 outside investors under period 2 market closure. Further, let $r_{1c}$ and $r_{3c}$ denote the period 1 and period 3 risk premia when the market is closed in period 2. One can easily show that under period 2 market closure, the period 1 demands of the market makers change to $S_1 = (r_{1c} - r_{3c})/2\theta\sigma^2$. The market closure forces the period 2 investors into period 3, where their demands are modified to $S_{2c} = r_{3c}/(\theta\sigma^2)$. Expressions for the other holdings remain unchanged, except for the fact that $r_1$ and $r_3$ in the relevant expressions in equations (4) and (6) are replaced by $r_{1c}$ and $r_{3c}$ respectively. The set of equations determining the equilibrium risk premia are thus modified to

$$
\begin{bmatrix}
1/3 + 1/2p & -1/2p \\
1/3 & 2 + 1/p
\end{bmatrix}
\begin{bmatrix}
r_{1c} \\
r_{3c}
\end{bmatrix}
= 
\begin{bmatrix}
k \\
3k
\end{bmatrix}, 
$$

(11)

After solving the above system, one can state the following lemma.

**Lemma 3:** When the market is closed in period 2, the risk premia in periods 1 and 3 are given by

$$
\begin{bmatrix}
r_{1c} \\
r_{3c}
\end{bmatrix}
= 
\begin{bmatrix}
3kp(4p + 5)/D_c \\
kp(4p + 9)/D_c
\end{bmatrix},
$$

(12)

where $D_c = 4p^2 + 9p + 3$.

Comparing $r_1$ and $r_3$ (from (10)) with $r_{1c}$ and $r_{3c}$ (from (12) above), and performing some tedious algebra, we obtain the following result.
Proposition 1: Under the assumptions of this section, the risk premia in periods 1 and 3 are lower when the market is closed in period 2 than when it is open in all three periods, i.e.

1. \( r_{1c} < r_1 \)
2. \( r_{3c} < r_3 \).

Thus, counter-intuitively, closing the market in period 2 causes a reduction in both period 1 and period 3 market risk premia. Preventing the market makers from trading in period 2 forces them to carry an unhedged position for a longer time, which should presumably lead to higher risk premia. However, the premium in period 3 decreases because of the additional risk capacity in period 3, which is induced by the batching of the period 2 and period 3 traders in this period. Further, the market makers are willing to provide more liquidity to the period 1 traders, knowing that the period 2 and period 3 traders will be batched together in period 3. This factor more than offsets the market maker’s cost of bearing an unhedged position.

The explanation given above indicates that the market makers may be willing to purchase a larger quantity of the risky security in period 1 if they know that the market will close in period 2. Using (4) and (10) to obtain an expression for \( S_1 \) that contains only exogenous variables, and then using (12) to produce a similar equation for \( S_1 \) under market closure yields the proposition below, confirming the above intuition.

Proposition 2: The equilibrium value of \( S_1 \) is lower (i.e., market makers take on larger inventories in the risky security in period 1) when the market is closed in period 2 than when it is open in all three periods.

The knowledge that the market will close induces more aggressive buying by the market makers. As described above, this behavior results from the fact that they do not hedge the endowment risk of the period 2 traders until period 3, in which they trade with both the period 2 and period 3 traders. Therefore, the market makers are willing to bear more risk in period 1.
B. Welfare Comparisons

A primary question addressed by this paper is the impact of closing the market in period 2 on investor welfare. To calculate the change in a trader's welfare from market closure, let $\text{EU}(x)$ represent a trader's expected utility given the event $x$ occurs. Then take $\log(\text{EU}(\text{market closes in period two}))$ minus $\log(\text{EU}(\text{the market remains open all three periods}))$ as a measure of the change in an agent's welfare when the market closes in period 2. Some algebra then shows that these measures for the outside investors (in terms of the risk premia $r_t$, $t = 1, 2, 3$, $r_{1a}$, and $r_{3o}$) are given by

$$\Delta W_1 = a \left[ \frac{r_1^2 - r_1^2}{6k} - (r_{1o} - r_1) \right]$$  \hspace{1cm} (13)$$

$$\Delta W_2 = a \left[ \frac{r_3^2 - r_2^2}{2k} - (r_{3o} - r_2) \frac{k}{2} \right]$$  \hspace{1cm} (14)$$

and

$$\Delta W_3 = a \left[ \frac{r_3^2 - r_3^2}{2k} - (r_{3o} - r_3) \right]$$  \hspace{1cm} (15)$$

Notice that in equation (14), there is an extra term affecting the period 2 traders' welfare. When the market closure takes effect, it forces the period 2 outside investors to remain unhedged against the $e_2$ shock. The $k/2$ term acts to capture the resulting utility loss. Since market closure does not alter the trading dates for the other investors, their welfare calculations lack an analogous term.

In addition to examining the welfare changes associated with outside investors arriving in particular periods, it is also of interest to examine whether the aggregate (net) welfare of the outside investors is enhanced or reduced as a result of market closure. To address the latter issue, we therefore examine the quantity $\Delta W_1 + \Delta W_2 + \Delta W_3$, which we term "net investor welfare." Substituting for the
various premia in equations (13)-(15) and performing a complex, but straightforward algebraic exercise yields the following proposition.

**Proposition 3:** Under the assumptions of this section, closing the market in period 2 increases the welfare of the period 1 and period 3 outside investors, and lowers the welfare of the period 2 outside investors, i.e.,

(1) $\Delta W_1 > 0$

(2) $\Delta W_2 < 0$

(3) $\Delta W_3 > 0$.

Further, net investor welfare increases upon market closure in period 2, i.e., $\Delta W_1 + \Delta W_2 + \Delta W_3 > 0$, if and only if $p > 0.685$.

The intuition behind the results in the above proposition is the following. From proposition 1, the risk premia in periods 1 and 3 decrease, which makes the period 1 and 3 traders unambiguously better off. For the period 2 traders, market closure has both a negative and a positive impact. As mentioned earlier, being forced to carry an unhedged position till period 3 tends to decrease these traders' utility. On the other hand, they face superior risk sharing opportunities by trading along with the period 3 traders. The first effect dominates the second effect. For a sufficiently large $p$, net investor welfare also increases because the higher the risk aversion of market makers relative to that of the outside investors, the higher the benefit of better risk sharing. Notice that the welfare losses induced by a move from a regime with a mandated closure to a regime without one are borne by traders who arrive both before and after the closure (i.e., period 2).

III. The General Case

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8Though individual market makers may be less risk averse than individual outside investors (because of higher wealth levels and therefore larger risk bearing capacities), it is evident that outside investors outnumber market makers as a group. Because of these offsetting effects, the value of $p$ may not be too different from unity.
This section attempts to gain more insight regarding the parameter space under which market
closure decreases trading costs and enhances social welfare by relaxing the assumptions that the
endowments and the variances of the endowment shocks are constant across time. Let the total period
1 endowment \((S_0 + a_1)\) be given by \(a\), and the endowments of the period 2 and 3 traders by \((e-1)a\) and
\((f-e)a\) respectively. Further, let \(\sigma_1^2 = \sigma^2, \sigma_2^2 = ma^2, \sigma_3^2 = na^2\) and preserve the notational definition that
\(\phi a \sigma^2 = k\). (These parameterizations simplify the exposition which follows.) One can easily derive the
following lemma, stated without proof.

Lemma 3: The equilibrium risk premia under the above specification are given by

\[
\begin{bmatrix}
  r_1 \\
  r_2 \\
  r_3
\end{bmatrix} = \begin{bmatrix}
  \frac{kp(m+n+1)(m^2e(p+1)+m(n[p(e+e+f)+(p+1)^2]+n(nf+2p+1)))/D}{D} \\
  \frac{kp(m+n)(m^2e(p+1)+m(n[p(e+e+f)+(p+1)+e](p+1)]+n(nf+p(f-1)+f))/D}{D} \\
  \frac{kpn[m^2[2p(f-e)+f]+m[2n(p(f-e)+f)+p^2(f-e)+p(2f-e-1)+f]+n(nf+p(f-1)+f))/D}{D}
\end{bmatrix}
\]

(16)

where

\[D = m^2(p+1)(2p+1)+m[2n(p^2+3p+1)+(p+1)^2]+n(3p+1)+p+1(2p+1)].\]

When there is no trade in period 2, the premia in periods 1 and 3 are given by

\[
\begin{bmatrix}
  r_{1e} \\
  r_{3e}
\end{bmatrix} = \begin{bmatrix}
  \frac{kp[m(2p+1)+nf+2p+1]m(m+n+1))]/D_e}{D_e} \\
  \frac{kpn[p(f-1)+f]+nf+p(f-1)+f)/D_e}{D_e}
\end{bmatrix}
\]

(17)

where \(D_e = m(p+1)(2p+1)+n(3p+1)+(p+1)(2p+1).\) The welfare expressions (13-15) are now
modified to

\[
\Delta W_1 = a\left[\frac{r_{1e}^2-r_1^2}{2(1+m+n)k}(r_{1e}-r_1)\right]
\]

(18)

\[
\Delta W_2 = a\left[\frac{r_{3e}^2-r_2^2}{k(m+n)}-(e-1)(r_{3e}-r_2)-\frac{(e-1)^2nk}{2}\right]
\]

(19)
\[ \Delta W_3 = a \left[ \frac{r_{3c}^2 - r_3^2}{2kn} - (f-e)(r_{3c} - r_3) \right]. \]

As the expressions in Lemma 3 are difficult to analyze and interpret, it is useful to consider special cases. We first examine the conditions under which the risk premia in periods 1 and 3 decrease as a result of market closure, in the special case of homogeneously distributed information shocks, i.e., \( m=n=1 \). Comparing the risk premia in equations (16) and (17), the following proposition can be stated.

**Proposition 4:** If \( m = n = 1 \), then,

(1) \( r_{1c} < r_1 \) if and only if

\[
e > \frac{p^3(f+13) + p^2(f+43) + 30p + 6}{2(4p^3 + 13p^2 + 2p + 3)}
\]

(2) \( r_{3c} < r_3 \) if and only if

\[
e < \frac{2p^3(f+1) + 2p^2(8f+5) + 9p(3f+2) + 9f + 6}{4p^3 + 29p^2 + 48p + 15}.
\]

The proposition indicates that if the period 2 traders have large endowments relative to those of the period 3 traders, closing the market in period 2 tends to reduce the period 1 risk premium and raise the period 3 risk premium. The effect results from the alteration in the hedging opportunities faced by the market makers as a result of market closure. When the period 2 traders have a large endowment, the market makers receive a large incremental benefit if trade is postponed. By hedging the period 2 trader’s endowment in period 3 instead of period 2, the market makers face a reduced level of risk, which results in a reduction in the period 1 risk premium. However, the reduced period 1 risk premium implies that the market makers purchase a larger amount of the security in period 1. Therefore, the traders enter
period 3 with a larger supply of the risky security; this effect tends to increase the final period's risk premium. Of course, a factor which tends to reduce the period 3 risk premium is the batching of the round 2 and the round 3 traders. However, for a sufficiently large market maker position at the end of period 1, the former effect dominates. Notice that the tendency for the market closure to lower both the period 1 and period 3 premia is strongest when e lies in an intermediate range, i.e., does not take on extreme values relative to f.

We now relax the restriction that the information shocks have identical variances (i.e., that m=n=1) but reimpose the assumption that the endowments brought to the market in each period are equal. Thus, in the notation of this section, e=2 and f=3.

During a 24-hour period, information is not generated at a constant rate. For example, one expects less news to arrive overnight relative to news flows during the day. Therefore, it is of interest to examine the desirability of closing the market during periods of high versus low informational arrival. Such comparisons can be accomplished by varying the parameter m. Owing to the analytical complexity of the impact of changing m on risk premia and welfare, we provide a numerical simulation for this case.

Proposition 3 suggests that net investor welfare tends to increase upon suspension of trade if p is sufficiently high. Figure 1, which plots the threshold value of p above which net investor welfare increases as a function of m, shows that the lower the m, the larger the range of p under which net investor welfare increases upon market closure. Thus, Figure 1 indicates that, on net, the outside investors are most likely to benefit if trade is suspended during periods with low levels of news arrival. Thus, it appears that closing the market overnight has a strong tendency to make outside investors better off relative to a regime with continuous trading.
IV. Concluding Remarks and Implications

There has been a tremendous impetus towards round-the-clock trading in recent months. We have analyzed the effects of mandated market closure on risk premia and investor welfare in an intertemporal model of risk sharing. In our three-period framework, some agents, termed market makers, act as intertemporal transferrers of risk, while outside investors with unhedged endowments serve, in effect, as purchasers of liquidity. The disadvantage of a market closure is that it forces some outside investors to hold unhedged positions for a certain period of time. However, there exists an offsetting advantage, in that the traders arriving during the halt trade together with those trading subsequent to the halt, which provides superior risk sharing opportunities.

Our analysis finds that when all information shocks are identically distributed, and the
endowments brought to the market are positive and equal across time, then closing the market in period 2 unambiguously reduces the risk premium in periods 1 and 3. From a welfare perspective, the outside investors arriving in periods 1 and 3 always benefit in this case, while those arriving in period 2 are worse off. When the welfare benefits are aggregated, it is found that net investor welfare increases under a large parameter set. Moving away from the homogeneous case, the analysis subsequently focuses upon two effects that influence the conditions under which closing the market in period two reduces market risk premia and increases net investor welfare. First, the tendency for market closure to lower risk premia is strongest when the endowment of the period 2 outside investors lies in an intermediate range, i.e., does not take on extreme values relative to the endowments of other market participants. Second, simulations examining the rate of information flow indicate that the greatest benefits from closing the market are likely to occur when there is relatively little news arrival. This suggests that suspending trade overnight is likely to lead to a higher net welfare relative to 24-hour trading.

Some testable implications of our analysis are worth mentioning. Our model indicates that due to the risk sharing benefit discussed above, market maker inventories are likely to be higher prior to relatively lengthy closures than relatively short ones. This implication can be tested using inventory data before weekend and weekday closures. Further, our model predicts that risk premia (which can be proxied by bid-ask spreads - see Ho and Stoll (1983)) before and after the closure should fall when trading hours are extended; this can be tested using data before and after the extended trading facilities were recently introduced by the NASDAQ and the NYSE (see the introduction).

Finally, we note that there may, of course, be other rationales for an exchange to be open round-the-clock (e.g., preventing the migration of order flow to overseas exchanges). We feel, however, that the vast majority of outside investors (e.g., individual investors) are unlikely to have access to international markets. Therefore, even taking an international perspective on the issue, our analysis forms a valid consideration in developing closure policy. In any case, our paper suggests that the benefits, if
any, of 24-hour trading are likely to be bought at a cost the market incurs in terms of higher risk premia and lower investor welfare.
References


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