PRIOR – NIER's input-output based cost push price model

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Abstract

The present paper introduces an input-output based cost push price model for the Swedish economy (PRIOR). The model is used at NIER for both short term forecasts and long run structural analysis of the Swedish economy. The economy is divided into about 30 products. For each product it is possible to calculate the impact on the consumer price of any change in wages, productivity, taxes, subsidies and import prices. This paper presents calculated price effects on consumer prices of a 10 percent change in hourly labour costs, taxes and import prices respectively, under different profitability assumptions. A novelty in this version of the price model is the explicit inclusion of trade margins. The model also gives the user a wider array of options regarding the profitability assumption than has been available in earlier model versions.

Jel classification codes: C67, E37

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Contents

1 Introduction

1.1 Background and main assumptions

PRIOR is a cost-push input-output price model that can be used to calculate the effect of changes in wages, taxes, import prices, et cetera, on the product prices for final users. It is a development and improvement of earlier versions that have been used by both the NIER and the Swedish Ministry of Finance since at least the 1970s (see Roström, 1981 and Markowski, et al. 2011).

The model has a flexible setup. It can be used for a wide range of purposes. Almost any variable can be set exogenously or be endogenously determined, as the user wishes. This means that the model can be used to calculate the effect on prices and deflators under a wide range of circumstances, such as:

- The effect on consumer prices of a change in the oil price with all other prices endogenously determined.
- The effect on the GDP deflator of a change in the hourly wage.
- The effect on the household consumption deflator of a certain rise in some or all import prices.
- The effect on consumer prices of goods resulting from an increase in product taxes on petroleum.

All of the above can be calculated under different assumptions regarding profitability: Fixed gross profit margins (gross profit/revenue), fixed return to capital or fixed profit shares of value added. The user is free to choose between these three alternative assumptions regarding profitability in production.

The model assumes Leontief technology; it is based on the assumtpion that there is no substitution between products, that production volumes are constant, and that prices are determined by costs. The consumtion choice is also fixed, the price elasticity of substitution in consumption is assumed to be zero. It is the same type of cost-push price model based on the industry technology assumption as described in Miller and Blair (2009). The model is implemented as a system of linear equations using the GAMS software package. By using the GAMS programming language, flexibility in the choice of assumtpions regarding profits and which products have endogenously or exogenously determined prices is facilitated. The program code is easy to follow and data output on the variables of interest is readily supplied. The model is static and is solved only once for each experiment at hand.

As a default the model is based on the cost and final demand structure in the base year. This is usually the most recent year with published supply and use tables from the national accounts. The user can deviate from the base year cost structure by changing the coefficients, weights or other parameters if desired. The model does not have a time dimension and is only solved for a single year (instantaneously). However, by setting exogenous price and productivity changes that represent long run expected growth rates, the model results can be interpreted as representative of long run rates of change, provided that there is no substitution in consumption or production.

1.2 The structure of this document

The introduction continues with some notation and teminology. Section 2 describes the main variables and parameters in the model. Section 3 contains a description of the cost-push model under some simplifying assumptions. Section 4 shows how model parameters are caclulated using actual data. Section 5 expands on the model description by introducing more complexities such as taxes, imports and trade margins. This section provides a full description of the model equations. Section 6 shows the results in three different applications of the model: changed wages, changed taxes and changed import prices. Section 7 concludes.

1.3 Notation, terminology and operators

Uppercase boldfont denotes a matrix. For example, $\mathbf{A}$ is a matrix of input coefficients. Lowercase boldfont denotes as column vector. For example, $\mathbf{q}$ is the commodity output vector. A prime ($\mathbf{A}'$) denotes the transpose of a matrix or vector. A hat ($\hat{\mathbf{q}}$) denotes a diagonalized vector.

The name 'commodity' refers to any product, a good or service. The word is used because it is standard in the input-output literature. It is synonymous with the word product. Industry refers to any production sector, manufacturing, services, etc. The section on calculation of model parameters using actual data goes into more detail regarding the product/industry distinction.

$\odot$ Is the Hadamard product, elementwise multiplication:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \odot \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} ab_{11} & ab_{12} \\ ab_{21} & ab_{22} \end{pmatrix}$$

Generally, uppercase non-boldfont letters refer to values or totals, such as:

Y_i is value added in constant prices in industry *i*.

H_i^{BY} are the number of hours worked by employees and the self employed in the base year in industry *i*.

K_i^{BY} is the capital stock in current prices in the the base year in industry *i*.

Q_i is the gross volume of output in industry *i*.

The superscript BY indicates that the value refers to actual base year data.

Lowercase letters generally refer to variables or parameters per unit, such as the wage rates by industry w_i.

NORMALIZATION

All model prices are normalized to 1 in the base year. There are several reasons for this normalization. It is standard in input-otput price models. It is easy to calculate relative price changes compared to the baseline. It is easy to verify the model setup by replicating the initial values without any changes. It is also recommended that starting values for variables in GAMS be close to 1 for numerical reasons.

For most data in the model, the data is spilt into a model shifting variable normalized to 1 as initial value, and a base-year data parameter. For instance, the wage rate is the product of the shifting variable w_i and the base-year actual wage rate $actualw_i^{BY}$. In most cases we are not interested in the actual levels for variables, only in relative rates of change. So the actual base-year data is mostly used to calculate the model parameters.

2 Main model variables and parameters

We begin the model description by simplifying to a symmetric industry/commodity setup. In sections 2 and 3, there is no distinction between industries and commodities. Section 4 below describes how the model parameters are calculated without this simplification.

2.1 Constant parameters

Parameters take the form of scalars, vectors or matrices that are usually not changed in the model and that come from base-year data. The main parameters that usually remain unchanged in the model are the input use coefficients and the value-added shares. These are all fractions dervided from base year data.

$\mathbf{A}$ Is the *n* by *n* matrix of input use coefficients as a share of total output of a commodity.

$\mathbf{v}$ Is a $n \times 1$ column vector of value-added shares by commodity/industry. It shows the value added as a share of gross output for each industry/commodity. Each element is equal to 1 minus the total sum of the input use coefficients in the production of the commodity.

$$\hat{\mathbf{v}} = \begin{pmatrix} v_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & v_n \end{pmatrix}$$

This is the diagonalized vector of value-added shares: The value-added shares of each commodity lie in the diagonal of a matrix that is otherwise filled with zeroes.

2.2 Variables

NORMALIZED PRICE VARIABLES

Prices are normalized to 1 as initial values. They have no corresponding base-year data.

$\mathbf{p}_{bp}$ is a column vector of product prices at basic prices (excluding taxes and subsidies). There are *n* products indexed by $i = \{1, 2, ..., n\}$. So this vector has the dimensions $n \times 1$. These prices are normalized to 1 as initial values.

$\mathbf{p}_v$ is a $n \times 1$ column vector of implicit value-added prices/deflators for industries/commodities. These prices are normalized to 1 as initial values.

$p_{K,i}$ is a price index for the capital stock in industry *i*. The base year value $p_{K,i}^{BY}$ is normalized to 1.

NORMALIZED SHIFTING VARIABLES WITH CORRESPONDING DATA PARAMETERS

The variables below have actual corresponding data parameters. They can be described as shifting variables. They are all normalized to 1 as initial values, but they can be set (shifted) to other values by the user. Depending on the assumptions of the user and the question at hand, some of them can be exogenously fixed, and others can be endogenous. The actual final level values in the model solution can be found by multiplying the actual base year parameter value with the model shifting variable.

w_i is the labour cost per hour in the production of commodity *i*. The actual hourly labour cost in actual data (SEK per hour) in the base year is denoted by $actualw_i^{BY}$.

pr_i is the labour productivity level in constant prices in the production of commodity *i*. It represents value added in volume terms per hour worked. The actual productivity by commodity in actual data (SEK per hour worked) in the base year is denoted by $actualpr_i^{BY}$. As an example, if the user wishes to calculate the effect of a ten percent increase in productivity throughout the economy, pr_i should be set to be 1.10 for all products.

ky_i is the capital-to-value added ratio in constant prices. This is calculated by dividing the capital stock in volume by the value added volume. A higher value implies a more capital intensive production activity. It is the inverse of capital productivity. The actual capital intensity in actual data (SEK capital stock per SEK value added) is denoted $actualky_i^{BY}$.

r_i is an implicit index for the gross rate of return to capital in the production of commodity *i*. The actual gross rate of return to capital in the base year is denoted $actualr_i^{BY}$. Note that the gross operating profit in the base year is equal to the product of this actual rate of return in the base year and the capital stock valued at current prices in the base year.

DERIVED MODEL VARIABLES

The unit labour and unit capital costs are derived by combining normalized model variables with model parameters. In practice, the base year values can be calculated directly from the data, as shown in section 4. These derived variables are not normalized to 1 in the base year.

$\mathbf{u}_l$ is the vector of unit labour costs in the production of commodities. $\mathbf{u}_l^{BY}$ are the actual unit labour costs in the base year.

$\mathbf{u}_k$ is the vector of unit capital costs in the production of commodities. $\mathbf{u}_k^{BY}$ are the actual unit capital costs in the base year.

2.3 Interpretation of model variables

Since all prices and model shifting variables are normalized to 1 as initial values, changes relative to the base year can be calculated in percent using the expression:

$$100 \cdot (x-1)$$

This equation can be used for any variable x in the model whose initial value has been normalized to 1. As an example, an exogenous wage increase of 10 per cent corresponds to setting $w = 1.1$. If this number is entered in exogenously and the model is solved, an endogenous price variable in the model solution having a value of 1.2 means that that particular price has risen by 20 percent as a result of the assumption of a 10 percent invrease in wages, compared to the no-change baseline scenario.

If all exogenous model variables are held fixed at the starting value of 1, all endogenous variables will also have the value 1 after the model is solved. This should be interpreted as no change relative to the initial base year data. In that case the model replicates the base year data. It is easy to verify that the model is internally consistent by setting all exogenous model variables equal to 1 and then solving the model. If any endogenous variable in the subsequent solution differs from 1, this indicates that something is wrong with the model equations.

3 The basic cost push price model

We begin first with a description of a closed economy model without taxes or trade margins, and with the simplifying assumption that wach industry only produces one commodity. Recall that $\mathbf{p}_{bp}$ is a vector of product prices:

$$\mathbf{p}_{bp} = \begin{pmatrix} p_{bp,1} \\ p_{bp,2} \\ \vdots \\ p_{bp,n} \end{pmatrix}$$

Prices are detemined by the price of value added and the price of inputs

$$\mathbf{p}'_{bp} = \mathbf{p}'_{bp}\mathbf{A} + \mathbf{p}'_{v}\hat{\mathbf{v}}$$

Lets take a look at how the last term in this equation can be expressed in terms of actual base year data. Note that

$$p_{v,i}^{BY} \cdot Y_i^{BY} = actualw_i^{BY} \cdot H_i^{BY} + actualr_i^{BY} \cdot p_K^{BY} \cdot K_i^{BY}$$

So

$$p_{v,i}^{BY} = \frac{actualw_i^{BY} \cdot H_i^{BY}}{Y_i^{BY}} + \frac{actualr_i^{BY} \cdot p_K^{BY} \cdot K_i^{BY}}{Y_i^{BY}}$$

This equation shows that the value added deflator is equal to the labour cost share plus the profit share. These shares sum to unity, because the base year value added deflator has been normalized to 1. It should be noted that hours worked for the self employed are included in the calculation of the total labour cost. This gives a better measure of the economically relevant labour costs, which are higher than the labour costs for employees only. It yields an adjusted labour cost share in vaue added in line with standard economic analysis of the labour share.

Note that labour productivity is equal to value added divided by hours worked, and that the capital stock divided by value added is the capital intensity. The base year price of capital is normalized to 1, so we can write:

$$p_{v,i}^{BY} = \frac{actualw_i^{BY}}{actualpr_i^{BY}} + actualr_i^{BY} \cdot actualky_i^{BY}$$

So far we have examined the value-added price deflator expressed using base year data. Now we move on to examine the model variables.

We introduce the shifting variables described above for the wage rate (hourly labour cost) (w_i), labour productivity (pr_i), gross return to capital capital (r_i), capital price ($p_{K,i}$) and capital intensity (ky_i).

The model variable for the value added deflator is (note that there is no superscript BY on $p_{v,i}$ below):

$$p_{v,i} = \frac{w_i \cdot actualw_i^{BY}}{pr_i \cdot actualpr_i^{BY}} + r_i \cdot actualr_i^{BY} \cdot p_{K,i} \cdot ky_i \cdot actualky_i^{BY}$$

The shifting variables are all normalized to 1 as initial values, and by construction therefore $p_{v,i}$ is also equal to 1 as initial value. As described above *w* or *r* for any commodity can be set to be either exogenous or endogenous as the user wishes. Depending on the question at hand, the model user can choose to exogenously fix *w* or *r* for any combination of industries. This makes the model very flexible. There is not complete freedom though: If too many variables are endogenous there will be more undetermined variables than equations and no unique solution will exist. The model user has to ensure that the experimental setup of exogenous and endogenous assumptions has a unique solution. If too many variables are assumed to be endogenous, it will soon become obvious that there is no unique solution to the model.

The first term in the above equation is the regular unit labour cost that most economists are familiar with:

$$ulc_i = \frac{w_i \cdot actualw_i^{BY}}{pr_i \cdot actualpr_i^{BY}}$$

The unit labour cost variable can also be written as an expression combining the base-year value of unit labour costs with the shifting variables:

$$ulc_i = \frac{w_i}{pr_i} ulc_i^{BY}$$

In practice, this is the expression that is used directly in the model.

The the second term in the equation determining the value added deflator is the unit capital cost variable:

$$ukc_i = r_i \cdot actualr_i^{BY} \cdot p_{K,i} \cdot ky_i \cdot actualky_i^{BY}$$

The unit capital cost variable can be written as an expression combining the base year value of unit capital costs with three shifting variables:

$$ukc_i = r_i \cdot p_{K,i} \cdot ky_i \cdot ukc_i^{BY}$$

Again, this is the expression that is used in the model. Note that in practice any identical change in any of r_i, $p_{K,i}$ or ky_i yields identical and indistinguishable results.

If the two model unit cost variables are stacked in the column vectors $\mathbf{u}_l$ and $\mathbf{u}_k$, then the column vector of value added deflators is given by

$$\mathbf{p}_v = \mathbf{u}_l + \mathbf{u}_k$$

And the full equation for product prices is given by

$$\mathbf{p}'_{bp} = \mathbf{p}'_{bp}\mathbf{A} + \mathbf{u}'_l \mathbf{v} + \mathbf{u}'_k \mathbf{v} \qquad (1)$$

How the base year unit cost parameters are calculated from actual data is shown in section 4.

3.1 Endogenous versus exogenous prices and profitability assumptions

If all prices in the economy are assumed to be cost driven, then the price response caused by changes in wages, productivity and/or the price of capital can be calculated using the expressions above. If some or all prices are assumed to be exogenously determined, for instance on world markets, one can calculate the effect on profits. However, profits are not necesseraly completely residual or completely exogenous. The user choose between three different assumtions regarding how profits are determined. The PRIOR model allows the user to choose between the following profitability assumptions for industries with endogenous prices.

1. Fixed return to capital
2. Fixed capital and labour cost shares of value added (in nominal terms)
3. Fixed gross operating profit margins (fixed gross profits as a share of the value of gross output).

For industries with exogenous prices, profits can also be residually determined, in which case the wage rate must be exogneously fixed. All of these three variants can be expressed as assumptions on how the implicit gross return to capital shifting variables r_i are determined in the model.

ALTERNATIVE 1: FIXED RETURN TO CAPITAL ($r = 1$)

In the fixed return to capital model setting, the shifting variable r_i is exogenously fixed to 1 for all industries *i*. This means that the return to capital is held constant and equal to the base year value for each industry. This assumption is consistent with an internationally determined cost of capital. It means that irrespective of any other changes in the economy, the rate of return of invested capital is unaffected. If the user so wishes, he or she can also set r_i for some or all industries exogenously equal to some value other than 1 to mimic the effect of changed costs of capital.

ALTERNATIVE 2: FIXED LABOUR-COST SHARE IN VALUE ADDED ($r = p_v$)

In this setting, the relative shares in value added are assumed to be constant.[1] If the price of capital and the capital intensity are held fixed, then r must change at the same rate as the value-added deflator, p_v. To see why, note that a fixed capital share in value added requires that the capital share of value added in current prices, as expressed below, remains constant (The product index *i* has been excluded for simplicity).

[1] This is a standard equilibrium result for the Cobb-Douglas production function. This price model is not based on the Cobb-Douglas production function.

$$\text{capital share of value added} = \frac{r \cdot actualr^{BY} \cdot p_K \cdot ky \ \cdot actualky^{BY}}{p_v}$$

If neither production technology nor the volume of output or price of capital changes, any change in the value-added deflator must give rise to a corresponding change in *r*. Note that this implies that the return to capital increases if there is an increase in the unit labour cost, given that neither production volumes or capital or labour input changes.[2]

ALTERNATIVE 3: FIXED OPERATING MARGIN ($r = p_{pb}$)

In this setting, the gross profit as a share of the value of gross output is assumed to be constant for all industries. This is in line with an assumption that companies set prices with an aim to maintaining the gross operating margins (profits divided by sales). In this case, $r = p_{bp}$. To see why, note that constant gross operating margins requires that the expression below be constant. (The industry index *i* has been excluded for simplicity):

$$\text{Gross operating profit margin} = \frac{r \cdot actualr^{BY} \cdot p_K \cdot ky \cdot actualky^{BY} \cdot Y}{p_{bp} \cdot Q}$$

If neither production volume (Q) or production technology (and therefore value added volume (Y), capital input share (ky) or the price of capital (p_K) is assumed to change, then any change in the product price p_{bp} must be accomanied by a corresponding change in r in order for the gross operating profit margin to remain unchanged. This implies that any rise in wages leads to an increase in the return to capital and an increase in the profit share of value added.

3.2 A Flexible model

The model allows the user to choose which products have prices that are endogenously determined by the price equation (equation 1), and which products have product prices determined exogenously. Exogenous prices may be realistic for products that are internationally traded. For endogenous prices, it is the cost-push price equation (equation 1) that determines the product price. For products with exogenous prices however, it is the other way around. It is the product price that residually determines the profitability rate (or the wage rate).

Flexibility regarding the profitability assumtpion means that the user can tailor the experiment as she or he prefers. However, the profitability assumption is common to all industries with endogenous prices. In the current model version, profitability in all

[2] If the price of capital is allowed to vary one could argue that the return to capital could remain constant and that it is instead the price of capital and/or the capital intensity that adjusts to ensure that the capital share of value added remains constant. This is possible in the model, and it yields identical results in terms of product prices.

industries with endogenous prices must be determined by one of the three choices outlined above.

4 Calculation of model parameters using actual data

4.1 The commodity industry distinction

The description so far has not distinguished between industries and commodities. However, actual data is only available in a commodity-by-industry format. The actual data must be transformed to a commodity-by-commodity format to fit the model. Industry-by-commodity supply and use tables from the national accounts covering commodity-by-industry transactions are used to caclulate the symmetric commodity-by-commodity input use coefficients, under the assumtpion of industry technology. The assumption of industry technology implies that each industry is assumed to use the same products as inputs regardless of the products that are outputs. We need to introduce some more notation to be able to describe the method that is used to calculate the model parameters. This section broadly follows the notation in chapter 5 in Miller and Blair (2009).

U is the commodity-by-industry transactions matrix, (commonly called the use matrix).

q is the vector of commodity outputs.

x is the vector of industry outputs.

M is the commodity-by-industry output matrix (sometimes called the make matrix).

D is the market shares matrix**.**

C is the commodity mix matrix.

B is the commodity-by-industry matrix of input coefficients.

Then the symmetric commodity-by-commodity input matrix **A** is found by first calculating a market shares matrix **D**:

$$\mathbf{D} = \mathbf{M}'\hat{\mathbf{q}}^{-1}$$

We also calculate a commodity-by-industry matrix of input coefficients

$$\mathbf{B} = \mathbf{U}'\hat{\mathbf{x}}^{-1}$$

Then the symmetric commodity-by-commodity matrix of input coefficients is

$$\mathbf{A} = \mathbf{B}\mathbf{D}$$

This is the input coefficient matrix derived under the assumption of industry technology as described in Miller and Blair (2009).

The value-added shares by commodities **v** are given by one minus the sum of the input use coefficients for each product:

$$\mathbf{v} = \mathbf{1} - \mathbf{A}'\mathbf{1}$$

The factor cost data (i.e labour costs and capital inputs) are also available by industry and not by product. The factor cost data must somehow be transformed to the commodity dimension. This is done using the commodity mix matrix **C**. The commodity mix matrix is found by dividing the output matrix by industry output instead of commodity output:

$$\mathbf{C} = \mathbf{M}'\hat{\mathbf{x}}^{-1}$$

The unit factor cost vectors are transformed to the commodity dimension by premultiplication with the commodity mix matrix. Let the $\boldsymbol{\lambda}_l^{BY}$ be the vector of total actual labour costs by industry (including inputed labour costs for the self employed) in the base year. Then the vector of unit labour costs by commodity $\mathbf{u}_l^{BY}$ in the base year is given by:

$$\mathbf{u}_l^{BY} = \left(\mathbf{vq}\right)^{-1}\mathbf{C}\boldsymbol{\lambda}_l^{BY}$$

The unit capital cost vector by commodity is calculated in an analogous manner. The gross operating surplus is equal to value added minus labour costs, so the vector of unit capital costs by commodity in the base year is given by:

$$\mathbf{u}_k^{BY} = \left(\mathbf{vq}\right)^{-1}\mathbf{C}\left(\mathbf{y} - \boldsymbol{\lambda}_l^{BY}\right)$$

These are the only parameters needed in a basic price model setup without imports, taxes and trade margins. The following section describes the full extended model including imports, taxes, subsidies and trade margins.

5 The full model with imports, taxes and trade margins

5.1 Imported inputs

In order to account for imported inputs, we must separate the input coefficient matrix A into separate matrices for the use of imported inputs, and the use of domestically produced inputs. These are denoted by $\mathbf{A}_{imp}$ and $\mathbf{A}_{pro}$ respectively. They are calculated in the same manner as the total input coefficient matrix using supply-use tables as described above, but with separate use matrices for imports and domestic production transactions. Let $\mathbf{p}_{imp}$ be the vector of import prices by commodity. The equation for domestic product prices at basic prices can now be expanded taking into account the distinction between imported and domestically produced inputs:

$$\mathbf{p}'_{bp} = \mathbf{p}'_{bp}\mathbf{A}_{pro} + \mathbf{p}'_{imp}\mathbf{A}_{imp} + \mathbf{p}'_{v}\hat{\mathbf{v}}$$

5.2 Excise taxes and product subsidies on input use

Some taxes are levied on the volume of inputs purchased, for example alcohol tax and environmental taxes. Similarly, the use of some inputs are benefited with a subsidy. Including taxes and subsidies makes the model more realistic, and allows us to calculate the effect on prices and profits from changes in taxes and/or subsidies. The national accounts provide data on actual taxes paid, and subsidies received, on input use. The data is available in the commodity-by-industry dimensions just lika all other transactions data. Using this data, we can construct two symmetric commodity-by-commodity input coefficient matrices measuring the taxes paid per unit of input purchased, and subsidies recieved, respectively. It is calculated in a manner analogous to that described in section 4.1 above. We denote the tax matrix by $\mathbf{T}_{ins}$, and the subsidy matrix by $\mathbf{SUB}_{ins}$. Each element in these matrices are the taxes and subsidies on input use of the commodities (in rows) per unit of *output* of commodities (in columns). The tax and subsidy rates can be expressed in terms of output because of the Leontief production function assumption: Any change in output is assumed to lead to a proportional increase in input use. Note that the coefficients are specific to both the input product and the purpose of use. The tax rates may differ between the products for which the input is used due to exemptions and other tax legislation. We can update the equation for market prices including these taxes and subsidies:

$$\mathbf{p}'_{bp} = \mathbf{p}'_{bp}\mathbf{A}_{pro} + \mathbf{p}'_{imp}\mathbf{A}_{imp} + \mathbf{T}_{ins} - \mathbf{SUB}_{ins} + \mathbf{p}'_{v}\hat{\mathbf{v}}$$

5.3 Trade margins on inputs purchased

Trade margins are the trade sector's markups on the price of products. They are the price differential between the price the trade sector pays to the producer of the product, and the price excluding taxes that they charge customers. One can make different assumptions regarding how trade margins respond to changes in output and volume. Perhaps the most reasonable is that the margins are proportional to the volume sold, rather than the value sold. So that if the basic price of purchased products change, the trade markup per unit sold does not. At least in the short run this is a realistic assumption.

This is the default assumption made in the PRIOR model, that trade margins are proportional to the volume sold, not the value sold. We assume that a fixed trade margin volume is made in each purchase. This trade markup has an implicit price index that determines the total markup. The implicit price index for the trade margin can be assumed to depend on costs in the trade sector, or it can be assumed to be fixed. Let $\mathbf{A}_{insmar}$ represent the coefficients for trade margin markups on purchased inputs in the base year. We introduce one trade margin deflator, that is equal to the product price of the trade product, p_{trade} and is normalized to 1 in the base year. The model user can change this deflator exogenously, let it be endogenously determined by costs in the model, as for any other product, or let it be unchanged.

The equation for product prices including trade margins with the trade margin deflator following the price of the trade product is:

$$\mathbf{p}'_{bp} = \mathbf{p}'_{bp}\mathbf{A}_{pro} + \mathbf{p}'_{imp}\mathbf{A}_{imp} + \mathbf{T}_{ins} - \mathbf{SUB}_{ins} + p_{trade}\mathbf{1}'\mathbf{A}_{insmar} + \mathbf{p}'_{v}\hat{\mathbf{v}}$$

The user can also assume that the trade margin price for each product follows the product price (excluding trade margins and taxes. This last option allows the user to assume that the trade margin ir proportional to the value sold, not the volume sold. In this case the equation for product prices is:

$$\mathbf{p}'_{bp} = \mathbf{p}'_{bp}\mathbf{A}_{pro} + \mathbf{p}'_{imp}\mathbf{A}_{imp} + \mathbf{T}_{ins} - \mathbf{SUB}_{ins} + \mathbf{p}'_{bp}\mathbf{A}_{insmar} + \mathbf{p}'_{v}\hat{\mathbf{v}}$$

5.4 Value-added tax on input use

Most firms do not pay value-added tax (VAT) on inputs purchased. VAT on inputs is deductible in most cases. But some firms, such as financial firms, do, because their sales are exempted from VAT. The national accounts provide data on actual non-deductible VAT paid on input use, by purchased commodity and purchasing industry.

Using this data, we can construct a matrix measuring the VAT rate on the value of inputs purchased in the base year. VAT is usually paid on the value including product taxes (but excluding subsidies). Let $\mathbf{VAT}_{ins}$ be a commodity-by-commodity matrix whose elements are one plus the value-added tax *rates* on input use, as a fraction of the *value* of the inputs at basic prices (including trade margins) plus product taxes, but excluding product subsidies. For example, if the financial sector pays a 25 percent VAT rate on electricity use, then that particular element in the matrix will be 1.25. If VAT is deductible, or there is no VAT, the element in the VAT matrix representing that particular transaction is 1. Now the equation for market prices including all product taxes and VAT on input use is:

$$\begin{aligned}\mathbf{p}'_{bp} &= \mathbf{p}'_{bp}(\mathbf{A}_{pro} \odot \mathbf{VAT}_{ins}) + \mathbf{p}'_{imp}(\mathbf{A}_{imp} \odot \mathbf{VAT}_{ins}) + (\mathbf{T}_{ins} \odot \mathbf{VAT}_{ins}) \\ &+ p_{trade}\mathbf{1}'(\mathbf{A}_{insmar} \odot \mathbf{VAT}_{ins}) + \mathbf{p}'_{v}\hat{\mathbf{v}} - \mathbf{SUB}_{ins}\end{aligned} \quad (2)$$

The $\odot$ denotes elemetwise multiplication.

Equation 2 is a full description of the determinants of basic product prices for products with endogenously determined prices.

5.5 Market prices (purchasers' prices in final demand)

We have so far fully described the equation determining product prices at basic prices. These are the prices recieved by the firm selling the product. We now continue to describe how market prices, sometimes called purchasers' prices, are determined. These are the prices that the customer pays for the products. The customer can be either a household, an exporter, the public sector or an investing firm. Prices can differ depending on who is purchasing the product.

We divide the purchasers into several categories differentiated by purpose, denoted by an index *fd* (final demand). These can be different categories of household consumption, investments, exports, and so on. We denote the market price of product *i* in final demand *fd* by $p_{mp,i,fd}$. Let $\mathbf{P}_{mp}$ be a product by final demand matrix containing all these prices. The row dimension contains the products and the column dimension contains the final demand category (the purpose of use).

If there were no taxes, direct imports, or trade margins, the market prices would be equal to the basic prices:

$$\mathbf{P}_{mp} = \mathbf{1}'\mathbf{p}_{bp}$$

Where **1** is the unit vector.

However, prices on direct imports, VAT and other product taxes also affect the market prices paid by the final users. Direct imports are imports used directly in final demand, as compared to imports used as inputs to production.

To allow for user specific product taxes, imports and trade margins we introduce coefficient matrices $\mathbf{\Gamma}_{SUP}$ representing the fraction of purchase value in the base year corresponding to these sources of supply. For example, $\mathbf{\Gamma}_{imp}$ is a matrix with dimensions products by final use, that represents the value of final use supplied by direct imports. An element with the value 0.4 in the product row 'vechicles' in the column 'household consumption' means that in the base year, 40 percent of the value of consumption of vehicles was directly imported. $\mathbf{\Gamma}_{pro}$ is similarly the share of purchases that are domestically produced. For each source of supply we reuse the price vector by product. For imports, for example, we assume that the price changes are identical for input purchases and for final demand, so the price vector for imports is $\mathbf{p}_{imp}$ that we introduced earlier.

The equation for purchasers' prices in final demand excluding VAT, product taxes and subsidies and trade margins is:

$$\mathbf{P}_{mp} = \hat{\mathbf{p}}_{imp}\mathbf{\Gamma}_{imp} + \hat{\mathbf{p}}_{pro}\mathbf{\Gamma}_{pro}$$

Recall that a hat denotes a diagonalized vector. We now include trade margins. Trade margins only have one price, that is, as before either endgenously determined by cost, or set exogenously by the model user:

$$\mathbf{P}_{mp} = \hat{\mathbf{p}}_{imp}\boldsymbol{\Gamma}_{imp} + \hat{\mathbf{p}}_{bp}\boldsymbol{\Gamma}_{pro} + p_{trade}\boldsymbol{\Gamma}_{mar}$$

This setup implicitly assumes that the trade margins are the same independently of whether the product is imported or domestically produced.

Product taxes and subsidies on the volumes purchased are treated in a similar manner as before. Let $\boldsymbol{\Gamma}_{tax}$ be the matrix containing the product tax per unit purchased. The elements of this matrix are derived from national-account data. If the household consumption of tobbacco is 100 SEK at market prices, and product taxes on tobacco in household consumption is 20 SEK, then the corresponding element in the $\boldsymbol{\Gamma}_{tax}$ matrix is 0.2. The equation for purchasers' prices including product taxes and subsidies is:

$$\mathbf{P}_{mp} = \hat{\mathbf{p}}_{imp}\boldsymbol{\Gamma}_{imp} + \hat{\mathbf{p}}_{bp}\boldsymbol{\Gamma}_{pro} + p_{trade}\boldsymbol{\Gamma}_{mar} + \boldsymbol{\Gamma}_{tax} - \boldsymbol{\Gamma}_{sub}$$

INCLUDING VAT IN FINAL USE

Most of VAT is paid at purchase by the final user. Let $\mathbf{VAT}_{fd}$ be a product by final demand category matrix whose elements are one plus the value-added tax for the purchase of some product for some purpose in final demand, as a share of the value of that purchase excluding VAT, including trade margins and excise tax (but excluding product subsidies).

$$\begin{aligned}\mathbf{P}_{mp} &= \hat{\mathbf{p}}_{imp}\left(\boldsymbol{\Gamma}_{imp} \odot \mathbf{VAT}_{fd}\right) + \hat{\mathbf{p}}_{bp}\left(\boldsymbol{\Gamma}_{pro} \odot \mathbf{VAT}_{fd}\right) \\ &+ p_{trade}\left(\boldsymbol{\Gamma}_{mar} \odot \mathbf{VAT}_{fd}\right) + \boldsymbol{\Gamma}_{tax} \odot \mathbf{VAT}_{fd} - \boldsymbol{\Gamma}_{sub}\end{aligned} \quad (3)$$

One more thing to note is that consumption abroad by Swedish residents is a separate product in the model whose price must be set exogenously. There is no cost equation for the price of consumption abroad.

Equation 3, together with the equation for product prices at basic prices (equation 2) fully describes how the market prices for final use are determined in the model.

5.6 Final demand deflators

We may also be interested in the average price change for some particular purpose, for example household consumption. Let $p_{mp,fd}$ be this average price. These are the final demand deflators in the national accounts. If the final demand category is household consumption, this is the household consumption deflator. Let $\mathbf{p}_{fdmp}$ be a column vector containing all these deflators. In order to compute the vector of final demand deflators on the basis of the market price of each product, in each use, we compute the weighed sum of the product prices in the category. Let $\psi_{i,fd}$ be the share of product *i* in use *fd*. These values are calculated by value terms in base year data. For example, if total household cosumption is 100 SEK in the base year, and 10 SEK is spent on vehicles, then $\psi_{vehicles,household\ consumption} = 0.1$.

Let $\mathbf{\Psi}_{mp}$ be a product-by-final demand matrix containing all of these shares. Then the vector of final demand deflators can be computed as follows:

$$\mathbf{p}'_{fdmp} = \mathbf{P}'_{mp}\mathbf{\Psi}_{mp}$$

6 Use of the model

The model can be used for a wide array of questions, some of which are described below. All of the examples below assume that the product price of owner occupied dwellings isfixed, and that trade margins are endogenously determined by the price of the trade product. Data is from the year 2016.[3]

6.1 The effect of changed wages

The effect of an exogenous change in labour costs on prices is easily modelled by fixing the model wage variable to some number different from 1 in any or all sectors. Setting w = 1.1 in all sectors i equivalent to a rise in the hourly labour cost by 10 percent. By holding all other exogenous variables fixed at the value 1, and choosing an appropriate profitability assumption, the resulting change in product prices shows the partial equilibrium effect of the labour cost increase on product prices under the assumtpion of zero substitution effects in both production and consumption. Using data from 2016, the results with differing profitability assumptions are shown in Table 1. Both the capital intensity and the price of capital are held constant in all variants. The prices of consumption abroad and energy minerals (coal and crude oil) are all held constant in the calculations.

Table 1 Effect of 10 percent higher hourly labour cost

Percentage change in deflator relative to unchanged labour costs

Profitability assumption	Private consumption deflator[1]	Value added deflator[2]	GDP deflator
Fixed rate of return	3.45	6.38	6.96
Fixed profit share in value added	5.30	10.00	9.96
Fixed operating profit margin	4.76	8.87	9.19

Notes: [1] Exluding consumption of non-profit institutions serving households. [2] Business sector excluding owner occupied dwellings.

Source: NIER.

[3] Using data prior to the revision of the national accounts published in september 2019.

6.2 The effect of changed taxes

All of the parameters in the model can be changed by the user. In particular it may be of interest to analyse the effect on prices of changed taxes or subsidies. Table 2 shows the effect of a 10 percent rise in all excise taxes. For example, if the excise tax on petrol was 3 SEK per litre in the base year, the experiment shows an the effect of an increase of the tax to 3.3 SEK. All assumptions are identical to the example on section 6.1. The VAT rate is unchanged in this experiment.

Table 2 Effect of 10 percent higher excise taxes

Percentage change in deflator relative to unchanged excise taxes

Profitability assumption	Private consumption deflator[1]	Value added deflator[2]	GDP deflator
Fixed rate of return	0.47	0.00	0.20
Fixed profit share in value added	0.47	0.00	0.20
Fixed operating profit margin	0.51	0.07	0.25

Notes: [1] Exluding consumption of non-profit institutions serving households. [2] Business sector excluding owner occupied dwellings.

Source: NIER.

6.3 The effect of exchange rate fluctuations

Import prices are an important driver of inflation. Import prices are affected by exchange rate fluctuations. We can use PRIOR to calculate the effect on different prices of an exchange rate depreciation, under a number of assumptions. The assumptions are: full price pass through of the exchange rate change on import prices, no substitution, no indirect effect on domestic prices for products that are sold in competition with their imported substitutes, and no effect on domestic wages. Table 3 shows the effect of 10 percent higher import prices on all imported products under these assumptions.[4]

[4] The GDP deflator falls slightly in the first two solutions because GDP includes the production of owner occupied dwelling services, whose value added deflator falls because the product price is assumed to be exogenously fixed. A fixed product price and increased input prices leads to a fall in the value added deflator for owner occupied dwellings, and therefore also leading to a fall in the GDP deflator.

Table 3 Effect of 10 percent higher import prices

Percentage change in deflator relative to unchanged import prices

Profitability assumption	Private consumption deflator[1]	Value added deflator[2]	GDP-deflator
Fixed rate of return	3,20	0,00	-0,25
Fixed profit share in value added	3,20	0,00	-0,25
Fixed operating profit margin	3,69	1,05	0,46

Notes: [1] Exluding consumption of non-profit institutions serving households. [2] Business sector excluding owner occupied dwellings.

Source: NIER.

6.4 Modelling technological change

The model is static and is solved only once. However, it can be coaxed to mimic the effects of technological change. The user can, for example, simultaneously alter both exogenous price changes and changes in input use coefficients to reflect substitution effects related to the price change.

It is also possible to calculate the effects of technological change through capital intensity. Assume that a new production technology becomes available, and that this technology means that a greater volume of capital can be used to reach a higher level of labour productivity. This can be modelled by exogenously increasing the capital intensity parameter ky_i and simultaneously increasing the labour productivity parameter pr_i. This effectively mimics a shift to a more capital intensive production technology with higher labour productivity.

7 Conclusion

The price model described here is an effective tool that allows the user to calculate the effect on prices in the economy caused by a wide array of different exogenous changes, be they wage changes, changes in productivity, taxes or import prices. Under the assumption of Leontief production technology and no substitution in consumption, the model calculates the effect on both basic prices and market prices in final use. The model can also be used to calculate wage growth consistent with a certain target for the increase in the private consumption deflator.

The program code is easy to follow and data output on the variables of interest is readily supplied. The model described here provides a level of flexibility and sophistication that supersedes the earlier versions. Despite the large amount of data and parameters included, the core of the model is remarkably simple.

The model shows that the effect of changed excise taxes on consumer prices is moderate. Under the assumption of a fixed rate of return to capital, similar sized changes in wages or import prices yield approximately similar effects on consumer prices. A 10 percent increase in import prices or hourly labour costs lead to an increase in consumer prices by slightly more than 3 percent. Under the assumtion of a fixed capital income share of value added, or fixed operating margins, roughly half of an increase in wages is passed through to consumer prices. To what extent labour costs influence consumer price inflation is to some degree a question of how profits are determined in the economy.

8 References

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9 Appendix

9.1 Products and industries

The products and industries are symmetric, (except for consumption abroad) and are the same as in the NIERs input-output model of the Swedish Economy (see Forsfält and Glans, 2015, for details). All of the codes below correspond to sets used in the GAMS program.

Table 4 Products and industries

PRIOR code	SNI 2007 (NACE Rev. 2) code	Description
JOFI	A01, A03	Agriculture and fishing
SKBR	A02	Forestry
ENMI	B05-06	Extraction of energy minerals
GRUV	B, excl. B05-06	Mining and quarrying
C10_12	C10-12	Manufacture of food products, etc.
C13_15	C13-15	Manufacture of textiles, etc.
C16_18	C16-18	Manufacture of wood and paper products, etc.
C19	C19	Manufacture of refined petroleum products
C20_21	C20-21	Manufacture of chemicals and pharmaceutical products
C22	C22	Manufacture of rubber and plastic products
C23	C23	Manufacture of other non-metallic mineral products
C24	C24	Manufacture of basic metals
C25	C25	Manufacture of fabricated metal products
C26_27	C26-27	Manufacture of electronic products and electrical equipment, etc.
C28	C28	Manufacture of machinery and equipment n.e.c.
C29_30	C29-30	Manufacture of motor vehicles and other transport equipment
C31_33	C31-33	Manufacture, other; repair and installation
ELVA	D, E	Electricity; water supply; sewerage, waste management, etc.
BYGG	F	Construction
HATJ	G	Wholesale and retail trade; repair of motor vehicles
FRKT	H	Transportation and storage
HOTR	I	Accommodation and food service activities
IKTJ	J	Information and communication
FITJ	K	Financial and insurance activities
FASH	L68201A, L68201B, L68A	Renting and operating of own or leased real estate
FATJ	L, excl. L68201A, L68201B, L68A	Real estate services
FOTJ	M, N	Professional, scientific and technical activities; support, etc.
O	O	Public administration and defence; compulsory social security
VUTB	P, Q	Education; human health and social work activities
FRTJ	R, S, T	Arts, entertainment and recreation; households' own production, etc.
KOHUI	X9901	Household consumption abroad (not an industry)

Note: Products are identical to industries but in accordance with SPIN 2015, with the exception of consumption abroad. Some SNI codes are not stricly speaking SNI codes, but are rather the codes used by the Swedish national accounts, such as X9901 and L68A.

Source: NIER.

9.2 Final use categories

The final use categories at a disaggregated level are show in the table below.

Table 5 Final use categories

PRIOR Code	Description
FBZM	Input use central government
FBBM	Input use local government
FBLM	Input use regional government
FBY	Input use NPISH
DKZM	Social transfers in kind, central government
DKBM	Social transfers in kind, local government
DKLM	Social transfers in kind, regional government
OMKOEP	Consumption of own production, government
YKOEP	Consumption of own production, NPISH
KOH	Private household consumption
FIN	Gross fixed investments, business sector
FIZM	Gross fixed investments, central government
FIBM	Gross fixed investments, local government
FILM	Gross fixed investments, regional government
FIY	Gross fixed investments, NPISH
LI	Inventory investments
EX	Exports

Source: NIER

9.3 Supply codes

The supply side in the supply use tables can be divided into domestic production, imports, taxes, etc. The codes used in the model are as follows:

Table 6 Supply codes

PRIOR Code	Description
PRO	Domestic business sector production
OFF	Public sector and NPISH sales
TUL	Custom duties
IMP	Imports
TRE	Merchanting
MAR	Trade margins
SKA	Excise taxes
SUB	Subsidies
MOM	VAT
MOT	Purchasers' prices (total)

Source: NIER.